

Team Note of A University

A Team

Compiled on January 10, 2026

Contents

1 Basic Implementation	2	16
1.1 Main Template	2	16
2 Math	2	16
2.1 Basic Arithmetic	2	16
2.2 Binomial Coefficient	2	16
2.3 Chinese Remainder Theorem	3	17
2.4 FFT & NTT	3	17
2.5 Linear Sieve	5	17
2.6 Miller-Rabin & Pollard-Rho	5	18
3 Data Structure	5	18
3.1 Erasable Priority Queue	5	18
3.2 Non-Recursive Segment Tree	5	19
3.3 Merge Sort Tree	6	19
3.4 Persistent Segment Tree	7	19
3.5 Sweep-line Mo's	7	20
4 Graph	8	20
4.1 Bellman Ford	8	20
4.2 LCA	8	20
4.3 HLD	8	20
4.4 Centroid Decomposition	9	21
4.5 Bipartite Matching	10	21
4.6 Dinic	11	21
4.7 MCMF	12	21
4.8 Circulation	13	22
4.9 SCC	13	22
4.10 BCC	14	22
5 DP Optimization	14	22
5.1 Convex Hull Trick	14	22
5.2 Linear CHT	15	23
5.3 D&C optimization	15	23
5.4 Monotone Queue optimization	15	24
5.5 Aliens Trick	16	24
5.6 Sum Over Subsets	16	24
6 Geometry	16	16
6.1 Geometry Template	16	16
6.2 Convex Hull	16	16
6.3 Rotating Calipers	17	17
6.4 Point in Convex Polygon	17	17
6.5 Point in Polygon	17	17
6.6 Polar Sort	17	17
6.7 Polygon Area	17	17
6.8 Segment Intersection	18	18
7 String	18	18
7.1 Aho-Corasick	18	18
7.2 Hashing	19	19
7.3 KMP	19	19
7.4 Manacher	19	19
7.5 Suffix Array	19	20
7.6 Z-algorithm	20	20
8 STL & pbds	20	20
8.1 Hash map	20	20
8.2 Ordered Set	20	20
8.3 Permutation & Combination	20	21
8.4 Priority Queue	21	21
8.5 Rope	21	21
8.6 Trie	21	21
9 Misc	21	21
9.1 Custom Hash	21	21
9.2 Fast I/O	22	22
9.3 Random	22	22
9.4 Ternary Search	22	22
10 Checklist + Useful Info	22	22
10.1 Useful Stuff	22	22
10.2 자주 쓰이는 문제 접근법	23	23
10.3 DP 최적화 접근	24	24

1 Basic Implementation

1.1 Main Template

Usage: Basic CP template with headers and defines.

```
// #pragma GCC optimize ("O3,unroll-loops")
// #pragma GCC target ("avx,avx2,fma") // SIMD
#include <bits/stdc++.h>
#define fastio cin.tie(0)->sync_with_stdio(0)
#define all(x) (x).begin(),(x).end()
#define rall(x) (x).rbegin(),(x).rend()
#define compress(v) sort(all(v)), v.erase(unique(all(v)), v.end())
#define sz(x) (int)(x).size()
using namespace std;
typedef long long ll;
const ll INF = 1e18;
const int MOD = 998244353;
const int SIZE = 524288;
```

2 Math

2.1 Basic Arithmetic

Usage: Modular inverse, Extended Euclidean.

```
ll modmul(ll a, ll b, ll m) { return (__int128)a * b % m; }
ll modpow(ll b, ll e, ll m) {
    ll ans = 1;
    for (; e; b = modmul(b, b, m), e /= 2)
        if (e & 1) ans = modmul(ans, b, m);
    return ans;
}
ll xgcd(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll x1, y1, g = xgcd(b, a % b, x1, y1);
    return x = y1, y = x1 - a / b * y1, g;
}
ll modinv(ll a, ll m) {
    ll x, y;
    ll g = xgcd(a, m, x, y);
    if (g != 1) return -1;
    return (x%m + m) % m;
}
```

2.2 Binomial Coefficient

Time Complexity: $O(1)/O(\sum p^k)$

```
// when M is big prime; Init: O(MAXN), Query: O(1)
ll modmul(ll a, ll b, ll m)
ll modpow(ll b, ll e, ll m)
const int M = 1e9+7, MAXN = 4000000;
```

```
ll fac[MAXN+5], finv[MAXN+5];
void init() {
    fac[0] = 1;
    for (int i = 1; i <= MAXN; i++) {
        fac[i] = modmul(fac[i-1], i, M);
    }
    finv[MAXN] = modpow(fac[MAXN], M-2, M);
    for (int i = MAXN-1; i >= 0; i--) {
        finv[i] = modmul(finv[i+1], i+1, M);
    }
}
ll nCk(int n, int k) {
    ll r = modmul(fac[n], finv[n-k], M);
    return modmul(r, finv[k], M);
}
// O(Sum of p^k) per query. (M = product of p^k)
ll modmul(ll a, ll b, ll m)
ll modpow(ll b, ll e, ll m)
ll xgcd(ll a, ll b, ll &x, ll &y)
ll modinv(ll a, ll m)
ll count(ll n, ll p) {
    ll cnt = 0;
    while (n > 0) { cnt += n/p; n /= p; }
    return cnt;
}
ll calc(ll n, ll p, ll pe, const auto& ft) {
    if (n == 0) return 1;
    ll v = ft[pe], res = modpow(v, n/pe, pe);
    res = modmul(res, ft[n%pe], pe);
    return modmul(res, calc(n/p, p, pe, ft), pe);
}
ll nCk_pe(ll n, ll k, ll p, ll pe, int e) {
    if (k < 0 || k > n) return 0;
    ll pc = count(n, p) - count(k, p) - count(n-k, p);
    if (pc >= e) return 0;
    vector<ll> ft(pe+1); ft[0] = 1;
    for(int i = 1; i <= pe; i++) {
        ft[i] = ft[i-1];
        if (i%p != 0) ft[i] = modmul(ft[i], i, pe);
    }
    ll den = modmul(calc(k, p, pe, ft), calc(n-k, p, pe, ft), pe);
    ll res = modmul(calc(n, p, pe, ft), modinv(den, pe), pe);
    res = modmul(res, modpow(p, pc, pe), pe);
    return res;
}
ll nCk(ll n, ll k, int m) {
    if (k < 0 || k > n) return 0;
    if (k == 0 || k == n) return 1;
    ll t = m, res = 0;
    auto add = [&](ll p, ll pe, ll e) {
        ll rem = nCk_pe(n, k, p, pe, e);
        ll tm = modmul(rem, m/p, pe);
        res += tm;
    };
    for (int i = 1; i <= pe; i++) {
        add(i, i, 1);
    }
}
```

```

        tm = modmul(tm, modinv(m/pe, pe), m);
        res = (res + tm) % m;
    };
    for (ll i = 2; i*i <= t; i++) {
        if (t%i == 0) {
            ll p = i, pe = 1, e = 0;
            while (t%i == 0) {
                pe *= i; t /= i; e++;
            }
            add(p, pe, e);
        }
    }
    if (t > 1) add(t, t, 1);
    return res;
}

```

2.3 Chinese Remainder Theorem

Usage: Solve system of linear congruences.

Time Complexity: $O(\log N)$

```

ll xgcd(ll a, ll b, ll &x, ll &y)
pair<ll,ll> CRT(ll a1, ll m1, ll a2, ll m2) {
    ll x, y, g = xgcd(m1, m2, x, y);
    if ((a2 - a1) % g) return { -1, -1 };
    ll md = m2 / g, k = (a2 - a1) / g % md * (x % md) % md;
    return { a1 + (k < 0 ? k + md : k) * m1, m1 / g * m2 };
}
pair<ll,ll> CRT(const vector<ll>& a, const vector<ll>& m) {
    ll ra = a[0], rm = m[0];
    for (int i = 1; i < (int)m.size(); i++) {
        auto [aa, mm] = CRT(ra, rm, a[i], m[i]);
        if (mm == -1) return { -1, -1 };
        ra = aa; rm = mm;
    }
    return { ra, rm };
}

```

2.4 FFT & NTT

Usage: Fast Fourier/Number Theoretic Transform for convolutions.

Time Complexity: $O(N \log N)$

```

template<int M> struct MINT {
    int v;
    MINT(ll _v = 0) { v = _v % M; if (v < 0) v += M; }
    MINT operator+(const MINT& o) const { return MINT(v + o.v); }
    MINT operator-(const MINT& o) const { return MINT(v - o.v); }
    MINT operator*(const MINT& o) const { return MINT((ll)v * o.v); }
    MINT& operator*=(const MINT& o) { return *this = *this * o; }
    friend MINT pw(MINT a, ll b) {
        MINT r = 1; for (; b; b >>= 1, a *= a) if (b & 1) r *= a;
    }
}

