

Team Note of A Team

A, B, C

Compiled on January 11, 2026

Contents

1 Basic Implementation	1
1.1 Main Template	1
2 Math	1
2.1 Basic Arithmetic	1
2.2 Binomial Coefficient	2
2.3 Chinese Remainder Theorem	2
2.4 FFT & NTT	2
2.5 Linear Sieve	3
2.6 Miller-Rabin & Pollard-Rho	4
3 Data Structure	4
3.1 Erasable Priority Queue	4
3.2 Non-Recursive Segment Tree	4
3.3 1D & 2D Fenwick Tree	5
3.4 Merge Sort Tree	5
3.5 Persistent Segment Tree	5
3.6 Sweep-line Mo's	5
3.7 Link-Cut Tree	6
4 Graph	7
4.1 Bellman Ford	7
4.2 SPFA (SLF Optimized)	7
4.3 LCA	8
4.4 HLD	8
4.5 Centroid Decomposition	8
4.6 Bipartite Matching	9
4.7 Dinic	9
4.8 MCMF	10
4.9 Circulation	10
4.10 SCC	10
4.11 2-sat	11
4.12 BCC	11
4.13 Find 3 or 4 cycle	11
4.14 Push Relabel	12
4.15 General Matching	12
4.16 Weighed General Matching	13

5 DP Optimization	15
5.1 Convex Hull Trick	15
5.2 Linear CHT	15
5.3 D&C optimization	15
5.4 Monotone Queue optimization	15
5.5 Aliens Trick	16
5.6 Sum Over Subsets	16
5.7 Berlekamp Massey	16
6 Geometry	17
6.1 Geometry Template	17
6.2 Convex Hull	17
6.3 Rotating Calipers	17
6.4 Point in Convex Polygon	17
6.5 Point in Polygon	18
6.6 Sort Points	18
6.7 Linear Minkowski Sum	18
6.8 Polygon Area	18
6.9 Smallest Enclosing Circle	18
6.10 Geometric Intersections	18
6.11 Half Plane Intersection	19
7 String	20
7.1 Aho-Corasick	20
7.2 Hashing	20
7.3 KMP	20
7.4 Manacher	21
7.5 Suffix Array	21
7.6 Z-algorithm	21
7.7 Eertree	21
7.8 Suffix Automaton	21
8 STL & pbds	22
8.1 Hash map (pb_ds)	22
8.2 Ordered Set (pb_ds)	22
8.3 Permutation & Combination	22
8.4 Priority Queue (pb_ds)	22
8.5 Rope	22
8.6 Trie (pb_ds)	22
9 Misc	23
9.1 Custom Hash	23
9.2 Fast I/O	23
9.3 Random	23
9.4 Ternary Search	23
9.5 Some tricks	23
10 Checklist + Useful Info	24
10.1 Highly Composite Numbers, Large Prime	24
10.2 Useful Stuff	24
10.3 자주 쓰이는 문제 접근법	24
10.4 DP 최적화 접근	24
10.5 Graph Matching(Graph with $ V \leq 500$)	25
10.6 MinCut 모델링	25

1 Basic Implementation**1.1 Main Template**

```
// #pragma GCC optimize ("O3,unroll-loops")
// #pragma GCC target ("avx,avx2,fma") // simd
#include <bits/stdc++.h>
#define fastio cin.tie(0)->sync_with_stdio(0)
#define all(x) (x).begin(),(x).end()
#define rall(x) (x).rbegin(),(x).rend()
#define compress(v) sort(all(v)), v.erase(unique(all(v)), v.end())
#define sz(x) (int)(x).size()
using namespace std;
typedef long long ll;
typedef unsigned long long ull;
const ll INF = 1e18;
const int MOD = 998244353;
const int SIZE = 524288;
```

2 Math**2.1 Basic Arithmetic**

```
ll modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (1L)M);
}
ll modpow(ll b, ll e, ll m) {
    ll ans = 1;
    for (; e; b = modmul(b, b, m), e /= 2)
        if (e & 1) ans = modmul(ans, b, m);
    return ans;
}
ll xgcd(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll x1, y1, g = xgcd(b, a % b, x1, y1);
    return x = y1, y = x1 - a / b * y1, g;
}
ll modinv(ll a, ll m) {
    ll x, y;
    ll g = xgcd(a, m, x, y);
    if (g != 1) return -1;
```

```
    return (x%m + m) % m;
}
```

2.2 Binomial Coefficient

Time Complexity: first: $O(1)$ / second: $O(\sum p^k)$

```
// when M is big prime; Init: O(MAXN), Query: O(1)
ll modmul(ll a, ll b, ll m)
ll modpow(ll b, ll e, ll m)
const int M = 1e9+7, MAXN = 4000000;
ll fac[MAXN+5], finv[MAXN+5];
void init() {
    fac[0] = 1;
    for (int i = 1; i <= MAXN; i++)
        fac[i] = modmul(fac[i-1], i, M);
    finv[MAXN] = modpow(fac[MAXN], M-2, M);
    for (int i = MAXN-1; i >= 0; i--)
        finv[i] = modmul(finv[i+1], i+1, M);
}
ll nCk(int n, int k) {
    ll r = modmul(fac[n], finv[n-k], M);
    return modmul(r, finv[k], M);
}
// O(Sum of p^k) per query. (M = product of p^k)
ll modmul(ll a, ll b, ll m)
ll modpow(ll b, ll e, ll m)
ll xgcd(ll a, ll b, ll &x, ll &y)
ll modinv(ll a, ll m)
ll count(ll n, ll p) {
    ll cnt = 0; while (n > 0) { cnt += n/p; n /= p; }
    return cnt;
}
ll calc(ll n, ll p, ll pe, const auto& ft) {
    if (n == 0) return 1;
    ll v = ft[pe], res = modpow(v, n/pe, pe);
    res = modmul(res, ft[n%pe], pe);
    return modmul(res, calc(n/p, p, pe, ft), pe);
}
ll nCk_pe(ll n, ll k, ll p, ll pe, ll e) {
    if (k < 0 || k > n) return 0;
    ll pc = count(n, p) - count(k, p) - count(n-k, p);
    if (pc >= e) return 0;
    vector<ll> ft(pe+1); ft[0] = 1;
    for (int i = 1; i <= pe; i++) {
        ft[i] = ft[i-1];
        if (i%p != 0) ft[i] = modmul(ft[i], i, pe);
    }
    ll den = modmul(calc(k, p, pe, ft), calc(n-k, p, pe,
        ft), pe);
    ll res = modmul(calc(n, p, pe, ft), modinv(den, pe),
        pe);
```

```
    res = modmul(res, modpow(p, pc, pe), pe);
    return res;
}
ll nCk(ll n, ll k, int m) {
    if (k < 0 || k > n) return 0;
    if (k == 0 || k == n) return 1;
    ll t = m, res = 0;
    auto add = [&](ll p, ll pe, ll e) {
        ll rem = nCk_pe(n, k, p, pe, e);
        ll tm = modmul(rem, m/pe, m);
        tm = modmul(tm, modinv(m/pe, pe), m);
        res = (res + tm) % m;
    };
    for (ll i = 2; i*i <= t; i++) {
        if (t%i == 0) {
            ll p = i, pe = 1, e = 0;
            while (t%i == 0) { pe *= i; t /= i; e++; }
            add(p, pe, e);
        }
    }
    if (t > 1) add(t, t, 1);
    return res;
}
```

2.3 Chinese Remainder Theorem

Usage: Solve system of linear congruences.

Time Complexity: $O(\log N)$

```
ll xgcd(ll a, ll b, ll &x, ll &y)
pair<ll, ll> CRT(ll a1, ll m1, ll a2, ll m2) {
    ll x, y, g = xgcd(m1, m2, x, y);
    if ((a2 - a1) % g) return {-1, -1};
    ll md = m2 / g, k = (a2 - a1) / g % md * (x % md) % md;
    return {a1 + (k < 0 ? k + md : k) * m1, m1 / g * m2};
}
pair<ll, ll> CRT(const vector<ll>& a, const vector<ll>& m) {
    ll ra = a[0], rm = m[0];
    for (int i = 1; i < (int)m.size(); i++) {
        auto [aa, mm] = CRT(ra, rm, a[i], m[i]);
        if (mm == -1) return {-1, -1};
        ra = aa; rm = mm;
    }
    return {ra, rm};
```

2.4 FFT & NTT

Usage: Fast Fourier/Number Theoretic Transform for convolutions.

Time Complexity: $O(N \log N)$

```
template<int M> struct MINT {
    int v;
    MINT(ll _v = 0) { v = _v % M; if (v < 0) v += M; }
    MINT operator+(const MINT& o) const { return MINT(v + o.v); }
    MINT operator-(const MINT& o) const { return MINT(v - o.v); }
    MINT operator*(const MINT& o) const { return MINT((ll)v * o.v); }
    MINT& operator*=(const MINT& o) { return *this = *this * o; }
    friend MINT pw(MINT a, ll b) {
        MINT r = 1; for (; b; b >>= 1, a *= a) if (b & 1) r *= a;
        return r;
    }
    friend MINT inv(MINT a) { return pw(a, M - 2); }
};
namespace fft {
    using cpx = complex<double>;
    void rev_bit(int n, vector<auto>& a) {
        for (int i = 1, j = 0; i < n; i++) {
            int bit = n >> 1; for (; j & bit; bit >>= 1) j ^= bit;
            if (i < j) swap(a[i], a[j]);
        }
    }
    void FFT(vector<cpx>& a, bool inv_f) {
        int n = a.size(); rev_bit(n, a);
        for (int len = 2; len <= n; len <<= 1) {
            double ang = 2 * acos(-1) / len * (inv_f ? -1 : 1);
            cpx wlen(cos(ang), sin(ang));
            for (int i = 0; i < n; i += len) {
                cpx w(1);
                for (int j = 0; j < len / 2; j++) {
                    cpx u = a[i + j], v = a[i + j + len / 2] * w;
                    a[i + j] = u + v; a[i + j + len / 2] = u - v;
                    w *= wlen;
                }
            }
            if (inv_f) for (auto& x : a) x /= n;
        }
        vector<ll> multiply(const vector<ll>& a, const vector<ll>& b) {
```

```

int n = 1; while (n < a.size() + b.size()) n <<= 1;
vector<cpx> fa(n), fb(n);
for (int i=0; i<a.size(); i++) fa[i] = cpx(a[i], 0);
for (int i=0; i<b.size(); i++) fb[i] = cpx(b[i], 0);
FFT(fa, 0); FFT(fb, 0);
for (int i=0; i<n; i++) fa[i] *= fb[i];
FFT(fa, 1); vector<ll> res(n);
for (int i=0; i<n; i++) res[i] =
    llround(fa[i].real());
return res;
}

vector<ll> multiply_mod(const vector<ll>& a, const
vector<ll>& b, ll mod) {
    int n = 1; while (n < a.size() + b.size()) n <<= 1;
    vector<cpx> v1(n), v2(n), r1(n), r2(n);
    for (int i = 0; i < a.size(); i++) v1[i] = cpx(a[i]
        >> 15, a[i] & 32767);
    for (int i = 0; i < b.size(); i++) v2[i] = cpx(b[i]
        >> 15, b[i] & 32767);
    FFT(v1, 0); FFT(v2, 0);
    for (int i = 0; i < n; i++) {
        int j = i ? n - i : i;
        cpx a1 = (v1[i]+conj(v1[j]))*cpx(0.5, 0), a2 =
            (v1[i]-conj(v1[j]))*cpx(0, -0.5);
        cpx b1 = (v2[i]+conj(v2[j]))*cpx(0.5, 0), b2 =
            (v2[i]-conj(v2[j]))*cpx(0, -0.5);
        r1[i] = a1 * b1 + a1 * b2 * cpx(0, 1); r2[i] = a2
        * b1 + a2 * b2 * cpx(0, 1);
    }
    FFT(r1, 1); FFT(r2, 1);
    vector<ll> res(n);
    for (int i = 0; i < n; i++) {
        ll av = ((ll)round(r1[i].real()) % mod, cv =
            ((ll)round(r2[i].imag()) % mod;
        ll bv = ((ll)round(r1[i].imag()) +
            ((ll)round(r2[i].real())) % mod;
        res[i] = (av << 30) + (bv << 15) + cv; res[i] =
        (res[i] % mod + mod) % mod;
    }
    return res;
}

template<int W, int M> void NTT(vector<MINT<M>>& a,
bool inv_f) {
    int n = a.size(); rev_bit(n, a);
    for (int len = 2; len <= n; len <= 1) {
        MINT<M> wlen = pw(MINT<M>(W), (M - 1) / len);
        if (inv_f) wlen = inv(wlen);
        for (int i = 0; i < n; i += len) {
            MINT<M> w = 1;
            for (int j = 0; j < len / 2; j++) {
                MINT<M> u = a[i + j], v = a[i + j + len / 2]
                    * w;
                a[i + j] = u + v; a[i + j + len / 2] = u - v;
                w *= wlen;
            }
        }
        if (inv_f) { MINT<M> rn = inv(MINT<M>(n)); for
            (auto& x : a) x *= rn; }
    }
}

template<int W, int M> struct Poly {
    using T = MINT<M>; vector<T> a;
    Poly(const vector<T>& _a = {}) : a(_a) { norm(); }
    void norm() { while (a.size() && a.back().v == 0)
        a.pop_back(); }
    int deg() const { return (int)a.size() - 1; }
    T operator[](int i) const { return i < a.size() ?
        a[i] : T(0); }
    Poly operator*(const Poly& o) const {
        if (a.empty() || o.a.empty()) return {};
        int n = 1, sz = a.size() + o.a.size() - 1;
        while (n < sz) n <= 1;
        vector<T> fa(n), fb(n); copy(all(a), fa.begin());
        copy(all(o.a), fb.begin());
        fft::NTT<W, M>(fa, 0); fft::NTT<W, M>(fb, 0);
        for (int i = 0; i < n; i++) fa[i] *= fb[i];
        fft::NTT<W, M>(fa, 1); return fa;
    }
    Poly inv(int n) const {
        Poly r({ ::inv(a[0]) });
        for (int i = 1; i < n; i <= 1) {
            Poly tmp(vector<T>(a.begin(), a.begin() +
                min((int)a.size(), i * 2)));
            r = (r * (Poly({T(2)})) - r * tmp)); r.a.resize(i
                * 2);
        }
        r.a.resize(n); return r;
    }
    Poly operator/(Poly o) const {
        if (deg() < o.deg()) return {};
        int n = deg() - o.deg() + 1;
        Poly ra = a, rb = o.a; reverse(all(ra.a));
        reverse(all(rb.a));
        Poly q = (ra * rb.inv(n)); q.a.resize(n);
        reverse(all(q.a)); return q;
    }
    Poly operator%(Poly o) const {
        if (deg() < o.deg()) return *this;
        Poly r = *this - (*this / o) * o; r.norm(); return
        r;
    }
};

Poly operator-(const Poly& o) const {
    vector<T> res(max(a.size(), o.a.size()));
    for (int i = 0; i < res.size(); i++) res[i] =
        (*this)[i] - o[i];
    return res;
}

using mint = MINT<998244353>;
using poly = Poly<3, 998244353>;
mint Kitamasa(poly c, poly a, ll n) {
    if (n <= a.deg()) return a[n];
    poly f; for (int i = 0; i <= c.deg(); i++)
        f.a.push_back(mint(0) - c[c.deg() - i]);
    f.a.push_back(1); poly res({1}); x({0, 1});
    for (; n; n >= 1, x = (x * x) % f)
        if (n & 1) res = (res * x) % f;
    mint ans = 0;
    for (int i = 0; i <= a.deg(); i++)
        ans = ans + a[i] * res[i];
    return ans;
}

int main() {
    vector<ll> A = {1, 2, 1}; // 1+2*x+x^2
    vector<ll> B = {1, 1}; // 1+x
    vector<ll> C = fft::multiply(A, B); // {1, 3, 3, 1}
    vector<ll> D = fft::multiply_mod(A, B, 1e9+7);
    poly p1({1, 2, 1}), p2({1, 1}); // NTT base
    p1 * p2; p1 / p2; p1 % p2; // polynomial operation
    // ex. A_n = 1*A_{n-1} + 1*A_{n-2}
    poly coef({1, 1}); // {c0, c1} 순서 (A_{n-2}, A_{n-1} 계수)
    poly init({0, 1}); // {A0, A1} 초기값
    cout << Kitamasa(coef, init, 1e9).v;
}



## 2.5 Linear Sieve



Usage: Find primes and multiplicative functions in linear time.  

    Time Complexity:  $O(N)$



```

struct Sieve {
 // sp: 최소 소인수, e: i의 최소 소인수 지수, phi: 오
 // 일러 피 함수(1~i 중 i와 서로소인 개수), mu:뫼비우스
 // 함수, tau: 약수 개수, sigma: 약수의 합
 vector<int> sp(n+1), e(n+1), phi(n+1), mu(n+1),
 tau(n+1), sigma(n+1), tmp(n+1);
 phi[1] = mu[1] = tau[1] = sigma[1] = 1;
 for (int i = 2; i <= n; i++) {
 if (!sp[i]) {
 sp[i]=i; primes.push_back(i);

```


```

```

e[i]=1; phi[i]=i-1; mu[i]=-1; tau[i]=2;
sigma[i]=tmp[i]=i+1;
}
for (int p : primes) {
    if (i*p > n || p > sp[i]) break;
    int m = i*p; sp[m] = p;
    if (i%p == 0) {
        e[m] = e[i]+1; phi[m] = phi[i]*p; mu[m] = 0;
        tau[m] = tau[i]/(e[i]+1)*(e[m]+1);
        tmp[m] = tmp[i]*p+1; sigma[m] =
        sigma[i]/tmp[i]*tmp[m];
        break;
    } else {
        e[m] = 1; phi[m] = phi[i]*(p-1); mu[m] =
        -mu[i];
        tau[m] = tau[i]*2; tmp[m] = p+1; sigma[m] =
        sigma[i]*(p+1);
    }
}
}
};

2.6 Miller-Rabin & Pollard-Rho
```

Usage: Primality test and integer factorization.

