Standard Model Overview

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The Standard Model is a $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory which describes our physical world. It is currently our best approximation to the physics that describes our universe, although we know that it does not encapsulate all of physics. The SM is (up to a global symmetry, i.e. modulo $\mathbb{Z}/6\mathbb{Z}$) completely characterized by its gauge symmetry and matter fields. The SM is typically split up into a few sectors when it is studied.

Electroweak theory is the sector of the Standard Model (SM) that deals with the gauge group $SU(2)_L \times U(1)_Y$. The $SU(2)$ piece acts on the left handed fermion fields in the SM, and the $U(1)_Y$ factor is the hypercharge. The unique physics in this sector primarily comes from the spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$. This gives rise to masses for fermions and the gauge bosons of the broken symmetry, which are the $W_\mu^\pm$ and Z_μ bosons. The unbroken symmetry is electromagnetism and manifests at low energies (less than the vev of the Higgs), and it is what we manifestly see in our everyday life. This symmetry breaking also provides constraints between the masses of the electroweak gauge bosons, the vev of the Higgs, and the mass of the Higgs.

Quantum chromodynamics (QCD) is the SM sector which describes the $SU(3)_C$ factor of the gauge group. The SM's fermion content coupled with the nature of the $SU(3)$ gauge theory which makes up QCD gives it some rather strange properties that are not seen in other sectors or in QED. First, QCD had *dimensional transmutation*, in which a scale Λ_{QCD} is generated by the theory seemingly out of dimensionless couplings and numbers. Λ_{QCD} is defined by being the scale at which the running coupling $\alpha(\mu)$ diverges, i.e. where the Landau pole in N_f flavor QCD is. Secondly, QCD has *asymptotic freedom*; it is non-perturbative at low energies ($\alpha(\mu)$ is too large to have a well defined perturbative expansion), but at high energies the $\alpha(\mu)$ flows to zero and becomes small, which allows one to compute QCD observables in perturbation theory. The final major feature QCD contains is *confinement*; its potential scales as $V(r) \sim r$, and so particles will clump together to minimize this potential: a lone quark can never be found, it will always be confined into a bound state with other quarks. These bound states are called *hadrons*, and studying QCD reveals a rich spectrum of such particles.

The **flavor sector** of the SM describes how the different copies of fermion fields interact. Flavor is the quantum number of the SM which distinguishes the different species of particles, i.e. which distinguishes the d quark from the s quark and the electron from the muon. The interesting physics in the flavor sector comes from quantifying the difference between these flavor eigenstates, which the SM couplings are built up from, between the mass eigenstates, which are the physical states which propagate from point to point. This manifests itself as a unitary rotation between the mass and flavor bases, and the *CKM* matrix V_{CKM} describes how different this rotation is for up-type quarks vs. down-type quarks. There is also a corresponding analogue for the lepton sector, called the *PMNS* matrix, yet that is not well understood because it is directly related to neutrino oscillations. This mixing between the flavor and mass eigenstates allows for flavor-changing decays in the SM, and the irremovable phase in the CKM matrix directly leads to CP violation in the electroweak interactions.

The SM has three generations of fermions, as follows:

Generation	u-type quark	d-type quark	e-type lepton	ν-type lepton
1	Up quark, u	Down quark, d	Electron, e	Electron neutrino, ν_e
2	Charm quark, c	Strange quark, s	Muon, μ	Muon neutrino, ν_μ
3	Top quark, t	Bottom quark, b	Tau, τ	Tau neutrino, ν_τ

Table 1: Standard Model fermions.

Note the mass hierarchy in the SM is more complicated than this generational picture suggests; although $m_u < m_d$, we instead have $m_s < m_c$ and $m_b < m_t$ in the second and third generations. The mass hierarchy for all SM particles and some of the most common hadrons is:

ν	e	u	d	s	μ	π^0	$\pi^\pm$
≈ 0	0.511 MeV	2.2 MeV	4.7 MeV	93 MeV	110 MeV	134 MeV	139 MeV

p^+	n^0	c	τ	b	W	Z	H	t
938 MeV	939 MeV	1.3 GeV	1.8 GeV	4.7 GeV	80 GeV	91 GeV	125 GeV	172 GeV

Table 2: Mass hierarchy of the SM and some light QCD bound states. The pion is π, neutron is n^0, and proton is p^+. The units to the left of the c quark are MeV, and the units to the right are GeV. Note that $\Lambda_{\text{QCD}} \approx 150 - 200$ MeV; the particles in the upper row are lighter than Λ_{QCD}, while the particles in the lower row are heavier than it. Masses are sourced from the Particle Data Group's Review of Particle Physics.

