## **Appendix**

Part A. Velocities and accelerations of the centres of mass of the robot links.

Angular velocity:

$$\mathbf{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}, \tag{A.1}$$

$$\mathbf{\omega}_{2} = \begin{bmatrix} \dot{q}_{1} \, \mathbf{c}_{q_{3}} \\ -\dot{q}_{1} \, \mathbf{s}_{q_{3}} \\ \dot{q}_{2} \end{bmatrix}, \tag{A.2}$$

$$\mathbf{\omega}_{3} = \begin{bmatrix} \dot{q}_{1} \, \mathbf{c}_{q_{3}} \\ -\dot{q}_{1} \, \mathbf{s}_{q_{3}} \\ \dot{q}_{3} \end{bmatrix}, \tag{A.3}$$

$$\mathbf{\omega}_{4} = \begin{bmatrix} -\dot{q}_{3} \, \mathbf{s}_{q_{4}} + \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \\ -\dot{q}_{3} \, \mathbf{c}_{q_{4}} - \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{s}_{q_{4}} \\ \dot{q}_{4} - \dot{q}_{1} \, \mathbf{s}_{q_{3}} \end{bmatrix}, \tag{A.4}$$

$$\mathbf{\omega}_{5} = \begin{bmatrix} \dot{q}_{3} \, \mathbf{s}_{q_{4}} \, \mathbf{c}_{q_{5}} - \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \, \mathbf{c}_{q_{5}} - \mathbf{s}_{q_{5}} \left( \dot{q}_{4} - \dot{q}_{1} \, \mathbf{s}_{q_{3}} \right) \\ -\dot{q}_{3} \, \mathbf{s}_{q_{4}} \, \mathbf{s}_{q_{5}} - \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \, \mathbf{s}_{q_{5}} - \mathbf{c}_{q_{5}} \left( \dot{q}_{4} - \dot{q}_{1} \, \mathbf{s}_{q_{3}} \right) \\ \dot{q}_{5} + \dot{q}_{3} \, \mathbf{c}_{q_{4}} + \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{s}_{q_{4}} \end{bmatrix},$$
(A.5)

$$\boldsymbol{\omega}_{6} = \begin{bmatrix} \mathbf{s}_{q_{6}} \left( \dot{q}_{5} + \dot{q}_{3} \, \mathbf{c}_{q_{4}} + \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{s}_{q_{4}} \right) + \mathbf{c}_{q_{6}} \left( \mathbf{c}_{q_{5}} \left( \dot{q}_{3} \, \mathbf{s}_{q_{4}} - \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \right) - \mathbf{s}_{q_{5}} \left( \dot{q}_{4} - \dot{q}_{1} \, \mathbf{s}_{q_{3}} \right) \right) \\ \mathbf{c}_{q_{6}} \left( \dot{q}_{5} + \dot{q}_{3} \, \mathbf{c}_{q_{4}} + \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{s}_{q_{4}} \right) - \mathbf{s}_{q_{6}} \left( \mathbf{c}_{q_{5}} \left( \dot{q}_{3} \, \mathbf{s}_{q_{4}} - \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \right) - \mathbf{s}_{q_{5}} \left( \dot{q}_{4} - \dot{q}_{1} \, \mathbf{s}_{q_{3}} \right) \right) \\ \dot{q}_{6} + \mathbf{c}_{q_{5}} \left( \dot{q}_{4} - \dot{q}_{1} \, \mathbf{s}_{q_{3}} \right) + \mathbf{s}_{q_{5}} \left( \dot{q}_{3} \, \mathbf{s}_{q_{4}} - \dot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \right) \end{bmatrix}$$
 (A.6)

Angular acceleration:

$$\boldsymbol{\varepsilon}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix} \tag{A.1}$$

$$\mathbf{\varepsilon}_{2} = \begin{bmatrix} \ddot{q}_{1} \, \mathbf{c}_{q_{2}} - \dot{q}_{1} \, \dot{q}_{2} \, \mathbf{s}_{q_{2}} \\ -\ddot{q}_{1} \, \mathbf{s}_{q_{2}} - \dot{q}_{1} \, \dot{q}_{2} \, \mathbf{c}_{q_{2}} \\ \ddot{q}_{2} \end{bmatrix}$$
(A.2)

$$\mathbf{\varepsilon}_{3} = \begin{bmatrix} \ddot{q}_{1} \, \mathbf{c}_{q_{3}} - \dot{q}_{1} \, \dot{q}_{3} \, \mathbf{s}_{q_{3}} \\ - \ddot{q}_{1} \, \mathbf{s}_{q_{3}} - \dot{q}_{1} \, \dot{q}_{3} \, \mathbf{c}_{q_{3}} \\ \ddot{q}_{3} \end{bmatrix}$$
(A.3)

$$\boldsymbol{\varepsilon}_{4} = \begin{bmatrix} -\left(\mathbf{s}_{q_{1}} \, \mathbf{s}_{q_{4}} + \mathbf{c}_{q_{1}} \, \mathbf{c}_{q_{4}} \, \mathbf{s}_{q_{3}}\right) \boldsymbol{\sigma}_{2} - \left(\mathbf{c}_{q_{1}} \, \mathbf{s}_{q_{4}} - \mathbf{c}_{q_{4}} \, \mathbf{s}_{q_{3}}\right) \boldsymbol{\sigma}_{1} - \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \, \boldsymbol{\sigma}_{3} \\ \mathbf{c}_{q_{3}} \, \mathbf{s}_{q_{4}} \, \boldsymbol{\sigma}_{3} - \left(\mathbf{c}_{q_{1}} \, \mathbf{c}_{q_{4}} + \mathbf{s}_{q_{1}} \, \mathbf{s}_{q_{3}} \, \mathbf{s}_{q_{4}}\right) \boldsymbol{\sigma}_{1} - \left(\mathbf{c}_{q_{4}} \, \mathbf{s}_{q_{1}} - \mathbf{c}_{q_{1}} \, \mathbf{s}_{q_{3}} \, \mathbf{s}_{q_{4}}\right) \boldsymbol{\sigma}_{2} \\ \ddot{q}_{4} - \ddot{q}_{1} \, \mathbf{s}_{q_{3}} - \dot{q}_{1} \, \dot{q}_{3} \, \mathbf{c}_{q_{3}}$$

$$(A.4)$$

where:

$$\sigma_{1} = \ddot{q}_{3} c_{q_{1}} - \dot{q}_{1} \dot{q}_{3} s_{q_{1}} + \ddot{q}_{4} c_{q_{3}} s_{q_{1}} - \dot{q}_{3} \dot{q}_{4} s_{q_{1}} s_{q_{3}} + \dot{q}_{1} \dot{q}_{4} c_{q_{1}} c_{q_{3}}$$

$$\sigma_{2} = \ddot{q}_{3} s_{q_{1}} + \dot{q}_{1} \dot{q}_{3} c_{q_{1}} - \ddot{q}_{4} c_{q_{1}} c_{q_{3}} + \dot{q}_{1} \dot{q}_{4} c_{q_{3}} s_{q_{1}} + \dot{q}_{3} \dot{q}_{4} c_{q_{1}} s_{q_{3}}$$

$$\sigma_{3} = \ddot{q}_{4} s_{q_{3}} - \ddot{q}_{1} + \dot{q}_{3} \dot{q}_{4} c_{q_{3}}$$
(A.5)

$$\boldsymbol{\epsilon}_{5} = \begin{bmatrix} \ddot{q}_{3} \, \mathbf{c}_{q_{5}} \, \mathbf{s}_{q_{4}} - \dot{q}_{4} \dot{q}_{5} \, \mathbf{c}_{q_{5}} - \ddot{q}_{4} \, \mathbf{s}_{q_{5}} + \ddot{q}_{1} \, \mathbf{s}_{q_{3}} \, \mathbf{s}_{q_{5}} - \dot{q}_{3} \, \dot{q}_{5} \, \mathbf{s}_{q_{4}} \, \mathbf{s}_{q_{5}} - \ddot{q}_{1} \, \mathbf{c}_{q_{3}} \, \mathbf{c}_{q_{4}} \, \mathbf{c}_{q_{5}} + \dot{q}_{3} \, \dot{q}_{4} \, \mathbf{c}_{q_{5}} \\ \dot{q}_{4} \, \dot{q}_{5} \, \mathbf{s}_{q_{5}} - \ddot{q}_{4} \, \mathbf{c}_{q_{5}} + \ddot{q}_{1} \, \mathbf{c}_{q_{5}} \, \mathbf{s}_{q_{3}} - \ddot{q}_{3} \, \mathbf{s}_{q_{4}} \, \mathbf{s}_{q_{5}} - \dot{q}_{1} \dot{q}_{5} \, \mathbf{s}_{q_{3}} \, \mathbf{s}_{q_{5}} + \ddot{q}_{1} \, \dot{q}_{3} \, \mathbf{c}_{q_{5}} \, \mathbf{c}_{q_{5}} \\ \left( -\dot{q}_{1} \, \dot{q}_{3} \, \right) \mathbf{s}_{q_{3}} \, \mathbf{s}_{q_{4}} + \ddot{q}_{1} \, \mathbf{s}_{q_{4}} \, \mathbf{c}_{q_{3}} + \left( -\dot{q}_{3} \, \dot{q}_{4} \right) \mathbf{s}_{q_{4}} \\ + \dot{q}_{1} \, \dot{q}_{3} \, \mathbf{c}_{q_{3}} \, \mathbf{s}_{q_{5}} + \dot{q}_{1} \, \dot{q}_{5} \, \mathbf{c}_{q_{5}} \, \mathbf{s}_{q_{4}} \, \mathbf{s}_{q_{5}} - \dot{q}_{1} \, \dot{q}_{3} \, \mathbf{c}_{q_{5}} \, \mathbf{s}_{q_{4}} \, \mathbf{s}_{q_{5}} \\ - \dot{q}_{3} \, \dot{q}_{4} \, \mathbf{c}_{q_{4}} \, \mathbf{s}_{q_{5}} - \dot{q}_{3} \, \dot{q}_{5} \, \mathbf{c}_{q_{5}} \, \mathbf{s}_{q_{4}} + \dot{q}_{1} \, \dot{q}_{5} \, \mathbf{c}_{q_{5}} \, \mathbf{s}_{q_{4}} \, \mathbf{c}_{q_{5}} - \dot{q}_{1} \, \dot{q}_{3} \, \mathbf{c}_{q_{4}} \, \mathbf{s}_{q_{5}} \, \mathbf{s}_{q_{4}} \, \mathbf{s}_{q_{5}} \\ + \left( \dot{q}_{1} \, \dot{q}_{4} \, \right) \mathbf{c}_{q_{5}} \, \mathbf{c}_{q_{5}} \, + \ddot{q}_{1} \, \dot{q}_{5} \, \mathbf{c}_{q_{5}} \, \mathbf{c}_$$

$$\boldsymbol{\varepsilon}_{6} = \begin{bmatrix} \sigma_{1} \, \mathbf{s} (q_{6}) + \sigma_{2} \, \mathbf{c} (q_{6}) \\ \mathbf{c}_{q_{6}} \, \sigma_{2} - \mathbf{s}_{q_{6}} \, \sigma_{1} - \dot{q}_{6} \left( \mathbf{c}_{q_{6}} \, \sigma_{3} + \mathbf{s}_{q_{6}} \, \sigma_{4} \right) \\ \ddot{q}_{6} + \sigma_{5} \, \mathbf{c}_{q_{5}} + \sigma_{6} \, \mathbf{s}_{q_{5}} \end{bmatrix}$$
(A.7)

where:

$$\begin{split} &\sigma_{1} = \ddot{q}_{3} \, c_{q_{5}} \, s_{q_{4}} - \dot{q}_{4} \dot{q}_{5} \, c_{q_{5}} - \dot{q}_{4} \, s_{q_{5}} + \dot{q}_{1} \, s_{q_{3}} \, s_{q_{5}} - \dot{q}_{1} \dot{q}_{5} \, s_{q_{4}} \, s_{q_{5}} - \ddot{q}_{1} \, c_{q_{3}} \, c_{q_{4}} \, c_{q_{5}} + \dot{q}_{3} \, \dot{q}_{4} \, c_{q_{5}} \\ &\quad + \dot{q}_{1} \, \dot{q}_{3} \, c_{q_{3}} \, s_{q_{5}} + \dot{q}_{1} \, \dot{q}_{5} \, c_{q_{5}} \, s_{q_{3}} + \dot{q}_{1} \, \dot{q}_{3} \, c_{q_{4}} \, c_{q_{5}} \, s_{q_{3}} + \dot{q}_{1} \, \dot{q}_{4} \, c_{q_{3}} \, c_{q_{5}} \, s_{q_{4}} + \dot{q}_{1} \dot{q}_{5} \, c_{q_{3}} \, c_{q_{4}} \, s_{q_{5}} \\ &\sigma_{2} = \ddot{q}_{5} + \ddot{q}_{3} \, c_{q_{4}} - \dot{q}_{3} \, \dot{q}_{4} \, s_{q_{4}} + \ddot{q}_{1} \, c_{q_{3}} \, s_{q_{4}} - \dot{q}_{1} \dot{q}_{3} \, s_{q_{3}} - \dot{q}_{1} \dot{q}_{3} \, s_{q_{3}} + \dot{q}_{1} \dot{q}_{4} \, c_{q_{3}} \, c_{q_{4}} \\ &\sigma_{3} = c_{q_{5}} \left( \dot{q}_{3} \, s_{q_{4}} - \dot{q}_{1} \, c_{q_{3}} \, c_{q_{4}} \right) - s_{q_{5}} \left( \dot{q}_{4} - \dot{q}_{1} \, s_{q_{3}} \right) \\ &\sigma_{4} = \dot{q}_{5} + \dot{q}_{3} \, c_{q_{4}} + \dot{q}_{1} \, c_{q_{3}} \, s_{q_{4}} \\ &\sigma_{5} = \ddot{q}_{4} - \ddot{q}_{1} \, s_{q_{3}} - \dot{q}_{1} \, \dot{q}_{3} \, c_{q_{3}} + \dot{q}_{3} \dot{q}_{5} \, s_{q_{4}} - \dot{q}_{1} \, \dot{q}_{5} \, c_{q_{3}} \, c_{q_{4}} \\ &\sigma_{6} = \left( -\dot{q}_{4} \, \dot{q}_{5} \, \right) + \ddot{q}_{3} \, s_{q_{4}} + \dot{q}_{1} \, \dot{q}_{5} \, s_{q_{3}} - \ddot{q}_{1} \, c_{q_{3}} \, c_{q_{4}} + \dot{q}_{3} \, \dot{q}_{4} \, c_{q_{4}} + \dot{q}_{1} \dot{q}_{3} \, c_{q_{4}} + \dot{q}_{1} \, \dot{q}_{4} \, c_{q_{3}} \, s_{q_{4}} \\ &\sigma_{6} = \left( -\dot{q}_{4} \, \dot{q}_{5} \, \right) + \ddot{q}_{3} \, s_{q_{4}} + \dot{q}_{1} \, \dot{q}_{5} \, s_{q_{3}} - \ddot{q}_{1} \, c_{q_{3}} \, c_{q_{4}} + \dot{q}_{3} \, \dot{q}_{4} \, c_{q_{4}} + \dot{q}_{1} \, \dot{q}_{3} \, c_{q_{4}} + \dot{q}_{1} \, \dot{q}_{4} \, c_{q_{3}} \, s_{q_{4}} \\ &\sigma_{6} = \left( -\dot{q}_{4} \, \dot{q}_{5} \, \right) + \ddot{q}_{3} \, s_{q_{4}} + \dot{q}_{1} \, \dot{q}_{5} \, s_{q_{3}} - \ddot{q}_{1} \, c_{q_{3}} \, c_{q_{4}} + \dot{q}_{3} \, \dot{q}_{4} \, c_{q_{4}} + \dot{q}_{1} \, \dot{q}_{3} \, c_{q_{4}} + \dot{q}_{1} \, \dot{q}_{4} \, c_{q_{3}} \, s_{q_{4}} \\ &\sigma_{6} = \left( -\dot{q}_{4} \, \dot{q}_{5} \, \right) + \ddot{q}_{3} \, s_{q_{4}} + \dot{q}_{1} \, \dot{q}_{5} \, s_{q_{5}} - \ddot{q}_{1} \, c_{q_{5}} \, c_{q_{5}} + \dot{q}_{1} \, \dot{q}_{5} \, c_{q_{5}} + \dot{q}_{1} \, \dot{q}_{5} \, c_{q$$