Time Complexity: $O(\log^3 N)/O(N^{1/4})$

```

ll modmul(ll a, ll b, ll m)
ll modpow(ll b, ll e, ll m)
bool isPrime(ll n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ll A[] = {2, 325, 9375, 28178, 450775, 9780504,
    1795265022},
    s = __builtin_ctzll(n-1), d = n >> s;
    for (ll a : A) { // ^ count trailing zeroes
        ll p = modpow(a%n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}
ll pollard(ll n) {
    auto f = [n](ll x) { return modmul(x, x, n) + 3; };
    ll x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd
        = q;
        x = f(x), y = f(f(y));
    }
}

```

```

    return __gcd(prd, n);
}
vector<ll> factor(ll n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ll x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
vector<ll> res = factor(N); // factor of N

```

3 Data Structure

3.1 Erasable Priority Queue

```

template <typename T = int, typename Compare =
std::less<T>>
struct EraseablePQ {
    priority_queue<T, vector<T>, Compare> q, del;
    void flush() {
        while (!del.empty() && !q.empty() && q.top() ==
        del.top()) {
            q.pop(); del.pop();
        }
    }
    void push(const T& x) { q.push(x); flush(); }
    void erase(const T& x) { del.push(x); flush(); }
    void pop() { flush(); if (!q.empty()) q.pop();
    flush(); }
    const T& top() { flush(); return q.top(); }
    int size() const { return int(q.size() - del.size()); }
    bool empty() { flush(); return q.empty(); }
};

```

3.2 Non-Recursive Segment Tree

```

template<typename Node>
struct SegTree {
    int n, lg, size;
    Node e; // 항등원
    vector<Node> tree;
    function<Node(Node, Node)> func;
    int log2(int n) {
        int res = 0;
        while (n > (1 << res)) res++;
        return res;
    }
    SegTree(int n, const Node& e, auto func) : n(n),
    lg(log2(n)), size(1<<lg), e(e), tree(size<<1, e),
    func(func) {}
}

```

```

SegTree(const vector<Node>& v, const Node& e, auto
func) : n(sz(v)), lg(log2(n)), size(1<<lg), e(e),
tree(size<<1, e), func(func) {
    for (int i = 0; i < n; i++) {
        tree[i+size] = v[i];
    }
    for (int i = size-1; i > 0; i--) {
        tree[i] = func(tree[i<<1], tree[i<<1 | 1]);
    }
}
void add(int i, const Node& val) {
    tree[--i |= size] += val;
    while (i >= 1) {
        tree[i] = func(tree[i<<1], tree[i<<1 | 1]);
    }
}
void update(int i, const Node& val) {
    tree[--i |= size] = val;
    while (i >= 1) {
        tree[i] = func(tree[i<<1], tree[i<<1 | 1]);
    }
}
Node query(int i) { return tree[--i | size]; }
Node query(int l, int r) {
    Node L = e, R = e;
    for (--l |= size, --r |= size; l <= r; l >= 1, r
    >= 1) {
        if (l & 1) L = func(L, tree[l++]);
        if (~r & 1) R = func(tree[r--], R);
    }
    return func(L, R);
}
int find_kth(Node k) {
    int node = 1, st = 1, en = size;
    while (st != en) {
        int mid = (st + en) / 2; node <<= 1;
        if (tree[node] >= k) en = mid;
        else k -= tree[node], node |= 1, st = mid+1;
    }
    return st;
}
int main() {
    // 1. Range Sum Query (RSQ)
    vector<int> v = {1, 2, 3, 4, 5};
    SegTree<int> rsq(v, 0, [] (int a, int b) { return a+b;
    });
    rsq.update(3, 10);
    int sum = rsq.query(2, 4);
    // 2. Range Minimum Query (RMQ)
}

```

```

const int INF = 1e9;
SegTree<int> rmq(N, INF, [](int a, int b) { return min(a, b); });
// 3. Binary Search on Tree (Order Statistic)
// - Requirement: The tree must represent frequency or counts.
// - Find the smallest index i such that
prefix_sum(1...i) >= k
int idx = rsq.find_kth(7);
}

```

3.3 1D & 2D Fenwick Tree

Usage: Point updates and subgrid sums on a 2D plane.

Time Complexity: 1D: $O(\log N)$, 2D: $O(\log N \log M)$

```

template <typename T = int, typename Compare =
std::less<T>>
struct EraseablePQ {
    priority_queue<T, vector<T>, Compare> q, del;
    void flush() {
        while (!del.empty() && !q.empty() && q.top() == del.top()) {
            q.pop(); del.pop();
        }
    }
    void push(const T& x) { q.push(x); flush(); }
    void erase(const T& x) { del.push(x); flush(); }
    void pop() { flush(); if (!q.empty()) q.pop(); }
    flush(); }
    const T& top() { flush(); return q.top(); }
    int size() const { return int(q.size() - del.size()); }
    bool empty() { flush(); return q.empty(); }
};

```

3.4 Merge Sort Tree

Usage: Count/rank of elements in range $[L, R]$.

Time Complexity: $O(\log^2 N)$ per query

```

template <typename T>
struct MergeSortTree {
    int sz;
    vector<vector<T>> tree; // Space:  $O(N \log N)$ 
    MergeSortTree(int n) {
        sz = 1;
        while (sz < n) sz *= 2;
        tree.resize(sz*2);
    }
    void add(int x, T v) { tree[x+sz].push_back(v); }
    void build() { // Build:  $O(N \log N)$ 
}

```

```

for (int i = sz-1; i > 0; i--) {
    tree[i].resize(sz(tree[i*2]) + sz(tree[i*2+1]));
    merge(all(tree[i*2]), all(tree[i*2+1]),
          tree[i].begin());
}
int query(int l, int r, T k) { // Query:  $O(\log^2 N)$ 
    int res = 0;
    for (l += sz, r += sz; l <= r; l /= 2, r /= 2) {
        if (l & 1) {
            res += tree[l].end() -
                upper_bound(all(tree[l]), k); l++;
        }
        if (!(r & 1)) {
            res += tree[r].end() -
                upper_bound(all(tree[r]), k); r--;
        }
        /*
         * Count < k: lower_bound(all(v)) - v.begin()
         * Count <= k: upper_bound(all(v)) - v.begin()
         * Count >= k: v.end() - lower_bound(all(v))
         * Count > k: v.end() - upper_bound(all(v))
        */
    }
    return res;
}

```

3.5 Persistent Segment Tree

Usage: Accessing previous versions and range k-th element.

Time Complexity: $O(\log N)$ per query

```

struct PSTNode{
    PSTNode *l, *r; int v;
    PSTNode(): l = r = nullptr, v = 0; }
    PSTNode *root[101010];
    PST(): memset(root, 0, sizeof root); // constructor
    void init(PSTNode *node, int s, int e){
        if(s == e) return;
        int m = s + e >> 1;
        node->l = new PSTNode(); node->r = new PSTNode();
        init(node->l, s, m); init(node->r, m+1, e);
    }
    void update(PSTNode *prv, PSTNode *now, int s, int e,
               int x){
        if (s == e) { now->v = prv ? prv->v + 1 : 1; return; }
        int m = s + e >> 1;
        if (x <= m) {
            now->l = new PSTNode(); now->r = prv->r;
}

```

```

update(prv->l, now->l, s, m, x);
}
else {
    now->r = new PSTNode(); now->l = prv->l;
    update(prv->r, now->r, m+1, e, x);
}
int t1 = now->l ? now->l->v : 0;
int t2 = now->r ? now->r->v : 0;
now->v = t1 + t2;
}
int kth(PSTNode *prv, PSTNode *now, int s, int e, int k){
    if (s == e) return s;
    int m = s + e >> 1, diff = now->l->v - prv->l->v;
    if (k <= diff) return kth(prv->l, now->l, s, m, k);
    else return kth(prv->r, now->r, m+1, e, k-diff);
}

```

3.6 Sweepline Mo's

Usage: Optimized Mo's for $O(1)$ update and $O(1)$ query via offline sweepline.

Time Complexity: $O(N\sqrt{Q})$

```

const int MAXN = 200005, BSIZ = 450;
struct SqrtDecomp {
    ll lz_v[BSIZ+5], lz_c[BSIZ+5], v_arr[MAXN],
    c_arr[MAXN];
    ll total_v = 0, total_c = 0;
    void clear() { memset(this, 0, sizeof(*this)); }
    void update(int idx, ll v) { //  $O(\sqrt{N})$ 
        total_v += v; total_c++;
        int b = idx / BSIZ;
        for (int i = idx; i < (b + 1) * BSIZ && i < MAXN;
             i++) {
            v_arr[i] += v; c_arr[i]++;
        }
        for (int i = b + 1; i <= BSIZ; i++) {
            lz_v[i] += v; lz_c[i]++;
        }
    }
    ll query(int idx, ll v) { //  $O(1)$ 
        if (idx < 0) return (total_c * v - total_v); // 필
        오 시 수정
        ll cur_v = lz_v[idx / BSIZ] + v_arr[idx];
        ll cur_c = lz_c[idx / BSIZ] + c_arr[idx];
        return (cur_c * v - cur_v) + ((total_v - cur_v) -
            (total_c - cur_c) * v);
    }
}
sd;
struct MoSweep {

```

```

struct Query {
    int l, r, id; ll ans;
    bool operator<(const Query& o) const {
        if (l / BSIZ != o.l / BSIZ) return l < o.l;
        return (l / BSIZ) & 1 ? r < o.r : r > o.r;
    }
};

struct Delta { int q_idx, l, r; bool is_sub; };
int n, q;
ll A[MAXN], pref[MAXN], result[MAXN], rnk[MAXN];
vector<Query> queries, sweep[MAXN];
void init(int _n) {
    n = _n; queries.clear();
    for(int i = 0; i<=n; i++) sweep[i].clear();
}
void add_query(int l, int r, int id) {
    queries.push_back({l, r, id, 0});
}
void build() {
    sort(queries.begin(), queries.end());
    sd.clear();
    for (int i = 1; i <= n; i++) {
        pref[i] = sd.query(rnk[i], A[i]);
        sd.update(rnk[i], A[i]);
    }
    int s = 1, e = 0;
    for (int i = 0; i < (int)queries.size(); i++) {
        int nl = queries[i].l, nr = queries[i].r;
        if (e < nr) sweep[s - 1].push_back({i, e + 1, nr,
            true}), e = nr;
        if (s > nl) sweep[e].push_back({i, nl, s - 1,
            false}), s = nl;
        if (e > nr) sweep[s - 1].push_back({i, nr + 1, e,
            false}), e = nr;
        if (s < nl) sweep[e].push_back({i, s, nl - 1,
            true}), s = nl;
    }
}

void solve() {
    sd.clear();
    for (int i = 1; i <= n; i++) {
        sd.update(rnk[i], A[i]);
        for (auto& d : sweep[i]) {
            ll tmp = 0;
            for (int k = d.l; k <= d.r; k++) tmp += d.q_idx;
            sd.query(rnk[k], A[k]);
            queries[d.q_idx].ans += (d.is_sub ? -tmp :
                tmp);
        }
    }
    int s = 1, e = 0;
    for (int i = 0; i < (int)queries.size(); i++) {

```

```

        while (e < queries[i].r) queries[i].ans += pref[++e];
        while (s > queries[i].l) queries[i].ans -= pref[--s];
        while (e > queries[i].r) queries[i].ans -= pref[e--];
        while (s < queries[i].l) queries[i].ans += pref[s++];
        if (i > 0) queries[i].ans += queries[i - 1].ans;
        result[queries[i].id] = queries[i].ans;
    }
}
int main() {
    int n, q; cin >> n >> q;
    engine.init(n);
    vector<pair<ll, int>> v(n);
    for (int i = 1; i <= n; i++) {
        cin >> engine.A[i];
        v[i - 1] = {engine.A[i], i};
    }
    sort(v.begin(), v.end());
    for (int i = 0; i < n; i++) engine.rnk[v[i].second] = i;
    for (int i = 0; i < q; i++) {
        int l, r; cin >> l >> r;
        engine.add_query(l, r, i);
    }
    engine.build();
    engine.solve();
    for (int i = 0; i < q; i++) cout << engine.result[i]
    << "\n";
    return 0;
}

```

3.7 Link-Cut Tree

Usage: Dynamic tree structure supporting link, cut, and path operations using auxiliary Splay trees.

Time Complexity: Amortized $O(\log N)$

```

struct Node {
    Node *l, *r, *p;
    bool flip; int sz;
    T now, sum, lz;
    Node() {
        l = r = p = nullptr; sz = 1;
        flip = false; now = sum = lz = 0;
    }
    bool IsLeft() const { return p && this == p->l; }
    bool IsRoot() const { return !p || (this != p->l &&
        this != p->r); }
}

```

```

friend int GetSize(const Node *x) { return x ? x->sz
: 0; }
friend T GetSum(const Node *x) { return x ? x->sum :
0; }
void Rotate() {
    p->Push(); Push();
    if (IsLeft())
        r && (r->p = p), p->l = r, r = p;
    else
        l && (l->p = p), p->r = l, l = p;
    if (!p->IsRoot())
        (p->IsLeft() ? p->p->l : p->p->r) = this;
    auto t = p; p = t->p;
    t->p = this; t->Update();
    Update();
}
void Update() {
    sz = 1 + GetSize(l) + GetSize(r);
    sum = now + GetSum(l) + GetSum(r);
}
void Update(const T &val) {
    now = val; Update();
}
void Push() {
    Update(now + lz);
    if (flip)
        swap(l, r);
    for (auto c : {l, r}) if (c)
        c->flip ^= flip, c->lz += lz;
    lz = 0; flip = false;
}
Node *rt;
Node *Splay(Node *x, Node *g = nullptr) {
    for (g || (rt = x); x->p != g; x->Rotate()) {
        if (!x->p->IsRoot()) x->p->p->Push();
        x->p->Push(); x->Push();
        if (x->p->p != g)
            (x->IsLeft() ^ x->p->IsLeft()) ? x :
            x->p)->Rotate();
    }
    x->Push(); return x;
}
Node *Kth(int k) {
    for (auto x = rt;; x = x->r) {
        for (; x->Push(), x->l && x->l->sz > k; x = x->l);
        if (x->l) k -= x->l->sz;
        if (!k--) return Splay(x);
    }
}
Node *Gather(int s, int e) {

```

```

auto t = Kth(e + 1);
return Splay(t, Kth(s - 1))->l;
}
Node *Flip(int s, int e) {
    auto x = Gather(s, e);
    x->flip ^= 1;
    return x;
}
Node *Shift(int s, int e, int k) {
    if (k >= 0) { // shift to right
        k %= e - s + 1;
        if (k)
            Flip(s, e), Flip(s, s + k - 1), Flip(s + k, e);
    } else { // shift to left
        k = -k;
        k %= e - s + 1;
        if (k)
            Flip(s, e), Flip(s, e - k), Flip(e - k + 1, e);
    }
    return Gather(s, e);
}
int Idx(Node **x) { return x->l->sz; }
//////////////// Link Cut Tree Start ///////////////////
Node *Splay(Node **x) {
    for (; !x->IsRoot(); x->Rotate()) {
        if (!x->p->IsRoot()) x->p->p->Push();
        x->p->Push(); x->Push();
        if (!x->p->IsRoot())
            (x->IsLeft() ^ x->p->IsLeft()) ? x :
                x->p)->Rotate();
    }
    x->Push();
    return x;
}
void Access(Node **x) {
    Splay(x); x->r = nullptr;
    x->Update();
    for (auto y = x; x->p; Splay(x))
        y = x->p, Splay(y), y->r = x, y->Update();
}
int GetDepth(Node **x) {
    Access(x); x->Push();
    return GetSize(x->l);
}
Node *GetRoot(Node **x) {
    Access(x);
    for (x->Push(); x->l; x->Push())
        x = x->l;
    return Splay(x);
}
Node *GetPar(Node **x) {
    Access(x);
    for (x->Push(); x->l; x->Push())
        x = x->l;
    return Splay(x);
}
Access(x); x->Push();
if (!x->l) return nullptr;
x = x->l;
for (x->Push(); x->r; x->Push())
    x = x->r;
return Splay(x);
}
void Link(Node *p, Node *c) {
    Access(c); Access(p);
    c->l = p; p->p = c;
    c->Update();
}
void Cut(Node *c) {
    Access(c);
    c->l->p = nullptr;
    c->l = nullptr;
    c->Update();
}
Node *GetLCA(Node *x, Node *y) {
    Access(x); Access(y); Splay(x);
    return x->p ? x->p : x;
}
Node *Ancestor(Node **x, int k) {
    k = GetDepth(x) - k;
    assert(k >= 0);
    for (;;) x->Push() {
        int s = GetSize(x->l);
        if (s == k) return Access(x), x;
        if (s < k) k -= s + 1, x = x->r;
        else x = x->l;
    }
}
void MakeRoot(Node **x) {
    Access(x); Splay(x);
    x->flip ^= 1; x->Push();
}
bool IsConnect(Node **x, Node *y) { return GetRoot(x) ==
    GetRoot(y); }
void PathUpdate(Node **x, Node *y, T val) {
    Node *root = GetRoot(x); // original root
    MakeRoot(x); Access(y); Splay(x);
    x->lz += val; x->Push();
    MakeRoot(root); // Revert
    Node *lca = GetLCA(x, y);
    Access(lca); Splay(lca);
    lca->Push(); lca->Update(lca->now - val);
}
T VertexQuery(Node **x, Node *y) {
    Node *l = GetLCA(x, y); T ret = l->now;
    Access(x); Splay(l);
    if (l->r) ret = ret + l->r->sum;
}

