Profit Distribution of DeFi Strategy

1 Introduction

Investor's investments are recorded on deposit-basis, one deposit can go into one product. Everything is then aggregated for the dashboard.

2 Model

The state of the DeFi desk at *the end of the day* t (or "at t" from now on) is modeled as the following *accounting equation*:

$$A(t) = L(t) + F(t) \tag{1}$$

where

$$\begin{aligned} A(t) &= \text{the assets under management at } t \text{ given by (3)} \\ L(t) &= \textstyle\sum_{k \in K} L_k(t) \\ L_k(t) &= \text{the value of investor } k\text{'s position at } t \text{ given by (5)} \\ F(t) &= \text{the fees cumulated but not withdrawn by } t \text{ given by (10)} \\ K &= \text{the set of all investors.} \end{aligned} \tag{2}$$

The choice of notation is suggestive with A representing the *assets* and the value of customer positions L representing the *liabilities* from the desk's point of view.

2.1 Assets under management

The *assets under management* within the strategy, or AUM for short, is given by the equation:

$$A(t) = \sum_{p \in A} A_p(t) \tag{3}$$

where

$$A_p(t) = \text{the value of individual position } p \text{ at } t.$$

The value of an *individual position* at t is calculated differently based on where the funds are deployed. For example, the value of a position p in *curve.fi* and *harvest.finance* would be calculated as follows:

$$A_p(t) = q_{\mathrm{H}}(t) \cdot r_{\mathrm{H/C}}(t) \cdot r_{\mathrm{C/X}}(t) + \frac{q_{\mathrm{F}}(t) \cdot r_{\mathrm{F/USD}}(t)}{r_{\mathrm{X/USD}}(t)} \tag{4}$$

where

$$\begin{aligned} q_{\mathrm{H}}(t) &= \text{the amount of HLP token held at } t \\ r_{\mathrm{H/C}}(t) &= \text{the price of HLP token in terms of CLP token at } t \\ r_{\mathrm{C/X}}(t) &= \text{the price of CLP token in terms of X at } t \\ q_{\mathrm{F}}(t) &= \text{the amount of Farm token held at } t \\ r_{\mathrm{F/USD}}(t) &= \text{the price of Farm token in terms of USD at } t \\ r_{\mathrm{X/USD}}(t) &= \text{the price of X and USD at } t \end{aligned}$$

and HLP stands for Harvest Liquidity Pool, CLP for Curve Liquidity Pool, and X is the base currency of the strategy.

Other positions may have other ways of measuring the value. The value of some of the positions may be tracked infrequently or manually or both.

2.2 Value of investor position

The value of the investor k's position is given by

$$L_k(t) = \sum_{\tau \le t} d_k(\tau) - \sum_{\tau \le t} w_k(\tau) + \sum_{\tau \le t} \pi_k(\tau) - \sum_{\tau \le t} f_k(\tau) \tag{5}$$

where

$$\begin{aligned} d_k(t) &= \text{deposits made by the investor } k \text{ at } t && (6) \\ w_k(t) &= \text{withdrawals made by the investor } k \text{ at } t && (7) \\ \pi_k(t) &= \text{daily P/L distributed to the investor } k \text{ at } t \text{ given by (9)} \\ f_k(t) &= \text{fees charged from the investor } k \text{ at } t \text{ given by (11).} \end{aligned}$$

The deposits $d_k(\cdot)$ and withdrawals $w_k(\cdot)$ are exogenous to the system in the sense that they are completely determined by the investor's decision to deposit and withdraw money. The distributed profit (or loss) $\pi_k(\cdot)$ and fees $f_k(\cdot)$ are endogenous to the system.

2.3 Distribution of profit and loss

The $A(t)$ given by (3) must equate to the total deposits $D(t)$, total withdrawals $W(t)$, and the total *undistributed* profits $\Pi_A(t)$:

$$A(t) = D(t) - W(t) + \Pi_A(t)$$

where

$$D(t) = \sum_{k \in K, \tau \leq t} d_k(\tau) \qquad \text{and} \qquad W(t) = \sum_{k \in K, \tau \leq t} w_k(\tau).$$

Noting that $A(\cdot)$, $D(\cdot)$, and $W(\cdot)$ are all exogenous to the system we can rearrange the equation to give the total accumulated undistributed profit:

$$\Pi_A(t) = A(t) - D(t) + W(t). \tag{8}$$

What remains is the distribution of $\Pi_A(t)$ between the investors. The distribution takes place at every t so that the daily change in the accumulated total profit or loss $\Delta\Pi_A(t)$ is distributed between the investors:

$$\Delta\Pi_A(t) = \sum_{k \in K} \pi_k(t)$$

There are multiple ways of attributing the P/L between the investors so that the above equation holds. We choose to attribute on the *pro rata* basis based on the value of investors' positions at $t - 1$:

$$\pi_k(t) = \Delta\Pi_A(t) \times \frac{\cancel{L_k(t-1)}}{L(t-1)}. \tag{9}$$

2.4 Fees

The total accumulated fees, i.e. our company's share of the Desk's liabilities, is given by the equation

$$F(t) = \sum_{k \in K, \tau \leq t} f_k(\tau) \tag{10}$$

The fees are charged from each investor k at the end of each month based on the profits attributed to the investor during that month:

$$f_k(t) = \begin{cases} r_{\text{fee}} \cdot \max\left(\sum_{\tau : P_m(\tau;t)} \pi_k(\tau), 0\right) & , P_e(t) \\ 0 & , \neg P_e(t) \end{cases} \tag{11}$$

where

$$r_{\text{fee}} = \text{the fee rate} \tag{12}$$

$$P_e(t) = \text{is true iff } t \text{ falls on the last day of the month} \tag{13}$$

$$P_m(\tau; t) = \text{is true iff } \tau \text{ belongs to the same month with } t \tag{14}$$

3 Calculation order

The natural order of calculating different quantities in the model is the following:

1. The assets under management: $A(t)$
2. The daily change in the total undistributed profits: $\Delta\Pi_A(t)$

3. The daily profit attributed to each investor: $\pi_k(t)$

4. The fees charged from each investor: $f_k(t)$

It is also worth noting that—while the assets under management $A(t)$ are independent of the liabilities $L(t)$ in this model—in reality the deposits and withdrawals affect $A(t)$. The quiet but important underlying assumption is that $A(t)$ is measured so that it includes all the deposits arriving at t and that all withdrawals at t have already departed from it.

4 Technical implementation

1. track_aum.py is used to calculate individual positions and derive AUM for each strategy from those

2. AUM is compared to deposits, withdrawals and accumulated profit to find total undistributed profit