Linear acceleration:

$$\mathbf{a}_{S1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{A.9}$$

$$a_{S_{2}} = \begin{bmatrix} \left(x_{S2} \left(c_{q^{2}}^{2}-1\right) - a_{1} s_{q^{2}} - c_{q^{2}} s_{q^{2}} y_{S2}\right) \dot{q}_{1}^{2} - x_{S2} \dot{q}_{2}^{2} - y_{S2} \ddot{q}_{2} \\ \left(-y_{S2} c_{q^{2}}^{2} - a_{1} c_{q^{2}} - \frac{1}{2} x_{S2} s(2q_{2})\right) \dot{q}_{1}^{2} - y_{S2} \dot{q}_{2}^{2} + x_{S2} \ddot{q}_{2} \\ \left(2 c_{q^{2}} x_{S2} - 2 s_{q^{2}} y_{S2}\right) \dot{q}_{1} \dot{q}_{2} + \left(a_{1} + c_{q^{2}} y_{S2} + s_{q^{2}} x_{S2}\right) \ddot{q}_{1} \end{bmatrix}$$

$$a_{S_{3}} = \begin{bmatrix} \left(x_{S3} \left(\sigma_{3} - \frac{1}{2}\right) - a_{1} s_{q_{3}} - \frac{1}{2} a_{2} \left(c_{q_{2} - q_{3}} - c_{q_{2} + q_{3}}\right) - \frac{1}{2} y_{S3} s_{2q_{3}}\right) \dot{q}_{1}^{2} - \\ \left(-\frac{1}{2} x_{S3} s_{2q_{3}} - a_{1} c_{q^{3}} - \frac{1}{2} a_{2} \left(s_{q_{2} - q_{3}} + s_{q_{2} + q_{3}}\right) - y_{S3} \left(\sigma_{3} + \frac{1}{2}\right)\right) \dot{q}_{1}^{2} + \\ \left(2 a_{2} c_{q_{2}}\right) \dot{q}_{1} \dot{q}_{2} + \left(2 c_{q^{3}} x_{S^{3}} - 2 s_{q_{3}} y_{S^{3}}\right) \dot{q}_{1} \dot{q}_{3} + \\ \left(2 a_{2} c_{q^{2}}\right) \dot{q}_{1} \dot{q}_{2} + \left(2 c_{q^{3}} x_{S^{3}} - 2 s_{q_{3}} y_{S^{3}}\right) \dot{q}_{1} \dot{q}_{3} + \\ \sigma_{2} \dot{q}_{2}^{2} + \sigma_{1} \ddot{q}_{2} - x_{S3} \dot{q}_{3}^{2} - y_{S3} \ddot{q}_{3} \\ \sigma_{1} \dot{q}_{2}^{2} + \sigma_{2} \ddot{q}_{2} - y_{S3} \dot{q}_{3}^{2} + x_{S3} \ddot{q}_{3} \\ \left(a_{1} + a_{2} s_{q^{2}} + c_{q^{3}} y_{S^{3}} + s_{q^{3}} x_{S^{3}}\right) \ddot{q}_{1} \end{bmatrix}$$
(A.11)

where: x

$$\sigma_1 = -a_2 \sin(q_2 - q_3)$$

$$\sigma_2 = a_2 \cos(q_2 - q_3)$$

$$\sigma_3 = \frac{1}{2} \cos(2q_3)$$
(A.12)

$$a_{S_4} = \begin{bmatrix} \sigma_1 \dot{q}_1^2 - 2a_2 c_{q2} s_{q4} \dot{q}_1 \dot{q}_2 + \sigma_2 \dot{q}_1 \dot{q}_3 - a_3 c_{q4} \dot{q}_3^2 - \sigma_3 \ddot{q}_1 - \left( c_{q4} d_4 + c_{q4} z_{S4} \right) \ddot{q}_3 - \\ \sigma_4 \dot{q}_1^2 - 2a_2 c_{q2} c_{q4} \dot{q}_1 \dot{q}_2 + \sigma_5 \dot{q}_1 \dot{q}_3 + a_3 s_{q4} \dot{q}_3^2 - \sigma_6 \ddot{q}_1 + \left( d_4 s_{q4} + s_{q4} z_{S4} \right) \ddot{q}_3 + \\ \left( -\frac{1}{2} a_3 s_{2q_3} - a_1 c_{q3} - d_4 c_{q3}^2 - c_{q3}^2 z_{S4} - a_2 c_{q3} s_{q2} \right) \dot{q}_1^2 + \\ a_2 \left( c_{q2} c_{q3} c_{q4} + c_{q4} s_{q2} s_{q3} \right) \dot{q}_2^2 + a_2 \left( c_{q2} c_{q4} s_{q3} - c_{q3} c_{q4} s_{q2} \right) \ddot{q}_2 \right] \\ a_2 \left( c_{q2} c_{q3} s_{q4} + s_{q2} s_{q3} s_{q4} \right) \dot{q}_2^2 - a_2 \left( c_{q2} s_{q3} s_{q4} - c_{q3} s_{q2} s_{q4} \right) \ddot{q}_2 \\ a_2 \left( c_{q2} s_{q3} - c_{q3} s_{q2} \right) \dot{q}_2^2 + a_2 \left( c_{q2} c_{q3} + s_{q2} s_{q3} \right) \ddot{q}_2 - \left( d_4 + z_{S4} \right) \dot{q}_3^2 + a_3 \ddot{q}_3 \end{bmatrix}$$

$$(A.13)$$

where:

$$\begin{split} &\sigma_{1} = a_{3} \left( c_{q3}^{2} c_{q4} - c_{q4} \right) - a_{1} c_{q4} s_{q3} - a_{2} c_{q4} s_{q2} s_{q3} - c_{q3} c_{q4} s_{q3} z_{S4} - d_{4} c_{q3} c_{q4} s_{q3} \right. \\ &\sigma_{2} = 2 d_{4} s_{q3} s_{q4} - 2 a_{3} c_{q3} s_{q4} + 2 s_{q3} s_{q4} z_{S4} \\ &\sigma_{3} = a_{1} s_{q4} + c_{q3} d_{4} s_{q4} + a_{2} s_{q2} s_{q4} + a_{3} s_{q3} s_{q4} + c_{q3} s_{q4} z_{S4} \\ &\sigma_{4} = a_{3} \left( s_{q4} - c_{q3}^{2} s_{q4} \right) + a_{1} s_{q3} s_{q4} + d_{4} c_{q3} s_{q3} s_{q4} + a_{2} s_{q2} s_{q3} s_{q4} + c_{q3} s_{q3} s_{q4} z_{S4} \\ &\sigma_{5} = 2 d_{4} c_{q4} s_{q3} - 2 a_{3} c_{q3} c_{q4} + 2 c_{q4} s_{q3} z_{S4} \\ &\sigma_{6} = a_{1} c_{q4} + c_{q3} c_{q4} d_{4} + a_{2} c_{q4} s_{q2} + a_{3} c_{q4} s_{q3} + c_{q3} c_{q4} z_{S4} \end{split} \tag{A.14}$$

$$a_{S_{5}} = \begin{bmatrix} \sigma_{1}\dot{q}_{1}^{2} + \left(2a_{2}c_{q2}c_{q5}s_{q4}\right)\dot{q}_{1}\dot{q}_{2} + \left(2a_{3}c_{q3}c_{q5}s_{q4} - 2c_{q5}d_{4}s_{q3}s_{q4}\right)\dot{q}_{1}\dot{q}_{3} + \sigma_{2}\ddot{q}_{1} + \\ \sigma_{5}\dot{q}_{1}^{2} + \left(-2a_{2}c_{q2}s_{q4}s_{q5}\right)\dot{q}_{1}\dot{q}_{2} + \left(2d_{4}s_{q3}s_{q4}s_{q5} - 2a_{3}c_{q3}s_{q4}s_{q5}\right)\dot{q}_{1}\dot{q}_{3} - \sigma_{6}\ddot{q}_{1} - \\ \sigma_{9}\dot{q}_{1}^{2} + \left(2a_{2}c_{q2}c_{q4}\right)\dot{q}_{1}\dot{q}_{2} + \left(2a_{3}c_{q3}c_{q4} - 2c_{q4}d_{4}s_{q3}\right)\dot{q}_{1}\dot{q}_{3} + \sigma_{10}\ddot{q}_{1} - \\ \sigma_{3}\dot{q}_{2}^{2} - \sigma_{4}\ddot{q}_{2} + \left(d_{4}s_{q5} + a_{3}c_{q4}c_{q5}\right)\dot{q}_{3}^{2} + \left(c_{q4}c_{q5}d_{4} - a_{3}s_{q5}\right)\ddot{q}_{3} \\ \sigma_{7}\dot{q}_{2}^{2} - \sigma_{8}\ddot{q}_{2} + \left(c_{q5}d_{4} - a_{3}c_{q4}s_{q5}\right)\dot{q}_{3}^{2} + \left(-a_{3}c_{q5} - c_{q4}d_{4}s_{q5}\right)\ddot{q}_{3} \\ \sigma_{11}\dot{q}_{2}^{2} + \sigma_{12}\ddot{q}_{2} + \left(-a_{3}s_{q4}\right)\dot{q}_{3}^{2} + \left(-d_{4}s_{q4}\right)\ddot{q}_{3} \end{bmatrix}$$

$$(A.15)$$

where:

$$\begin{split} &\sigma_{1} = d_{4} \Big( s_{q5} c_{q3}^{\ 2} + c_{q4} c_{q5} s_{q3} c_{q3} \Big) + a_{2} \Big( c_{q3} s_{q2} s_{q5} + c_{q4} c_{q5} s_{q2} s_{q3} \Big) + \\ &a_{3} \Big( - c_{q4} c_{q5} c_{q3}^{\ 2} + s_{q3} s_{q5} c_{q3} + c_{q4} c_{q5} \Big) + a_{1} \Big( c_{q3} s_{q5} + c_{q4} c_{q5} s_{q3} \Big), \\ &\sigma_{2} = a_{1} c_{q5} s_{q4} + a_{2} c_{q5} s_{q2} s_{q4} + a_{3} c_{q5} s_{q3} s_{q4} + c_{q3} c_{q5} d_{4} s_{q4}, \\ &\sigma_{3} = a_{2} \Big( c_{q3} s_{q2} s_{q5} - c_{q2} s_{q3} s_{q5} + c_{q4} c_{q5} s_{q2} s_{q4} + c_{q2} c_{q3} c_{q4} c_{q5} \Big), \\ &\sigma_{4} = a_{2} \Big( c_{q2} c_{q3} s_{q5} + s_{q2} s_{q3} s_{q5} + c_{q2} c_{q4} c_{q5} s_{q2} s_{q3} + c_{q2} c_{q3} c_{q4} c_{q5} s_{q2} \Big), \\ &\sigma_{5} = d_{4} \Big( c_{q3}^{\ 2} c_{q5} - c_{q3} c_{q4} s_{q3} s_{q5} \Big) + a_{2} \Big( c_{q3} c_{q5} s_{q2} - c_{q4} s_{q2} s_{q3} s_{q5} \Big) + \\ &a_{3} \Big( c_{q4} s_{q5} c_{q3}^{\ 2} + c_{q5} s_{q3} c_{q3} - c_{q4} s_{q5} \Big) + a_{1} \Big( c_{q3} c_{q5} - c_{q4} s_{q2} s_{q3} s_{q5} \Big), \\ &\sigma_{6} = a_{1} s_{q4} s_{q5} + c_{q3} d_{4} s_{q4} s_{q5} + a_{2} s_{q2} s_{q4} s_{q5} + a_{3} s_{q3} s_{q4} s_{q5}, \\ &\sigma_{7} = a_{2} \Big( c_{q2} c_{q5} s_{q3} - c_{q3} c_{q5} s_{q2} + c_{q4} s_{q2} s_{q3} s_{q5} + c_{q2} c_{q3} c_{q4} s_{q5} \Big), \\ &\sigma_{8} = a_{2} \Big( c_{q2} c_{q5} s_{q5} - c_{q3} s_{q4} s_{q5} - c_{q4} s_{q2} s_{q3} s_{q5} + c_{q2} c_{q3} c_{q4} s_{q5} \Big), \\ &\sigma_{9} = -a_{3} \Big( s_{q4} - c_{q3}^{\ 2} s_{q4} \Big) - a_{1} s_{q3} s_{q4} - c_{q3} s_{q5} + c_{q2} s_{q3} s_{q4} + a_{2} s_{q2} s_{q3} s_{q4}, \\ &\sigma_{10} = a_{1} c_{q4} + c_{q3}^{\ 2} c_{q4} d_{4} + a_{2} c_{q4} s_{q2} + a_{3} c_{q4} s_{q3}, \\ &\sigma_{11} = a_{2} \Big( c_{q2} c_{q3} s_{q4} + s_{q2} s_{q3} s_{q4} + a_{3} c_{q4} s_{q3} \Big), \\ &\sigma_{12} = a_{2} \Big( c_{q2} c_{q3} s_{q4} + c_{q3} s_{q2} s_{q4} \Big). \end{aligned}$$

$$\mathbf{a}_{S_6} = \begin{bmatrix} a_{S6x} \\ a_{S6y} \\ a_{S6z} \end{bmatrix} \tag{A.17}$$

where:

$$a_{S6x} = \left(a_3 \left(c_{q3}^2 s_{q4} s_{q6} - s_{q4} s_{q6} + c_{q4} c_{q5} c_{q6} + c_{q3} c_{q6} s_{q3} s_{q5} - c_{q3}^2 c_{q4} c_{q5} c_{q6}\right) - \\ z_{S6} \left(-c_{q6} s_{q5}^2 c_{q3}^2 c_{q4}^2 c_{q5} + s_{q4} s_{q5} s_{q6} c_{q3}^2 c_{q4} - c_{q6} s_{q5} c_{q3}^2 c_{q5} - 2 c_{q6} s_{q3} c_{q3} c_{q4} c_{q5}^2 + c_{q5}^2 c_{q6} c_{q5}^2 c_{q5} + c_{q6} s_{q5}^2 c_{q5}^2 + c_{q6} s_{q5} c_{q3}^2 c_{q5} - 2 c_{q6} s_{q3} c_{q3} c_{q4} c_{q5}^2 + c_{q5}^2 c_{q6} c_{q5}^2 c_{q5}$$

$$\begin{split} a_{S6y} &= \left( -a_3 (c_{q6} s_{q4} + c_{q4} c_{q5} s_{q5} - c_{q3}^{-2} c_{q6} s_{q4} + c_{q3} s_{q5} s_{q5} s_{q5} - c_{q3}^{-2} c_{q4} c_{q5} s_{q5} \right) - \\ &- a_1 (c_{q6} s_{q3} s_{q4} + c_{q3} s_{q5} s_{q6} + c_{q4} c_{q5} s_{q3} s_{q6}) - z_{S6} (s_{q5} s_{q6} c_{q3}^{-2} c_{q4}^{-2} c_{q5} + c_{q6} s_{q4} s_{q5} c_{q4}^{-2} + s_{q5} s_{q6} c_{q3}^{-2} c_{q5}^{-2} + c_{q6} s_{q4} s_{q5} c_{q5}^{-2} - s_{q4} s_{q5} c_{q5}^{-2} - s_{q4} s_{q6} c_{q3}^{-2} c_{q4}^{-2} + c_{q6} s_{q3} s_{q4} c_{q5}^{-2} - s_{q5} s_{q6} c_{q3}^{-2} c_{q4}^{-2} + c_{q6} s_{q3} s_{q4}^{-2} c_{q5}^{-2} - s_{q4}^{-2} c_{q5}^{-2} - s_{q4}^{-2} c_{q5}^{-2} - s_{q4}^{-2} c_{q6}^{-2} c_{q5}^{-2} - s_{q4}^{-2} c_{q6}^{-2} c_{q5}^{-2} c_{q4}^{-2} c_{q5}^{-2} - s_{q4}^{-2} c_{q6}^{-2} c_{q5}^{-2} c_{q4}^{-2} c_{q6}^{-2} - c_{q5}^{-2} c_{q5}^{-2} c_{q4}^{-2} c_{q6}^{-2} c_{q5}^{-2} c_$$

$$\sigma_{1} = c_{q4} c_{q5} s_{q2} s_{q3} s_{q6}$$

$$\sigma_{2} = c_{q4} c_{q5} c_{q6} s_{q2} s_{q3}$$

$$\sigma_{3} = s_{q4} s_{q6} - c_{q4} c_{q5} c_{q6}$$

$$\sigma_{4} = c_{q4} c_{q6} - c_{q5} s_{q4} s_{q6}$$

$$\sigma_{5} = c_{q6} s_{q4} + c_{q4} c_{q5} s_{q6}$$

$$\sigma_{6} = c_{q4} s_{q6} + c_{q5} c_{q6} s_{q4}$$
(A.21)

$$\mathbf{a}_{ST} = \begin{bmatrix} a_{STx} \\ a_{STy} \\ a_{STz} \end{bmatrix} \tag{A.22}$$

where:

$$a_{STx} = \left(a_3 \left(c_{q3}^2 s_{q4} s_{q6} - s_{q4} s_{q6} + c_{q4} c_{q5} c_{q6} + c_{q3} c_{q6} s_{q3} s_{q5} - c_{q3}^2 c_{q4} c_{q5} c_{q6}\right) - \left(d_7 + z_{cT}\right) \sigma_2 \right. \\ + a_1 \left(c_{q3} c_{q6} s_{q5} - s_{q3} s_{q4} s_{q6} + c_{q4} c_{q5} c_{q6} s_{q3}\right) + x_{cT} \left(c_{q3}^2 c_{q4}^2 c_{q5}^2 c_{q6}^2 + c_{q3}^2 c_{q4}^2 c_{q6}^2 - c_{q3}^2 c_{q4}^2 - 2 s_{q4} s_{q6} c_{q3}^2 c_{q4} c_{q5} c_{q6} + c_{q3}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 + c_{q3}^2 c_{q4}^2 c_{q5}^2 c_{q6}^2 + c_{q5}^2 c_{q6}^2 - c_{q5}^2 c_{q6}^2 + c_{q6}^2 - 1\right) - \\ y_{cT} \sigma_3 + d_4 \left(c_{q3}^2 c_{q6} s_{q5} - c_{q3}^2 s_{q3} s_{q4} s_{q5} c_{q6}^2 c_{q5}^2 c_{q6}^2 s_{q3}^2 \right) \dot{q}_1^2 + 2 \left(2 c_{q2} c_{q4} s_{q6} + 2 c_{q2} c_{q5} c_{q6} s_{q4}^2 c_{q5}^2 c_{q5}^2 - c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5$$

$$a_{2}(c_{q2}c_{q3}S_{q4}S_{q6}+c_{q2}c_{q6}S_{q3}S_{q5}-c_{q3}c_{q6}S_{q2}S_{q5}+\\S_{q2}S_{q3}S_{q4}S_{q6}-c_{q2}c_{q3}c_{q4}c_{q5}c_{q6}-c_{q4}c_{q5}c_{q6}S_{q2}S_{q3})\dot{q}_{2}^{2}-\\a_{2}(c_{q3}S_{q2}S_{q4}S_{q6}-c_{q2}S_{q3}S_{q4}S_{q6}-c_{q6}S_{q2}S_{q3}S_{q5}+c_{q2}C_{q3}c_{q6}S_{q5}+\\c_{q2}c_{q4}c_{q5}c_{q6}S_{q3}-c_{q3}C_{q4}C_{q5}c_{q6}S_{q2})\ddot{q}_{2}+\\((d_{7}+z_{c7})(-c_{q5}c_{q6}S_{q5}c_{q4}^{2}+s_{q4}S_{q5}c_{q6}C_{q4}+c_{q5}c_{q6}S_{q5})+y_{c7}\sigma_{6}+\\x_{c7}(-c_{q4}^{2}c_{q5}^{2}c_{q6}^{2}-c_{q4}^{2}c_{q6}^{2}+c_{q4}^{2}+2S_{q4}S_{q5}c_{q6}C_{q4}+c_{q5}c_{q6}S_{q5})+y_{c7}\sigma_{6}+\\x_{c7}(-c_{q4}^{2}c_{q5}^{2}c_{q6}^{2}-c_{q4}^{2}c_{q6}^{2}+c_{q4}^{2}+2S_{q4}S_{q5}c_{q6}c_{q4}+c_{q5}c_{q6}^{2}+c_{q5}^{2}-c_{q6}^{2}-c_{q5}^{2}-c_{q6}^{2}-c_{q4}^{2}+c_{q4}^{2}+2S_{q4}S_{q5}c_{q6}^{2}+c_{q5}^{2}-c_{q5}^{2}-c_{q6}^{2}-c_{q5}^{2}-c_{q6}^{2}-c_{q4}^{2}+c_{q5}^{2}-c$$

$$\left( \left( d_7 + z_{cT} \right) \left( 2c_{q3}c_{q4}s_{q6}c_{q5}^2 - 2s_{q3}s_{q5}s_{q6}c_{q5} - 2c_{q3}c_{q4}s_{q6} \right) - x_{cT} \left( 2s_{q3}s_{q6}c_{q5}^2 c_{q6} + 2c_{q3}s_{q4}s_{q5}c_{q6}^2 - 2s_{q3}s_{q6}c_{q5} - 2c_{q3}s_{q4}s_{q5} \right) + x_{cT} \left( 2s_{q3}s_{q6}c_{q5}^2 c_{q6} + 2c_{q3}s_{q4}s_{q5}c_{q6}^2 - 2s_{q3}s_{q6}c_{q6} - 2c_{q3}s_{q4}s_{q5} \right) + x_{cT} \left( 2s_{q3}s_{q4}s_{q5}c_{q5}^2 c_{q6}^2 + 2s_{q3}c_{q4}^2 c_{q5}^2 c_{q6}^2 + 2s_{q3}c_{q4}^2 c_{q5}^2 c_{q6}^2 + 2s_{q3}c_{q4}^2 c_{q5}^2 c_{q6}^2 \right) + x_{cT} \left( 2s_{q3}s_{q4}s_{q5}c_{q5}^2 c_{q6}^2 + 2s_{q3}c_{q4}s_{q5}c_{q5}^2 c_{q6}^2 \right) + x_{cT} \left( 2s_{q3}s_{q4}s_{q5}c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}^2 c_{q5}^2 c_{q6}^2 c_{q5}^2 c_{q5}$$