```

```

Access(y); Splay(l);
if (l->r) ret = ret + l->r->sum;
return ret;
}
Node *GetQueryResultNode(Node *u, Node *v) {
    if (!IsConnect(u, v)) return 0;
    MakeRoot(u); Access(v);
    auto ret = v->l;
    while (ret->mx != ret->now) {
        if (ret->l && ret->mx == ret->l->mx)
            ret = ret->l;
        else ret = ret->r;
    }
    Access(ret); return ret;
}

```

4 Graph

4.1 Bellman Ford

Usage: SSSP with negative weights/cycles.
Time Complexity: $O(VE)$

```

auto bellman = [&](int s) -> bool {
    fill(all(d), INF); d[s] = 0; bool chk = 0;
    for (int i = 0; i < n; i++) { chk = 0;
        for (int u = 1; u <= n; u++) {
            if (d[u] == INF) continue;
            for (auto [w, v] : adj[u]) if (d[v] > d[u] + w) {
                d[v] = d[u] + w; chk = 1; if (i == n - 1)
                    return 0;
            }
        }
        if (!chk) break;
    }
    return 1;
};

```

4.2 SPFA (SLF Optimized)

Usage: SSSP with negative weights. Returns false if negative cycle detected.
Time Complexity: Avg $O(E)$, Worst $O(VE)$

```

auto spfa = [&](int s) -> bool {
    vector<int> c(n+1), inq(n+1); fill(all(d), INF);
    deque<int> q; q.push_back(s); d[s] = 0; inq[s] = 1;
    while (!q.empty()) {
        int u = q.front(); q.pop_front(); inq[u] = 0;
        for (auto [w, v] : adj[u]) if (d[v] > d[u] + w) {
            d[v] = d[u] + w; if (inq[v]) continue;
            if (sz(q) && d[v] < d[q.front()])
                q.push_front(v);
        }
    }
};

```

```

    else q.push_back(v);
    inq[v] = 1; if (++c[v] >= n) return 0;
}
}
return 1;
}

```

4.3 LCA

Usage: Lowest Common Ancestor using binary lifting.

Time Complexity: $O(\log N)$

```

int N, Q, D[101010], P[22][101010];
vector<int> G[101010];
void Connect(int u, int v){
    G[u].push_back(v); G[v].push_back(u);
}
void DFS(int v, int b=-1){
    for(auto i : G[v]) if(i != b) D[i] = D[v] + 1, P[0]
        [i] = v, DFS(i, v);
}
int LCA(int u, int v){
    if(D[u] < D[v]) swap(u, v);
    int diff = D[u] - D[v];
    for(int i=0; diff; i++, diff>>=1) if(diff & 1) u =
        P[i][u];
    if(u == v) return u;
    for(int i=21; i>=0; i--) if(P[i][u] != P[i][v]) u =
        P[i][u], v = P[i][v];
    return P[0][u];
}
// 1. Connect로 간선 추가 2. DFS(1) 호출 3. 아래 코드
실행
for(int i=1; i<22; i++) for(int j=1; j<=N; j++) P[i][j] =
P[i-1][P[i-1][j]];
// 4. LCA(u, v)로 최소 공통 조상 구할 수 있음

```

4.4 HLD

Usage: Heavy-Light Decomposition for path queries on trees.

Time Complexity: $O(\log^2 N)$

```

struct HLD{
    vector<int> dep, par, sz, in, out, top;
    int n, idx;
    vector<vector<int>> adj, graph;
    HLD (int n_) : n(n_), dep(n+1), par(n+1), sz(n+1),
        in(n+1), out(n+1), top(n+1), adj(n+1), graph(n+1) {}
    void addEdge(int u, int v) { adj[u].push_back(v);
        adj[v].push_back(u); }
    void dfs(int v = 1, int pre = -1) {
        for (int u : adj[v]) {

```

```

            if (u == pre) continue;
            graph[v].push_back(u);
            dfs(u, v);
        }
        void dfs1(int v = 1) {
            sz[v] = 1;
            for (int &u : graph[v]) {
                dep[u] = dep[v] + 1;
                par[u] = v;
                dfs1(u);
                sz[v] += sz[u];
                if (sz[u] > sz[graph[v][0]]) swap(u, graph[v]
                    [0]);
            }
        }
        void dfs2(int v = 1) {
            in[v] = ++idx;
            for (int u : graph[v]) {
                top[u] = (u == graph[v][0]) ? top[v] : u;
                dfs2(u);
            }
            out[v] = idx;
        }
        void calculate(){
            dfs(); dfs1(); dfs2();
        }
    array<vector<array<int,2>>,2> getPath(int u, int v) {
        vector<array<int,2>> v1, v2;
        while (top[u] != top[v]) {
            if (dep[top[u]] > dep[top[v]]) {
                ll xx = top[u];
                v1.push_back({in[xx], in[u]});
                u = par[xx];
            }else {
                ll xx = top[v];
                v2.push_back({in[xx], in[v]});
                v = par[xx];
            }
            if (dep[u] < dep[v]) {
                v2.push_back({in[u], in[v]});
            }else {
                v1.push_back({in[v], in[u]});
            }
        }
        return {v1, v2};
    }
    // auto pp = hld.getPath(u, v);
    // Node res1 = id;
    // Node res2 = id;
    // for (auto p2 : pp[0]){
    //     res1 = seg.merge(seg.query(p2[0], p2[1]+1),
    //         res1);

```

```

    }
    // for (auto p2 : pp[1]){
    //     res2 = seg.merge(seg.query(p2[0], p2[1]+1),
    //         res2);
    // }
    // swap(res1.lsum, res1.rsum);
    // auto res = seg.merge(res1, res2);
}
}

```

4.5 Centroid Decomposition

Usage: Divide and conquer on trees for path/distance problems.

Time Complexity: $O(N \log N)$ build

```

struct CentroidTree {
    vector<vector<int>> adj, c_adj; // adj: 원본트리 /
    c_adj: 센트로이드트리
    vector<int> sz, par, vis; int N;
    CentroidTree(int n) : N(n), adj(n+1), c_adj(n+1),
        sz(n+1), par(n+1), vis(n+1) {}
    void add_edge(int u, int v) { adj[u].push_back(v);
        adj[v].push_back(u); }
    int get_sz(int curr, int prev) {
        sz[curr] = 1;
        for (int next : adj[curr])
            if (next != prev && !vis[next]) sz[curr] +=
                get_sz(next, curr);
        return sz[curr];
    }
    int get_cent(int curr, int prev, int to_sz) {
        for (int next : adj[curr])
            if (next != prev && !vis[next] && sz[next] >
                to_sz / 2)
                return get_cent(next, curr, to_sz);
        return curr;
    }
    void static_solve(int u) { /* 현재 센트로이드를 포함
        하는 모든 경로를 계산하는 로직 */ }
    int build(int curr, int p = -1) {
        int cent = get_cent(curr, -1, get_sz(curr, -1));
        static_solve(cent); // 정적 분할 정복 문제일 때 사용
        vis[cent] = 1; par[cent] = p;
        for (int next : adj[cent]) {
            if (!vis[next]) {
                int child = build(next, cent);
                c_adj[cent].push_back(child); // 센트로이드 계
                층 연결
            }
        }
    }
}

```

```

}
return cent;
}

```

4.6 Bipartite Matching

Usage: Maximum matching in bipartite graphs.

Time Complexity: $O(E\sqrt{V})$

```

struct BiMatch { // Hopcroft-Karp
    vector<vector<int>> graph, grev;
    vector<int> mA, mB, dist, work;
    vector<bool> visA, visB, fA, fB; // vertex i can be
    excluded from some max matching
    int ns, ms;
    BiMatch(int n, int m) : ns(n), ms(m), graph(n+1),
    grev(m+1), mA(n+1), mB(m+1), dist(n+1), work(n+1) {}
    void add(int a, int b) { graph[a].push_back(b);
    grev[b].push_back(a); }
    void bfs() {
        fill(all(dist), -1);
        queue<int> q;
        for (int i = 1; i <= ns; i++) if (!mA[i]) {
            dist[i] = 0; q.push(i);
        }
        while (!q.empty()) {
            int i = q.front(); q.pop();
            for (auto j : graph[i]) {
                int k = mB[j];
                if (k && dist[k] == -1) {
                    dist[k] = dist[i] + 1; q.push(k);
                }
            }
        }
    }
    bool dfs(int cur) {
        for (int& i = work[cur]; i < sz(graph[cur]); i++) {
            int nb = graph[cur][i], ori = mB[nb];
            if (!ori || dist[ori] == dist[cur] + 1 &&
            dfs(ori)) {
                mA[cur] = nb; mB[nb] = cur; return true;
            }
        }
        return false;
    }
    int match() {
        int ans = 0;
        while (true) {
            fill(all(work), 0); bfs();
            int cnt = 0;
            for (int i = 1; i <= ns; i++) {

```

```

                if (!mA[i] && dfs(i)) cnt++;
            }
            if (!cnt) break;
            ans += cnt;
        }
        return ans;
    }
    void chkEss() {
        fA.assign(ns+1, 0); fB.assign(ms+1, 0);
        visA.assign(ns+1, 0); visB.assign(ms+1, 0);
        queue<int> q;
        for (int i = 1; i <= ns; i++) if (!mA[i]) fA[i] =
        visA[i] = 1, q.push(i);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (int v : graph[u]) if (!visB[v]) {
                visB[v] = 1; int r = mB[v];
                if (r && !fA[r]) {
                    fA[r] = visA[r] = 1; q.push(r);
                }
            }
        }
        for (int i = 1; i <= ms; i++) if (!mB[i]) fB[i] =
        1, q.push(i);
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (int u : grev[v]) {
                int r = mA[u];
                if (r && !fB[r]) {
                    fB[r] = 1; q.push(r);
                }
            }
        }
        pair<vector<int>, vector<int>> vertex() { // find
            minimum vertex cover
            chkEss();
            vector<int> va, vb;
            for (int i = 1; i <= ns; i++) if (!visA[i])
            va.push_back(i);
            for (int i = 1; i <= ms; i++) if (visB[i])
            vb.push_back(i);
            return {va, vb};
        }
        /* struct BiMatch {
            vector<vector<int>> graph;
            vector<int> mA, mB, vis;
            int ns, ms;
            BiMatch(int n, int m) : ns(n), ms(m), graph(n+1),
            mA(n+1), mB(m+1), vis(n+1) {}
```

```

        void add(int a, int b) { graph[a].push_back(b); }
        bool dfs(int cur) {
            vis[cur] = 1;
            for (auto i : graph[cur]) {
                int ori = mB[i];
                if (ori == 0 || (!vis[ori] && dfs(ori))) {
                    mA[cur] = i; mB[i] = cur; return true;
                }
            }
            return false;
        }
        int match() {
            int res = 0;
            for (int i = 1; i <= ns; i++) {
                if (mA[i]) continue;
                fill(all(vis), 0);
                if (dfs(i)) res++;
            }
            return res;
        }
    }; */
}

4.7 Dinic
```

Usage: Efficient maximum flow algorithm.

Time Complexity: $O(V^2E)$

```

const ll INF = 1e18;
struct Dinic {
    struct Edge { int to; ll cap; int rev; };
    vector<vector<Edge>> graph;
    vector<int> level, work; int n;
    Dinic(int n) : n(n), graph(n+1), level(n+1),
    work(n+1) {}
    void add(int u, int v, ll cap) {
        graph[u].push_back({v, cap, sz(graph[v])});
        graph[v].push_back({u, 0, sz(graph[u])-1});
    }
    bool bfs(int s, int t) {
        fill(all(level), -1); level[s] = 0;
        queue<int> q; q.push(s);
        while (!q.empty()) {
            int cur = q.front(); q.pop();
            for (auto [nxt, cap, rev] : graph[cur]) {
                if (cap > 0 && level[nxt] == -1) {
                    level[nxt] = level[cur]+1;
                    q.push(nxt);
                }
            }
        }
        return (level[t] != -1);
    }
}
```

```

}
ll dfs(int cur, int t, ll flow) {
    if (cur == t) return flow;
    for (int& i = work[cur]; i < sz(graph[cur]); i++) {
        auto& [nxt, cap, rev] = graph[cur][i];
        if (cap > 0 && level[nxt] == level[cur]+1) {
            ll push = dfs(nxt, t, min(flow, cap));
            if (push > 0) {
                cap -= push; graph[nxt][rev].cap += push;
                return push;
            }
        }
    }
    return 0;
}
ll flow(int s, int t) {
    ll ans = 0;
    while (bfs(s, t)) {
        fill(all(work), 0);
        while (auto flow = dfs(s, t, INF)) ans += flow;
    }
    return ans;
}
vector<bool> mincut(int s) {
    vector<bool> vis(n+1); vis[s] = true;
    queue<int> q; q.push(s);
    while (!q.empty()) {
        int cur = q.front(); q.pop();
        for (auto [nxt, cap, rev] : graph[cur]) {
            if (cap > 0 && !vis[nxt]) {
                vis[nxt] = true; q.push(nxt);
            }
        }
    }
    return vis;
}
};

4.8 MCMF

```

Usage: Minimum Cost Maximum Flow using SPFA.

Time Complexity: $O(F \cdot E \log V)$

```

const ll INF = 1e18;
struct MCMF {
    struct Edge { int to; ll cap, cost; int rev; };
    vector<vector<Edge>> graph;
    vector<ll> dist;
    vector<int> parent, edge;
    vector<bool> vis;
    int n;
};

```

```

MCMF(int n) : n(n), graph(n+1), dist(n+1),
parent(n+1), edge(n+1), vis(n+1) {}
void add(int u, int v, ll cap, ll cost) {
    graph[u].push_back({v, cap, cost, sz(graph[v])});
    graph[v].push_back({u, 0, -cost, sz(graph[u])-1});
}
bool spfa(int s, int t) {
    fill(all(dist), INF); fill(all(parent), -1);
    fill(all(vis), false);
    queue<int> q; q.push(s);
    dist[s] = 0; vis[s] = true;
    while (!q.empty()) {
        int cur = q.front(); q.pop();
        vis[cur] = false;
        for (int i = 0; i < sz(graph[cur]); i++) {
            auto& [nxt, cap, cost, rev] = graph[cur][i];
            if (cap > 0 && dist[nxt] > dist[cur] + cost) {
                dist[nxt] = dist[cur] + cost;
                parent[nxt] = cur; edge[nxt] = i;
                if (!vis[nxt]) {
                    vis[nxt] = true; q.push(nxt);
                }
            }
        }
    }
    return dist[t] != INF;
}
pair<int,ll> flow(int s, int t) {
    int res = 0; ll cost = 0;
    while (spfa(s, t)) {
        ll fl = INF;
        for (int v = t; v != s; v = parent[v]) {
            int u = parent[v], idx = edge[v];
            fl = min(fl, graph[u][idx].cap);
        }
        for (int v = t; v != s; v = parent[v]) {
            int u = parent[v], idx = edge[v], ridx =
graph[u][idx].rev;
            graph[u][idx].cap -= fl;
            graph[v][ridx].cap += fl;
            cost += (ll)fl * graph[u][idx].cost;
        }
        res += fl;
    }
    return {res, cost};
}

```

4.9 Circulation

Usage: Flow with lower and upper bounds.

```

const ll INF = 1e18;
struct Dinic {};// or MCMF
struct Circulation {
    vector<ll> demand, low;
    vector<pair<int,int>> edge;
    int n, S, T; Dinic dn;
    Circulation(int n) : n(n), S(n+1), T(n+2),
    demand(n+3, 0), dn(n+2) {}
    void add_demand(int u, ll d) { demand[u] += d; }
    int add(int u, int v, ll l, ll r) {
        demand[u] -= l; demand[v] += l;
        dn.add(u, v, r - l); low.push_back(l);
        edge.push_back({u, sz(dn.graph[u])-1});
    }
    ll solve() {
        ll sum = 0, res = 0;
        for (int i = 1; i <= n; i++) sum += demand[i];
        if (sum != 0) return false;
        for (int i = 1; i <= n; i++) {
            if (demand[i] > 0) {
                dn.add(S, i, demand[i]); res += demand[i];
            }
            else if (demand[i] < 0) dn.add(i, T, -demand[i]);
        }
        ll f = dn.flow(S, T);
        return (f != res ? -1 : f);
    }
    ll get_flow(int i) { // get actual flow
        auto [u, idx] = edge[i];
        int v = dn.graph[u][idx].to, rev = dn.graph[u][idx].rev;
        return dn.graph[v][rev].cap + low[i];
    }
};

```

4.10 SCC

Usage: Strongly Connected Components.

