AML_Lab3_Group8

Hyungyum Kim, Milda Poceviciute, Boxi Zhang, Fanny Karelius

2018 - 10 - 8

Question 1

A State Space Model was implemented with the following features:

Transition model:

$$p(z_t|z_{t-1}) = \frac{1}{3}[N(z_t|z_{t-1}, 1) + N(z_t|z_{t-1} + 1, 1) + N(z_t|z_{t-1} + 2, 1)]$$

Emission model:

$$p(x_t|z_t) = \frac{1}{3}[N(x_t|z_t, 1) + N(x_t|z_t + 1, 1) + N(x_t|z_t - 1, 1)]$$

Initial model:

$$p(z_1) = Uniform(0, 100)$$

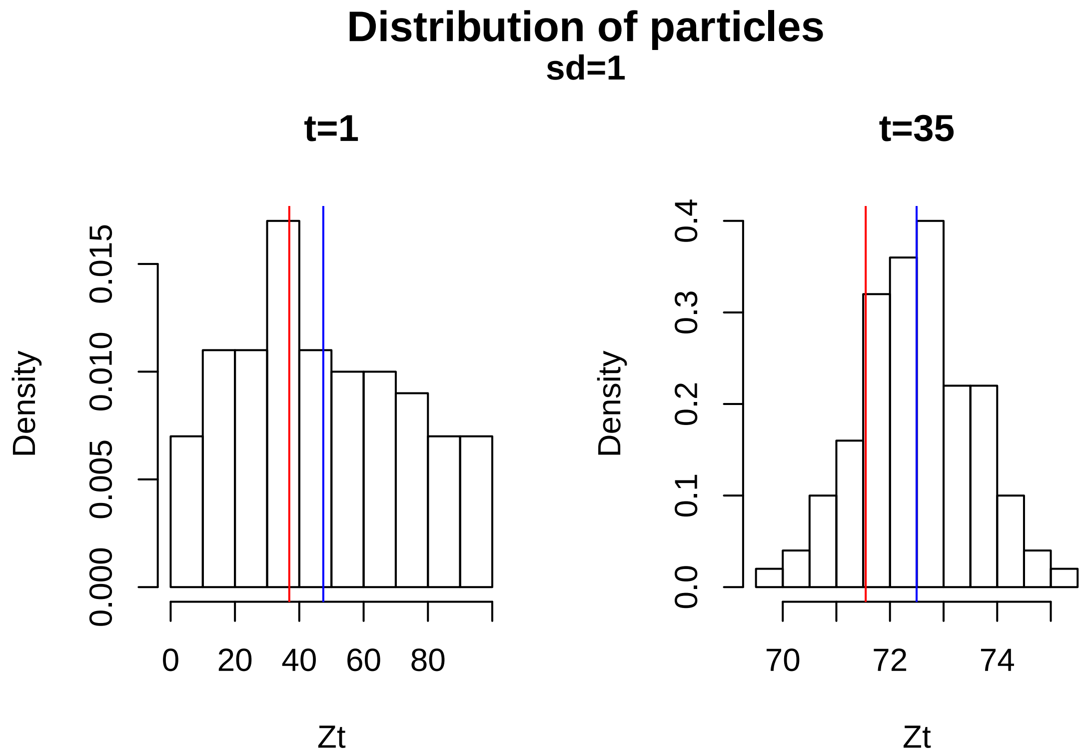
The functions are as follows:

```
# M - number of particals Iter - number of time steps T z is
# latent variable x is the observation

# Function for sampling from a mixture model
sampleMix <- function(mean1, mean2, mean3, sdX) {
    u <- sample(1:3, 1)
    means <- c(mean1, mean2, mean3)
    return(rnorm(1, mean = means[u], sd = sdX))
}

## Function that creates the SSM model
create_SSM <- function(z0, TT, sdX = 1) {
    Zt <- c(z0)

    Xt <- sampleMix(mean1 = z0, mean2 = (z0 - 1), mean3 = (z0 +
        1), sd = 1)

    for (i in 2:TT) {
        Zt[i] <- sampleMix(mean1 = Zt[i - 1], mean2 = (Zt[i -
            1] + 1), mean3 = (Zt[i - 1] + 2), sdX = 1)
        Xt[i] <- sampleMix(mean1 = Zt[i], mean2 = (Zt[i] - 1),
            mean3 = (Zt[i] + 1), sdX = sdX)
    }
    return(list(sates = Zt, observations = Xt))

}

generateX <- function(Zt, TT, sdX = 1) {
    Xt <- c()
    for (i in 1:TT) {
        Xt[i] <- sampleMix(mean1 = Zt[i], mean2 = (Zt[i] - 1),
            mean3 = (Zt[i] + 1), sdX = sdX)
```

