

# HW5\_Task2

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## 1 Applied Machine Learning Homework 5

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### 2 Task 2

Train a **multilayer perceptron (fully connected)** on the **Fashion MNIST dataset** using the **traditional train/test split** as given by `fashion_mnist.load_data` in `keras`. Use a separate **10000 samples (from the training set)** for **model selection** and to compute **learning curves (accuracy vs epochs, not vs n\_samples)**. Compare a “vanilla” model with a model using **drop-out** (potentially a bigger model), and to a model using **batch normalization** and **residual connections** (but not dropout). Visualize learning curves for all models.

```
In [0]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

```
In [2]: from keras.datasets import fashion_mnist
(X_train, y_train), (X_test, y_test) = fashion_mnist.load_data()
```

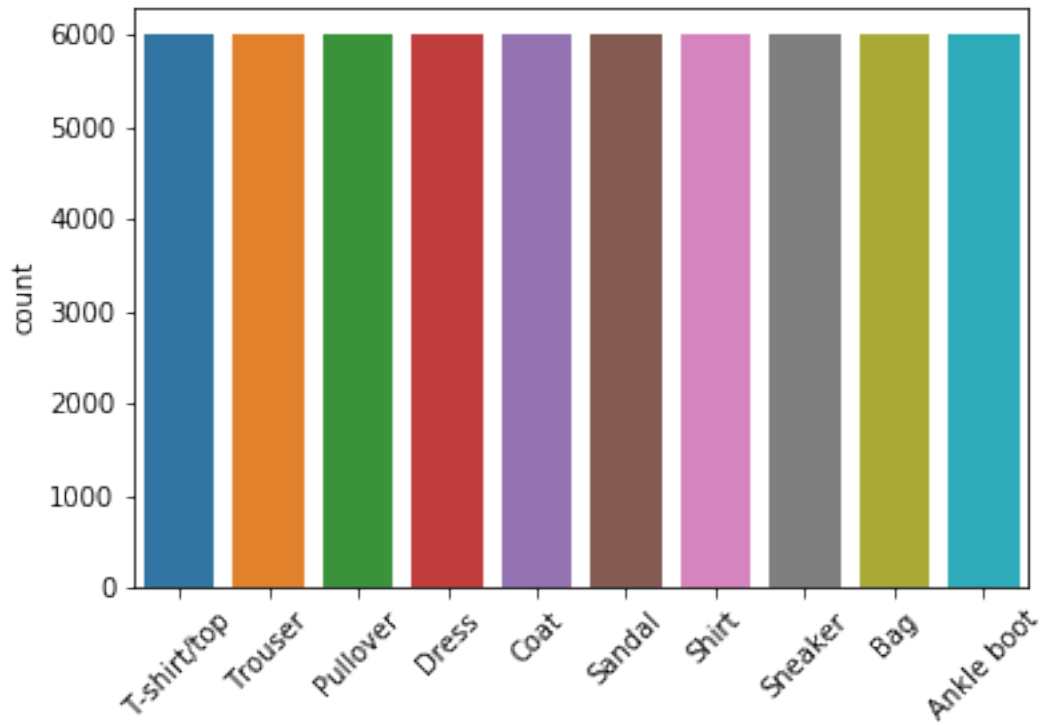
Using TensorFlow backend.

```
Downloading data from http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-
32768/29515 [=====] - 0s 3us/step
Downloading data from http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-
26427392/26421880 [=====] - 2s 0us/step
Downloading data from http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-i
8192/5148 [=====] - 0s 0us/step
Downloading data from http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-i
4423680/4422102 [=====] - 1s 0us/step
```

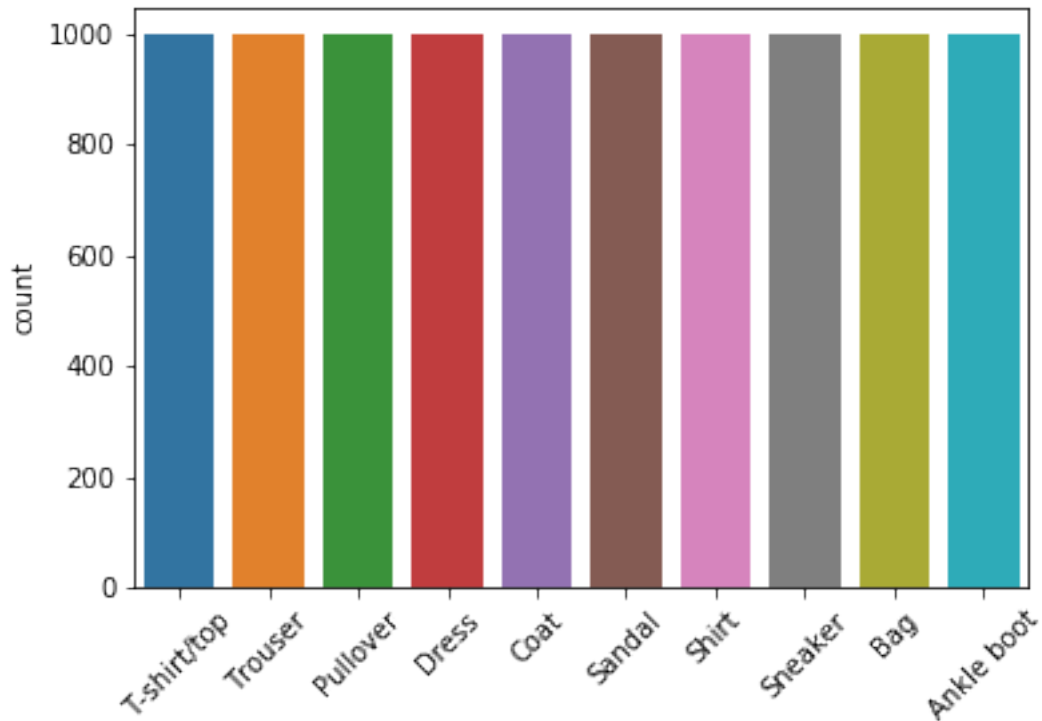
```
In [0]: # scale
X_train = X_train/255
X_test = X_test/255
```

Confirm that we have a balanced data set:

```
In [4]: ax = sns.countplot(y_train)
        _ = ax.set_xticklabels(['T-shirt/top', 'Trouser', 'Pullover',
                                'Dress', 'Coat', 'Sandal', 'Shirt',
                                'Sneaker', 'Bag', 'Ankle boot'],
                                rotation=45)
```



```
In [5]: ax = sns.countplot(y_test)
        _ = ax.set_xticklabels(['T-shirt/top', 'Trouser', 'Pullover',
                                'Dress', 'Coat', 'Sandal', 'Shirt',
                                'Sneaker', 'Bag', 'Ankle boot'],
                                rotation=45)
```



Check GPU status

```
In [6]: # tf
import tensorflow as tf
sess = tf.Session(config=tf.ConfigProto(log_device_placement=True))
from keras import backend
backend.tensorflow_backend._get_available_gpus()
```

```
Out[6]: ['/job:localhost/replica:0/task:0/device:GPU:0']
```

### 3 Base 'Vanilla' Model

```
In [0]: from keras import Sequential
from keras.layers import Dense, Flatten, Dropout, BatchNormalization, Input, add, Activation
from keras import regularizers
from keras import Model
```

```
In [8]: # 1 layer model
# initiate model
model_vanilla1 = Sequential()

# flatten layer
model_vanilla1.add(Flatten(input_shape = (28,28)))
```

```

# first layer
model_vanilla1.add(Dense(128,
                        input_dim = 784,
                        activation='relu'))

# output layer
model_vanilla1.add(Dense(10, activation='softmax'))

# compile
model_vanilla1.compile(optimizer = 'adam',
                      loss = 'sparse_categorical_crossentropy',
                      metrics = ['accuracy'])
model_vanilla1.summary()

```

WARNING:tensorflow:From /usr/local/lib/python3.6/dist-packages/tensorflow/python/framework/op\_def\_registry.py:101: Instructions for updating:  
Colocations handled automatically by placer.

Layer (type)	Output Shape	Param #
flatten_1 (Flatten)	(None, 784)	0
dense_1 (Dense)	(None, 128)	100480
dense_2 (Dense)	(None, 10)	1290

=====  
 Total params: 101,770  
 Trainable params: 101,770  
 Non-trainable params: 0  
 =====

