

# Aquarium

?? correct • solved at: ???:?? by  
??  
University of ??

Author: **Freek Henstra**

## Overview

- We need to line some piranhas up for a group photo.
- Putting a finger in the tank causes adjacent piranhas to start swimming toward the finger.
- Find the number of seconds of finger exposure needed to put every fish in its specified place.

# Aquarium - Solution

## Techniques

- Intervals
- Greedy algorithms
- Amortisation

## Algorithm

- Each fish needs to move by  $X_i$  places.
  - The number of finger placements to the right must be  $X_i$  more than the number to the left.
- Assume the number of placements to the left of the leftmost fish is  $-X_0$ .
  - Then the next number must be **0**, the next after that must be  $0+X_1$ , then  $0+X_1+X_2$ , etcetera.
  - We need all values to be non-negative. Take the minimum of value and subtract it from everything.
- Now we need to check if this number of placements is feasible
  - The answer is guaranteed to be at most  $O(N*K^2)$ .
  - So we can just go from left to right making all valid moves.

# B — Big Brother

## Problem

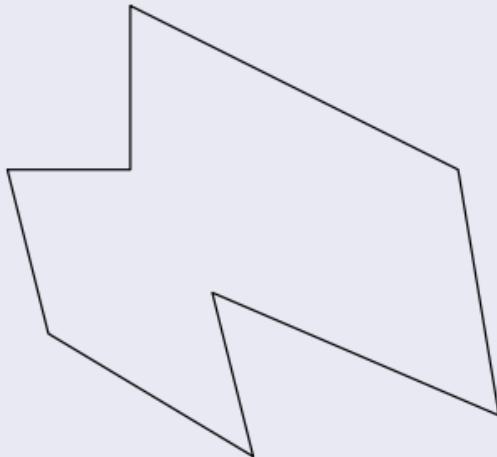
Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution



# B — Big Brother

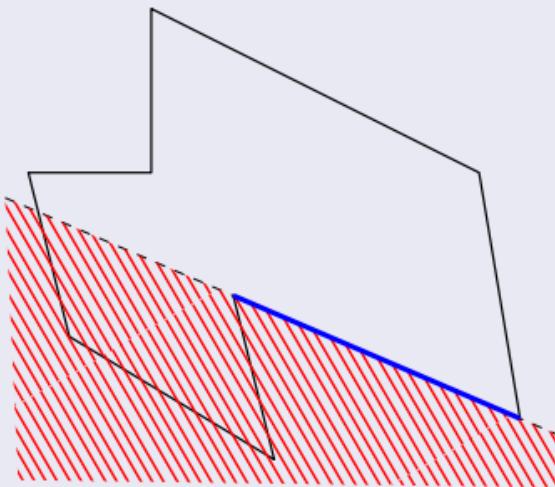
## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:

In order for points along that wall not to be obscured, we cannot be behind that wall.



# B — Big Brother

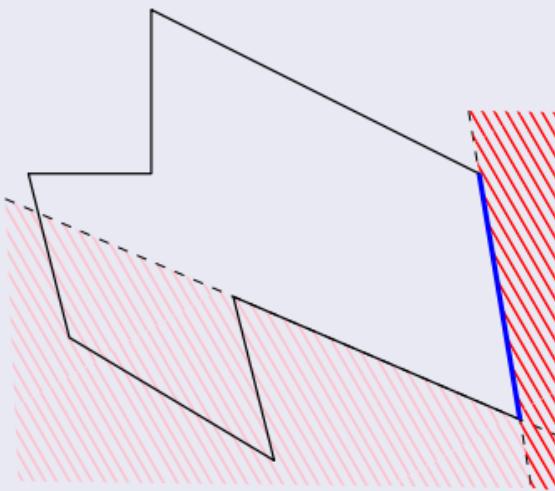
## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:

In order for points along that wall not to be obscured, we cannot be behind that wall.



# B — Big Brother

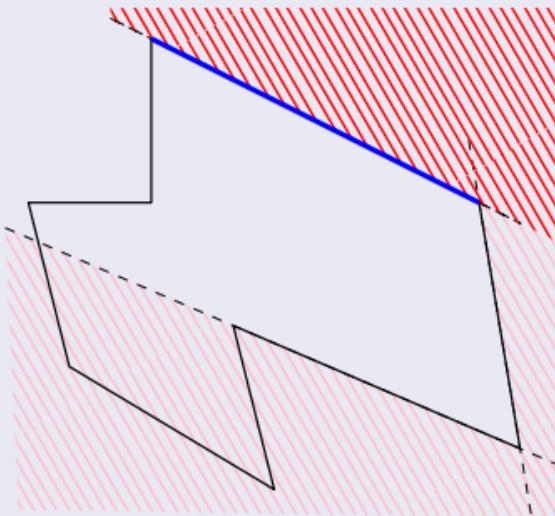
## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:

In order for points along that wall not to be obscured, we cannot be behind that wall.



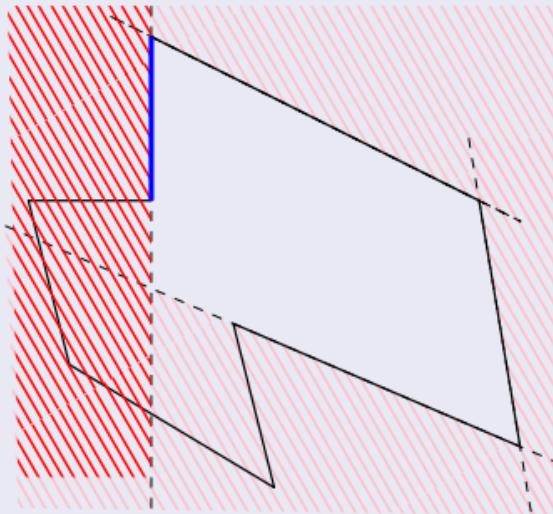
# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:  
In order for points along that wall not to be obscured, we cannot be behind that wall.



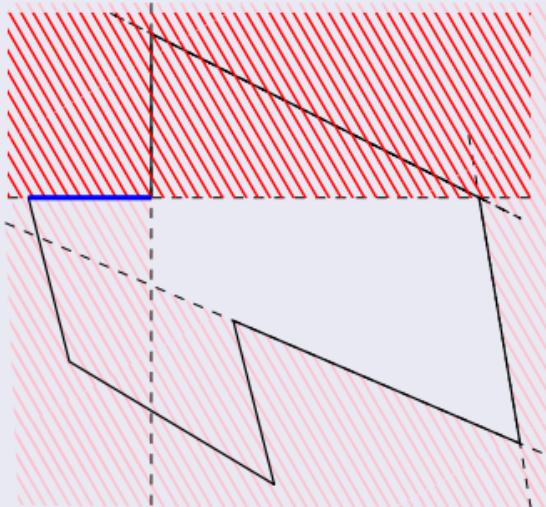
# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:  
In order for points along that wall not to be obscured, we cannot be behind that wall.



