Implementing a GARCH Model on Apple Stock

Authors: Preston O'Connor, Anthony Yasan, Matthew Jacob, Khoa Dao, Nick Wierzbowski

Date: 5/2/2025

Introduction

Our time series model predicts volatility in Apple's stock price, and we specifically used a GARCH (generalized autoregressive conditional heteroskedasticity) model to do so. Our data is from Yahoo finance covering the time frame from the start of 2019 to the end of 2024, which obviously includes some pretty high volatility periods such as COVID 19 and the 2022 Bull Market.

The packages used include quantmod for financial modelling, lmtest for diagnostics, dplyr for data manipulation, PerformanceAnalytics for more econometric tools, tidyverse for visualization, xts, feasts, fable, forecast and tseries for time series based calculations, rugarch for GARCH models, lubridate for working with dates, and gridextra for visualization.

Our model seems to predict the volatility fairly well despite the extenuating circumstances in the chosen time period.

Data Description

Data Source

The stock data for Apple Inc. (AAPL) used in this analysis was obtained from Yahoo Finance. The dataset includes daily trading information from January 1, 2019 to December 31, 2024. Key variables include the opening price, high and low prices of the day, closing price, trading volume, and adjusted closing price. This information was accessed using the `getSymbols()` function from the `quantmod` package in R, allowing for efficient retrieval and preprocessing of financial time series data.

- open: Opening price of the Stock
- High: Highest price of the stock during the day
- low: Lowest price of the stock during the day
- close: Closing price of the stock (needed for the GARCH model)
- volume: Number of Shares traded
- Adjusted: Stock's closing price corrected for dividends, splits, and other corporate actions to reflect true value over time

Clean Stock Data

```
colSums(is.na(AAPL))
```

```
    AAPL.Open      AAPL.High       AAPL.Low     AAPL.Close    AAPL.Volume
            0              0              0              0              0
AAPL.Adjusted
            0
```

Calculate Returns and Transform into Log values

```
data <- cbind(Price = AAPL$AAPL.Close,
 ↪  Return=CalculateReturns(AAPL$AAPL.Close,
method = 'log'))
colnames(data) <- c('Price','Return')
head(data)
```

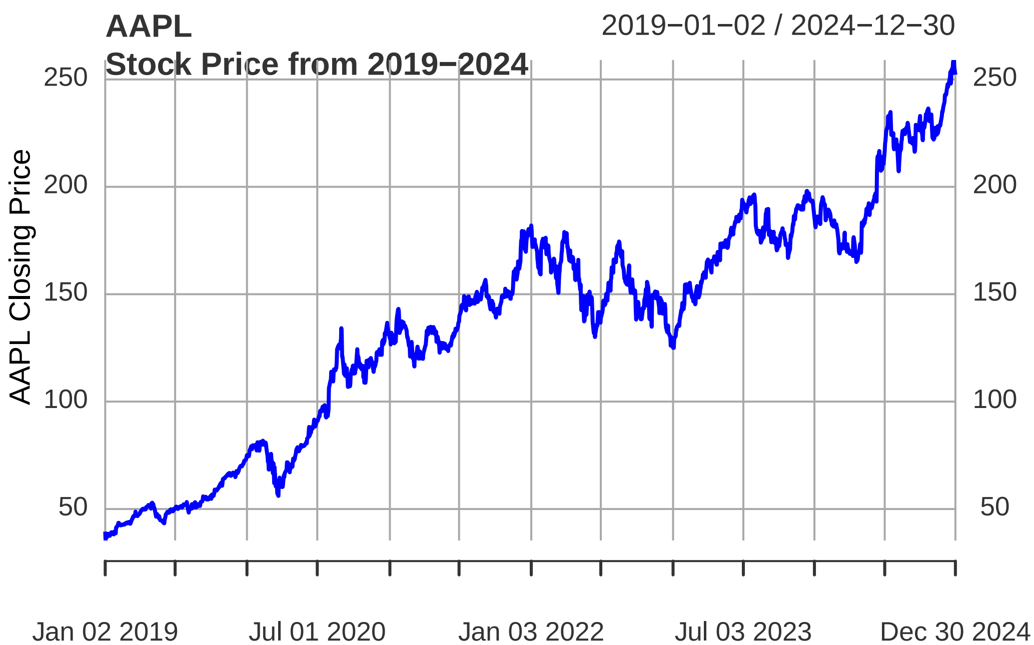
```
             Price       Return
2019-01-02 39.4800           NA
2019-01-03 35.5475 -0.104924356
2019-01-04 37.0650  0.041803244
2019-01-07 36.9825 -0.002228313
2019-01-08 37.6875  0.018883696
2019-01-09 38.3275  0.016839164
```

- We need to take the log returns because of the nature of stock prices being non-stationary, while returns are usually stationary, which is a requirement for GARCH modeling. Log returns also allow changes over time to be additive, making analysis and forecasting simpler; furthermore, log returns better capture volatility patterns and make percent changes much easier to interpret.

- above are the small day-to-day percentage changes in Apple's stock Price, shown in the log scale/
  - Positive number: price went up that day
  - Negative Number: Price went down that day

AAPL Price over Time

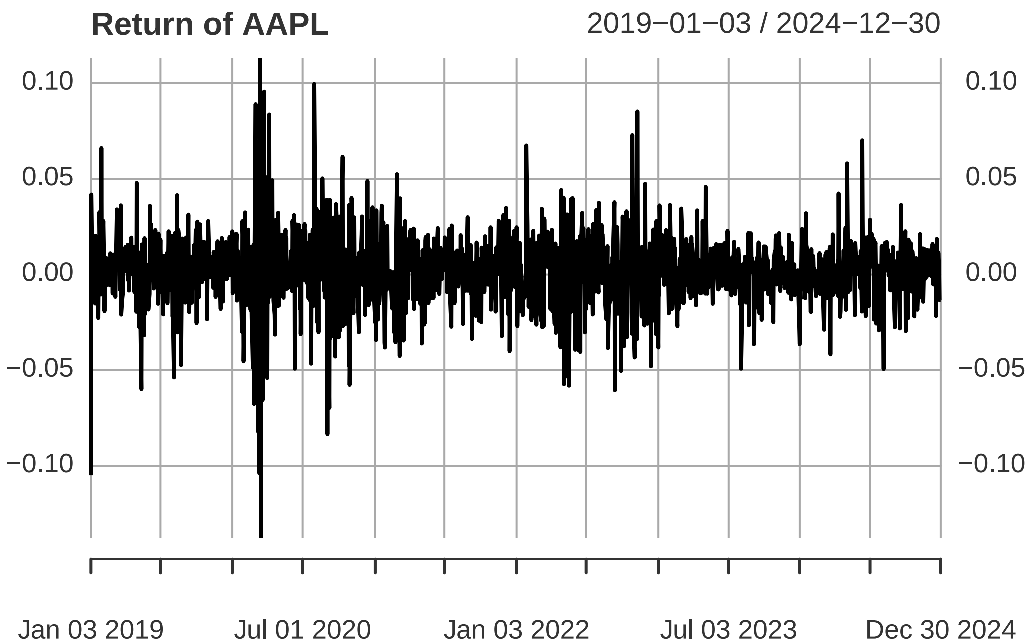
```
plot(na.omit(data$Price), ylab='AAPL Closing Price',main='AAPL
Stock Price from 2019-2024',col='blue')
```