```

```

    return r;
}
friend MINT inv(MINT a) { return pw(a, M - 2); }
};

namespace fft {
    using cpx = complex<double>;
    void rev_bit(int n, vector<auto>& a) {
        for (int i = 1, j = 0; i < n; i++) {
            int bit = n >> 1; for (; j & bit; bit >>= 1) j ^= bit; j ^= bit;
            if (i < j) swap(a[i], a[j]);
        }
    }
    void FFT(vector<cpx>& a, bool inv_f) {
        int n = a.size(); rev_bit(n, a);
        for (int len = 2; len <= n; len <= 1) {
            double ang = 2 * acos(-1) / len * (inv_f ? -1 : 1);
            cpx wlen(cos(ang), sin(ang));
            for (int i = 0; i < n; i += len) {
                cpx w(1);
                for (int j = 0; j < len / 2; j++) {
                    cpx u = a[i + j], v = a[i + j + len / 2] * w;
                    a[i + j] = u + v; a[i + j + len / 2] = u - v; w *= wlen;
                }
            }
            if (inv_f) for (auto& x : a) x /= n;
        }
    }
    vector<ll> multiply(const vector<ll>& a, const vector<ll>& b) {
        int n = 1; while (n < a.size() + b.size()) n <= 1;
        vector<cpx> fa(n), fb(n);
        for(int i=0; i<a.size(); i++) fa[i] = cpx(a[i], 0);
        for(int i=0; i<b.size(); i++) fb[i] = cpx(b[i], 0);
        FFT(fa, 0); FFT(fb, 0);
        for(int i=0; i<n; i++) fa[i] *= fb[i];
        FFT(fa, 1); vector<ll> res(n);
        for(int i=0; i<n; i++) res[i] = llround(fa[i].real());
        return res;
    }
    vector<ll> multiply_mod(const vector<ll>& a, const vector<ll>& b, ll mod) {
        int n = 1; while (n < a.size() + b.size()) n <= 1;
        vector<cpx> v1(n), v2(n), r1(n), r2(n);
        for (int i = 0; i < a.size(); i++) v1[i] = cpx(a[i] >> 15, a[i] & 32767);
        for (int i = 0; i < b.size(); i++) v2[i] = cpx(b[i] >> 15, b[i] & 32767);
        FFT(v1, 0); FFT(v2, 0);
        for (int i = 0; i < n; i++) {
            int j = i ? n - i : i;
            cpx a1 = (v1[i] + conj(v1[j])) * cpx(0.5, 0), a2 = (v1[i] - conj(v1[j])) * cpx(0, -0.5);
            cpx b1 = (v2[i] + conj(v2[j])) * cpx(0.5, 0), b2 = (v2[i] - conj(v2[j])) * cpx(0, -0.5);
            r1[i] = a1 * b1 + a2 * b2 * cpx(0, 1); r2[i] = a2 * b1 + a2 * b2 * cpx(0, 1);
        }
    }
}
```

```

    }
    FFT(r1, 1); FFT(r2, 1);
    vector<ll> res(n);
    for (int i = 0; i < n; i++) {
        ll av = ((ll)round(r1[i].real()) % mod, cv = ((ll)round(r2[i].imag()) % mod;
        ll bv = (((ll)round(r1[i].imag()) + ((ll)round(r2[i].real())) % mod;
        res[i] = (av << 30) + (bv << 15) + cv; res[i] = (res[i] % mod + mod) % mod;
    }
    return res;
}

template<int W, int M> void NTT(vector<MINT<M>>& a, bool inv_f) {
    int n = a.size(); rev_bit(n, a);
    for (int len = 2; len <= n; len <= 1) {
        MINT<M> wlen = pw(MINT<M>(W), (M - 1) / len);
        if (inv_f) wlen = inv(wlen);
        for (int i = 0; i < n; i += len) {
            MINT<M> w = 1;
            for (int j = 0; j < len / 2; j++) {
                MINT<M> u = a[i + j], v = a[i + j + len / 2] * w;
                a[i + j] = u + v; a[i + j + len / 2] = u - v; w *= wlen;
            }
        }
        if (inv_f) { MINT<M> rn = inv(MINT<M>(n)); for (auto& x : a) x *= rn; }
    }
}

template<int W, int M> struct Poly {
    using T = MINT<M>; vector<T> a;
    Poly(const vector<T>& _a = {}) : a(_a) { norm(); }
    void norm() { while (a.size() && a.back().v == 0) a.pop_back(); }
    int deg() const { return (int)a.size() - 1; }
    T operator[](int i) const { return i < a.size() ? a[i] : T(0); }
    Poly operator*(const Poly& o) const {
        if (o.empty() || o.a.empty()) return {};
        int n = 1, sz = a.size() + o.a.size() - 1;
        while (n < sz) n <= 1;
        vector<T> fa(n), fb(n); copy(all(a), fa.begin()); copy(all(o.a), fb.begin());
        fft::NTT<W, M>(fa, 0); fft::NTT<W, M>(fb, 0);
        for (int i = 0; i < n; i++) fa[i] *= fb[i];
        fft::NTT<W, M>(fa, 1); return fa;
    }
    Poly inv(int n) const {
        Poly r({ ::inv(a[0]) });
        for (int i = 1; i < n; i <= 1) {
            Poly tmp(vector<T>(a.begin(), a.begin() + min((int)a.size(), i * 2)));
            r = (r * (Poly({T(2)})) - r * tmp); r.a.resize(i * 2);
        }
        r.a.resize(n); return r;
    }
    Poly operator/(Poly o) const {

```

```

        if (deg() < o.deg()) return {};
        int n = deg() - o.deg() + 1;
        Poly ra = a, rb = o.a; reverse(all(ra.a)); reverse(all(rb.a));
        Poly q = (ra * rb.inv(n)); q.a.resize(n); reverse(all(q.a)); return q;
    }
    Poly operator%(Poly o) const {
        if (deg() < o.deg()) return *this;
        Poly r = *this - (*this / o) * o; r.norm(); return r;
    }
    Poly operator-(const Poly& o) const {
        vector<T> res(max(a.size(), o.a.size()));
        for (int i = 0; i < res.size(); i++) res[i] = (*this)[i] - o[i];
        return res;
    }
};

using mint = MINT<998244353>;
using poly = Poly<3, 998244353>;
mint Kitamasa(poly c, poly a, ll n) {
    if (n <= a.deg()) return a[n];
    poly f; for (int i = 0; i <= c.deg(); i++) f.a.push_back(mint(0) - c[c.deg() - i]);
    f.a.push_back(1); poly res({1}), x({0, 1});
    for (; n; n >= 1, x = (x * x) % f) if (n & 1) res = (res * x) % f;
    mint ans = 0; for (int i = 0; i <= a.deg(); i++) ans = ans + a[i] * res[i];
    return ans;
}

int main() {
    vector<ll> A = {1, 2, 1}; // 1+2*x+x^2
    vector<ll> B = {1, 1}; // 1+x
    vector<ll> C = fft::multiply(A, B); // {1, 3, 3, 1}
    vector<ll> D = fft::multiply_mod(A, B, 1000000007);
    poly p1({1, 2, 1}), p2({1, 1}); // NTT base
    p1 * p2; p1 / p2; p1 % p2; // polynomial operation
    // ex. A_n = 1*A_{n-1} + 1*A_{n-2}
    poly coeffs({1, 1}); // {c0, c1} 순서 (A_{n-2}, A_{n-1} 계수)
    poly initial({0, 1}); // {A0, A1} 초기값
    cout << Kitamasa(coeffs, initial, 1000000000).v;
}
```

Should be **tested**.

2.5 Linear Sieve

Usage: Find primes and multiplicative functions in linear time.
Time Complexity: $O(N)$

```

vector<int> Linear_sieve(int N) {
    vector<int> sieve(N+1, 1), prime(N+1);
    int pcnt = 0;
    for (int i = 2; i <= N; i++) {
        if (sieve[i]) prime[pcnt++] = i;
        for (int j = 0; i*prime[j] <= N; j++) {

```

```

        sieve[i*prime[j]] = 0;
        if (i%prime[j] == 0) break;
    }
    prime.resize(pcnt); return prime;
}

2.6 Miller-Rabin & Pollard-Rho

Usage: Primality test and integer factorization.
Time Complexity:  $O(\log^3 N)/O(N^{1/4})$ 

ll modmul(ll a, ll b, ll m)
ll modpow(ll b, ll e, ll m)
bool isPrime(ll n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ll A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ll a : A) { // ^ count trailing zeroes
        ll p = modpow(a%n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}
ll pollard(ll n) {
    auto f = [n](ll x) { return modmul(x, x, n) + 3; };
    ll x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}
vector<ll> factor(ll n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ll x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
vector<ll> res = factor(N); // factor of N

```

3 Data Structure

3.1 Erasable Priority Queue

Usage: Priority queue supporting arbitrary element deletion.

```

template <typename T = int, typename Compare = std::less<T>>
struct EraseablePQ {
    priority_queue<T, vector<T>, Compare> q, del;
    void flush() {
        while (!del.empty() && !q.empty() && q.top() == del.top()) {
            q.pop(); del.pop();
        }
    }
    void push(const T& x) { q.push(x); flush(); }
    void erase(const T& x) { del.push(x); flush(); }
    void pop() { flush(); if (!q.empty()) q.pop(); flush(); }
    const T& top() { flush(); return q.top(); }
    int size() const { return int(q.size()) - del.size(); }
    bool empty() { flush(); return q.empty(); }
};

3.2 Non-Recursive Segment Tree

Usage: Segment tree for performance.
Time Complexity:  $O(\log N)$ / query

template<typename Node>
struct SegTree {
    int n, lg, size;
    Node e; // 항등원
    vector<Node> tree;
    function<Node(Node, Node)> func;
    int log2(int n) {
        int res = 0;
        while (n > (1 << res)) res++;
        return res;
    }
    SegTree(int n, const Node& e, auto func) : n(n), lg(log2(n)), size(1<<lg), e(e),
        tree(size<<1, e), func(func) {}
    SegTree(const vector<Node>& v, const Node& e, auto func) : n(sz(v)), lg(log2(n)),
        size(1<<lg), e(e), tree(size<<1, e), func(func) {
        for (int i = 0; i < n; i++) {
            tree[i+size] = v[i];
        }
        for (int i = size-1; i > 0; i--) {
            tree[i] = func(tree[i<<1], tree[i<<1 | 1]);
        }
    }
    void add(int i, const Node& val) {
        tree[-i |= size] += val;
        while (i >= 1) {
            tree[i] = func(tree[i<<1], tree[i<<1 | 1]);
        }
    }
    void update(int i, const Node& val) {
        tree[~i |= size] = val;
        while (i >= 1) {

```

```

        tree[i] = func(tree[i<<1], tree[i<<1 | 1]);
    }
}

Node query(int i) { return tree[--i | size]; }

Node query(int l, int r) {
    Node L = e, R = e;
    for (l |= size, r |= size; l <= r; l >= 1, r >= 1) {
        if (l & 1) L = func(L, tree[l++]);
        if (~r & 1) R = func(tree[r--], R);
    }
    return func(L, R);
}

int find_kth(Node k) {
    int node = 1, st = 1, en = size;
    while (st != en) {
        int mid = (st + en) / 2; node <= 1;
        if (tree[node] >= k) en = mid;
        else k -= tree[node], node |= 1, st = mid+1;
    }
    return st;
}

int main() {
    // 1. Range Sum Query (RSQ)
    vector<int> v = {1, 2, 3, 4, 5};
    SegTree<int> rsq(v, 0, [](int a, int b) { return a+b; });
    rsq.update(3, 10);
    int sum = rsq.query(2, 4);
    // 2. Range Minimum Query (RMQ)
    const int INF = 1e9;
    SegTree<int> rmq(N, INF, [](int a, int b) { return min(a, b); });
    // 3. Binary Search on Tree (Order Statistic)
    // - Requirement: The tree must represent frequency or counts.
    // - Find the smallest index i such that prefix_sum(1...i) >= k
    int idx = rsq.find_kth(7);
}

```

3.3 Merge Sort Tree

Usage: Count/rank of elements in range $[L, R]$.