Time Complexity: $O(V + E)$

```

struct SCC {
    int n, cnt, timer;
    vector<vector<int>> adj;
    vector<int> dfn, low, id;
    vector<bool> ins; stack<int> st;
    SCC(int n) : n(n), adj(n), dfn(n, -1), low(n, -1),
    id(n, -1), ins(n), cnt(0), timer(0) {}
    void add_edge(int u, int v) { adj[u].push_back(v); }
    void dfs(int u) {
        dfn[u] = low[u] = ++timer;

```

```

st.push(u); ins[u] = true;
for (int v : adj[u]) {
    if (dfn[v] == -1) {
        dfs(v); low[u] = min(low[u], low[v]);
    } else if (ins[v]) {
        low[u] = min(low[u], dfn[v]);
    }
}
if (low[u] == dfn[u]) {
    while (true) {
        int v = st.top(); st.pop();
        ins[v] = false; id[v] = cnt;
        if (u == v) break;
    }
    cnt++;
}
void build() {
    for (int i = 0; i < n; i++)
        if (dfn[i] == -1) dfs(i);
}

```

4.11 2-sat

Usage: Solves 2-SAT in $O(V + E)$; x_i is true if $SCC(x_i) < SCC(\neg x_i)$.

Time Complexity: $O(V + E)$

```

struct TwoSat {
    int n; SCC scc;
    vector<bool> result;
    TwoSat(int n) : n(n), scc(2 * n + 2), result(n + 1) {}
    // (x_a == is_a) OR (x_b == is_b) 추가
    void add_clause(int a, bool is_a, int b, bool is_b) {
        int u = a << 1 | !is_a, v = b << 1 | !is_b;
        scc.add_edge(u ^ 1, v);
        scc.add_edge(v ^ 1, u);
    }
    bool satisfiable() {
        scc.build();
        for (int i = 1; i <= n; i++) {
            if (scc.scc_id[i << 1] == scc.scc_id[i << 1 | 1])
                return false;
            result[i] = scc.scc_id[i << 1] < scc.scc_id[i << 1 | 1];
        }
        return true;
    }
    bool get(int i) { return result[i]; }
}

```

4.12 BCC

Usage: Biconnected Components, Cut-vertices, and Bridges.
Time Complexity: $O(V + E)$

```

// 1-based, 다른 거 호출하기 전에 tarjan 먼저 호출
vector<int> G[MAX_V]; int In[MAX_V], Low[MAX_V],
P[MAX_V];
void addEdge(int s, int e){ G[s].push_back(e);
G[e].push_back(s); }
void tarjan(int n){ // Pre-Process
    int pv = 0;
    function<void(int,int)> dfs = [&pv,&dfs](int v, int b){
        In[v] = Low[v] = ++pv; P[v] = b;
        for(auto i : G[v]){
            if(i == b) continue;
            if(!In[i]) dfs(i, v), Low[v] = min(Low[v],
Low[i]);
            else Low[v] = min(Low[v], In[i]);
        }
    };
    for(int i=1; i<=n; i++) if(!In[i]) dfs(i, -1);
}
vector<int> cutVertex(int n){
    vector<int> res; array<char,MAX_V> isCut;
    isCut.fill(0);
    function<void(int)> dfs = [&dfs,&isCut](int v){
        int ch = 0;
        for(auto i : G[v]){
            if(P[i] != v) continue; dfs(i); ch++;
            if(P[v] == -1 && ch > 1) isCut[v] = 1;
            else if(P[v] != -1 && Low[i] >= In[v])
                isCut[v]=1;
        }
    };
    for(int i=1; i<=n; i++) if(P[i] == -1) dfs(i);
    for(int i=1; i<=n; i++) if(isCut[i])
        res.push_back(i);
    return move(res);
}
vector<PII> cutEdge(int n){
    vector<PII> res;
    function<void(int)> dfs = [&dfs,&res](int v){
        for(int t=0; t<G[v].size(); t++){
            int i = G[v][t]; if(t != 0 && G[v][t-1] == G[v]
[t]) continue;
            if(P[i] != v) continue; dfs(i);
            if((t+1 == G[v].size() || i != G[v][t+1]) &&
Low[i] > In[v]) res.emplace_back(min(v,i),
max(v,i));
        }
    };
    for(int i=0; i<n; i++) pos[i] = i;
    for(auto [u,v] : edges)
        gph[pos[u]].push_back(pos[v]),
        gph[pos[v]].push_back(pos[u]);
    for(int i=0; i<n; i++){
        for(auto j : gph[i]){
            if(i > j) continue;

```

```

        };
        for(int i=1; i<=n; i++) sort(G[i].begin(),
G[i].end()); // multi edge -> sort
        for(int i=1; i<=n; i++) if(P[i] == -1) dfs(i);
        return move(res); // sort(all(res));
    }
    vector<int> BCC[MAX_V]; // BCC[v] = components which
contains v
    void vertexDisjointBCC(int n){ // allow multi edge, not
allow self loop
        int cnt = 0; array<char,MAX_V> vis; vis.fill(0);
        function<void(int,int)> dfs = [&dfs,&vis,&cnt](int v,
int c){
            vis[v] = 1; if(c > 0) BCC[v].push_back(c);
            for(auto i : G[v]){
                if(vis[i]) continue;
                if(In[v] <= Low[i]) BCC[v].push_back(++cnt),
dfs(i, cnt);
                else dfs(i, c);
            }
        };
        for(int i=1; i<=n; i++) if(!vis[i]) dfs(i, 0);
        for(int i=1; i<=n; i++) if(BCC[i].empty())
            BCC[i].push_back(++cnt);
    }
}

```

4.13 Find 3 or 4 cycle

Usage: Finds all C_3 and C_4 cycles using degree ordering.
Time Complexity: $O(M\sqrt{M})$

```

vector<tuple<int,int,int>> Find3Cycle(int n, const
vector<pair<int,int>> &edges){ // N+Msqrtn
    int m = edges.size();
    vector<int> deg(n), pos(n), ord; ord.reserve(n);
    vector<vector<int>> gph(n), que(m+1), vec(n);
    vector<vector<tuple<int,int,int>>> tri(n);
    vector<tuple<int,int,int>> res;
    for(auto [u,v] : edges) deg[u]++;
    for(int i=0; i<n; i++) que[deg[i]].push_back(i);
    for(int i=m; i>=0; i--) ord.insert(ord.end(),
que[i].begin(), que[i].end());
    for(int i=0; i<n; i++) pos[ord[i]] = i;
    for(auto [u,v] : edges)
        gph[pos[u]].push_back(pos[v]),
        gph[pos[v]].push_back(pos[u]);
    for(int i=0; i<n; i++){
        for(auto j : gph[i]){
            if(i > j) continue;

```

```

    for(int x=0, y=0; x<vec[i].size() &&
        y<vec[j].size(); ) {
        if(vec[i][x] == vec[j][y])
            res.emplace_back(ord[i], ord[j], ord[vec[i]
                [x]]), x++, y++;
        else if(vec[i][x] < vec[j][y]) x++; else y++;
    }
    vec[j].push_back(i);
}

for(auto &[u,v,w] : res){
    if(pos[u] < pos[v]) swap(u, v);
    if(pos[u] < pos[w]) swap(u, w);
    if(pos[v] < pos[w]) swap(v, w);
    tri[u].emplace_back(u, v, w);
}
res.clear();
for(int i=n-1; i>=0; i--) res.insert(res.end(),
tri[ord[i]].begin(), tri[ord[i]].end());
return res;
}

bitset<500> B[500]; // N3/w
long long Count3Cycle(int n, const
vector<pair<int,int>> &edges) {
    long long res = 0;
    for(int i=0; i<n; i++) B[i].reset();
    for(auto [u,v] : edges) B[u].set(v), B[v].set(u);
    for(int i=0; i<n; i++) for(int j=i+1; j<n; j++)
        if(B[i].test(j)) res += (B[i] & B[j]).count();
    return res / 3;
}

// O(n + m * sqrt(m) + th) for graphs without loops or
multiedges
void Find4Cycle(int n, const vector<array<int, 2>>
&edge, auto process, int th = 1) {
    int m = (int)edge.size();
    vector<int> deg(n), order, pos(n);
    vector<vector<int>> appear(m+1), adj(n), found(n);
    for(auto [u, v]: edge) ++deg[u], ++deg[v];
    for(auto u=0; u<n; u++) appear[deg[u]].push_back(u);
    for(auto d=m; d>=0; d--) order.insert(order.end(),
appear[d].begin(), appear[d].end());
    for(auto i=0; i<n; i++) pos[order[i]] = i;
    for(auto i=0; i<m; i++) {
        int u = pos[edge[i][0]], v = pos[edge[i][1]];
        adj[u].push_back(v), adj[v].push_back(u);
    }
    T res = 0; vector<int> cnt(n);
    for(auto u=0; u<n; u++) {
        for(auto v: adj[u]) if(u < v) for(auto w: adj[v])
            if(u < w) cnt[w] = 0;
    }
}

```

```

    for(auto v: adj[u]) if(u < v) for(auto w: adj[v])
        if(u < w) res += cnt[w]++;
    }

    for(auto u=0; u<n; u++){
        for(auto v: adj[u]) if(u < v) for(auto w: adj[v])
            if(u < w) found[w].clear();
        for(auto v: adj[u]) if(u < v) for(auto w: adj[v])
            if(u < w) {
                for(auto x: found[w]){
                    if(!th--) return;
                    process(order[u], order[v], order[w],
                        order[x]);
                }
                found[w].push_back(v);
            }
    }
}

4.14 Push Relabel

Usage: Max flow via preflow-push and labeling
Time Complexity:  $O(V^3)$ 

const ll INF = 1e18;
struct HLPP {
    struct Edge {
        int to; ll cap; int rev;
    };
    vector<vector<Edge>> graph;
    vector<ll> ex;
    vector<int> level, work;
    vector<vector<int>> B;
    int n, high = 0, cnt = 0;
} HLPP(int n) : n(n+1), graph(n+1), level(n+1),
work(n+1), ex(n+1), B(n+1) {}

void add(int u, int v, ll cap) {
    graph[u].push_back({v, cap, sz(graph[v])});
    graph[v].push_back({u, cap, sz(graph[u])-1});
}

void push(int u) {
    if (level[u] >= n) return;
    B[level[u]].push_back(u);
    high = max(high, level[u]);
}

void relabel(int t) {
    cnt = 0;
    for (auto& b : B) b.clear();
    fill(all(level), n);
    queue<int> q; q.push(t);
    level[t] = 0;
    while (!q.empty()) {
        int u = q.front(); q.pop();

```

```

        int h = level[u] + 1;
        for (auto& e : graph[u]) {
            if (graph[e.to][e.rev].cap > 0 && h <
                level[e.to]) {
                level[e.to] = h; q.push(e.to);
                if (ex[e.to] > 0) push(e.to);
            }
        }
    }

    void discharge(int u) {
        auto& excess = ex[u];
        int h = n * 2, sz = sz(graph[u]);
        for (int& i = work[u], m = sz; m-- ; i = (i-1 + sz)
% sz) {
            auto& e = graph[u][i];
            if (!e.cap) continue;
            if (level[u] != level[e.to] + 1) {
                h = min(h, level[e.to] + 1);
                continue;
            }
            auto f = min(e.cap, excess);
            e.cap -= f;
            excess -= f;
            if (!ex[e.to]) push(e.to);
            ex[e.to] += f;
            graph[e.to][e.rev].cap += f;
            if (!excess) return;
        }
        cnt++; level[u] = h;
        if (level[u] < n && ex[u] > 0) push(u);
    }

    ll flow(int s, int t) {
        relabel(t);
        ex[s] = INF; ex[t] = -INF;
        push(s);
        for (; ~high; high--) {
            while (!B[high].empty()) {
                int u = B[high].back();
                B[high].pop_back();
                if (level[u] == high) discharge(u);
                if (cnt > n/8) relabel(t);
            }
        }
        return ex[t] + INF;
    }
}

```

4.15 General Matching

Usage: Maximum Unweighted Matching.

Time Complexity: $O(N^3)$

```

struct GeneralMatch {
    vector<vector<int>> graph;
    vector<int> vis, parent, orig, matched, aux;
    queue<int> q; int n, t = 0;
    GeneralMatch(int n) : n(n) {
        auto init = [&](auto&... vecs) { (vecs.resize(n+1),
            ...); };
        init(graph, vis, parent, orig, matched, aux);
    }
    void add(int a, int b) { graph[a].push_back(b);
        graph[b].push_back(a); }
    void augment(int u, int v) {
        while (v) {
            int pv = parent[v], nv = matched[pv];
            matched[v] = pv; matched[pv] = v;
            v = nv;
        }
    }
    int lca(int v, int w) {
        ++t;
        while (1) {
            if (v) {
                if (aux[v] == t) return v;
                aux[v] = t; v = orig[parent[matched[v]]];
            }
            swap(v, w);
        }
    }
    void blossom(int v, int w, int a) {
        while (orig[v] != a) {
            parent[v] = w; w = matched[v];
            if (vis[w] == 1) { q.push(w), vis[w] = 0; }
            orig[v] = orig[w] = a; v = parent[w];
        }
    }
    bool bfs(int u, int ban = 0) {
        fill(all(vis), -1); iota(all(orig), 0);
        if (ban) vis[ban] = 2;
        q = queue<int>(); q.push(u); vis[u] = 0;
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (auto w : graph[v]) {
                if (vis[w] == -1) {
                    parent[w] = v; vis[w] = 1;
                    if (!matched[w]) { augment(u, w); return
                        true; }
                    vis[matched[w]] = 0; q.push(matched[w]);
                }
            else if (vis[w] == 0 && orig[v] != orig[w]) {
                int a = lca(orig[v], orig[w]);
            }
        }
    }
}
```

```

        blossom(w, v, a); blossom(v, w, a);
    }
}
return false;
}
int match() {
    int ans = 0;
    for (int i = 1; i <= n; i++) {
        if (!matched[i] && bfs(i)) ans++;
    }
    return ans;
}
bool chk(int u) { // find max matching except u
    if (!matched[u]) return true;
    auto backup = matched;
    int v = matched[u];
    matched[u] = matched[v] = 0;
    bool res = bfs(v, u);
    matched = backup;
    return res;
}
}
```

4.16 Weighed General Matching

Usage: Maximum Weighted Matching.
Time Complexity: $O(N^3)$

```

const ll INF = 1e18;
struct WeightedGeneralMatch {
    struct Edge { int u = 0, v = 0; ll w = 0; };
    int n, nx;
    vector<vector<Edge>> adj; vector<ll> label;
    vector<int> matched, slack, root, parent, state, vis;
    vector<vector<int>> node, via;
    queue<int> q; int time = 0;
    WeightedGeneralMatch(int n) : n(n), nx(n) {
        int size = 2*n+1;
        auto init = [&](auto&... vecs) {
            (vecs.resize(size), ...); };
        init(label, matched, slack, root, parent, state,
            vis, node);
        adj.resize(size, vector<Edge>(size));
        via.resize(size, vector<int>(n+1));
        for (int u = 1; u <= n; u++)
            for (int v = 1; v <= n; v++)
                adj[u][v] = { u, v, 0 };
    }
    void add(int u, int v, ll w) { adj[u][v].w = adj[v]
        [u].w = w; }
}
```

```

    ll calc(Edge e) { return label[e.u] + label[e.v] -
        adj[e.u][e.v].w * 2; }
    void minSlack(int u, int x) {
        if (!slack[x] || calc(adj[u][x]) <
            calc(adj[slack[x]][x]))
            slack[x] = u;
    }
    void updSlack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; u++) {
            if (adj[u][x].w > 0 && root[u] != x &&
                state[root[u]] == 0) {
                minSlack(u, x);
            }
        }
    }
    void updRoot(int x, int b) {
        root[x] = b;
        if (x > n) for (auto i : node[x]) updRoot(i, b);
    }
    void enqueue(int x) {
        if (x <= n) q.push(x);
        else for (auto i : node[x]) enqueue(i);
    }
    int getIdx(int b, int xr) {
        int idx = find(all(node[b]), xr) - node[b].begin();
        if (idx % 2 == 1) {
            reverse(node[b].begin() + 1, node[b].end());
            return sz(node[b]) - idx;
        }
        return idx;
    }
    void rematch(int u, int v) {
        matched[u] = adj[u][v].v;
        if (u <= n) return;
        Edge e = adj[u][v];
        int xr = via[u][e.u], pr = getIdx(u, xr);
        for (int i = 0; i < pr; i++)
            rematch(node[u][i], node[u][i ^ 1]);
        rematch(xr, v);
        rotate(node[u].begin(), node[u].begin() + pr,
            node[u].end());
    }
    void augment(int u, int v) {
        while (true) {
            int xnv = root[matched[u]];
            rematch(u, v);
            if (!xnv) return;
            rematch(xnv, root[parent[xnv]]);
            u = root[parent[xnv]]; v = xnv;
        }
    }
}
```

```

}

int findLCA(int u, int v) {
    time++;
    while (u || v) {
        if (u == 0) { swap(u, v); continue; }
        if (vis[u] == time) return u;
        vis[u] = time; u = root[matched[u]];
        if (u) u = root[parent[u]];
        swap(u, v);
    }
    return 0;
}

void addBlossom(int u, int lca, int v) {
    int b = n+1;
    while (b <= nx && root[b]) b++;
    if (b > nx) nx++;
    label[b] = 0, state[b] = 0;
    matched[b] = matched[lca];
    node[b].clear(); node[b].push_back(lca);
    for (int x = u, y; x != lca; x = root[parent[y]]) {
        node[b].push_back(x), node[b].push_back(y =
            root[matched[x]]);
        enqueue(y);
    }
    reverse(node[b].begin() + 1, node[b].end());
    for (int x = v, y; x != lca; x = root[parent[y]]) {
        node[b].push_back(x), node[b].push_back(y =
            root[matched[x]]);
        enqueue(y);
    }
    updRoot(b, b);
    for (int x = 1; x <= nx; x++)
        adj[b][x].w = adj[x][b].w = 0;
    for (int x = 1; x <= n; x++) via[b][x] = 0;
    for (int xs : node[b]) {
        for (int x = 1; x <= nx; x++) {
            if (adj[b][x].w == 0 || calc(adj[xs][x]) <
                calc(adj[b][x])) {
                adj[b][x] = adj[xs][x];
                adj[x][b] = adj[x][xs];
            }
        }
        for (int x = 1; x <= n; x++)
            if (via[xs][x])
                via[b][x] = xs;
    }
    updSlack(b);
}

void expandBlossom(int b) {
    for (auto i : node[b]) updRoot(i, i);
    int xr = via[b][adj[b][parent[b]].u], pr =
        getIdx(b, xr);
}