- Overall, an Upward trend Which quintupled from $40 to $250 by the end of 2024
- Several noticeable dips in the stocks and large recoveries, particularly during 2022
- Accelerated gain period during 2023 to 2024, indicating bullish market activity

AAPL Log Return

```
plot(na.omit(data$Return),main='Return of AAPL')
```



- High volatility in early 2020 (Covid-19 Market Shock), mid 2022 and slightly in 2024
- Our returns tend to cluster around the 0, but we can see there are frequent spikes above 5% and below -5% showcasing that the market is experiencing some turbulence

ADF Stationary Test

```r
adf.test(na.omit(data$Price))
```

```

	Augmented Dickey-Fuller Test

data:  na.omit(data$Price)
Dickey-Fuller = -2.593, Lag order = 11, p-value = 0.3273
alternative hypothesis: stationary
```

```r
adf.test(na.omit(data$Return))
```

```
Warning in adf.test(na.omit(data$Return)): p-value smaller than printed p-value
```

```
	Augmented Dickey-Fuller Test

data:  na.omit(data$Return)
Dickey-Fuller = -10.841, Lag order = 11, p-value = 0.01
alternative hypothesis: stationary
```

- Price series: The ADF test shows a high p-value (0.3273), meaning the price data is not stationary ; it has a trend or changing variance over time.
- Return series: The ADF test gives a very low statistic (–10.841), indicating the returns are stationary ; they fluctuate around a stable mean, making them suitable for modeling.

Analysis

Train and Test Set Split (80/20)

```r
train_size <- floor(0.8 * nrow(na.omit(data)))
train_data <- na.omit(data)[1:train_size,]
test_data <- na.omit(data)[(train_size+1):nrow(na.omit(data)),]

cat("Training set size:", nrow(train_data), "\n")
```

```
Training set size: 1206
```

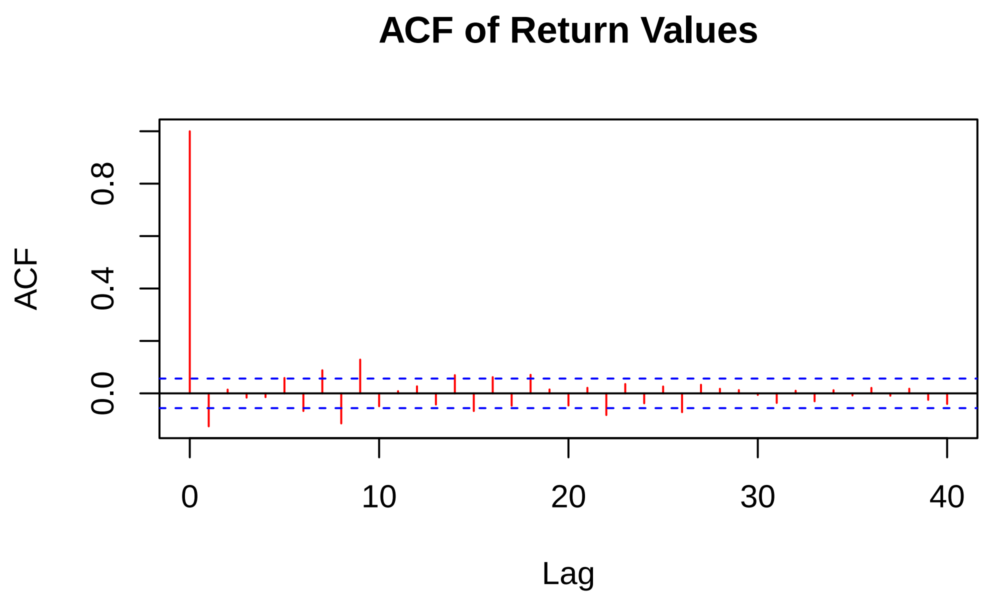
```r
cat("Test set size:", nrow(test_data), "\n")
```

```
Test set size: 302
```

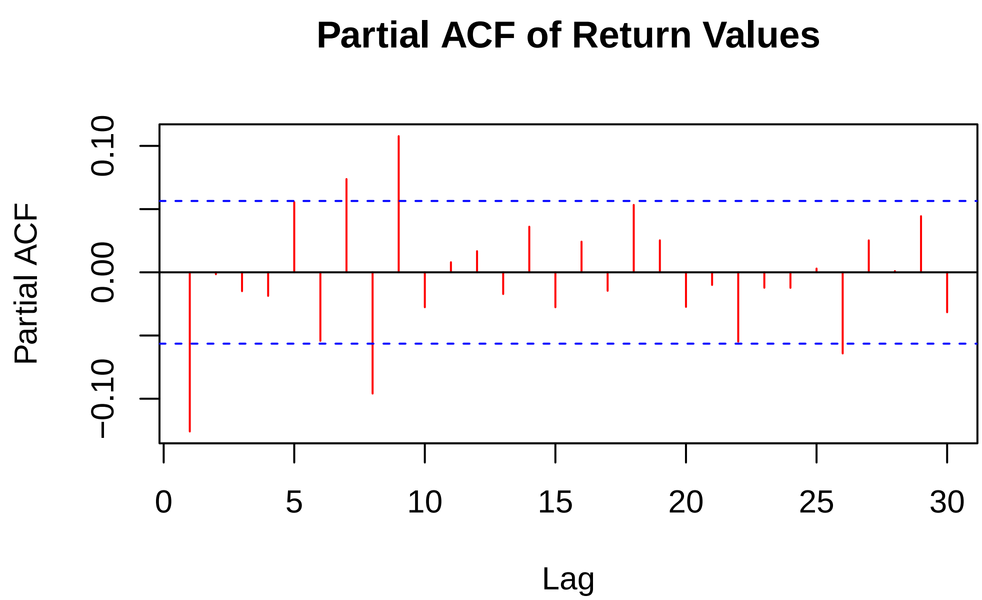
- unlike our typical models we take the first 80% of the data for training and try to see how well we can predict the last 20% of the data

ACF/PACF plots

```r
acf(na.omit(train_data$Return), lag.max = 40, main='ACF of Return Values',
↪  col='red')
```