Time Complexity: $O(\log^2 N)$ / query

```

template <typename T>
struct MergeSortTree {
    int sz;
    vector<vector<T>> tree; // Space: O(N log N)
    MergeSortTree(int n) {
        sz = 1;
        while (sz < n) sz <= 1;
        tree.resize(sz*2);
    }
    void add(int x, T v) { tree[x+sz].push_back(v); }
}

```

```

void build() { // Build: O(N log N)
    for (int i = sz-1; i > 0; i--) {
        tree[i].resize(sz(tree[i*2]) + sz(tree[i*2+1]));
        merge(all(tree[i*2]), all(tree[i*2+1]), tree[i].begin());
    }
}

int query(int l, int r, T k) { // Query: O(log^2 N)
    int res = 0;
    for (l += sz, r += sz; l <= r; l >= 1, r >= 1) {
        if (l & 1) {
            res += tree[l].end() - upper_bound(all(tree[l]), k); l++;
        }
        if (!(r & 1)) {
            res += tree[r].end() - upper_bound(all(tree[r]), k); r--;
        }
        /*
        - Count < k: lower_bound(all(v)) - v.begin()
        - Count <= k: upper_bound(all(v)) - v.begin()
        - Count >= k: v.end() - lower_bound(all(v))
        - Count > k: v.end() - upper_bound(all(v))
        */
    }
    return res;
}

```

Should be **tested**.

3.4 Persistent Segment Tree

Usage: Accessing previous versions and range k-th element.

Time Complexity: $O(\log N)$ / query

```

struct PSTNode{
    PSTNode *l, *r; int v;
    PSTNode(){ l = r = nullptr; v = 0; }
};

PSTNode *root[101010];
PST(){ memset(root, 0, sizeof root); } // constructor
void init(PSTNode *node, int s, int e){
    if(s == e) return;
    int m = s + e >> 1;
    node->l = new PSTNode; node->r = new PSTNode;
    init(node->l, s, m); init(node->r, m+1, e);
}

void update(PSTNode *prv, PSTNode *now, int s, int e, int x){
    if (s == e) { now->v = prv ? prv->v + 1 : 1; return; }
    int m = s + e >> 1;
    if (x <= m) {
        now->l = new PSTNode; now->r = prv->r;
        update(prv->l, now->l, s, m, x);
    }
}

```

```

else {
    now->r = new PSTNode; now->l = prv->l;
    update(prv->r, now->r, m+1, e, x);
}
int t1 = now->l ? now->l->v : 0;
int t2 = now->r ? now->r->v : 0;
now->v = t1 + t2;
}

int kth(PSTNode *prv, PSTNode *now, int s, int e, int k){
    if (s == e) return s;
    int m = s + e >> 1, diff = now->l->v - prv->l->v;
    if (k <= diff) return kth(prv->l, now->l, s, m, k);
    else return kth(prv->r, now->r, m+1, e, k-diff);
}

```

Should be **tested**.

3.5 Sweepline Mo's

Usage: Optimized Mo's for $O(1)$ update and $O(1)$ query via offline sweepline.
Time Complexity: $O(N\sqrt{Q})$

```

const int MAXN = 200005, BSIZ = 450;
struct SqrtDecomp {
    ll lz_v[BSIZ+5], lz_c[BSIZ+5], v_arr[MAXN], c_arr[MAXN];
    ll total_v = 0, total_c = 0;
    void clear() { memset(this, 0, sizeof(*this)); }
    void update(int idx, ll v) { // O(sqrt N)
        total_v += v; total_c++;
        int b = idx / BSIZ;
        for (int i = idx; i < (b + 1) * BSIZ && i < MAXN; i++) {
            v_arr[i] += v; c_arr[i]++;
        }
        for (int i = b + 1; i <= BSIZ; i++) {
            lz_v[i] += v; lz_c[i]++;
        }
    }
    ll query(int idx, ll v) { // O(1)
        if (idx < 0) return (total_c * v - total_v); // 필요 시 수정
        ll cur_v = lz_v[idx / BSIZ] + v_arr[idx];
        ll cur_c = lz_c[idx / BSIZ] + c_arr[idx];
        return (cur_c * v - cur_v) + ((total_v - cur_v) - (total_c - cur_c) * v);
    }
} sd;
struct MoSweep {
    struct Query {
        int l, r, id; ll ans;
        bool operator<(const Query& o) const {
            if (l / BSIZ != o.l / BSIZ) return l < o.l;
            return (l / BSIZ) & 1 ? r < o.r : r > o.r;
        }
    };
};

```

```

struct Delta { int q_idx, l, r; bool is_sub; };
int n, q;
ll A[MAXN], pref[MAXN], result[MAXN], rnk[MAXN];
vector<Query> queries, sweep[MAXN];
void init(int _n) {
    n = _n; queries.clear();
    for(int i = 0; i <= n; i++) sweep[i].clear();
}
void add_query(int l, int r, int id) { queries.push_back({l, r, id, 0}); }
void build() {
    sort(queries.begin(), queries.end());
    sd.clear();
    for (int i = 1; i <= n; i++) {
        pref[i] = sd.query(rnk[i], A[i]);
        sd.update(rnk[i], A[i]);
    }
    int s = 1, e = 0;
    for (int i = 0; i < (int)queries.size(); i++) {
        int nl = queries[i].l, nr = queries[i].r;
        if (e < nr) sweep[s - 1].push_back({i, e + 1, nr, true}), e = nr;
        if (s > nl) sweep[e].push_back({i, nl, s - 1, false}), s = nl;
        if (e > nr) sweep[s - 1].push_back({i, nr + 1, e, false}), e = nr;
        if (s < nl) sweep[e].push_back({i, s, nl - 1, true}), s = nl;
    }
}
void solve() {
    sd.clear();
    for (int i = 1; i <= n; i++) {
        sd.update(rnk[i], A[i]);
        for (auto& d : sweep[i]) {
            ll tmp = 0;
            for (int k = d.l; k <= d.r; k++) tmp += sd.query(rnk[k], A[k]);
            queries[d.q_idx].ans += (d.is_sub ? -tmp : tmp);
        }
    }
    int s = 1, e = 0;
    for (int i = 0; i < (int)queries.size(); i++) {
        while (e < queries[i].r) queries[i].ans += pref[++e];
        while (s > queries[i].l) queries[i].ans -= pref[--s];
        while (e > queries[i].r) queries[i].ans -= pref[e--];
        while (s < queries[i].l) queries[i].ans += pref[s++];
        if (i > 0) queries[i].ans += queries[i - 1].ans;
        result[queries[i].id] = queries[i].ans;
    }
}
engine;
int main() {
    int n, q; cin >> n >> q;
    engine.init(n);
    vector<pair<ll,int>> v(n);
    for (int i = 1; i <= n; i++) {
        cin >> engine.A[i];
    }
}

```

```

        v[i - 1] = {engine.A[i], i};
    }
    sort(v.begin(), v.end());
    for (int i = 0; i < n; i++) engine.rnk[v[i].second] = i;
    for (int i = 0; i < q; i++) {
        int l, r; cin >> l >> r;
        engine.add_query(l, r, i);
    }
    engine.build();
    engine.solve();
    for (int i = 0; i < q; i++) cout << engine.result[i] << "\n";
    return 0;
}

```

4 Graph

4.1 Bellman Ford

Usage: SSSP with negative weights/cycles.
Time Complexity: $O(VE)$

```

int N, M; ll D[555]; // 주의: 웬만하면 ll로 잡는 게 좋음
vector<tuple<int,int,ll>> E; // {from, to, weight}
void AddEdge(int s, int e, int w){
    E.emplace_back(s, e, w);
}
bool Run(int s){ // 도달 가능한 음수 사이클 있으면 false 반환
    memset(D, 0x3f, sizeof D);
    ll INF = D[0];
    D[s] = 0;
    for(int iter=1; iter<=N; iter++){
        bool changed = false;
        for(auto [u, v, w] : E){
            if(D[u] == INF) continue;
            if(D[v] > D[u] + w) D[v] = D[u] + w, changed = true;
        }
        if(iter == N && changed) return false;
    }
    return true;
}

```

4.2 LCA

Usage: Lowest Common Ancestor using binary lifting.
Time Complexity: $O(\log N)$

```

int N, Q, D[101010], P[22][101010];
vector<int> G[101010];
void Connect(int u, int v){
    G[u].push_back(v); G[v].push_back(u);
}
void DFS(int v, int b=-1){

```

```

        for(auto i : G[v]) if(i != b) D[i] = D[v] + 1, P[0][i] = v, DFS(i, v);
    }
    int LCA(int u, int v){
        if(D[u] < D[v]) swap(u, v);
        int diff = D[u] - D[v];
        for(int i=0; diff; i++, diff>>=1) if(diff & 1) u = P[i][u];
        if(u == v) return u;
        for(int i=21; i>=0; i--) if(P[i][u] != P[i][v]) u = P[i][u], v = P[i][v];
        return P[0][u];
    }
    // 1. Connect로 간선 추가 2. DFS(1) 호출 3. 아래 코드 실행
    for(int i=1; i<22; i++) for(int j=1; j<=N; j++) P[i][j] = P[i-1][P[i-1][j]];
    // 4. LCA(u, v)로 최소 공통 조상 구할 수 있음
}

```

4.3 HLD

Usage: Heavy-Light Decomposition for path queries on trees.
Time Complexity: $O(\log^2 N)$

```

struct HLD{
    vector<int> dep, par, sz, in, out, top;
    int n, idx;
    vector<vector<int>> adj, graph;
    HLD(int n_) : n(n_), dep(n+1), par(n+1), sz(n+1), in(n+1), out(n+1), top(n+1),
    adj(n+1), graph(n+1) {}
    void addEdge(int u, int v) { adj[u].push_back(v); adj[v].push_back(u); }
    void dfs(int v = 1, int pre = -1) {
        for (int u : adj[v]) {
            if (u == pre) continue;
            graph[v].push_back(u);
            dfs(u, v);
        }
    }
    void dfs1(int v = 1) {
        sz[v] = 1;
        for (int &u : graph[v]) {
            dep[u] = dep[v] + 1;
            par[u] = v;
            dfs1(u);
            sz[v] += sz[u];
            if (sz[u] > sz[graph[v][0]]) swap(u, graph[v][0]);
        }
    }
    void dfs2(int v = 1) {
        in[v] = ++idx;
        for (int u : graph[v]) {
            top[u] = (u == graph[v][0]) ? top[v] : u;
            dfs2(u);
        }
        out[v] = idx;
    }
    void calculate(){

```

```

        dfs(); dfs1(); dfs2();
    }
    array<vector<array<int,2>>,2> getPath(int u, int v) {
        vector<array<int,2>> v1, v2;
        while (top[u] != top[v]) {
            if (dep[top[u]] > dep[top[v]]) {
                ll xx = top[u];
                v1.push_back({in[xx], in[u]});
                u = par[xx];
            } else {
                ll xx = top[v];
                v2.push_back({in[xx], in[v]});
                v = par[xx];
            }
            if (dep[u] < dep[v])
                v2.push_back({in[u], in[v]});
            else
                v1.push_back({in[v], in[u]});
        }
        return {v1, v2};
        // auto pp = hld.getPath(u, v);
        // Node res1 = id;
        // Node res2 = id;
        // for (auto p2 : pp[0]){
        //     res1 = seg.merge(seg.query(p2[0], p2[1]+1), res1);
        // }
        // for (auto p2 : pp[1]){
        //     res2 = seg.merge(seg.query(p2[0], p2[1]+1), res2);
        // }
        // swap(res1.lsum, res1.rsum);
        // auto res = seg.merge(res1, res2);
    }
}

```

Should be **tested**.