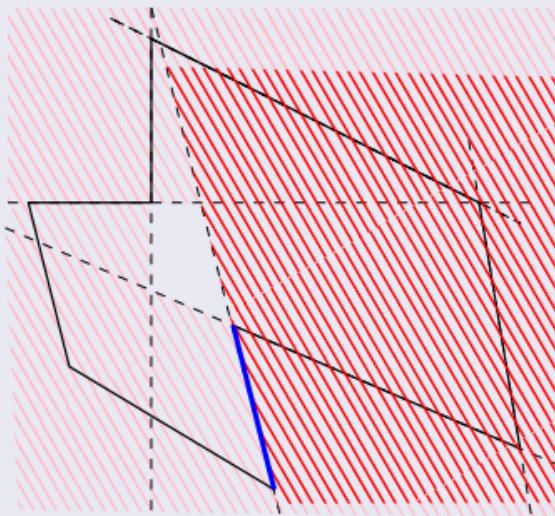
# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:  
In order for points along that wall not to be obscured, we cannot be behind that wall.



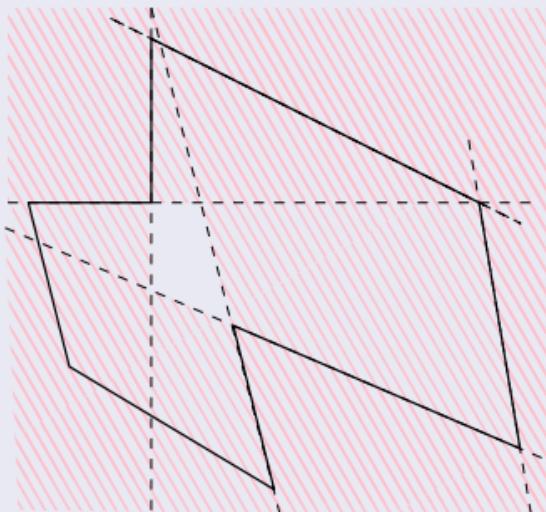
# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:  
In order for points along that wall not to be obscured, we cannot be behind that wall.



# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Solution

Line segments of the polygon induce *half-planes*:

In order for points along that wall not to be obscured, we cannot be behind that wall.

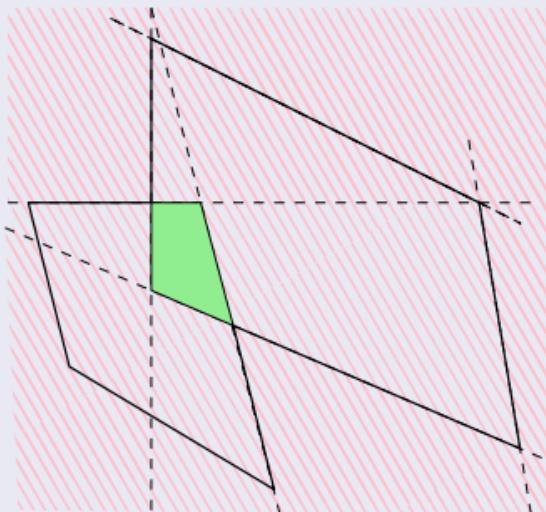
Seeing all parts of the room



Not “behind” any wall



Inside the intersection of the  $n$  half-planes.



# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.
  - ➋ Recursively compute intersection  $K_1$  of half-spaces in  $H_1$

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.
  - ➋ Recursively compute intersection  $K_1$  of half-spaces in  $H_1$
  - ➌ Recursively compute intersection  $K_2$  of half-spaces in  $H_2$

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.
  - ➋ Recursively compute intersection  $K_1$  of half-spaces in  $H_1$
  - ➌ Recursively compute intersection  $K_2$  of half-spaces in  $H_2$
  - ➍ Compute intersection of  $K_1$  and  $K_2$

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.
  - ➋ Recursively compute intersection  $K_1$  of half-spaces in  $H_1$
  - ➌ Recursively compute intersection  $K_2$  of half-spaces in  $H_2$
  - ➍ Compute intersection of  $K_1$  and  $K_2$
- Analysis:
  - ➊  $K_1$  and  $K_2$  are convex regions (possibly unbounded), can compute their intersection in  $O(n)$  time.

# B — Big Brother

## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.
  - ➋ Recursively compute intersection  $K_1$  of half-spaces in  $H_1$
  - ➌ Recursively compute intersection  $K_2$  of half-spaces in  $H_2$
  - ➍ Compute intersection of  $K_1$  and  $K_2$
- Analysis:
  - ➊  $K_1$  and  $K_2$  are convex regions (possibly unbounded), can compute their intersection in  $O(n)$  time.
  - ➋ Get recurrence  $T(n) = 2T(n/2) + O(n)$ , yields  $O(n \log n)$  time complexity.

# B — Big Brother

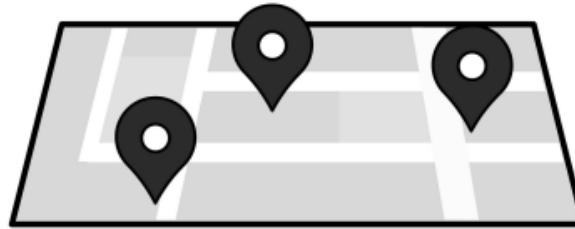
## Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

## Finding intersection of half-spaces

- Divide and conquer algorithm for intersection of half-spaces:
  - ➊ Split the half-spaces into two groups  $H_1$  and  $H_2$  of roughly  $n/2$  half-spaces each.
  - ➋ Recursively compute intersection  $K_1$  of half-spaces in  $H_1$
  - ➌ Recursively compute intersection  $K_2$  of half-spaces in  $H_2$
  - ➍ Compute intersection of  $K_1$  and  $K_2$
- Analysis:
  - ➊  $K_1$  and  $K_2$  are convex regions (possibly unbounded), can compute their intersection in  $O(n)$  time.
  - ➋ Get recurrence  $T(n) = 2T(n/2) + O(n)$ , yields  $O(n \log n)$  time complexity.

Statistics at 4-hour mark: 36 submissions, 11 accepted, first after 00:28



# Kleptocrat

? correct • solved at: ???:?? by

??

??

Author: **Jorke de Vlas**

## Overview

- Given a connected undirected graph, find a path from A to N that minimizes XOR of the values on the edges.

# Kleptocrat - Solution

## Techniques

- Linear Algebra
- Cycles

## Algorithm

- Observation: walking back and forth does not change the XOR-value since  $x \oplus x = 0$ .
- When there is a cycle starting from  $c$  with XOR-value  $v$ , we may walk from  $a$  to  $c$ , around the cycle, back to  $a$  and then to  $b$  giving a value of  $w \oplus v$  where  $w$  was the value of a path from  $a$  to  $b$ .
- This is an equation over  $\mathbb{F}_2$  and the  $v_i$  can be reduced with Gaussian Elimination giving 64 values.