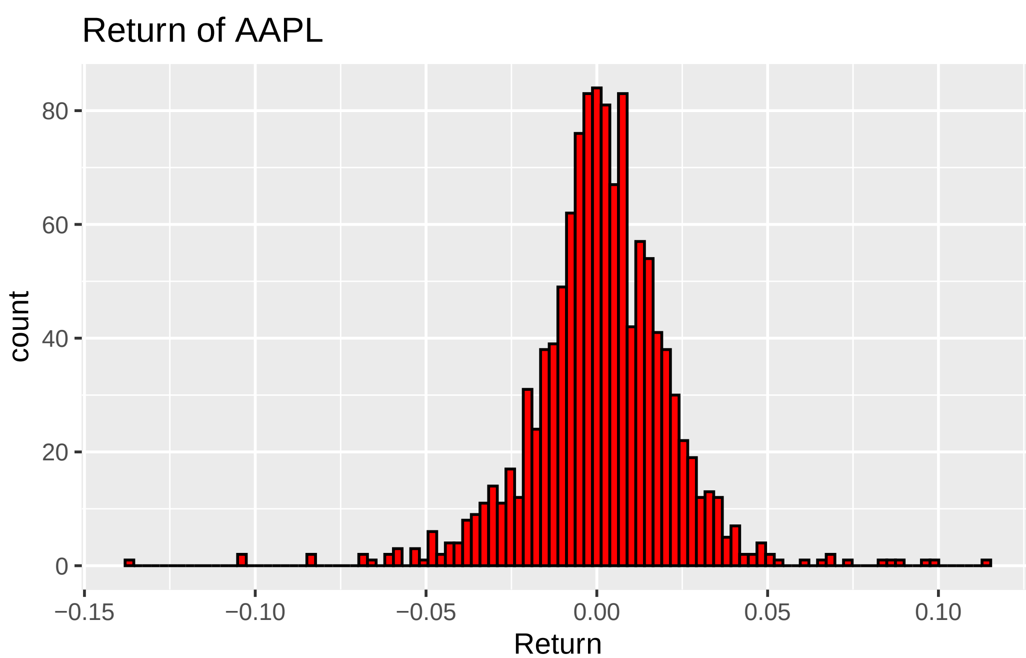

```
pacf(na.omit(train_data$Return), main='Partial ACF of Return Values',
↪  col='red')
```



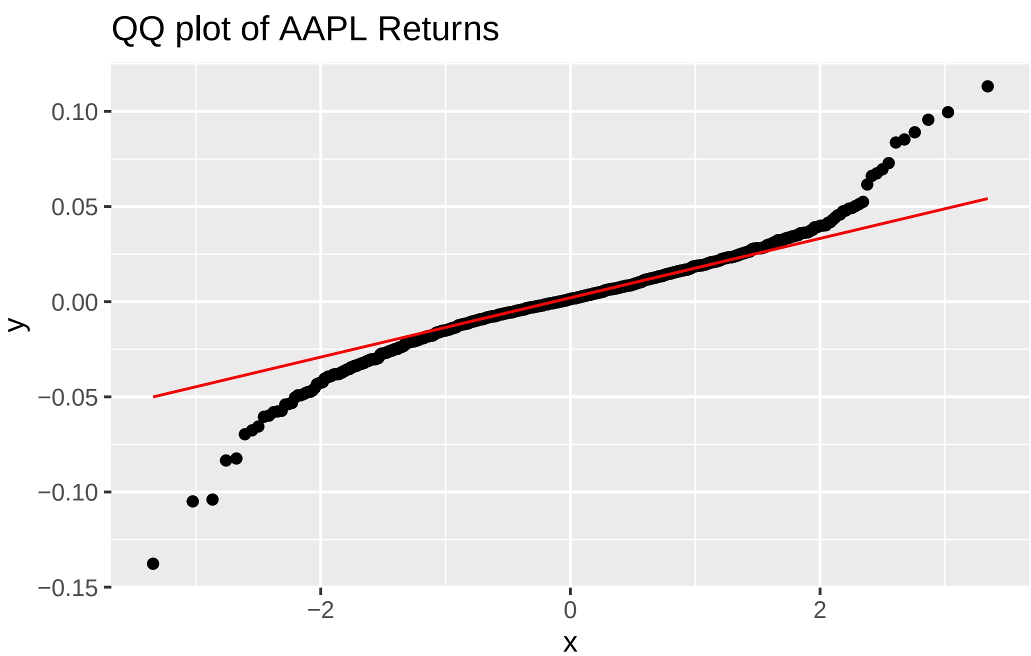
- ACF of Returns
  - Our return series does not exhibit strong autocorrelation; this is typical for financial returns
  - Supports the idea that returns are weakly dependent over time
  - Volatility may still may be auto correlated, garch model may still be useful
- Partial ACF: how much each lag contributes to returns
  - most of our bars are staying within the blue lines; so, returns have very little direct dependency at specific lags (previous day returns)
  - a few small spikes, Lag 9, may hint at minor short term structure, but nothing we can leverage for a clear AR model

Histogram and QQ plot

```
ggplot(aes(Return), data=as.data.frame(train_data)) +
  geom_histogram(bins=100, col='black', fill='red') +
  ggtitle('Return of AAPL')
```



```r
ggplot(data=as.data.frame(train_data), aes(sample = Return)) +
  stat_qq() +
  stat_qq_line(col='red') +
  ggtitle('QQ plot of AAPL Returns')
```



- Histogram: Most of the returns are very close to 0, meaning small day-to-day changes are common(somewhat expected). There are fewer returns in the far left and right, showing rare but large losses or gains in the market. The shape is roughly bell-shaped but with a few outliers, suggesting a perfectly normal distribution
- QQ Plot: The points follow the red line in the middle fairly closely, so we have a fairly normal distribution in the center. The points bend away at the ends, showing fat tails, showcasing more extreme returns than a normal distribution would expect. this means our Apple stock has higher risk of big moves (for increase/decrease) than a normal model predicts

Normality test

```r
jarque.bera.test(na.omit(train_data$Return))
```

```
    Jarque Bera Test

data:  na.omit(train_data$Return)
X-squared = 1381.6, df = 2, p-value < 2.2e-16
```

- An extremely small p-value, we reject normality
    - AAPL returns do not follow a normal distribution

Box-Ljung test

```r
Box.test(na.omit(train_data$Return), type="Ljung-Box")
```

```
    Box-Ljung test

data:  na.omit(train_data$Return)
X-squared = 19.172, df = 1, p-value = 1.194e-05
```

- Due to the very small value, we reject the idea of no autocorrelation
    - AAPL shows signs of autocorrelation which is useful for the time series model