4.4 Centroid Decomposition

Usage: Divide and conquer on trees for path/distance problems.

Time Complexity: $O(N \log N)$ build

```

struct CentroidTree {
    vector<vector<int>> adj, c_adj; // adj: 원본트리 / c_adj: 센트로이드트리
    vector<int> sz, par, vis; int N;
    CentroidTree(int n) : N(n), adj(n+1), c_adj(n+1), sz(n+1), par(n+1), vis(n+1) {}
    void add_edge(int u, int v) { adj[u].push_back(v); adj[v].push_back(u); }
    int get_sz(int curr, int prev) {
        sz[curr] = 1;
        for (int next : adj[curr])
            if (next != prev && !vis[next]) sz[curr] += get_sz(next, curr);
        return sz[curr];
    }
}

```

```

    }
    int get_cent(int curr, int prev, int to_sz) {
        for (int next : adj[curr])
            if (next != prev && !vis[next] && sz[next] > to_sz / 2)
                return get_cent(next, curr, to_sz);
        return curr;
    }
    void static_solve(int u) { /* 현재 센트로이드를 포함하는 모든 경로를 계산하는 로직 */ }
    int build(int curr, int p = -1) {
        int cent = get_cent(curr, -1, get_sz(curr, -1));
        static_solve(cent); // 정적 분할 정복 문제일 때 사용
        vis[cent] = 1; par[cent] = p;
        for (int next : adj[cent]) {
            if (!vis[next]) {
                int child = build(next, cent);
                c_adj[cent].push_back(child); // 센트로이드 계층 연결
            }
        }
        return cent;
    }
}

```

Should be **tested**.

4.5 Bipartite Matching

Usage: Maximum matching in bipartite graphs.

Time Complexity: $O(E\sqrt{V})$

```

struct BiMatch { // Hopcroft-Karp
    vector<vector<int>> graph, grev;
    vector<int> mA, mB, dist, work;
    vector<bool> visA, visB, fA, fB; // vertex i can be excluded from some max matching
    int ns, ms;
    BiMatch(int n, int m) : ns(n), ms(m), graph(n+1), grev(m+1), mA(n+1), mB(m+1),
    dist(n+1), work(n+1) {}
    void add(int a, int b) { graph[a].push_back(b); grev[b].push_back(a); }
    void bfs() {
        fill(all(dist), -1);
        queue<int> q;
        for (int i = 1; i <= ns; i++) if (!mA[i]) {
            dist[i] = 0; q.push(i);
        }
        while (!q.empty()) {
            int i = q.front(); q.pop();
            for (auto j : graph[i]) {
                int k = mB[j];
                if (k && dist[k] == -1) {
                    dist[k] = dist[i] + 1; q.push(k);
                }
            }
        }
    }
}

```

```

        }
    }

    bool dfs(int cur) {
        for (int& i = work[cur]; i < sz(graph[cur]); i++) {
            int nb = graph[cur][i], ori = mB[nb];
            if (!ori || dist[ori] == dist[cur] + 1 && dfs(ori)) {
                mA[cur] = nb; mB[nb] = cur; return true;
            }
        }
        return false;
    }

    int match() {
        int ans = 0;
        while (1) {
            fill(all(work), 0); bfs();
            int cnt = 0;
            for (int i = 1; i <= ns; i++) {
                if (!mA[i] && dfs(i)) cnt++;
            }
            if (!cnt) break;
            ans += cnt;
        }
        return ans;
    }

    void chkEss() {
        fA.assign(ns+1, 0); fB.assign(ms+1, 0);
        visA.assign(ns+1, 0); visB.assign(ms+1, 0);
        queue<int> q;
        for (int i = 1; i <= ns; i++) if (!mA[i]) fA[i] = visA[i] = 1, q.push(i);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (int v : graph[u]) if (!visB[v]) {
                visB[v] = 1; int r = mB[v];
                if (r && !fA[r]) {
                    fA[r] = visA[r] = 1; q.push(r);
                }
            }
        }
        for (int i = 1; i <= ms; i++) if (!mB[i]) fB[i] = 1, q.push(i);
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (int u : grev[v]) {
                int r = mA[u];
                if (r && !fB[r]) {
                    fB[r] = 1; q.push(r);
                }
            }
        }
    }

    pair<vector<int>, vector<int>> vertex() { // find minimum vertex cover
        chkEss();
    }
}

```

```

vector<int> va, vb;
for (int i = 1; i <= ns; i++) if (!visA[i]) va.push_back(i);
for (int i = 1; i <= ms; i++) if (visB[i]) vb.push_back(i);
return {va, vb};

};

/* struct BiMatch {
vector<vector<int>> graph;
vector<int> mA, mB, vis;
int ns, ms;
BiMatch(int n, int m) : ns(n), ms(m), graph(n+1), mA(n+1), mB(m+1), vis(n+1) {}
void add(int a, int b) { graph[a].push_back(b); }
bool dfs(int cur) {
    vis[cur] = 1;
    for (auto i : graph[cur]) {
        int ori = mB[i];
        if (ori == 0 || (!vis[ori] && dfs(ori))) {
            mA[cur] = i; mB[i] = cur; return true;
        }
    }
    return false;
}
int match() {
    int res = 0;
    for (int i = 1; i <= ns; i++) {
        if (mA[i]) continue;
        fill(all(vis), 0);
        if (dfs(i)) res++;
    }
    return res;
}
}; */


```

4.6 Dinic

Usage: Efficient maximum flow algorithm.

Time Complexity: $O(V^2E)$

```

const ll INF = 1e18;
struct Dinic {
    struct Edge { int to; ll cap; int rev; };
    vector<vector<Edge>> graph;
    vector<int> level, work; int n;
    Dinic(int n) : n(n), graph(n+1), level(n+1), work(n+1) {}
    void add(int u, int v, ll cap) {
        graph[u].push_back({v, cap, sz(graph[v])});
        graph[v].push_back({u, 0, sz(graph[u])-1});
    }
    bool bfs(int s, int t) {
        fill(all(level), -1); level[s] = 0;
        queue<int> q; q.push(s);
        while (!q.empty()) {

```

```

        int cur = q.front(); q.pop();
        for (auto [nxt, cap, rev] : graph[cur]) {
            if (cap > 0 && level[nxt] == -1) {
                level[nxt] = level[cur]+1;
                q.push(nxt);
            }
        }
    }
    return (level[t] != -1);
}
ll dfs(int cur, int t, ll flow) {
    if (cur == t) return flow;
    for (int& i = work[cur]; i < sz(graph[cur]); i++) {
        auto& [nxt, cap, rev] = graph[cur][i];
        if (cap > 0 && level[nxt] == level[cur]+1) {
            ll push = dfs(nxt, t, min(flow, cap));
            if (push > 0) {
                cap -= push; graph[nxt][rev].cap += push;
                return push;
            }
        }
    }
    return 0;
}
ll flow(int s, int t) {
    ll ans = 0;
    while (bfs(s, t)) {
        fill(all(work), 0);
        while (auto flow = dfs(s, t, INF)) ans += flow;
    }
    return ans;
}
vector<bool> mincut(int s) {
    vector<bool> vis(n+1); vis[s] = true;
    queue<int> q; q.push(s);
    while (!q.empty()) {
        int cur = q.front(); q.pop();
        for (auto [nxt, cap, rev] : graph[cur]) {
            if (cap > 0 && !vis[nxt]) {
                vis[nxt] = true; q.push(nxt);
            }
        }
    }
    return vis;
}

```

4.7 MCMF

Usage: Minimum Cost Maximum Flow using SPFA.

Time Complexity: $O(F \cdot E \log V)$

```

const ll INF = 1e18;
struct MCMF {
    struct Edge { int to; ll cap, cost; int rev; };
    vector<vector<Edge>> graph;
    vector<ll> dist;
    vector<int> parent, edge;
    vector<bool> vis;
    int n;
    MCMF(int n) : n(n), graph(n+1), dist(n+1), parent(n+1), edge(n+1), vis(n+1) {}
    void add(int u, int v, ll cap, ll cost) {
        graph[u].push_back({v, cap, cost, sz(graph[v])});
        graph[v].push_back({u, 0, -cost, sz(graph[u])-1});
    }
    bool spfa(int s, int t) {
        fill(all(dist), INF); fill(all(parent), -1); fill(all(vis), false);
        queue<int> q; q.push(s);
        dist[s] = 0; vis[s] = true;
        while (!q.empty()) {
            int cur = q.front(); q.pop();
            vis[cur] = false;
            for (int i = 0; i < sz(graph[cur]); i++) {
                auto& [nxt, cap, cost, rev] = graph[cur][i];
                if (cap > 0 && dist[nxt] > dist[cur] + cost) {
                    dist[nxt] = dist[cur] + cost;
                    parent[nxt] = cur; edge[nxt] = i;
                    if (!vis[nxt]) {
                        vis[nxt] = true; q.push(nxt);
                    }
                }
            }
        }
        return dist[t] != INF;
    }
    pair<int,ll> flow(int s, int t) {
        int res = 0; ll cost = 0;
        while (spfa(s, t)) {
            ll fl = INF;
            for (int v = t; v != s; v = parent[v]) {
                int u = parent[v], idx = edge[v];
                fl = min(fl, graph[u][idx].cap);
            }
            for (int v = t; v != s; v = parent[v]) {
                int u = parent[v], idx = edge[v], ridx = graph[u][idx].rev;
                graph[u][idx].cap -= fl;
                graph[v][ridx].cap += fl;
                cost += (ll)fl * graph[u][idx].cost;
            }
            res += fl;
        }
        return {res, cost};
    }
};

```

4.8 Circulation

Usage: Flow with lower and upper bounds.

```
const ll INF = 1e18;
struct Dinic {};
struct Circulation {
    vector<ll> demand, low;
    vector<pair<int,int>> edge;
    int n, S, T; Dinic dn;
    Circulation(int n) : n(n), S(n+1), T(n+2), demand(n+3, 0), dn(n+2) {};
    void add_demand(int u, ll d) { demand[u] += d; }
    int add(int u, int v, ll l, ll r) {
        demand[u] -= l; demand[v] += l;
        dn.add(u, v, r - l); low.push_back(l);
        edge.push_back({u, sz(dn.graph[u])-1});
        return sz(edge)-1;
    }
    ll solve() {
        ll sum = 0, res = 0;
        for (int i = 1; i <= n; i++) sum += demand[i];
        if (sum != 0) return false;
        for (int i = 1; i <= n; i++) {
            if (demand[i] > 0) {
                dn.add(S, i, demand[i]); res += demand[i];
            } else if (demand[i] < 0) dn.add(i, T, -demand[i]);
        }
        ll f = dn.flow(S, T);
        return (f != res ? -1 : f);
    }
    ll get_flow(int i) { // get actual flow
        auto [u, idx] = edge[i];
        int v = dn.graph[u][idx].to, rev = dn.graph[u][idx].rev;
        return dn.graph[v][rev].cap + low[i];
    }
};
```

4.9 SCC

Usage: Strongly Connected Components (Tarjan's or Kosaraju's).

