

# BlackHoleImages: A Wolfram Mathematica packet for analytical black hole imaging

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## Summary

Black holes are astrophysical objects with gravitational fields so extreme that they bend even the trajectories of light. Specifically, in the geometrical optics limit, the paths of particles and fields moving at the speed of light follow null geodesics in the curved spacetime geometry. Consequently, the presence of a black hole can lead to strong lensing effects on the images emerging from their surroundings. Additionally, the passage of light through the gravitational field leads to changes in the frequency and intensity of the light, which is particularly important to understand the observational imprints of accretion disks forming around black holes.

BlackHoleImages is a Wolfram Mathematica packet designed to compute the aforementioned effects in the field of isolated rotating black holes in Einstein's General Relativity, which are known as the Kerr family of spacetimes. The packet can be used for computing:

- photon orbits (null geodesics) in the fields of black holes,
- generating images of stellar backgrounds distorted by black holes, and
- generating images of accretion disks near black holes.

The packet contains three packages, `KerrNullGeodesics`, `AlphaDiskModel`, and `KerrImages`. `KerrNullGeodesics` implements the analytical solutions for null geodesics in Kerr space-times as described in detail by Gralla & Lupsasca (2020). The `AlphaDiskModel` then implements the standard  $\alpha$ -disk model for the accretion disk near a black hole as proposed by Shakura & Sunyaev (1973), specifically its relativistic incarnation by Novikov & Thorne (1973) and Page & Thorne (1974). Finally, the `KerrImages` package uses the previous two packages to provide functions that compute the disk images and the lensed images of faraway backgrounds.

We expect this packet to be used primarily for investigating the observational characteristics of accretion disks of black holes, including dynamical processes in the disk, since the time delay of each geodesic is also computable in this packet. The packet can also be used for educational purposes, such as plotting null geodesics in the Kerr metric or illustrating the lensing effects, or for any other purpose where the computation of photon trajectories is necessary.

## Statement of need

Black holes are one of the most intriguing objects in modern physics. They were one of the earliest predictions of Einstein's general relativity and started to be indirectly through observations in various electromagnetic bands starting from the 1960s (Narayan & McClintock, 2015). Since most observations were and still are unable to resolve the spatial scales of the so-called event horizon of the black hole, we need to understand the radiation that emerges from the interactions of the black hole with its surroundings and the emergent spectrum. This was provided by the seminal  $\alpha$  accretion disk model of Shakura & Sunyaev (1973) and Novikov

40 & Thorne (1973). As a result, the community of astronomers and astrophysicists identified  
41 the first candidates for black holes in the Universe.

42 Currently, the evidence for dark compact objects consistent with black holes is overwhelming.  
43 Throughout the 1990s, two teams of astronomers followed stars orbiting very close to our  
44 Galactic center and deduced on the presence of a dark compact object with the mass of<sup>1</sup>  
45  $4 \cdot 10^6 M_{\odot}$  Ghez & others (2008); the leaders of the teams later shared the 2020 Nobel prize  
46 in physics (Nobel Prize Outreach, 2020). In 2015, the LIGO collaboration detected the first  
47 merger of black holes by gravitational waves (LIGO and Virgo collaborations, 2016) and more  
48 than 180 events were detected to date (LVK Collaboration, 2025). In 2017, the Event Horizon  
49 Telescope collaboration reconstructed the image of the accretion flow in the immediate vicinity  
50 of the massive black hole in the center of the M87 galaxy by the methods of very long baseline  
51 interferometry (Event Horizon Telescope collaboration, 2019a). They followed up this result  
52 in 2020 by presenting a similar image of the accretion flow from the center of our very own  
53 Galaxy the Milky Way (Event Horizon Telescope collaboration, 2022).

54 The images of the massive black holes from M87 and the Milky Way present a dark feature  
55 in their centers known as the “shadow” of the black hole. It corresponds to a region of the  
56 sky where the paths of the photons are bent so much that they end behind the black hole  
57 horizon, the point from beyond which not even light can escape. While this qualitative picture  
58 is correct, the precise quantitative science inferred from the observations of the Event Horizon  
59 Telescope requires the detailed modeling of the accretion flow, its radiation, and the emerging  
60 photon orbits (Event Horizon Telescope collaboration, 2019b). This is one of the important  
61 use cases our packet is aiming for.

62 Another important application of the packet is the interpretation of timing and spectral  
63 features of unresolved accreting black holes as observed in the burgeoning field of time-domain  
64 astronomy (Burns et al., 2025). Transients such as quasi-periodic eruptions, oscillations, or  
65 outflows emerging from systems containing accreting black holes are detected at an increasing  
66 rate. They often contain subtle timing features that require precise modelling of the accretion  
67 flows and other physical components, but possibly also the lensing effects by the black holes.  
68 Our packet can be easily extended to compute the lightcurves and/or images emerging from  
69 these systems.

70 Last but not least, the topic of black holes captures the imagination of the public. In the  
71 2014 movie *Interstellar* directed by Christopher Nolan, the general movie audience was able  
72 to observe a faithful simulation of a black hole image for the first time (James et al., 2015).  
73 Our packet provides an intermediate step for relativity students and other interested parties to  
74 be able to dissect how such an image is made through a user-friendly Wolfram Mathematica  
75 packet.

76 On the more technical side, this is also one of the first open source releases of an implementation  
77 of the *analytical* solution for null geodesics in Kerr space-time (the only other implementation  
78 we know of is (Omwoyo, 2025)). Other implementations typically use numerical differential  
79 equation solvers (e.g. the EinsteinPy implementation (Shivotam & Bapat, 2020)). The  
80 arbitrary precision capabilities of *Wolfram Mathematica* thus allow to use this packet as an  
81 exact validation and reference solution for such machine precision codes. Using the symbolic  
82 calculus capabilities of Mathematica provided a good platform to implement the relatively  
83 complicated analytical solution as a starting point. The packet could now also be used as  
84 a reference when porting the solutions to more optimized lower-level implementations. For  
85 example, the Mathematica code *KerrGeodesics* (Warburton et al., 2023) for time-like geodesics  
86 (trajectories of massive test particles) in Kerr space-time was recently rewritten in Python as  
87 *KerrGeoPy* (Park & Nasipak, 2024), our packet could follow a similar route.

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<sup>1</sup> $M_{\odot} \approx 2 \times 10^30$  kg is the solar mass.

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