Model Evaluation and Prediction

- Implementing multiple Garche models into our set up. the reason for the multiple GARCHE models is to check for different orders that might better capture the dynamics of the mean returns, We can also leverage some combinations model volatility clustering and also a mdel lets us compare for the best performance set up

```r
# Model 1: ARMA(0,0) - eGARCH(1,1) - Student t
AAPL_garch_1 <- ugarchspec(
  mean.model = list(armaOrder = c(0, 0)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_1 <- ugarchfit(spec = AAPL_garch_1, data =
 ↪  na.omit(train_data$Return))
fit_garch_1
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001654    0.000527   3.1406 0.001686
omega  -0.334085    0.049383  -6.7652 0.000000
alpha1 -0.085173    0.024406  -3.4899 0.000483
beta1   0.958542    0.006213 154.2753 0.000000
gamma1  0.196792    0.035032   5.6175 0.000000
shape   6.385595    1.127530   5.6633 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001654    0.000614   2.6933 0.007075
omega  -0.334085    0.038608  -8.6533 0.000000
alpha1 -0.085173    0.028150  -3.0257 0.002481
beta1   0.958542    0.004932 194.3684 0.000000
gamma1  0.196792    0.043950   4.4776 0.000008
shape   6.385595    1.002002   6.3728 0.000000

LogLikelihood : 3143.318

Information Criteria
------------------------------------

Akaike       -5.2028
Bayes        -5.1775
Shibata      -5.2029
Hannan-Quinn -5.1933

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
```

```
Lag[1]                        1.389  0.2387
Lag[2*(p+q)+(p+q)-1][2]       2.007  0.2616
Lag[4*(p+q)+(p+q)-1][5]       3.300  0.3551
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.001514  0.9690
Lag[2*(p+q)+(p+q)-1][5]  0.690405  0.9247
Lag[4*(p+q)+(p+q)-1][9]  1.723692  0.9353
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2740 0.500 2.000  0.6006
ARCH Lag[5]    0.5192 1.440 1.667  0.8779
ARCH Lag[7]    1.2206 2.315 1.543  0.8752

Nyblom stability test
------------------------------------
Joint Statistic:  2.1443
Individual Statistics:
mu     0.7227
omega  0.2344
alpha1 0.2043
beta1  0.2419
gamma1 0.2192
shape  0.3416

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.49 1.68 2.12
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.9284 0.3534
Negative Sign Bias  0.1822 0.8554
Positive Sign Bias  0.6403 0.5221
Joint Effect        0.9580 0.8114
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     8.295       0.9834
2    30    14.249       0.9900
3    40    19.207       0.9967
4    50    30.401       0.9830


Elapsed time : 0.124007
```

```r
# Model 2: ARMA(1,1) - eGARCH(1,1) - Student t
AAPL_garch_2 <- ugarchspec(
  mean.model = list(armaOrder = c(1, 1)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_2 <- ugarchfit(spec = AAPL_garch_2, data =
  ↪  na.omit(train_data$Return))
fit_garch_2
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001814    0.000406   4.4642  8.0e-06
ar1     0.485102    0.081308   5.9663  0.0e+00
ma1    -0.537429    0.079323  -6.7752  0.0e+00
omega  -0.329248    0.045883  -7.1758  0.0e+00
alpha1 -0.076723    0.022581  -3.3977  6.8e-04
```

```
beta1   0.959205    0.005669 169.2135  0.0e+00
gamma1  0.197004    0.034637   5.6877  0.0e+00
shape   6.350924    1.123153   5.6545  0.0e+00

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001814    0.000430   4.2153 0.000025
ar1     0.485102    0.018090  26.8155 0.000000
ma1    -0.537429    0.021206 -25.3433 0.000000
omega  -0.329248    0.028356 -11.6112 0.000000
alpha1 -0.076723    0.024692  -3.1072 0.001888
beta1   0.959205    0.003395 282.5709 0.000000
gamma1  0.197004    0.041462   4.7515 0.000002
shape   6.350924    0.989677   6.4172 0.000000

LogLikelihood : 3145.574

Information Criteria
------------------------------------

Akaike       -5.2033
Bayes        -5.1695
Shibata      -5.2034
Hannan-Quinn -5.1905

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1574  0.6916
Lag[2*(p+q)+(p+q)-1][5]    0.9176  1.0000
Lag[4*(p+q)+(p+q)-1][9]    2.7544  0.9243
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.005052  0.9433
Lag[2*(p+q)+(p+q)-1][5]  0.731682  0.9168
Lag[4*(p+q)+(p+q)-1][9]  1.811683  0.9263
d.o.f=2

Weighted ARCH LM Tests
```

```
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2781 0.500 2.000  0.5979
ARCH Lag[5]    0.5369 1.440 1.667  0.8727
ARCH Lag[7]    1.3617 2.315 1.543  0.8486

Nyblom stability test
------------------------------------
Joint Statistic:  2.7681
Individual Statistics:
mu     0.8818
ar1    0.3284
ma1    0.2944
omega  0.2178
alpha1 0.1913
beta1  0.2245
gamma1 0.1945
shape  0.3956

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.70894 0.4785
Negative Sign Bias 0.02849 0.9773
Positive Sign Bias 0.38772 0.6983
Joint Effect       0.63208 0.8890


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     14.07       0.7798
2    30     18.48       0.9340
3    40     30.48       0.8335
4    50     49.72       0.4444


Elapsed time : 0.1589301
```

```r
# Model 3: ARMA(2,2) - eGARCH(1,1) - Student t
AAPL_garch_3 <- ugarchspec(
  mean.model = list(armaOrder = c(2, 2)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_3 <- ugarchfit(spec = AAPL_garch_3, data =
 ↪  na.omit(train_data$Return))
fit_garch_3
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(2,0,2)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001800    0.000362   4.9688 0.000001
ar1     0.762548    0.035372  21.5580 0.000000
ar2    -0.505034    0.197822  -2.5530 0.010681
ma1    -0.795867    0.035930 -22.1504 0.000000
ma2     0.478691    0.194429   2.4620 0.013815
omega  -0.314399    0.044024  -7.1416 0.000000
alpha1 -0.079762    0.022536  -3.5394 0.000401
beta1   0.960978    0.005432 176.9184 0.000000
gamma1  0.199010    0.034952   5.6938 0.000000
shape   6.447827    1.144301   5.6347 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001800    0.000339   5.3057 0.000000
ar1     0.762548    0.012672  60.1751 0.000000
ar2    -0.505034    0.168214  -3.0023 0.002679
ma1    -0.795867    0.008637 -92.1511 0.000000
ma2     0.478691    0.166693   2.8717 0.004083
```

```
omega  -0.314399    0.026438 -11.8921 0.000000
alpha1 -0.079762    0.025182  -3.1674 0.001538
beta1   0.960978    0.003111 308.9346 0.000000
gamma1  0.199010    0.041006   4.8532 0.000001
shape   6.447827    0.966671   6.6701 0.000000

LogLikelihood : 3147.537

Information Criteria
------------------------------------

Akaike       -5.2032
Bayes        -5.1610
Shibata      -5.2033
Hannan-Quinn -5.1873

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.1374  0.7109
Lag[2*(p+q)+(p+q)-1][11]    3.2472  1.0000
Lag[4*(p+q)+(p+q)-1][19]    7.9950  0.7950
d.o.f=4
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1067  0.7439
Lag[2*(p+q)+(p+q)-1][5]    0.7012  0.9227
Lag[4*(p+q)+(p+q)-1][9]    1.7128  0.9364
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1034 0.500 2.000  0.7478
ARCH Lag[5]    0.3168 1.440 1.667  0.9358
ARCH Lag[7]    1.1582 2.315 1.543  0.8865

Nyblom stability test
------------------------------------
Joint Statistic:  2.9411
```

```
Individual Statistics:
mu     0.88891
ar1    0.39482
ar2    0.05272
ma1    0.37741
ma2    0.04454
omega  0.21197
alpha1 0.29881
beta1  0.21933