# Kleptocrat - Solution (cont.)

## Techniques

- Linear Algebra
- Cycles

## Algorithm

- See  $v_i$  as vectors in  $F_2^{64}$  by writing  $v_i$  in base 2.
- The linear combinations form a subspace of dimension at most 64: find a basis, which has at most 64 elements.
- Now, given an initial path  $w$ , for  $i$  from 63 to 0, look if  $w$  has a 1 in the  $i$ th binary digit and check if there is a basis element with  $i$  as most significant digit, in which XOR  $w$  with this value.

# D — Dams in Distress

## Problem

We get a rooted tree forming a system of  $n \leq 200\,000$  dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for  $w$  units of water to reach the root?

## Solution

- ① For each dam  $i$  compute how much water  $f(i)$  is needed if we add water at  $i$ .

# D — Dams in Distress

## Problem

We get a rooted tree forming a system of  $n \leq 200\,000$  dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for  $w$  units of water to reach the root?

## Solution

- ① For each dam  $i$  compute how much water  $f(i)$  is needed if we add water at  $i$ .
- ② For the root,  $f(i) = w$ .

# D — Dams in Distress

## Problem

We get a rooted tree forming a system of  $n \leq 200\,000$  dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for  $w$  units of water to reach the root?

## Solution

- ① For each dam  $i$  compute how much water  $f(i)$  is needed if we add water at  $i$ .
- ② For the root,  $f(i) = w$ .
- ③ For non-root with parent  $p_i$ , capacity  $c_i$  and currently  $u_i$  water in it:
  - Need to add  $c_i - u_i$  water to break the dam, this causes  $c_i$  water to go upstream.
  - Need to add  $f(p_i)$  water at  $p_i$ , so need to add  $f(p_i) - c_i$  more water if  $f(p_i) > c_i$ .

# D — Dams in Distress

## Problem

We get a rooted tree forming a system of  $n \leq 200\,000$  dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for  $w$  units of water to reach the root?

## Solution

- ① For each dam  $i$  compute how much water  $f(i)$  is needed if we add water at  $i$ .
- ② For the root,  $f(i) = w$ .
- ③ For non-root with parent  $p_i$ , capacity  $c_i$  and currently  $u_i$  water in it:
  - Need to add  $c_i - u_i$  water to break the dam, this causes  $c_i$  water to go upstream.
  - Need to add  $f(p_i)$  water at  $p_i$ , so need to add  $f(p_i) - c_i$  more water if  $f(p_i) > c_i$ .

Gives equation  $f(i) = c_i - u_i + \max(0, f(p_i) - c_i)$ .

# D — Dams in Distress

## Problem

We get a rooted tree forming a system of  $n \leq 200\,000$  dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for  $w$  units of water to reach the root?

## Solution

- ① For each dam  $i$  compute how much water  $f(i)$  is needed if we add water at  $i$ .
- ② For the root,  $f(i) = w$ .
- ③ For non-root with parent  $p_i$ , capacity  $c_i$  and currently  $u_i$  water in it:
  - Need to add  $c_i - u_i$  water to break the dam, this causes  $c_i$  water to go upstream.
  - Need to add  $f(p_i)$  water at  $p_i$ , so need to add  $f(p_i) - c_i$  more water if  $f(p_i) > c_i$ .Gives equation  $f(i) = c_i - u_i + \max(0, f(p_i) - c_i)$ .
- ④ Complexity  $O(n)$  – compute top-down so  $f(p_i)$  is known when computing  $f(i)$ .

# D — Dams in Distress

## Problem

We get a rooted tree forming a system of  $n \leq 200\,000$  dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for  $w$  units of water to reach the root?

## Solution

- ① For each dam  $i$  compute how much water  $f(i)$  is needed if we add water at  $i$ .
- ② For the root,  $f(i) = w$ .
- ③ For non-root with parent  $p_i$ , capacity  $c_i$  and currently  $u_i$  water in it:
  - Need to add  $c_i - u_i$  water to break the dam, this causes  $c_i$  water to go upstream.
  - Need to add  $f(p_i)$  water at  $p_i$ , so need to add  $f(p_i) - c_i$  more water if  $f(p_i) > c_i$ .Gives equation  $f(i) = c_i - u_i + \max(0, f(p_i) - c_i)$ .
- ④ Complexity  $O(n)$  – compute top-down so  $f(p_i)$  is known when computing  $f(i)$ .

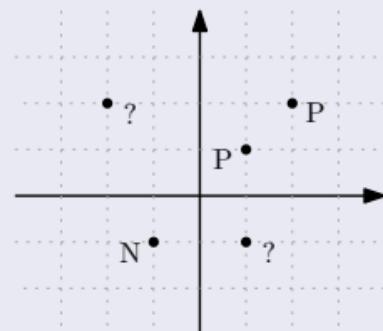
Statistics at 4-hour mark: 270 submissions, 90 accepted, first after 00:32

# E — Exhaustive Experiment

## Problem

We have  $n$  points which are potentially faulty. We can test points but tests only tell us if there is a faulty point within a cone above the test point. Given test results what is minimum number of faulty points?

## Solution (1/2)

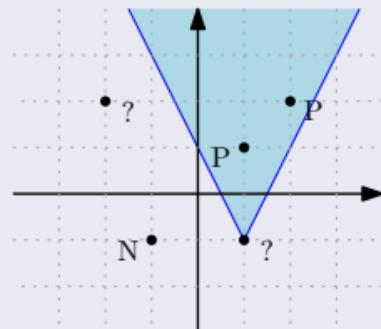


# E — Exhaustive Experiment

## Problem

We have  $n$  points which are potentially faulty. We can test points but tests only tell us if there is a faulty point within a cone above the test point. Given test results what is minimum number of faulty points?

## Solution (1/2)



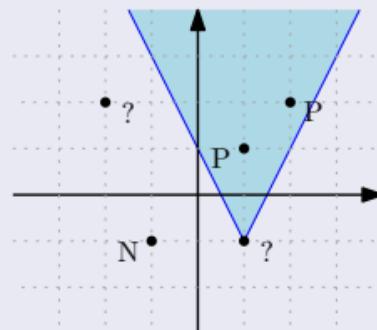
# E — Exhaustive Experiment

## Problem

We have  $n$  points which are potentially faulty. We can test points but tests only tell us if there is a faulty point within a cone above the test point. Given test results what is minimum number of faulty points?