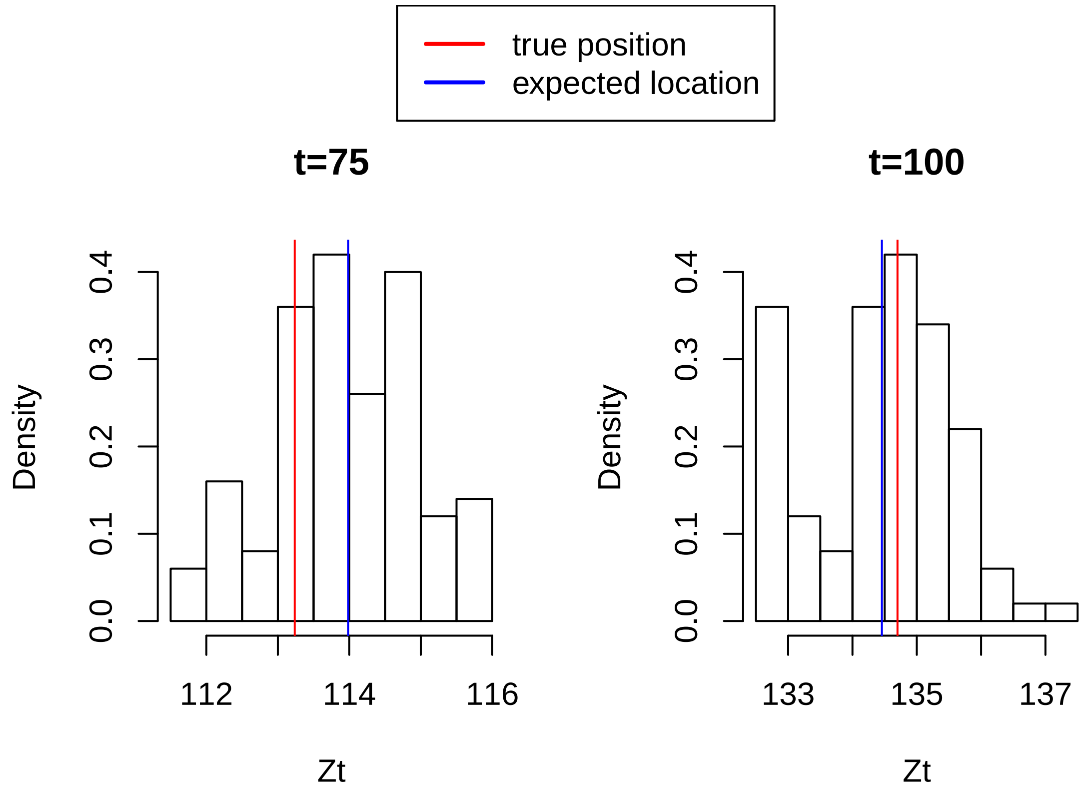
```r
    }
    return(Xt)
}

## Particle filter function
particles <- function(Obs, M, Iter, sdX = 1, correction = TRUE) {
    Xt <- Obs

    # Initialise particles for the latent variable
    Zt <- matrix(ncol = Iter, nrow = M)
    wt <- matrix(ncol = Iter, nrow = M)
    Zt_bar <- matrix(ncol = Iter, nrow = M)
    # fill the initial time step
    Zt[, 1] <- runif(M, 0, 100)
    # Add arbitrary values in order for the indexing to work
    # later
    Zt_bar[, 1] <- rep.int(1, M)
    wt[, 1] <- rep.int(1, M)

    for (t in 2:Iter) {
        # Prediction
        for (m in 1:M) {
            # sample from the transition model (current Z_t depends on
            # the Z_{t-1})
            Zt_bar[m, t] <- sampleMix(mean1 = Zt[m, t - 1], mean2 = (Zt[m,
                t - 1] + 1), mean3 = (Zt[m, t - 1] + 2), sdX = 1)
            # Calculate weights
            wt[m, t] <- (dnorm(Xt[t], mean = Zt_bar[m, t], sd = sdX) +
                dnorm(Xt[t], mean = (Zt_bar[m, t] - 1), sd = sdX) +
                dnorm(Xt[t], mean = (Zt_bar[m, t] + 1), sd = sdX))/3
        }
        # Correction
        if (correction) {
            Zt[, t] <- sample(Zt_bar[, t], M, replace = TRUE,
                prob = wt[, t])
        } else {
            Zt[, t] <- Zt_bar[, t]
        }

    }
    return(list(Zt = Zt, Zt_bar = Zt_bar, wt = wt))

}
```

Afterwards the states $z_{1:T}$ (using the initial and transition models) and observations $x_{1:T}$ of sensor readings (using the emission model) were simulated for $T = 100$ time steps. The observations were used in the particle filter algorithm with 100 particles in order to predict the possible states (location of the robot).

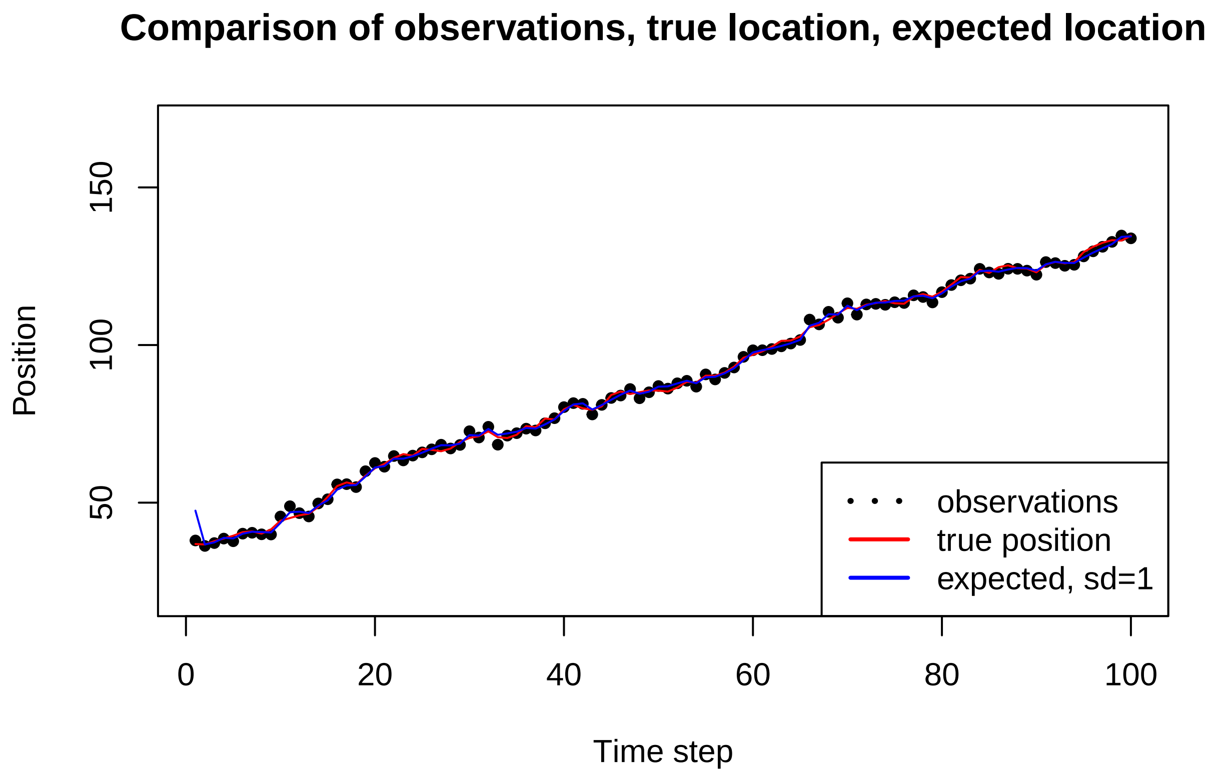
We chose time points \$t=\$1, 35, 75, and 100 for which we plot the distribution of the particles (histograms), and indicate on them what is the true value of the z_t and the expected value at that particular time step.