```

In [10]: vanilla1 = model_vanilla1.fit(X_train,
                                       y_train,
                                       epochs=50,
                                       validation_split=10000/60000)

```

WARNING:tensorflow:From /usr/local/lib/python3.6/dist-packages/tensorflow/python/ops/math\_ops.py:101: Instructions for updating:  
Use tf.cast instead.  
Train on 50000 samples, validate on 10000 samples

Epoch 1/50  
50000/50000 [=====] - 5s 94us/step - loss: 0.5173 - acc: 0.8194 - val\_loss: 0.3885 - val\_acc: 0.8603

Epoch 2/50  
50000/50000 [=====] - 4s 89us/step - loss: 0.3885 - acc: 0.8603 - val\_loss: 0.3461 - val\_acc: 0.8744

Epoch 3/50  
50000/50000 [=====] - 4s 89us/step - loss: 0.3461 - acc: 0.8744 - val\_loss: 0.3461 - val\_acc: 0.8744

Epoch 4/50

50000/50000 [=====] - 4s 88us/step - loss: 0.3189 - acc: 0.8836 - val.  
 Epoch 5/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.3021 - acc: 0.8898 - val.  
 Epoch 6/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.2831 - acc: 0.8954 - val.  
 Epoch 7/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.2728 - acc: 0.8985 - val.  
 Epoch 8/50  
 50000/50000 [=====] - 4s 89us/step - loss: 0.2594 - acc: 0.9042 - val.  
 Epoch 9/50  
 50000/50000 [=====] - 4s 89us/step - loss: 0.2519 - acc: 0.9064 - val.  
 Epoch 10/50  
 50000/50000 [=====] - 4s 89us/step - loss: 0.2422 - acc: 0.9087 - val.  
 Epoch 11/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.2329 - acc: 0.9141 - val.  
 Epoch 12/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.2244 - acc: 0.9157 - val.  
 Epoch 13/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.2191 - acc: 0.9176 - val.  
 Epoch 14/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.2122 - acc: 0.9198 - val.  
 Epoch 15/50  
 50000/50000 [=====] - 5s 99us/step - loss: 0.2065 - acc: 0.9239 - val.  
 Epoch 16/50  
 50000/50000 [=====] - 5s 99us/step - loss: 0.1991 - acc: 0.9257 - val.  
 Epoch 17/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.1928 - acc: 0.9280 - val.  
 Epoch 18/50  
 50000/50000 [=====] - 4s 86us/step - loss: 0.1894 - acc: 0.9288 - val.  
 Epoch 19/50  
 50000/50000 [=====] - 4s 87us/step - loss: 0.1819 - acc: 0.9323 - val.  
 Epoch 20/50  
 50000/50000 [=====] - 5s 97us/step - loss: 0.1776 - acc: 0.9336 - val.  
 Epoch 21/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.1747 - acc: 0.9351 - val.  
 Epoch 22/50  
 50000/50000 [=====] - 4s 89us/step - loss: 0.1703 - acc: 0.9368 - val.  
 Epoch 23/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.1655 - acc: 0.9375 - val.  
 Epoch 24/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.1605 - acc: 0.9393 - val.  
 Epoch 25/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.1583 - acc: 0.9410 - val.  
 Epoch 26/50  
 50000/50000 [=====] - 4s 88us/step - loss: 0.1538 - acc: 0.9426 - val.  
 Epoch 27/50  
 50000/50000 [=====] - 4s 87us/step - loss: 0.1491 - acc: 0.9442 - val.  
 Epoch 28/50

```

50000/50000 [=====] - 4s 88us/step - loss: 0.1465 - acc: 0.9453 - val.
Epoch 29/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1419 - acc: 0.9471 - val.
Epoch 30/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1390 - acc: 0.9477 - val.
Epoch 31/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1368 - acc: 0.9492 - val.
Epoch 32/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1324 - acc: 0.9505 - val.
Epoch 33/50
50000/50000 [=====] - 5s 99us/step - loss: 0.1318 - acc: 0.9507 - val.
Epoch 34/50
50000/50000 [=====] - 5s 98us/step - loss: 0.1252 - acc: 0.9531 - val.
Epoch 35/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1271 - acc: 0.9524 - val.
Epoch 36/50
50000/50000 [=====] - 4s 89us/step - loss: 0.1226 - acc: 0.9542 - val.
Epoch 37/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1209 - acc: 0.9545 - val.
Epoch 38/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1173 - acc: 0.9567 - val.
Epoch 39/50
50000/50000 [=====] - 4s 89us/step - loss: 0.1155 - acc: 0.9561 - val.
Epoch 40/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1109 - acc: 0.9582 - val.
Epoch 41/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1097 - acc: 0.9589 - val.
Epoch 42/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1087 - acc: 0.9594 - val.
Epoch 43/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1065 - acc: 0.9602 - val.
Epoch 44/50
50000/50000 [=====] - 4s 87us/step - loss: 0.1066 - acc: 0.9597 - val.
Epoch 45/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1026 - acc: 0.9618 - val.
Epoch 46/50
50000/50000 [=====] - 4s 88us/step - loss: 0.1010 - acc: 0.9630 - val.
Epoch 47/50
50000/50000 [=====] - 4s 88us/step - loss: 0.0998 - acc: 0.9618 - val.
Epoch 48/50
50000/50000 [=====] - 4s 89us/step - loss: 0.0974 - acc: 0.9641 - val.
Epoch 49/50
50000/50000 [=====] - 4s 87us/step - loss: 0.0951 - acc: 0.9645 - val.
Epoch 50/50
50000/50000 [=====] - 4s 89us/step - loss: 0.0957 - acc: 0.9646 - val.

```

```
In [30]: print('Vanilla modeltest score:',
```

```

model_vanilla1.evaluate(X_test, y_test))

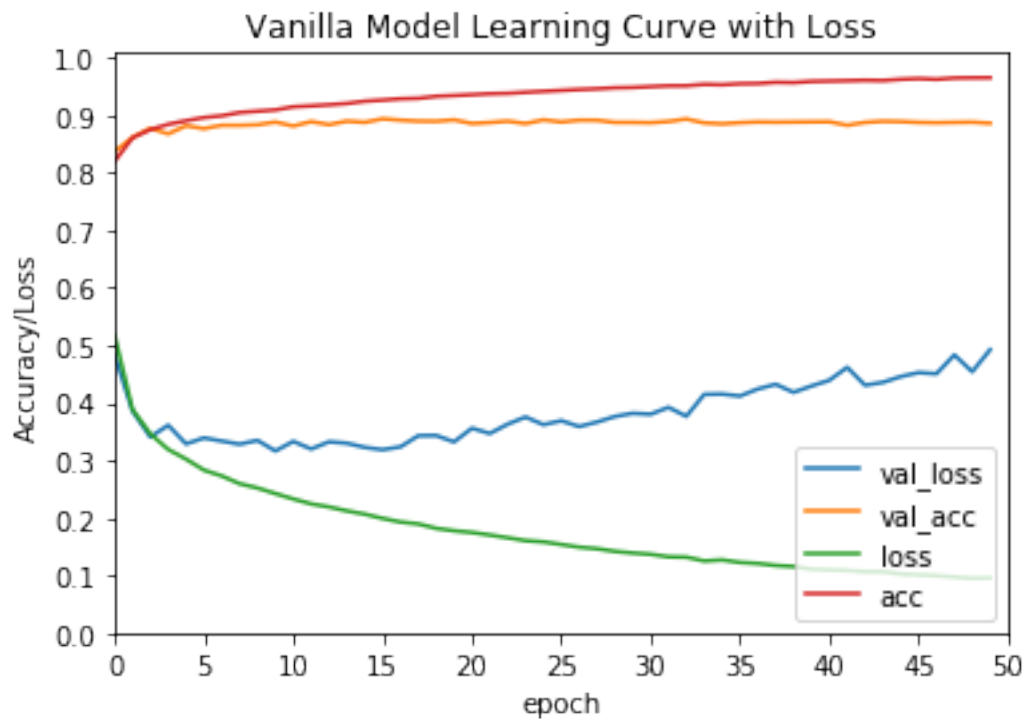
10000/10000 [=====] - 0s 40us/step
Vanilla model test score: [0.5406438440233469, 0.8806]

```

```

In [27]: _ = pd.DataFrame(vanilla1.history).plot(
        title = 'Vanilla Model Learning Curve with Loss',
        xticks = range(0,51,5),
        yticks = [0.1* x for x in range(0,11)]
    )
    _ = plt.legend(loc = 4)
    _ = plt.xlabel('epoch')
    _ = plt.ylabel('Accuracy/Loss')

```



**Quick observation** Validation accuracy starts to flatten at around 5 epochs, while the validation loss bottoms out at about epoch 5, and thereafter steadily increases until the end.

**Larger and Deeper Vanilla Model** 6 layers with 512 cells

```

In [28]: # 6 layers model with 512 cells
        # initiate model
        model_vanilla2 = Sequential()

```

```

# flatten layer
model_vanilla2.add(Flatten(input_shape = (28,28)))

# first layer
model_vanilla2.add(Dense(512,
                          input_dim = 784,
                          activation='relu'))

# second
model_vanilla2.add(Dense(512,
                          activation='relu'))

# third
model_vanilla2.add(Dense(512,
                          activation='relu'))

# fourth
model_vanilla2.add(Dense(512,
                          activation='relu'))

# fifth
model_vanilla2.add(Dense(512,
                          activation='relu'))

# sixth
model_vanilla2.add(Dense(512,
                          activation='relu'))

# output layer
model_vanilla2.add(Dense(10, activation='softmax'))

# compile
model_vanilla2.compile(optimizer = 'adam',
                       loss = 'sparse_categorical_crossentropy',
                       metrics = ['accuracy'])
model_vanilla2.summary()

```

Layer (type)	Output Shape	Param #
flatten_2 (Flatten)	(None, 784)	0
dense_3 (Dense)	(None, 512)	401920
dense_4 (Dense)	(None, 512)	262656
dense_5 (Dense)	(None, 512)	262656



```

-----
dense_6 (Dense)                (None, 512)                262656
-----
dense_7 (Dense)                (None, 512)                262656
-----
dense_8 (Dense)                (None, 512)                262656
-----
dense_9 (Dense)                (None, 10)                 5130
=====
Total params: 1,720,330
Trainable params: 1,720,330
Non-trainable params: 0
-----

```

```

In [29]: vanilla2 = model_vanilla2.fit(X_train,
                                         y_train,
                                         epochs=50,
                                         validation_split= 10000/60000)

```

Train on 50000 samples, validate on 10000 samples