```

```

for (int i = 0; i < pr; i+=2) {
    int xs = node[b][i], xns = node[b][i + 1];
    parent[xs] = adj[xns][xs].u;
    state[xs] = 1; state[xns] = slack[xs] = 0;
    updSlack(xns); enqueue(xns);
}
state[xr] = 1; parent[xr] = parent[b];
for (int i = pr+1; i < sz(node[b]); i++) {
    int xs = node[b][i];
    state[xs] = -1; updSlack(xs);
}
root[b] = 0;
}

bool inspect(Edge e) {
    int u = root[e.u], v = root[e.v];
    if (state[v] == -1) {
        parent[v] = e.u; state[v] = 1;
        int nu = root[matched[v]];
        slack[v] = slack[nu] = 0;
        state[nu] = 0; enqueue(nu);
    }
    else if (state[v] == 0) {
        int lca = findLCA(u, v);
        if (!lca) {
            augment(u, v); augment(v, u);
            return true;
        }
        else addBlossom(u, lca, v);
    }
    return false;
}

bool matching() {
    fill(state.begin(), state.begin() + nx + 1, -1);
    fill(slack.begin(), slack.begin() + nx + 1, 0);
    q = queue<int>();
    for (int x = 1; x <= nx; x++) {
        if (root[x] == x && !matched[x]) {
            parent[x] = 0; state[x] = 0;
            enqueue(x);
        }
    }
    if (q.empty()) return false;
    while (true) {
        while (!q.empty()) {
            int u = q.front(); q.pop();
            if (state[root[u]] == 1) continue;
            for (int v = 1; v <= n; v++) {
                if (adj[u][v].w > 0 && root[u] != root[v]) {
                    if (calc(adj[u][v]) == 0) {
                        if (inspect(adj[u][v])) return true;
                    }
                    else minSlack(u, root[v]);
                }
            }
        }
    }
}

```

```

}

ll d = INF;
for (int b = n+1; b <= nx; b++) {
    if (root[b] == b && state[b] == 1) {
        d = min(d, label[b] / 2);
    }
}
for (int x = 1; x <= nx; x++) {
    if (root[x] == x && slack[x]) {
        if (state[x] == -1) d = min(d,
            calc(adj[slack[x]][x]));
        else if (state[x] == 0) d = min(d,
            calc(adj[slack[x]][x]) / 2);
    }
}
for (int u = 1; u <= n; u++) {
    if (state[root[u]] == 0) {
        if (label[u] <= d) return false;
        label[u] -= d;
    }
    else if (state[root[u]] == 1) label[u] += d;
}
for (int b = n + 1; b <= nx; b++) {
    if (root[b] == b) {
        if (state[root[b]] == 0) label[b] += d * 2;
        else if (state[root[b]] == 1) label[b] -= d * 2;
    }
}
q = queue<int>();
for (int x = 1; x <= nx; x++) {
    if (root[x] == x && slack[x] && root[slack[x]] != x && calc(adj[slack[x]][x]) == 0) {
        if (inspect(adj[slack[x]][x])) return true;
    }
}
for (int b = n + 1; b <= nx; b++) {
    if (root[b] == b && state[b] == 1 && label[b] == 0) expandBlossom(b);
}
return false;
}

pair<ll,int> match() {
    fill(matched.begin(), matched.begin() + n+1, 0);
    nx = n;
    ll cost = 0; int res = 0;
}

```

```

for (int u = 0; u <= n; u++) {
    root[u] = u;
    node[u].clear();
}
ll maxw = 0;
for (int u = 1; u <= n; u++) {
    for (int v = 1; v <= n; v++) {
        via[u][v] = (u == v ? 0 : 0);
        maxw = max(maxw, adj[u][v].w);
    }
}
for (int u = 1; u <= n; u++) label[u] = maxw;
while (matching()) res++;
for (int u = 1; u <= n; u++) {
    if (matched[u] && matched[u] < u) {
        cost += adj[u][matched[u]].w;
    }
}
return {cost, res};
}

```

5 DP Optimization

5.1 Convex Hull Trick

Usage: $dp[i] = \min(dp[j] + b[j] * a[i])$, $b[j] \geq b[j+1]$

Time Complexity: $O(N \log N)$

```

// O(logN) Dynamic CHT: Slopes(k) and queries(x) can be
// in any order (no sorting required)
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) { // y = kx + m
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
    }
};

```

```

if (x != begin() && isect(--x, y)) isect(x, y =
erase(y));
while ((y = x) != begin() && (--x)->p >= y->p)
isect(x, erase(y));
}
ll query(ll x) {
assert(!empty());
auto l = *lower_bound(x);
return l.k * x + l.m;
}
} CHT; // add(-k, -m), -query(x) for Lower hull(min)
int main() {
dp[0] = 0; CHT.add(a[0], dp[0]);
for (int i = 1; i < n; i++) { // dp[i] = Max
j < i a[j]*b[i] + dp[j])
dp[i] = CHT.query(b[i]);
CHT.add(a[i], dp[i]);
}
cout << dp[n-1] << "\n";
}

```

5.2 Linear CHT

Usage: CHT when slopes/queries are monotonic.

Time Complexity: $O(N)$

```

// O(1) CHT: Both slopes (k) and queries (x) must be
// monotonic (sorted).
struct PLL {
    ll x, y;
    PLL(const ll x = 0, const ll y = 0) : x(x), y(y) {}
    bool operator<= (const PLL& i) const { return 1. * x
    / y <= 1. * i.x / i.y; }
};

struct ConvexHull {
    static ll F(const PLL& i, const ll x) { return i.x *
    x + i.y; }
    static PLL C(const PLL& a, const PLL& b) { return {
        a.y - b.y, b.x - a.x }; }
    deque<PLL> S;
    void add(const ll a, const ll b) {
        while (S.size() > 1 && C(S.back(), PLL(a, b)) <=
        C(S[S.size() - 2], S.back())) S.pop_back();
        S.push_back(PLL(a, b));
        /* when x is monotonic decreasing
        while (S.size() > 1 && C(S[0], S[1]) <= C(PLL(a,
        b), S[0])) S.pop_front()
        S.push_front(PLL(a, b)); */
    }
    ll query(const ll x) {
        while (S.size() > 1 && F(S[0], x) <= F(S[1], x))
        S.pop_front(); // upper hull(max)
    }
};

```

```

// while (S.size() > 1 && F(S[0], x) >= F(S[1], x))
S.pop_front(); // lower hull(min)
return F(S[0], x);
}
} CHT;

```

5.3 D&C optimization

Usage: $dp[t][i] = \min(dp[t-1][j] + c[j][i])$, c is Monge

Time Complexity: $O(KN \log N)$

```

ll dp[MAX_K][MAX_N];
// 1부터 r까지 구간의 비용을 계산하는 함수 (문제에 맞게
// 구현)
ll get_cost(int l, int r) {/* return sum[l][r] + c; */}
// k: 현재 단계(구간 개수 등), pL, pR: 최적의 j를 찾을
// 탐색 범위
void dnc(int k, int l, int r, int pL, int pR) {
    if (l > r) return;
    int opt = pL, mid = (l + r) / 2;
    dp[k][mid] = -1e18 // 최솟값 문제면 INF, 최댓값 문제
    //면 -INF
    for (int j = pL; j <= min(mid, pR); j++) {
        ll val = (j == 0 ? 0 : dp[k - 1][j - 1]) +
        get_cost(j, mid);
        if (val > dp[k][mid]) {
            dp[k][mid] = val;
            opt = j;
        }
    }
    dnc(k, l, mid - 1, pL, opt);
    dnc(k, mid + 1, r, opt, pR);
}
// usage: for (int i = 1; i <= T; i++) dnc(i, 0, n-1,
// 0, n-1);

```

5.4 Monotone Queue optimization

Usage: $dp[i] = \min(dp[j] + c[j][i])$, c is Monge, find cross

Time Complexity: $O(N \log N)$

```

ll f(int j, int i); // j에서 i로 전이할 때의 값 (dp[j]
+ cost(j, i))
void solve() {
    auto cross = [&](ll p, ll q) {
        ll lo = max(p, q), hi = n + 1;
        while (lo + 1 < hi) {
            ll mid = (lo + hi) / 2;

```

```

if (f(p, mid) > f(q, mid)) hi = mid; // min 기준:
f(p) > f(q)면 q가 우세
else lo = mid;
}
return hi;
};

deque<pair<ll, ll>> dq; // {candidate_index,
start_pos}
dq.push_back({0, 1}); // 초기값: 0번이 1번 위치부터
최적이라고 가정
for (int i = 1; i <= n; i++) {
    while (dq.size() > 1 && dq[1].second <= i)
        dq.pop_front();
    dp[i] = f(dq[0].first, i);
    while (!dq.empty()) {
        ll p = dq.back().first;
        ll pos = cross(p, i);
        if (pos <= dq.back().second) dq.pop_back();
        else {
            if (pos <= n) dq.push_back({i, pos});
            break;
        }
    }
    if (dq.empty()) dq.push_back({i, 1});
}

```

5.5 Aliens Trick

Usage: $dp[t][i] = \min(dp[t-1][j] + c[j+1][i])$, c is

Monge, find lambda w/ half bs

Time Complexity: $O(T \log X)$

/* n: 원소 개수 (경로 복원 끝점), k: 정확히 골라야 하는 개수

* lo, hi: 패널티 이분탐색 범위 (0 ~ 최대 가치)

* f(c): 패널티가 c 일 때 $\{2*(가치합), prv\}$ 반환 (가치는 c 를 뺀 값) */

```

template<class T, bool GET_MAX = false>
pair<T, vector<int>> AliensTrick(int n, int k, auto f,
T lo, T hi) {
    T l = lo, r = hi;
    while (l < r) {
        T m = (l + r + (GET_MAX ? 1 : 0)) >> 1;
        vector<int> prv = f(m * 2 + (GET_MAX ? -1 :
1)).second;
        int cnt = 0; for (int i = n; i; i = prv[i]) cnt++;
        if (cnt <= k) (GET_MAX ? l : r) = m;
        else (GET_MAX ? r : l) = m + (GET_MAX ? -1 : 1);
    }
    T opt_val = f(l * 2).first / 2 - k * l;
    auto get_path = [&](T c) {

```

```

        vector<int> p{n};
        for (auto prv = f(c).second; p.back(); )
            p.push_back(prv[p.back()]);
        reverse(p.begin(), p.end()); return p;
    };
    auto p1 = get_path(l * 2 + (GET_MAX ? 1 : -1));
    auto p2 = get_path(l * 2 - (GET_MAX ? 1 : -1));
    if (p1.size() == k + 1) return {opt_val, p1};
    if (p2.size() == k + 1) return {opt_val, p2};
    for (int i = 1, j = 1; i < p1.size(); i++) {
        while (j < p2.size() && p2[j] < p1[i - 1]) j++;
        if (p1[i] <= p2[j] && i - j == k + 1 -
            (int)p2.size()) {
            vector<int> res(p1.begin(), p1.begin() + i);
            res.insert(res.end(), p2.begin() + j, p2.end());
            return {opt_val, res};
        }
    }
    return {opt_val, {}}; // Should not reach here
}

```

5.6 Sum Over Subsets

Usage: $dp[mask] = \sum(A[i])$, i is in mask

Time Complexity: $O(N2^N)$

```

for (int i = 0; i < (1<<n); i++)
    f[i] = a[i];
for (int j = 0; j < n; j++)
    for (int i = 0; i < (1<<n); i++)
        if (i & (1<<j)) f[i] += f[i ^ (1<<j)];

```

5.7 Berlekamp Massey

Usage: Linear recurrence N -th term. Sparse matrix determinant.

Time Complexity: N -th term: $O(K^2 \log N) / \text{Det}: O(N(N+E))$

```

mt19937_64
rng(chrono::high_resolution_clock::now().time_since_epoch());
int randint(int lb, int ub) { return
uniform_int_distribution<int>(lb, ub)(rng); }
const int mod = 998244353;
ll ipow(ll x, ll p) {
    ll ret = 1, piv = x;
    while (p) {
        if (p & 1) ret = ret * piv % mod;
        piv = piv * piv % mod; p >>= 1;
    }
    return ret;
}

```

```

vector<int> berlekamp_massey(vector<int> x) {
    vector<int> ls, cur; int lf, ld;
    for (int i = 0; i < sz(x); i++) {
        ll t = 0;
        for (int j = 0; j < sz(cur); j++) t = (t +
            1ll*x[i-j-1]*cur[j]) % mod;
        if ((t - x[i]) % mod == 0) continue;
        if (cur.empty()) {
            lf = i; ld = (t-x[i]) % mod;
            cur.resize(i+1); continue;
        }
        ll k = -(x[i]-t) * ipow(ld, mod-2) % mod;
        vector<int> c(i-lf+1); c.push_back(k);
        for (auto& j : ls) c.push_back(-j*k % mod);
        if (sz(c) < sz(cur)) c.resize(sz(cur));
        for (int j = 0; j < sz(cur); j++) c[j] =
            (c[j]+cur[j]) % mod;
        if (i-lf+sz(ls) >= sz(cur)) {
            tie(ls, lf, ld) = make_tuple(cur, i, (t-x[i]) %
                mod);
        }
        cur = c;
    }
    for (auto& i : cur) i = (i % mod + mod) % mod;
    return cur;
}

int get_nth(vector<int> rec, vector<int> dp, ll n) {
    int m = sz(rec); vector<int> s(m), t(m); s[0] = 1;
    if (m != 1) t[1] = 1; else t[0] = rec[0];
    auto mul = [&rec](vector<int> v, vector<int> w) {
        int m = sz(v); vector<int> t(2*m);
        for (int j = 0; j < m; j++) {
            for (int k = 0; k < m; k++) {
                t[j+k] += 1ll*v[j]*w[k] % mod;
                if (t[j+k] >= mod) t[j+k] -= mod;
            }
        }
    };
    for (int j = 2*m-1; j >= m; j--) {
        for (int k = 1; k <= m; k++) {
            t[j-k] += 1ll*t[j]*rec[k-1] % mod;
            if (t[j-k] >= mod) t[j-k] -= mod;
        }
    }
    t.resize(m); return t;
};

while (n) {
    if (n & 1) s = mul(s, t);
    t = mul(t, t); n >>= 1;
}
ll ret = 0;
for (int i = 0; i < m; i++) ret += 1ll*s[i]*dp[i] %
mod;

```

```

    return ret % mod;
}

int guess_nth_term(vector<int> x, ll n) {
    if (n < sz(x)) return x[n];
    vector<int> v = berlekamp_massey(x);
    if (v.empty()) return 0;
    return get_nth(v, x, n);
}

struct elem { int x, y, v; }; // A_(x, y) <- v,
// 0-based. no duplicate
vector<int> get_min_poly(int n, vector<elem> M) {
    // smallest poly P such that A^i = sum_{j < i} {A^j
    // \times P_j}
    vector<int> rnd1, rnd2;
    for (int i = 0; i < n; i++) {
        rnd1.push_back(randint(1, mod - 1));
        rnd2.push_back(randint(1, mod - 1));
    }
    vector<int> gobs;
    for (int i = 0; i < 2*n+2; i++) {
        int tmp = 0;
        for (int j = 0; j < n; j++) {
            tmp += 111 * rnd2[j] * rnd1[j] % mod;
            if (tmp >= mod) tmp -= mod;
        }
        gobs.push_back(tmp); vector<int> nxt(n);
        for (auto& i : M) {
            nxt[i.x] += 111 * i.v * rnd1[i.y] % mod;
            if (nxt[i.x] >= mod) nxt[i.x] -= mod;
        }
        rnd1 = nxt;
    }
    auto sol = berlekamp_massey(gobs);
    reverse(all(sol)); return sol;
}

ll det(int n, vector<elem> M) {
    vector<int> rnd;
    for (int i = 0; i < n; i++) rnd.push_back(randint(1, mod-1));
    for (auto& i : M) i.v = 111 * i.v * rnd[i.y] % mod;
    auto sol = get_min_poly(n, M)[0]; if (n%2 == 0) sol =
    mod-sol;
    for (auto& i : rnd) sol = 111 * sol * ipow(i, mod-2)
    % mod;
    return sol;
}

```

6 Geometry

6.1 Geometry Template

Usage: Basic point, line, and circle operations.

```

const double EPS = 1e-9;
template<typename T>
struct Point {
    T x, y;
    bool operator<(const Point& p) const { return x==p.x
    ? y<p.y : x<p.x; }
    bool operator==(const Point& p) const { return x==p.x
    && y==p.y; }
    Point operator+(const Point& p) const { return
    {x+p.x, y+p.y}; }
    Point operator-(const Point& p) const { return
    {x-p.x, y-p.y}; }
    Point operator*(T n) const { return {x*n, y*n}; }
    Point operator/(T n) const { return {x/n, y/n}; }
    T operator*(const Point& p) const { return x*p.x +
    y*p.y; }
    T operator/(const Point& p) const { return x*p.y -
    y*p.x; }
    T dist2() const { return x*x + y*y; }
    Point<double> d() const { return {(double)x,
    (double)y}; }
};

using P = Point<ll>;
using Pd = Point<double>;
int ccw(P a, P b, P c) {
    ll res = (b - a) / (c - a);
    return (res > 0) - (res < 0);
}

bool isInter(P a, P b, P c, P d) {
    int ab = ccw(a, b, c) * ccw(a, b, d), cd = ccw(c, d,
    a) * ccw(c, d, b);
    if (ab == 0 && cd == 0) {
        if (b < a) swap(a, b); if (d < c) swap(c, d);
        return !(b < c || d < a);
    }
    return ab <= 0 && cd <= 0;
}

```

6.2 Convex Hull

Usage: Finds the convex hull using Monotone Chain. Returns vertices in CCW order.

