Lattice Gauge Theory

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1 Introduction

The essential idea of lattice gauge theory is to numerically evaluate the path integral for a quantum theory in order to determine correlation functions. From these correlation functions, we can get the physics.

2 Basic Definitions

Instead of working with gauge connections $A_\mu(x)$, the fundamental gauge fields that we work with are the parallel transporters $U(x, y)$. When quantized, we call these fields $U_\mu(n)$ the **link fields**. Under a gauge transformation $\Omega(n)$, the fields transform as:

$$\psi(n) \mapsto \Omega(n)\psi(n) \tag{1}$$

$$U_\mu(n) \mapsto \Omega(n)U_\mu(n)\Omega(n+\hat{\mu})^\dagger \tag{2}$$

This allows a nice definition of the **gauge covariant derivative**. We consider both the forward difference and the backwards differences:

$$\overrightarrow{D}\psi(n) = \frac{1}{2a}\left(U_\mu(n)\psi(n+\hat{\mu}) - U_\mu(n-\hat{\mu})^\dagger\psi(n-\hat{\mu})\right) \tag{3}$$

$$\overline{\psi}(n)\overleftarrow{D} = \frac{1}{2a}\left(\overline{\psi}(n+\hat{\mu})U_\mu(n)^\dagger - \overline{\psi}(n-\hat{\mu})U_\mu(n-\hat{\mu})\right) \tag{4}$$

We generally will consider the difference between these two operators:

$$\overleftrightarrow{D} := \overrightarrow{D} - \overleftarrow{D} \tag{5}$$

2.1 Translational invariance on the lattice

Propagators on the lattice are translationally invariant in the infinite volume limit. What that means is that if I want to compute something like:

$$S_1(p) = \frac{1}{V}\sum_{x,y} e^{-ip\cdot(x-y)}S(x, y) \tag{6}$$

I can choose an origin for the sum over y. This eliminates the sum and will give the same results with infinite statistics, but for practical calculations will make the results noisier. So, I can choose y to be at the point 0, which will give me:

$$S_2(p) = \frac{1}{V}\sum_{x} e^{-ipx}S(x, 0) \tag{7}$$

When evaluating $S_1(p)$ and $S_2(p)$ on the same configuration, the result will come out to be different. However, when we evaluate these on different configurations, they should give the same signal, albeit $S_2(p)$ will be noisier.

3 Lattice Units

4 Clover Improvement

5 QLUA Snippets