```

Epoch 1/50
50000/50000 [=====] - 8s 153us/step - loss: 0.5558 - acc: 0.7994 - va
Epoch 2/50
50000/50000 [=====] - 7s 142us/step - loss: 0.4151 - acc: 0.8527 - va
Epoch 3/50
50000/50000 [=====] - 7s 143us/step - loss: 0.3781 - acc: 0.8652 - va
Epoch 4/50
50000/50000 [=====] - 7s 143us/step - loss: 0.3495 - acc: 0.8744 - va
Epoch 5/50
50000/50000 [=====] - 8s 155us/step - loss: 0.3254 - acc: 0.8822 - va
Epoch 6/50
50000/50000 [=====] - 8s 151us/step - loss: 0.3159 - acc: 0.8876 - va
Epoch 7/50
50000/50000 [=====] - 8s 157us/step - loss: 0.2969 - acc: 0.8938 - va
Epoch 8/50
50000/50000 [=====] - 7s 142us/step - loss: 0.2927 - acc: 0.8929 - va
Epoch 9/50
50000/50000 [=====] - 7s 143us/step - loss: 0.2780 - acc: 0.8997 - va
Epoch 10/50
50000/50000 [=====] - 7s 142us/step - loss: 0.2698 - acc: 0.9004 - va
Epoch 11/50
50000/50000 [=====] - 7s 142us/step - loss: 0.2635 - acc: 0.9035 - va
Epoch 12/50
50000/50000 [=====] - 7s 142us/step - loss: 0.2587 - acc: 0.9074 - va
Epoch 13/50
50000/50000 [=====] - 7s 142us/step - loss: 0.2476 - acc: 0.9104 - va
Epoch 14/50

```

50000/50000 [=====] - 7s 142us/step - loss: 0.2461 - acc: 0.9104 - va  
 Epoch 15/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.2349 - acc: 0.9144 - va  
 Epoch 16/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.2272 - acc: 0.9160 - va  
 Epoch 17/50  
 50000/50000 [=====] - 7s 149us/step - loss: 0.2243 - acc: 0.9180 - va  
 Epoch 18/50  
 50000/50000 [=====] - 8s 154us/step - loss: 0.2142 - acc: 0.9209 - va  
 Epoch 19/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.2083 - acc: 0.9232 - va  
 Epoch 20/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.2131 - acc: 0.9206 - va  
 Epoch 21/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.2084 - acc: 0.9241 - va  
 Epoch 22/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.2001 - acc: 0.9265 - va  
 Epoch 23/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1968 - acc: 0.9275 - va  
 Epoch 24/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1892 - acc: 0.9290 - va  
 Epoch 25/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.1892 - acc: 0.9311 - va  
 Epoch 26/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1875 - acc: 0.9300 - va  
 Epoch 27/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1783 - acc: 0.9330 - va  
 Epoch 28/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.1845 - acc: 0.9336 - va  
 Epoch 29/50  
 50000/50000 [=====] - 8s 161us/step - loss: 0.1702 - acc: 0.9372 - va  
 Epoch 30/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.1759 - acc: 0.9362 - va  
 Epoch 31/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1716 - acc: 0.9366 - va  
 Epoch 32/50  
 50000/50000 [=====] - 7s 142us/step - loss: 0.1712 - acc: 0.9383 - va  
 Epoch 33/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1631 - acc: 0.9398 - va  
 Epoch 34/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1648 - acc: 0.9390 - va  
 Epoch 35/50  
 50000/50000 [=====] - 7s 140us/step - loss: 0.1543 - acc: 0.9431 - va  
 Epoch 36/50  
 50000/50000 [=====] - 7s 140us/step - loss: 0.1549 - acc: 0.9425 - va  
 Epoch 37/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1593 - acc: 0.9412 - va  
 Epoch 38/50

```

50000/50000 [=====] - 7s 142us/step - loss: 0.1525 - acc: 0.9441 - va
Epoch 39/50
50000/50000 [=====] - 7s 142us/step - loss: 0.1509 - acc: 0.9449 - va
Epoch 40/50
50000/50000 [=====] - 8s 156us/step - loss: 0.1502 - acc: 0.9454 - va
Epoch 41/50
50000/50000 [=====] - 7s 144us/step - loss: 0.1530 - acc: 0.9444 - va
Epoch 42/50
50000/50000 [=====] - 7s 141us/step - loss: 0.1437 - acc: 0.9469 - va
Epoch 43/50
50000/50000 [=====] - 7s 140us/step - loss: 0.1504 - acc: 0.9456 - va
Epoch 44/50
50000/50000 [=====] - 7s 140us/step - loss: 0.1508 - acc: 0.9461 - va
Epoch 45/50
50000/50000 [=====] - 7s 141us/step - loss: 0.1357 - acc: 0.9496 - va
Epoch 46/50
50000/50000 [=====] - 7s 141us/step - loss: 0.1331 - acc: 0.9515 - va
Epoch 47/50
50000/50000 [=====] - 7s 149us/step - loss: 0.1299 - acc: 0.9518 - va
Epoch 48/50
50000/50000 [=====] - 7s 148us/step - loss: 0.1288 - acc: 0.9517 - va
Epoch 49/50
50000/50000 [=====] - 7s 141us/step - loss: 0.2142 - acc: 0.9419 - va
Epoch 50/50
50000/50000 [=====] - 7s 141us/step - loss: 0.1390 - acc: 0.9492 - va

```

```

In [31]: print('Vanilla model 2 test score:',
              model_vanilla2.evaluate(X_test, y_test))

```

```

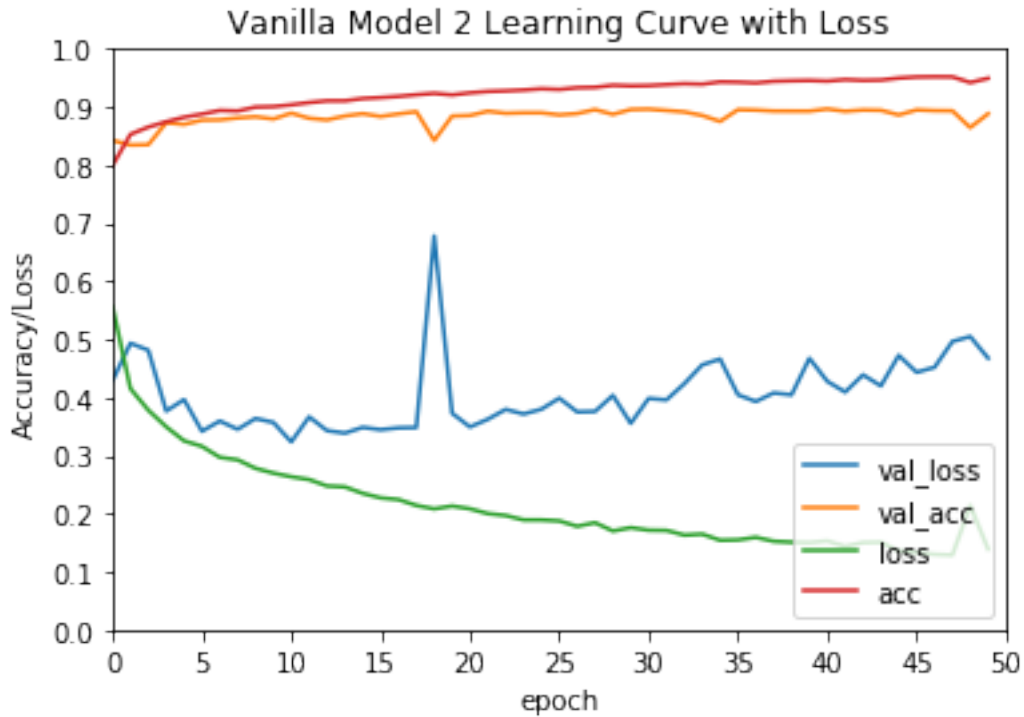
10000/10000 [=====] - 0s 46us/step
Vanilla model 2 test score: [0.49062124730870127, 0.8858]

```

```

In [32]: _ = pd.DataFrame(vanilla2.history).plot(
          title = 'Vanilla Model 2 Learning Curve with Loss',
          xticks = range(0,51,5),
          yticks = [0.1* x for x in range(0,11)]
        )
_ = plt.legend(loc = 4)
_ = plt.xlabel('epoch')
_ = plt.ylabel('Accuracy/Loss')

```



**Quick observation** For the deeper and larger vanilla model, the validation accuracy also starts to flatten at around the fifth epoch, while the validation loss seems to bottom out at around 15. It then has a very odd spike, but seems to correct itself and then steadily rise after that.

## 4 Dropout Model

```
In [33]: # 1 layer
         # initiate model
         model_dropout1 = Sequential()

         # flatten layer
         model_dropout1.add(Flatten(input_shape = (28,28)))

         # first layer
         model_dropout1.add(Dense(128,
                                   input_dim = 784,
                                   activation='relu'))

         # drop out layer
         model_dropout1.add(Dropout(rate = 0.5))

         # output layer
```

```

model_dropout1.add(Dense(10, activation='softmax'))

# compile
model_dropout1.compile(optimizer = 'adam',
                        loss = 'sparse_categorical_crossentropy',
                        metrics = ['accuracy'])

model_dropout1.summary()

```

WARNING:tensorflow:From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow\_backend.py:307: Instructions for updating:  
Please use `rate` instead of `keep\_prob`. Rate should be set to `rate = 1 - keep\_prob`.

Layer (type)	Output Shape	Param #
flatten_3 (Flatten)	(None, 784)	0
dense_10 (Dense)	(None, 128)	100480
dropout_1 (Dropout)	(None, 128)	0
dense_11 (Dense)	(None, 10)	1290

Total params: 101,770  
 Trainable params: 101,770  
 Non-trainable params: 0

```

In [34]: dropout1 = model_dropout1.fit(X_train,
                                         y_train,
                                         epochs=50,
                                         validation_split= 10000/60000)

```

Train on 50000 samples, validate on 10000 samples