## Solution (1/2)

- ① First let us simplify the geometry: scale  $x$ -coordinate by 2 and rotate 45 degrees



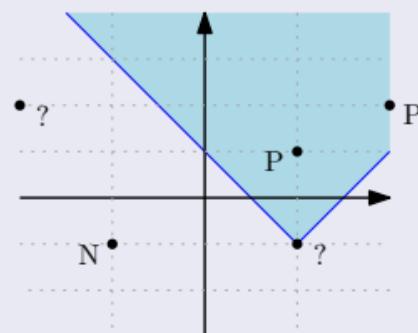
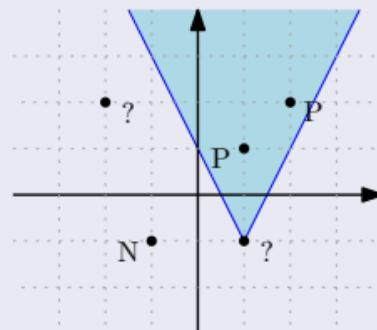
# E — Exhaustive Experiment

## Problem

We have  $n$  points which are potentially faulty. We can test points but tests only tell us if there is a faulty point within a cone above the test point. Given test results what is minimum number of faulty points?

## Solution (1/2)

- ① First let us simplify the geometry: scale  $x$ -coordinate by 2 and rotate 45 degrees



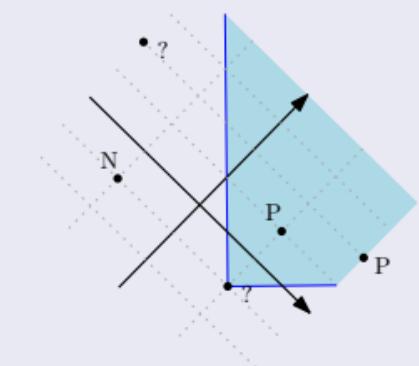
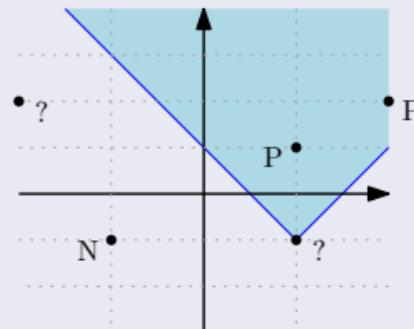
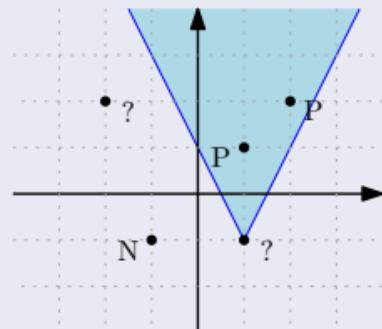
# E — Exhaustive Experiment

## Problem

We have  $n$  points which are potentially faulty. We can test points but tests only tell us if there is a faulty point within a cone above the test point. Given test results what is minimum number of faulty points?

## Solution (1/2)

- ① First let us simplify the geometry: scale  $x$ -coordinate by 2 and rotate 45 degrees



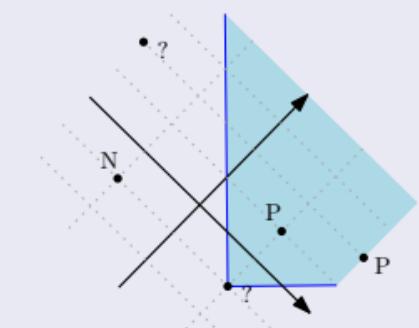
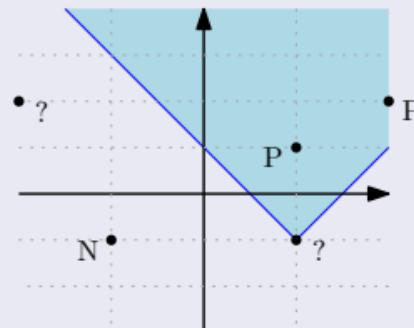
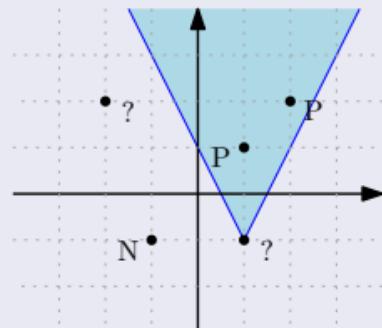
# E — Exhaustive Experiment

## Problem

We have  $n$  points which are potentially faulty. We can test points but tests only tell us if there is a faulty point within a cone above the test point. Given test results what is minimum number of faulty points?

## Solution (1/2)

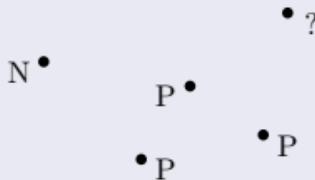
- ① First let us simplify the geometry: scale  $x$ -coordinate by 2 and rotate 45 degrees



- ② Now cone of points affected becomes a quadrant!

# E — Exhaustive Experiment

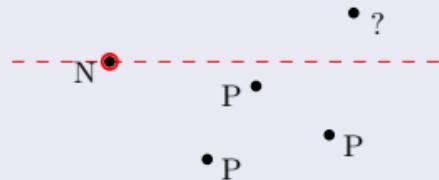
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.

# E — Exhaustive Experiment

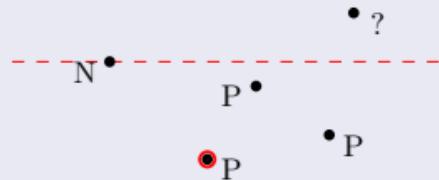
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen

# E — Exhaustive Experiment

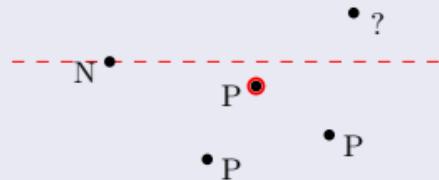
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen

# E — Exhaustive Experiment

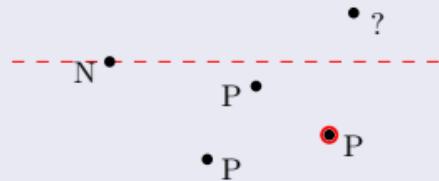
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen

# E — Exhaustive Experiment

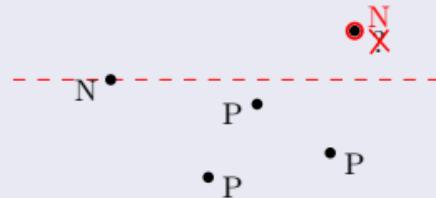
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen

# E — Exhaustive Experiment

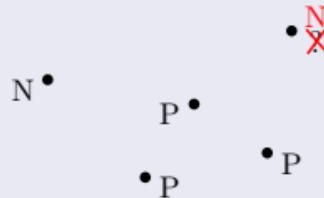
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ➋ Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)

# E — Exhaustive Experiment

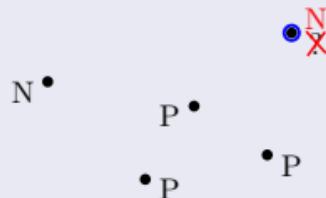
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ➋ Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:

# E — Exhaustive Experiment

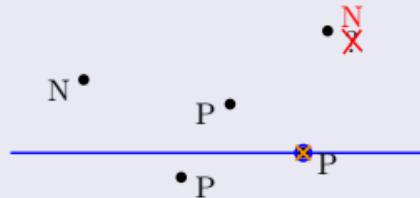
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.