Time Complexity: $O(N \log N)$

```

// Monotone Chain 알고리즘, O(N log N)
// 반시계 방향(CCW)으로 정렬된 볼록 꺽질 반환

```

```

// 일직선상의 점을 제외하려면 ccw <= 0, 포함하려면 ccw
// < 0
vector<P> ConvexHull(vector<P> ps) {
    if (sz(ps) <= 2) return ps;
    sort(all(ps)); vector<P> v(sz(ps)+2);
    int s = 0, t = 0;
    for (int i = 2; i-->0; s = --t, reverse(all(ps))) {
        for (P p : ps) {
            while (t >= s+2 && ccw(v[t-2], v[t-1], p) <= 0)
                t--;
            v[t++] = p;
        }
    }
    v.resize(t - (t > 1)); return v;
}

```

6.3 Rotating Calipers

Usage: Calculates the diameter (farthest pair of points) of a convex hull(CCW order).

Time Complexity: $O(N)$

```

// 가장 먼 두 점을 구하는 함수, O(N)
// hull: 반시계 방향(CCW)으로 정렬된 볼록 다각형
pair<P, P> Calipers(const vector<P>& hull) {
    int n = sz(hull); if (n < 2) return {hull[0], hull[0]};
    ll mx = 0; P a = hull[0], b = hull[1];
    for (int i = 0, j = 1; i < n; i++) {
        P vec_i = hull[(i + 1) % n] - hull[i];
        while ((vec_i / (hull[(j + 1) % n] - hull[j])) > 0)
        {
            ll now = (hull[i] - hull[j]).dist2();
            if (now > mx) mx = now, a = hull[i], b = hull[j];
            j = (j + 1) % n;
        }
        ll now = (hull[i] - hull[j]).dist2();
        if (now > mx) mx = now, a = hull[i], b = hull[j];
    }
    return {a, b};
}

```

6.4 Point in Convex Polygon

Usage: Checks if a point is inside or on the boundary of a convex polygon (CCW sorted).

Time Complexity: $O(\log N)$

```

// CCW 정렬된 다각형 내부/경계 판별, O(log N)
bool PointInConvexPolygon(const vector<P>& v, P p) {
    int n = v.size(); if (n < 3) return false;
    // ccw <= 0 || ccw >= 0: exclude boundary

```

```

if (ccw(v[0], v[1], p) < 0 || ccw(v[0], v.back(), p)
> 0) return false;
int l = 1, r = n - 1;
while (l + 1 < r) {
    int mid = (l + r) / 2;
    if (ccw(v[0], v[mid], p) >= 0) l = mid;
    else r = mid;
}
return ccw(v[1], v[l + 1], p) >= 0; // > 0: exclude
boundary
}

```

6.5 Point in Polygon

Usage: Ray casting algorithm for general polygons.

Time Complexity: $O(N)$

```

// 다각형 내부 포함 판별 (Ray Casting), O(N) (CCW/CW 순
서 무관)
bool PointInPolygon(const vector<P>& poly, P p) {
    int n = poly.size(); bool inside = false;
    for (int i = 0; i < n; i++) {
        P a = poly[i], b = poly[(i + 1) % n];
        if ((b - a) / (p - a) == 0 && (a - p) * (b - p) <
        0) return true; // false: exclude boundary
        if ((a.y > p.y) != (b.y > p.y)) {
            double ix = (double)(b.x-a.x) * (p.y-a.y) /
            (double)(b.y-a.y) + a.x;
            if (p.x < ix) inside = !inside;
        }
    }
    return inside;
}

```

6.6 Sort Points

Usage: Angular sort. 1. Relative to pivot (Convex Hull). 2. Relative to origin.

```

// Sorts points by angle relative to the bottom-left
point (CCW).
void SortByAngle(vector<P>& v) {
    if (v.size() < 2) return;
    swap(v[0], *min_element(v.begin(), v.end(), [&](const
P& a, const P& b) {
        return a.y != b.y ? a.y < b.y : a.x < b.x;
}));
    sort(v.begin() + 1, v.end(), [&](const P& a, const P&
b) {
        ll cp = (a - v[0]) / (b - v[0]); if (cp != 0)
        return cp > 0;
        return (a - v[0]).dist2() < (b - v[0]).dist2();
    });
}

```

```

    });
}

// Sorts points by angle around origin (0,0) from range
[0, 360].
void SortAroundOrigin(vector<P>& v) {
    auto half = [](<const P& p) { return p.y > 0 || (p.y
== 0 && p.x > 0); };
    sort(v.begin(), v.end(), [&](const P& a, const P& b)
{
    if (half(a) != half(b)) return half(a) > half(b);
    return (a / b) > 0;
});
}

```

6.7 Linear Minkowski Sum

Usage: Minkowski Sum of Two convex(must be CCW order).

Time Complexity: $O(N + M)$

```

// Minkowski Sum of Convex Polygons (CCW only), O(N +
M)
vector<P> Minkowski(vector<P> p, vector<P> q) {
    if (p.empty() || q.empty()) return {};
    rotate(p.begin(), min_element(all(p)), p.end());
    rotate(q.begin(), min_element(all(q)), q.end());
    p.push_back(p[0]); p.push_back(p[1]);
    q.push_back(q[0]); q.push_back(q[1]);
    vector<P> res; int i = 0, j = 0;
    while (i < p.size() - 2 || j < q.size() - 2) {
        res.push_back(p[i] + q[j]);
        ll cp = (p[i + 1] - p[i]) / (q[j + 1] - q[j]);
        if (cp >= 0 && i < p.size() - 2) i++;
        if (cp <= 0 && j < q.size() - 2) j++;
    }
    return res;
}

```

6.8 Polygon Area

Usage: Calculates $2 \times$ Area of a polygon (Shoelace formula).

Time Complexity: $O(N)$

```

// 다각형 넓이의 2배를 반환 (Shoelace Formula), O(N)
ll PolygonArea2(const vector<P>& poly) {
    ll area = 0; int n = poly.size();
    for (int i = 0; i < n; i++) area += poly[i] / poly[(i
+ 1) % n];
    return abs(area);
}

```

6.9 Smallest Enclosing Circle

Usage: Welzl's algorithm to find the Minimum Enclosing Circle. Returns center, radius.

Time Complexity: Expected $O(N)$

```

mt19937_64
rng(chrono::steady_clock::now().time_since_epoch().count())
const double EPS = 1e-9;
Pd getC(Pd a, Pd b) { return (a + b) / 2.0; }
Pd getc(Pd a, Pd b, Pd c) {
    Pd aa = b - a, bb = c - a;
    double c1 = aa * aa * 0.5, c2 = bb * bb * 0.5, d = aa
    / bb;
    return {a.x+(c1 * bb.y - c2 * aa.y)/d, a.y+(c2 * aa.x
    - c1 * bb.x)/d};
}
pair<Pd,double> solve(vector<Pd> v) {
    shuffle(all(v), rng);
    Pd p = {0, 0}; double r = 0;
    auto dist = [&](Pd p1, Pd p2) { return
sqrt((p1-p2).dist2()); };
    for (int i = 0; i < sz(v); i++) if (dist(p, v[i]) > r
+ EPS) {
        p = v[i]; r = 0;
        for (int j = 0; j < i; j++) if (dist(p, v[j]) > r +
EPS) {
            p = getC(v[i], v[j]); r = dist(p, v[i]);
            for (int k = 0; k < j; k++) if (dist(p, v[k]) > r +
EPS) {
                p = getC(v[i], v[j], v[k]); r = dist(p, v[k]);
            }
        }
    }
    return {p, r};
}

```

6.10 Geometric Intersections

Usage: Intersection primitives: Segment, Line-Circle, Line-Hull, and Circle-Polygon area.

Time Complexity: $O(1)/O(1)/O(\log N)/O(N)$

```

const double EPS = 1e-9;
using T = __int128_t; // T <= 0(COORD^4)
// [1] 선분 교차 (정밀 좌표) / Param: 선분 ab, 선분 cd
// Return: {flag, xn, xd, yn, yd} -> 교점 (xn/xd,
yn/yd)
// Flag: 0(안만남), 1(교점=끝점), 4(교차), -1(무수히 겹
침)
tuple<int, T, T, T, T> segmentInter(P a, P b, P c, P d)
{
}

```

```

if (!isInter(a, b, c, d)) return {0, 0, 0, 0, 0};
T det = (b - a) / (d - c);
if (det == 0) {
    if (b < a) swap(a, b);
    if (d < c) swap(c, d);
    if (b == c) return {1, b.x, 1, b.y, 1};
    if (d == a) return {1, d.x, 1, d.y, 1};
    return {-1, 0, 0, 0, 0};
}
T p = (c - a) / (d - c), q = det;
T xp = a.x * q + (b.x - a.x) * p, xq = q;
T yp = a.y * q + (b.y - a.y) * p, yq = q;
if (xq < 0) {xp = -xp; xq = -xq;};
if (yq < 0) {yp = -yp; yq = -yq;};
T g_x = __gcd(xp, xq); xp /= g_x; xq /= g_x;
T g_y = __gcd(yp, yq); yp /= g_y; yq /= g_y;
int f = 4;
if ((xp == a.x * xq && yp == a.y * yq) || (xp == b.x
* xq && yp == b.y * yq)) f = 1;
if ((xp == c.x * xq && yp == c.y * yq) || (xp == d.x
* xq && yp == d.y * yq)) f = 1;
return {f, xp, xq, yp, yq};

// [2] 원-직선 교차 / Param: 직선 ab, 원(c, r)
// Return: 교점 좌표 목록 (a -> b 방향 순서 정렬됨)
vector<Pd> lineCircle(P a, P b, P c, double r) {
    P ab = b - a;
    Pd p = a.d() + ab.d() * ((c - a) * ab /
    (double)ab.dist2());
    double h2 = r * r - (p - c.d()).dist2();
    if (h2 < -EPS) return {};
    if (abs(h2) < EPS) return {p};
    Pd h = ab.d() * (sqrt(h2) / sqrt(ab.dist2()));
    return {p - h, p + h};

// [3] 원-다각형 교차 넓이
// Param: 원(c, r), 다각형 poly (CCW/CW 무관) Return:
// 교차하는 영역의 넓이
double areaCirclePoly(P c, double r, const vector<P>&
poly) {
    auto tri = [&](Pd p, Pd q) {
        Pd d = q - p;
        double a = d * p / d.dist2(), b = (p.dist2() - r *
        r) / d.dist2();
        double det = a * a - b;
        if (det <= EPS) return r * r * atan2(p / q, p * q);
        double t1 = -a - sqrt(max(0.0, det)), t2 = -a +
        sqrt(max(0.0, det));
        if (t2 < -EPS || t1 > 1.0 + EPS) return r * r *
        atan2(p / q, p * q);
    }
}

```

```

Pd u = p + d * max(0.0, t1), v = p + d * min(1.0,
t2);
return r * r * atan2(p / u, p * u) + u / v + r * r
* atan2(v / q, v * q);
};

double res = 0; int n = poly.size();
for(int i = 0; i < n; i++)
    res += tri((poly[i] - c).d(), (poly[(i + 1) % n] -
c).d());
return res * 0.5;

}

// [4] 볼록 다각형-직선 교차 (O(log N)) Param: 볼록 다
각형 h (CCW), 직선 ab
// Return: {i, j} -> 직선이 변(i, i+1)과 변(j, j+1)을
교차함. 만만다면 {-1, -1}
int extrVertex(const vector<P>& h, P dir) {
    int n = h.size();
    auto cmp = [&](int i, int j) { return ccw({0,0}, dir,
h[i%n] - h[j%n]); };
    auto isExtr = [&](int i) { return cmp(i, i - 1 + n)
>= 0 && cmp(i, i + 1) > 0; };
    if (isExtr(0)) return 0;
    int l = 0, r = n;
    while (l + 1 < r) {
        int m = (l + r) / 2;
        if (isExtr(m)) return m;
        if (cmp(l + 1, l) > 0) {
            if (cmp(m + 1, m) > 0 && cmp(l, m) > 0) r = m;
            else l = m;
        } else {
            if (cmp(m + 1, m) <= 0 && cmp(l, m) < 0) l = m;
            else r = m;
        }
    }
    return l;
}

array<int,2> hullLineInter(const vector<P>& h, P a, P
b) {
    P dir = b - a, per = {-dir.y, dir.x};
    int n = h.size();
    int i1 = extrVertex(h, per), i2 = extrVertex(h, per *
-1);
    if (ccw(a, b, h[i1]) * ccw(a, b, h[i2]) > 0) return
{-1, -1};
    auto search = [&](int s, int e) {
        if (ccw(a, b, h[s]) == 0) return s;
        int l = s, r = e;
        if (r < l) r += n;
        while (l + 1 < r) {
            int m = (l + r) / 2;

```

```

            if (ccw(a, b, h[m % n]) == ccw(a, b, h[s])) l =
m; else r = m;
        }
        return l % n;
    };
    return {search(i1, i2), search(i2, i1)};
}



## 6.11 Half Plane Intersection



Usage: Intersection of half-planes defined by lines (left side is valid). Returns a convex polygon.



Time Complexity:  $O(N \log N)$



```

// 각 선분의 '왼쪽' 영역들의 교집합(볼록 다각형)을 반
// 환 // 영역이 없거나 닫히지 않는 경우(Unbounded) 빈 벡터 반
// 환 가능성 있음
const double EPS = 1e-9;
struct Line {
 double a, b, c; // ax + by <= c
 Line(Pd p1, Pd p2) {
 a = p1.y - p2.y; // -dy
 b = p2.x - p1.x; // dx
 c = a * p1.x + b * p1.y;
 }
 Pd slope() const { return {a, b}; }
};

Pd intersect(Line u, Line v) { // 평행하지 않은 두 직선
 double det = u.a * v.b - u.b * v.a;
 return {(u.c * v.b - u.b * v.c) / det, (u.a * v.c -
u.c * v.a) / det};
}

bool bad(Line l, Pd p) {
 return l.a * p.x + l.b * p.y > l.c + EPS;
}

vector<Pd> HPI(vector<Line> lines) {
 sort(all(lines), [&](const Line& u, const Line& v) {
 Pd p1 = u.slope(), p2 = v.slope();
 bool f1 = p1.y > 0 || (p1.y == 0 && p1.x > 0);
 bool f2 = p2.y > 0 || (p2.y == 0 && p2.x > 0);
 if (f1 != f2) return f1 > f2;
 if (abs(p1 / p2) > EPS) return (p1 / p2) > 0;
 return u.c < v.c;
 });
 deque<Line> dq;
 for (auto& l : lines) {
 if (!dq.empty() && abs(dq.back().slope() /
l.slope()) < EPS) continue;
 while (sz(dq) >= 2 && bad(l, intersect(dq.back(),
dq[sz(dq) - 2]))) dq.pop_back();
 }
}

```


```

```

while (sz(dq) >= 2 && bad(l, intersect(dq[0], dq[1]))) dq.pop_front();
dq.push_back(l);
}
while (sz(dq) > 2 && bad(dq[0], intersect(dq.back(), dq[sz(dq)-2]))) dq.pop_back();
while (sz(dq) > 2 && bad(dq.back(), intersect(dq[0], dq[1]))) dq.pop_front();
vector<Pd> res; if (sz(dq) < 3) return {};
for (int i = 0; i < sz(dq); i++) {
    res.push_back(intersect(dq[i], dq[(i + 1) % sz(dq)]));
}
return res;
}

```

7 String

7.1 Aho-Corasick

Usage: Multi-pattern matching using trie and failure links.