Distribution of particles
sd=1
t=1
t=35
Density
0.000
0.005
0.010
0.015
0
20
40
60
80
Zt
0.0
0.1
0.2
0.3
0.4
70
72
74



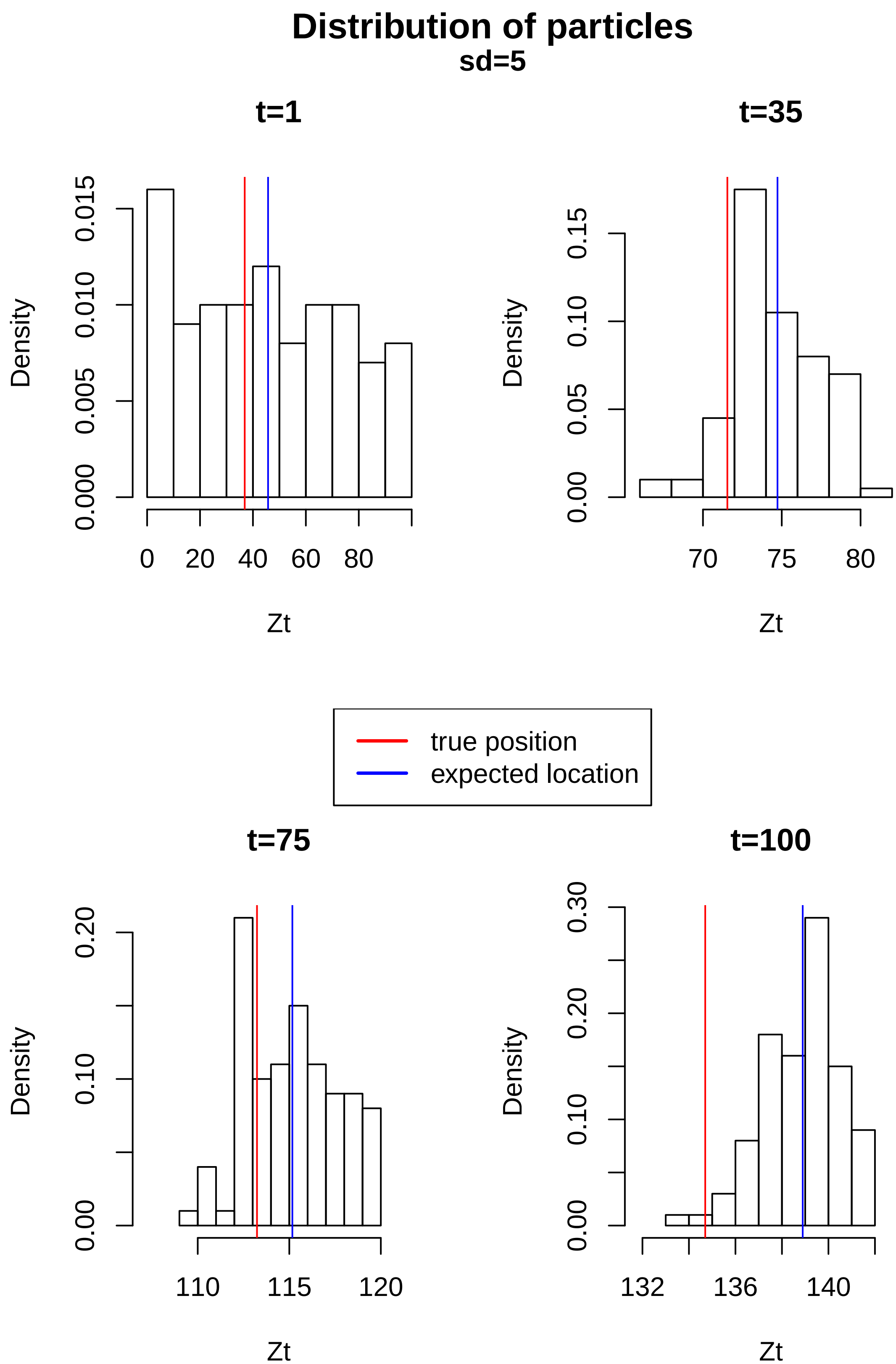
From the histograms, we conclude that the particle algorithm worked pretty well when the standard deviation was 1. We can see that the algorithm converged, hence the spread of the distribution deacreased significantly. In all four plots, the red line falls on a region very close to the blue line: this indicates that the expected position by the particle algortihm is very close to the true position at that time step.



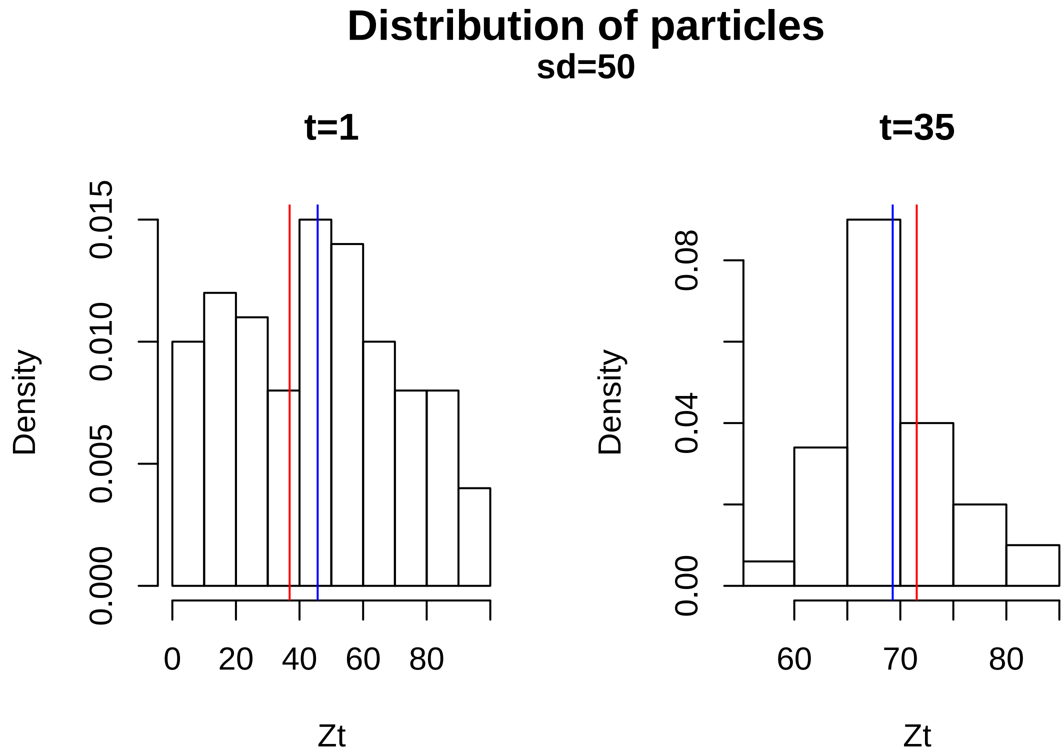
The plot above shows the average of particles in each time step, the corresponding actual state and observations. We conclude that algortihm converged pretty fast: after a few time steps, the true position and predicted positions become very close to each other, with quite small fluctuations in the distance between them. Hence, the prediction seems to be very accurate.