# E — Exhaustive Experiment

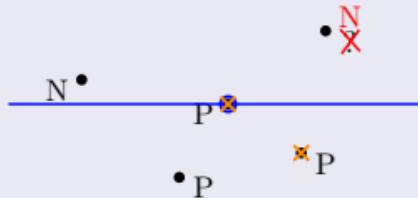
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ➊ Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ➋ Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ➊ Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.

# E — Exhaustive Experiment

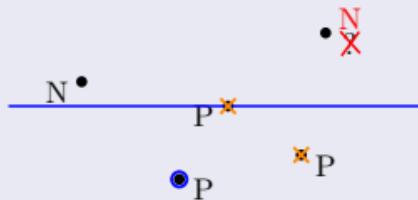
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.

# E — Exhaustive Experiment

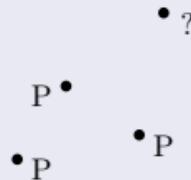
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.

# E — Exhaustive Experiment

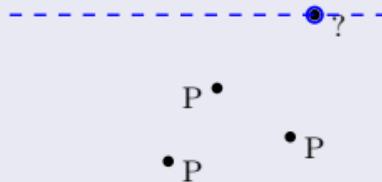
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.

# E — Exhaustive Experiment

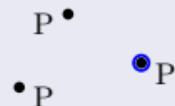
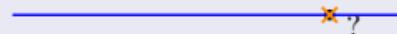
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked  $P \Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.

# E — Exhaustive Experiment

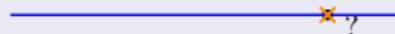
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked  $P \Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.
  - ② When explanation for positive test needed, use best *potentially faulty* one seen so far.

# E — Exhaustive Experiment

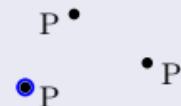
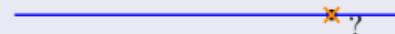
## Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.
  - ② When explanation for positive test needed, use best *potentially faulty* one seen so far.

# E — Exhaustive Experiment

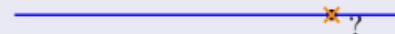
## Solution (2/2)

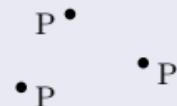


- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.
  - ② When explanation for positive test needed, use best *potentially faulty* one seen so far.

# E — Exhaustive Experiment

## Solution (2/2)

\_\_\_\_\_? 

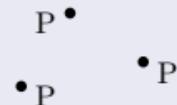


- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.
  - ② When explanation for positive test needed, use best *potentially faulty* one seen so far.
- Time complexity  $O(n \log n)$  for sorting then  $O(n)$ .

# E — Exhaustive Experiment

## Solution (2/2)

\_\_\_\_\_? \_\_\_\_\_?



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  - ① Sweep from left to right, maintaining minimum  $y$ -coordinate of a negative point seen
  - ② Points above current minimum must also be negative (if marked P  $\Rightarrow$  “impossible”)
- Greedily find smallest possible number of positive points:
  - ① Sweep from right to left, maintaining maximum  $y$ -coordinate of a faulty point, and maximum  $y$ -coordinate of a *potentially faulty* point.
  - ② When explanation for positive test needed, use best *potentially faulty* one seen so far.
- Time complexity  $O(n \log n)$  for sorting then  $O(n)$ .

Statistics at 4-hour mark: 18 submissions, 3 accepted, first after 01:14

## Problem

We have one movie and  $n$  critics with opinions  $x_1, \dots, x_n$  on how good it is.

If current review average of the movie exceeds a reviewers opinion they will score it 0, otherwise they will score it  $m$ .

Order the critics so that the film ends up getting review average exactly  $k/n$ .

## Problem

We have one movie and  $n$  critics with opinions  $x_1, \dots, x_n$  on how good it is.

If current review average of the movie exceeds a reviewers opinion they will score it 0, otherwise they will score it  $m$ .

Order the critics so that the film ends up getting review average exactly  $k/n$ .

## Initial observations

- ① Each review is either *positive* (score  $m$ ) or *negative* (score 0).

## Problem

We have one movie and  $n$  critics with opinions  $x_1, \dots, x_n$  on how good it is.

If current review average of the movie exceeds a reviewers opinion they will score it 0, otherwise they will score it  $m$ .

Order the critics so that the film ends up getting review average exactly  $k/n$ .

## Initial observations

- ① Each review is either *positive* (score  $m$ ) or *negative* (score 0).
- ② If there are  $p$  positive reviews, final review average is  $pm/n$ .

# F — Film Critics

## Problem

We have one movie and  $n$  critics with opinions  $x_1, \dots, x_n$  on how good it is.

If current review average of the movie exceeds a reviewers opinion they will score it 0, otherwise they will score it  $m$ .

Order the critics so that the film ends up getting review average exactly  $k/n$ .

## Initial observations

- ① Each review is either *positive* (score  $m$ ) or *negative* (score 0).
- ② If there are  $p$  positive reviews, final review average is  $pm/n$ .
- ③ So  $k$  must be divisible by  $m$  (otherwise impossible),  
and we need exactly  $p = k/m$  positive reviews.

## Solution

- ① Key insights:

## Solution

- ① Key insights: we may assume that
  - the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.

## Solution

① Key insights: we may assume that

- the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.
- the positive reviewers will come in increasing order of  $x_i$   
(and negative reviews in decreasing order)

## Solution

- ① Key insights: we may assume that
  - the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.
  - the positive reviewers will come in increasing order of  $x_i$  (and negative reviews in decreasing order)
- ② Build the answer iteratively. Each iteration, two candidates for next reviewer:
  - lowest remaining  $x_i$  from among the  $p$  largest, or
  - largest remaining  $x_i$  from among the  $n - p$  smallest

## Solution

- ➊ Key insights: we may assume that
  - the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.
  - the positive reviewers will come in increasing order of  $x_i$  (and negative reviews in decreasing order)
- ➋ Build the answer iteratively. Each iteration, two candidates for next reviewer:
  - lowest remaining  $x_i$  from among the  $p$  largest, or
  - largest remaining  $x_i$  from among the  $n - p$  smallest
- ➌ Pick one that yields the desired outcome (positive or negative) given the current review average.

## Solution

- ➊ Key insights: we may assume that
  - the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.
  - the positive reviewers will come in increasing order of  $x_i$  (and negative reviews in decreasing order)
- ➋ Build the answer iteratively. Each iteration, two candidates for next reviewer:
  - lowest remaining  $x_i$  from among the  $p$  largest, or
  - largest remaining  $x_i$  from among the  $n - p$  smallest
- ➌ Pick one that yields the desired outcome (positive or negative) given the current review average.
- ➍ If no such choice then ordering is **impossible** (can happen if one of the groups has become empty and the other has remaining  $x_i$  too low/high for desired result).