gamma1 0.25240
shape  0.28481

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.29 2.54 3.05
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.1775 0.2392
Negative Sign Bias  0.1201 0.9044
Positive Sign Bias  0.6620 0.5081
Joint Effect        1.6408 0.6502


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     12.71       0.8533
2    30     18.73       0.9281
3    40     32.01       0.7787
4    50     44.00       0.6756


Elapsed time : 0.214525
```

```r
# Model 4: ARMA(1,2) - eGARCH(1,1) - Student t
AAPL_garch_4 <- ugarchspec(
  mean.model = list(armaOrder = c(1, 2)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
```

```r
fit_garch_4 <- ugarchfit(spec = AAPL_garch_4, data =
 ↪  na.omit(train_data$Return))
fit_garch_4
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(1,0,2)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001824    0.000385   4.7409 0.000002
ar1     0.329707    0.038142   8.6443 0.000000
ma1    -0.377530    0.037806  -9.9860 0.000000
ma2    -0.025717    0.014456  -1.7790 0.075234
omega  -0.320645    0.045894  -6.9866 0.000000
alpha1 -0.079771    0.022419  -3.5582 0.000373
beta1   0.960198    0.005667 169.4411 0.000000
gamma1  0.201353    0.035351   5.6958 0.000000
shape   6.459419    1.149408   5.6198 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001824    0.000393   4.6465 0.000003
ar1     0.329707    0.010907  30.2286 0.000000
ma1    -0.377530    0.011876 -31.7893 0.000000
ma2    -0.025717    0.007844  -3.2787 0.001043
omega  -0.320645    0.027268 -11.7591 0.000000
alpha1 -0.079771    0.024307  -3.2817 0.001032
beta1   0.960198    0.003244 295.9620 0.000000
gamma1  0.201353    0.041273   4.8786 0.000001
shape   6.459419    0.976039   6.6180 0.000000

LogLikelihood : 3147.231
```

```
Information Criteria
------------------------------------

Akaike       -5.2044
Bayes        -5.1663
Shibata      -5.2045
Hannan-Quinn -5.1900

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                     0.01661  0.8974
Lag[2*(p+q)+(p+q)-1][8]    1.86158  1.0000
Lag[4*(p+q)+(p+q)-1][14]   5.34158  0.8552
d.o.f=3
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.04131  0.8389
Lag[2*(p+q)+(p+q)-1][5]   0.58046  0.9446
Lag[4*(p+q)+(p+q)-1][9]   1.66414  0.9411
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.09631 0.500 2.000  0.7563
ARCH Lag[5]   0.32793 1.440 1.667  0.9327
ARCH Lag[7]   1.25273 2.315 1.543  0.8693

Nyblom stability test
------------------------------------
Joint Statistic:  2.9361
Individual Statistics:
mu     0.95278
ar1    0.47354
ma1    0.44437
ma2    0.05368
omega  0.21988
alpha1 0.28081
```

```
beta1  0.22825
gamma1 0.22747
shape  0.29101

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.1 2.32 2.82
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.64246 0.5207
Negative Sign Bias 0.08319 0.9337
Positive Sign Bias 0.33692 0.7362
Joint Effect       0.62779 0.8900


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     7.201       0.9931
2    30    11.910       0.9979
3    40    26.371       0.9387
4    50    26.255       0.9968


Elapsed time : 0.1779392
```

```r
# Model 5: ARMA(2,1) - eGARCH(1,1) - Student t
AAPL_garch_5 <- ugarchspec(
  mean.model = list(armaOrder = c(2, 1)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_5 <- ugarchfit(spec = AAPL_garch_5, data =
↪  na.omit(train_data$Return))
fit_garch_5
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*
```

```
Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(2,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001811    0.000402   4.5014 0.000007
ar1     0.339641    0.186459   1.8215 0.068527
ar2    -0.031966    0.029397  -1.0874 0.276861
ma1    -0.378026    0.182076  -2.0762 0.037875
omega  -0.319841    0.045574  -7.0180 0.000000
alpha1 -0.080356    0.022643  -3.5489 0.000387
beta1   0.960295    0.005630 170.5799 0.000000
gamma1  0.200961    0.035320   5.6898 0.000000
shape   6.459747    1.149482   5.6197 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001811    0.000420   4.3080 0.000016
ar1     0.339641    0.072230   4.7022 0.000003
ar2    -0.031966    0.026930  -1.1870 0.235228
ma1    -0.378026    0.077498  -4.8779 0.000001
omega  -0.319841    0.027128 -11.7901 0.000000
alpha1 -0.080356    0.025114  -3.1997 0.001376
beta1   0.960295    0.003227 297.5523 0.000000
gamma1  0.200961    0.041016   4.8996 0.000001
shape   6.459747    0.978929   6.5988 0.000000

LogLikelihood : 3146.849

Information Criteria
------------------------------------

Akaike       -5.2037
Bayes        -5.1657
Shibata      -5.2038
Hannan-Quinn -5.1894

Weighted Ljung-Box Test on Standardized Residuals
```

```
------------------------------------
                         statistic p-value
Lag[1]                     0.03862  0.8442
Lag[2*(p+q)+(p+q)-1][8]    1.88392  1.0000
Lag[4*(p+q)+(p+q)-1][14]   5.41510  0.8449
d.o.f=3
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1251  0.7235
Lag[2*(p+q)+(p+q)-1][5]    0.6621  0.9300
Lag[4*(p+q)+(p+q)-1][9]    1.7310  0.9346
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.08171 0.500 2.000  0.7750
ARCH Lag[5]   0.30516 1.440 1.667  0.9389
ARCH Lag[7]   1.21279 2.315 1.543  0.8767

Nyblom stability test
------------------------------------
Joint Statistic:  2.8386
Individual Statistics:
mu     0.94821
ar1    0.38392
ar2    0.04908
ma1    0.36192
omega  0.22057
alpha1 0.28755
beta1  0.22899
gamma1 0.23420
shape  0.28426

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.1 2.32 2.82
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
```

```
                   t-value   prob sig
Sign Bias           0.7472 0.4551
Negative Sign Bias  0.1150 0.9085
Positive Sign Bias  0.4343 0.6642
Joint Effect        0.8535 0.8366


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     6.736       0.9955
2    30    14.149       0.9906
3    40    17.085       0.9991
4    50    20.368       0.9999


Elapsed time : 0.1635001
```

```
# Model 6: ARMA(3,1) - eGARCH(1,1) - Student t
AAPL_garch_6 <- ugarchspec(
  mean.model = list(armaOrder = c(3, 1)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_6 <- ugarchfit(spec = AAPL_garch_6, data =
↪  na.omit(train_data$Return))
fit_garch_6
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(3,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
```

```
mu      0.001816    0.000392   4.631925 0.000004
ar1    -0.037259    0.069841  -0.533490 0.593694
ar2    -0.042210    0.023121  -1.825596 0.067911
ar3    -0.040136    0.022874  -1.754618 0.079325
ma1    -0.000626    0.080432  -0.007778 0.993794
omega  -0.316906    0.045463  -6.970612 0.000000
alpha1 -0.078195    0.022472  -3.479733 0.000502
beta1   0.960656    0.005613 171.143066 0.000000
gamma1  0.202002    0.035334   5.716861 0.000000
shape   6.473037    1.155382   5.602510 0.000000

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.001816    0.000402   4.520410 0.000006
ar1    -0.037259    0.045947  -0.810928 0.417407
ar2    -0.042210    0.017257  -2.445878 0.014450
ar3    -0.040136    0.018049  -2.223773 0.026164
ma1    -0.000626    0.033952  -0.018426 0.985299
omega  -0.316906    0.026540 -11.940790 0.000000
alpha1 -0.078195    0.024825  -3.149917 0.001633