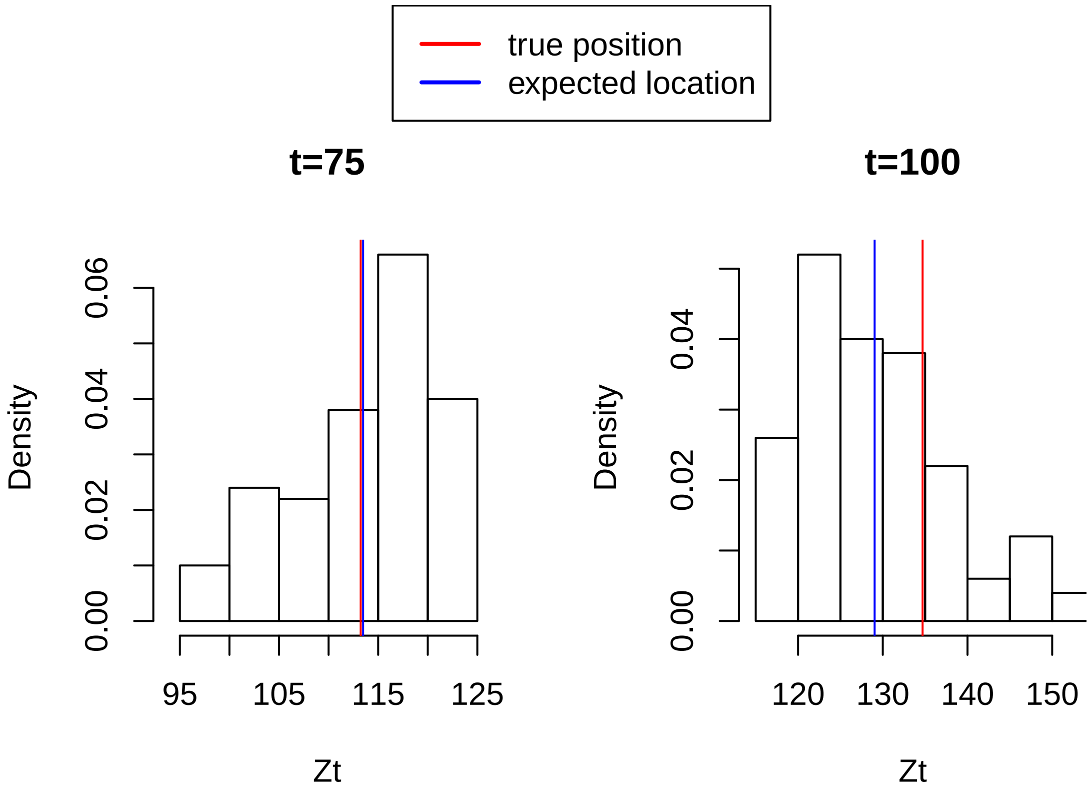
Question 2

In this question the SSM was modified by changing the emission models. Firstly, the standard deviation was set to 5, and later it was increased to 50. The states $z_{1:T}$ were kept from Question 1, but new observations $x_{1:T}$ were generated. The particle filter algorithm was applied to the two new sets of observations.

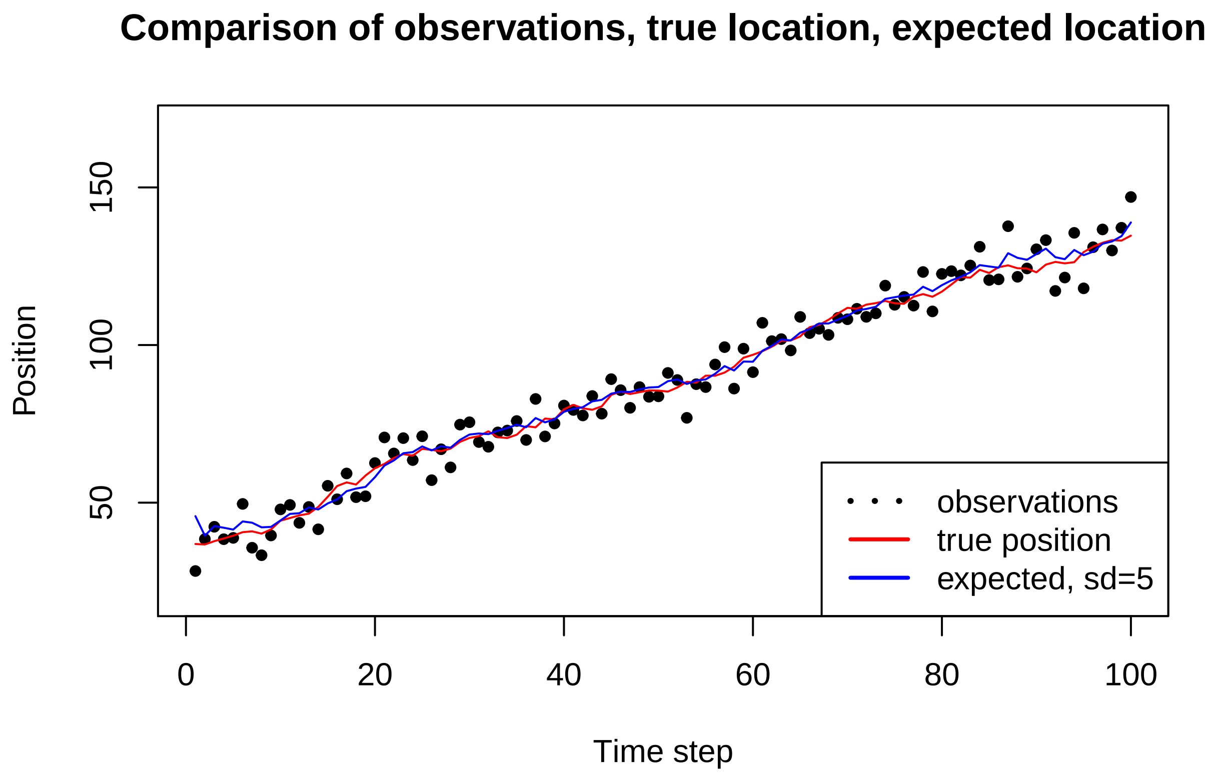

Distribution of particles
sd=5
t=1
t=35
t=75
t=100
Density
Zt
true position
expected location
0.000
0.005
0.010
0.015
0
20
40
60
80
0.00
0.05
0.10
0.15
70
75
80
0.00
0.10
0.20
110
115
120
0.00
0.10
0.20
0.30
132
136
140