Time Complexity: $O(\sum |P| + |T|)$

```

struct AhoCorasick {
    struct Node {
        Node *nxt[26], *fail;
        vector<int> out; // 패턴의 인덱스 저장
        int terminal;
        Node() : fail(nullptr), terminal(-1) { fill(nxt, nxt + 26, nullptr); }
        ~Node() {
            for (int i = 0; i < 26; i++) if (nxt[i]) delete nxt[i];
        }
        void insert(const char* s, int id) {
            if (*s == 0) { terminal = id; out.push_back(id); return; }
            int curr = *s - 'a';
            if (!nxt[curr]) nxt[curr] = new Node();
            nxt[curr]->insert(s + 1, id);
        }
    };
    Node* root;
    AhoCorasick() { root = new Node(); }
    ~AhoCorasick() { delete root; }
    void insert(const string& s, int id) {
        root->insert(s.c_str(), id);
    }
    void build() {
        queue<Node*> q;
        root->fail = root;
        for (int i = 0; i < 26; i++) {
            if (root->nxt[i])

```

```

                root->nxt[i]->fail = root;
                q.push(root->nxt[i]);
            } else {
                root->nxt[i] = root; // DFA optimization
            }
        }
        while (!q.empty()) {
            Node* curr = q.front(); q.pop();
            for (int i = 0; i < 26; i++) {
                if (curr->nxt[i]) {
                    Node* next = curr->nxt[i];
                    next->fail = curr->fail->nxt[i];
                    next->out.insert(next->out.end(),
                        next->fail->out.begin(),
                        next->fail->out.end());
                    q.push(next);
                } else {
                    curr->nxt[i] = curr->fail->nxt[i]; // DFA
                    optimization
                }
            }
        }
    }
    vector<pair<int, int>> query(const string& s) {
        vector<pair<int, int>> res;
        Node* curr = root;
        for (int i = 0; i < s.size(); i++) {
            curr = curr->nxt[s[i] - 'a'];
            for (int id : curr->out) res.emplace_back(i, id);
        }
        return res;
    }
};

int main() {
    AhoCorasick ac;
    vector<string> patterns = {"he", "she", "hers",
        "his"};
    for(int i = 0; i < patterns.size(); i++)
        ac.insert(patterns[i], i); // 패턴과 ID(0~N-1) 삽입
    ac.build(); // 실패 함수/DFA 빌드 (필수)
    string text = "ushers";
    auto res = ac.query(text); // 탐색: {끝 인덱스, 패턴
    ID} 쌍 반환
    for (auto& [idx, id] : res) {
        // patterns[id] 가 text의 idx에서 끝남을 의미
    }
}

```

7.2 Hashing

Usage: Rolling hash for string matching.

Time Complexity: $O(N)$

```

// 전처리 O(N), 부분 문자열의 해시값을 O(1)에 구함
// Hashing<917, 998244353> H; H.build("ABCDABCD");
// assert(H.get(1, 4) == H.get(5, 8));
// 주의: get 함수의 인자는 1-based 닫힌 구간
// 주의: M은 10억 근처의 소수, P는 M과 서로소
// 1e5+3, 1e5+13, 131'071, 524'287, 1'299'709,
1'301'021
// 1e9-63, 1e9+7, 1e9+9, 1e9+103
template<long long P, long long M> struct Hashing {
    vector<long long> h, p;
    void build(const string &s){
        int n = s.size();
        h = p = vector<long long>(n+1); p[0] = 1;
        for(int i=1; i<=n; i++) h[i] = (h[i-1] * P +
            s[i-1]) % M;
        for(int i=1; i<=n; i++) p[i] = p[i-1] * P % M;
    }
    long long get(int s, int e) const {
        long long res = (h[e] - h[s-1] * p[e-s+1]) % M;
        return res >= 0 ? res : res + M;
    }
};

```

7.3 KMP

Usage: Single pattern matching using prefix function.

Time Complexity: $O(N + M)$

```

template <typename T>
struct KMP {
    T P; vector<int> pi;
    KMP(const T& P) : P(P), pi(sz(P)) {
        for (int i = 1, j = 0; i < sz(P); i++) {
            while (j > 0 && P[i] != P[j]) j = pi[j-1];
            if (P[i] == P[j]) pi[i] = ++j;
        }
    }
    vector<int> find(const T& S) {
        vector<int> res;
        int n = sz(S), m = sz(P), j = 0;
        for (int i = 0; i < n; i++) {
            while (j > 0 && S[i] != P[j]) j = pi[j-1];
            if (S[i] == P[j]) {
                if (j == m-1) {
                    res.push_back(i-m+1); j = pi[j];
                } else j++;
            }
        }
    }
};

```

```

    return res;
}

int minPeriod() {
    int m = sz(P); if (m == 0) return 0;
    int len = m - pi[m-1]; if (m % len == 0) return
    len;
    return m;
}

```

7.4 Manacher

Usage: Find all palindromic substrings in linear time.

Time Complexity: $O(N)$

```

// 각 문자를 중심으로 하는 최장 팰린드롬의 반경을 반환
// Manacher("abaaba") = {0,1,0,3,0,1,6,1,0,3,0,1,0}
// # a # b # a # a # b # a #
// 0 1 0 3 0 1 6 1 0 3 0 1 0
vector<int> Manacher(const string &inp){
    int n = inp.size() * 2 + 1;
    vector<int> ret(n);
    string s = "#";
    for(auto i : inp) s += i, s += "#";
    for(int i=0, p=-1, r=-1; i<n; i++){
        ret[i] = i <= r ? min(r-i, ret[2*p-i]) : 0;
        while(i-ret[i]-1 >= 0 && i+ret[i]+1 < n &&
        s[i-ret[i]-1] == s[i+ret[i]+1]) ret[i]++;
        if(i+ret[i] > r) r = i+ret[i], p = i;
    }
    return ret;
}

```

7.5 Suffix Array

Usage: Suffix array and LCP array.

Time Complexity: $O(N \log N)$

```

// calculates suffix array with O(n*logn)
auto get_sa(const string& s) {
    const int n = s.size(), m = max(256, n) + 1;
    vector<int> sa(n), r(n << 1), nr(n << 1), cnt(m),
    idx(n);
    for (int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];
    for (int d = 1; d < n; d <= 1) {
        auto cmp = [&](int a, int b) { return r[a] < r[b]
        || r[a] == r[b] && r[a+d] < r[b+d];};
        for (int i = 0; i < m; ++i) cnt[i] = 0;
        for (int i = 0; i < n; ++i) cnt[r[i+d]]++;
        for (int i = 1; i < m; ++i) cnt[i] += cnt[i - 1];
        for (int i = n - 1; ~i; --i) idx[--cnt[r[i+d]]] =
        i;
    }
}

```

```

for (int i = 0; i < m; ++i) cnt[i] = 0;
for (int i = 0; i < n; ++i) cnt[r[i]]++;
for (int i = 1; i < m; ++i) cnt[i] += cnt[i - 1];
for (int i = n - 1; ~i; --i) sa[--cnt[r[idx[i]]]] =
idx[i];
nr[sa[0]] = 1;
for (int i = 1; i < n; ++i) nr[sa[i]] = nr[sa[i -
1]] + cmp(sa[i - 1], sa[i]);
for (int i = 0; i < n; ++i) r[i] = nr[i];
if (r[sa[n - 1]] == n) break;
}
return sa;
}

// calculates lcp array. it needs suffix array &
original sequence with O(n)
auto get_lcp(const string& s, const auto& sa) {
    const int n = s.size(); vector<int> lcp(n - 1, 0), isa(n,
    0);
    for (int i = 0; i < n; i++) isa[sa[i]] = i;
    for (int i = 0, k = 0; i < n; i++) if (isa[i]) {
        for (int j = sa[isa[i] - 1]; s[i + k] == s[j + k];
        k++);
        lcp[isa[i] - 1] = k ? k-- : 0;
    }
    return lcp;
}

```

7.6 Z-algorithm

Usage: Longest common prefix between S and its suffixes.

Time Complexity: $O(N)$

```

// Z[i] = LongestCommonPrefix(S[0:N], S[i:N])
//      = S[0:N]과 S[i:N]이 앞에서부터 몇 글자 겹치는지
vector<int> Z(const string &s){
    int n = s.size();
    vector<int> z(n);
    z[0] = n;
    for(int i=1, l=0, r=0; i<n; i++){
        if(i < r) z[i] = min(r-i-1, z[i-1]);
        while(i+z[i] < n && s[i+z[i]] == s[z[i]]) z[i]++;
        if(i+z[i] > r) r = i+z[i], l = i;
    }
    return z;
}

```

7.7 Eertree

Usage: Manages all distinct palindromic substrings using length and suffix links.

Time Complexity: $O(N \log \Sigma)$

```

// Z[i] = LongestCommonPrefix(S[0:N], S[i:N])
//      = S[0:N]과 S[i:N]이 앞에서부터 몇 글자 겹치는지
vector<int> Z(const string &s){
    int n = s.size();
    vector<int> z(n);
    z[0] = n;
    for(int i=1, l=0, r=0; i<n; i++){
        if(i < r) z[i] = min(r-i-1, z[i-1]);
        while(i+z[i] < n && s[i+z[i]] == s[z[i]]) z[i]++;
        if(i+z[i] > r) r = i+z[i], l = i;
    }
    return z;
}

```

7.8 Suffix Automaton

Usage: Builds a DFA representing all substrings. Supports substring counting and pattern matching.

Time Complexity: $O(N \cdot \Sigma)$

```

template<typename T, size_t S, T init_val>
struct initialized_array : public array<T, S> {
    initialized_array(){ this->fill(init_val); }
};

template<class Char_Type, class Adjacency_Type>
struct suffix_automaton{
    // Begin States
    // len: length of the longest substring in the class
    // link: suffix link
    // firstpos: minimum value in the set endpos
    vector<int> len{0}, link{-1}, firstpos{-1},
    is_clone{false};
    vector<Adjacency_Type> next{{}};
    ll ans{0LL}; // 서로 다른 부분 문자열 개수
    // End States
    void set_link(int v, int lnk){
        if(link[v] != -1) ans -= len[v] - len[link[v]];
        link[v] = lnk;
        if(link[v] != -1) ans += len[v] - len[link[v]];
    }
    int new_state(int l, int sl, int fp, bool c, const
    Adjacency_Type &adj){
        int now = len.size(); len.push_back(l);
        link.push_back(-1);
        set_link(now, sl); firstpos.push_back(fp);
        is_clone.push_back(c); next.push_back(adj); return
        now;
    }
    int last = 0;
    void extend(const vector<Char_Type> &s){
        last = 0; for(auto c: s) extend(c); }
    void extend(Char_Type c){
```

```

int cur = new_state(len[last] + 1, -1, len[last],
false, {}), p = last;
while(~p && !next[p][c]) next[p][c] = cur, p =
link[p];
if(!~p) set_link(cur, 0);
else{
    int q = next[p][c];
    if(len[p] + 1 == len[q]) set_link(cur, q);
    else{
        int clone = new_state(len[p] + 1, link[q],
firstpos[q], true, next[q]);
        while(~p && next[p][c] == q) next[p][c] =
clone, p = link[p];
        set_link(cur, clone); set_link(q, clone);
    }
}
last = cur;
}
int size() const { return (int)len.size(); } // # of
states
}; suffix_automaton<int, initialized_array<int,26,0>>
T;
// for(auto c : s) if((x=T.next[x][c]) == 0) return
false;

```

8 STL & pbds

8.1 Hash map (pb_ds)

Usage: Faster hash table using pb_ds.

Time Complexity: $O(1)$

```

// faster than unordered_map
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int,int> hashmap; // cannot use
hashmap.count()

```

8.2 Ordered Set (pb_ds)

Usage: Set supporting order_of_key and find_by_order.

Time Complexity: $O(\log N)$

```

// k번째 원소 확인 및 x보다 작은 원소개수 확인을
O(logN)에 수행
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using ordered_set = tree<T, null_type, less<T>,
rb_tree_tag, tree_order_statistics_node_update>;
// ordered_set<int> os;

```

```

// os.find_by_order(k): k번째 원소의 iterator 반환
(0-indexed, 없으면 os.end())
// os.order_of_key(x) : x보다 작은 원소의 개수 반환
template <typename T>
using ordered_multiset = tree<T, null_type,
less_equal<T>, rb_tree_tag,
tree_order_statistics_node_update>;
auto m_find(ordered_multiset<int> &os, int val) { //
multiset 전용 find 함수
    int idx = os.order_of_key(val); auto it =
os.find_by_order(idx);
    if (it != os.end() && *it == val) return it;
    return os.end();
} // os.erase(m_find(os, val))

```

8.3 Permutation & Combination

Usage: next_permutation and mask-based combinations.

```

#include <algorithm>
/* 1. Permutation */
sort(all(v))
do {
    // process v
} while (next_permutation(all(v)));
/* 2. Combination (nCr): Use a mask vector */
vector<int> mask(n, 0);
fill(mask.end()-r, mask.end(), 1); // pick r elements
do {
    for (int i = 0; i < n; i++) {
        if (mask[i]) { /* v[i] is selected */
    }
} while (next_permutation(all(mask)));
/* 3. Partial Permutation (nPk) */
sort(all(v));
do {
    for(int i = 0; i < k; i++) { /* use v[i] */
reverse(v.begin()+k, v.end());
} while (next_permutation(all(v)));

```

8.4 Priority Queue (pb_ds)

Usage: Meldable heap supporting modify/erase via point_iterator.

Time Complexity: $O(\log N)$

```

// 큐 병합, 임의 값 수정 및 삭제 가능
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
template <typename T>
using pbds_pq = __gnu_pbds::priority_queue<T, less<T>,
pairing_heap_tag>;

```

```

int main() {
    pbds_pq<int> pq1, pq2;
    auto it = pq1.push(10); pq2.push(100);
    pq1.join(pq2); // O(1), pq1: {10, 100}, pq2: {}
    pq1.modify(it, 50); // O(logN), pq1 : {50, 100}
    pq1.erase(it); // O(logN), pq1: {100}
    pq1.top(); pq1.empty(); pq1.size(); pq1.pop(); //
    same
}

```

8.5 Rope

Usage: Persistent sequence supporting fast insertion, deletion and slicing.

Time Complexity: $O(\log N)$

```

#include<ext/rope>
using namespace __gnu_cxx;
int main() {
    string str; rope r(str.c_str()); // vector<T> v;
    rope<T> r(all(v));
    r.insert(pos, str); r.erase(pos, len); // Insert &
    Erase O(logN)
    r.replace(pos, len, str); // Replace O(logN)
    rope r2 = r; // O(1)
    r2 = r.substr(pos, len); // O(logN)
    r += r2; // Append O(logN)
    r[idx]; // O(logN), but for(auto i : r) is O(N)
    cout << r; // O(N)
}

```

8.6 Trie (pb_ds)

Usage: Prefix tree implementation from pb_ds.

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/trie_policy.hpp>
using namespace __gnu_pbds;
typedef trie<string, null_type,
trie_string_access_traits<>, pat_trie_tag,
trie_prefix_search_node_update> trie_set;
int main() {
    trie_set t; t.insert("apple"); t.insert("app");
    t.insert("banana");
    if (t.find("app") != t.end()) { /* found app */
    auto [st, en] = t.prefix_range("ba");
    for (auto it = st; it != en; it++) { cout << *it <<
    "\n"; /* banana */
    *t.lower_bound("app"); // app
    *t.upper_bound("app"); // apple
    t.split("b", t2); // t: {app, apple}, t2: {banana}
    t.erase("apple");
}

```

9 Misc

9.1 Custom Hash

Usage: Custom hash for pair, vector.

```
struct custom_hash {
    template <class T>
    void combine(size_t& seed, const T& v) const {
        seed ^= hash<T>{}(v) + 0x9e3779b9 + (seed << 6) +
        (seed >> 2);
    }
    template <class T1, class T2>
    size_t operator()(const pair<T1, T2>& p) const {
        size_t seed = 0; combine(seed, p.first);
        combine(seed, p.second);
        return seed;
    }
    template <class T>
    size_t operator()(const vector<T>& v) const {
        size_t seed = 0; for (const auto& i : v)
            combine(seed, i);
        return seed;
    }
};
```

9.2 Fast I/O

Usage: Fast integer I/O using fread/fwrite.

```
#include <unistd.h>
constexpr int rbuf_sz = 1 << 20, wbuf_sz = 1 << 20;
int main() {
    char r[rbuf_sz], *pr = r; read(0, r, rbuf_sz);
    auto read_char = [&] {
        if (pr - r == rbuf_sz) read(0, pr = r, rbuf_sz);
        return *pr++;
    };
    auto read_int = [&] {
        int ret = 0, flag = 0; char c = read_char();
        while (c == ' ' || c == '\n') c = read_char();
        if (c == '-') flag = 1, c = read_char();
        while (c != ' ' && c != '\n') ret = 10 * ret + c -
            '0', c = read_char();
        if (flag) ret = -ret;
        return ret;
    };
    char w[wbuf_sz], *pw = w;
    auto write_char = [&](char c) {
        if (pw - w == wbuf_sz) write(1, w, pw - w), pw = w;
        *pw++ = c;
    };
    auto write_int = [&](int x) {
```

```
        if (pw - w + 40 > wbuf_sz) write(1, w, pw - w), pw =
        = w;
        if (x < 0) *pw++ = '-', x = -x;
        char t[10], *pt = t;
        do *pt++ = x % 10 + '0'; while (x /= 10);
        do *pw++ = --pt; while (pt != t);
    };
}
```

9.3 Random

Usage: Better random for mt19937.