```

Epoch 1/50
50000/50000 [=====] - 5s 96us/step - loss: 0.6433 - acc: 0.7741 - val_
Epoch 2/50
50000/50000 [=====] - 5s 92us/step - loss: 0.4833 - acc: 0.8257 - val_
Epoch 3/50
50000/50000 [=====] - 5s 92us/step - loss: 0.4494 - acc: 0.8379 - val_
Epoch 4/50
50000/50000 [=====] - 5s 92us/step - loss: 0.4260 - acc: 0.8448 - val_
Epoch 5/50
50000/50000 [=====] - 5s 92us/step - loss: 0.4111 - acc: 0.8506 - val_
Epoch 6/50
50000/50000 [=====] - 5s 92us/step - loss: 0.4002 - acc: 0.8541 - val_
Epoch 7/50

```

50000/50000 [=====] - 5s 92us/step - loss: 0.3900 - acc: 0.8578 - val.  
 Epoch 8/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3797 - acc: 0.8610 - val.  
 Epoch 9/50  
 50000/50000 [=====] - 5s 91us/step - loss: 0.3762 - acc: 0.8624 - val.  
 Epoch 10/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3712 - acc: 0.8644 - val.  
 Epoch 11/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3625 - acc: 0.8681 - val.  
 Epoch 12/50  
 50000/50000 [=====] - 5s 98us/step - loss: 0.3551 - acc: 0.8687 - val.  
 Epoch 13/50  
 50000/50000 [=====] - 6s 111us/step - loss: 0.3545 - acc: 0.8687 - val.  
 Epoch 14/50  
 50000/50000 [=====] - 5s 100us/step - loss: 0.3512 - acc: 0.8696 - val.  
 Epoch 15/50  
 50000/50000 [=====] - 5s 99us/step - loss: 0.3489 - acc: 0.8724 - val.  
 Epoch 16/50  
 50000/50000 [=====] - 5s 105us/step - loss: 0.3407 - acc: 0.8728 - val.  
 Epoch 17/50  
 50000/50000 [=====] - 5s 96us/step - loss: 0.3387 - acc: 0.8743 - val.  
 Epoch 18/50  
 50000/50000 [=====] - 5s 91us/step - loss: 0.3349 - acc: 0.8759 - val.  
 Epoch 19/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3352 - acc: 0.8756 - val.  
 Epoch 20/50  
 50000/50000 [=====] - 5s 94us/step - loss: 0.3307 - acc: 0.8764 - val.  
 Epoch 21/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3260 - acc: 0.8797 - val.  
 Epoch 22/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.3241 - acc: 0.8788 - val.  
 Epoch 23/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3219 - acc: 0.8795 - val.  
 Epoch 24/50  
 50000/50000 [=====] - 5s 92us/step - loss: 0.3179 - acc: 0.8827 - val.  
 Epoch 25/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.3163 - acc: 0.8798 - val.  
 Epoch 26/50  
 50000/50000 [=====] - 5s 94us/step - loss: 0.3129 - acc: 0.8836 - val.  
 Epoch 27/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.3145 - acc: 0.8816 - val.  
 Epoch 28/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.3113 - acc: 0.8844 - val.  
 Epoch 29/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.3102 - acc: 0.8825 - val.  
 Epoch 30/50  
 50000/50000 [=====] - 5s 93us/step - loss: 0.3109 - acc: 0.8839 - val.  
 Epoch 31/50

```

50000/50000 [=====] - 5s 94us/step - loss: 0.3050 - acc: 0.8863 - val.
Epoch 32/50
50000/50000 [=====] - 5s 99us/step - loss: 0.3041 - acc: 0.8851 - val.
Epoch 33/50
50000/50000 [=====] - 5s 105us/step - loss: 0.3016 - acc: 0.8862 - val.
Epoch 34/50
50000/50000 [=====] - 5s 97us/step - loss: 0.2996 - acc: 0.8872 - val.
Epoch 35/50
50000/50000 [=====] - 5s 92us/step - loss: 0.2989 - acc: 0.8860 - val.
Epoch 36/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2967 - acc: 0.8889 - val.
Epoch 37/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2959 - acc: 0.8886 - val.
Epoch 38/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2947 - acc: 0.8898 - val.
Epoch 39/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2921 - acc: 0.8898 - val.
Epoch 40/50
50000/50000 [=====] - 5s 95us/step - loss: 0.2931 - acc: 0.8897 - val.
Epoch 41/50
50000/50000 [=====] - 5s 105us/step - loss: 0.2866 - acc: 0.8918 - val.
Epoch 42/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2854 - acc: 0.8908 - val.
Epoch 43/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2869 - acc: 0.8904 - val.
Epoch 44/50
50000/50000 [=====] - 5s 92us/step - loss: 0.2829 - acc: 0.8923 - val.
Epoch 45/50
50000/50000 [=====] - 5s 91us/step - loss: 0.2830 - acc: 0.8923 - val.
Epoch 46/50
50000/50000 [=====] - 5s 93us/step - loss: 0.2816 - acc: 0.8918 - val.
Epoch 47/50
50000/50000 [=====] - 5s 92us/step - loss: 0.2833 - acc: 0.8914 - val.
Epoch 48/50
50000/50000 [=====] - 5s 92us/step - loss: 0.2836 - acc: 0.8930 - val.
Epoch 49/50
50000/50000 [=====] - 5s 95us/step - loss: 0.2783 - acc: 0.8942 - val.
Epoch 50/50
50000/50000 [=====] - 5s 104us/step - loss: 0.2791 - acc: 0.8949 - val.

```

```

In [35]: print('Dropout model 1 test score:',
              model_dropout1.evaluate(X_test, y_test))

```

```

10000/10000 [=====] - 0s 44us/step
Dropout model 1 test score: [0.36197602721452715, 0.883]

```

```

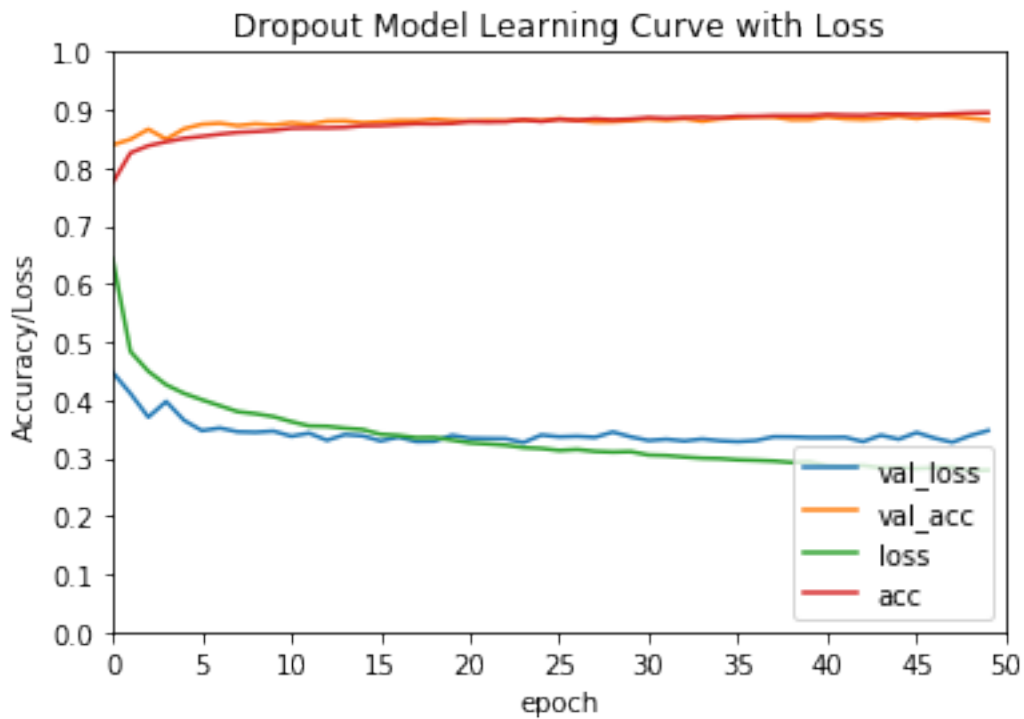
In [37]: _ = pd.DataFrame(dropout1.history).plot(

```

```

    title = 'Dropout Model Learning Curve with Loss',
    xticks = range(0,51,5),
    yticks = [0.1* x for x in range(0,11)]
)
_ = plt.legend(loc = 4)
_ = plt.xlabel('epoch')
_ = plt.ylabel('Accuracy/Loss')

```



**Quick observation** Both training and validation accuracy flatten at a level around 0.85~0.9. In comparison with the vanilla model, the training accuracy doesn't appear to be overfitted which reached 1, and also saw the validation loss steadily rise as it was approaching 1.

### Larger and deeper dropout model

```

In [38]: # initiate model
model_dropout2 = Sequential()

# flatten layer
model_dropout2.add(Flatten(input_shape = (28,28)))

# first layer
model_dropout2.add(Dense(512,
                        input_dim = 784,

```



```

        activation='relu'))

# drop out layer
model_dropout2.add(Dropout(rate = 0.5))

# second layer
model_dropout2.add(Dense(512,
        activation='relu'))

# drop out layer
model_dropout2.add(Dropout(rate = 0.5))

# third layer
model_dropout2.add(Dense(512,
        activation='relu'))

# drop out layer
model_dropout2.add(Dropout(rate = 0.5))

# fourth layer
model_dropout2.add(Dense(512,
        activation='relu'))

# drop out layer
model_dropout2.add(Dropout(rate = 0.5))

# fifth layer
model_dropout2.add(Dense(512,
        activation='relu'))

# drop out layer
model_dropout2.add(Dropout(rate = 0.5))

# sixth layer
model_dropout2.add(Dense(512,
        activation='relu'))

# drop out layer
model_dropout2.add(Dropout(rate = 0.5))

# output layer
model_dropout2.add(Dense(10, activation='softmax'))

# compile
model_dropout2.compile(optimizer = 'adam',
        loss = 'sparse_categorical_crossentropy',
        metrics = ['accuracy'])

```

```
model_dropout2.summary()
```

Layer (type)	Output Shape	Param #
flatten_4 (Flatten)	(None, 784)	0
dense_12 (Dense)	(None, 512)	401920
dropout_2 (Dropout)	(None, 512)	0
dense_13 (Dense)	(None, 512)	262656
dropout_3 (Dropout)	(None, 512)	0
dense_14 (Dense)	(None, 512)	262656
dropout_4 (Dropout)	(None, 512)	0
dense_15 (Dense)	(None, 512)	262656
dropout_5 (Dropout)	(None, 512)	0
dense_16 (Dense)	(None, 512)	262656
dropout_6 (Dropout)	(None, 512)	0
dense_17 (Dense)	(None, 512)	262656
dropout_7 (Dropout)	(None, 512)	0
dense_18 (Dense)	(None, 10)	5130
Total params: 1,720,330		
Trainable params: 1,720,330		
Non-trainable params: 0		