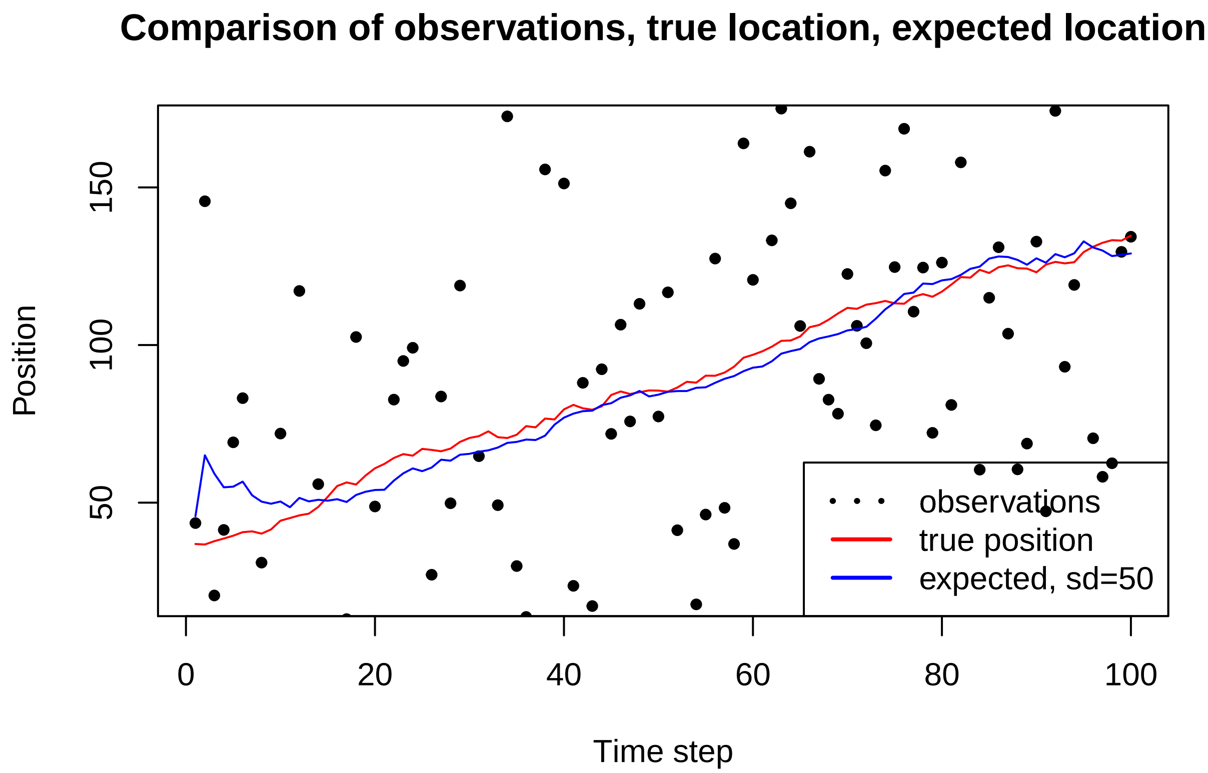
The histogram plots above show the distribution of the particles when standard deviation is 5 (for time steps 1, 35, 75, and 100). This time the algorithm performed worse: the distance between the true position and expected location has increased. In general, the spread of the distribution has increased: this means that more uncertainty has been introduced to the predictions. However, the overall performance cannot be deduced by looking at only 4 time points: each new step introduces some new uncertainty, and the results at it might be worse then the results from the previous time step.





The histogram plots above show the distribution of the particles when standard deviation is 50 (for time steps 1, 35, 75, and 100). This time the spread of distribution is much wider, and the predicted location is not always close to the true position. It is reasonable to assume that the performance of the particle filter worsens as the standard deviation of the observations from the true states is increased: there is more uncertainty involved.

