Propagators

Patrick Oare

The propagator is simply the inverse of the quadratic term in the Lagrangian. Let's do a few examples.

1 Abelian case (photon propagator)

After applying Fadeev-Popov to fix the gauge, we have the gauge-fixed Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \tag{1}$$

We consider the parts of $\mathcal{L}$ which are pure gauge, and we expand them out, integrating by parts to make this more compact:

$$\begin{aligned}\mathcal{L}_{gauge} &= -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2\xi}(\partial_\mu A^\mu)(\partial_\nu A^\nu) \\ &= \frac{1}{2}A^\mu \left(g_{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial_\mu\partial_\nu \right) A^\nu \\ &= \frac{1}{2}A^\mu D_{\mu\nu} A^\nu \end{aligned} \tag{2}$$

where we have defined the operator $D_{\mu\nu}$ by:

$$D_{\mu\nu} := g_{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial_\mu\partial_\nu \tag{3}$$

The differential operator $D_{\mu\nu}$ is the term which we will need to invert (i.e. we need to find a Green's function for $D_{\mu\nu}$) by solving the equation:

$$D_{\mu\nu}\Pi^{\nu\alpha}(x) = \delta_\mu^\alpha \delta^4(x) \tag{4}$$

The Green's function $\Pi^{\mu\nu}(x)$ is the **propagator**. We solve this by taking it to momentum space:

$$\left(g_{\mu\nu}k^2 - \left(1 - \frac{1}{\xi}\right)k_\mu k_\nu \right)\tilde{\Pi}^{\nu\alpha} = \delta_\mu^\alpha \tag{5}$$

One can then verify the result in any QFT textbook works for $\tilde{\Pi}^{\mu\nu}$. The easiest way to invert this equation is to write it as a 4×4 matrix in Lorentz space, then find the inverse of the matrix.

2 Non-abelian case