```
In [39]: dropout2 = model_dropout2.fit(X_train,
                                         y_train,
                                         epochs=50,
                                         validation_split=10000/60000)
```

Train on 50000 samples, validate on 10000 samples

Epoch 1/50

50000/50000 [=====] - 9s 189us/step - loss: 1.0096 - acc: 0.6020 - va

Epoch 2/50

50000/50000 [=====] - 8s 166us/step - loss: 0.7129 - acc: 0.7362 - va  
 Epoch 3/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6647 - acc: 0.7648 - va  
 Epoch 4/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6460 - acc: 0.7766 - va  
 Epoch 5/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6257 - acc: 0.7862 - va  
 Epoch 6/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6193 - acc: 0.7891 - va  
 Epoch 7/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6091 - acc: 0.7930 - va  
 Epoch 8/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6066 - acc: 0.7988 - va  
 Epoch 9/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.5988 - acc: 0.7996 - va  
 Epoch 10/50  
 50000/50000 [=====] - 8s 159us/step - loss: 0.5929 - acc: 0.8025 - va  
 Epoch 11/50  
 50000/50000 [=====] - 9s 174us/step - loss: 0.6040 - acc: 0.8004 - va  
 Epoch 12/50  
 50000/50000 [=====] - 8s 167us/step - loss: 0.6011 - acc: 0.7978 - va  
 Epoch 13/50  
 50000/50000 [=====] - 8s 159us/step - loss: 0.5947 - acc: 0.8013 - va  
 Epoch 14/50  
 50000/50000 [=====] - 9s 173us/step - loss: 0.6035 - acc: 0.7991 - va  
 Epoch 15/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6087 - acc: 0.8009 - va  
 Epoch 16/50  
 50000/50000 [=====] - 8s 156us/step - loss: 0.5955 - acc: 0.8003 - va  
 Epoch 17/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.5906 - acc: 0.8056 - va  
 Epoch 18/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.5906 - acc: 0.8059 - va  
 Epoch 19/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.5963 - acc: 0.8027 - va  
 Epoch 20/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.5928 - acc: 0.8050 - va  
 Epoch 21/50  
 50000/50000 [=====] - 9s 176us/step - loss: 0.5989 - acc: 0.8028 - va  
 Epoch 22/50  
 50000/50000 [=====] - 8s 165us/step - loss: 0.5789 - acc: 0.8107 - va  
 Epoch 23/50  
 50000/50000 [=====] - 8s 159us/step - loss: 0.5870 - acc: 0.8048 - va  
 Epoch 24/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.5931 - acc: 0.7998 - va  
 Epoch 25/50  
 50000/50000 [=====] - 8s 159us/step - loss: 0.6003 - acc: 0.7986 - va  
 Epoch 26/50

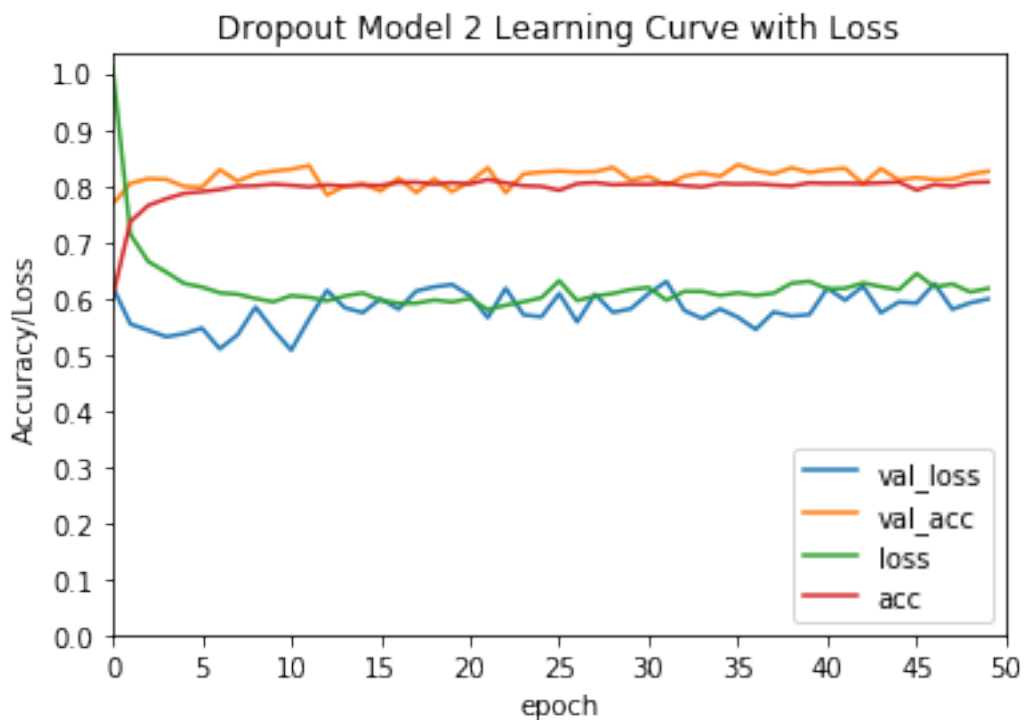
50000/50000 [=====] - 8s 157us/step - loss: 0.6305 - acc: 0.7918 - va.  
 Epoch 27/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.5952 - acc: 0.8034 - va.  
 Epoch 28/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6034 - acc: 0.8053 - va.  
 Epoch 29/50  
 50000/50000 [=====] - 8s 159us/step - loss: 0.6078 - acc: 0.8016 - va.  
 Epoch 30/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6149 - acc: 0.8025 - va.  
 Epoch 31/50  
 50000/50000 [=====] - 9s 174us/step - loss: 0.6187 - acc: 0.8021 - va.  
 Epoch 32/50  
 50000/50000 [=====] - 8s 167us/step - loss: 0.5958 - acc: 0.8045 - va.  
 Epoch 33/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6117 - acc: 0.7998 - va.  
 Epoch 34/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6114 - acc: 0.7977 - va.  
 Epoch 35/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6045 - acc: 0.8038 - va.  
 Epoch 36/50  
 50000/50000 [=====] - 9s 171us/step - loss: 0.6091 - acc: 0.8027 - va.  
 Epoch 37/50  
 50000/50000 [=====] - 9s 177us/step - loss: 0.6045 - acc: 0.8032 - va.  
 Epoch 38/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6084 - acc: 0.8011 - va.  
 Epoch 39/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6261 - acc: 0.7992 - va.  
 Epoch 40/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6295 - acc: 0.8048 - va.  
 Epoch 41/50  
 50000/50000 [=====] - 9s 179us/step - loss: 0.6160 - acc: 0.8039 - va.  
 Epoch 42/50  
 50000/50000 [=====] - 8s 162us/step - loss: 0.6170 - acc: 0.8042 - va.  
 Epoch 43/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6265 - acc: 0.8034 - va.  
 Epoch 44/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6201 - acc: 0.8049 - va.  
 Epoch 45/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6144 - acc: 0.8062 - va.  
 Epoch 46/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6433 - acc: 0.7921 - va.  
 Epoch 47/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6195 - acc: 0.8019 - va.  
 Epoch 48/50  
 50000/50000 [=====] - 8s 158us/step - loss: 0.6248 - acc: 0.7988 - va.  
 Epoch 49/50  
 50000/50000 [=====] - 8s 157us/step - loss: 0.6105 - acc: 0.8056 - va.  
 Epoch 50/50

50000/50000 [=====] - 8s 158us/step - loss: 0.6169 - acc: 0.8065 - va

```
In [40]: print('Dropout model 2 test score:',  
             model_dropout2.evaluate(X_test, y_test))
```

10000/10000 [=====] - 0s 44us/step  
Dropout model 2 test score: [0.6149588119506836, 0.8166]

```
In [42]: _ = pd.DataFrame(dropout2.history).plot(  
         title = 'Dropout Model 2 Learning Curve with Loss',  
         xticks = range(0,51,5),  
         yticks = [0.1* x for x in range(0,11)]  
         )  
_ = plt.legend(loc = 4)  
_ = plt.xlabel('epoch')  
_ = plt.ylabel('Accuracy/Loss')
```



**Quick observation** The larger and deeper net with dropout can effectively control the overfitting, as can be seen in this chart. The training and validation scores are much closer together, while the validation loss does not exhibit the steady increase in the later epochs, as was noticed earlier. This generalizability came at a cost in terms of accuracy, though, as the ended around 0.8.

## 5 Batch Normalization and Residual Connection Model

```
In [43]: # 1 layer model, which can't adopt res connect in this
# initiate model
model_batch1 = Sequential()

# flatten layer
model_batch1.add(Flatten(input_shape = (28,28)))

# first layer
model_batch1.add(Dense(128,
                        input_dim = 784,
                        activation='relu'))

# batchnormalize layer
model_batch1.add(BatchNormalization())

# output layer
model_batch1.add(Dense(10, activation='softmax'))

# compile
model_batch1.compile(optimizer = 'adam',
                     loss = 'sparse_categorical_crossentropy',
                     metrics = ['accuracy'])

model_batch1.summary()
```

```
-----
Layer (type)                 Output Shape              Param #
=====
flatten_5 (Flatten)          (None, 784)               0
-----
dense_19 (Dense)              (None, 128)              100480
-----
batch_normalization_1 (Batch (None, 128)          512
-----
dense_20 (Dense)              (None, 10)               1290
=====
Total params: 102,282
Trainable params: 102,026
Non-trainable params: 256
-----
```

```
In [44]: batch1 = model_batch1.fit(X_train,
                                   y_train,
                                   epochs=50,
                                   validation_split=10000/60000)
```

Train on 50000 samples, validate on 10000 samples