$$\begin{split} a_{STy} &= \left( -x_{cT} \sigma_2 - y_{cT} \sigma_1 - d_4 \left( c_{q_3}^2 c_{q_5} - c_{q_3} c_{q_4} s_{q_3} s_{q_5} \right) + \left( d_7 + z_{cT} \right) \left( -c_{q_3}^2 c_{q_4}^2 c_{q_4}^2 c_{q_5}^2 \right) \\ &+ c_{q_3}^2 c_{q_4}^2 - c_{q_3}^2 c_{q_5}^2 + 2 s_{q_3} s_{q_5} c_{q_5} c_{q_5} c_{q_5} c_{q_5} + c_{q_5}^2 - 1 \right) - a_2 \left( c_{q_5} c_{q_5} c_{q_5} s_{q_2} - c_{q_4} s_{q_2} s_{q_5} s_{q_5} \right) - a_3 \left( c_{q_4} s_{q_5}^2 c_{q_4}^2 + c_{q_5} s_{q_3} c_{q_5} - c_{q_4} s_{q_5} - c_{q_4} s_{q_5} s_{q_5} \right) \right) \dot{q}_1^2 + \\ &+ \left( 2 a_2 c_{q_2} s_{q_4} s_{q_5} \right) \dot{q}_1 \dot{q}_2 + \left( -x_{cT} \left( 2 c_{q_3} s_{q_5} s_{q_5} c_{q_4}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} c_{q_4}^2 \right) \\ &+ \left( 2 a_2 c_{q_2} s_{q_4} s_{q_5} \right) \dot{q}_1 \dot{q}_2 + \left( -x_{cT} \left( 2 c_{q_3} s_{q_5} s_{q_5} c_{q_4}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} c_{q_4}^2 \right) \\ &+ \left( 2 c_{q_2} s_{q_4} s_{q_5} \right) \dot{q}_1 \dot{q}_2 + \left( -x_{cT} \left( 2 c_{q_3} s_{q_5} s_{q_5} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} c_{q_4}^2 \right) \\ &+ \left( 2 c_{q_2} s_{q_3} s_{q_4} c_{q_5}^2 - 2 c_{q_5} s_{q_3} s_{q_4} - 2 c_{q_3} s_{q_5} s_{q_5} \right) + y_{cT} \left( -2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} c_{q_4}^2 \right) \\ &+ \left( 2 c_{q_3} s_{q_4} c_{q_5}^2 - 2 c_{q_5} s_{q_3} s_{q_4} s_{q_5} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} - 2 s_{q_3} s_{q_5} s_{q_5} \right) \\ &+ \left( d_7 + z_{cT} \right) \left( -2 c_{q_3} c_{q_4} c_{q_5}^2 + 2 s_{q_3} s_{q_4} s_{q_5} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} - 2 c_{q_3} s_{q_4} s_{q_5} \right) \\ &+ \left( d_7 + z_{cT} \right) \left( 2 c_{q_3} c_{q_4} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} - 2 c_{q_3} s_{q_4} s_{q_5} \right) \\ &+ \left( d_7 + z_{cT} \right) \left( 2 c_{q_3} c_{q_4} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} + 2 c_{q_5} c_{q_5} s_{q_5} s_{q_5} s_{q_5} \right) \\ &+ \left( d_7 + z_{cT} \right) \left( 2 c_{q_3} c_{q_4} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} s_{q_5} - 2 c_{q_3} c_{q_5} c_{q_5} s_{q_5} s_{q_5} \right) \\ &+ \left( d_7 + z_{cT} \right) \left( 2 c_{q_3} c_{q_4} c_{q_5}^2 + 2 c_{q_3} c_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q_5} \right) \\ &+ \left( c_7 + c_{c_5} c_{q_5} c_{q_5} c_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q_5} s_{q$$

$$\begin{split} &\sigma_{3} = s_{q6} c_{q3}^{\ 2} c_{q4}^{\ 2} c_{q5}^{\ 2} c_{q6} + s_{q6} c_{q3}^{\ 2} c_{q4}^{\ 2} c_{q6} + 2 s_{q4} c_{q3}^{\ 2} c_{q4} c_{q5} c_{q6}^{\ 2} - s_{q4} c_{q3}^{\ 2} c_{q4} c_{q5} + s_{q6} c_{q3}^{\ 2} c_{q6}^{\ 2} - 2 s_{q6} c_{q3}^{\ 2} c_{q6}^{\ 2} - 2 s_{q3} s_{q5} s_{q6} c_{q3}^{\ 2} c_{q6}^{\ 2} - 2 s_{q3} s_{q4} s_{q5} c_{q6}^{\ 2} + s_{q6} c_{q5}^{\ 2} c_{q6}^{\ 2} - 2 s_{q6} c_{q5}^{\ 2} c_{q6}^{\ 2} + \frac{1}{2} s_{2q_6} \\ &\sigma_{4} = 2 c_{q3} c_{q4}^{\ 2} c_{q6} s_{q5}^{\ 2} - 2 c_{q5}^{\ 2} s_{q3} s_{q4} s_{q6}^{\ 2} + 2 c_{q4} c_{q5} c_{q6} s_{q3}^{\ 2} - 2 c_{q3} c_{q4} c_{q5} s_{q4} s_{q5} s_{q6} \\ &\sigma_{5} = 2 c_{q4} c_{q5} s_{q3} s_{q6}^{\ 2} + 2 c_{q3} c_{q4}^{\ 2} s_{q5}^{\ 2} s_{q6}^{\ 2} + 2 c_{q5}^{\ 2} c_{q6} s_{q3}^{\ 2} s_{q4}^{\ 2} c_{q6}^{\ 2} s_{q4}^{\ 2} c_{q6}^{\ 2} + 2 c_{q5}^{\ 2} c_{q6}^{\ 2} s_{q5}^{\ 2} c_{q6}^{\ 2} - s_{q4} c_{q5}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} \\ &\sigma_{6} = s_{q6} c_{q4}^{\ 2} c_{q5}^{\ 2} c_{q6}^{\ 2} + s_{q6} c_{q4}^{\ 2} c_{q6}^{\ 2} + 2 c_{q4}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q4}^{\ 2} c_{q5}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} \\ &\sigma_{6} = s_{q6} c_{q4}^{\ 2} c_{q5}^{\ 2} c_{q6}^{\ 2} + s_{q6}^{\ 2} c_{q6}^{\ 2} + 2 c_{q4}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q4}^{\ 2} c_{q5}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} \\ &\sigma_{6} = s_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} + s_{q6}^{\ 2} c_{q6}^{\ 2} + 2 c_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} \\ &\sigma_{6} = s_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} + s_{q6}^{\ 2} c_{q6}^{\ 2} + s_{q6}^{\ 2} c_{q6}^{\ 2} + s_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q6}^{\ 2} \\ &\sigma_{6} = s_{q6}^{\ 2} c_{q6}^{\ 2} c_{q6}^{\ 2} + s_{q6}^{\ 2} c_{q6}^{\ 2} - s_{q6}^{\ 2} c_{q$$

## Part B. Elementy dynamiki robota

Elements of M matrix:

$$\mathbf{M} = \begin{bmatrix} M_{11} & \cdots & M_{16} \\ \vdots & \ddots & \vdots \\ M_{61} & \cdots & M_{66} \end{bmatrix}$$
 (B.1)

$$\begin{split} M_{11} &= p_4 + p_1 \, \mathsf{C}_{q2}^2 + p_5 \big( 2s_{q4} \, \mathsf{S}_{q6} - 2c_{q4} \, \mathsf{C}_{q5} \, \mathsf{C}_{q6} + \mathsf{C}_{q3}^2 - 2c_{q3}^2 \, \mathsf{S}_{q4} \, \mathsf{S}_{q6} - 2c_{q3} \, \mathsf{C}_{q6} \, \mathsf{S}_{q5} - \\ & 2c_{q3} \, \mathsf{C}_{q6} \, \mathsf{S}_{q3} \, \mathsf{S}_{q5} + 2c_{q3} \, \mathsf{S}_{q3} \, \mathsf{S}_{q5} \, \mathsf{S}_{q6} \big) + 2p_6 \, \big( c_{q4} \, \mathsf{C}_{q5} \, \mathsf{S}_{q6} + 2c_{q3}^2 \, \mathsf{S}_{q4} \, \mathsf{S}_{q5} \, \mathsf{C}_{q4} \, \mathsf{C}_{q5} \\ & 2c_{q3}^2 \, c_{q6} \, \mathsf{S}_{q4} + 2c_{q3} \, \mathsf{S}_{q5} \, \mathsf{S}_{q6} \big) + 2p_7 \, \big( c_{q3}^2 \, c_{q4} \, \mathsf{S}_{q5} \, \mathsf{S}_{q5} \, \mathsf{S}_{q5} \, \mathsf{S}_{2q} \, \mathsf{S}_{2q} \, \mathsf{S}_{q5} \, \mathsf{S}_{q6} - \\ & 2c_{q3}^2 \, c_{q6} \, \mathsf{S}_{q4} \, \mathsf{S}_{q3} \, \mathsf{S}_{q6} \, + \mathsf{C}_{q5}^2 \, \mathsf{S}_{q6} \, \mathsf{C}_{q3}^2 \, \mathsf{C}_{q4} \, \mathsf{S}_{q5} \, \mathsf{S}_{q5} \, \mathsf{S}_{q5} \, \mathsf{S}_{q5} \, \mathsf{S}_{q6} - \\ & c_{q5} \, \mathsf{S}_{q5} \, \mathsf{S}_{46} \, \mathsf{C}_{q4}^2 \, \mathsf{S}_{q3} \, \mathsf{S}_{q6} \, \mathsf{C}_{q5}^2 \, \mathsf{C}_{q4}^2 \, \mathsf{S}_{35} \, \mathsf{S}_{36} \, \mathsf{C}_{q5}^2 \, \mathsf{C}_{q4}^2 \, \mathsf{S}_{35} \, \mathsf{S}_{36} \, \mathsf{C}_{q5}^2 \, \mathsf{C}_{q6}^2 \, \mathsf{C}_{q6}^2 \, \mathsf{C}_{q6}^2 \, \mathsf{C}_{q5}^2 \, \mathsf{C}_{q5}^2 \, \mathsf{C}_{q6}^2 \, \mathsf{C}_{q5}^2 \, \mathsf{C}_{q5}^2$$

$$M_{12} = M_{21} = p_{45} c_{q_2} - p_{46} s_{q_2} - p_{16} (c_{q_2} c_{q_4} s_{q_6} + c_{q_2} c_{q_5} c_{q_6}) - p_{17} (c_{q_2} c_{q_4} c_{q_6} + c_{q_2} c_{q_5} s_{q_4} s_{q_6}) - p_{18} c_{q_2} s_{q_4} s_{q_5}$$
(B.3)

```
M_{13} = M_{31} = p_5 \left( c_{q_5} c_{q6} s_{q_3} s_{q4} - c_{q_3} c_{q5} c_{q6} s_{q_4} - c_{q_4} c_{q_3} s_{q_6} \right) - p_7 c_{q_3} s_{q_4} s_{q5} +
                     p_6\left(c_{a_3}c_{a_5}s_{a_4}s_{a_6}-c_{a_6}c_{a_4}c_{a_3}\right)+p_{11}\left(c_{a_5}^2s_{a_3}s_{a_4}s_{a_6}+c_{a_3}c_{a_6}s_{a_5}-c_{a_5}s_{a_5}\right)
                    2c_{a_3}c_{a_4}^2c_{a_6}s_{a_5} - s_{a_3}s_{a_4}s_{a_6} - c_{a_4}c_{a_5}^2c_{a_6}^2c_{a_3}s_{a_4} + c_{a_3}c_{a_4}c_{a_5}s_{a_4}s_{a_5}s_{a_6} -
                    c_{q4} c_{q5} c_{q6} s_{q3} + p_{12} (c_{q3} c_{q5} c_{q6} s_{q6} + c_{q4} c_{q6} s_{q3} s_{q5} s_{q6} - 2 c_{q3} c_{q4}^2 c_{q5} c_{q6} s_{q6} -
                    c_{q3}\,c_{q4}\,c_{q6}^{\ 2}\,s_{q4}+c_{q5}\,c_{q6}^{\ 2}\,s_{q4}\,s_{q3}\,s_{q5}\,\big)+p_{13}\,\big(c_{q5}\,s_{q3}\,s_{q4}\,s_{q5}-c_{q3}\,c_{q4}\,c_{q5}^{\ 2}\,s_{q4}\,\big)+
                    p_{14} \left(2c_{a3}c_{a5}c_{a6}^2 - 4c_{a3}c_{a4}^2c_{a5}c_{a6}^2 + c_{a4}c_{a6}s_{a3}s_{a5} - 2c_{a5}c_{a6}s_{a3}s_{a4}s_{a5}s_{a6} + c_{a4}c_{a5}c_{a6}s_{a3}s_{a5} - 2c_{a5}c_{a6}s_{a3}s_{a4}s_{a5}s_{a6} + c_{a4}c_{a5}c_{a6}s_{a3}s_{a5} + c_{a4}c_{a5}c_{a6}s_{a5}c_{a6}s_{a5}s_{a6}s_{a5}s_{a6} + c_{a4}c_{a5}c_{a6}s_{a5}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a6}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{a5}s_{
                                                                                                                                                                                                                                                                                                                                                (B.4)
                    c_{a_3} c_{a_4} c_{a_6} s_{a_4} s_{a_6} + 2 c_{a_3} c_{a_4} c_{a_6} s_{a_4} s_{a_6} + p_{15} (c_{a_3} s_{a_5} s_{a_6} - 2 c_{a_3} s_{a_6} s_{a_5} c_{a_4}^2 -
                    c_{a5}^2 c_{a6} s_{a3} s_{a4} - c_{a6} s_{a3} s_{a4} - c_{a6} s_{a3} s_{a4} - c_{a4} c_{a5} s_{a3} s_{a6} - 2 c_{a3} c_{a4} c_{a5} c_{a6} s_{a4} s_{a5} + c_{a6} c_{a5} c_{a6} s_{a5} c_{a6} c_{a5} c_{a6} c_{a6} c_{a5} c_{a5} c_{a6} c_{a5} c_{a5} c_{a6} c_{a5} c_{a5} c_{a6} c_{a5} 
                     p_{21} c_{q4} s_{q3} s_{q6} + p_{22} (c_{q4} c_{q6} s_{q3} - c_{q5} s_{q3} s_{q4} s_{q6}) + p_{25} s_{q3} s_{q4} s_{q5} +
                     p_{33}(c_{q3}c_{q5}+c_{q4}s_{q3}s_{q5})+p_{34}c_{q4}c_{q3}s_{q4}+2p_{40}(c_{q5}s_{q3}s_{q4}+2c_{q5}c_{q4}c_{q3}s_{q4}s_{q5})+
                    p_{41}\left(c_{a_4}c_{a_5}c_{a_3}-c_{a_3}c_{a_5}+2c_{a_3}c_{a_4}^2c_{a_5}\right)+p_{50}c_{a_3}-p_{44}c_{a_3}-2p_{42}c_{a_3}c_{a_4}^2+
                     p_{43} c_{q_4} s_{q_3} + p_{48} s_{q_4} s_{q_3} - 2p_{33} c_{q_3} c_{q_4}^2 c_{q_5}
M_{14} = M_{41} = p_5(c_{q4}c_{q5}c_{q6}s_{q3}-c_{q6}s_{q3}s_{q4}) - p_6(c_{q4}c_{q5}s_{q3}s_{q6}-c_{q6}s_{q3}s_{q4}) +
                  p_7 c_{q4} s_{q3} s_{q5} + p_{11} (2 c_{q3} c_{q5} c_{q6} s_{q4} - c_{q3} c_{q4} s_{q6} - 2 c_{q5} s_{q3} s_{q5} s_{q6} + 2 c_{q5}^2 c_{q3} c_{q4} s_{q6}) +
                  p_{12} \left( c_{a3} c_{a4} c_{a5} c_{a6}^2 s_{a5} - c_{a6}^2 s_{a3} + c_{a5}^2 c_{a6}^2 s_{a3} - c_{a3} c_{a6} s_{a4} s_{a5} s_{a6} \right)
                  +p_{13}(c_{a5}^2s_{a3}+c_{a3}c_{a4}c_{a5}s_{a5}-c_{a6}^2)+2p_{14}(2c_{a6}s_{a3}s_{a6}-2c_{a3}c_{a6}^2s_{a4}s_{a5}-
                  2c_{a5}^2c_{a6}s_{a3}s_{a6} - 2c_{a3}c_{a4}c_{a5}c_{a6}s_{a5}s_{a6} + p_{15}(c_{a3}c_{a5}s_{a4}s_{a6} + c_{a4}s_{a2}s_{a5} +
                  2c_{q5}c_{q6}s_{q3}s_{q5}-c_{q5}^2c_{q3}c_{q4}c_{q6} + p_{16}(c_{q4}c_{q5}c_{q6}s_{q2}-s_{q2}s_{q4}s_{q6}) -
                                                                                                                                                                                                                                                                                                                                                (B.5)
                  p_{17}(c_{q6} s_{q2} s_{q4} + c_{q4} c_{q5} s_{q2} s_{q6}) + p_{21}(c_{q3} c_{q4} c_{q5} c_{q6} - c_{q3} s_{q4} s_{q6}) -
                   p_{22}(c_{q3}c_{q6}s_{q4}+c_{q3}c_{q4}c_{q5}s_{q6})+p_{23}(c_{q4}c_{q5}c_{q6}-s_{q4}s_{q6})+
                   p_{24}(c_{q4}s_{q6}-c_{q4}c_{q5}s_{q6})+p_{25}c_{q3}c_{q4}s_{q5}+2p_{32}c_{q4}s_{q5}-p_{33}c_{q3}s_{q4}s_{q5}+
                   p_{48} c_{a3} c_{a4} - p_{35} s_{a3} + 2p_{40} (c_{a5}^2 c_{a3} c_{a4} - 2c_{a5} s_{a3} s_{a5}) -
                  p_{41} c_{q5} c_{q3} s_{q4} - p_{43} c_{q3} s_{q4}
M_{15} = M_{51} = p_6 S_{q3} S_{q4} S_{q5} S_{q6} - p_5 S_{q3} S_{q4} C_{q6} S_{q5} + p_7 C_{q5} S_{q3} S_{q4} -
                  p_{11}(c_{q3}c_{q4}c_{q6}s_{q5}+2c_{q5}c_{q6}s_{q3})+
                  p_{12} \left( c_{q6} \, s_{q3} \, s_{q5} \, s_{q6} - c_{q3} \, c_{q5} \, c_{q6} \, s_{q6} \, c_{q4} + c_{q3} \, c_{q4} \, c_{q5} \, c_{q6}^2 \, s_{q5} \right) +
                  p_{14} \left( c_{a6}^2 s_{a3} s_{a5} + 2 c_{a3} c_{a6} s_{a4} s_{a6} - 2 c_{a3} c_{a4} c_{a5} c_{a6}^2 \right) +
                                                                                                                                                                                                                                                                                                                                                (B.6)
                  p_{15}(c_{a5}s_{a2}s_{a4}-c_{a3}c_{a4}s_{a5}s_{a6}-c_{a5}s_{a3}s_{a6})-p_{21}c_{a3}c_{a6}s_{a4}s_{a5}-
                  p_{16} S_{q2} S_{q4} S_{q5} C_{q6} + p_{17} S_{q2} S_{q4} S_{q5} S_{q6} +
                   +p_{22}c_{q3}s_{q4}s_{q5}s_{q6}-p_{23}c_{q6}s_{q4}s_{q5}+p_{24}s_{q4}s_{q5}s_{q6}+p_{25}c_{q3}c_{q5}s_{q4}+
                  p_{32} c_{q5} s_{q4} + p_{33} s_{q3} s_{q5} - p_{33} c_{q3} c_{q4} c_{q5} + p_{36} c_{q3} s_{q4} + p_{41} (c_{q5} s_{q3} + c_{q3} c_{q4} s_{q5})
M_{16} = M_{61} = p_5 (c_{q4} c_{q6} s_{q3} - c_{q5} s_{q3} s_{q4} s_{q6}) - p_6 (c_{q4} s_{q3} s_{q6} + c_{q5} c_{q6} s_{q3} s_{q4}) +
                  +p_{11}(c_{q3}c_{q4}c_{q5}s_{q6}-s_{q3}s_{q5}s_{q6}+c_{q3}c_{q6}s_{q4})+
                  p_{15}(c_{a3} c_{a4} c_{a6} + c_{a6} c_{a3} c_{a5} - c_{a3} c_{a4} c_{a5} c_{a6}) +
                                                                                                                                                                                                                                                                                                                                                (B.7)
                   p_{16}(c_{q4}c_{q6}s_{q2}-p_{16}c_{q5}s_{q2}s_{q4}s_{q6})-p_{17}(c_{q5}c_{q6}s_{q2}s_{q4}+c_{q4}s_{q2}s_{q6})+
                   p_{21}(c_{q3}c_{q4}c_{q6}-c_{q3}c_{q5}s_{q4}s_{q6})-p_{22}(c_{q3}c_{q4}s_{q6}+c_{q3}c_{q5}c_{q6}s_{q4})+
                   p_{23}\left(c_{q4}\,c_{q6}-c_{q5}\,s_{q4}\,s_{q6}\right)-p_{24}\left(c_{q5}\,c_{q6}\,s_{q4}+c_{q4}\,s_{q6}\right)-p_{37}\left(c_{q5}\,s_{q3}+c_{q3}\,c_{q4}\,s_{q5}\right)
M_{22} = p_{51}
                                                                                                                                                                                                                                                                                                                                                (B.8)
```