```
#include <random>
#include <chrono>
mt19937_64
rng(chrono::steady_clock::now().time_since_epoch().count())
uniform_int_distribution<int>(l, r)(rng); // [l, r]
uniform_real_distribution<double>(l, r)(rng); // [l, r]
shuffle(all(v), rng) // shuffle vector
vector<double> w = { 40, 10, 50 };
discrete_distribution<int>(all(w))(rng); // 0: 40%, 1:
10%, 2: 50%
```

9.4 Ternary Search

Usage: Finding extremum of unimodal functions.

Time Complexity: $O(\log N)$

```
ll ternary_search(ll lo, ll hi, auto f) {
    while (hi - lo >= 3) {
        ll p = lo + (hi-lo) / 3, q = hi - (hi-lo) / 3;
        if (f(p) < f(q)) hi = q; // for max: f(p) > f(q)
        else lo = p;
    }
    ll res = lo;
    for (ll i = lo+1; i <= hi; i++) if (f(i) < f(res))
        res = i;
    return res;
}
double ternary_search(double lo, double hi, auto f, int
it=100) {
    while (it--) {
        double p = (lo*2 + hi) / 3., q = (lo + hi*2) / 3.;
        if (f(p) < f(q)) hi = q; // for max: f(p) > f(q)
        else lo = p;
    }
    return (lo+hi) / 2.;
```

9.5 Some tricks

Usage: Collection of bitwise hacks and optimization techniques.

```
__builtin_popcount(x); // 켜진 비트(1)의 총 개수
__builtin_clz(x); // 왼쪽(MSB)부터 연속된 0의 개수
__builtin_ctz(x); // 오른쪽(LSB)부터 연속된 0의 개수
// popcount를 유지하면서 다음으로 큰 수
bool next_combination(ll& bit, int N) {
    ll x = bit & -bit, y = bit + x;
    bit = (((bit & ~y) / x) >> 1) | y;
    return (bit < (1LL << N));
}
// v(>0)보다 크고 popcount가 같은 가장 작은 정수
ll next_perm(ll v) {
    ll t = v | (v - 1);
    return (t + 1) | (((~t & -~t) - 1) >>
        (_builtin_ctz(v) + 1));
}
// mask의 모든 부분집합을 내림차순으로 순회 (0 제외),
// 0(3^N)
for (int submask = mask; submask > 0; submask =
    (submask-1) & mask);
// mask를 포함하는 모든 상위집합을 오름차순으로 순회
for (int supmask = mask; supmask < (1 << n); supmask =
    (supmask+1) | mask);
// 런타임 변수 n에 맞는 크기의 bitset을 사용
const int MAXLEN = 200005; // 최대 범위
template <int len = 1>
void solve(int n) {
    if (len < n) { solve<min(len * 2, MAXLEN)>(n);
        return; }
    bitset<len> bs;
    // do stuff
}
// bitset 고속순회 (켜져있는 비트만 순회)
void bitset_iterate(bitset<1000>& bs) {
    int idx = bs._Find_first();
    while (idx < bs.size()) {
        // do stuff
        idx = bs._Find_next(idx);
    }
}
// 1부터 n까지의 수에서 숫자 i가 등장하는 총 횟수
ll count_digit_fraq(ll n, int i) {
    ll ret = 0;
    for (ll j = 1; j <= n; j *= 10) {
        ll div = j * 10, quote = n / div, rem = n % div;
        if (i == 0) ret += (quote - 1) * j;
        else ret += quote * j;
        if (rem >= i * j) {
            if (rem < (i + 1) * j) ret += rem - i * j + 1;
            else ret += j;
        }
    }
}
```

```

    return ret;
}

// 특정 날짜(년, 월, 일)의 요일 / 0: Sat, 1: Sun, ...
int get_day_of_week(int y, int m, int d) {
    if (m <= 2) y--, m += 12; int c = y / 100; y %= 100;
    int w = ((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1) %
7;
    if (w < 0) w += 7; return w;
}

```

10 Checklist + Useful Info

10.1 Highly Composite Numbers, Large Prime

< 10^k	number	divs
2	2	2
3	3	3
4	4	3 2
5	5	5
6	6	4 3
7	7	7
8	8	6 4
9	9	9
10	10	5 2
11	11	11
12	12	6 4
13	13	13
14	14	7 2
15	15	5 3
16	16	8 4
17	17	17
18	18	9 6

< 10^k	number	divs
1	1	1
2	2	2
3	3	3
4	4	3 2
5	5	5
6	6	4 3
7	7	7
8	8	6 4
9	9	9
10	10	5 2
11	11	11
12	12	6 4
13	13	13
14	14	7 2
15	15	5 3
16	16	8 4
17	17	17
18	18	9 6

< 10^k	prime	# of prime	< 10^k	prime
1	2	1	2	3
2	3	1	5	7
3	5	1	7	11
4	7	1	11	13
5	11	1	13	17
6	13	1	17	19
7	17	1	19	23
8	19	1	23	29
9	23	1	29	31
10	29	1	31	37
11	31	1	37	41
12	37	1	41	43
13	41	1	43	47
14	43	1	47	53
15	47	1	53	59
16	53	1	59	61
17	59	1	61	67
18	61	1	67	71
19	67	1	71	73
20	71	1	73	79
21	73	1	79	83
22	79	1	83	89
23	83	1	89	97
24	89	1	97	101
25	101	1	101	103
26	103	1	103	107
27	107	1	107	109
28	109	1	109	113
29	113	1	113	127
30	127	1	127	131
31	131	1	131	137
32	137	1	137	149
33	149	1	149	151
34	151	1	151	157
35	157	1	157	163
36	163	1	163	173
37	173	1	173	179
38	179	1	179	181
39	181	1	181	191
40	191	1	191	197
41	197	1	197	199
42	199	1	199	211
43	211	1	211	223
44	223	1	223	227
45	227	1	227	229
46	229	1	229	233
47	233	1	233	239
48	239	1	239	241
49	241	1	241	251
50	251	1	251	257
51	257	1	257	263
52	263	1	263	269
53	269	1	269	271
54	271	1	271	281
55	281	1	281	283
56	283	1	283	291
57	291	1	291	293
58	293	1	293	297
59	297	1	297	301
60	301	1	301	307
61	307	1	307	311
62	311	1	311	313
63	313	1	313	317
64	317	1	317	323
65	323	1	323	329
66	329	1	329	331
67	331	1	331	337
68	337	1	337	341
69	341	1	341	343
70	343	1	343	347
71	347	1	347	351
72	351	1	351	353
73	353	1	353	357
74	357	1	357	361
75	361	1	361	367
76	367	1	367	371
77	371	1	371	373
78	373	1	373	377
79	377	1	377	381
80	381	1	381	383
81	383	1	383	387
82	387	1	387	391
83	391	1	391	397
84	397	1	397	401
85	401	1	401	403
86	403	1	403	407
87	407	1	407	409
88	409	1	409	411
89	411	1	411	413
90	413	1	413	417
91	417	1	417	419
92	419	1	419	421
93	421	1	421	423
94	423	1	423	427
95	427	1	427	429
96	429	1	429	431
97	431	1	431	433
98	433	1	433	437
99	437	1	437	439
100	439	1	439	441

10.2 Useful Stuff

- Catalan Number
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900

$$C_n = \binom{2n}{n} / (n+1);$$

- 길이가 $2n$ 인 올바른 꽂호 수식의 수
- $n+1$ 개의 리프를 가진 풀 바이너리 트리의 수
- $n+2$ 각형을 n 개의 삼각형으로 나누는 방법의 수

- Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는? 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 셀다(단, “아무것도 하지 않는다”라는 operation도 있어야 함!) 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

- 알고리즘 게임

- Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0이 아니면 첫번째, 0이면 두번째 플레이어가 승리.
- Grundy Number : 어떤 상황의 Grundy Number는, 가능한 다음 상황들의 Grundy Number를 모두 모은 다음, 그 집합에 포함되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러 개의 state들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.
- Subtraction Game : 한 번에 k 개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 $k+1$ 로 나눈 나머지를 XOR 합하여 판단한다.
- Index-k Nim : 한 번에 최대 k 개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 $k+1$ 로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.
- Misere Nim : 모든 돌 무더기가 1이면 N이 홀수일 때 후공승, 그렇지 않은 경우 XOR 합 0이면 후공승
- Pick's Theorem

격자점으로 구성된 simple polygon이 주어짐. I는 polygon 내부의 격자점 수, B는 polygon 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다. $A = I + B/2 - 1$

- 가장 가까운 두 점 : 분할정복으로 가까운 6개의 점만 확인
- 홀의 결혼 정리 : 이분그래프(L-R)에서, 모든 L을 매칭하는 필요충분 조건 = L에서 임의의 부분집합 S를 골랐을 때, 반드시 $(S의 크기) \leq (S와 연결되어 있는 모든 R의 크기)$ 이다.
- 소수 : 10 007, 10 009, 10 111, 31 567, 70 001, 1 000 003, 1 000 033, 4 000 037, 99 999 989, 999 999 937, 1 000 000 007, 1 000 000 009, 9 999 999 967, 99 999 999 977
- 소수 개수 : (1e5 이하: 9592), (1e7 이하: 664 579), (1e9 이하: 50 847 534)
- 10^{15} 이하의 정수 범위의 나눗셈 한 번은 오차가 없다.
- N의 약수의 개수 = $O(N^{1/3})$, N의 약수의 합 = $O(N \log \log N)$
- $\phi(mn) = \phi(m)\phi(n), \phi(pr^n) = pr^n - pr^{n-1}, a^{\phi(n)} \equiv 1 \pmod{n}$ if coprime
- Euler characteristic : v - e + f (면, 외부 포함) = 1 + c (컴포넌트)

$$\bullet \text{ Euler's phi } \phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

- Lucas' Theorem $\binom{m}{n} = \prod \binom{m_i}{n_i} \pmod{p}$ m_i, n_i 는 p^i 의 계수
- 스케줄링에서 데드라인이 빠른 걸 쓰는게 이득. 늦은 스케줄이 안 들어갈 때 가장 시간 소모가 큰 스케줄 1개를 제거하면 이득.

10.3 자주 쓰이는 문제 접근법

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (brute force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해보면서)
- 문제를 단순화할 수 있을까? / 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까? / 문제를 분해할 수 있을까?
- 뒤에서부터 생각해서 문제를 풀 수 있을까? / 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)
- 구간을 통째로 가져간다 : 플로우 + 적당한 자료구조 $(i, i+1, k, 0), (s, e, 1, w), (N, T, k, 0)$
- 말도 안 되는 것 / 당연하다고 생각한 것 다시 생각해 보기
- 특수 조건을 꼭 활용 / 여사건으로 생각하기
- 게임이론 - 거울 전략 혹은 mex DP 연계
- 겁먹지 말고 경우 나누어 생각 / 해법에서 역순으로 가능한가?
- 딱 맞는 시간복잡도에 집착하지 말자 / 문제에 의미 있는 작은 상수 이용
- 스몰투라지, 트라이, 해싱, 루트질 같은 트릭 생각
- 너무 추상화하기보단 풀려야 하는 방식으로 생각하기
- 잘못된 방법으로 파고들지 말고 버리자 / 제발 터널 비전에 빠지지 말자
- 헬프 콜은 적극적으로 / 혼자 멘탈 나가지 않기

10.4 DP 최적화 접근

- $C[i, j] = A[i] * B[j]$ 이고 A, B가 단조증가, 단조감소이면 Monge
- 1..r의 값들의 sum이나 min은 Monge
- 식 정리해서 일차(HT) 혹은 비슷한(MQ) 함수를 발견, 구현 힘들면 Li-Chao
- $a \leq b \leq c \leq d$ 에서 $A[a, c] + A[b, d] \leq A[a, d] + A[b, c]$
- Monge 성질을 보이기 어려우면 N^2 나이브 짜서 opt의 단조성을 확인하고 찍맞
- 식이 간단하거나 변수가 독립적이면 DP 테이블을 세그 위에 옮겨서 해결
- 침착하게 점화식부터 세우고 Monge인지 판별
- Monge에 침착하지 말고 단조성이나 볼록성만 보여도 됨

10.5 Graph Matching(Graph with $|V| \leq 500$)

- Game on a Graph :** s 에 토큰이 있음. 플레이어는 각자의 턴마다 토큰을 인접한 정점으로 옮기고 못 옮기면 짐. s 를 포함하지 않는 최대 매칭이 존재함 \leftrightarrow 후공이 이김
- Chinese Postman Problem :** 모든 간선을 방문하는 최소 가중치 Walk를 구하는 문제.
Floyd를 돌린 다음, 홀수 정점들을 모아서 최소 가중치 매칭(홀수 정점은 짝수 개 존재)
- Unweighted Edge Cover :** 모든 정점을 덮는 가장 작은 (minimum cardinality/weight) 간선 집합을 구하는 문제
 $|V| - |M|$, 길이 3짜리 경로 없음, star graph 여러 개로 구성
- Weighted Edge Cover :** $\sum_{v \in V} w(v)) - \sum_{(u,v) \in M} (w(u) + w(v) - d(u,v))$, $w(x)$ 는 x 와 인접한 간선의 최소 가중치
- NEERC'18 B :** 각 기계마다 2명의 노동자가 다뤄야 하는 문제.
기계마다 두 개의 정점을 만들고 간선으로 연결하면 정답은 $|M| - |\text{기계}|$ 임. 정답에 $1/2$ 씩 기여한다는 점을 생각해보면 좋음.
- Min Disjoint Cycle Cover :** 정점이 중복되지 않으면서 모든 정점을 덮는 길이 3 이상의 사이클을 집합을 찾는 문제.
모든 정점은 2개의 서로 다른 간선, 일부 간선은 양쪽 끝점과 매칭되어야 하므로 플로우를 생각할 수 있지만 용량 2짜리 간선에 유량을 1만큼 흘릴 수 있으므로 플로우는 불가능.
각 정점과 간선을 2개씩 $((v, v'), (e_{i,u}, e_{i,v}))$ 로 복사하자. 모든 간선 $e = (u, v)$ 에 대해 e_u 와 e_v 를 잇는 가중치 w 짜리 간선을 만들고 (like NEERC18), $(u, e_{i,u}), (u', e_{i,u}), (v, e_{i,v}), (v', e_{i,v})$ 를 연결하는 가중치 0짜리 간선을 만들자. Perfect 매칭이 존재함 \leftrightarrow Disjoint Cycle Cover 존재. 최대 가중치 매칭 찾은 뒤 모든 간선 가중치 합에서 매칭 빼면 됨.
- Two Matching:** 각 정점이 최대 2개의 간선과 인접할 수 있는 최대 가중치 매칭 문제.
각 컴포넌트는 정점 하나/경로/사이클이 되어야 함. 모든 서로 다른 정점 쌍에 대해 가중치 0짜리 간선 만들고, 가중치 0짜리 (v, v') 간선 만들면 Disjoing Cycle Cover 문제가 됨. 정점 하나만 있는 컴포넌트는 self-loop, 경로 형태의 컴포넌트는 양쪽 끝점을 연결한다고 생각하면 편함.

10.6 MinCut 모델링

- N 개의 boolean 변수 v_1, \dots, v_n 을 정해서 비용을 최소화하는 문제 = true인 점은 T , false인 점은 F 와 연결되게 분할하는 민컷 문제
 - v_i 가 T일 때 비용 발생: i 에서 F로 가는 비용 간선
 - v_i 가 F일 때 비용 발생: i 에서 T로 가는 비용 간선
 - v_i 가 T이고 v_j 가 F일 때 비용 발생: i 에서 j 로 가는 비용 간선

- $v_i \neq v_j$ 일 때 비용 발생: i 에서 j 로, j 에서 i 로 가는 비용 간선
- v_i 가 T면 v_j 도 T여야 함: i 에서 j 로 가는 무한 간선
- v_i 가 F면 v_j 도 F여야 함: j 에서 i 로 가는 무한 간선
- 5/6번 + v_i 와 v_j 가 달라야 한다는 조건이 있으면 MAX-2SAT
- Maximum Density Subgraph (NEERC'06H, BOJ 3611 팀의 난이도)
 - $\text{density} \geq x$ 인 subgraph가 있는지 이분 탐색
 - 정점 N 개, 간선 M 개, 차수 D_i 개
 - 그래프의 간선마다 용량 1인 양방향 간선 추가
 - 소스에서 정점으로 용량 M , 정점에서 싱크로 용량 $M - D_i + 2x$
 - min cut에서 S와 붙어 있는 애들이 1개 이상이면 x 이상이고, 그게 subgraph의 정점들
 - while($r-l \geq 1.0/(n^2)$) 으로 해야 함. 너무 많이 돌리면 실수 오차