```
Epoch 1/50
50000/50000 [=====] - 7s 140us/step - loss: 0.4980 - acc: 0.8264 - va
Epoch 2/50
50000/50000 [=====] - 6s 122us/step - loss: 0.4037 - acc: 0.8571 - va
Epoch 3/50
50000/50000 [=====] - 7s 138us/step - loss: 0.3725 - acc: 0.8670 - va
Epoch 4/50
50000/50000 [=====] - 7s 136us/step - loss: 0.3536 - acc: 0.8733 - va
Epoch 5/50
50000/50000 [=====] - 6s 124us/step - loss: 0.3396 - acc: 0.8768 - va
Epoch 6/50
50000/50000 [=====] - 6s 124us/step - loss: 0.3270 - acc: 0.8811 - va
Epoch 7/50
50000/50000 [=====] - 6s 123us/step - loss: 0.3181 - acc: 0.8822 - va
Epoch 8/50
50000/50000 [=====] - 7s 146us/step - loss: 0.3093 - acc: 0.8867 - va
Epoch 9/50
50000/50000 [=====] - 7s 137us/step - loss: 0.3039 - acc: 0.8888 - va
Epoch 10/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2971 - acc: 0.8907 - va
Epoch 11/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2915 - acc: 0.8918 - va
Epoch 12/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2822 - acc: 0.8961 - va
Epoch 13/50
50000/50000 [=====] - 6s 125us/step - loss: 0.2770 - acc: 0.8986 - va
Epoch 14/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2715 - acc: 0.9007 - va
Epoch 15/50
50000/50000 [=====] - 6s 130us/step - loss: 0.2650 - acc: 0.9024 - va
Epoch 16/50
50000/50000 [=====] - 7s 142us/step - loss: 0.2637 - acc: 0.9019 - va
Epoch 17/50
50000/50000 [=====] - 6s 127us/step - loss: 0.2604 - acc: 0.9042 - va
Epoch 18/50
50000/50000 [=====] - 6s 125us/step - loss: 0.2516 - acc: 0.9061 - va
Epoch 19/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2465 - acc: 0.9081 - va
Epoch 20/50
50000/50000 [=====] - 6s 125us/step - loss: 0.2487 - acc: 0.9094 - va
Epoch 21/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2429 - acc: 0.9114 - va
Epoch 22/50
50000/50000 [=====] - 6s 124us/step - loss: 0.2365 - acc: 0.9117 - va
Epoch 23/50
50000/50000 [=====] - 6s 125us/step - loss: 0.2337 - acc: 0.9131 - va
Epoch 24/50
```

50000/50000 [=====] - 6s 125us/step - loss: 0.2320 - acc: 0.9148 - va  
 Epoch 25/50  
 50000/50000 [=====] - 6s 125us/step - loss: 0.2263 - acc: 0.9161 - va  
 Epoch 26/50  
 50000/50000 [=====] - 7s 130us/step - loss: 0.2233 - acc: 0.9170 - va  
 Epoch 27/50  
 50000/50000 [=====] - 7s 134us/step - loss: 0.2219 - acc: 0.9172 - va  
 Epoch 28/50  
 50000/50000 [=====] - 7s 138us/step - loss: 0.2147 - acc: 0.9193 - va  
 Epoch 29/50  
 50000/50000 [=====] - 7s 136us/step - loss: 0.2117 - acc: 0.9203 - va  
 Epoch 30/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.2098 - acc: 0.9214 - va  
 Epoch 31/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.2083 - acc: 0.9232 - va  
 Epoch 32/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.2064 - acc: 0.9228 - va  
 Epoch 33/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.2036 - acc: 0.9241 - va  
 Epoch 34/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.1972 - acc: 0.9263 - va  
 Epoch 35/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1998 - acc: 0.9255 - va  
 Epoch 36/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1959 - acc: 0.9279 - va  
 Epoch 37/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.1960 - acc: 0.9276 - va  
 Epoch 38/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1920 - acc: 0.9293 - va  
 Epoch 39/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1904 - acc: 0.9289 - va  
 Epoch 40/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.1912 - acc: 0.9279 - va  
 Epoch 41/50  
 50000/50000 [=====] - 7s 141us/step - loss: 0.1898 - acc: 0.9303 - va  
 Epoch 42/50  
 50000/50000 [=====] - 7s 133us/step - loss: 0.1853 - acc: 0.9308 - va  
 Epoch 43/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.1845 - acc: 0.9298 - va  
 Epoch 44/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1856 - acc: 0.9309 - va  
 Epoch 45/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1855 - acc: 0.9309 - va  
 Epoch 46/50  
 50000/50000 [=====] - 6s 123us/step - loss: 0.1783 - acc: 0.9331 - va  
 Epoch 47/50  
 50000/50000 [=====] - 6s 124us/step - loss: 0.1839 - acc: 0.9319 - va  
 Epoch 48/50



```

50000/50000 [=====] - 6s 123us/step - loss: 0.1758 - acc: 0.9341 - va
Epoch 49/50
50000/50000 [=====] - 6s 125us/step - loss: 0.1748 - acc: 0.9338 - va
Epoch 50/50
50000/50000 [=====] - 6s 124us/step - loss: 0.1723 - acc: 0.9359 - va

```

```

In [45]: print('Batch normalized model 1 test score:',
              model_batch1.evaluate(X_test, y_test))

```

```

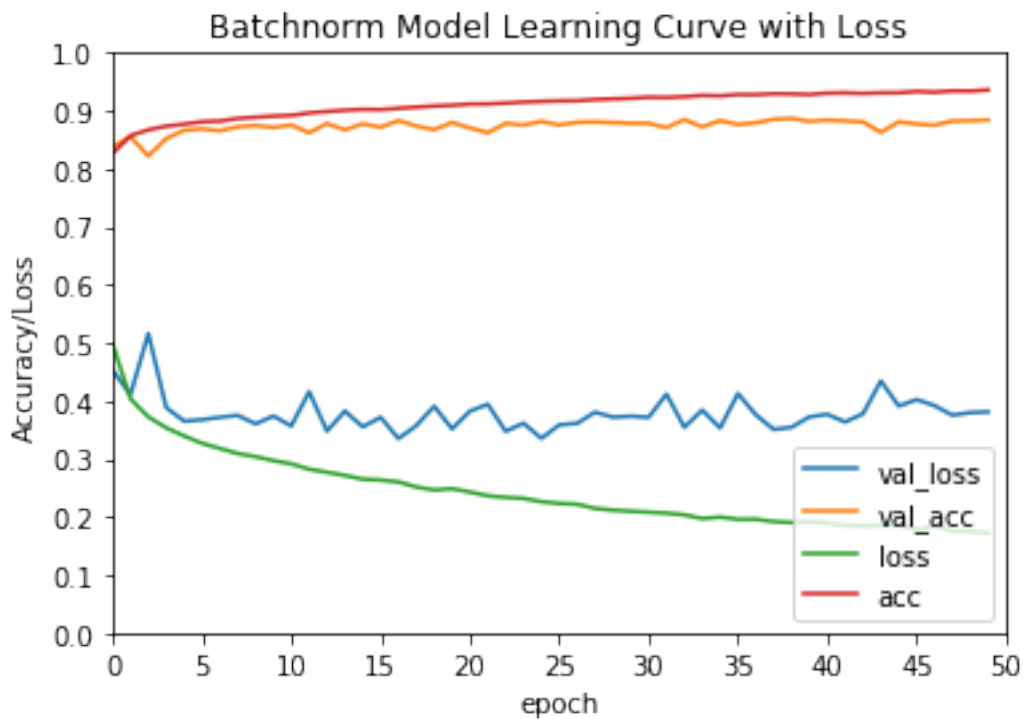
10000/10000 [=====] - 0s 45us/step
Batch normalized model 1 test score: [0.3997763850092888, 0.8806]

```

```

In [46]: _ = pd.DataFrame(batch1.history).plot(
          title = 'Batchnorm Model Learning Curve with Loss',
          xticks = range(0,51,5),
          yticks = [0.1* x for x in range(0,11)]
        )
_ = plt.legend(loc = 4)
_ = plt.xlabel('epoch')
_ = plt.ylabel('Accuracy/Loss')

```



## Larger and deeper dropout batch norm. and residual connection model

```
In [47]: # 6 layer model
num_class = 10

# define input layer
inputs = Input(shape=(28,28,1))

# flatten
flat = Flatten()(inputs)

# 1st Dense layer
L1_1 = Dense(512, activation='relu')(flat)
L1_2 = BatchNormalization()(L1_1)

# 2nd layer
L2_1 = Dense(512, activation='relu')(L1_2)
L2_2 = BatchNormalization()(L2_1)

# 3rd layer
L3_1 = Dense(512, activation='relu')(L2_2)
L3_2 = BatchNormalization()(L3_1)

# skip1
skip1 = add([L1_2, L3_2])

# 4th layer
L4_1 = Dense(512, activation='relu')(skip1)
L4_2 = BatchNormalization()(L4_1)

# 5th layer
L5_1 = Dense(512, activation='relu')(L4_2)
L5_2 = BatchNormalization()(L5_1)

# skip2
skip2 = add([skip1, L5_2])

# 6th layer
L6_1 = Dense(512, activation='relu')(skip2)
L6_2 = BatchNormalization()(L6_1)

# output layer
dense = Dense(num_class, activation='softmax')(L6_2)

# compile
model_batch2 = Model(inputs = inputs, outputs = dense)
model_batch2.compile(optimizer = 'adam',
                    loss = 'sparse_categorical_crossentropy',
```

