Practice

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- Data = $\{(0,0), (1,1), (1,2), (2,1)\}$

  $f(x; w_0, w_1, w_2) = w_2x^2 + w_1x + w_0$

- Determine w_0, w_1, w_2 to minimize

  - $E(w_0,w_1,w_2) = \sum_{(x,y)\in Data} (y - f(x; w_0,w_1,w_2))^2$

    $= (0-w_0)^2 + (1-(w_2+w_1+w_0))^2 + (2-(w_2+w_1+w_0))^2 + (1-(4w_2+2w_1+w_0))^2$

    $= w_0^2 + 1+w_2^2+w_1^2+w_0^2 -2w_2-2w_1-2w_0 + 2w_1w_2 + 2w_0w_2 + 2w_0w_1$

    $+ 4+w_2^2+w_1^2+w_0^2 - 4w_2-4w_1-4w_0 + 2w_1w_2 + 2w_0w_2 + 2w_0w_1$

    $+ 1+16w_2^2+4w_1^2+w_0^2 - 8w_2-4w_1-2w_0 + 16w_1w_2 + 8w_0w_2 + 4w_0w_1$

    $= 18w_2^2 + 6w_1^2 + 4w_0^2 - 14w_2 - 10w_1 - 8w_0 + 20w_1w_2 + 12w_0w_2 + 8w_0w_1$

  - $\frac{\partial E}{\partial w_0} = 8w_0 - 8 + 12w_2 + 8w_1 = 0$

    $\frac{\partial E}{\partial w_1} = 12w_1 - 10 + 20w_2 + 8w_0 = 0$

    $\frac{\partial E}{\partial w_2} = 36w_2 - 14 + 20w_1 + 12w_0 = 0$

    $\Rightarrow$

    $2w_0 + 2w_1 + 3w_2 = 2 \quad \cdots ①$

    $4w_0 + 6w_1 + 10w_2 = 5 \quad \cdots ②$

    $6w_0 + 10w_1 + 18w_2 = 7 \quad \cdots ③$

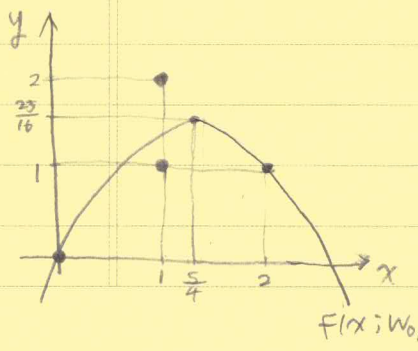
  - ② − 2×① ; $2w_1 + 4w_2 = 1 \quad \cdots ④$

    ③ − 3×① ; $4w_1 + 9w_2 = 1 \quad \cdots ⑤$

    ⑤ − 2×④ ; $w_2 = -1$, $w_1 = \frac{5}{2}$, ① → $2w_0 + 5 - 3 = 2$, $w_0 = 0$

- $f(x; w_0,w_1,w_2) = -x^2 + \frac{5}{2}x$

- Graph



$y = -x^2 + \frac{5}{2}x = -(x^2 - \frac{5}{2}x + \frac{25}{16}) + \frac{25}{16}$

$= -(x - \frac{5}{4})^2 + \frac{25}{16} \quad \rightarrow \quad (\frac{5}{4}, \frac{25}{16})$