Lattice Gauge Theory

Patrick Oare

1 Introduction

The essential idea of lattice gauge theory is to numerically evaluate the path integral for a quantum theory in order to determine correlation functions. From these correlation functions, we can get the physics.

2 Basic Definitions

Instead of working with gauge connections $A_\mu(x)$, the fundamental gauge fields that we work with are the parallel transporters $U(x, y)$. When quantized, we call these fields $U_\mu(n)$ the **link fields**. Under a gauge transformation $\Omega(n)$, the fields transform as:

$$\psi(n) \mapsto \Omega(n)\psi(n) \tag{1}$$

$$U_\mu(n) \mapsto \Omega(n)U_\mu(n)\Omega(n+\hat{\mu})^\dagger \tag{2}$$

This allows a nice definition of the **gauge covariant derivative**. We consider both the forward difference and the backwards differences:

$$\overrightarrow{D}\psi(n) = \frac{1}{2a}\left(U_\mu(n)\psi(n+\hat{\mu}) - U_\mu(n-\hat{\mu})^\dagger\psi(n-\hat{\mu})\right) \tag{3}$$

$$\overline{\psi}(n)\overleftarrow{D} = \frac{1}{2a}\left(\overline{\psi}(n+\hat{\mu})U_\mu(n)^\dagger - \overline{\psi}(n-\hat{\mu})U_\mu(n-\hat{\mu})\right) \tag{4}$$

We generally will consider the difference between these two operators:

$$\overleftrightarrow{D} := \overrightarrow{D} - \overleftarrow{D} \tag{5}$$

3 Lattice Units

4 Clover Improvement

5 QLUA Snippets