```

        metrics = ['accuracy'])
model_batch2.summary()

```

Layer (type)	Output Shape	Param #	Connected to
input_1 (InputLayer)	(None, 28, 28, 1)	0	
flatten_6 (Flatten)	(None, 784)	0	input_1[0][0]
dense_21 (Dense)	(None, 512)	401920	flatten_6[0][0]
batch_normalization_2 (BatchNor	(None, 512)	2048	dense_21[0][0]
dense_22 (Dense)	(None, 512)	262656	batch_normalization_2[0][0]
batch_normalization_3 (BatchNor	(None, 512)	2048	dense_22[0][0]
dense_23 (Dense)	(None, 512)	262656	batch_normalization_3[0][0]
batch_normalization_4 (BatchNor	(None, 512)	2048	dense_23[0][0]
add_1 (Add)	(None, 512)	0	batch_normalization_2[0][0] batch_normalization_4[0][0]
dense_24 (Dense)	(None, 512)	262656	add_1[0][0]
batch_normalization_5 (BatchNor	(None, 512)	2048	dense_24[0][0]
dense_25 (Dense)	(None, 512)	262656	batch_normalization_5[0][0]
batch_normalization_6 (BatchNor	(None, 512)	2048	dense_25[0][0]
add_2 (Add)	(None, 512)	0	add_1[0][0] batch_normalization_6[0][0]
dense_26 (Dense)	(None, 512)	262656	add_2[0][0]
batch_normalization_7 (BatchNor	(None, 512)	2048	dense_26[0][0]
dense_27 (Dense)	(None, 10)	5130	batch_normalization_7[0][0]
Total params: 1,732,618			
Trainable params: 1,726,474			
Non-trainable params: 6,144			

```
In [49]: X_train_ = X_train.reshape(X_train.shape[0], 28, 28, 1)
```

```

batch2 = model_batch2.fit(X_train_,
                           y_train,
                           epochs=50,
                           validation_split=10000/60000)

```

Train on 50000 samples, validate on 10000 samples

```

Epoch 1/50
50000/50000 [=====] - 20s 401us/step - loss: 0.5506 - acc: 0.8056 - va
Epoch 2/50
50000/50000 [=====] - 18s 366us/step - loss: 0.4273 - acc: 0.8462 - va
Epoch 3/50
50000/50000 [=====] - 19s 379us/step - loss: 0.3911 - acc: 0.8565 - va
Epoch 4/50
50000/50000 [=====] - 19s 386us/step - loss: 0.3614 - acc: 0.8686 - va
Epoch 5/50
50000/50000 [=====] - 19s 371us/step - loss: 0.3401 - acc: 0.8759 - va
Epoch 6/50
50000/50000 [=====] - 18s 364us/step - loss: 0.3203 - acc: 0.8821 - va
Epoch 7/50
50000/50000 [=====] - 18s 364us/step - loss: 0.3081 - acc: 0.8872 - va
Epoch 8/50
50000/50000 [=====] - 18s 368us/step - loss: 0.2918 - acc: 0.8905 - va
Epoch 9/50
50000/50000 [=====] - 19s 387us/step - loss: 0.2831 - acc: 0.8946 - va
Epoch 10/50
50000/50000 [=====] - 18s 363us/step - loss: 0.2708 - acc: 0.8992 - va
Epoch 11/50
50000/50000 [=====] - 18s 363us/step - loss: 0.2619 - acc: 0.9020 - va
Epoch 12/50
50000/50000 [=====] - 18s 365us/step - loss: 0.2472 - acc: 0.9075 - va
Epoch 13/50
50000/50000 [=====] - 20s 406us/step - loss: 0.2409 - acc: 0.9093 - va
Epoch 14/50
50000/50000 [=====] - 19s 378us/step - loss: 0.2339 - acc: 0.9127 - va
Epoch 15/50
50000/50000 [=====] - 18s 366us/step - loss: 0.2265 - acc: 0.9151 - va
Epoch 16/50
50000/50000 [=====] - 18s 362us/step - loss: 0.2188 - acc: 0.9164 - va
Epoch 17/50
50000/50000 [=====] - 19s 388us/step - loss: 0.2128 - acc: 0.9187 - va
Epoch 18/50
50000/50000 [=====] - 18s 366us/step - loss: 0.2064 - acc: 0.9202 - va
Epoch 19/50
50000/50000 [=====] - 19s 379us/step - loss: 0.1995 - acc: 0.9225 - va
Epoch 20/50
50000/50000 [=====] - 18s 367us/step - loss: 0.1909 - acc: 0.9280 - va
Epoch 21/50
50000/50000 [=====] - 19s 373us/step - loss: 0.1879 - acc: 0.9276 - va

```

```

Epoch 22/50
50000/50000 [=====] - 19s 384us/step - loss: 0.1804 - acc: 0.9302 - va
Epoch 23/50
50000/50000 [=====] - 18s 365us/step - loss: 0.1736 - acc: 0.9335 - va
Epoch 24/50
50000/50000 [=====] - 18s 367us/step - loss: 0.1692 - acc: 0.9345 - va
Epoch 25/50
50000/50000 [=====] - 18s 366us/step - loss: 0.1658 - acc: 0.9371 - va
Epoch 26/50
50000/50000 [=====] - 20s 390us/step - loss: 0.1613 - acc: 0.9383 - va
Epoch 27/50
50000/50000 [=====] - 18s 368us/step - loss: 0.1568 - acc: 0.9396 - va
Epoch 28/50
50000/50000 [=====] - 18s 366us/step - loss: 0.1513 - acc: 0.9411 - va
Epoch 29/50
50000/50000 [=====] - 18s 366us/step - loss: 0.1450 - acc: 0.9439 - va
Epoch 30/50
50000/50000 [=====] - 20s 402us/step - loss: 0.1397 - acc: 0.9456 - va
Epoch 31/50
50000/50000 [=====] - 18s 365us/step - loss: 0.1401 - acc: 0.9471 - va
Epoch 32/50
50000/50000 [=====] - 18s 369us/step - loss: 0.1362 - acc: 0.9474 - va
Epoch 33/50
50000/50000 [=====] - 18s 365us/step - loss: 0.1322 - acc: 0.9494 - va
Epoch 34/50
50000/50000 [=====] - 19s 378us/step - loss: 0.1272 - acc: 0.9510 - va
Epoch 35/50
50000/50000 [=====] - 19s 388us/step - loss: 0.1251 - acc: 0.9523 - va
Epoch 36/50
50000/50000 [=====] - 18s 370us/step - loss: 0.1200 - acc: 0.9539 - va
Epoch 37/50
50000/50000 [=====] - 18s 367us/step - loss: 0.1203 - acc: 0.9534 - va
Epoch 38/50
50000/50000 [=====] - 18s 367us/step - loss: 0.1145 - acc: 0.9553 - va
Epoch 39/50
50000/50000 [=====] - 20s 391us/step - loss: 0.1118 - acc: 0.9562 - va
Epoch 40/50
50000/50000 [=====] - 18s 364us/step - loss: 0.1065 - acc: 0.9589 - va
Epoch 41/50
50000/50000 [=====] - 18s 363us/step - loss: 0.1065 - acc: 0.9583 - va
Epoch 42/50
50000/50000 [=====] - 18s 366us/step - loss: 0.1062 - acc: 0.9593 - va
Epoch 43/50
50000/50000 [=====] - 20s 390us/step - loss: 0.1022 - acc: 0.9607 - va
Epoch 44/50
50000/50000 [=====] - 18s 365us/step - loss: 0.1026 - acc: 0.9606 - va
Epoch 45/50
50000/50000 [=====] - 18s 366us/step - loss: 0.0975 - acc: 0.9622 - va

```

```

Epoch 46/50
50000/50000 [=====] - 18s 364us/step - loss: 0.0945 - acc: 0.9631 - va
Epoch 47/50
50000/50000 [=====] - 21s 414us/step - loss: 0.0935 - acc: 0.9641 - va
Epoch 48/50
50000/50000 [=====] - 19s 374us/step - loss: 0.0889 - acc: 0.9661 - va
Epoch 49/50
50000/50000 [=====] - 18s 365us/step - loss: 0.0861 - acc: 0.9674 - va
Epoch 50/50
50000/50000 [=====] - 18s 366us/step - loss: 0.0883 - acc: 0.9662 - va

```

```

In [50]: X_test_ = X_test.reshape(X_test.shape[0], 28, 28, 1)
         print('Batch normalized model 2 test score:',
               model_batch2.evaluate(X_test_, y_test))

```

```

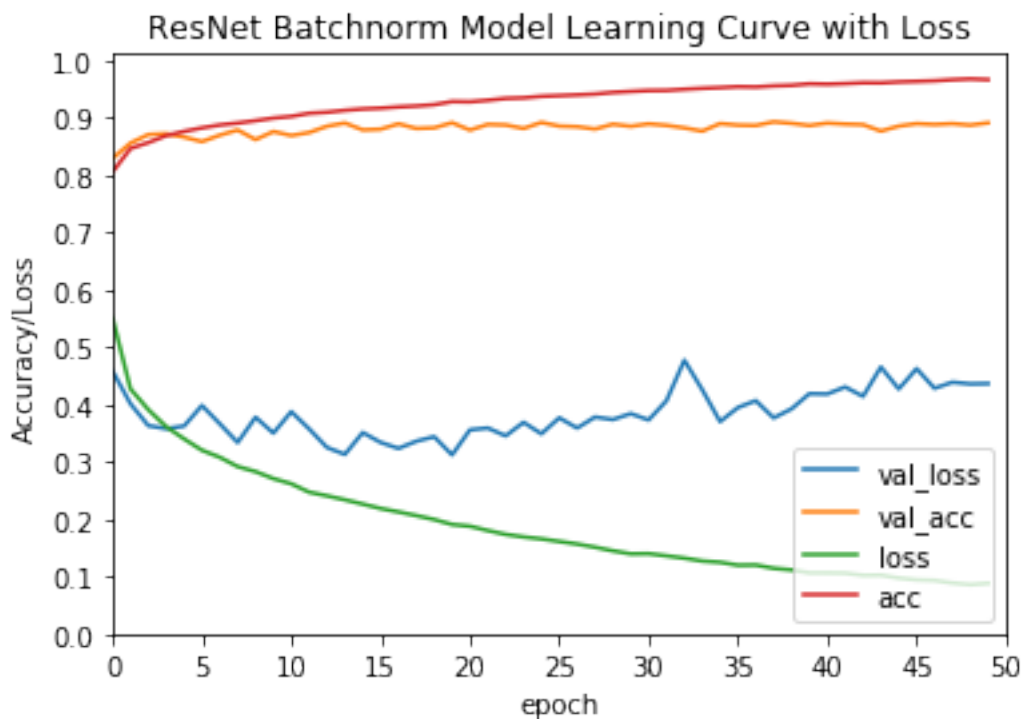
10000/10000 [=====] - 1s 76us/step
Batch normalized model 2 test score: [0.4340573483712971, 0.8855]

```

