Lecture 4

Outline:

1) ΛCDM parameters & the spherical spectral density C_ℓ^{TT} for the CMB temperature map $\{T(\hat{n}) : \hat{n} \in S^2\}$

2) Accoustic Peaks & silk damping.

The CMB temperature C_ℓ^{TT}

Under standard models of physics the temperature on the surface of last scattering $\{T(x) : x \in \mathbb{R}^3\}$ is an isotropic GRF with euclidean spectral density $C_{|k|}^{TT,\mathbb{R}^3}$.

We observe (with noise & foreground contamination) $T(\hat{n})$, $\hat{n} \in S^2$, which is therefore a isotropic GRF on S^2 with spherical spectral density

$$C_\ell^{TT} = \frac{2}{\pi} \int_0^\infty j_\ell^2(r)\, C_r^{TT,\mathbb{R}^3}\, r^2 dr$$

where the exact form of C_ℓ^{TT} can be derived under different models of physics.

The standard model of Big bang cosmology is call LCDM (Λ cold dark matter)

a cosmological constant associated with dark energy

The simplest LCDM model has 6 varying parameters

Approx Planck estimates

$\Omega_c h^2$: dark matter density.
0.118 ± 0.002

$\Omega_b h^2$: baryon density.
0.0220 ± 0.0002

τ: Thompson optical depth.
0.09 ± 0.02

θ_s: angular scale of the sound horizon
where $100\,\theta_s$ is estimated 1.0413 ± 0.0006

A_s: where $10^9 A_s e^{-2\tau}$ is estimated 1.87 ± 0.02

n_s: 0.968 ± 0.006

Where A_s & n_s parameterize the $\mathbb{R}^3$ spectral density which seeded energy density $\rho(x)$ fluctuations (before recombination).

$$\rho(x) = \bar{\rho} + \delta\rho(x)$$

$$\delta(x) := \frac{\delta\rho(x)}{\bar{\rho}}$$

often denoted $P(|k|) \propto \Delta_R^2(|k|)$ with $k_* = 0.05\ \mathrm{Mpc}^{-1}$

$$C_{|k|}^{\delta\delta} \propto A_s |k|^{n_s - 4} \propto \frac{1}{|k|^3} \underbrace{\left[A_s \left(\frac{|k|}{k_*} \right)^{n_s - 1} \right]}$$

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$T(\hat{n})$ observations constrain LCDM parameters

- C_ℓ^{TT} varies with LCDM parameters.
- C_ℓ^{TT} also characterizes the angular auto covariance function of $T(\hat{n}_1)$

$$\mathrm{cov}\big(T(\hat{n}_1), T(\hat{n}_2)\big) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_\ell(\hat{n}_1 \cdot \hat{n}_2) C_\ell^{ff}$$

Since the fdd of T are Gaussian one can immediately write down the log likelihood of the LCDM parameter vector $\theta = (\Omega_c h^2, \Omega_b h^2, \ldots, n_s)$ given observations of the form

$$d(\hat{n}) = T(\hat{n}) + \underbrace{\varepsilon(\hat{n})}_{\text{Noise and contaminants.}}$$

So long as one has a model for N one can estimate θ.

e.g. Let $\vec{d} = \underbrace{\begin{pmatrix} T(\hat{n}_1) \\ \vdots \\ T(\hat{n}_m) \end{pmatrix}}_{:= \vec{T}} + \underbrace{\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_m \end{pmatrix}}_{:= \vec{\varepsilon}}$, m pixels

Simplified assumption $\rightarrow$ where $\vec{\varepsilon} \sim N(0, \Sigma^{\varepsilon\varepsilon})$ &

$\vec{T} \sim N(0, \Sigma^{TT}(\theta))$ ← depends on θ.

$$\Sigma^{TT}_{ij}(\theta) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_\ell(\hat{n}_i \cdot \hat{n}_j) C_\ell^{TT}$$

$$\therefore \ \mathrm{loglike}(\theta | \vec{d}) = -\frac{1}{2} \vec{d}^{\,T} \left(\Sigma^{TT}(\theta) + \Sigma^{\varepsilon\varepsilon}\right)^{-1} \vec{d} - \frac{1}{2} \log\det\left(2\pi \left(\Sigma^{TT}(\theta) + \Sigma^{\varepsilon\varepsilon}\right)\right)$$

Difficulty #1: $\Sigma^{TT}(\theta) + \Sigma^{\varepsilon\varepsilon}$ is a huge matrix ... something like 10 mill $\times$ 10 mill for Planck.

Difficulty #2: $\vec{\varepsilon}$ is usually highly Non-Gaussian.

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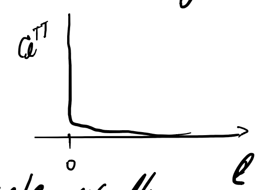
Qualitative description of C_ℓ^{TT} & how C_ℓ^{TT} depends on LCDM parameters

- The fluctuations of T are very small and very smooth.
  $\therefore$ C_ℓ^{TT} looks like



to see the tails, people usually plot on log scale or/and plot $\frac{\ell(\ell+1)C_\ell^{TT}}{2\pi}$.

