Standard Model Formulas

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Non-Abelian Gauge Theories

Classical Yang-Mills Theory

Consider a Yang-Mills Theory with gauge group G generated by $\{T_a\}_{a=1}^{N}$, gauge field $A_\mu(x) = A_\mu^a(x)T^a$, and field strength $F_{\mu\nu}(x) = F_{\mu\nu}^a(x)T^a$. The gauge field has coupling g. The field $\psi(x)$ has spinor degrees of freedom ψ^α and group degrees of freedom ψ_j. The gauge field, field strength, and covariant derivative lie in the Lie algebra $\mathfrak{g}$ of G.

- Structure constants:
$$[T^a, T^b] = if^{abc}T^c$$
- General element of the gauge group:
$$U = \exp(i\omega^a T^a) \sim 1 + i\omega^a T^a$$
- Generators of $SU(2)$ (σ^a are the 3 Pauli matrices):
$$T^a = \frac{\sigma^a}{2}$$
- Generators of $SU(3)$ (λ^a are the 8 Gell-Mann Matrices):
$$T^a = \frac{\lambda^a}{2}$$
- Gauge covariant derivative:
$$D_\mu = \partial_\mu - igA_\mu$$
- Field strength:
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] = \frac{i}{g}[D_\mu, D_\nu]$$
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$
- Comparator:
$$V(y, x) = P\exp\left(i\int_x^y A_\mu dx^\mu\right)$$
- Transformation Laws:
$$\psi \to U\psi$$
$$D_\mu \to UD_\mu U^\dagger \sim \partial_\mu - i(\partial_\mu\omega^a)T^a + g[\omega, A_\mu]$$
$$A_\mu \to UA_\mu U^\dagger + iU\partial_\mu U^\dagger \sim A_\mu + \frac{1}{g}\partial_\mu\omega + i[\omega, A_\mu]$$
$$V(y, x) \to U(y)V(y, x)U(x)^\dagger$$

- Classical Yang-Mills Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi} \left(i \gamma^\mu D_\mu - m \right) \psi$$

Fadeev-Popov Quantization

- Auxiliary Field:

$$\delta(\partial_\mu A^\mu) = \int DB \exp\left(-i \int d^d x B \partial_\mu A^\mu \right)$$

- Ghost fields:

$$\det(i \not{D}) = \int Dc D\overline{c} \exp\left(i \int d^d x \overline{c} \partial_\mu D^\mu c \right)$$

- Fadeev-Popov Lagrangian:

$$\mathcal{L} = \mathcal{L}_{YM} + B^a \partial_\mu A^{\mu a} + \frac{\xi}{2} (B^a)^2 + \overline{c}^a \partial_\mu D^\mu c^a$$

The Standard Model

- Fields: $SU(3) \times SU(2) \times U(1)$ Yang-Mills theory.

	$SU(3)$	$SU(2)$	$U(1)$
Generators	λ^A	t^a	Y
# Generators	8	3	1
Gauge Field	G^A_μ	W^a_μ	B_μ

Table 1: Standard Model Gauge Group.

- First generation transformation laws. Under $SU(2)$, note that $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ and $\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$.

	q_L	u_R	d_R	ℓ_L	e_R
$SU(3)$	3	3	3	1	1
$SU(2)$	2	1	1	2	1
$U(1)$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1

Table 2: Dimension of representation for first generation.

Electroweak Sector

- Generators of $SU(2)$:

$$t^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad t^2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad t^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Plus and minus matrices:

$$t^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad t^- = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$$

- Charge operator:
$$Q = t^3 + Y$$
- $W^\pm$ fields:
$$W^\mp = \frac{1}{\sqrt{2}}\left(W^1 \pm iW^2\right)$$
- W_μ gauge fields:
$$W_\mu^a t^a = \frac{1}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix}$$
- Weak mixing angle:
$$\tan\theta_w = \frac{g'}{g}$$
- Electromagnetic field:
$$A_\mu = \cos\theta_w B_\mu + \sin\theta_w W_\mu^3$$
- Z boson:
$$Z_\mu = -\sin\theta_w B_\mu + \cos\theta_w W_\mu^3$$
- Electromagnetic coupling:
$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$
- Covariant derivative with new field definitions:
$$iD_\mu \supset gW_\mu^a t^a + g'B_\mu Y = eQA_\mu + \frac{g}{\cos\theta_w}(t^3 - Q\sin^2\theta_w)Z_\mu + \frac{g}{\sqrt{2}}(W_\mu^+ t^+ - W_\mu^- t^-)$$
- Masses of Z and W boson:
$$M_Z = \frac{M_W}{\cos\theta_w}$$
- Fermi coupling:
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8\cos^2\theta_w M_Z^2}$$
- Propagators of $W^\pm$ and Z bosons with mass M (with $\xi = 1$):
$$D_{\mu\nu} = \frac{-g_{\mu\nu}}{k^2 - M^2 + i\epsilon}$$
- Electroweak Lagrangian:
$$\mathcal{L} = \mathcal{L}_{321} + \mathcal{L}_{EWSB} + \mathcal{L}_{FSB}$$
- 3-2-1 Lagrangian:
$$\mathcal{L}_{321} = TODO$$
- Electroweak symmetry breaking Lagrangian:
$$\mathcal{L}_{EWSB} = (D_\mu H^\dagger)(D^\mu H) - (-\mu^2 H^\dagger H + \lambda(H^\dagger H)^2)$$
- Flavor symmetry breaking Lagrangian:
$$\mathcal{L}_{FSB} = TODO$$

Miscellaneous

- Chiral projection operators:

$$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$$

- Clifford Algebra:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

- Adjoint of γ^μ:

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$

- Properties of γ_5:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\{\gamma^\mu, \gamma_5\} = 0$$

$$(\gamma_5)^2 = 0$$

$$(\gamma^5)^\dagger = \gamma_5$$