CEE 498 Applied Machine Learning - HW5

Rini Jasmine Gladstone (rjg7) and Maksymillian Podraza(podraza3)

April 13, 2020

1 Problem 1

1.1 Part a

We used the following image for our analysis.

We normalized the image so that the darkest pixel takes the value (0,0,0) and the lightest pixel takes the value (1,1,1).

```
image_paths = ["RobertMixed03.jpg", "smallstrelitzia.jpg", "
    smallsunset.jpg"]

def load_images():
    image_array = []
    try:
        for image_path in image_paths:
            img = Image.open(image_path)
            image_array.append(img)
    except IOError:
        pass
    return image_array

```

```
images = load_images()

working_image = images[0]

def image_matrix_create(image):
    image_matrix = []
    img_w, img_h = image.size
    image_data = list(image.getdata())
    for y in range(img_h):
        image_matrix.append(image_data[y*img_w:(y+1)*img_w])
    image_df = pd.DataFrame(image_matrix)
    return image_df

def rgb_extract(image):
    rgb = []
    for i in range(3):
        rgb.append(image.apply(lambda x: [y[i] for y in x]))
    return rgb

def rgb_normalize(image):
    normalized = []
    for i in range(3):
        x = image[i].values
        min_max_scaler = preprocessing.MinMaxScaler()
        x_scaled = min_max_scaler.fit_transform(x)
        df = pd.DataFrame(x_scaled)
        normalized.append(df)
    return normalized

r = image_matrix_create(working_image)
rgb_matrix_list = rgb_extract(r)
rgb_normalized_list = rgb_normalize(rgb_matrix_list)
```

Then, we wrote the function for EM.

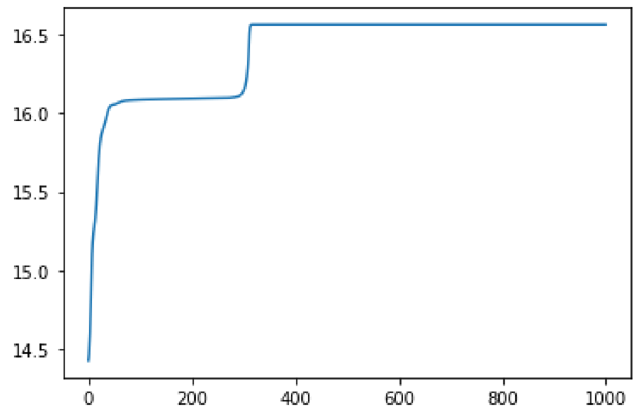
```
def EM_function(X,cov,k,niter):
        weights = np.ones((k)) / k
        means = np.random.choice(X.values.flatten(), (k,X.shape[1]))
        eps=1e-8
        likelihood = []
        log_likelihoods = []
        for step in range(niter):
            likelihood = []
            # Expectation step
            for j in range(k):
                likelihood.append(multivariate_normal.pdf(x=X, mean=means[j], cov=cov[j],allow_singular=True))
            likelihood = np.array(likelihood)
            assert likelihood.shape == (k, len(X))
            b = []
            # Maximization step
            for j in range(k):
                # use the current values for the parameters to evaluate the posterior
                # probabilities of the data have been generated by each gaussian
                b.append((likelihood[j] * weights[j]) / (np.sum([
```

```
    likelihood[i] * weights[i] for i in range(k)], axis=0)+eps))
                # update mean and variance
                means[j] = np.sum(b[j].reshape(len(X),1) * X, axis
    =0) / (np.sum(b[j]+eps))
                cov[j] = np.dot((b[j].reshape(len(X),1) * (X -
    means[j])).T, (X - means[j])) / (np.sum(b[j])+eps)
                # update the weights
                weights[j] = np.mean(b[j])
                assert cov.shape == (k, X.shape[1], X.shape[1])
                assert means.shape == (k, X.shape[1])
            log_likelihoods.append(np.log(np.sum([k*
    multivariate_normal(means[i],cov[j],allow_singular=True).pdf(X)
     for k,i,j in zip(weights,range(len(means)),range(len(cov)))]))
    )
            if (step+1)%100==0 and (step+1)>=100:
                print(step+1)
        return means,cov,weights,b,log_likelihoods,likelihood
```

We set the initial covariance matrix (cov) to be identity matrix and ran the following code setting k = 10, 20 and 50.

```
rvalues=rgb_normalized_list[0].values.flatten()
gvalues=rgb_normalized_list[1].values.flatten()
bvalues=rgb_normalized_list[2].values.flatten()
list_of_tuples = list(zip(rvalues, gvalues, bvalues))
data=pd.DataFrame(list_of_tuples, columns = ['R', 'G', 'B'])
#Number of clusters
k=50
#Initializing the covariance matrix
cov = []
for i in range(k):
    cov.append(np.eye(data.shape[1]))
cov = np.array(cov)
means,cov,weights,b,log_likelihoods,likelihood=EM_function(data,cov
    ,k,1000)
#Plotting the log-likelihood
plt.plot(log_likelihoods)
likelihood = np.array(likelihood)
#Finding the closest cluster for each pixel
predictions = np.argmax(likelihood, axis=0)
data['cluster']=predictions
data['mean_R']=np.zeros(data.shape[0])
data['mean_G']=np.zeros(data.shape[0])
data['mean_B']=np.zeros(data.shape[0])
for i in range(data.shape[0]):
    data.at[i,'mean_R']=means[data.at[i,'cluster']][0]
    data.at[i,'mean_G']=means[data.at[i,'cluster']][1]
    data.at[i,'mean_B']=means[data.at[i,'cluster']][2]
for i in range(k):
    data['weight_c'+str(i)]=b[i]
```