Time Complexity: $O(V + E)$

```
int N, M, C[10101]; // C[i] = i번 정점이 속한 SCC 번호
vector<int> G[10101], R[10101], V;
vector<vector<int>> S; // 각 SCC에 속한 정점 목록
void AddEdge(int s, int e){
    G[s].push_back(e);
    R[e].push_back(s);
}
void DFS1(int v){
    C[v] = -1;
    for(auto i : G[v]) if(!C[i]) DFS1(i);
```

```
    V.push_back(v);
}
void DFS2(int v, int c){
    C[v] = c; S.back().push_back(v);
    for(auto i : R[v]) if(C[i] == -1) DFS2(i, c);
}
int GetSCC(){ // SCC 개수 반환
    for(int i=1; i<=N; i++) if(!C[i]) DFS1(i);
    reverse(V.begin(), V.end());
    int cnt = 0;
    for(auto i : V) if(C[i] == -1) S.emplace_back(), DFS2(i, cnt++);
    return cnt;
} // 각 SCC는 위상 정렬 순서대로 번호 매겨져 있음
```

4.10 BCC

Usage: Biconnected Components, Cut-vertices, and Bridges.

Time Complexity: $O(V + E)$

```
// 1-based, 다른 거 호출하기 전에 tarjan 먼저 호출해야 함
vector<int> G[MAX_V]; int In[MAX_V], Low[MAX_V], P[MAX_V];
void addEdge(int s, int e){ G[s].push_back(e); G[e].push_back(s); }
void tarjan(int n){ // Pre-Process
    int pv = 0;
    function<void(int,int)> dfs = [&pv,&dfs](int v, int b){
        In[v] = Low[v] = ++pv; P[v] = b;
        for(auto i : G[v]){
            if(i == b) continue;
            if(!In[i]) dfs(i, v), Low[v] = min(Low[v], Low[i]);
            else Low[v] = min(Low[v], In[i]);
        }
    };
    for(int i=1; i<=n; i++) if(!In[i]) dfs(i, -1);
}
vector<int> cutVertex(int n){
    vector<int> res; array<char,MAX_V> isCut; isCut.fill(0);
    function<void(int)> dfs = [&dfs,&isCut](int v){
        int ch = 0;
        for(auto i : G[v]){
            if(P[i] != v) continue; dfs(i); ch++;
            if(P[v] == -1 && ch > 1) isCut[v] = 1;
            else if(P[v] != -1 && Low[i] >= In[v]) isCut[v]=1;
        }
    };
    for(int i=1; i<=n; i++) if(P[i] == -1) dfs(i);
    for(int i=1; i<=n; i++) if(isCut[i]) res.push_back(i);
    return move(res);
}
vector<PII> cutEdge(int n){
    vector<PII> res;
    function<void(int)> dfs = [&dfs,&res](int v){
        for(int t=0; t<G[v].size(); t++){
            if(G[v][t] == P[v]) continue;
```

```

    int i = G[v][t]; if(t != 0 && G[v][t-1] == G[v][t]) continue;
    if(P[i] != v) continue; dfs(i);
    if((t+1 == G[v].size() || i != G[v][t+1]) && Low[i] > In[v])
        res.emplace_back(min(v,i), max(v,i));
}
for(int i=1; i<=n; i++) sort(G[i].begin(), G[i].end()); // multi edge -> sort
for(int i=1; i<=n; i++) if(P[i] == -1) dfs(i);
return move(res); // sort(all(res));
}

vector<int> BCC[MAX_V]; // BCC[v] = components which contains v
void vertexDisjointBCC(int n){ // allow multi edge, not allow self loop
    int cnt = 0; array<char,MAX_V> vis; vis.fill(0);
    function<void(int,int)> dfs = [&dfs,&vis,&cnt](int v, int c){
        vis[v] = 1; if(c > 0) BCC[v].push_back(c);
        for(auto i : G[v]){
            if(vis[i]) continue;
            if(In[v] <= Low[i]) BCC[v].push_back(++cnt), dfs(i, cnt);
            else dfs(i, c);
        }
    };
    for(int i=1; i<=n; i++) if(!vis[i]) dfs(i, 0);
    for(int i=1; i<=n; i++) if(BCC[i].empty()) BCC[i].push_back(++cnt);
}

```

Should be **tested**.

5 DP Optimization

5.1 Convex Hull Trick

Usage: $dp[i] = \min(dp[j] + b[j] * a[i]), b[j] \geq b[j+1]$
 Time Complexity: $O(N \log N)$

```

// O(logN) Dynamic CHT: Slopes(k) and queries(x) can be in any order (no sorting
// required)
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
}

```

```

void add(ll k, ll m) { // y = kx + m
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (-x)->p >= y->p) isect(x, erase(y));
}
ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
}
} CHT; // add(-k, -m), -query(x) for Lower hull(min)
int main() {
    dp[0] = 0; CHT.add(a[0], dp[0]);
    for (int i = 1; i < n; i++) { // dp[i] = Max j < i (a[j]*b[i] + dp[j])
        dp[i] = CHT.query(b[i]);
        CHT.add(a[i], dp[i]);
    }
    cout << dp[n-1] << "\n";
}

```

5.2 Linear CHT

Usage: CHT when slopes/queries are monotonic.
 Time Complexity: $O(N)$

```

// O(1) CHT: Both slopes (k) and queries (x) must be monotonic (sorted).
struct PLL {
    ll x, y;
    PLL(const ll x = 0, const ll y = 0) : x(x), y(y) {}
    bool operator<=(const PLL& i) const { return 1. * x / y <= 1. * i.x / i.y; }
};
struct ConvexHull {
    static ll F(const PLL& i, const ll x) { return i.x * x + i.y; }
    static PLL C(const PLL& a, const PLL& b) { return {a.y - b.y, b.x - a.x}; }
    deque<PLL> S;
    void add(const ll a, const ll b) {
        while (S.size() > 1 && C(S.back(), PLL(a, b)) <= C(S[S.size() - 2],
            S.back())) S.pop_back();
        S.push_back(PLL(a, b));
        /* when x is monotonic decreasing
        while (S.size() > 1 && C(S[0], S[1]) <= C(PLL(a, b), S[0])) S.pop_front()
        S.push_front(PLL(a, b)); */
    }
    ll query(const ll x) {
        while (S.size() > 1 && F(S[0], x) <= F(S[1], x)) S.pop_front(); // upper
        // hull(max)
        // while (S.size() > 1 && F(S[0], x) >= F(S[1], x)) S.pop_front(); // lower
        // hull(min)
        return F(S[0], x);
    }
} CHT;

```

```

int main() { // sorted a, b
    dp[0] = 0; CHT.add(a[0], dp[0]);
    for (int i = 1; i < n; i++) { // dp[i] = Max j<i (a[j]*b[i] + dp[j])
        dp[i] = CHT.query(b[i]);
        CHT.add(a[i], dp[i]);
    }
    cout << dp[n-1] << "\n";
}

```

5.3 D&C optimization

Usage: $dp[t][i] = \min(dp[t-1][j] + c[j][i])$, c is Monge

Time Complexity: $O(KN \log N)$

```

ll dp[MAX_K][MAX_N];
// 1부터 r까지 구간의 비용을 계산하는 함수 (문제에 맞게 구현)
ll get_cost(int l, int r) { /* return sum[l][r] + C; */ }
// k: 현재 단계(구간 개수 등), pL, pR: 최적의 j를 찾을 탐색 범위
void dnc(int k, int l, int r, int pL, int pR) {
    if (l > r) return;
    int opt = pL, mid = (l + r) / 2;
    dp[k][mid] = -1e18 // 최솟값 문제면 INF, 최댓값 문제면 -INF
    for (int j = pL; j <= min(mid, pR); j++) {
        ll val = (j == 0 ? 0 : dp[k - 1][j - 1]) + get_cost(j, mid);
        if (val > dp[k][mid]) {
            dp[k][mid] = val;
            opt = j;
        }
    }
    dnc(k, l, mid - 1, pL, opt);
    dnc(k, mid + 1, r, opt, pR);
}

// usage: for (int i = 1; i <= T; i++) dnc(i, 0, n-1, 0, n-1);

```

Should be **tested**.

5.4 Monotone Queue optimization

Usage: $dp[i] = \min(dp[j] + c[j][i])$, c is Monge, find cross

Time Complexity: $O(N \log N)$

```

ll f(int j, int i); // j에서 i로 전이할 때의 값 (dp[j] + cost(j, i))
void solve() {
    auto cross = [&](ll p, ll q) {
        ll lo = max(p, q), hi = n + 1;
        while (lo + 1 < hi) {
            ll mid = (lo + hi) / 2;
            if (f(p, mid) > f(q, mid)) hi = mid; // min 기준: f(p) > f(q)면 q가 우세
            else lo = mid;
        }
        return hi;
    };
}

```

```

deque<pair<ll, ll>> dq; // {candidate_index, start_pos}
dq.push_back({0, 1}); // 초기값: 0번이 1번 위치부터 최적이라고 가정
for (int i = 1; i <= n; i++) {
    while (dq.size() > 1 && dq[1].second <= i) dq.pop_front();
    dp[i] = f(dq[0].first, i);
    while (!dq.empty()) {
        ll p = dq.back().first;
        ll pos = cross(p, i);
        if (pos <= dq.back().second) dq.pop_back();
        else {
            if (pos <= n) dq.push_back({i, pos});
            break;
        }
    }
    if (dq.empty()) dq.push_back({i, 1});
}

```

Should be **tested**.