## Solution

- ➊ Key insights: we may assume that
  - the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.
  - the positive reviewers will come in increasing order of  $x_i$  (and negative reviews in decreasing order)
- ➋ Build the answer iteratively. Each iteration, two candidates for next reviewer:
  - lowest remaining  $x_i$  from among the  $p$  largest, or
  - largest remaining  $x_i$  from among the  $n - p$  smallest
- ➌ Pick one that yields the desired outcome (positive or negative) given the current review average.
- ➍ If no such choice then ordering is **impossible** (can happen if one of the groups has become empty and the other has remaining  $x_i$  too low/high for desired result).
- ➎ Time complexity  $O(n \log n)$  for sorting then  $O(n)$ .

## Solution

- ➊ Key insights: we may assume that
  - the  $p$  highest  $x_i$ 's will give a positive review, and the others a negative review.
  - the positive reviewers will come in increasing order of  $x_i$  (and negative reviews in decreasing order)
- ➋ Build the answer iteratively. Each iteration, two candidates for next reviewer:
  - lowest remaining  $x_i$  from among the  $p$  largest, or
  - largest remaining  $x_i$  from among the  $n - p$  smallest
- ➌ Pick one that yields the desired outcome (positive or negative) given the current review average.
- ➍ If no such choice then ordering is **impossible** (can happen if one of the groups has become empty and the other has remaining  $x_i$  too low/high for desired result).
- ➎ Time complexity  $O(n \log n)$  for sorting then  $O(n)$ .

Statistics at 4-hour mark: 62 submissions, 21 accepted, first after 00:43

# G — Gig Combinatorics

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form "12+3" ("2+" means 1 or more twos)

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+3” (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.

# G — Gig Combinatorics

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+3”  
 (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.
- ② Let  $p(i)$  be number of subsequences of form “12+” on the first  $i$  digits.

# G — Gig Combinatorics

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+” (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.
- ② Let  $p(i)$  be number of subsequences of form “12+” on the first  $i$  digits.
  - If  $i$ th digit is 2 then  $p(i) = 2 \cdot p(i - 1) + \text{ones}(i)$

# G — Gig Combinatorics

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+” (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.
- ② Let  $p(i)$  be number of subsequences of form “12+” on the first  $i$  digits.
  - If  $i$ th digit is 2 then  $p(i) = 2 \cdot p(i - 1) + \text{ones}(i)$
  - Otherwise  $p(i) = p(i - 1)$

# G — Gig Combinatorics

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+” (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.
- ② Let  $p(i)$  be number of subsequences of form “12+” on the first  $i$  digits.
  - If  $i$ th digit is 2 then  $p(i) = 2 \cdot p(i - 1) + \text{ones}(i)$
  - Otherwise  $p(i) = p(i - 1)$
- ③ Answer is sum of  $p(i)$  over all positions  $i$  where there we have a 3.

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+3”  
 (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.
- ② Let  $p(i)$  be number of subsequences of form “12+” on the first  $i$  digits.
  - If  $i$ th digit is 2 then  $p(i) = 2 \cdot p(i - 1) + \text{ones}(i)$
  - Otherwise  $p(i) = p(i - 1)$
- ③ Answer is sum of  $p(i)$  over all positions  $i$  where there we have a 3.
- ④ Time complexity  $O(n)$ .

## Problem

Given sequence of  $n \leq 10^6$  digits 1/2/3, count how many subsequences have the form “12+” (“2+” means 1 or more twos)

## Solution

- ① Let  $\text{ones}(i)$  be number of ones up to position  $i$  in the sequence.
- ② Let  $p(i)$  be number of subsequences of form “12+” on the first  $i$  digits.
  - If  $i$ th digit is 2 then  $p(i) = 2 \cdot p(i - 1) + \text{ones}(i)$
  - Otherwise  $p(i) = p(i - 1)$
- ③ Answer is sum of  $p(i)$  over all positions  $i$  where there we have a 3.
- ④ Time complexity  $O(n)$ .

Statistics at 4-hour mark: 437 submissions, 78 accepted, first after 00:04

# H — Hiring and Firing

## Problem

A company hires and fires workers over up to 100 000 days. Assign an HR employee to each day so that for each worker a different HR employee is assigned the day they are hired and the day they are fired. Minimize number of HR employees used.

## Solution (1/3)

# H — Hiring and Firing

## Problem

A company hires and fires workers over up to 100 000 days. Assign an HR employee to each day so that for each worker a different HR employee is assigned the day they are hired and the day they are fired. Minimize number of HR employees used.

## Solution (1/3)

- ① The hiring/firings form a graph: each day is a vertex, and each worker is an edge between the day they are hired and the day they are fired.

# H — Hiring and Firing

## Problem

A company hires and fires workers over up to 100 000 days. Assign an HR employee to each day so that for each worker a different HR employee is assigned the day they are hired and the day they are fired. Minimize number of HR employees used.

## Solution (1/3)

- ① The hiring/firings form a graph: each day is a vertex, and each worker is an edge between the day they are hired and the day they are fired.
- ② We seek the chromatic number of this graph.

# H — Hiring and Firing

## Problem

A company hires and fires workers over up to 100 000 days. Assign an HR employee to each day so that for each worker a different HR employee is assigned the day they are hired and the day they are fired. Minimize number of HR employees used.

## Solution (1/3)

- ① The hiring/firings form a graph: each day is a vertex, and each worker is an edge between the day they are hired and the day they are fired.
- ② We seek the chromatic number of this graph.
- ③ Graph coloring is NP-hard in general and even in planar graphs.

# H — Hiring and Firing

## Problem

A company hires and fires workers over up to 100 000 days. Assign an HR employee to each day so that for each worker a different HR employee is assigned the day they are hired and the day they are fired. Minimize number of HR employees used.

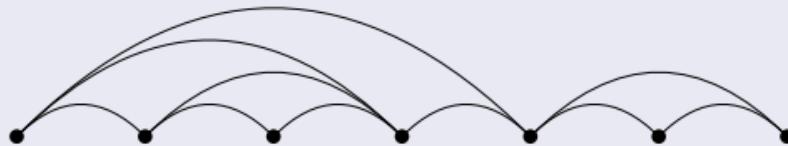
## Solution (1/3)

- ① The hiring/firings form a graph: each day is a vertex, and each worker is an edge between the day they are hired and the day they are fired.
- ② We seek the chromatic number of this graph.
- ③ Graph coloring is NP-hard in general and even in planar graphs.  
But these graphs are special... what do they look like?

# H — Hiring and Firing

## Solution (2/3)

- 1 Mark the days on a timeline and draw an arc for each worker:



# H — Hiring and Firing

## Solution (2/3)

- 1 Mark the days on a timeline and draw an arc for each worker:



- 2 Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily

## Solution (2/3)

- 1 Mark the days on a timeline and draw an arc for each worker:

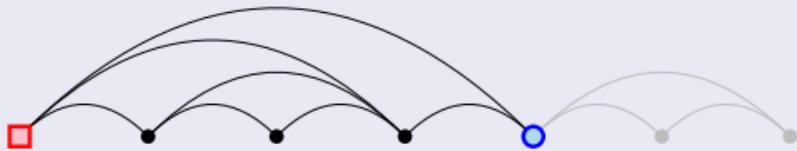


- 2 Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- 3 Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- 4 No-crossing property  $\Rightarrow$  separation into two independent subproblems.