We ran the code for 1000 iterations. The convergence of log likelhood values for 10 clusters is as shown below.



The values converge around 400 iterations.

The following are the images obtained by replacing each pixel with the mean of its cluster center.

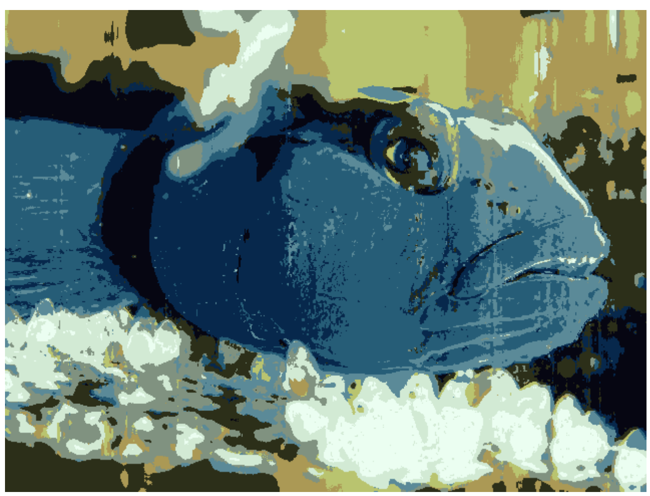


Figure 1: 10 Clusters

Figure 2: 20 Clusters

Figure 3: 50 Clusters

The code for generating the above images is as shown below.

```
def rescale_matrix(matrix, mode="int"):
    new_matrix = matrix
    for col in range(640):
        new_matrix.loc[:, col] *= 255

    if mode == "int":
        new_matrix.astype(int)
    return new_matrix

```

```
def print_matrix_image(original_image, image):
    im2 = Image.new(original_image.mode, original_image.size)
    img = image.values.flatten()
    im2.putdata(img)
    im2.show()

def matricize_list(image, img_w=640, img_h=480):
    image_matrix = []
    for y in range(img_h):
        image_matrix.append(image[y*img_w:(y+1)*img_w])
    image_df = pd.DataFrame(image_matrix)
    return image_df

def rescale_weight_matrix(matrix, mode="int"):
    new_matrix = matrix
    for col in range(640):
        new_matrix.loc[:, col] *= 1000

    if mode == "int":
        new_matrix.astype(int)
    return new_matrix

matrix_mean_R = matricize_list(list(data["mean_R"]))
matrix_mean_G = matricize_list(list(data["mean_G"]))
matrix_mean_B = matricize_list(list(data["mean_B"]))

matrix_mean_rescaled_R = rescale_matrix(matrix_mean_R)
matrix_mean_rescaled_G = rescale_matrix(matrix_mean_G)
matrix_mean_rescaled_B = rescale_matrix(matrix_mean_B)

matrix_mean_R = matrix_mean_R.astype(int)
matrix_mean_G = matrix_mean_G.astype(int)
matrix_mean_B = matrix_mean_B.astype(int)
matrix_mean_RGB = pd.DataFrame(np.rec.fromarrays((matrix_mean_R.values, matrix_mean_G.values, matrix_mean_B.values)).tolist(),
                          columns=matrix_mean_R.columns,
                          index=matrix_mean_R.index)

print_matrix_image(working_image, matrix_mean_RGB)
```

1.2 Part b

We constructed the figures showing the weights linking each pixel to each cluster center. Following is the code which executes this.

```
def print_matrix_weight(working_image, weight_image_matrix):
    im2 = Image.new("L", working_image.size)
    img = weight_image_matrix.values.flatten()
    im2.putdata(img)
    im2.show()

def weight_matrix_creation():
    for i in range(10):
        weight_image_matrix = matricize_list(list(data["weight_c"+str(i)]))
```

```
        weight_matrix_rescaled = rescale_weight_matrix(
    weight_image_matrix)
        print_matrix_weight(working_image, weight_matrix_rescaled)

weight_image_matrix = matricize_list(list(data["weight_c9"]))    ##
    c9 is for cluster 10. for cluster i, it would be c i-1
weight_matrix_rescaled = rescale_weight_matrix(weight_image_matrix)
print_matrix_weight(working_image, weight_matrix_rescaled)
```

The images generated are as shown below.