5.5 Aliens Trick

Usage: $dp[t][i] = \min(dp[t-1][j] + c[j+1][i])$, c is Monge, find lambda w/ half bs

Time Complexity: $O(T \log X)$

```

/* n: 원소 개수 (경로 끝점), k: 정확히 골라야 하는 개수
 * lo, hi: 패널티 이분탐색 범위 (0 ~ 최대 가치)
 * f(c): 패널티가 c일 때 {2*(가치합), prv} 반환 (가치는 c를 뺀 값) */
template<class T, bool GET_MAX = false>
pair<T, vector<int>> AliensTrick(int n, int k, auto f, T lo, T hi) {
    T l = lo, r = hi;
    while (l < r) {
        T m = (l + r + (GET_MAX ? 1 : 0)) >> 1;
        vector<int> prv = f(m * 2 + (GET_MAX ? -1 : 1)).second;
        int cnt = 0; for (int i = n; i; i = prv[i]) cnt++;
        if (cnt <= k) (GET_MAX ? l : r) = m;
        else (GET_MAX ? r : l) = m + (GET_MAX ? -1 : 1);
    }
    T opt_val = f(l * 2).first / 2 - k * l;
    auto get_path = [&](T c) {
        vector<int> p{n};
        for (auto prv = f(c).second; p.back(); ) p.push_back(prv[p.back()]);
        reverse(p.begin(), p.end()); return p;
    };
    auto p1 = get_path(l * 2 + (GET_MAX ? 1 : -1));
    auto p2 = get_path(l * 2 - (GET_MAX ? 1 : -1));
    if (p1.size() == k + 1) return {opt_val, p1};
    if (p2.size() == k + 1) return {opt_val, p2};
    for (int i = 1, j = 1; i < p1.size(); i++) {
        while (j < p2.size() && p2[j] < p1[i - 1]) j++;
        if (p1[i] <= p2[j] && i - j == k + 1 - (int)p2.size()) {
            vector<int> res(p1.begin(), p1.begin() + i);
            return {opt_val, res};
        }
    }
}

```

```

        res.insert(res.end(), p2.begin() + j, p2.end());
        return {opt_val, res};
    }
}
return {opt_val, {}}; // Should not reach here
}

```

Should be **tested**.

5.6 Sum Over Subsets

Usage: $dp[mask] = \text{sum}(A[i])$, i is in mask

Time Complexity: $O(N2^N)$

```

for (int i = 0; i < (1<<n); i++)
    f[i] = a[i];
for (int j = 0; j < n; j++)
    for (int i = 0; i < (1<<n); i++)
        if (i & (1<<j)) f[i] += f[i ^ (1<<j)];
}

```

6 Geometry

6.1 Geometry Template

Usage: Basic point, line, and circle operations.

```

#define x first
#define y second
using Point = pair<ll, ll>;
Point operator + (Point p1, Point p2){ return {p1.x + p2.x, p1.y + p2.y}; }
Point operator - (Point p1, Point p2){ return {p1.x - p2.x, p1.y - p2.y}; }
ll operator * (Point p1, Point p2){ return p1.x * p2.x + p1.y * p2.y; } // 내적
ll operator / (Point p1, Point p2){ return p1.x * p2.y - p2.x * p1.y; } // 외적
int Sign(ll v){ return (v > 0) - (v < 0); } // 양수면 +1, 음수면 -1, 0이면 0 반환
ll Dist(Point p1, Point p2){ return (p2 - p1) * (p2 - p1); } // 두 점 거리 제곱
ll SignedArea(Point p1, Point p2, Point p3){ return (p2 - p1) / (p3 - p1); }
int CCW(Point p1, Point p2, Point p3){ return Sign(SignedArea(p1, p2, p3)); }

```

6.2 Convex Hull

Usage: Monotone Chain or Graham Scan.

Time Complexity: $O(N \log N)$

```

// 모든 점을 포함하는 가장 작은 불록 다각형, O(N log N)
vector<Point> ConvexHull(vector<Point> points){ // Graham scan
    if(points.size() <= 1) return points;
    swap(points[0], *min_element(all(points)));
    sort(points.begin() + 1, points.end(), [&](auto a, auto b){
        int dir = CCW(points[0], a, b);
        if(dir != 0) return dir > 0;
        return Dist(points[0], a) < Dist(points[0], b);
    });
}

```

```

vector<Point> hull;
for(auto p : points){
    while(hull.size() >= 2 && CCW(hull[hull.size() - 2], hull.back(), p) <= 0)
        hull.pop_back();
    hull.push_back(p);
}
return hull;
}
vector<Point> convexHull(vector<Point> points) { // Monotone chain
    if (sz(points) <= 1) return points;
    sort(all(points), [&](Point p1, Point p2) {
        return p1.x == p2.x ? p1.y < p2.y : p1.x < p2.x;
    });
    Polygon v(sz(points)+1);
    int s = 0, t = 0;
    for (int i = 2; i--; s = --t, reverse(all(points))) {
        for (auto p : points) {
            while (t >= s+2 && ccw(v[t-2], v[t-1], p) <= 0) t--;
            v[t++] = p;
        }
    }
    v.resize(t - (t==2 && v[0].x == v[1].x && v[0].y == v[1].y));
    return v;
}

```

6.3 Rotating Calipers

Usage: Maximum distance/diameter of a convex hull.

Time Complexity: $O(N)$

```

// 가장 먼 두 점을 구하는 함수, O(N)
// 주의: hull은 반시계 방향으로 정렬된 불록 다각형이어야 함
pair<Point, Point> Calipers(vector<Point> hull){
    int n = hull.size(); ll mx = 0; Point a, b;
    for(int i=0, j=0; i<n; i++){
        while(j + 1 < n && (hull[i+1] - hull[i]) / (hull[j+1] - hull[j]) >= 0){
            ll now = Dist(hull[i], hull[j]);
            if(now > mx) mx = now, a = hull[i], b = hull[j];
            j++;
        }
        ll now = Dist(hull[i], hull[j]);
        if(now > mx) mx = now, a = hull[i], b = hull[j];
    }
    return {a, b};
}

```

6.4 Point in Convex Polygon

Usage: Binary search based inclusion test.

Time Complexity: $O(\log N)$

```
// 다각형 내부 또는 경계 위에 p가 있으면 true, O(log N)
```

```
// 주의: v는 반시계 방향으로 정렬된 볼록 다각형이어야 함
bool PointInConvexPolygon(const vector<Point> &v, const Point &pt){
    if(CCW(v[0], v[1], pt) < 0) return false; int l = 1, r = v.size() - 1;
    while(l < r){
        int m = l + r + 1 >> 1;
        if(CCW(v[0], v[m], pt) >= 0) l = m; else r = m - 1;
    }
    if(l == v.size() - 1) return CCW(v[0], v.back(), pt) == 0 && v[0] <= pt && pt <= v.back();
    return CCW(v[0], v[l], pt) >= 0 && CCW(v[l], v[l+1], pt) >= 0 && CCW(v[l+1], v[0], pt) >= 0;
}
```

6.5 Point in Polygon

Usage: Ray casting algorithm for general polygons.

Time Complexity: $O(N)$

```
// 다각형 내부 또는 경계 위에 p가 있으면 true, O(N)
bool PointInPolygon(const vector<Point> &v, Point p){
    int n = v.size(), cnt = 0;
    Point p2(p.x+1, 1'000'000'000 + 1); // 좌표 범위보다 큰 수
    for(int i=0; i<n; i++){
        int j = i + 1 < n ? i + 1 : 0;
        if(min(v[i], v[j]) <= p && p <= max(v[i], v[j]) && CCW(v[i], v[j], p) == 0)
            return true;
        if(SegmentIntersection(v[i], v[j], p, p2)) cnt++;
    }
    return cnt % 2 == 1;
}
```

6.6 Polar Sort

Usage: Sorting points by angle.

Time Complexity: $O(N \log N)$

```
int QuadrantID(const Point p){
    static int arr[9] = { 5, 4, 3, 6, -1, 2, 7, 0, 1 };
    return arr[Sign(p.x)*3+Sign(p.y)+4];
}
sort(points.begin(), points.end(), [&](auto p1, auto p2){
    if(QuadrantID(p1) != QuadrantID(p2)) return QuadrantID(p1) < QuadrantID(p2);
    else return p1 / p2 > 0; // 반시계
});
```

6.7 Polygon Area

Usage: Shoelace formula.

Time Complexity: $O(N)$

```
// 다각형의 넓이의 2배를 반환, 항상 정수, O(N)
ll PolygonArea(const vector<Point> &v){
```

```
ll res = 0;
for(int i=0; i<v.size(); i++) res += v[i] * v[(i+1)%v.size()];
return abs(res);
}
```

6.8 Segment Intersection

Usage: Check if two line segments intersect.

Time Complexity: $O(1)$

```
// 선분 교차 - 선분 ab와 선분 cd가 만나면 true
bool Cross(Point s1, Point e1, Point s2, Point e2){
    int ab = CCW(s1, e1, s2) * CCW(s1, e1, e2);
    int cd = CCW(s2, e2, s1) * CCW(s2, e2, e1);
    if(ab == 0 && cd == 0){
        if(s1 > e1) swap(s1, e1);
        if(s2 > e2) swap(s2, e2);
        return !(e1 < s2 || e2 < s1);
    }
    return ab <= 0 && cd <= 0;
}
// 교차하지 않으면 0, 교점이 무한히 많으면 -1
// 교점이 1개면 1 반환하고 res에 교점 저장
int Cross(Point s1, Point e1, Point s2, Point e2, pair<double,double> &res){
    if(!Cross(s1, e1, s2, e2)) return 0;
    ll det = (e1 - s1) / (e2 - s2);
    if(!det){
        if(s1 > e1) swap(s1, e1);
        if(s2 > e2) swap(s2, e2);
        if(e1 == s2){ res = s2; return 1; }
        if(e2 == s1){ res = s1; return 1; }
        return -1;
    }
    res.x = s1.x + (e1.x - s1.x) * ((s2 - s1) / (e2 - s2) * 1.0 / det);
    res.y = s1.y + (e1.y - s1.y) * ((s2 - s1) / (e2 - s2) * 1.0 / det);
    return 1;
}
```

7 String

7.1 Aho-Corasick

Usage: Multi-pattern matching using trie and failure links.