$$\begin{split} M_{23} &= M_{32} = p_{16} \left( c_{q2} \, c_{q6} \, s_{q3} \, s_{q5} - c_{q2} \, c_{q3} \, c_{q4} \, c_{q5} \, c_{q6} - c_{q4} \, c_{q5} \, c_{q6} \, s_{q2} \, s_{q3} + \\ & c_{q2} \, c_{q3} \, s_{q4} \, s_{q6} + s_{q2} \, s_{q3} \, s_{q4} \, s_{q6} - c_{q3} \, c_{q6} \, s_{q2} \, s_{q5} \right) + \\ & p_{17} \left( c_{q3} \, s_{q2} \, s_{q5} \, s_{q6} - c_{q2} \, s_{q3} \, s_{q5} \, s_{q6} + c_{q5} \, s_{q3} + c_{q6} \, s_{q2} \, s_{q3} \, s_{q4} + \\ & c_{q2} \, c_{q3} \, c_{q4} \, c_{q5} \, s_{q6} + c_{q2} \, c_{q3} \, c_{q6} \, s_{q4} \right) + \\ & p_{18} \left( c_{q3} \, c_{q5} \, s_{q2} - c_{q2} \, c_{q5} \, s_{q3} - c_{q2} \, c_{q4} \, c_{q5} \, s_{q3} - c_{q4} \, s_{q2} \, s_{q3} \, s_{q5} \right) + \\ & p_{19} \left( s_{q2} \, s_{q3} + c_{q2} \, c_{q3} \right) + p_{20} \left( c_{q3} \, s_{q2} - c_{q2} \, s_{q3} \right) \end{split}$$

$$M_{24} = M_{42} = p_{17} \left( c_{q5} \, s_{q4} \, s_{q6} \, s_{q_2-q_3} - c_{q4} \, c_{q6} \, s_{q_2-q_3} \right) - p_{16} \left( c_{q4} \, s_{q6} \, s_{q_2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q_2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q_2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q_2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q_2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, c_{q6} \, s_{q4} \, s_{q5} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q4} \, s_{q5} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, s_{q4} \, s_{q5} \, s_{q2-q_3} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q2-q_3} + c_{q5} \, s_{q4} \, s_{q5} \, s_{q4} \, s_{q5} \, s_{q5} \right) - p_{18} \left( c_{q4} \, s_{q6} \, s_{q4} \, s_{q5} \, s_{q5} \, s_{q5} \, s_{q5} \right) - p_{18} \left( c_{q4} \, s_{q5} \, s_{$$

$$M_{25} = M_{52} = p_{16} \left( c_{q2} c_{q6} c_{q4} s_{q3} s_{q5} - c_{q2} c_{q3} c_{q5} c_{q6} - c_{q4} c_{q3} c_{q6} s_{q2} s_{q5} - c_{q5} c_{q6} c_{q4} s_{q3} c_{q5} c_{q6} + c_{q5} c_{q6} c_{q4} c_{q5} c_{q6} c_{q4} c_{q5} c_{q6} + c_{q5} c_{q6} c_{q6} c_{q5} c_{q6} c_{q$$

$$M_{26} = M_{62} = p_{16} \left( c_{q2} c_{q4} c_{q5} c_{q6} s_{q3} + c_{q2} c_{q4} c_{q5} s_{q3} s_{q6} + c_{q2} c_{q6} s_{q3} s_{q4} - c_{q3} c_{q6} s_{q2} s_{q4} + c_{q2} c_{q3} s_{q5} s_{q6} + s_{q2} s_{q3} s_{q5} s_{q6} - c_{q3} c_{q4} c_{q5} s_{q2} s_{q6} \right) +$$

$$p_{17} \left( c_{q2} c_{q3} c_{q6} s_{q5} - c_{q2} s_{q3} s_{q6} + c_{q3} s_{q2} s_{q4} s_{q6} + c_{q6} s_{q2} s_{q3} s_{q5} - c_{q3} c_{q4} c_{q5} c_{q6} s_{q2} \right)$$
(B.12)

$$\begin{split} M_{33} &= p_{47} + 2p_5 \left( \mathbf{s}_{q4} \, \mathbf{s}_{q6} - \mathbf{c}_{q4} \, \mathbf{c}_{q5} \, \mathbf{c}_{q6} \right) + 2p_6 \left( \mathbf{c}_{q4} \, \mathbf{c}_{q5} \, \mathbf{s}_{q6} + \mathbf{c}_{q4} \, \mathbf{c}_{q5} \, \mathbf{s}_{q6} + \mathbf{c}_{q6} \, \mathbf{s}_{q4} \right) - \\ & 2p_7 \, \mathbf{c}_{q4} \, \mathbf{s}_{q5} + 2p_{11} \left( \mathbf{c}_{q4} \, \mathbf{c}_{q6} \, \mathbf{s}_{q4} \, \mathbf{s}_{q5} - \mathbf{c}_{q5} \, \mathbf{s}_{q5} \, \mathbf{s}_{q6} + \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q5} \, \mathbf{s}_{q5} \, \mathbf{s}_{q6} \right) + \\ & p_{12} \left( 2\mathbf{c}_{q4} \, \mathbf{c}_{q5} \, \mathbf{c}_{q6} \, \mathbf{s}_{q4} \, \mathbf{s}_{q6} - \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q5}^{\ 2} \, \mathbf{c}_{q6}^{\ 2} + \mathbf{c}_{q5}^{\ 2} \, \mathbf{c}_{q6}^{\ 2} \, \mathbf{c}_{q6}^{\ 2} + \mathbf{c}_{q4}^{\ 2} \right) + \\ & p_{13} \left( \mathbf{c}_{q5}^{\ 2} - \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q5}^{\ 2} \right) + p_{14} \left( \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q5}^{\ 2} \, \mathbf{c}_{q6} \, \mathbf{s}_{q6} + \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q6} \, \mathbf{s}_{q6} - 2 \, \mathbf{c}_{q5}^{\ 2} \, \mathbf{c}_{q6} \, \mathbf{s}_{q6} + \\ & 2\mathbf{c}_{q4} \, \mathbf{c}_{q5} \, \mathbf{c}_{q6}^{\ 2} \, \mathbf{s}_{q4} \right) + 2p_{15} \left( \mathbf{c}_{q4} \, \mathbf{s}_{q4} \, \mathbf{s}_{q5} \, \mathbf{s}_{q6} + \mathbf{c}_{q5} \, \mathbf{c}_{q6} \, \mathbf{s}_{q5} + \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q5} \, \mathbf{c}_{q6} \, \mathbf{s}_{q5} \right) - \\ & 2p_{21} \, \mathbf{c}_{q6} \, \mathbf{s}_{q5} + 2p_{22} \, \mathbf{s}_{q5} \, \mathbf{s}_{q6} + 2p_{25} \, \mathbf{c}_{q5} + 2p_{33} \, \mathbf{c}_{q4} \, \mathbf{c}_{q5} \, \mathbf{s}_{q4} + \\ & 2p_{40} \left( \mathbf{c}_{q4}^{\ 2} \, \mathbf{c}_{q5} \, \mathbf{s}_{q5} - \mathbf{s}_{2q5} \right) - 2p_{41} \, \mathbf{c}_{q4} \, \mathbf{s}_{q4} \, \mathbf{s}_{q5} \, \mathbf{s}_{p4} + p_{42} \, \mathbf{s}_{2q4} \end{aligned}$$

$$\begin{split} M_{34} &= M_{43} = p_{11} \left( s_{q4} \, s_{q6} - 2 \, c_{q5}^{\ 2} \, s_{q4} \, s_{q6} + p_{11} \, c_{q4} \, c_{q5} \, c_{q6} \right) - p_{12} \left( c_{q4} \, c_{q6} \, s_{q5} \, s_{q6} - c_{q6} \, c_{q6}^{\ 2} \, s_{q4} \, s_{q5} \right) - p_{13} \, c_{q5} \, s_{q4} \, s_{q5} + p_{14} \left( c_{q5} \, c_{q6} \, s_{q4} \, s_{q5} \, s_{q6} - c_{q4} \, c_{q6}^{\ 2} \, s_{q5} \right) \\ &+ p_{15} \left( c_{q4} \, c_{q5} \, s_{q6} - c_{q6} \, s_{q4} + c_{q5}^{\ 2} \, c_{q6} \, s_{q4} \right) - p_{21} \left( c_{q4} \, s_{q6} + c_{q5} \, c_{q6} \, s_{q4} \right) + \\ & p_{22} \left( c_{q5} \, s_{q4} \, s_{q6} - c_{q4} \, c_{q6} \right) - p_{25} \, s_{q4} \, s_{q5} - p_{33} \, c_{q4} \, s_{q5} - 2 p_{40} \, c_{q5}^{\ 2} \, s_{q4} - p_{48} \, s_{q4} \end{split}$$

$$(B.14)$$

$$M_{35} = M_{53} = (-p_5)c_{q5}c_{q6} + p_6c_{q5}s_{q6} - p_7s_{q5} + p_{11}c_{q6}s_{q4}s_{q5} + p_{12}(c_{q5}c_{q6}s_{q4}s_{q6} - c_{q5}c_{q6}s_{q4}s_{q6} - c_{q5}c_{q6}s_{q4}c_{q5}^2) + p_{14}(c_{q6}^2c_{q5}s_{q4} + c_{q4}c_{q6}s_{q6}) + p_{15}s_{q4}s_{q5}s_{q6} - p_{21}c_{q4}c_{q6}s_{q5} + c_{q6}c_{q5}c_{q5}c_{q6}c_{q5}c_{q5}c_{q6}c_{q5}c_{q6$$

$$M_{36} = M_{63} = p_5 \, s_{q5} \, s_{q6} + p_6 \, c_{q6} \, s_{q5} + p_{11} \left( c_{q4} \, c_{q6} - c_{q5} \, s_{q4} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5} \, s_{q6} \right) + p_{15} \left( c_{q4} \, s_{q6} + c_{q5} \, s_{q6} + c_{q5}$$

$$M_{44} = p_{35} + 2p_{11} c_{q5} s_{q5} s_{q6} + p_{12} (c_{q_6}^2 - c_{q_5}^2 c_{q_6}^2) - p_{13} c_{q_5}^2 + p_{14} (c_{q_6}^2 c_{q6} s_{q6} - s_{2q_6}) + 2p_{15} c_{q5} c_{q6} s_{q5} + p_{40} s_{2q_5}$$
(B.17)

$$M_{45} = M_{54} = p_{11} c_{q5} c_{q6} - p_{12} c_{q6} s_{q5} s_{q6} - p_{14} c_{q6}^2 s_{q5} + p_{15} c_{q5} s_{q6} - p_{33} s_{q5} - p_{41} c_{q5}$$
(B.18)

$$M_{46} = M_{64} = p_{11}s_{q5}s_{q6} - p_{15}\left(c_{q6}s_{q5} + 2c_{q3}^2c_{q5}c_{q6}s_{q5}\right)$$
(B.19)

$$M_{55} = p_{36} + p_{14} s_{2a_c} - p_{12} c_{a_c}^{2}$$
(B.20)

$$M_{56} = M_{65} = p_{11}c_{66} + p_{15}s_{66} \tag{B.21}$$

$$M_{66} = p_{37}$$
 (B.22)

Parameters:

$$p_1 = (I_{2xx} - I_{2yy} - m_2 x_{S2}^2 - a_2^2 (m_3 + m_4 + m_5 + m_6 + m_T) + m_2 y_{S2}^2)$$
 (B.23)

$$p_2 = a_2^2 \left( m_3 + m_4 + m_5 + m_6 + m_T \right) \tag{B.24}$$

$$p_3 = 2a_1 m_2 y_{s2} (B.25)$$

$$p_{4} = I_{2,yy} + I_{1zz} + I_{3,yy} + m_{2} x_{S2}^{2} + m_{3} x_{S3}^{2} + p_{2} + m_{T} x_{cT}^{2} + 2d_{7} m_{T} z_{cT} + m_{T} z_{cT}^{2} + m_{T} z_{cT}^{2}$$

$$p_{\scriptscriptstyle E} = a_{\scriptscriptstyle 2} \, m_{\scriptscriptstyle T} \, x_{\scriptscriptstyle cT} \tag{B.27}$$

$$p_6 = a_3 m_T y_{cT} (B.28)$$

$$p_{\tau} = a_2 d_{\tau} m_{\tau} + a_2 m_{\epsilon} z_{s\epsilon} + a_2 m_{\tau} z_{s\tau}$$
 (B.29)

$$p_{o} = a_{2}^{2} (m_{A} + m_{c} + m_{C} + m_{T})$$
 (B.30)

$$p_{10} = I_{5xx} + I_{4zz} + I_{6yy} + I_{Tyy}$$
 (B.31)

$$p_{11} = I_{6vz} + I_{Tvz} - d_7 m_T y_{cT} - m_T y_{cT} Z_{cT}$$
(B.32)

$$p_{12} = (-m_T)x_{cT}^2 + m_T y_{cT}^2 + I_{6xx} - I_{6yy} + I_{Txx} - I_{Tyy}$$
(B.33)

$$p_{13} = m_T d_7^2 + 2m_T d_7 z_{cT} - m_T y_{cT}^2 + m_6 z_{S6}^2 + m_T z_{cT}^2 + I_{5xx} - I_{5yy} + I_{6yy} - I_{6zz} + I_{Tyy} - I_{Tzz}$$
(B.34)

$$p_{14} = I_{6xy} + I_{Txy} - m_T x_{cT} y_{cT}$$
(B.35)

$$p_{15} = I_{6xz} + I_{Txz} - d_7 m_T x_{cT} - m_T x_{cT} z_{cT}$$
(B.36)

$$p_{16} = a_2 \, m_T \, x_{cT} \tag{B.37}$$

$$p_{17} = a_2 m_T y_{cT} (B.38)$$

$$p_{18} = a_2 d_7 m_T + a_2 m_6 z_{S6} + a_2 m_T z_{cT}$$
 (B.39)

$$p_{19} = a_2 a_3 m_4 + a_2 a_3 m_5 + a_2 a_3 m_6 + a_2 a_3 m_T + a_2 m_3 x_{s3}$$
 (B.40)

$$p_{20} = a_2 d_4 m_4 + a_2 d_4 m_5 + a_2 d_4 m_6 + a_2 d_4 m_7 + a_2 m_3 y_{53} + a_2 m_4 z_{54}$$
 (B.41)

$$p_{21} = d_4 m_T x_{cT} (B.42)$$

$$p_{22} = d_4 m_T y_{cT} (B.43)$$

$$p_{23} = a_1 m_T x_{cT} (B.44)$$

$$p_{24} = a_1 m_T y_{cT} (B.45)$$

$$p_{25} = d_A d_7 m_T + d_A m_6 Z_{56} + d_A m_T Z_{57}$$
 (B.46)

$$p_{26} = a_1^2 (m_2 + m_3 + m_4 + m_5 + m_6 + m_T)$$
(B.47)

$$p_{27} = a_3 d_4 (m_4 + m_5 + m_6 + m_7) + a_3 m_4 z_{54} + m_3 x_{53} y_{53} - I_{3xy}$$
(B.48)

$$p_{28} = 2a_1(a_2m_3 + a_2m_4 + a_2m_5 + a_2m_6 + a_2m_7 + m_2x_{s2})$$
(B.49)

$$p_{29} = 2a_1 \left( d_4 m_4 + d_4 m_5 + d_4 m_6 + d_4 m_7 + m_3 y_{S3} + m_4 z_{S4} \right)$$
 (B.50)

$$p_{30} = I_{3xx} - I_{3yy} - I_{5xx} + I_{4yy} - I_{4zz} + I_{5zz} + p_{12} - m_3 x_{s3}^2 + m_4 z_{s4}^2 + m_3 y_{s3}^2 + 2d_4 m_4 z_{s4} - a_3^2 (m_T + m_6 + m_5 + m_4) + d_4^2 (m_T + m_6 + m_5 + m_4)$$
(B.51)

$$p_{31} = 2a_1(a_3m_4 + a_3m_5 + a_3m_6 + a_3m_7 + m_3x_{53})$$
(B.52)

$$p_{32} = a_1 \left( d_7 m_T + m_6 z_{S6} + m_T z_{cT} \right) \tag{B.53}$$

$$p_{33} = I_{5xz} - I_{6xy} - I_{Txy} + m_T x_{cT} y_{cT}$$
(B.54)

$$p_{34} = m_T d_7^2 + 2m_T d_7 z_{cT} - m_T x_{cT}^2 + m_6 z_{S6}^2 + m_T z_{cT}^2 - I_{4xx} + I_{4yy} + I_{6xx} - I_{5yy} + I_{5zz} - I_{6zz} + I_{Txx} - I_{Tzz}$$
(B.55)

$$p_{35} = (I_{5xx} + I_{4zz} + I_{6yy} + I_{Tyy} + m_T z_{cT}^2 + m_6 z_{S6}^2 + d_7^2 m_T + m_T x_{cT}^2 + 2d_7 m_T z_{cT})$$
 (B.56)

$$p_{36} = m_T d_7^2 + 2m_T d_7 z_{cT} + m_T y_{cT}^2 + m_6 z_{56}^2 + m_T z_{cT}^2 + I_{6yy} + I_{5zz} + I_{7yy}$$
 (B.57)

$$p_{27} = m_T x_{cT}^2 + m_T y_{cT}^2 + I_{677} + I_{777}$$
(B.58)

$$p_{38} = a_2 \left( d_7 m_T + m_6 Z_{56} + m_T Z_{cT} \right) \tag{B.59}$$

$$p_{20} = d_A^2 (m_T + m_E + m_E + m_A)$$
 (B.60)

$$p_{40} = I_{5xy} {(B.61)}$$

$$p_{41} = I_{5vz} (B.62)$$

$$p_{42} = I_{4xy} (B.63)$$

$$p_{43} = I_{4yz} (B.64)$$

$$p_{44} = I_{3yz} (B.65)$$

$$p_{45} = I_{2xz} {(B.66)}$$

$$p_{46} = I_{2yz} (B.67)$$

$$p_{47} = m_T x_{cT}^2 + m_T y_{cT}^2 + I_{6zz} + I_{Tzz} + I_{4xx} + I_{3zz} + I_{5yy} + a_3^2 (m_4 + m_5 + m_6 + m_T) + m_3 x_{c3}^2 + m_4 z_{c4}^2 + m_3 y_{c3}^2 + 2d_4 m_4 z_{c4} + d_4^2 (m_T + m_6 + m_5 + m_4)$$
(B.68)

$$p_{48} = I_{4xz} - I_{5xy} \tag{B.69}$$

$$p_{49} = I_{2xy} - m_2 x_{S2} y_{S2} \tag{B.70}$$

$$p_{50} = I_{3xz} + I_{4xy} (B.71)$$

$$p_{51} = a_2^2 (m_3 + m_4 + m_5 + m_6 + m_T) + I_{2zz} + m_2 x_{S2}^2 + m_2 y_{S2}^2$$
(B.72)