beta1   0.960656    0.003151 304.853358 0.000000
gamma1  0.202002    0.041118   4.912792 0.000001
shape   6.473037    0.973139   6.651710 0.000000

LogLikelihood : 3147.498

Information Criteria
------------------------------------

Akaike       -5.2031
Bayes        -5.1609
Shibata      -5.2033
Hannan-Quinn -5.1872

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.0493  0.8243
Lag[2*(p+q)+(p+q)-1][11]    3.1276  1.0000
Lag[4*(p+q)+(p+q)-1][19]    7.7838  0.8232
d.o.f=4
H0 : No serial correlation
```

```
Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1521  0.6966
Lag[2*(p+q)+(p+q)-1][5]    0.6433  0.9335
Lag[4*(p+q)+(p+q)-1][9]    1.6727  0.9403
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.08802 0.500 2.000  0.7667
ARCH Lag[5]   0.33255 1.440 1.667  0.9315
ARCH Lag[7]   1.17808 2.315 1.543  0.8830

Nyblom stability test
------------------------------------
Joint Statistic:  3.3542
Individual Statistics:
mu     0.96962
ar1    0.48529
ar2    0.06233
ar3    0.21197
ma1    0.46898
omega  0.21394
alpha1 0.27712
beta1  0.22229
gamma1 0.24919
shape  0.30442

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.29 2.54 3.05
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.6798 0.4968
Negative Sign Bias  0.2010 0.8407
Positive Sign Bias  0.4200 0.6746
Joint Effect        0.8194 0.8448
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     8.992       0.9736
2    30    18.030       0.9437
3    40    25.907       0.9466
4    50    42.176       0.7441


Elapsed time : 0.186758
```

```r
# Model 7: ARMA(3,2) - eGARCH(1,1) - Student t
AAPL_garch_7 <- ugarchspec(
  mean.model = list(armaOrder = c(3, 2)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_7 <- ugarchfit(spec = AAPL_garch_7, data =
 ↪  na.omit(train_data$Return))
fit_garch_7
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(3,0,2)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001808    0.000462   3.91329 0.000091
ar1     0.492764    0.125068   3.93996 0.000081
ar2    -0.557283    0.249593  -2.23277 0.025564
ar3    -0.039413    0.060605  -0.65033 0.515481
ma1    -0.537422    0.146735  -3.66254 0.000250
ma2     0.541315    0.259871   2.08302 0.037250
omega  -0.312158    0.043828  -7.12237 0.000000
```

```
alpha1 -0.077361    0.020754  -3.72757 0.000193
beta1   0.961243    0.005362 179.25957 0.000000
gamma1  0.199783    0.034637   5.76797 0.000000
shape   6.432412    1.139213   5.64636 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001808    0.001562   1.15730 0.247149
ar1     0.492764    0.111023   4.43838 0.000009
ar2    -0.557283    0.236314  -2.35823 0.018363
ar3    -0.039413    0.170212  -0.23155 0.816886
ma1    -0.537422    0.241924  -2.22145 0.026321
ma2     0.541315    0.250363   2.16213 0.030609
omega  -0.312158    0.025980 -12.01551 0.000000
alpha1 -0.077361    0.034293  -2.25591 0.024076
beta1   0.961243    0.003416 281.43009 0.000000
gamma1  0.199783    0.038898   5.13603 0.000000
shape   6.432412    0.982085   6.54975 0.000000

LogLikelihood : 3148.133

Information Criteria
------------------------------------

Akaike       -5.2025
Bayes        -5.1561
Shibata      -5.2027
Hannan-Quinn -5.1850

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                   9.136e-04  0.9759
Lag[2*(p+q)+(p+q)-1][14] 4.136e+00  1.0000
Lag[4*(p+q)+(p+q)-1][24] 1.043e+01  0.7761
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1242  0.7245
Lag[2*(p+q)+(p+q)-1][5]    0.6611  0.9302
```

```
Lag[4*(p+q)+(p+q)-1][9]    1.6324  0.9440
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1264 0.500 2.000  0.7222
ARCH Lag[5]    0.3567 1.440 1.667  0.9248
ARCH Lag[7]    1.1473 2.315 1.543  0.8885

Nyblom stability test
------------------------------------
Joint Statistic:  3.1431
Individual Statistics:
mu     0.93435
ar1    0.44166
ar2    0.05439
ar3    0.07231
ma1    0.46293
ma2    0.05367
omega  0.20819
alpha1 0.27980
beta1  0.21579
gamma1 0.24979
shape  0.31179

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.49 2.75 3.27
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.6660 0.5056
Negative Sign Bias  0.1470 0.8832
Positive Sign Bias  0.3442 0.7308
Joint Effect        0.7514 0.8611


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     9.788       0.9580
```

```
2    30    20.070        0.8907
3    40    34.929        0.6561
4    50    37.698        0.8800


Elapsed time : 0.2329779
```

```r
# Model 8: ARMA(1,3) - eGARCH(1,1) - Student t
AAPL_garch_8 <- ugarchspec(
  mean.model = list(armaOrder = c(1, 3)),
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  distribution.model = "std"
)
fit_garch_8 <- ugarchfit(spec = AAPL_garch_8, data =
 ↪  na.omit(train_data$Return))
fit_garch_8
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model  : ARFIMA(1,0,3)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.001828    0.000397   4.604102 0.000004
ar1    -0.050087    0.051823  -0.966497 0.333796
ma1     0.003102    0.045600   0.068021 0.945769
ma2    -0.036589    0.026967  -1.356817 0.174839
ma3    -0.036639    0.027237  -1.345182 0.178566
omega  -0.318710    0.045937  -6.937937 0.000000
alpha1 -0.078163    0.022554  -3.465644 0.000529
beta1   0.960439    0.005673 169.310229 0.000000
gamma1  0.202306    0.035484   5.701350 0.000000
shape   6.474666    1.156139   5.600248 0.000000
```

```
Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001828    0.000423   4.32595 0.000015
ar1    -0.050087    0.024307  -2.06057 0.039344
ma1     0.003102    0.005599   0.55402 0.579566
ma2    -0.036589    0.021667  -1.68870 0.091277
ma3    -0.036639    0.023093  -1.58661 0.112602
omega  -0.318710    0.026830 -11.87907 0.000000
alpha1 -0.078163    0.024863  -3.14379 0.001668
beta1   0.960439    0.003201 300.00150 0.000000
gamma1  0.202306    0.041256   4.90365 0.000001
shape   6.474666    0.973343   6.65199 0.000000

LogLikelihood : 3147.693

Information Criteria
------------------------------------

Akaike       -5.2035
Bayes        -5.1612
Shibata      -5.2036
Hannan-Quinn -5.1876

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.007948  0.9290
Lag[2*(p+q)+(p+q)-1][11]  3.116884  1.0000
Lag[4*(p+q)+(p+q)-1][19]  7.692067  0.8348
d.o.f=4
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.06069  0.8054
Lag[2*(p+q)+(p+q)-1][5]   0.56006  0.9480
Lag[4*(p+q)+(p+q)-1][9]   1.60761  0.9463
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
```

```
ARCH Lag[3]    0.1060 0.500 2.000  0.7448
ARCH Lag[5]    0.3609 1.440 1.667  0.9237
ARCH Lag[7]    1.2230 2.315 1.543  0.8748

Nyblom stability test
------------------------------------
Joint Statistic:  3.3928
Individual Statistics:
mu     0.96248
ar1    0.58321
ma1    0.55900
ma2    0.07472
ma3    0.19001
omega  0.21453
alpha1 0.27280
beta1  0.22295
gamma1 0.23762
shape  0.31191

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.29 2.54 3.05
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.5556 0.5786
Negative Sign Bias  0.1815 0.8560
Positive Sign Bias  0.3169 0.7514
Joint Effect        0.5776 0.9015


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     11.74       0.8962
2    30     13.45       0.9938
3    40     21.99       0.9871
4    50     35.21       0.9307


Elapsed time : 0.1911979
```