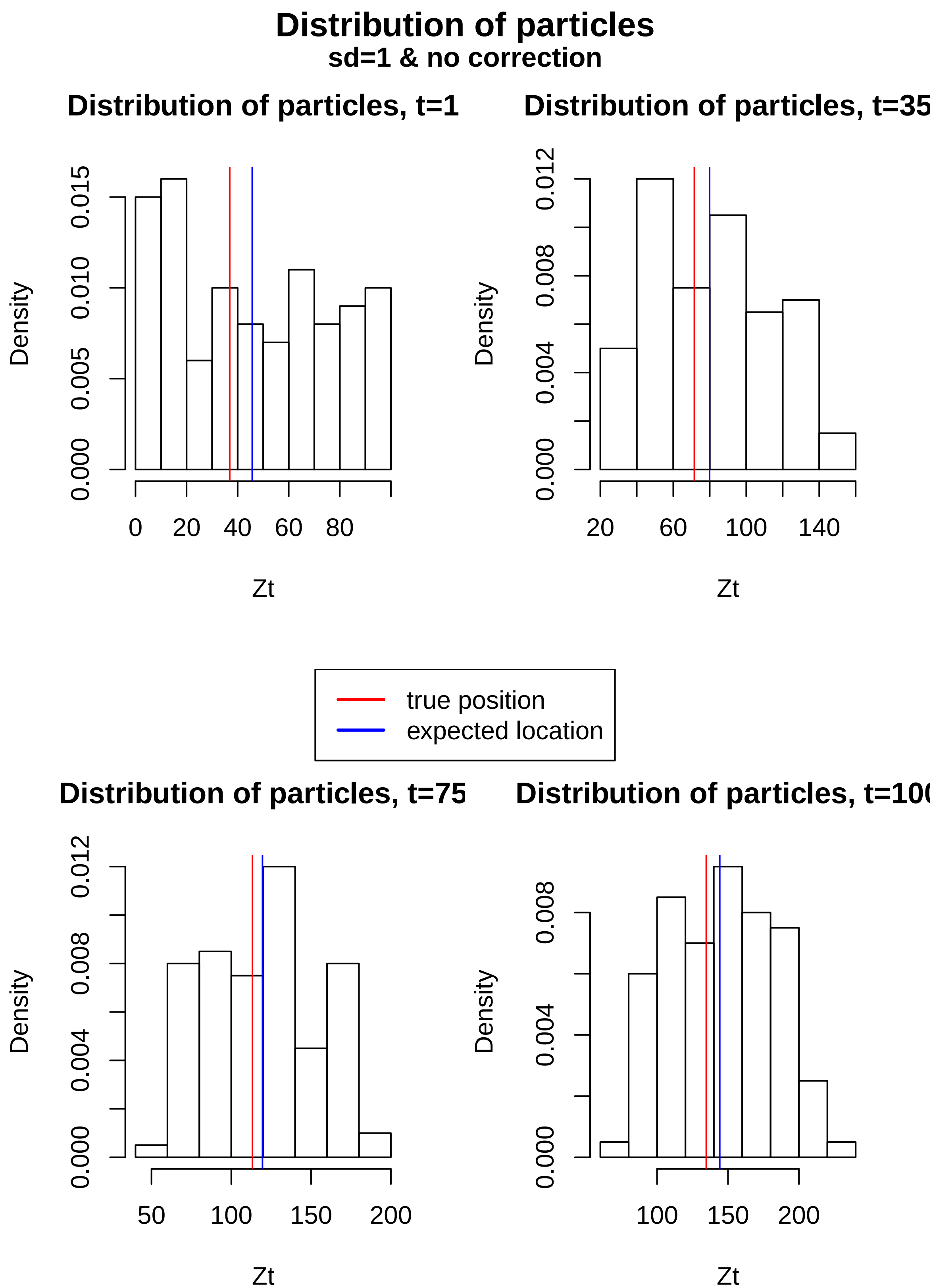

Comparison of observations, true location, expected location
Position
150
100
50
0
20
40
60
80
100
Time step
observations
true position
expected, sd=5



From the plots above, we see that the particle algorihtm with $sd = 50$ converges much slower than in the previous cases. Also, the observations and expected location deviate more from the true position than they did when standard deviation was 5. This is because $sd = 50$ adds even more uncertainty to the observations and thus the expected location than $sd = 5$.

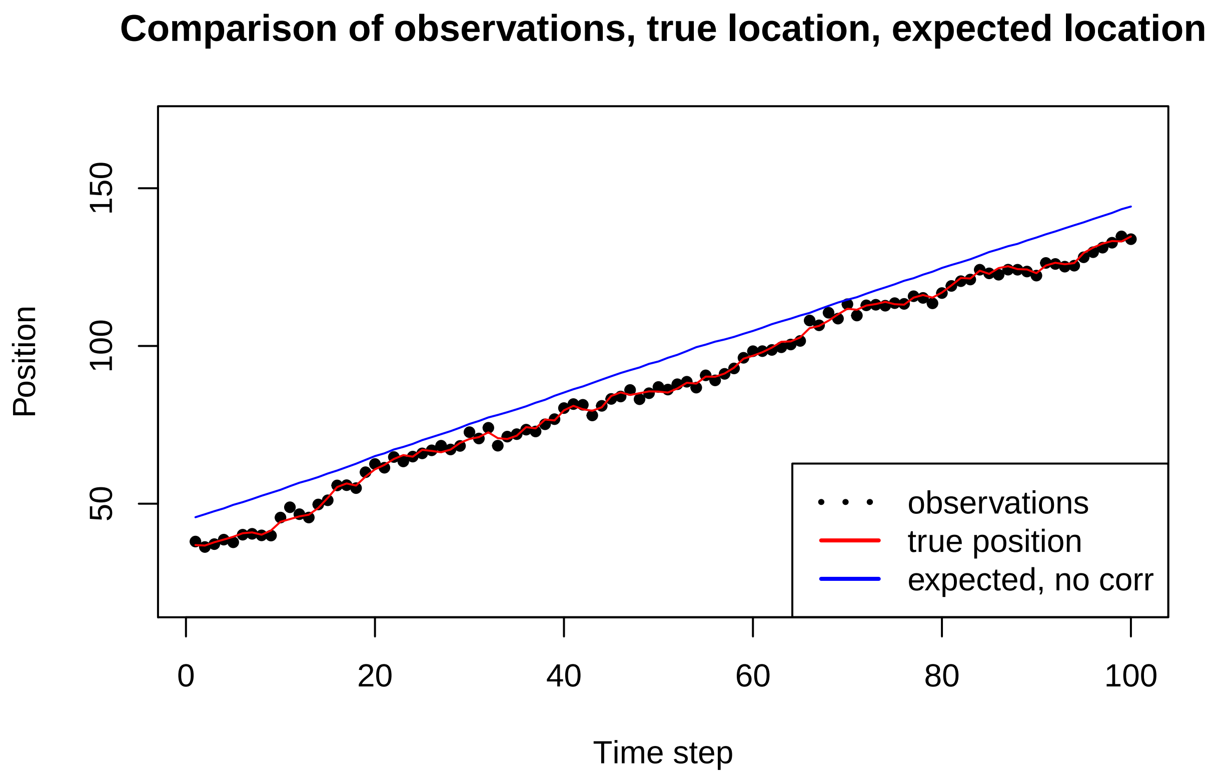
Question 3

Question 1 was repeated without correction, i.e. importance weights were always equal to 1.



Distribution of particles
sd=1 & no correction
Distribution of particles, t=1
Distribution of particles, t=35
Distribution of particles, t=75
Distribution of particles, t=10(
Density
Zt
0.000
0.005
0.010
0.015
0.004
0.008
0.012
0
20
40
60
80
100
140
50
150
200
true position
expected location

From the distribution plots, it seems that the particle filtering algorithm without correction step did not performed much worse than for the same model with correction.



When correction step is omitted, the algortihm prediction is no longer based on the observations of the sensors. This means that the algorithm just uses the transition model to predict what is the likely outcomes after the initial z_t values. Therefore, the spread of the distribution is very wide: this indicates that a lot of uncertainty is involved. Without correction, unlikely particles continues to the next time step. With correction, unlikely particles are less likely to be selected to continue to the next time step and therefore the expected location will deviate less from the true position.