Elements of C matrix:

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} C_{11} & \cdots & C_{16} \\ \vdots & \ddots & \vdots \\ C_{61} & \cdots & C_{66} \end{bmatrix}$$
 (B.73)

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(-\dot{a}_{6}(-3c_{a3}^{2}c_{a4}^{2}c_{a5}c_{a6}^{3}s_{a5}+2c_{a3}^{2}c_{a4}^{2}c_{a5}c_{a6}s_{a5}+c_{a3}^{2}c_{a4}^{2}c_{a5}s_{a6}+c_{a3}^{2}c_{a4}c_{a6}s_{a4}s_{a5}+
c_{q3}^2 c_{q5} s_{q5} s_{q6} + c_{q3} c_{q4} c_{q5}^2 s_{q3} s_{q6} + c_{q3} c_{q5} c_{q6} s_{q3} s_{q4} - c_{q5} s_{q5} s_{q6} - \dot{q}_3 \left(-2 c_{q3}^2 c_{q4} c_{q5}^2 c_{q6} + c_{q5}^2 c_{q6}^2 c_{q
2c_{93}^2c_{95}s_{94}s_{96} + 2c_{93}c_{94}^2c_{95}c_{96}^2s_{93}s_{95}s_{96} + 2c_{93}c_{94}^2c_{95}c_{96}s_{93}s_{95} - 2c_{93}c_{94}s_{93}s_{94}s_{95}s_{96} +
2c_{q3}c_{q5}c_{q6}s_{q3}s_{q5}+c_{q4}c_{q5}^2c_{q6}-c_{q5}s_{q4}s_{q6})-\dot{q}_4(2c_{q3}^2c_{q4}^2s_{q5}s_{q6}-c_{q3}^2s_{q5}s_{q6}+c_{q5}^2s_{q5}s_{q6})
2c_{a3}^2c_{a4}c_{a5}c_{a6}^2s_{a4}s_{a5}s_{a6} + 2c_{a3}^2c_{a4}c_{a5}c_{a6}s_{a4}s_{a5} + c_{a3}c_{a4}c_{a5}s_{a3}s_{a6} + c_{a3}c_{a5}^2c_{a6}s_{a3}s_{a4} ) -
\dot{q}_5\left(-2c_{a3}^2c_{a4}^2c_{a5}^2c_{a6}^2s_{a6}-2c_{a3}^2c_{a4}^2c_{a5}^2c_{a6}+c_{a3}^2c_{a4}^2c_{a6}^2s_{a6}+c_{a3}^2c_{a4}^2c_{a6}+\right)
c_{q3}^2 c_{q4} c_{q5} s_{q4} s_{q6} - 2 c_{q3}^2 c_{q5}^2 c_{q6} + c_{q3}^2 c_{q6} + 2 c_{q3} c_{q4} c_{q5} c_{q6} s_{q3} s_{q5} - c_{q3} s_{q3} s_{q4} s_{q5} s_{q6} +
2c_{q5}^2c_{q6}-c_{q6})p_{15}+(-\dot{q}_2(2c_{q2}c_{q3}c_{q6}s_{q5}-c_{q2}s_{q3}s_{q4}s_{q6}+2c_{q2}c_{q4}c_{q5}c_{q6}s_{q3})+
\dot{q}_5(2c_{q4}c_{q6}s_{q2}s_{q3}s_{q5}-2c_{q3}c_{q5}c_{q6}s_{q2})+\dot{q}_3(c_{q3}s_{q2}s_{q4}s_{q6}+2c_{q6}s_{q2}s_{q3}s_{q5}-
2c_{q3}c_{q4}c_{q5}c_{q6}s_{q2} + \dot{q}_4 (c_{q4}s_{q2}s_{q3}s_{q6} + 2c_{q5}c_{q6}s_{q2}s_{q3}s_{q4} ) + \dot{q}_6 (c_{q6}s_{q2}s_{q3}s_{q4} +
2c_{q3}s_{q2}s_{q5}s_{q6} + 2c_{q4}c_{q5}s_{q2}s_{q3}s_{q6})p_{16}(\dot{q}_2(2c_{q2}c_{q6}s_{q4} + c_{q2}c_{q3}s_{q5}s_{q6} +
2c_{q2}c_{q4}c_{q5}s_{q3}s_{q6} + \dot{q}_4 (2c_{q4}c_{q6}s_{q2} - 2c_{q5}s_{q2}s_{q3}s_{q4}s_{q6} ) + \dot{q}_6 (c_{q3}c_{q6}s_{q2}s_{q5} -
2s_{q2}s_{q4}s_{q6} + 2c_{q4}c_{q5}c_{q6}s_{q2}s_{q3} + \dot{q}_3(2c_{q3}c_{q4}c_{q5}s_{q2}s_{q6} - s_{q2}s_{q3}s_{q5}s_{q6}) +
\dot{q}_5(c_{q3}c_{q5}s_{q2}s_{q6}-2c_{q4}s_{q2}s_{q3}s_{q5}s_{q6})p_{17}+(c_{q2}c_{q3}c_{q5}\dot{q}_2-c_{q3}\dot{q}_5s_{q2}s_{q5}-
c_{q5}\dot{q}_{3}s_{q2}s_{q3}\big)p_{18} + \big(c_{q2}\dot{q}_{2}s_{q3} + c_{q3}\dot{q}_{3}s_{q2}\big)p_{19} + \big(c_{q2}c_{q3}\dot{q}_{2} - \dot{q}_{3}s_{q2}s_{q3}\big)p_{20} + \\
(\dot{q}_3 (2c_{q3}^2 s_{q4} s_{q6} - s_{q4} s_{q6} + c_{q4} c_{q5} c_{q6} + 2c_{q3} c_{q6} s_{q3} s_{q5} - 2c_{q3}^2 c_{q4} c_{q5} c_{q6} -
c_{q3}c_{q4}c_{q6}s_{q2}+c_{q3}c_{q4}c_{q5}s_{q6}+c_{q3}c_{q4}c_{q5}^2c_{q6}s_{q2} -\dot{q}_5(c_{q3}^2c_{q5}c_{q6}-c_{q3}c_{q4}c_{q6}s_{q3}s_{q5}+
c_{q4} s_{q3} s_{q5} s_{q6} + 2 c_{q4} c_{q5} c_{q6} s_{q2} s_{q3} s_{q5} + \dot{q}_6 (s_{q5} s_{q6} c_{q3}^2 + c_{q4} s_{q3} s_{q6} c_{q3} c_{q5} + c_{q6} s_{q3} s_{q4} c_{q3} -
c_{q4} s_{q2} s_{q3} s_{q6} c_{q5}^2 + c_{q4} c_{q6} s_{q3} c_{q5} + c_{q4} s_{q2} s_{q3} s_{q6} + \dot{q}_4 (c_{q3} c_{q4} s_{q3} s_{q6} + c_{q6} s_{q2} s_{q3} s_{q4} -
c_{q5} s_{q3} s_{q4} s_{q6} + c_{q3} c_{q5} c_{q6} s_{q3} s_{q4} - c_{q5}^2 c_{q6} s_{q2} s_{q3} s_{q4} + \dot{q}_2 (c_{q2} c_{q4} c_{q5}^2 c_{q6} s_{q3} -
c_{q2}c_{q4}c_{q6}s_{q3})p_{21}+(\dot{q}_3(2c_{q3}{}^2c_{q6}s_{q4}-c_{q4}c_{q5}s_{q6}-c_{q6}s_{q4}-2c_{q3}s_{q3}s_{q5}s_{q6}+
2c_{q3}^2c_{q4}c_{q5}s_{q6} + \dot{q}_6(c_{q3}^2c_{q6}s_{q5} - c_{q3}s_{q3}s_{q4}s_{q6} + c_{q3}c_{q4}c_{q5}c_{q6}s_{q3}) + \dot{q}_5(c_{q3}^2c_{q5}s_{q6} - c_{q5}s_{q6})
c_{q3}c_{q4}s_{q3}s_{q5}s_{q6} + \dot{q}_4 (c_{q3}c_{q4}c_{q6}s_{q3} - c_{q3}c_{q5}s_{q3}s_{q4}s_{q6} )) p_{22} + (\dot{q}_3c_{q3}s_{q4}s_{q6} + \dot{q}_4c_{q4}s_{q3}s_{q6} +
\dot{q}_6 \, c_{q6} \, s_{q3} \, s_{q4} \, p_{23} + \left( c_{q3} \, c_{q6} \, \dot{q}_3 \, s_{q4} - \dot{q}_2 \, s_{q2} + c_{q4} \, c_{q6} \, \dot{q}_4 \, s_{q3} - \dot{q}_6 \, s_{q3} \, s_{q4} \, s_{q6} \, \right) p_{24} +
\left(-\dot{q}_{5}\left(s_{q5}c_{q3}^{2}+c_{q4}c_{q5}s_{q3}c_{q3}\right)-\dot{q}_{3}\left(2c_{q3}^{2}c_{q4}s_{q5}+2c_{q3}c_{q5}s_{q3}-c_{q4}s_{q5}\right)+\right.
\dot{q}_4 c_{q3} s_{q3} s_{q4} s_{q5} p_{25} + (\dot{q}_3 (2 c_{q3}^2 - 1)) p_{27} + \frac{1}{2} \dot{q}_2 c_{q2} p_{28} - \frac{1}{2} \dot{q}_3 s_{q3} p_{29} - \frac{1}{2} \dot{q}_3 s_{q3} p_{30} +
\frac{1}{2}c_{q3}\dot{q}_{3}p_{31} + \left(\dot{q}_{4}S_{q3}S_{q4}S_{q5} - \dot{q}_{5}\left(c_{q3}S_{q5} + c_{q4}c_{q5}S_{q3}\right) - \dot{q}_{3}\left(c_{q5}S_{q3} + c_{q3}c_{q4}S_{q5}\right)\right)p_{32} +
\left(-\dot{q}_{4}\left(2c_{q3}^{2}c_{q4}^{2}c_{q5}-c_{q3}^{2}c_{q5}+c_{q3}c_{q4}s_{q3}s_{q5}\right)+\dot{q}_{5}\left(c_{q3}^{2}c_{q4}s_{q4}s_{q5}-c_{q3}c_{q5}s_{q3}s_{q4}\right)+\right.
\dot{q}_{3}\left(-2 s_{\mathsf{q}4} \, s_{\mathsf{q}5} \, {c_{\mathsf{q}3}}^{2}+2 c_{\mathsf{q}4} \, c_{\mathsf{q}5} \, s_{\mathsf{q}3} \, s_{\mathsf{q}4} \, c_{\mathsf{q}3}+s_{\mathsf{q}4} \, s_{\mathsf{q}5}\right)\right) p_{33}+\left(\dot{q}_{4} \, s_{\mathsf{q}4} \, {c_{\mathsf{q}3}}^{2} \, c_{\mathsf{q}4}+\dot{q}_{3} \, s_{\mathsf{q}3} \, {c_{\mathsf{q}3}}^{2}\right) p_{34}+\\
\left(\dot{q}_{4}\left(2c_{\mathsf{q}4}\,{s_{\mathsf{q}4}}\,{s_{\mathsf{q}5}}\,{c_{\mathsf{q}3}}^{2}c_{\mathsf{q}5}+2{s_{\mathsf{q}3}}\,{s_{\mathsf{q}4}}\,{c_{\mathsf{q}3}}\,{c_{\mathsf{q}5}}^{2}\right)+\dot{q}_{3}\left(-4{c_{\mathsf{q}3}}^{2}\,{c_{\mathsf{q}4}}\,{c_{\mathsf{q}5}}^{2}+2{s_{\mathsf{q}3}}\,{s_{\mathsf{q}5}}\,{c_{\mathsf{q}3}}\,{c_{\mathsf{q}4}}^{2}\,{c_{\mathsf{q}5}}+\right.
2s_{\alpha 3}s_{\alpha 5}c_{\alpha 3}c_{\alpha 5} + 2c_{\alpha 4}c_{\alpha 5}^{2} + \dot{q}_{5} \left(-2c_{\alpha 3}^{2}c_{\alpha 4}^{2}c_{\alpha 5}^{2} + c_{\alpha 3}^{2}c_{\alpha 4}^{2} - 2c_{\alpha 3}^{2}c_{\alpha 5}^{2} + c_{\alpha 3}^{2} + c_{\alpha 3}^{2}c_{\alpha 5}^{2}\right)
4s_{q3}s_{q5}c_{q3}c_{q4}c_{q5} + 2c_{q5}^2 - 1)p_{40} + (\dot{q}_4(4s_{q5}c_{q3}^2c_{q4}^2 - 2s_{q5}c_{q3}^2 + c_{q5}s_{q3}c_{q4}) +
\dot{q}_{5}\left(2c_{q3}^{2}c_{q4}c_{q5}s_{q4}-c_{q3}s_{q3}s_{q4}s_{q5}\right)-\dot{q}_{3}\left(-2c_{q3}^{2}c_{q5}s_{q4}+4c_{q3}c_{q4}s_{q3}s_{q4}s_{q5}+c_{q5}s_{q4}\right)\right)p_{41}
+\left(\dot{q}_{4}\left(c_{\mathsf{q3}}^{2}-2 c_{\mathsf{q3}}^{2} c_{\mathsf{q4}}^{2}\right)+2 c_{\mathsf{q3}} c_{\mathsf{q4}} \dot{q}_{3} s_{\mathsf{q3}} s_{\mathsf{q4}}\right) p_{42}+\left(\dot{q}_{3}\left(2 c_{\mathsf{q3}}^{2} s_{\mathsf{q4}}-s_{\mathsf{q4}}\right)+\dot{q}_{4} c_{\mathsf{q3}} c_{\mathsf{q4}} s_{\mathsf{q3}}\right) p_{43}
+(\dot{q}_3(c_{q4}-2c_{q3}^2c_{q4})+c_{q3}\dot{q}_4s_{q3}s_{q4})p_{48}+(\dot{q}_2(1-2c_{q2}^2))p_{49}
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$$\begin{aligned} C_{12} &= \left( -\frac{1}{2} \frac{\dot{q}}{1} S_{2a_2} \right) p_1 + \left( \frac{1}{2} \dot{q}_1 c_q z_q e_1 s_{qq} s_{qq} + \frac{1}{2} \dot{q}_2 c_q c_2 c_5 s_q e_1 \right) p_{13} + \left( \dot{q}_2 \left( c_{q5} c_{q6} s_{qq} + c_{q1} s_{q2} s_{q6} \right) - \left( \dot{q}_1 \left( 2 c_{q2} c_{q3} s_{q6} s_{q4} s_{q5} \right) + \dot{q}_5 \left( \frac{1}{2} c_{q2} c_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q6} s_{q4} s_{q5} \right) + \dot{q}_5 \left( \frac{1}{2} c_{q2} c_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q5} s_{q6} s_{q6} s_{q} \right) + \frac{1}{2} \dot{q}_4 c_{q5} c_{q5} c_{q5} s_{q5} - \frac{1}{2} c_{q2} c_{q6} s_{q4} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q2} c_{q5} s_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q5} s_{q5} s_{q5} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q2} c_{q5} s_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q5} s_{q5} s_{q5} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q2} c_{q5} c_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q5} c_{q5} s_{q5} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q2} c_{q5} c_{q5} s_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q5} c_{q5} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q2} c_{q5} s_{q5} s_{q5} + c_{q5} c_{q5} c_{q5} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q2} c_{q5} s_{q5} s_{q5} \right) - \dot{q}_6 c_{q2} c_{q5} c_{q5} s_{q5} + c_{q5} c_{q5} c_{q5} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} c_{q5} c_{q5} s_{q5} s_{q5} \right) - \dot{q}_6 c_{q5} c_{q5}$$

$$\begin{aligned} &2c_{33}c_{45}c_{46}c_{43}c_{44}^2c_{24}^2c_{46}^2+4s_{43}s_{46}c_{43}^2c_{44}c_{45}c_{66}^2+2s_{43}s_{46}c_{32}c_{66}^2+4s_{43}s_{46}c_{43}c_{46}^2+2s_{43}s_{46}c_{43}c_{66}^2+2s_{45}s_{46}c_{46}c_{46}^2-4s_{45}c_{46}^2+4s_{43}s_{46}c_{43}c_{66}^2+2s_{45}s_{45}c_{66}^2+2s_{45}c_{66}^2-4s_{45}c_{46}^2-4s_{45}c_{46}^2-4s_{45}c_{46}^2-2s_{45}c_{46}^2+4s_{45}c_{46}^2-2s_{45}c_{46}^2+2s_{45}c_{46}^2-2s_{45}c_{46}^2-4s_{45}c_{46}^2-2s_{45}c_{4$$