Time Complexity: $O(\sum |P| + |T|)$

```
struct AhoCorasick {
    struct Node {
        Node *nxt[26], *fail;
        vector<int> out; // 패턴의 인덱스 저장
        int terminal;
        Node() : fail(nullptr), terminal(-1) { fill(nxt, nxt + 26, nullptr); }
        ~Node() { }
```

```

        for (int i = 0; i < 26; i++) if (nxt[i]) delete nxt[i];
    }
    void insert(const char* s, int id) {
        if (*s == 0) { terminal = id; out.push_back(id); return; }
        int curr = *s - 'a';
        if (!nxt[curr]) nxt[curr] = new Node();
        nxt[curr]->insert(s + 1, id);
    }
};

Node* root;
AhoCorasick() { root = new Node(); }
~AhoCorasick() { delete root; }
void insert(const string& s, int id) { root->insert(s.c_str(), id); }
void build() {
    queue<Node*> q;
    root->fail = root;
    for (int i = 0; i < 26; i++) {
        if (root->nxt[i]) {
            root->nxt[i]->fail = root;
            q.push(root->nxt[i]);
        } else {
            root->nxt[i] = root; // DFA optimization
        }
    }
    while (!q.empty()) {
        Node* curr = q.front(); q.pop();
        for (int i = 0; i < 26; i++) {
            if (curr->nxt[i]) {
                Node* next = curr->nxt[i];
                next->fail = curr->fail->nxt[i];
                next->out.insert(next->out.end(), next->fail->out.begin(),
                                  next->fail->out.end());
                q.push(next);
            } else {
                curr->nxt[i] = curr->fail->nxt[i]; // DFA optimization
            }
        }
    }
}

vector<pair<int, int>> query(const string& s) {
    vector<pair<int, int>> res;
    Node* curr = root;
    for (int i = 0; i < s.size(); i++) {
        curr = curr->nxt[s[i] - 'a'];
        for (int id : curr->out) res.emplace_back(i, id);
    }
    return res;
}

int main() {
    AhoCorasick ac;
    vector<string> patterns = {"he", "she", "hers", "his"};
}

```

```

for(int i = 0; i < patterns.size(); i++)
    ac.insert(patterns[i], i); // 패턴과 ID(0~N-1) 삽입
ac.build(); // 실패 함수/DFA 빌드 (필수)
string text = "ushers";
auto res = ac.query(text); // 탐색: {끝 인덱스, 패턴 ID} 쌍 반환
for (auto& [idx, id] : res) {
    // patterns[id] 가 text의 idx에서 끝남을 의미
}
}

```

7.2 Hashing

Usage: Rolling hash for string matching.
Time Complexity: $O(N)$

```

// 전처리  $O(N)$ , 부분 문자열의 해시값을  $O(1)$ 에 구함
// Hashing<917, 998244353> H;
// H.build("ABCDABCD");
// assert(H.get(1, 4) == H.get(5, 8));
// 주의: get 함수의 인자는 1-based 닫힌 구간
// 주의: M은 10억 근처의 소수, P는 M과 서로소
// 1e5+3, 1e5+13, 131'071, 524'287, 1'299'709, 1'301'021
// 1e9-63, 1e9+7, 1e9+9, 1e9+103
template<long long P, long long M> struct Hashing {
    vector<long long> h, p;
    void build(const string &s){
        int n = s.size();
        h = p = vector<long long>(n+1); p[0] = 1;
        for(int i=1; i<=n; i++) h[i] = (h[i-1] * P + s[i-1]) % M;
        for(int i=1; i<=n; i++) p[i] = p[i-1] * P % M;
    }
    long long get(int s, int e) const {
        long long res = (h[e] - h[s-1] * p[e-s+1]) % M;
        return res >= 0 ? res : res + M;
    }
};

```

7.3 KMP

Usage: Single pattern matching using prefix function.
Time Complexity: $O(N + M)$

```

template <typename T>
struct KMP {
    T P; vector<int> pi;
    KMP(const T& P) : P(P), pi(sz(P)) {
        for (int i = 1, j = 0; i < sz(P); i++) {
            while (j > 0 && P[i] != P[j]) j = pi[j-1];
            if (P[i] == P[j]) pi[i] = ++j;
        }
    }
    vector<int> find(const T& S) {

```

```

vector<int> res;
int n = sz(S), m = sz(P), j = 0;
for (int i = 0; i < n; i++) {
    while (j > 0 && S[i] != P[j]) j = pi[j-1];
    if (S[i] == P[j]) {
        if (j == m-1) {
            res.push_back(i-m+1); j = pi[j];
        }
        else j++;
    }
}
return res;
}

int minPeriod() {
    int m = sz(P); if (m == 0) return 0;
    int len = m - pi[m-1]; if (m % len == 0) return len;
    return m;
}

```

7.4 Manacher

Usage: Find all palindromic substrings in linear time.

Time Complexity: $O(N)$

```

// 각 문자를 중심으로 하는 최장 팰린드롬의 반경을 반환
// Manacher("abaaba") = {0,1,0,3,0,1,6,1,0,3,0,1,0}
// # a # b # a # b # a #
// 0 1 0 3 0 1 6 1 0 3 0 1 0
vector<int> Manacher(const string &inp){
    int n = inp.size() * 2 + 1;
    vector<int> ret(n);
    string s = "#";
    for(auto i : inp) s += i, s += "#";
    for(int i=0, p=-1, r=-1; i<n; i++){
        ret[i] = i <= r ? min(r-i, ret[2*p-i]) : 0;
        while(i-ret[i]-1 >= 0 && i+ret[i]+1 < n && s[i-ret[i]-1] == s[i+ret[i]+1])
            ret[i]++;
        if(i+ret[i] > r) r = i+ret[i], p = i;
    }
    return ret;
}

```

7.5 Suffix Array

Usage: Suffix array and LCP array construction.

Time Complexity: $O(N \log N)$

```

// LCP는 1-based
pair<vector<int>, vector<int>> SuffixArray(const string &s) { // O(N log N)
    int n = s.size(), m = max(n, 256);
    vector<int> sa(n), lcp(n), pos(n), tmp(n), cnt(m);

```

```

auto counting_sort = [&](){
    fill(cnt.begin(), cnt.end(), 0);
    for(int i=0; i<n; i++) cnt[pos[i]]++;
    partial_sum(cnt.begin(), cnt.end(), cnt.begin());
    for(int i=n-1; i>=0; i--) sa[--cnt[pos[tmp[i]]]] = tmp[i];
};

for(int i=0; i<n; i++) sa[i] = i, pos[i] = s[i], tmp[i] = i;
counting_sort();
for(int k=1; ; k<=n-1){
    int p = 0; for(int i=n-k; i<n; i++) tmp[p++] = i;
    for(int i=0; i<n; i++) if(sa[i] >= k) tmp[p++] = sa[i] - k;
    counting_sort(); tmp[sa[0]] = 0;
    for(int i=1; i<n; i++){
        tmp[sa[i]] = tmp[sa[i-1]];
        if(sa[i-1]+k < n && sa[i]+k < n && pos[sa[i-1]] == pos[sa[i]] &&
           pos[sa[i-1]+k] == pos[sa[i]+k]) continue;
        tmp[sa[i]] += 1;
    }
    swap(pos, tmp); if(pos[sa.back()] + 1 == n) break;
}
for(int i=0, j=0; i<n; i++, j=max(j-1, 0)){
    if(pos[i] == 0) continue;
    while(sa[pos[i]-1]+j < n && sa[pos[i]]+j < n && s[sa[pos[i]-1]+j] ==
          s[sa[pos[i]]+j]) j++;
    lcp[pos[i]] = j;
}
return {sa, lcp};
}

```

7.6 Z-algorithm

Usage: Longest common prefix between S and its suffixes.

Time Complexity: $O(N)$

```

// Z[i] = LongestCommonPrefix(S[0:N], S[i:N])
//      = S[0:N]과 S[i:N]이 앞에서부터 몇 글자 겹치는지
vector<int> Z(const string &s){
    int n = s.size();
    vector<int> z(n);
    z[0] = n;
    for(int i=1, l=0, r=0; i<n; i++){
        if(i < r) z[i] = min(r-i-1, z[i-1]);
        while(i+z[i] < n && s[i+z[i]] == s[z[i]]) z[i]++;
        if(i+z[i] > r) r = i+z[i], l = i;
    }
    return z;
}

```

8 STL & pbds

8.1 Hash map

Usage: Faster hash table using pb_ds.

Time Complexity: $O(1)$

```
// faster than unordered_map
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

gp_hash_table<int,int> hashmap;
// cannot use hashmap.count()
```

8.2 Ordered Set

Usage: Set supporting order_of_key and find_by_order.

Time Complexity: $O(\log N)$

```
// k번째 원소 확인(find_by_order) 및 x보다 작은 원소 개수 확인(order_of_key)을
// O(logN)에 수행
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;
// ordered_set<int> os;
// os.order_of_key(x) : x보다 작은 원소의 개수 반환
// os.find_by_order(k) : k번째 원소의 iterator 반환 (0-indexed, 없으면 os.end())
template <typename T>
using ordered_multiset = tree<T, null_type, less_equal<T>, rb_tree_tag,
tree_order_statistics_node_update>;
auto m_find(ordered_multiset<int> &os, int val) { // multiset 전용 find 함수
    int idx = os.order_of_key(val);
    auto it = os.find_by_order(idx);
    if (it != os.end() && *it == val) return it;
    return os.end();
} // os.erase(m_find(os, val))
```

8.3 Permutation & Combination

Usage: next_permutation and mask-based combinations.

```
#include <algorithm>
/* 1. Permutation */
sort(all(v))
do {
    // process v
} while (next_permutation(all(v)));
/* 2. Combination (nCr): Use a mask vector */
vector<int> mask(n, 0);
fill(mask.end()-r, mask.end(), 1); // pick r elements
```

```
do {
    for (int i = 0; i < n; i++) {
        if (mask[i]) { /* v[i] is selected */
    }
} while (next_permutation(all(mask)));
/* 3. Partial Permutation (nPk) */
sort(all(v));
do {
    for(int i = 0; i < k; i++) { /* use v[i] */
        reverse(v.begin()+k, v.end());
} while (next_permutation(all(v)));
```

8.4 Priority Queue

Usage: Meldable/Thin heaps from pb_ds.