Appendix

```r
knitr::opts_chunk$set(echo = FALSE, tidy.opts = list(width.cutoff = 60),
    tidy = TRUE)
set.seed(987654321)
# M - number of particals Iter - number of time steps T z is
# latent variable x is the observation

# Function for sampling from a mixture model
sampleMix <- function(mean1, mean2, mean3, sdX) {
    u <- sample(1:3, 1)
    means <- c(mean1, mean2, mean3)
    return(rnorm(1, mean = means[u], sd = sdX))
}

## Function that creates the SSM model
create_SSM <- function(z0, TT, sdX = 1) {
    Zt <- c(z0)

    Xt <- sampleMix(mean1 = z0, mean2 = (z0 - 1), mean3 = (z0 +
        1), sd = 1)

    for (i in 2:TT) {
        Zt[i] <- sampleMix(mean1 = Zt[i - 1], mean2 = (Zt[i -
            1] + 1), mean3 = (Zt[i - 1] + 2), sdX = 1)
        Xt[i] <- sampleMix(mean1 = Zt[i], mean2 = (Zt[i] - 1),
            mean3 = (Zt[i] + 1), sdX = sdX)
    }
    return(list(sates = Zt, observations = Xt))

}

generateX <- function(Zt, TT, sdX = 1) {
    Xt <- c()
    for (i in 1:TT) {
        Xt[i] <- sampleMix(mean1 = Zt[i], mean2 = (Zt[i] - 1),
            mean3 = (Zt[i] + 1), sdX = sdX)
    }
    return(Xt)
}

## Particle filter function
particles <- function(Obs, M, Iter, sdX = 1, correction = TRUE) {
    Xt <- Obs

    # Initialise particles for the latent variable
    Zt <- matrix(ncol = Iter, nrow = M)
    wt <- matrix(ncol = Iter, nrow = M)
    Zt_bar <- matrix(ncol = Iter, nrow = M)
    # fill the initial time step
    Zt[, 1] <- runif(M, 0, 100)
    # Add arbitrary values in order for the indexing to work
    # later
```

```r
    Zt_bar[, 1] <- rep.int(1, M)
    wt[, 1] <- rep.int(1, M)

    for (t in 2:Iter) {
        # Prediction
        for (m in 1:M) {
            # sample from the transition model (current Z_t depends on
            # the Z_{t-1})
            Zt_bar[m, t] <- sampleMix(mean1 = Zt[m, t - 1], mean2 = (Zt[m,
                t - 1] + 1), mean3 = (Zt[m, t - 1] + 2), sdX = 1)
            # Calculate weights
            wt[m, t] <- (dnorm(Xt[t], mean = Zt_bar[m, t], sd = sdX) +
                dnorm(Xt[t], mean = (Zt_bar[m, t] - 1), sd = sdX) +
                dnorm(Xt[t], mean = (Zt_bar[m, t] + 1), sd = sdX))/3
        }
        # Correction
        if (correction) {
            Zt[, t] <- sample(Zt_bar[, t], M, replace = TRUE,
                prob = wt[, t])
        } else {
            Zt[, t] <- Zt_bar[, t]
        }

    }
    return(list(Zt = Zt, Zt_bar = Zt_bar, wt = wt))

}

# Create the model
my_SSM <- create_SSM(runif(1, 0, 100), 100)

# Filter particles
results_q1 <- particles(my_SSM$observations, 100, 100)

# Expected location
exp_loc_q1 <- colMeans(results_q1$Zt)
par(mfrow = c(1, 2), oma = c(1, 1, 2, 1))
hist(results_q1$Zt[, 1], freq = FALSE, xlab = "Zt", main = "t=1")
abline(v = my_SSM$sates[1], col = "red")
abline(v = exp_loc_q1[1], col = "blue")

hist(results_q1$Zt[, 35], freq = FALSE, xlab = "Zt", main = "t=35")
abline(v = my_SSM$sates[35], col = "red")
abline(v = exp_loc_q1[35], col = "blue")

mtext("Distribution of particles", side = 3, line = 0, outer = TRUE,
    font = 2, cex = 1.3)
mtext("sd=1", side = 3, line = -1, outer = TRUE, font = 2, cex = 1.1)

hist(results_q1$Zt[, 75], freq = FALSE, xlab = "Zt", main = "t=75")
abline(v = my_SSM$sates[75], col = "red")
abline(v = exp_loc_q1[75], col = "blue")
```

```r
hist(results_q1$Zt[, 100], freq = FALSE, xlab = "Zt", main = "t=100")
abline(v = my_SSM$sates[100], col = "red")
abline(v = exp_loc_q1[100], col = "blue")

par(fig = c(0, 1, 0, 1), oma = c(0, 0, 0, 0), mar = c(0, 0, 0,
    0), new = TRUE)
plot(0, 0, type = "n", bty = "n", xaxt = "n", yaxt = "n")
legend("top", c("true position", "expected location"), col = c("red",
    "blue"), lty = c(1, 1), lwd = c(2, 2), xpd = TRUE)
plot(my_SSM$observations, pch = 20, ylab = "Position", xlab = "Time step",
    main = "Comparison of observations, true location, expected location",
    ylim = c(20, 170))
lines(my_SSM$sates, col = "red")
lines(exp_loc_q1, col = "blue")
legend("bottomright", c("observations", "true position", "expected, sd=1"),
    col = c("black", "red", "blue"), lty = c(3, 1, 1), lwd = c(3,
        rep(2, 2)))
# Create observations for the model with sd=5
my_SSM_2a <- list()
my_SSM_2a$sates <- my_SSM$sates
my_SSM_2a$observations <- generateX(my_SSM$sates, 100, sdX = 5)
# Filter particles, sd=5
results_q2a <- particles(my_SSM_2a$observations, 100, 100, sdX = 5)
# Expected location
exp_loc_q2a <- colMeans(results_q2a$Zt)

# Create the model, sd=50
my_SSM_2b <- list()
my_SSM_2b$sates <- my_SSM$sates
my_SSM_2b$observations <- generateX(my_SSM$sates, 100, sdX = 50)
# Filter particles, sd=50
results_q2b <- particles(my_SSM_2b$observations, 100, 100, sdX = 50)
# Expected location
exp_loc_q2b <- colMeans(results_q2b$Zt)
# Plot results from 2a, sd =5

par(mfrow = c(1, 2), oma = c(1, 1, 2, 1))
hist(results_q2a$Zt[, 1], freq = FALSE, xlab = "Zt", main = "t=1",
    xlim = c(min(min(results_q2a$Zt[, 1], my_SSM_2a$sates[1])) -
        2, max(max(results_q2a$Zt[, 1]), my_SSM_2a$sates[1]) +
        2))
abline(v = my_SSM_2a$sates[1], col = "red")
abline(v = exp_loc_q2a[1], col = "blue")

hist(results_q2a$Zt[, 35], freq = FALSE, xlab = "Zt", main = "t=35",
    xlim = c(min(min(results_q2a$Zt[, 35], my_SSM_2a$sates[35])) -
        2, max(max(results_q2a$Zt[, 35]), my_SSM_2a$sates[35]) +
        2))
abline(v = my_SSM_2a$sates[35], col = "red")