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C_{14} = (\dot{q}_3 \frac{1}{2} (c_{q3} s_{q4} s_{q6} - c_{q3} c_{q6} s_{q4} + c_{q4} c_{q5} c_{q6} s_{q3}) - \dot{q}_6 (\frac{1}{2} c_{q6} s_{q3} s_{q4} - \frac{1}{2} s_{q3} s_{q4} s_{q6} +
c_{q4}c_{q5}s_{q3}s_{q6}) -\dot{q}_4(c_{q4}c_{q6}s_{q3}+c_{q5}c_{q6}s_{q3}s_{q4})+\dot{q}_1(c_{q4}s_{q6}-c_{q4}s_{q6}c_{q3}^2+
c_{q5}c_{q6}s_{q4}) -\dot{q}_5c_{q4}c_{q6}s_{q3}s_{q5}) p_5 + (\dot{q}_1(2c_{q4}c_{q6} - c_{q5}s_{q4}s_{q6} - 2c_{q3}^2c_{q4}c_{q6} +
2c_{q3}^2c_{q5}s_{q4}s_{q6}) -\dot{q}_4(c_{q4}c_{q6}s_{q3}-c_{q5}s_{q3}s_{q4}s_{q6})+\dot{q}_6(s_{q3}s_{q4}s_{q6}-c_{q4}c_{q5}c_{q6}s_{q3})+
\dot{q}_5 c_{q4} s_{q3} s_{q5} s_{q6} p_6 + (\dot{q}_4 (2c_{q3} c_{q4} c_{q6} c_{q5} - 2c_{q3} s_{q4} s_{q6} c_{q5}^2 + c_{q3} s_{q4} s_{q6}) -
\dot{q}_3 \left(c_{a3}c_{a4}^2c_{a5}^2c_{a6}^2-c_{a3}s_{a5}s_{a6}c_{a4}^2c_{a5}+\frac{1}{2}s_{a3}s_{a6}c_{a4}c_{a5}^2-2c_{a3}s_{a4}s_{a5}c_{a4}c_{a6}-\right)
\frac{1}{2}c_{q3}c_{q5}^2c_{q6}^2 + \frac{1}{2}s_{q3}s_{q4}c_{q5}c_{q6} + \frac{3}{2}c_{q3}s_{q5}s_{q6}c_{q5} + \dot{q}_1(s_{q4}s_{q5}s_{q6}c_{q3}^2c_{q4}c_{q5} -
c_{q6}s_{q5}c_{q3}^2c_{q4}^2 + \frac{1}{2}c_{q6}s_{q5}c_{q3}^2 - c_{q6}s_{q3}c_{q3}c_{q4}c_{q5} + 2s_{q3}s_{q4}s_{q6}c_{q3}c_{q5}^2 -
\frac{1}{2} \, S_{q3} \, S_{q4} \, S_{q6} \, C_{q3} \, \Big) - \dot{q}_6 \, \frac{3}{2} \, C_{q3} \, C_{q5} \, S_{q4} \, S_{q6} + C_{q5} \, C_{q6} \, S_{q3} \, S_{q5} - C_{q3} \, C_{q4} \, C_{q5}^{\ \ 2} \, C_{q6} \, \Big) -
\dot{q}_5 \left(2s_{q3}s_{q6}c_{q5}^2 + 2c_{q3}c_{q4}s_{q5}s_{q6}c_{q5} - s_{q3}s_{q6} + \frac{1}{2}c_{q3}c_{q6}s_{q4}s_{q5}\right)p_{11} +
\left(-\dot{q}_{5}\left(\frac{1}{2}c_{a3}c_{a4}c_{a6}^{2}+c_{a5}c_{a6}^{2}s_{a3}s_{a5}-c_{a3}c_{a4}c_{a5}^{2}c_{a6}^{2}+\frac{1}{2}c_{a3}c_{a5}c_{a6}^{2}s_{a4}s_{a5}\right)-\right.
\dot{q}_6 \left( s_{q3} s_{q6} c_{q5}^2 c_{q6} + c_{q3} c_{q4} s_{q5} s_{q6} c_{q5} c_{q6} + c_{q3} s_{q4} s_{q5} c_{q6}^2 - s_{q3} s_{q6} c_{q6} \right)
-\frac{1}{2}c_{q3}s_{q4}s_{q5} -\dot{q}_4(c_{q3}c_{q5}s_{q4}s_{q5}c_{q6}^2+c_{q3}c_{q4}s_{q5}s_{q6}c_{q6})+
\dot{q}_3\left(2c_{q3}s_{q4}s_{q6}c_{q4}c_{q5}c_{q6}-c_{q3}c_{q4}^2c_{q6}^2+\frac{1}{2}c_{q3}c_{q5}^2c_{q6}^2\right)+\dot{q}_1\left(s_{q6}c_{q3}^2c_{q5}c_{q6}-c_{q5}^2c_{q6}^2\right)
2s_{a6}c_{a3}^2c_{a4}^2c_{a5}c_{a6} - s_{a4}c_{a3}^2c_{a4}c_{a6}^2 - s_{a4}c_{a3}c_{a4}c_{a5}^2c_{a6}^2 + s_{a3}s_{a5}s_{a6}c_{a3}c_{a4}c_{a6} +
S_{q3}S_{q4}S_{q5}C_{q3}C_{q5}C_{q6}^2 + S_{q3}S_{q4}C_{q5}C_{q6}))p_{12} + (\frac{1}{2}\dot{q}_6S_{2q6} - \dot{q}_1(c_{q3}^2C_{q4}C_{q5}^2S_{q4} - c_{q5}^2S_{q4}))p_{12} + (\frac{1}{2}\dot{q}_6S_{2q6} - \dot{q}_1(c_{q3}^2C_{q4}C_{q5}^2S_{q4} - c_{q5}^2S_{q4}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{q5}^2S_{
\tfrac{1}{2} c_{q3} c_{q5} s_{q3} s_{q4} s_{q5} \Big) - \dot{q}_5 \Big( - c_{q3} c_{q4} c_{q5}^2 + s_{q3} s_{q5} c_{q5} + \tfrac{1}{2} c_{q3} c_{q4} \Big) + \dot{q}_3 \Big( c_{q3} c_{q5}^2 -
(c_{a3}c_{a4}^2c_{a5}^2) - (c_{a3}c_{a5}\dot{q}_4s_{a4}s_{a5})p_{13} + (\dot{q}_1(2s_{a4}s_{a6}c_{a3}^2c_{a4}c_{a5}^2c_{a6} -
4c_{q3}^2c_{q4}^2c_{q5}c_{q6}^2+2c_{q3}^2c_{q5}c_{q6}^2+2s_{q3}s_{q5}c_{q3}c_{q4}c_{q6}^2-2s_{q3}s_{q4}s_{q5}s_{q6}c_{q3}c_{q5}c_{q6})-
\dot{q}_4 \left(4c_{q3}c_{q4}c_{q6}^2s_{q5}-4c_{q3}c_{q5}c_{q6}s_{q4}s_{q5}s_{q6}\right)+\dot{q}_3\frac{1}{2}c_{q3}c_{q6}s_{q6}-\frac{1}{2}c_{q6}s_{q3}s_{q4}s_{q5}+
3c_{a3}c_{a4}^2c_{a6}s_{a6} - 2c_{a3}c_{a5}^2c_{a6}s_{a6} + 2c_{a6}^2s_{a3}s_{a4}s_{a5} + 4c_{a3}c_{a4}c_{a5}c_{a6}^2s_{a4} +
c_{q4}c_{q5}c_{q6}s_{q3}s_{q5}s_{q6} + \dot{q}_{6} \left(2s_{q3}c_{q5}^{2} - 4s_{q3}c_{q5}^{2}c_{q6}^{2} - 4c_{q3}c_{q4}s_{q5}c_{q6}^{2} + 4c_{q5}c_{q6}^{2}c_{q6}^{2} + 4c_{q5}c_{q6}^{2}c_{q6}^{2}\right)
2c_{a3}c_{a4}s_{a5}c_{a5} + 4s_{a3}c_{a6}^2 + 4c_{a3}s_{a4}s_{a5}s_{a6}c_{a6} - 2s_{a3} - \dot{q}_5(4c_{a3}c_{a4}s_{a6}c_{a5}^2c_{a6} +
c_{q3}s_{q4}c_{q5}c_{q6}^2 - 4s_{q3}s_{q5}s_{q6}c_{q5}c_{q6} - 3c_{q3}c_{q4}s_{q6}c_{q6}) p_{14} + (-\dot{q}_3(2c_{q3}c_{q5}c_{q6}s_{q5} + 2c_{q6}s_{q5}c_{q6}s_{q5})
2c_{q3}c_{q4}s_{q4}s_{q5}s_{q6} - 2c_{q3}c_{q4}^2c_{q5}c_{q6}s_{q5} - \frac{1}{2}c_{q4}c_{q6}s_{q3} + \dot{q}_6(\frac{1}{2}c_{q3}c_{q4}s_{q6} - \frac{1}{2}c_{q4}c_{q6}s_{q3})
c_{q5}s_{q3}s_{q5}s_{q6} + \tfrac{1}{2}c_{q3}c_{q4}c_{q5}^2s_{q6} + c_{q3}c_{q5}c_{q6}s_{q4} + \dot{q}_4 \left(c_{q3}c_{q6}s_{q4}c_{q5}^2 + c_{q3}c_{q4}s_{q6}c_{q5} - c_{q5}c_{q6}s_{q4}c_{q5}\right) + \dot{q}_4 \left(c_{q3}c_{q6}s_{q4}c_{q5}^2 + c_{q3}c_{q4}s_{q6}c_{q5} - c_{q5}c_{q6}s_{q4}c_{q5}\right) + \dot{q}_4 \left(c_{q3}c_{q6}s_{q4}c_{q5}^2 + c_{q3}c_{q4}s_{q6}c_{q5} - c_{q5}c_{q6}s_{q4}c_{q5}\right) + \dot{q}_4 \left(c_{q3}c_{q6}s_{q4}c_{q5}^2 + c_{q3}c_{q4}s_{q6}c_{q5}\right) + \dot{q}_4 \left(c_{q3}c_{q6}s_{q6}^2 + c_{q5}c_{q6}^2 + 
S_{a2}S_{a4}S_{a5}) + \dot{q}_5 (C_{a4}C_{a5}S_{a2} - C_{a6}S_{a3} + 2C_{a5}^2C_{a6}S_{a3} + C_{a3}C_{a4}C_{a5}C_{a6}S_{a5}) -
\dot{q}_1 \left(2s_{a5}s_{a6}c_{a3}^2c_{a4}^2+2s_{a4}s_{a5}s_{a6}c_{a3}^2c_{a4}c_{a5}c_{a6}^2+2s_{a4}s_{a5}c_{a3}^2c_{a4}c_{a5}c_{a6}-\right)
S_{q5}S_{q6}C_{q3}^2 + S_{q3}S_{q6}C_{q3}C_{q4}C_{q5} + S_{q3}S_{q4}C_{q3}C_{q5}^2C_{q6} + \frac{1}{2}\dot{q}_2C_{q2}C_{q4}S_{q5} + p_{15} +
(\dot{q}_1(c_{a4}s_{a2}s_{a3}s_{a6} + 2c_{a5}c_{a6}s_{a2}s_{a3}s_{a4}) - \dot{q}_4(c_{a4}s_{a2}s_{a6} + c_{a5}c_{a6}s_{a2}s_{a4}) -
\dot{q}_6\left(c_{q6}s_{q2}s_{q4}+c_{q4}c_{q5}s_{q2}s_{q6}\right)+\frac{1}{2}c_{q2}c_{q4}c_{q5}c_{q6}\dot{q}_2-c_{q4}c_{q6}\dot{q}_5s_{q2}s_{q5}\right)p_{16}+
(\dot{q}_1(2c_{q4}c_{q6}s_{q2}-2c_{q5}s_{q2}s_{q3}s_{q4}s_{q6})-\dot{q}_4(c_{q4}c_{q6}s_{q2}-c_{q5}s_{q2}s_{q4}s_{q6})+
\dot{q}_{6}(s_{q2}s_{q4}s_{q6}-c_{q4}c_{q5}c_{q6}s_{q2})-\dot{q}_{2}\,c_{q2}c_{q4}c_{q5}\,s_{q6}+\dot{q}_{5}\,c_{q4}\,s_{q2}s_{q5}s_{q6})p_{17}-
                                                                                                                                                                                                                                                                                                                  (B.77)
\frac{1}{2}\dot{q}_2c_{q2}c_{q4}s_{q5}p_{18} + (\dot{q}_1(c_{q3}c_{q4}s_{q3}s_{q6} + c_{q6}s_{q2}s_{q3}s_{q4} - c_{q5}s_{q3}s_{q4}s_{q6} +
c_{q3}c_{q5}c_{q6}s_{q3}s_{q4}-c_{q5}^2c_{q6}s_{q2}s_{q3}s_{q4})-\dot{q}_6(c_{q3}c_{q6}s_{q4}+c_{q3}c_{q4}c_{q5}s_{q6})-
\dot{q}_4 (c_{q3} c_{q4} s_{q6} + c_{q3} c_{q5} c_{q6} s_{q4}) - \frac{1}{2} c_{q4} c_{q5} c_{q6} \dot{q}_3 s_{q3} - c_{q3} c_{q4} c_{q6} \dot{q}_5 s_{q5}) p_{21} +
\left(\dot{q}_{1}\left(c_{q3}c_{q4}c_{q6}s_{q3}-c_{q3}c_{q5}s_{q3}s_{q4}s_{q6}\right)-\dot{q}_{4}\left(c_{q3}c_{q4}c_{q6}-c_{q3}c_{q5}s_{q4}s_{q6}\right)+\right.
```

$$\begin{array}{l} \dot{q}_0\left(C_{q13}S_{q14}S_{q06}-C_{q13}C_{q14}C_{q5}S_{q04}\right) + \dot{q}_5C_{q13}C_{q14}S_{q5}S_{q6}\right) - Q_{q14}C_{q16}S_{3}S_{45} - \dot{q}_4\left(C_{q44}S_{q16} + C_{q54}C_{q46}S_{q44}\right)\right) p_{23} + \\ \dot{q}_6\left(\frac{1}{2}C_{q44}C_{q64} + \frac{1}{2}S_{q44}S_{q6} - C_{q44}C_{q54}S_{q6}\right) - Q_{q44}C_{q44}S_{q6} - C_{q35}S_{q44}S_{q6} + C_{q45}C_{q64}S_{q34}\right) + \\ \dot{q}_6\left(\frac{1}{2}C_{q44}C_{q64} + \frac{1}{2}S_{q44}S_{q64} - C_{q44}C_{q54}S_{q64}\right) - Q_{q44}C_{q44}S_{q64} - C_{q35}S_{q44}S_{q64}\right) + Q_{q44}C_{q46}C_{q45}S_{q44} + Q_{q45}C_{q44}C_{q45}S_{q45} + Q_{q45}C_{q44}C_{q45}S_{q45}S_{q45} + Q_{q45}C_{q44}C_{q45}S_{q45}$$