# H — Hiring and Firing

## Solution (2/3)

- 1 Mark the days on a timeline and draw an arc for each worker:



- 2 Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- 3 Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- 4 No-crossing property  $\Rightarrow$  separation into two independent subproblems.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- ④ No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- ⑤ Recurse on subproblems and repeat. Use a valid color for the middle vertex.

# H — Hiring and Firing

## Solution (2/3)

- 1 Mark the days on a timeline and draw an arc for each worker:



- 2 Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- 3 Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- 4 No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- 5 Recurse on subproblems and repeat. Use a valid color for the middle vertex.

# H — Hiring and Firing

## Solution (2/3)

- 1 Mark the days on a timeline and draw an arc for each worker:



- 2 Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- 3 Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- 4 No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- 5 Recurse on subproblems and repeat. Use a valid color for the middle vertex.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- ④ No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- ⑤ Recurse on subproblems and repeat. Use a valid color for the middle vertex.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- ④ No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- ⑤ Recurse on subproblems and repeat. Use a valid color for the middle vertex.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- ④ No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- ⑤ Recurse on subproblems and repeat. Use a valid color for the middle vertex.
- ⑥ In each subproblem, only left & right of current range have already been colored.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- ④ No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- ⑤ Recurse on subproblems and repeat. Use a valid color for the middle vertex.
- ⑥ In each subproblem, only left & right of current range have already been colored.  
So if we have three colors, will always be a choice available for the middle vertex.

# H — Hiring and Firing

## Solution (2/3)

- ① Mark the days on a timeline and draw an arc for each worker:



- ② Because of last-in-first-out order of hirings and firings, cannot be any crossing arcs.
- ③ Gives rise to a recursive structure that we can use to color the graph:  
start with longest edge from leftmost vertex and color its endpoints arbitrarily
- ④ No-crossing property  $\Rightarrow$  separation into two independent subproblems.
- ⑤ Recurse on subproblems and repeat. Use a valid color for the middle vertex.
- ⑥ In each subproblem, only left & right of current range have already been colored.  
So if we have three colors, will always be a choice available for the middle vertex.  
In other words, 3 colors is always enough!

## Solution (3/3)

- ① Check if graph is 2-colorable (or 1-colorable), standard algorithm.

## Solution (3/3)

- ① Check if graph is 2-colorable (or 1-colorable), standard algorithm.
- ② If not, construct a 3-coloring using the procedure described before  
(or in some other way, many similar strategies work)

## Solution (3/3)

- ① Check if graph is 2-colorable (or 1-colorable), standard algorithm.
- ② If not, construct a 3-coloring using the procedure described before  
(or in some other way, many similar strategies work)
- ③ Can be implemented in  $O(n)$  time.

## Solution (3/3)

- ① Check if graph is 2-colorable (or 1-colorable), standard algorithm.
- ② If not, construct a 3-coloring using the procedure described before  
(or in some other way, many similar strategies work)
- ③ Can be implemented in  $O(n)$  time.

Statistics at 4-hour mark: 22 submissions, 0 accepted, first after N/A

# I — Infection Estimation

## Problem

Estimate within a factor 2 the number of infected people in a population, using 50 tests of the form “choose  $k$  people at random and check if at least one of them is infected”.

# I — Infection Estimation

## Problem

Estimate within a factor 2 the number of infected people in a population, using 50 tests of the form “choose  $k$  people at random and check if at least one of them is infected”.

## Solution

- ① To reduce number of possible answers, only consider answers

$$100, \quad 100 \cdot 1.01, \quad 100 \cdot 1.01^2, \quad \dots \quad 100 \cdot 1.01^i, \quad \dots \quad 5 \cdot 10^6$$

(around  $\log_{1.01}(5 \cdot 10^6) \approx 1500$  different values)

# I — Infection Estimation

## Problem

Estimate within a factor 2 the number of infected people in a population, using 50 tests of the form “choose  $k$  people at random and check if at least one of them is infected”.

## Solution

- ① To reduce number of possible answers, only consider answers

$$100, \quad 100 \cdot 1.01, \quad 100 \cdot 1.01^2, \quad \dots \quad 100 \cdot 1.01^i, \quad \dots \quad 5 \cdot 10^6$$

(around  $\log_{1.01}(5 \cdot 10^6) \approx 1500$  different values)

- ② Maintain probability distribution of likelihood of each answer (initially uniform).

# I — Infection Estimation

## Problem

Estimate within a factor 2 the number of infected people in a population, using 50 tests of the form “choose  $k$  people at random and check if at least one of them is infected”.

## Solution

- ① To reduce number of possible answers, only consider answers

$$100, \quad 100 \cdot 1.01, \quad 100 \cdot 1.01^2, \quad \dots \quad 100 \cdot 1.01^i, \quad \dots \quad 5 \cdot 10^6$$

(around  $\log_{1.01}(5 \cdot 10^6) \approx 1500$  different values)

- ② Maintain probability distribution of likelihood of each answer (initially uniform).
- ③ Each round, choose  $k$  such that test result yes/no probability is close to 50-50.

# I — Infection Estimation

## Problem

Estimate within a factor 2 the number of infected people in a population, using 50 tests of the form “choose  $k$  people at random and check if at least one of them is infected”.

## Solution

- ① To reduce number of possible answers, only consider answers

$$100, \quad 100 \cdot 1.01, \quad 100 \cdot 1.01^2, \quad \dots \quad 100 \cdot 1.01^i, \quad \dots \quad 5 \cdot 10^6$$

(around  $\log_{1.01}(5 \cdot 10^6) \approx 1500$  different values)

- ② Maintain probability distribution of likelihood of each answer (initially uniform).
- ③ Each round, choose  $k$  such that test result yes/no probability is close to 50-50.
- ④ Given result, update likelihoods using Bayes's theorem

$$\Pr[\text{infected} = t \mid \text{yes}] = \frac{\Pr[\text{yes} \mid \text{infected} = t] \cdot \Pr[\text{infected} = t]}{\Pr[\text{yes}]}$$

# I — Infection Estimation

## Problem

Estimate within a factor 2 the number of infected people in a population, using 50 tests of the form “choose  $k$  people at random and check if at least one of them is infected”.