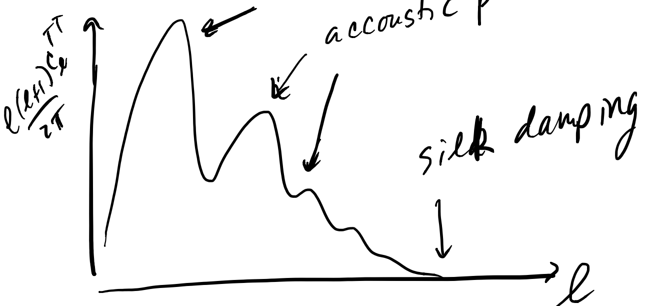
- Note: C_ℓ^{TT} is actually computed by including the integrated "line-of-sight" effects on the CMB photons after it is released from the surface of last scattering.

- Show ijulia Notebook which plots different C_ℓ^{TT} as one marginally varies the LCDM parameters.

- Show WMAP & Planck estimates of C_ℓ^{TT}.

Acoustic Peaks & Silk damping

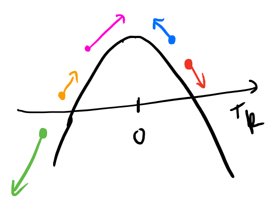
2 features of C_ℓ^{TT}:



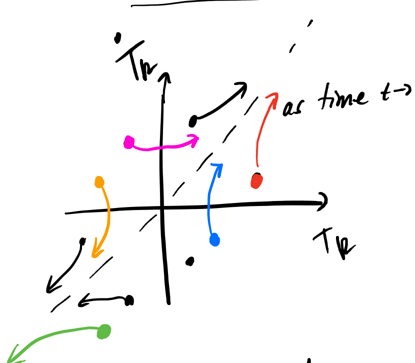
Acoustic peaks: From what I understand these follow from the conversion from an expansionary regeme of evolution to a gravity-pressure dominated regeme.

Fix wavenumber k_0 & consider $\{T_k : |k| = k_0\}$

Initial $\{T_k : |k| = k_0\}$

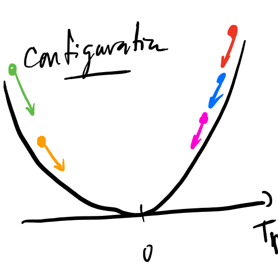


Phase plot of $\{T_k : |k| = k_0\}$

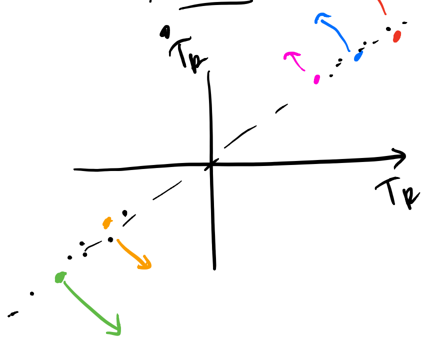


when gravity-pressure regeme kicks in one has a harmonic occilator

Configuration

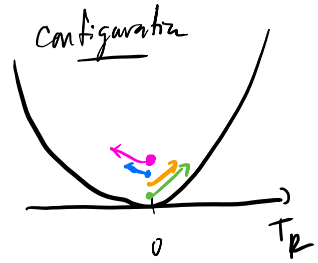


Phase plot

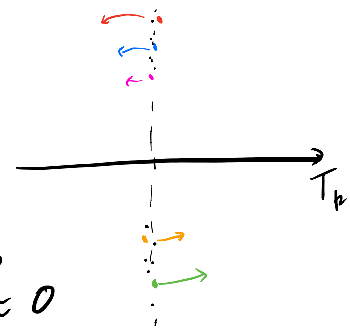


At recombination some wavenumbers look like

Configuration



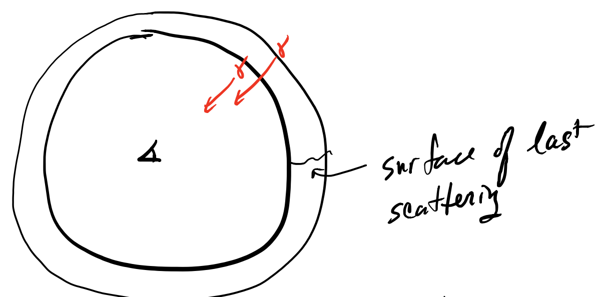
Phase plot



At this k_0, $C_{k_0}^{TT, R^3} \approx 0$

Silk damping

Since recombination didn't happen instantaniously the surface of last scattering has some depth



∴ we are observing a "average" temp T along the ban which smooths wiggles & acts like diffusion smoothing (i.e. attimating C_ℓ^{TT} at high wavenumber)