```
c_{a3}c_{a4}c_{a5}s_{a6}) -\dot{q}_1(s_{a6}c_{a3}^2c_{a4}^2c_{a6}^2-2s_{a6}c_{a3}^2c_{a4}^2c_{a5}^2c_{a6}^2-2c_{a3}^2c_{a4}^2c_{a5}^2c_{a6}+
                c_{a3}^{2}c_{a4}^{2}c_{a6} + s_{a4}s_{a6}c_{a3}^{2}c_{a4}c_{a5} - 2c_{a3}^{2}c_{a5}^{2}c_{a6} + c_{a3}^{2}c_{a6} + 2s_{a3}s_{a5}c_{a3}c_{a4}c_{a5}c_{a6} -
                s_{q3}s_{q4}s_{q5}s_{q6}c_{q3} + 2c_{q5}^2c_{q6} - c_{q6}) + \dot{q}_3(c_{q4}s_{q3}s_{q5}s_{q6} - c_{q3}c_{q4}^2c_{q5}s_{q6} + c_{q3}c_{q4}c_{q6}s_{q4} +
                c_{q5}c_{q6}s_{q3}s_{q4}s_{q5}-2c_{q3}c_{q4}c_{q5}^2c_{q6}s_{q4}+\frac{1}{2}\dot{q}_2c_{q2}c_{q5}s_{q4})p_{15}+(\dot{q}_2(\frac{1}{2}c_{q2}c_{q6}s_{q5}-
                \frac{1}{2}c_{q2}c_{q6}s_{q4}s_{q5}) -\dot{q}_1(2c_{q3}c_{q5}c_{q6}s_{q2}-2c_{q4}c_{q6}s_{q2}s_{q3}s_{q5})-\dot{q}_4c_{q4}c_{q6}s_{q2}s_{q5}-
                \dot{q}_5 c_{q5} c_{q6} s_{q2} s_{q4} + \dot{q}_6 s_{q2} s_{q4} s_{q5} s_{q6} p_{16} + (\dot{q}_1 (c_{q3} c_{q5} s_{q2} s_{q6} - 2 c_{q4} s_{q2} s_{q3} s_{q5} s_{q6}) +
                \dot{q}_2\,c_{q2}\,s_{q4}\,s_{q5}\,s_{q6}+c_{q4}\dot{q}_4\,s_{q2}\,s_{q5}\,s_{q6}+c_{q5}\dot{q}_5\,s_{q2}\,s_{q4}\,s_{q6}+c_{q6}\dot{q}_6\,s_{q2}\,s_{q4}\,s_{q5}\,)p_{17}-
                (\dot{q}_2\,c_{q2}\,c_{q5}\,s_{q4}-\dot{q}_1\,c_{q3}\,s_{q2}\,s_{q5})p_{18}+(-\dot{q}_1(c_{q5}c_{q6}c_{q3}^2-c_{q4}c_{q6}\,s_{q3}\,s_{q5}c_{q3}+c_{q4}\,s_{q3}\,s_{q5}\,s_{q6}+
                2c_{q4}c_{q5}c_{q6}s_{q2}s_{q3}s_{q5}) -c_{q3}c_{q4}c_{q6}\dot{q}_4s_{q5}-c_{q3}c_{q5}c_{q6}\dot{q}_5s_{q4}+\frac{1}{2}c_{q6}\dot{q}_3s_{q3}s_{q4}s_{q5}+
                (c_{q3}\dot{q}_6 s_{q4} s_{q5} s_{q6}))p_{21} + (\dot{q}_1 (c_{q3}^2 c_{q5} s_{q6} - c_{q3} c_{q4} s_{q3} s_{q5} s_{q6}) + c_{q3} c_{q4} \dot{q}_4 s_{q5} s_{q6} +
                c_{q3}c_{q5}\dot{q}_{5}s_{q4}s_{q6} + c_{q3}c_{q6}\dot{q}_{6}s_{q4}s_{q5})p_{22} + \left(\dot{q}_{6}s_{q4}s_{q5}s_{q6} - c_{q5}c_{q6}\dot{q}_{5}s_{q4} - c_{q4}c_{q6}\dot{q}_{4}s_{q5}\right)p_{23} +
                (\dot{q}_4c_{q4} s_{q5}s_{q6} + \dot{q}_5c_{q5}s_{q4}s_{q6} + \dot{q}_6c_{q6}s_{q4}s_{q5})p_{24} + (-\dot{q}_1(s_{q5}c_{q3}^2 + c_{q4}c_{q5}s_{q3}c_{q3}) -
                \dot{q}_{5}c_{\mathbf{q}3}\,s_{\mathbf{q}4}s_{\mathbf{q}5} + \dot{q}_{4}c_{\mathbf{q}3}\,c_{\mathbf{q}4}\,c_{\mathbf{q}5}\,\big)p_{25}\big(\tfrac{3}{2}\dot{q}_{4}c_{\mathbf{q}4}\,c_{\mathbf{q}5} - \dot{q}_{1}\big(c_{\mathbf{q}3}s_{\mathbf{q}5} + c_{\mathbf{q}4}c_{\mathbf{q}5}s_{\mathbf{q}3}\big) - \dot{q}_{5}s_{\mathbf{q}4}s_{\mathbf{q}5}\big)p_{32} + c_{\mathbf{q}4}c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\big)p_{32}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{\mathbf{q}5}\,c_{
                (\dot{q}_1(c_{q3}^2c_{q4}s_{q4}s_{q5}-c_{q3}c_{q5}s_{q3}s_{q4})+\dot{q}_3(c_{q3}s_{q5}c_{q4}^2+c_{q5}s_{q3}c_{q4})+
                \dot{q}_5(c_{q5}s_{q3}+c_{q3}c_{q4}s_{q5}))p_{33}+\frac{1}{2}(c_{q3}c_{q4}\dot{q}_4-\frac{1}{2}\dot{q}_3s_{q3}s_{q4})p_{36}+
                \left(\dot{q}_{6}\left(\frac{1}{2}S_{q3}S_{q5}-\frac{1}{2}c_{q3}c_{q4}c_{q5}\right)\right)p_{37}+\left(\dot{q}_{4}\left(2S_{q3}-4S_{q3}c_{q5}^{2}-2c_{q3}c_{q4}S_{q5}c_{q5}\right)+\right.
                (\dot{q}_3(4c_{q3}c_{q4}s_{q4}c_{q5}^2-2c_{q3}c_{q4}s_{q4}-s_{q3}s_{q4}s_{q5})+\dot{q}_1(c_{q3}^2c_{q4}^2-2c_{q3}^2c_{q4}^2c_{q5}^2-
                2c_{q3}^2c_{q5}^2+c_{q3}^2+4s_{q3}s_{q5}c_{q3}c_{q4}c_{q5}+2c_{q5}^2-1)p_{40}+
                \left(-\dot{q}_{1}\left(c_{\mathsf{q}3}\,s_{\mathsf{q}3}\,s_{\mathsf{q}4}\,s_{\mathsf{q}5}-2{c_{\mathsf{q}3}}^{2}\,c_{\mathsf{q}4}\,c_{\mathsf{q}5}\,s_{\mathsf{q}4}\right)-\dot{q}_{3}\left(c_{\mathsf{q}4}\,s_{\mathsf{q}3}\,s_{\mathsf{q}5}-{c_{\mathsf{q}3}}\,c_{\mathsf{q}4}^{2}\,c_{\mathsf{q}5}\right)-\dot{q}_{5}\left(s_{\mathsf{q}3}\,s_{\mathsf{q}5}-c_{\mathsf{q}3}\,c_{\mathsf{q}4}\,c_{\mathsf{q}5}\right)\right)p_{41}
C_{16} = (\dot{q}_1 (c_{q6} s_{q4} + c_{q4} c_{q5} s_{q6} + c_{q3} s_{q5} s_{q6} - c_{q3}^2 c_{q6} s_{q4} + c_{q3} c_{q6} s_{q3} s_{q5} + c_{q3} s_{q3} s_{q5} s_{q6}) -
               \dot{q}_4\left(\frac{1}{2}c_{q6}S_{q3}S_{q4}-\frac{1}{2}S_{q3}S_{q4}S_{q6}+c_{q4}c_{q5}S_{q3}S_{q6}\right)-\dot{q}_6\left(c_{q4}S_{q3}S_{q6}+c_{q5}c_{q6}S_{q3}S_{q4}\right)-
               \dot{q}_3 c_{q5} s_{q3} s_{q4} s_{q6} + \dot{q}_5 s_{q3} s_{q4} s_{q5} s_{q6} p_5 + (\dot{q}_4 (s_{q3} s_{q4} s_{q6} - c_{q4} c_{q5} c_{q6} s_{q3}) -
               \dot{q}_{6}\left(c_{q4}c_{q6}s_{q3}-c_{q5}s_{q3}s_{q4}s_{q6}\right)+\dot{q}_{1}\left(2c_{q3}^{2}s_{q4}s_{q6}-2s_{q4}s_{q6}+c_{q4}c_{q5}c_{q6}+\right.\\
               2c_{q3}c_{q6}s_{q5}-2c_{q3}^2c_{q4}c_{q5}c_{q6} + \dot{q}_5c_{q6}s_{q3}s_{q4}s_{q5} ) p_6 + (\dot{q}_1(\frac{1}{2}s_{q4}s_{q5}s_{q6}c_{q3}^2c_{q4}-
               \tfrac{1}{2} c_{q6} s_{q5} c_{q3}{}^2 c_{q4}{}^2 c_{q5} - c_{q6} s_{q5} c_{q3}{}^2 c_{q5} - 2 c_{q6} s_{q3} c_{q3} c_{q4} c_{q5}{}^2 + \tfrac{1}{2} c_{q6} s_{q3} c_{q3} c_{q4} +
               S_{a3}S_{a4}S_{a6}C_{a3}C_{a5} + \frac{1}{2}C_{a6}S_{a5}C_{a5} + (\dot{q}_3(C_{a3}S_{a5}S_{a6}C_{a4}^2 + C_{a3}C_{a6}S_{a4}S_{a6}C_{a4}C_{a5}^2 +
               \frac{1}{2}C_{q3}C_{q6}S_{q4}S_{q5}C_{q4}C_{q5} + \frac{1}{2}C_{q6}S_{q3}S_{q4}C_{q5}^2 - C_{q6}S_{q3}S_{q4} - C_{q3}S_{q5}S_{q6})
               \dot{q}_{6}\left(c_{q3}s_{q4}s_{q6}+c_{q6}s_{q3}s_{q5}-c_{q3}c_{q4}c_{q5}c_{q6}\right)-\dot{q}_{4}\left(\frac{3}{2}3c_{q3}c_{q5}s_{q4}s_{q6}+c_{q5}c_{q6}s_{q3}s_{q5}-\right.
               (c_{q3}c_{q4}c_{q5}^2c_{q6}) + \frac{1}{2}\dot{q}_5c_{q5}s_{q3}s_{q6})p_{11} + (\dot{q}_1(-s_{q6}c_{q3}^2c_{q4}^2c_{q6} -
               2s_{a4}c_{a3}^2c_{a4}c_{a5}c_{a6}^2 + s_{a4}c_{a3}^2c_{a4}c_{a5} - s_{a6}c_{a3}^2c_{a5}^2c_{a6} + 2s_{a6}c_{a3}^2c_{a6} -
               s_{q6}c_{q3}c_{q4}^2c_{q5}^2c_{q6} + 2s_{q3}s_{q5}s_{q6}c_{q3}c_{q4}c_{q5}c_{q6} + 2s_{q3}s_{q4}s_{q5}c_{q3}c_{q6}^2 - s_{q3}s_{q4}s_{q5}c_{q3} +
               S_{q3}S_{q6}C_{q4}C_{q5} + S_{q6}C_{q5}^2C_{q6} - \frac{1}{2}S_{2q6} -\frac{1}{2}S_{q3}S_{q5} - C_{q6}^2S_{q3}S_{q5} - \frac{1}{2}C_{q3}C_{q4}C_{q5} +
               c_{q3}c_{q4}c_{q5}c_{q6}^2 + c_{q3}c_{q4}c_{q5}c_{q6}s_{q5}s_{q6} - \dot{q}_4(s_{q3}s_{q6}c_{q5}^2c_{q6} + c_{q3}c_{q4}s_{q5}s_{q6}c_{q5}c_{q6} +
               c_{q3}s_{q4}s_{q5}c_{q6}^2 - s_{q3}s_{q6}c_{q6} - \frac{1}{2}c_{q3}s_{q4}s_{q5} - \dot{q}_3(2c_{q3}c_{q5}c_{q4}^2c_{q6}^2 - c_{q3}c_{q5}c_{q4}^2 -
                                                                                                                                                                                                                                                                                                                                 (B.79)
```

 $(\dot{q}_4(c_{q4}c_{q5}s_{q2}-c_{q6}s_{q3}+2c_{q5}^2c_{q6}s_{q3}+c_{q3}c_{q4}c_{q5}c_{q6}s_{q5})-\dot{q}_5(s_{q2}s_{q4}s_{q5}-s_{q3}s_{q5}s_{q6}+$ 

$$\begin{split} &S_{63}S_{q5}C_{q4}C_{q5}^2 - C_{q3}S_{q4}S_{q6}C_{q4}C_{q6}C_{q5}^2 + \frac{1}{2}S_{q3}S_{q5}C_{q4} - C_{q3}^2C_{q5}^2 + C_{q5}^2C_{q4}^2 + C_{q5}^2S_{q4}^2C_{q5}^2 + C_{q5}^2C_{q4}^2 + C_{q5}^2C_{q6}^2 + C_{q5}^2C_{q6}^2 + C_{q5}^2C_{q6}^2 + C_{q5}^2C_{q5}^2 + C_{q5}^2C_{q5}^2$$

$$\begin{aligned} &C_{23} = \left(\dot{q}_{5} \left( C_{62} C_{62} C_{65} C_{66} S_{64} - C_{63} C_{64} S_{62} + C_{62} C_{63} C_{64} S_{64} + C_{63} C_{66} S_{64} + C_{64} C_{66} S_{62} S_{63} S_{64} \right) + \\ &\dot{q}_{3} \left( C_{64} S_{62} S_{63} S_{64} + C_{62} C_{64} S_{64} + C_{62} C_{66} S_{64} + C_{63} C_{66} S_{62} S_{63} S_{64} \right) + \\ &\dot{q}_{3} \left( C_{64} S_{62} S_{63} S_{64} + C_{62} S_{64} S_{64} + C_{63} C_{66} S_{64} + C_{63} C_{66} S_{62} S_{63} S_{64} \right) + \\ &\dot{q}_{3} \left( C_{64} S_{64} C_{63} S_{66} S_{62} \right) + \dot{q}_{6} \left( C_{64} S_{62} S_{63} S_{64} + C_{64} S_{62} S_{62} S_{63} S_{64} + C_{62} C_{63} C_{64} S_{64} S_{64} \right) + \\ &\dot{q}_{5} \left( C_{63} S_{64} C_{65} S_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} + C_{64} C_{66} S_{64} S_{64} S_{64} + C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} + C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} C_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} C_{64} C_{64} S_{64} S_{64} S_{64} S_{64} S_{64} C_{64} S_{64} S_{64} S_{64} S_{64} C_{64} S_{64} S_{64} S_{64} C_{64} S_{64} S$$

$$\begin{split} C_{26} &= \left( \dot{q}_1, \left( \frac{1}{2} c_{q2} c_{q3} s_{q6} - c_{q2} c_{q4} c_{q6} + \frac{1}{2} c_{q2} c_{q5} s_{q4} s_{q6} \right) - \dot{q}_4 \left( c_{q5} c_{q4} c_{q6} s_{q2} - c_{q5} c_{q4} c_{q6} s_{q4} + \frac{1}{2} c_{q2} c_{q5} c_{q6} s_{q3} s_{q4} + c_{q2} c_{q5} s_{q3} s_{q4} s_{q6} - c_{q3} c_{q5} c_{q5} s_{q4} s_{q6} \right) + \dot{q}_1 \left( c_{q6} s_{q2} s_{q3} s_{q4} + c_{q2} s_{q3} s_{q6} s_{q6} + c_{q3} c_{q5} s_{q6} + c_{q4} c_{q5} c_{q5} c_{q6} + c_{q4} c_{q5} c_{q5} c_{q6} c_{q5} c_{q5} c_{q6} + c_{q5} c_{q5} c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c_$$

 $\dot{q}_1 \left(4 S_{a5} S_{a6} C_{a3}^2 C_{a4} C_{a5} C_{a6} + 4 S_{a4} S_{a5} C_{a3}^2 C_{a6}^2 + 2 S_{a3} S_{a6} C_{a3} C_{a4}^2 C_{a5}^2 C_{a6} + \right)$ 

$$\begin{aligned} & + 3c_{3}s_{4} + c_{43}s_{4} + c_{45}c_{45}c_{45}^{-2} + 2c_{45}s_{45}c_{46}c_{43}c_{45}^{-2} + 2c_{46}c_{45}s_{45}s_{45}s_{45} - 2c_{45}s_{45}c_{46}c_{45}c_{46}c_{45}c_{46}c_{45}c_{45}c_{46}c_{45}c_$$

 $(c_{q4}c_{q5}s_{q2}s_{q3}))p_{18}+\dot{q}_2(c_{q2}s_{q3}-c_{q3}s_{q2})p_{19}+\dot{q}_2(c_{q2}c_{q3}+s_{q2}s_{q3})p_{20}-$ 

 $\dot{q}_4 \left( \frac{1}{2} c_{q2} c_{q3} s_{q4} s_{q5} + \frac{1}{2} s_{q2} s_{q3} s_{q4} s_{q5} \right) p_{38}$ 

$$C_{33} = (\dot{q}_1 \left( C_{q4} S_{q6} + C_{q5} C_{q6} S_{q4} \right) + \dot{q}_6 \left( C_{q6} S_{q6} + C_{q6} C_{q5} S_{q6} \right) + C_{q4} C_{q6} S_{q5} S_{q5} \right) p_5 + \\ (\dot{q}_1 C_{q4} C_{q6} - Z_{c5} S_{q6} S_{q6}) - \dot{q}_6 \left( S_{q4} S_{q6} - Z_{c6} C_{q6} C_{q6} \right) - 2 d_5 C_{q6} S_{q5} S_{q6} \right) p_6 + \\ (\dot{q}_1 S_{q4} S_{q5} - C_{q4} C_{q5} \dot{q}_3 \right) p_7 + (\dot{q}_3 \left( 2 S_{q6} C_{q4}^2 C_{q5}^2 - S_{q6} C_{q4}^2 + C_{q6} S_{q4} C_{q6} - C_{q5} C_{q6} \right) p_6 + \\ (\dot{q}_1 S_{q4} S_{q5} - C_{q4} C_{q5} \dot{q}_3 \right) p_7 + (\dot{q}_3 \left( 2 S_{q6} C_{q4}^2 C_{q5}^2 - S_{q6} C_{q4}^2 + C_{q6} S_{q4} C_{q5} C_{q6} - C_{24} C_{q6} C$$

$$\begin{array}{l} c_{q1}c_{q3}^{-2}c_{q5}s_{q3}-2c_{q3}c_{q4}s_{q4}s_{q5}s_{q6}+2c_{q3}c_{q4}^{-2}c_{q5}c_{q6}s_{q5}-c_{q4}-2s_{q5}s_{q6}c_{q4}-2s_{q5}s_{q6}c_{q4}-2s_{q5}s_{q6}c_{q6}-c_{q6}-c_{q6}c_{q5}^{-2}c_{q4}c_{q6}c_{q5}^{-2}-c_{q4}c_{q6}c_{q6}^{-2}-c_{q4}^{-2}c_{q5}c_{q5}s_{q5}c_{q6}-c_{q4}c_{q5}c_{q6}-c_{q4}c_{q5$$

$$\begin{aligned} &C_{36} = \left( \dot{q}_{3} \left( C_{66} S_{49} + C_{49} C_{49} S_{89} \right) - \dot{q}_{1} \left( C_{62} C_{64} C_{69} - C_{62} C_{64} S_{48} S_{49} + \frac{1}{2} C_{65} S_{49} S_{49} + C_{66} S_{68} S_{67} \right) + \\ &C_{67} \dot{q}_{1}^{2} S_{49} + C_{66} \dot{q}_{1} S_{67} S_{67} \right) + \dot{q}_{1}^{2} \left( C_{61} C_{49} S_{64} S_{69} + C_{62} C_{67} S_{68} S_{44} \right) - \dot{q}_{2}^{2} \left( S_{64} S_{69} \right) + \\ &C_{67} \dot{q}_{1}^{2} S_{49} + C_{66} \dot{q}_{1} S_{67} S_{69} \right) + \dot{q}_{1}^{2} \left( C_{69} S_{64} C_{69} S_{67} S_{68} S_{48} \right) - \dot{q}_{3}^{2} \left( C_{68} S_{64} S_{64} + C_{63} C_{69} S_{68} S_{64} \right) - \dot{q}_{3}^{2} \left( C_{64} S_{64} C_{68} S_{64} C_{68} \right) - \dot{q}_{3}^{2} \left( C_{64} S_{64} C_{68} S_{64} C_{68} \right) - \dot{q}_{3}^{2} \left( C_{64} S_{64} C_{64} C_{64} C_{64} C_{64} \right) - \dot{q}_{3}^{2} \left( C_{64} S_{64} C_{64} C_{64} C_{64} C_{64} C_{64} \right) - \dot{q}_{3}^{2} \left( C_{64} S_{64} C_{64} C_{64} C_{64} C_{64} C_{64} C_{64} C_{64} \right) - \dot{q}_{3}^{2} \left( C_{64} S_{64} C_{64} C_{64$$