abline(v = exp_loc_q2a[35], col = "blue")

mtext("Distribution of particles", side = 3, line = 0, outer = TRUE,
    font = 2, cex = 1.3)
```

```r
mtext("sd=5", side = 3, line = -1, outer = TRUE, font = 2, cex = 1.1)

hist(results_q2a$Zt[, 75], freq = FALSE, xlab = "Zt", main = "t=75",
    xlim = c(min(min(results_q2a$Zt[, 75], my_SSM_2a$sates[75])) -
        2, max(max(results_q2a$Zt[, 75]), my_SSM_2a$sates[75]) +
        2))
abline(v = my_SSM_2a$sates[75], col = "red")
abline(v = exp_loc_q2a[75], col = "blue")

hist(results_q2a$Zt[, 100], freq = FALSE, xlab = "Zt", main = "t=100",
    xlim = c(min(min(results_q2a$Zt[, 100], my_SSM_2a$sates[100])) -
        2, max(max(results_q2a$Zt[, 100]), my_SSM_2a$sates[100]) +
        2))
abline(v = my_SSM_2a$sates[100], col = "red")
abline(v = exp_loc_q2a[100], col = "blue")

par(fig = c(0, 1, 0, 1), oma = c(0, 0, 0, 0), mar = c(0, 0, 0,
    0), new = TRUE)
plot(0, 0, type = "n", bty = "n", xaxt = "n", yaxt = "n")
legend("top", c("true position", "expected location"), col = c("red",
    "blue"), lty = c(1, 1), lwd = c(2, 2), xpd = TRUE)
# Plot results from 2b, sd =50

par(mfrow = c(1, 2), oma = c(1, 1, 2, 1))
hist(results_q2b$Zt[, 1], freq = FALSE, xlab = "Zt", main = "t=1",
    xlim = c(min(min(results_q2b$Zt[, 1], my_SSM_2b$sates[1])) -
        2, max(max(results_q2b$Zt[, 1]), my_SSM_2b$sates[1]) +
        2))
abline(v = my_SSM_2b$sates[1], col = "red")
abline(v = exp_loc_q2b[1], col = "blue")

hist(results_q2b$Zt[, 35], freq = FALSE, xlab = "Zt", main = "t=35",
    xlim = c(min(min(results_q2b$Zt[, 35], my_SSM_2b$sates[35])) -
        2, max(max(results_q2b$Zt[, 35]), my_SSM_2b$sates[35]) +
        2))
abline(v = my_SSM_2b$sates[35], col = "red")
abline(v = exp_loc_q2b[35], col = "blue")

mtext("Distribution of particles", side = 3, line = 0, outer = TRUE,
    font = 2, cex = 1.3)
mtext("sd=50", side = 3, line = -1, outer = TRUE, font = 2, cex = 1.1)

hist(results_q2b$Zt[, 75], freq = FALSE, xlab = "Zt", main = "t=75",
    xlim = c(min(min(results_q2b$Zt[, 75], my_SSM_2b$sates[75])) -
        2, max(max(results_q2b$Zt[, 75]), my_SSM_2b$sates[75]) +
        2))
abline(v = my_SSM_2b$sates[75], col = "red")
abline(v = exp_loc_q2b[75], col = "blue")

hist(results_q2b$Zt[, 100], freq = FALSE, xlab = "Zt", main = "t=100",
    xlim = c(min(min(results_q2b$Zt[, 100], my_SSM_2b$sates[100])) -
        2, max(max(results_q2b$Zt[, 100]), my_SSM_2b$sates[100]) +
        2))
```

```
abline(v = my_SSM_2b$sates[100], col = "red")
abline(v = exp_loc_q2b[100], col = "blue")

par(fig = c(0, 1, 0, 1), oma = c(0, 0, 0, 0), mar = c(0, 0, 0,
    0), new = TRUE)
plot(0, 0, type = "n", bty = "n", xaxt = "n", yaxt = "n")
legend("top", c("true position", "expected location"), col = c("red",
    "blue"), lty = c(1, 1), lwd = c(2, 2), xpd = TRUE)
plot(my_SSM_2a$observations, pch = 20, ylab = "Position", xlab = "Time step",
    main = "Comparison of observations, true location, expected location",
    ylim = c(20, 170))
lines(my_SSM$sates, col = "red")
lines(exp_loc_q2a, col = "blue")
legend("bottomright", c("observations", "true position", "expected, sd=5"),
    col = c("black", "red", "blue"), lty = c(3, 1, 1), lwd = c(3,
        rep(2, 2)))

plot(my_SSM_2b$observations, pch = 20, ylab = "Position", xlab = "Time step",
    main = "Comparison of observations, true location, expected location",
    ylim = c(20, 170))
lines(my_SSM$sates, col = "red")
lines(exp_loc_q2b, col = "blue")
legend("bottomright", c("observations", "true position", "expected, sd=50"),
    col = c("black", "red", "blue"), lty = c(3, 1, 1), lwd = c(3,
        rep(2, 2)))
# Filter particles
results_q3 <- particles(my_SSM$observations, 100, 100, correction = FALSE)

# Expected location
exp_loc_q3 <- colMeans(results_q3$Zt)

par(mfrow = c(1, 2), oma = c(1, 1, 2, 1))

hist(results_q3$Zt[, 1], freq = FALSE, xlab = "Zt", main = "Distribution of particles, t=1")
abline(v = my_SSM$sates[1], col = "red")
abline(v = exp_loc_q3[1], col = "blue")

hist(results_q3$Zt[, 35], freq = FALSE, xlab = "Zt", main = "Distribution of particles, t=35")
abline(v = my_SSM$sates[35], col = "red")
abline(v = exp_loc_q3[35], col = "blue")

mtext("Distribution of particles", side = 3, line = 0, outer = TRUE,
    font = 2, cex = 1.3)
mtext("sd=1 & no correction", side = 3, line = -1, outer = TRUE,
    font = 2, cex = 1.1)

hist(results_q3$Zt[, 75], freq = FALSE, xlab = "Zt", main = "Distribution of particles, t=75")
abline(v = my_SSM$sates[75], col = "red")
abline(v = exp_loc_q3[75], col = "blue")

hist(results_q3$Zt[, 100], freq = FALSE, xlab = "Zt", main = "Distribution of particles, t=100")
abline(v = my_SSM$sates[100], col = "red")
abline(v = exp_loc_q3[100], col = "blue")
```

```r
par(fig = c(0, 1, 0, 1), oma = c(0, 0, 0, 0), mar = c(0, 0, 0,
    0), new = TRUE)
plot(0, 0, type = "n", bty = "n", xaxt = "n", yaxt = "n")
legend("top", c("true position", "expected location"), col = c("red",
    "blue"), lty = c(1, 1), lwd = c(2, 2), xpd = TRUE)
plot(my_SSM$observations, pch = 20, ylab = "Position", xlab = "Time step",
    main = "Comparison of observations, true location, expected location",
    ylim = c(20, 170))
lines(my_SSM$sates, col = "red")
lines(exp_loc_q3, col = "blue")
legend("bottomright", c("observations", "true position", "expected, no corr"),
    col = c("black", "red", "blue"), lty = c(3, 1, 1), lwd = c(3,
        rep(2, 2)))
```