The mass of the pion is a good number to keep in mind for hadronic decay; as the lightest hadron, a process can only decay into hadrons if the incoming kinematics is sufficient for pion creation: since quarks must be confined, it is not enough for a process to occur if the kinematics simply allows u, d, or s quark creation.

The structure of the Standard Model is completely determined by the irreps of $SU(3)_c \times SU(2)_L \times U(1)_Y$ which the particles transform under. Here, the N-dimensional fundamental representation of $SU(N)$ is denoted by $\mathbf{N}$, and the irreps of the Lorentz group are denoted by (j_L, j_R) as an irrep of $SO(1,3) \cong SU(2) \times SU(2)$.

Particle	SU(3)	SU(2)	U(1)	Lorentz
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	1/6	$(1/2, 0)$
u_R	$\mathbf{3}$	1	2/3	$(0, 1/2)$
d_R	$\mathbf{3}$	1	$-1/3$	$(0, 1/2)$
$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	1	$\mathbf{2}$	$-\frac{1}{2}$	$(1/2, 0)$
e_R	1	1	-1	$(0, 1/2)$
ν_R	1	1	0	$(0, 1/2)$
H	1	$\mathbf{2}$	1/2	$(0, 0)$

Table 3: Charges of the particles in the SM (not including gauge bosons). All of the particles have been seen in nature except for the sterile right-handed neutrino, which may or may not exist.

The gauge pieces of the SM are simply from Yang-Mills theory. Let B_μ, $W^a_{\mu\nu}$, and $G^A_{\mu\nu}$ be the gauge fields for $U(1)_Y$, $SU(2)_L$, and $SU(3)$ respectively, with $a \in \{1, 2, 3\}$ and $A \in \{1, ..., 8\}$.

Denote their corresponding field strengths by the same letter, i.e.

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu] \tag{1}$$
$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{abc}W^b_\mu W^c_\nu \tag{2}$$

and let the covariant derivative be D_μ, which acts on a field ϕ as:

$$D_\mu\phi = \partial_\mu\phi - ig'B_\mu\phi - igW^a_\mu t^a\phi - ig_3G^A_\mu T^A\phi \tag{3}$$

where A sums over the gauge fields. Note that $t^a\phi$ and $T^A\phi$ will change based on what representation ϕ is in; if $\phi = \phi^i$ lives in the fundamental representation, then $t^a\phi = (t^a)^{ij}\phi^j$ where t^a (T^A) is represented by half the Pauli (Gell-Mann) matrices, but if $\phi = \phi^a$ lives in the adjoint, then $T^a\phi = [T^a, \phi^bT^b] = if^{abc}\phi^bT^c$, i.e. $D_\mu\phi^a = \partial_\mu\phi^a + gf^{abc}A^b_\mu\phi^c$.

Given this setup, the Standard Model Lagrangian is typically split up into four parts:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermi}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\nu_R} \tag{4}$$

Each of these sectors are relatively self-explanatory. We have:

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu a} - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A} + \theta_{\text{QCD}}\epsilon^{\mu\nu\alpha\beta}G_{\mu\nu}G_{\alpha\beta} \tag{5}$$
$$\mathcal{L}_{\text{Fermi}} = i\sum_\psi \overline{\psi}\not{D}\psi = i\sum_{\psi_L}\overline{\psi}_L\overline{\sigma}^\mu D_\mu\psi_L + i\sum_{\psi_R}\sigma^\mu D_\mu\psi_R \tag{6}$$
$$\mathcal{L}_{\text{Higgs}} = D_\mu HD^\mu H^\dagger + \mu^2H^\dagger H - \lambda(H^\dagger H)^2 \tag{7}$$
$$\mathcal{L}_{\text{Yukawa}} = -Y^d_{ij}\overline{Q}^i_L Hd^j_R - Y^u_{ij}\overline{Q}^i_L\epsilon H^* u^j_R - Y^e_{ij}\overline{\ell}^i_L He^j_R \tag{8}$$
$$\mathcal{L}_{\nu_R} = -Y^\nu_{ij}\overline{\ell}_L\epsilon H^*\nu^j_R - iM_{ij}(\nu^i_R)^c\nu^j_R + h.c. \tag{9}$$

Here $\not{D}\psi = \overline{\sigma}^\mu D_\mu\psi_L + \sigma^\mu D_\mu\psi_R$ with $\sigma^\mu = (1, \sigma^i)$ and $\overline{\sigma}^\mu = (1, -\sigma^i)$, $\epsilon^{ab} = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, and

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{10}$$

We will later gauge transform H to unitary gauge to make more apparent where the physical Higgs boson is. A few comments about this Lagrangian:

1. **The SM Lagrangian does not have any explicit mass terms for the fermion**. A Dirac mass term of the form $m_q\overline{q}q = m_q(\overline{q}_Rq_L + \overline{q}_Lq_R)$ violates $SU(2)_L$ symmetry since q_L and q_R transform differently, and a Majorana mass term which goes as $mqq = m(\epsilon^{ab}q_{L,a}q_{L,a} + \epsilon_{\dot{a}\dot{b}}q^{\dot{a}}_Rq^{\dot{b}}_R)$ also violates $SU(2)_L$ symmetry.