$$\begin{array}{l} \dot{q}_3\left(c_{q3}c_{q4}^2c_{q5}^2-c_{q4}c_{q5}s_{q3}s_{q5}\right) - \dot{q}_5\left(s_{q3}s_{q5}c_{q5}-c_{q3}c_{q4}c_{q5}^2+\frac{1}{2}c_{q3}c_{q4}\right)\right)p_{13} + \\ \dot{q}_1\left(4c_{q3}^2c_{q4}^2c_{q5}c_{q6}^2-2s_{q4}s_{q6}c_{q3}^2c_{q4}c_{q5}^2c_{q6}-2c_{q3}^2c_{q5}c_{q6}^2-2s_{q3}s_{q4}c_{q6}^2+2s_{q3}s_{q4}s_{q5}c_{q3}c_{q4}c_{q5}^2+2c_{q5}c_{q6}^2-2s_{q3}s_{q4}c_{q6}c_{q5}c_{q6}^2-2s_{q3}s_{q4}c_{q6}c_{q5}c_{q6}^2-2s_{q3}s_{q4}c_{q6}c_{q5}c_{q6}^2-2s_{q3}s_{q4}c_{q6}c_{q5}c_{q6}^2-2s_{q3}s_{q4}c_{q6}c_{q5}c_{q6}c_{q5}c_{q$$

 $S_{a4}C_{a3}^2C_{a4}C_{a6}^2 - S_{a4}C_{a3}C_{a4}C_{a5}^2C_{a6}^2 + S_{a3}S_{a5}S_{a6}C_{a3}C_{a4}C_{a6} + S_{a3}S_{a4}S_{a5}C_{a3}C_{a5}C_{a6}^2 +$ 

 $S_{q3}S_{q4}C_{q5}C_{q6})p_{12} + (\frac{1}{2}\dot{q}_6S_{2q_6} + \dot{q}_1(C_{q3}^2C_{q4}C_{q5}^2S_{q4} - \frac{1}{2}C_{q3}C_{q5}S_{q3}S_{q4}S_{q5}) +$ 

$$\begin{split} C_{43} &= \left( \dot{q}_1 \left( -\frac{1}{2} c_{43} c_{46} c_{84} - \frac{1}{2} c_{43} c_{45} c_{46} c_{46} c_{46} - \frac{1}{2} c_{44} c_{45} c_{65} c_{46} c_{36} \right) - \dot{q}_3 \left( c_{44} c_{86} + c_{65} c_{66} c_{34} \right) \right) p_5 + \left( -\dot{q}_1 \left( c_{43} c_{66} c_{84} + c_{43} c_{64} c_{65} c_{66} \right) - \dot{q}_3 \left( c_{44} c_{46} - 2 c_{45} c_{84} c_{86} \right) \right) p_6 + \left( c_{43} c_{46} \dot{q}_1 s_{45} - \dot{q}_3 s_{44} c_{45} c_{65} s_{44} + c_{45}^2 c_{66} c_{84} \right) + \dot{q}_3 \left( 2 c_{65} c_{84} c_{45} c_{84} c_{64} \right) \right) p_6 + \left( c_{43} c_{44} \dot{q}_1 s_{45} - \dot{q}_3 s_{45} c_{45} c_{45} \right) p_7 + \left( \dot{q}_6 \left( c_{46} c_{46} c_{84} c_{46} c_{84} c_{46} c_{84} \right) + \dot{q}_3 \left( 2 c_{45} c_{84} c_{45} c_{84} c_{46} \right) \right) p_6 + \left( c_{43} c_{44} \dot{q}_1 s_{45} c_{45} c_{45} c_{45} \right) p_7 + \left( \dot{q}_6 \left( c_{46} c_{46} c_{84} c_{46} c_{54} c_{46} c_{46} c_{44} \right) + \dot{q}_3 \left( 2 c_{45} c_{84} c_{45} c_{45} c_{46} \right) \right) p_6 + \left( c_{43} c_{44} \dot{q}_1 s_{45} c_{45} c_{44} c_{46} c_{45} c_{45} c_{45} \right) p_6 + \left( c_{43} c_{44} \dot{q}_1 s_{45} c_{45} c_{45} c_{45} c_{46} \right) p_6 + \left( c_{43} c_{44} \dot{q}_1 s_{45} c_{45} c_{46} c_{46} c_{45} c_{54} c_{45} c_{46} c_{45} c_{$$

$$\begin{split} C_{45} &= \left(\dot{q}_4 \left( + 2c_{9}^2 s_{q6} - s_{q6} \right) - \dot{q}_3 \left( c_{q4} c_{q6} s_{q5} - 2c_{q5} s_{q4} s_{q5} s_{q6} \right) - \dot{q}_1 \left( 2s_{q3} s_{q6} c_{q5}^2 + 2c_{q3} s_{q4} s_{q5} s_{q6} \right) - \dot{q}_3 \left( c_{q6} s_{q5} \right) p_{11} + \left( \dot{q}_1 \left( \frac{1}{2} c_{q3} c_{q4} c_{q5}^2 c_{q6}^2 \right) - \dot{q}_3 \left( c_{q4} s_{q6} c_{q5} - s_{q3} s_{q6} + \frac{3}{2} c_{q5} c_{q6} s_{q5} \right) p_{11} + \left( \dot{q}_1 \left( \frac{1}{2} c_{q3} c_{q4} c_{q5}^2 c_{q6}^2 \right) - \dot{q}_3 \left( \frac{1}{2} s_{q4} c_{q5}^2 - s_{q4} s_{q5} c_{q6} s_{q4} s_{q6} + c_{q3} c_{q5} c_{q6}^2 s_{q4} s_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} s_{q5} - c_{q6}^2 s_{q5} \right) - \dot{q}_3 \left( \frac{1}{2} s_{q4} c_{q5}^2 - s_{q4} c_{q5}^2 c_{q6}^2 c_{q4} s_{q6} c_{q5} c_{q6} + \frac{1}{2} s_{q4} c_{q6}^2 \right) - \dot{q}_5 c_{q5} c_{q6} s_{q4} + c_{q5} c_{q5}^2 s_{q5} \right) p_{12} + \frac{1}{2} \dot{q}_4 \sin \left( 2q_5 \right) + \dot{q}_3 \left( \frac{1}{2} s_{q4} - c_{q5}^2 s_{q4} \right) - \dot{q}_5 c_{q5} c_{q6} s_{q4} c_{q5}^2 c_{q5}^2 + \frac{1}{2} c_{q3} c_{q4} \right) p_{13} - \left( \dot{q}_3 \left( c_{q4} c_{q5} c_{q5} c_{q6}^2 - c_{q5}^2 c_{q6} s_{q4} s_{q6} \right) + \dot{q}_1 \left( 4c_{q3} c_{q4} s_{q6} c_{q5}^2 c_{q6}^2 + 3c_{q5} s_{q4} c_{q5}^2 c_{q5}^2 - 4s_{q3} s_{q5} c_{q5} c_{q5} c_{q6}^2 c_{q6}^2 - c_{q5}^2 c_{q6} s_{q4} s_{q6} c_{q5}^2 \right) + \dot{q}_1 \left( 4c_{q3} c_{q4} s_{q6} c_{q5}^2 c_{q6}^2 + 3c_{q3} s_{q4} c_{q5}^2 c_{q5}^2 - 4s_{q3} s_{q5} c_{q5} c_{q5} c_{q6}^2 c_{q5}^2 c_{q6}^2 - c_{q5}^2 c_{q6} s_{q4} s_{q6}^2 c_{q6}^2 \right) + \dot{q}_1 \left( 4c_{q3} c_{q4} c_{q5}^2 c_{q6}^2 + 3c_{q5} s_{q5}^2 c_{q6}^2 - 4c_{q5} s_{q5}^2 c_{q6}^2 - 4c_{q5} s_{q5}^2 s_{q6}^2 c_{q6}^2 - 4c_{q5}^2 c_{q6}^2 c_{q6}^2 c_{q6}^2 c_{q5}^2 c_{q6}^2 c_{q6}^$$

$$\begin{split} &C_{31} = \left(\dot{q}_1 \left( C_{q3} C_{q4} C_{q5} - C_{q4} C_{q6} S_{q5} - C_{q3} C_{q5} S_{q3} S_{q6} + C_{q3} C_{q5} C_{q6} S_{q3} \right) - \dot{q}_3 \left( C_{q3} C_{q6} S_{q4} S_{q4} S_{q5} \right) \right) p_5 + \left(\dot{q}_1 \left( C_{q4} S_{q5} S_{36} - 2 C_{44} S_{95} S_{66} C_{q2}^2 - 2 C_{25} S_{26} C_{q3} \right) + c_{34} S_{44} S_{43} S_{36} S_{40} \right) p_5 + \left(\dot{q}_1 \left( C_{q4} S_{q5} S_{36} G_{-2} C_{24} S_{45} S_{36} C_{q2}^2 - 2 C_{25} S_{26} C_{q3} \right) + \dot{q}_3 \left( S_{43} C_{q5} C_{q4} \right) p_7 + c_{34} \left( S_{46} S_{43} S_{44} S_{43} S_{36} S_{36} \right) p_5 + \left(\dot{q}_1 \left( S_{46} C_{q4} C_{45} C_{44} S_{45} S_{36} C_{46}^2 - 2 C_{45} S_{45} S_{45} S_{45} S_{45} S_{45} \right) + \dot{q}_3 \left( S_{43} C_{45} C_{25} S_{44} \right) p_7 + c_{34} \left( S_{46} S_{44} S_{45} S_{4$$

$$C_{52} = \frac{\dot{q}_1}{c_{q2}} c_{q5} c_{q6} c_{q6} c_{q6} c_{q6} c_{q6} c_{q6} c_{q6} c_{q3} c_{q6} c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c$$

$$\begin{split} C_{54} &= \left( \dot{q}_4 \left( \mathbf{s}_{q6} - 2\mathbf{c}_{q5}^2 \mathbf{s}_{q6} \right) + \dot{q}_3 \left( \mathbf{c}_{q4} \mathbf{c}_{q6} \mathbf{s}_{q5} - 2\mathbf{c}_{q5} \mathbf{s}_{q4} \mathbf{s}_{q5} \mathbf{s}_{q6} \right) + \dot{q}_1 2\mathbf{s}_{q3} \mathbf{s}_{q6} \mathbf{c}_{q5}^2 + \\ & 2\mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{s}_{q5} \mathbf{s}_{q6} \mathbf{c}_{q5} - \mathbf{s}_{q3} \mathbf{s}_{q6} + \frac{3}{2}\mathbf{c}_{q3} \mathbf{c}_{q6} \mathbf{s}_{q4} \mathbf{s}_{q5} - \dot{q}_6 \mathbf{c}_{q5} \mathbf{s}_{q6} \right) p_{11} + \\ & \left( \dot{q}_1 \left( \frac{1}{2} \mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{c}_{q6}^2 + \mathbf{c}_{q5} \mathbf{c}_{q6}^2 \mathbf{s}_{q3} \mathbf{s}_{q5} - \mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{c}_{q5}^2 \mathbf{c}_{q6}^2 + \mathbf{c}_{q3} \mathbf{c}_{q5} \mathbf{c}_{q6} \mathbf{s}_{q4} \mathbf{s}_{q6} - \\ & \frac{1}{2} \mathbf{c}_{q3} \mathbf{c}_{q5} \mathbf{c}_{q6}^2 \mathbf{s}_{q4} \mathbf{s}_{q5} \right) + \dot{q}_6 \left( \frac{1}{2} \mathbf{s}_{q5} - \mathbf{c}_{q6}^2 \mathbf{s}_{q5} \right) + \dot{q}_3 \left( \mathbf{s}_{q4} \mathbf{c}_{q6}^2 - \mathbf{s}_{q4} \mathbf{c}_{q5}^2 \mathbf{c}_{q6}^2 + \frac{1}{2} \mathbf{s}_{q4} \mathbf{c}_{q5}^2 \right) + \\ & \mathbf{c}_{q4} \mathbf{s}_{q6} \mathbf{c}_{q5} \mathbf{c}_{q6} \right) - \dot{q}_4 \mathbf{c}_{q5} \mathbf{c}_{q6}^2 \mathbf{s}_{q5} \right) p_{12} + \left( \dot{q}_1 \left( \frac{1}{2} \mathbf{c}_{q3} \mathbf{c}_{q4} - \mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{c}_{q5}^2 \mathbf{c}_{q6}^2 + \frac{1}{2} \mathbf{s}_{q4} \mathbf{c}_{q5}^2 \right) - \\ & \dot{q}_4 \mathbf{s}_{11} \left( 2\mathbf{q}_5 \right) - \dot{q}_3 \left( \frac{1}{2} \mathbf{s}_{q4} - \mathbf{c}_{q5}^2 \mathbf{s}_{q4} \right) \right) p_{13} + \left( \dot{q}_3 \left( \mathbf{c}_{q4} \mathbf{c}_{q5} \mathbf{c}_{q6}^2 - \mathbf{c}_{q5}^2 \mathbf{c}_{q6} \mathbf{s}_{q4} \mathbf{s}_{q6} \right) + \\ & \dot{q}_1 \left( 4\mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{s}_{q6} \mathbf{c}_{q5}^2 \mathbf{c}_{q6} + 3\mathbf{c}_{q3} \mathbf{s}_{q4} \mathbf{c}_{q5} \mathbf{c}_{q6}^2 - 4\mathbf{s}_{q3} \mathbf{s}_{q5} \mathbf{s}_{q6} \mathbf{c}_{q5} \mathbf{c}_{q6} \mathbf{s}_{q4} \mathbf{s}_{q6} \right) + \\ & \dot{q}_1 \left( 4\mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{s}_{q6} \mathbf{c}_{q5}^2 \mathbf{c}_{q6} + 3\mathbf{c}_{q3} \mathbf{s}_{q4} \mathbf{c}_{q5} \mathbf{c}_{q6}^2 - 4\mathbf{s}_{q3} \mathbf{s}_{q5} \mathbf{s}_{q6} \mathbf{c}_{q5} \mathbf{c}_{q5} \mathbf{c}_{q6} \mathbf{c}_{q3} \right) + \\ & \dot{q}_1 \left( 4\mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{s}_{q6} \mathbf{c}_{q5} \mathbf{c}_{q6} \mathbf{s}_{q5} \mathbf{s}_{q6} \right) p_{14} + \dot{q}_4 \left( \mathbf{c}_{q6} - 2\mathbf{c}_{q5}^2 \mathbf{c}_{q6} \mathbf{c}_{q3} \mathbf{c}_{q4} \mathbf{c}_{q6} \mathbf{c}_{q5} \right) + \\ & \dot{q}_1 \left( \mathbf{c}_{q6} \mathbf{c}_{q5} \mathbf{c}_{q5} \mathbf{c}_{q6} \mathbf{s}_{q4} \mathbf{s}_{q5} \right) p_{15} + \dot{q}_2 \left( \frac{1}{2} \mathbf{c}_{q2} \mathbf{c}_{q5} \mathbf{s}_{q3} \mathbf{c}_{q4} \mathbf{c}_{q6} \mathbf{s}_{q5} \mathbf{c}_{q5} \mathbf{c}_{q6} \mathbf{s}_{q3} \right) + \\ & \dot{q}_1 \mathbf{c}_{q4} \mathbf{c}_{q5} \mathbf{c}_{q5} \mathbf{c}_{q5} \mathbf{c}_{q5} \mathbf$$

$$C_{55} = \frac{1}{2}\dot{q}_6 \, s_{2a_c} \, p_{12} + \dot{q}_6 \, c_{2a_c} \, p_{14} \tag{B.102}$$

$$\begin{split} &C_{56} = \left(\dot{q}_1 \left(\frac{3}{2}c_{q5}s_{q3}s_{q6} + c_{q3}c_{q4}s_{q5}s_{q6}\right) - \dot{q}_6 \, s_{q6} - c_{q5} \, \dot{q}_4 \, s_{q6} - \dot{q}_3 \, s_{q4} \, s_{q5} \, s_{q6}\right) p_{11} \, + \\ & \left(\frac{1}{2}\dot{q}_5 \, s_{2q6} - \dot{q}_1 \left(\frac{1}{2}s_{q3}s_{q5} - c_{q6}^2 \, s_{q3} \, s_{q5} - \frac{1}{2}c_{q3} \, c_{q4} \, c_{q5} + c_{q3}c_{q4}c_{q5}c_{q6}^2 \, + \right. \\ & \left. \left(c_{q3}c_{q4}c_{q5}c_{q6}s_{q5}s_{q6}\right) - \dot{q}_3 \left(\frac{1}{2}c_{q5}s_{q4} - c_{q5}c_{q6}^2 \, s_{q4}\right) + \dot{q}_4 \left(\frac{1}{2}s_{q5} - c_{q6}^2 \, s_{q5}\right)\right) p_{12} \, + \\ & \left. \left(\dot{q}_5 \left(2c_{q6}^2 - 1\right) - \dot{q}_3 \left(\frac{1}{2}c_{q4} - c_{q4}c_{q6}^2 + c_{q5}s_{q4}s_{q6}c_{q6}\right) - \dot{q}_1 \left(c_{q3}s_{q4} - 2c_{q3}c_{q6}^2 \, s_{q4} + \right. \right. \\ & \left. \left(c_{q6}^2 \, s_{q3}^2 \, s_{q6}^2 - 2c_{q3}c_{q4}c_{q6} \, s_{q6}\right) + \dot{q}_4 \, c_{q6} \, s_{q5} \, s_{q6}\right) p_{14} + \left(\dot{q}_4 \left(2c_{q6}c_{q3}^2 \, c_{q5}^2 - c_{q6}^2 \, s_{q4}\right) + c_{q6}^2 \, \dot{q}_6 \, c_{q5}^2 + c_{q6}^2 \, c_{q5}^2 + c_{q6}^2 \, \dot{q}_6 \, c_{q5}^2 + c_{q6}^2 \, \dot{q}_6 \, c_{q5}^2 + c_{q6}^2 \, \dot{q}_6 \, c_{q5}^2 + c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, s_{q5}^2 + c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, s_{q5}^2 + c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, s_{q5}^2 + c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, s_{q5}^2 + c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, \dot{q}_6^2 \, c_{q6}^2 \, \dot{q}_6^2 \, \dot{q}_6^2$$

$$\begin{split} &C_{61} = \left( \dot{q}_2 \left( C_{63} \, C_{64} \, C_{67} \, C_{67} \, S_{67} \, S_{48} \, S_{67} \, + \frac{1}{2} \, C_{55} \, S_{63} \, S_{64} \, S_{69} \right) - \dot{q}_3 \left( C_{65} \, S_{64} \, + \, C_{67} \, S_{67} \, S_{64} \, + \, C_{67} \, S_{67} \, S_{67} \, S_{67} \, S_{67} \, S_{67} \right) - \dot{q}_3 \left( \frac{1}{2} \, C_{67} \, S_{67} \, S$$

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$$C_{62} = \left( \dot{q}_1 \left( c_{q2} c_{q4} c_{q6} - \frac{1}{2} c_{q2} c_{q5} s_{q6} - \frac{1}{2} c_{q2} c_{q5} s_{q4} s_{q6} \right) - \dot{q}_2 \left( c_{q6} s_{q2} s_{q3} s_{q4} - c_{q2} c_{q3} s_{q4} c_{q5} s_{q6} + c_{q2} c_{q3} c_{q6} s_{q4} + c_{q2} c_{q3} c_{q4} c_{q5} s_{q6} + c_{q2} c_{q3} c_{q4} c_{q5} s_{q6} + c_{q2} c_{q3} c_{q4} c_{q5} c_{q6} \dot{q}_3 - c_{q4} c_{q5} c_{q6} \dot{q}_4 c_{q5} c_{q6} \dot{q}_4 c_{q5} c_{q6} \dot{q}_4 c_{q5} c_{q6} \dot{q}_4 c_{q5} c_{q6} \dot{q}_5 c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c_{q6} c_{q5} c_{q6} c_{q5} c_{q6} c_{q5} c_{q5} c_{q6} c_{q5} c_{q6}$$

$$\begin{split} C_{64} &= \left( -\dot{q}_1 \left( \frac{1}{2} c_{q6} s_{q3} s_{q4} + \frac{1}{2} s_{q3} s_{q4} s_{q6} \right) \right) p_5 + \left( \dot{q}_1 \left( c_{q3} c_{q4} c_{q6} + \frac{1}{2} c_{q3} c_{q5} s_{q4} s_{q6} + c_{q5} c_{q6} s_{q4} \right) + \dot{q}_5 c_{q5} s_{q4} s_{q6} + c_{q5} c_{q6} s_{q3} s_{q5} c_{q6} c_{q5} c_{q6} c_{$$