## Solution

- ① To reduce number of possible answers, only consider answers

$$100, \quad 100 \cdot 1.01, \quad 100 \cdot 1.01^2, \quad \dots \quad 100 \cdot 1.01^i, \quad \dots \quad 5 \cdot 10^6$$

(around  $\log_{1.01}(5 \cdot 10^6) \approx 1500$  different values)

- ② Maintain probability distribution of likelihood of each answer (initially uniform).
- ③ Each round, choose  $k$  such that test result yes/no probability is close to 50-50.
- ④ Given result, update likelihoods using Bayes's theorem

$$\Pr[\text{infected} = t \mid \text{yes}] = \frac{\Pr[\text{yes} \mid \text{infected} = t] \cdot \Pr[\text{infected} = t]}{\Pr[\text{yes}]}$$

Statistics at 4-hour mark: 46 submissions, 6 accepted, first after 01:03



# Lost Map

?? correct • solved at: ???:?? by  
??  
??

Author: **Robin Lee**

## Overview

- Two map fragments (strings) are badly damaged and some items are replaced by "?"s
- Find how many ways you can overlay the two fragments so that no non-?" values conflict.

# Lost Map - Solution

## Techniques

- FFT
- Convolution

## Algorithm

- We can treat the "?" and non-?" values completely separately.
  - We make two separate binary strings
    - One encoding "?" as 0 values and everything else as 1 values
    - One encoding "?" as several -1s and everything else as its binary representation (takes LogK bits where K is the number of directions)
  - If we reverse one array, then the **convolution** of both versions of the binary strings from each map can be used to find the number of bits matching each time
    - We need the number of bits across both convolutions to exactly equal the length of the string.
- Pitfalls: FFT needs to be **fast**.

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

Simplifying assumptions:

- ①  $A$  and  $B$  have the same number of digits (or zero-pad)
- ② it is always the first integer we add to (try both options; take the best one)

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① If digits in  $i$ th position have  $a_i + b_i \geq 10$  there is a carry in  $i$ th position.

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① If digits in  $i$ th position have  $a_i + b_i \geq 10$  there is a carry in  $i$ th position.
- ② We must increment  $a_i$  and decrement  $b_i \pmod{10}$  until there is no longer carry.

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① If digits in  $i$ th position have  $a_i + b_i \geq 10$  there is a carry in  $i$ th position.
- ② We must increment  $a_i$  and decrement  $b_i \pmod{10}$  until there is no longer carry.
- ③ This happens when  $a_i$  turns to 0 (i.e. we add  $9 - a_i$ ).

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① If digits in  $i$ th position have  $a_i + b_i \geq 10$  there is a carry in  $i$ th position.
- ② We must increment  $a_i$  and decrement  $b_i \pmod{10}$  until there is no longer carry.
- ③ This happens when  $a_i$  turns to 0 (i.e. we add  $9 - a_i$ ).
- ④ If  $i$  is the leftmost digit causing carry, turning it to 0 will turn all remaining digits to 0 as well and get rid of any carries there.

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① If digits in  $i$ th position have  $a_i + b_i \geq 10$  there is a carry in  $i$ th position.
- ② We must increment  $a_i$  and decrement  $b_i \pmod{10}$  until there is no longer carry.
- ③ This happens when  $a_i$  turns to 0 (i.e. we add  $9 - a_i$ ).
- ④ If  $i$  is the leftmost digit causing carry, turning it to 0 will turn all remaining digits to 0 as well and get rid of any carries there.
- ⑤ Lets us compute  $A + X$  easily, subtract  $A$  to get  $X$ .

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① Caveat: turning the leftmost carry  $a_i$  into a 0 causes  $a_{i-1}$  to increase by 1, can cause a new carry in the previous digit.

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① Caveat: turning the leftmost carry  $a_i$  into a 0 causes  $a_{i-1}$  to increase by 1, can cause a new carry in the previous digit.
- ② Any preceding sequence of digits summing to 9 must also get their  $a_i$ 's turned to 0.  
Example:

$$A = 811\textcolor{blue}{7}654321\textcolor{red}{1}3$$

$$B = 111\textcolor{blue}{2}34567897$$

$$\text{Target } A + X = 812000000000$$

# K — Keep Calm and Carry Off

## Problem

Given two 1 000 000-digit integers  $A$  and  $B$ , find the smallest non-negative integer  $X$  such that  $A + X$  and  $B - X$  (or  $A - X$  and  $B + X$ ) can be added without carry.

## Solution

- ① Caveat: turning the leftmost carry  $a_i$  into a 0 causes  $a_{i-1}$  to increase by 1, can cause a new carry in the previous digit.
- ② Any preceding sequence of digits summing to 9 must also get their  $a_i$ 's turned to 0.  
Example:

$$A = 811\textcolor{blue}{7}654321\textcolor{red}{1}3$$

$$B = 111\textcolor{blue}{2}3456789\textcolor{red}{7}$$

$$\text{Target } A + X = 812000000000$$

Statistics at 4-hour mark: 115 submissions, 11 accepted, first after 00:44

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (1/2)

- ① Try to partition grid into three *disjoint* connected regions with following property:  
for every cell  $(i, j)$  there is a neighboring cell  $(i', j')$  belonging to a different region.

# L — Language Survey

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (1/2)

- ① Try to partition grid into three *disjoint* connected regions with following property:  
for every cell  $(i, j)$  there is a neighboring cell  $(i', j')$  belonging to a different region.
- ② If we manage to find this, problem becomes easy:

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (1/2)

- ① Try to partition grid into three *disjoint* connected regions with following property:  
for every cell  $(i, j)$  there is a neighboring cell  $(i', j')$  belonging to a different region.
- ② If we manage to find this, problem becomes easy:
  - Use the partition as a starting point.

# L — Language Survey

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (1/2)

- ① Try to partition grid into three *disjoint* connected regions with following property:  
for every cell  $(i, j)$  there is a neighboring cell  $(i', j')$  belonging to a different region.
- ② If we manage to find this, problem becomes easy:
  - Use the partition as a starting point.
  - For every cell  $(i, j)$  where two or more regions should overlap, extend the region from  $(i', j')$  into  $(i, j)$ .

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (2/2)

- ① How to find the starting point partition?

# L — Language Survey

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (2/2)

- ① How to find the starting point partition?

ACCCCCCCC

- ② If dimensions not too small, one possible pattern →

ACBBBBBBB

ACBCCCCCB

ACBCCCBCB

ACBBBBBCB

ACCCCCCCB

# L — Language Survey

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (2/2)

- ① How to find the starting point partition? ACCCCCCCC
- ② If dimensions not too small, one possible pattern → ACBBBBBBB  
ACBCCCCCB
- ③ Case when dimensions are small is left as exercise...
  - when at least two rows and columns the above works ACBCCCBCB
  - case with only one row or one column easy to solve directly ACBBBBBCB  
ACCCCCCCB

# L — Language Survey

## Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

## Solution (2/2)

- ① How to find the starting point partition? ACCCCCCCC
- ② If dimensions not too small, one possible pattern → ACBBBBBBB  
ACBCCCCCB
- ③ Case when dimensions are small is left as exercise...
  - when at least two rows and columns the above works ACBCCCBCB
  - case with only one row or one column easy to solve directly ACBBBBBCB  
ACCCCCCCB

Statistics at 4-hour mark: 5 submissions, 0 accepted, first after N/A