2. **Transformation properties of the Higgs**: The Yukawa couplings must be singlets under $SU(2)$ and $U(1)$. To verify the $U(1)$ properties, one can simply add the hypercharges. The $SU(2)$ properties are harder; for the up quark term, the hypercharge cancellation means we need the antiparticle field for Q_L and H. To make an $SU(2)$ singlet, one must notice the transformation properties of the ϵ tensor under $U \in SU(2)$:

$$\epsilon U\epsilon = -U^* \implies U\epsilon = \epsilon U^* \tag{11}$$

as $\epsilon^2 = -1$. Now if A, B are both fields in **2** of SU(2), then:

$$A^\dagger\epsilon B^* \mapsto A^\dagger U^\dagger\epsilon U^*B^* = A^\dagger U^\dagger U\epsilon B^* = A^\dagger\epsilon B^* \tag{12}$$

hence we see that $A^\dagger\epsilon B^*$ is a singlet under SU(2). Equivalently, we can use ϵ^{ab} or ϵ_{ab} to contract the color indices in Q^*_L and H^* into a singlet.

3. **Parameters in the SM**: Once the gauge symmetry is specified and the charges of the fermions are set, the theory is not yet complete. It needs experimental input in the form of input parameters like the coupling of each force and masses of some of the particles. There are 19 independent parameters in the Standard Model:

   (a) Gauge couplings (3 parameters).

   (b) Fermion masses (9 parameters).

   (c) Higgs vev v and mass m_H (2 parameters).

   (d) Angles $\theta_{12}, \theta_{13}, \theta_{23}$ in the CKM matrix (3 parameters).

   (e) Irremovable phase δ in the CKM matrix (1 parameter).

   (f) $\theta_{\text{QCD}} \approx 0$, which is the **strong CP problem** (1 parameter).

Here are some of the current problems with the Standard model.

- Neutrino masses: We know that neutrino masses exist because neutrino oscillations have been discovered, but we don't know the nature of the neutrino. There are two main ways to incorporate neutrino masses into the SM, and they depend on the nature of the neutrino. If the neutrino is a Majorana particle, then neutrino masses can be added via a dimension-5 operator:
$$\Delta\mathcal{L}^{(1)}_{\text{mass}} = \frac{c_5}{\Lambda}\epsilon^{ij}(\epsilon^{ab}\ell_{ia}H_b)(\epsilon^{cd}\ell_{jc}H_d) \tag{13}$$
where the color and spinor indices are contracted with ϵ^{ij}. On the other hand, if the neutrino is a Dirac particle, we can add in right-handed neutrinos as a field ν_R (see Table **??**), and the neutrino will gain a Dirac mass via a new Yukawa coupling:
$$\Delta\mathcal{L}^{(2)}_{\text{mass}} = (Y_\nu)_{ij}H\ell_i\nu_j^\dagger + h.c. \tag{14}$$
Neutrinoless double β decay is a process which may be able to tell us the nature of the neutrino; if this process is observed, the neutrino must be able to annihilate itself and will therefore be its own antiparticle, i.e. we would have evidence the neutrino is a Majorana particle.

- Strong CP: The strong CP problem is the question of why (to the precision of current experiments) the CP violating term in the QCD Lagrangian, $\theta G \wedge G$, vanishes. There is no reason this term should not be present in $\mathcal{L}_{\text{SM}}$ and it would generate CP violating interactions, yet no one has seen CP violation in nature in the QCD sector. A common solution to this is the **QCD axion**, which adds in a field to dynamically set the CP violating coupling to zero, ensuring that θ_{QCD} is zero.

- Hierarchy (fine tuning): Fine-tuning problems are related to relative sizes of quantities. They often appear when discussing the size of loop effects: typically there is no reason to assume that loop effects are small, and when loops are taken into account to compute quantities that we have measured to be small, there must be some precise calculation of loops that allow for this. See https://en.wikipedia.org/wiki/Hierarchy_problem for more detail.