Time Complexity: $O(\log N)$

```
// 큐 병합, 임의 값 수정 및 삭제 가능
#include <functional>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
template <typename T>
using pbds_pq = __gnu_pbds::priority_queue<T, less<T>, pairing_heap_tag>;
int main() {
    pbds_pq<int> pq1, pq2;
    auto it = pq1.push(10);
    pq2.push(100);
    pq1.join(pq2); // O(1), pq1: {10, 100}, pq2: {}
    pq1.modify(it, 50); // O(logN), pq1 : {50, 100}
    pq1.erase(it); // O(logN), pq1: {100}
    pq1.top(); pq1.empty(); pq1.size(); pq1.pop(); // same
}
```

8.5 Rope

Usage: Fast insertions/deletions in large strings.

Time Complexity: $O(\log N)$

```
#include<ext/rope>
using namespace __gnu_cxx;
int main() {
    string str; cin >> str;
    rope r(str.c_str());
    // vector<T> v; rope<T> r(v);
    r.insert(pos, str); // Insert O(logN)
    r.erase(pos, len); // Erase O(logN)
    r.replace(pos, len, str); // Replace O(logN)
    rope r2 = r; // O(1)
    r2 = r.substr(pos, len); // O(logN + len)
    r += r2; // Append O(logN + len(r2))
    r[idx]; // O(logN)
    for (auto i : r) // Traversal O(N)?
```

```
    cout << r;
}
```

8.6 Trie

Usage: Prefix tree implementation from pb_ds.

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/trie_policy.hpp>
using namespace __gnu_pbds;
typedef trie<string, null_type, trie_string_access_traits<>, pat_trie_tag,
trie_prefix_search_node_update> trie_set;
int main() {
    trie_set t; t.insert("apple");
    t.insert("app"); t.insert("banana");
    if (t.find("app") != t.end()) { /* found app */ }
    auto [st, en] = t.prefix_range("ba");
    for (auto it = st; it != en; it++) {
        cout << *it << "\n"; // banana
    }
    *t.lower_bound("app"); // app
    *t.upper_bound("app"); // apple
    t.split("b", t2); // t: {app, apple}, t2: {banana}
    t.erase("apple");
}
```

9 Misc

9.1 Custom Hash

Usage: Custom hash for pair, vector.

```
struct custom_hash {
    template <class T>
    void combine(size_t& seed, const T& v) const {
        seed ^= hash<T>{}(v) + 0x9e3779b9 + (seed << 6) + (seed >> 2);
    }
    template <class T1, class T2>
    size_t operator()(const pair<T1, T2>& p) const {
        size_t seed = 0;
        combine(seed, p.first);
        combine(seed, p.second);
        return seed;
    }
    template <class T>
    size_t operator()(const vector<T>& v) const {
        size_t seed = 0;
        for (const auto& i : v) combine(seed, i);
        return seed;
    }
};
```

9.2 Fast I/O

Usage: Fast integer I/O using fread/fwrite.

```
#include <unistd.h>
constexpr int rbuf_sz = 1 << 20, wbuf_sz = 1 << 20;
int main() {
    char r[rbuf_sz], *pr = r; read(0, r, rbuf_sz);
    auto read_char = [&] {
        if (pr - r == rbuf_sz) read(0, pr = r, rbuf_sz);
        return *pr++;
    };
    auto read_int = [&] {
        int ret = 0, flag = 0;
        char c = read_char();
        while (c == ' ' || c == '\n') c = read_char();
        if (c == '-') flag = 1, c = read_char();
        while (c != ' ' && c != '\n') ret = 10 * ret + c - '0', c = read_char();
        if (flag) ret = -ret;
        return ret;
    };
    char w[wbuf_sz], *pw = w;
    auto write_char = [&](char c) {
        if (pw - w == wbuf_sz) write(1, w, pw - w), pw = w;
        *pw++ = c;
    };
    auto write_int = [&](int x) {
        if (pw - w + 40 > wbuf_sz) write(1, w, pw - w), pw = w;
        if (x < 0) *pw++ = '-', x = -x;
        char t[10], *pt = t;
        do *pt++ = x % 10 + '0'; while (x /= 10);
        do *pw++ = *--pt; while (pt != t);
    };
}
```

9.3 Random

Usage: Better random for mt19937.

```
#include <random>
#include <chrono>
mt19937_64 rng(chrono::high_resolution_clock::now().time_since_epoch().count());

uniform_int_distribution<int>(l, r)(rng); // [l, r]
uniform_real_distribution<double>(l, r)(rng); // [l, r]
shuffle(all(v), rng) // shuffle vector
vector<double> w = { 40, 10, 50 };
discrete_distribution<int>(all(w))(rng); // 0: 40%, 1: 10%, 2: 50%
bernoulli_distribution(P)(rng); // True with probability P(0.0~1.0)
```

9.4 Ternary Search

Usage: Finding extremum of unimodal functions.

Time Complexity: $O(\log N)$

```

11 ternary_search(ll lo, ll hi, auto f) {
    while (hi - lo >= 3) {
        ll p = lo + (hi - lo) / 3, q = hi - (hi - lo) / 3;
        if (f(p) < f(q)) hi = q;
        else lo = p;
    }
    ll idx = lo; auto minv = f(lo);
    for (ll i = lo+1; i <= hi; i++) {
        auto v = f(i);
        if (v < minv) {
            minv = v; idx = i;
        }
    }
    return idx;
}

double ternary_search(double lo, double hi, auto f, int iter = 100) {
    for (int i = 0; i < iter; i++) {
        double p = (lo*2 + hi) / 3., q = (lo + hi*2) / 3.;
        if (f(p) < f(q)) hi = q;
        else lo = p;
    }
    return (lo+hi) / 2.;
}

```

10 Checklist + Useful Info

10.1 Useful Stuff

- Catalan Number
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900
 $C_n = \frac{(2n)!}{n!(n+1)!}$
 - 길이가 $2n$ 인 올바른 괄호 수식의 수
 - $n + 1$ 개의 리프를 가진 풀 바이너리 트리의 수
 - $n + 2$ 각형을 n 개의 삼각형으로 나누는 방법의 수
- Burnside's Lemma
경우의 수를 세는데, 특정 transform operation(회전, 반사, ...) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는? 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다(단, “아무것도 하지 않는다”라는 operation도 있어야 함!) 전체 경우의 수를 더한 후, operation의 수로 나눈다.(답이 맞다면 항상 나누어 떨어져야 한다)
- 알고리즘 게임
 - Nim Game의 해법: 각 더미의 돌의 개수를 모두 XOR 했을 때 0이 아니면 첫번째, 0이면 두번째 플레이어가 승리.
 - Grundy Number: 어떤 상황의 Grundy Number는, 가능한 다음 상황들의 Grundy Number를 모두 모은 다음, 그 집합에 포함되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러개의 state들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.
 - Subtraction Game: 한 번에 k 개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 $k+1$ 로 나눈 나머지를 XOR 합하여 판단한다.

- Index-k Nim : 한 번에 최대 k 개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 $k+1$ 로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.

- Pick's Theorem
격자점으로 구성된 simple polygon이 주어짐. I 는 polygon 내부의 격자점 수, B 는 polygon 선분 위 격자점 수, A 는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다. $A = I + B/2 - 1$
- 가장 가까운 두 점: 분할정복으로 가까운 6개의 점만 확인
- 홀의 결혼 정리: 이분그래프(L-R)에서, 모든 L을 매칭하는 필요충분 조건 = L에서 임의의 부분집합 S를 골랐을 때, 반드시 (S의 크기) \leq (S와 연결되어 있는 모든 R의 크기)이다.
- 소수: 10 007, 10 009, 10 111, 31 567, 70 001, 1 000 003, 1 000 033, 4 000 037, 99 999 989, 999 999 937, 1 000 000 007, 1 000 000 009, 9 999 999 967, 99 999 999 977
- 소수 개수: (1e5 이하: 9592), (1e7 이하: 664 579), (1e9 이하: 50 847 534)
- 10^{15} 이하의 정수 범위의 나눗셈 한번은 오차가 없다.
- N 의 약수의 개수 = $O(N^{1/3})$, N 의 약수의 합 = $O(N \log \log N)$
- $\phi(mn) = \phi(m)\phi(n)$, $\phi(pr^n) = pr^n - pr^{n-1}, a^{\phi(n)} \equiv 1 \pmod{n}$ if coprime
- Euler characteristic: $v - e + f$ (면, 외부 포함) = $1 + c$ (컴포넌트)
- Euler's phi $\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$
- Lucas' Theorem $\binom{m}{n} = \prod \binom{m_i}{n_i} \pmod{p}$ m_i, n_i 는 p^i 의 계수
- 스케줄링에서 데드라인이 빠른 걸 쓰는게 이득. 늦은 스케줄이 만들어질 때 가장 시간 소모가 큰 스케줄 1개를 제거하면 이득.

10.2 자주 쓰이는 문제 접근법

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (brute force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해결해보면서)
- 문제를 단순화할 수 없을까?
- 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까?
- 문제를 분해할 수 있을까?
- 뒤에서부터 생각해서 문제를 풀 수 있을까?
- 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)

- 특수 조건을 꼭 활용
- 예사건으로 생각하기
- 게임이론 - 거울 전략 혹은 mex DP 연계
- 겁먹지 말고 경우 나누어 생각
- 해법에서 역순으로 가능한가?
- 딱 맞는 시간복잡도에 집착하지 말자
- 문제에 의미있는 작은 상수 이용
- 스몰트루리지, 트라이, 해싱, 루트질 같은 트릭 생각
- 너무 추상화하기보단 풀려야 하는 방식으로 생각하기
- 잘못된 방법으로 파고들지 말고 버리자
- 제발 터널 비전에 빠지지 말자
- 헬프 콜은 적극적으로
- 혼자 맨날 나가지 않기

10.3 DP 최적화 접근

- $C[i, j] = A[i] * B[j]$ 이고 A, B 가 단조증가, 단조감소이면 Monge
- $l..r$ 의 값들의 sum이나 min은 Monge
- 식 정리해서 일차(HT) 혹은 비슷한(MQ) 함수를 발견, 구현 힘들면 Li-Chao
- $a \leq b \leq c \leq d$ 에서 $A[a, c] + A[b, d] \leq A[a, d] + A[b, c]$
- Monge 성질을 보이기 어려우면 N^2 나이브 짜서 opt의 단조성을 확인하고 찍맞
- 식이 간단하거나 변수가 독립적이면 DP 테이블을 세그 위에 올려서 해결
- 침착하게 점화식부터 세우고 Monge인지 판별
- Monge에 집착하지 말고 단조성이나 볼록성만 보여도 됨