```

In [51]: _ = pd.DataFrame(batch2.history).plot(
         title = 'ResNet Batchnorm Model Learning Curve with Loss',
         xticks = range(0,51,5),
         yticks = [0.1* x for x in range(0,11)]
         )
         _ = plt.legend(loc = 4)
         _ = plt.xlabel('epoch')
         _ = plt.ylabel('Accuracy/Loss')

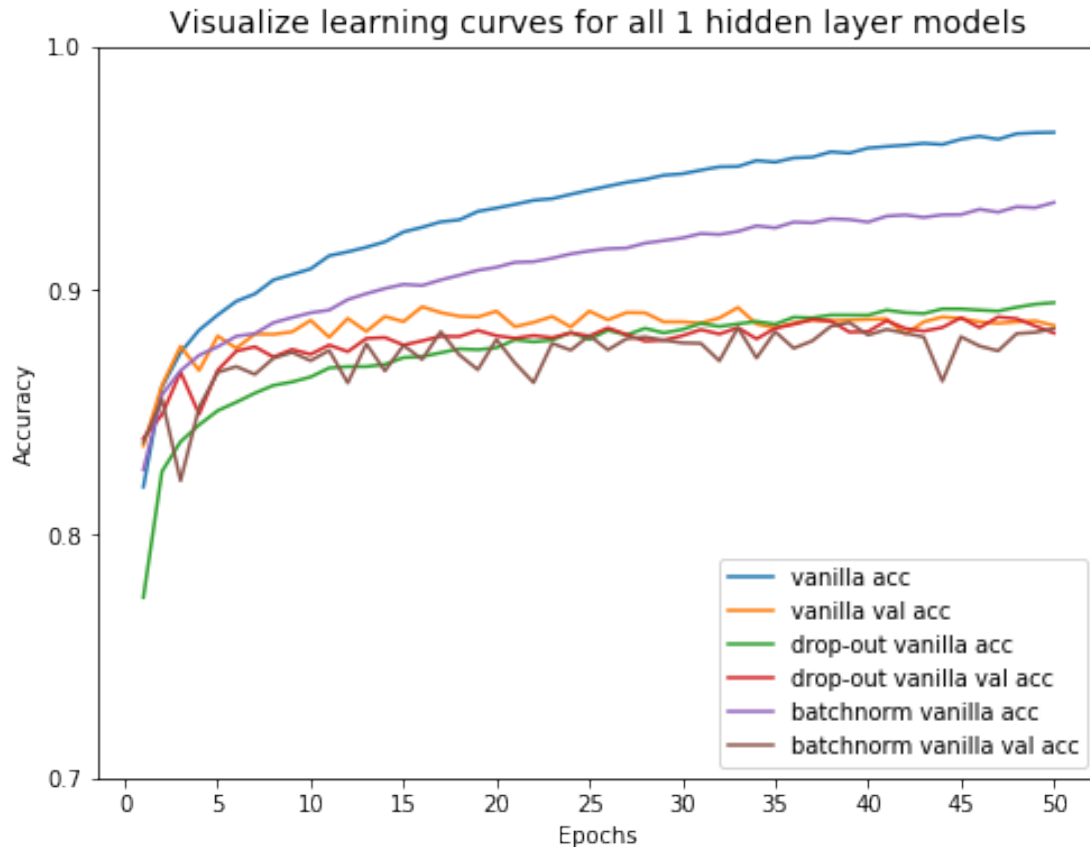
```



## 6 Visualization

```
In [61]: plt.figure(figsize=(8, 6))
_ = sns.lineplot(y = vanilla1.history['acc'],
                 x = range(1,51), label='vanilla acc')
_ = sns.lineplot(y = vanilla1.history['val_acc'],
                 x = range(1,51), label='vanilla val acc')
_ = sns.lineplot(y = dropout1.history['acc'],
                 x = range(1,51), label='drop-out vanilla acc')
_ = sns.lineplot(y = dropout1.history['val_acc'],
                 x = range(1,51), label='drop-out vanilla val acc')
_ = sns.lineplot(y = batch1.history['acc'],
                 x = range(1,51), label='batchnorm vanilla acc')
_ = sns.lineplot(y = batch1.history['val_acc'],
                 x = range(1,51), label='batchnorm vanilla val acc')

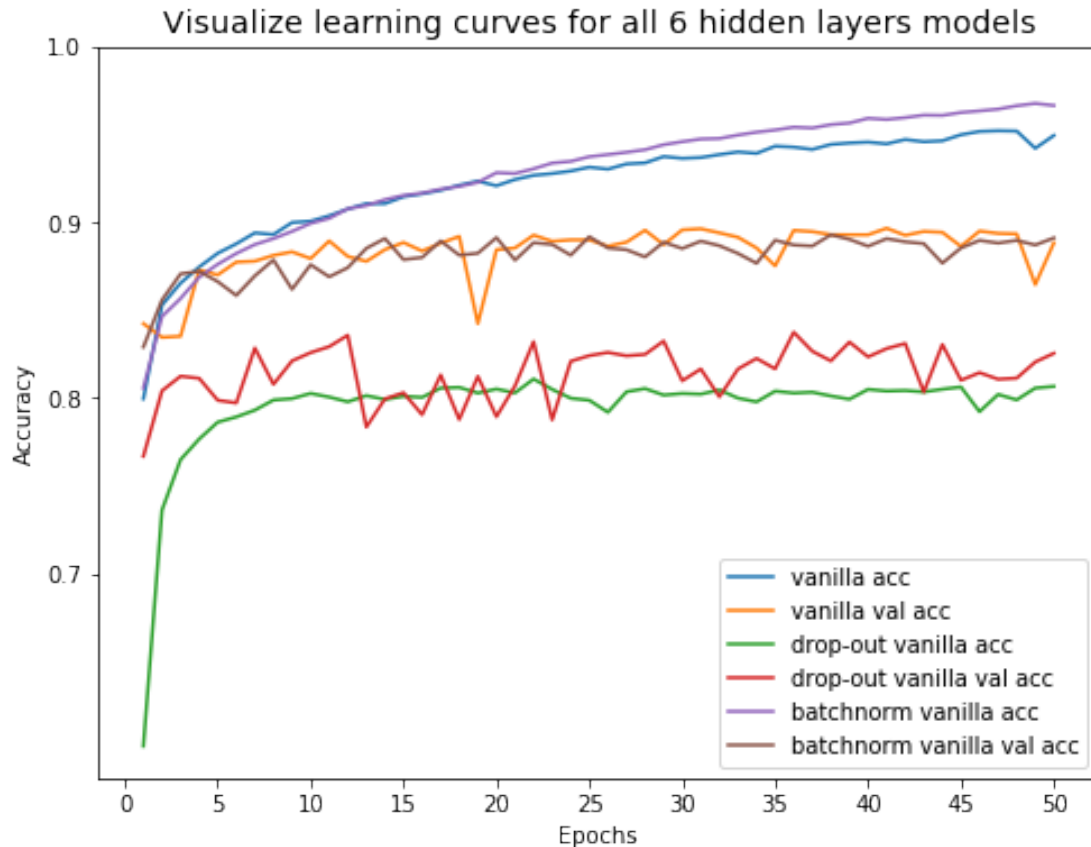
_ = plt.title('Visualize learning curves for all 1 hidden layer models',size=14)
_ = plt.xlabel('Epochs')
_ = plt.ylabel('Accuracy')
_ = plt.yticks([x/10 for x in range(7,11)])
_ = plt.xticks(range(0,51,5))
```



```
In [64]: plt.figure(figsize=(8, 6))
_ = sns.lineplot(y = vanilla2.history['acc'],
                 x = range(1,51), label='vanilla acc')
_ = sns.lineplot(y = vanilla2.history['val_acc'],
                 x = range(1,51), label='vanilla val acc')
_ = sns.lineplot(y = dropout2.history['acc'],
                 x = range(1,51), label='drop-out vanilla acc')
_ = sns.lineplot(y = dropout2.history['val_acc'],
                 x = range(1,51), label='drop-out vanilla val acc')
_ = sns.lineplot(y = batch2.history['acc'],
                 x = range(1,51), label='batchnorm vanilla acc')
_ = sns.lineplot(y = batch2.history['val_acc'],
                 x = range(1,51), label='batchnorm vanilla val acc')

_ = plt.title('Visualize learning curves for all 6 hidden layers models',size=14)
_ = plt.xlabel('Epochs')
_ = plt.ylabel('Accuracy')
_ = plt.yticks([x/10 for x in range(7,11)])
_ = plt.xticks(range(0,51,5))
```





## 7 Summary

In this task, we tested six models: a shallow (1 hidden layer) and a deeper (6 hidden layers) model for three model types:

1. Base model
2. Model with dropout
3. Model with batch normalization and residual connections (without dropout)

Our most successful model, in terms of validation set accuracy, was not surprisingly the deeper model with six hidden layers and batch normalization (without dropout). It approached the mid-to-upper 80s in accuracy. The vanilla model was nearly identical in validation set accuracy, and slightly below the training set accuracy of the batch normalization model.

Drop out did not seem to be an effective strategy, as accuracy scores were considerably below those of the base model and the batch normalization model. It should be noted, however, that it was very effective at reducing the degree of overfitting, as the training and validation scores were nearly identical.