AIC values

```
Model <- c('fit_garch_1', 'fit_garch_2', 'fit_garch_3', 'fit_garch_4',
           'fit_garch_5', 'fit_garch_6', 'fit_garch_7', 'fit_garch_8')

AIC_values <- c(
  infocriteria(fit_garch_1)[1],
  infocriteria(fit_garch_2)[1],
  infocriteria(fit_garch_3)[1],
  infocriteria(fit_garch_4)[1],
  infocriteria(fit_garch_5)[1],
  infocriteria(fit_garch_6)[1],
  infocriteria(fit_garch_7)[1],
  infocriteria(fit_garch_8)[1]
)

(model_table <- data.frame(Model, AIC_values))
```

```
        Model AIC_values
1 fit_garch_1  -5.202849
2 fit_garch_2  -5.203273
3 fit_garch_3  -5.203213
4 fit_garch_4  -5.204363
5 fit_garch_5  -5.203729
6 fit_garch_6  -5.203148
7 fit_garch_7  -5.202542
8 fit_garch_8  -5.203470
```

```
which.min(model_table$AIC_values)
```

```
[1] 4
```

- chose fit__garch__4 as it is the smallest Return from the model
  - ARMA(1,2) Captures both short-term autoregressive behavior and moving average shocks in the return series. Also accounts for the fat tails with the Student-t errors

```
print(convergence(fit_garch_4))
```

```
[1] 0
```

Forecast Future Returns

```
for_cast1 <- ugarchforecast(fit_garch_4, data = data, n.ahead = 20)
for_cast1
```

```
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: eGARCH
Horizon: 20
Roll Steps: 0
Out of Sample: 0

0-roll forecast [T0=2023-10-17]:
       Series   Sigma
T+1  0.002615 0.01384
T+2  0.002373 0.01398
T+3  0.002005 0.01412
T+4  0.001884 0.01425
T+5  0.001844 0.01437
T+6  0.001831 0.01450
T+7  0.001827 0.01462
T+8  0.001825 0.01473
T+9  0.001825 0.01484
T+10 0.001825 0.01495
T+11 0.001825 0.01506
T+12 0.001824 0.01516
T+13 0.001824 0.01525
T+14 0.001824 0.01535
T+15 0.001824 0.01544
T+16 0.001824 0.01553
T+17 0.001824 0.01561
T+18 0.001824 0.01569
T+19 0.001824 0.01577
T+20 0.001824 0.01585
```

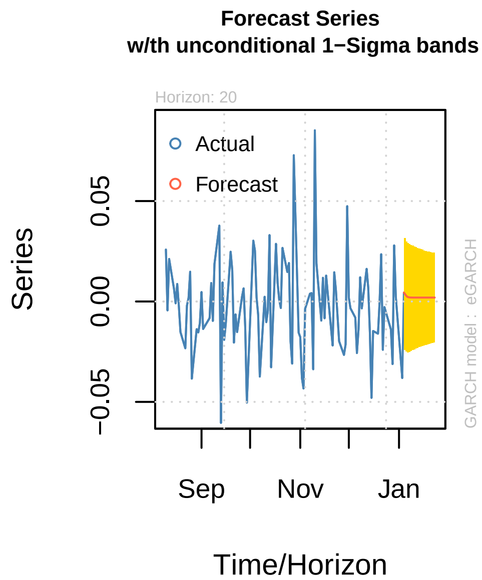
```
# Rolling forecast - fix this line
fit_roll <- ugarchfit(spec = AAPL_garch_4, data = na.omit(data$Return),
↪  out.sample = 500)
```

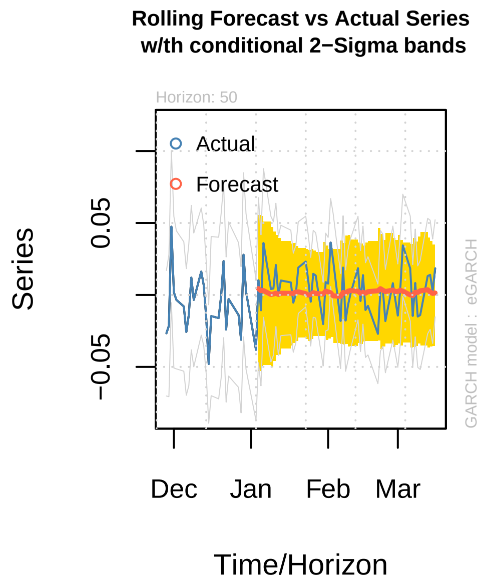
```r
# Generate rolling forecast
fore_roll <- ugarchforecast(fit_roll, n.ahead = 20, n.roll = 50)
fore_roll
```

```
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: eGARCH
Horizon: 20
Roll Steps: 50
Out of Sample: 20

0-roll forecast [T0=2023-01-03]:
       Series   Sigma
T+1  0.004441 0.02868
T+2  0.003360 0.02810
T+3  0.002376 0.02756
T+4  0.002078 0.02705
T+5  0.001988 0.02658
T+6  0.001961 0.02615
T+7  0.001953 0.02573
T+8  0.001950 0.02535
T+9  0.001950 0.02499
T+10 0.001949 0.02465
T+11 0.001949 0.02434
T+12 0.001949 0.02404
T+13 0.001949 0.02376
T+14 0.001949 0.02350
T+15 0.001949 0.02325
T+16 0.001949 0.02302
T+17 0.001949 0.02280
T+18 0.001949 0.02260
T+19 0.001949 0.02240
T+20 0.001949 0.02222
```

```r
# Plot rolling forecast results
par(mfrow = c(1, 2))
plot(fore_roll, which = 1)  # Forecasted mean
plot(fore_roll, which = 2)
```

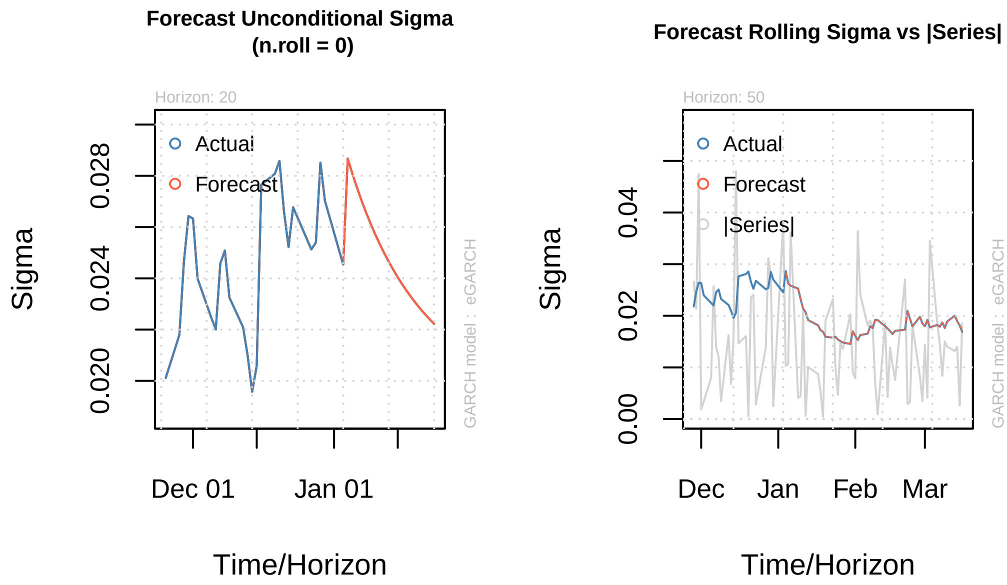




- The model converges well and was the best based on the AIC score
- It does a good job of modeling volatility in AAPL Returns
- The average return it predicts is close to zero, which is typical of the daily stock return
- Yellow band showcases that there is realistic ranges for future changes which is useful for checking our risk

Forecasting

```
par(mfrow=c(1,2))
plot(fore_roll,which=3)
plot(fore_roll,which=4)
```



- The drop in our forecast (red line) starts high and slowly declines, meaning the model expects to have less movement in prices soon
- We see the previous volatility; Expects a calm period after the fluctuation
- Provides us insight on the future market predictions and risk levels for possible future investments

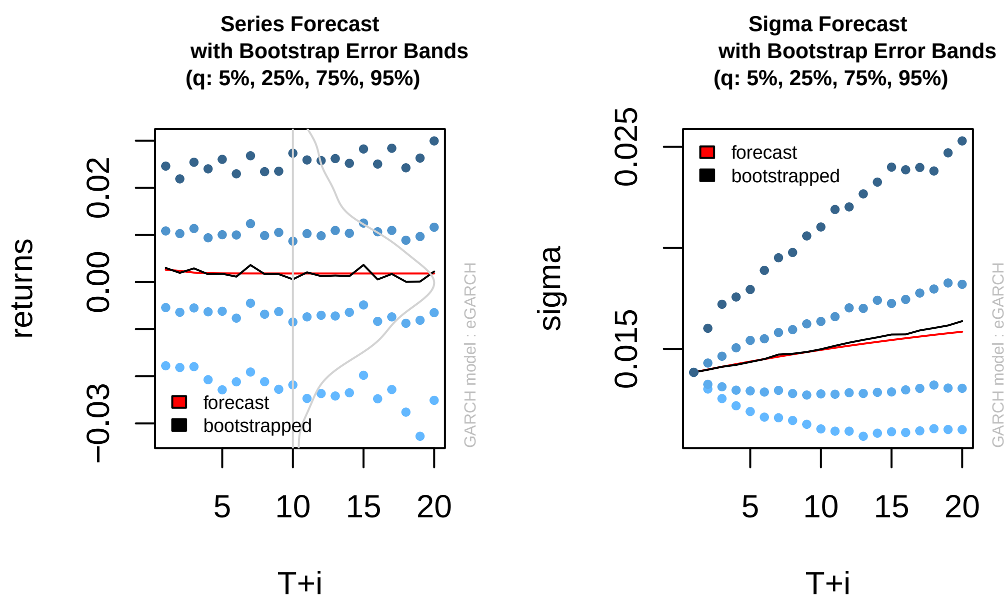
Bootstrap Forecasting

```
par(mfrow=c(1,2))
fore_boot <- ugarchboot(fit_garch_4,data = na.omit(data$Return),
method = c("Partial", "Full")[1], n.ahead = 20, n.bootpred = 500)
plot(fore_boot,which=2)
```

```
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
```

```
plot(fore_boot,which=3)
```

```
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
Warning in title(...): font width unknown for character 0x09 in encoding cp1252
```



- Left Plot: Forecasted Returns with Bootstrap Error Bands
  - Forecasted mean returns (red-line) shows the forecasted mean returns, which are close to zero, showcasing no strong expected trend
  - Uncertainty at different quantiles (dotted blue line) shows the returns could vary widely, even though we have a flat average
  - Range simulated outcomes(shaded grey outline) indicates the model captures random shocks and market noise
- Right Plot: Forecasted Volatility with Bootstrap Bands
  - Red line is the main forecast of future volatility and it shows a gradual linear increase, suggesting rising uncertainty

- Blue dots and black line show bootstrap error bands showing a very wide spread, so the forecasted risk has a lot of variation especially post 10 day

Conclusion & Summary

In this project, we used various GARCH models with different ARMA configurations to model and forecast the daily return volatility of Apple Inc. (AAPL) stock, using data from 2019 to 2024 from Yahoo Finance. After initial data wrangling and processing, we split the dataset into an 80:20 training-to-test ratio. We implemented eight candidate models, all assuming a Student-t distribution to capture the fat-tailed nature of returns. After evaluating their performance using AIC values, the ARMA(1,2)-eGARCH(1,1) model 4 emerged as the best. That model generally did a good job with modeling volatility in stock returns, and it showed some promise as a meaningful predictive model. Overall, our final model was able to capture some of the expected variations of financial returns, and it produced realistic forecasts of future volatility. Nonetheless, the model could be further improved by incorporating other variables to account for industry, market and greater economic changes.

References

- https://finance.yahoo.com/quote/AAPL/history/
- https://rpubs.com/Mahmud_Hasan/778532
- https://iopscience.iop.org/article/10.1088/1757-899X/548/1/012023/pdf