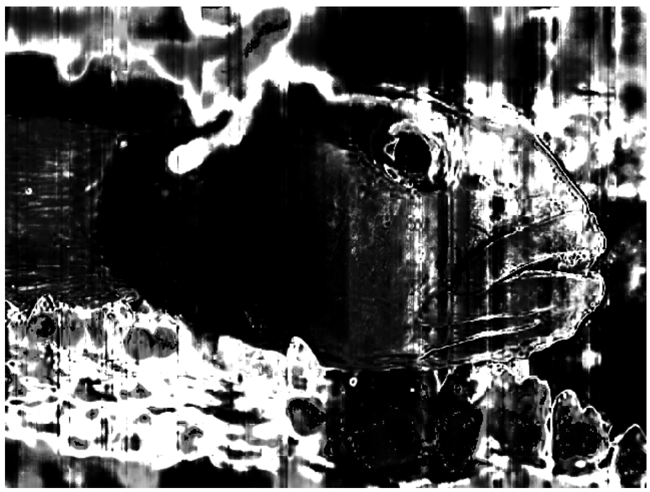


Figure 4: Cluster 1

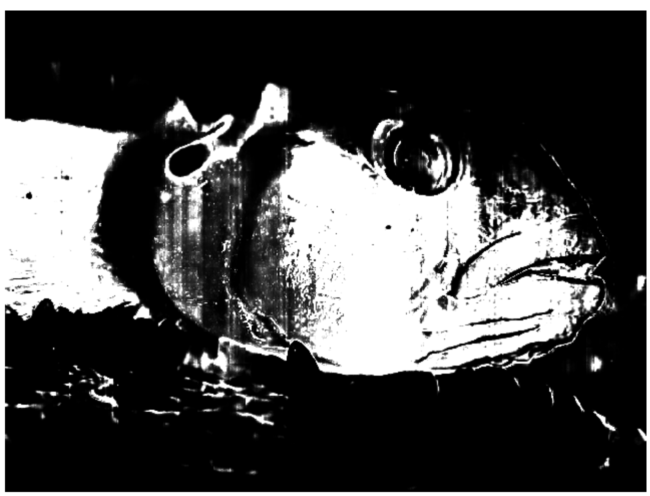


Figure 5: Cluster 2

Figure 6: Cluster 3

Figure 7: Cluster 4

Figure 8: Cluster 5

Figure 9: Cluster 6

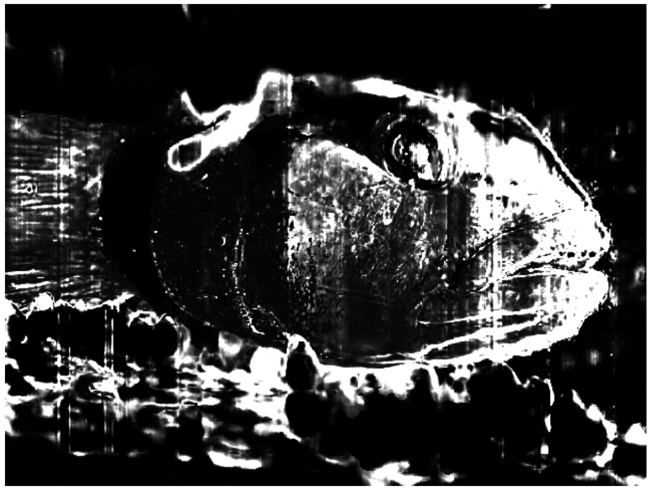


Figure 10: Cluster 7

Figure 11: Cluster 8

Figure 12: Cluster 9

Figure 13: Cluster 10

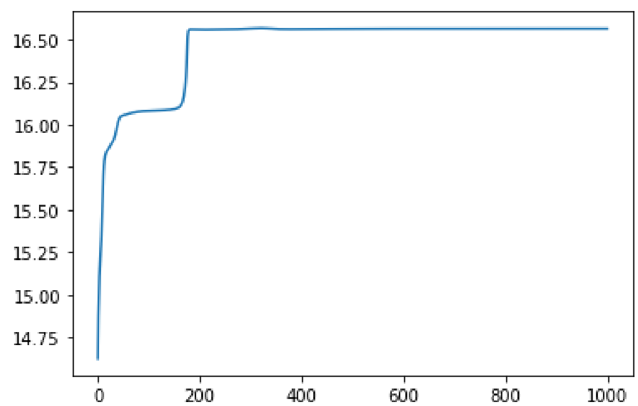
1.3 Part c

We change the covariance to $0.1XI$ and ran the code.

```
rvalues=rgb_normalized_list[0].values.flatten()
gvalues=rgb_normalized_list[1].values.flatten()
bvalues=rgb_normalized_list[2].values.flatten()
list_of_tuples = list(zip(rvalues, gvalues, bvalues))
data=pd.DataFrame(list_of_tuples, columns = ['R', 'G', 'B'])
#Number of clusters
```

```
k=50
#Initializing the covariance matrix
cov = []
for i in range(k):
    cov.append(0.1*np.eye(data.shape[1]))
cov = np.array(cov)
means,cov,weights,b,log_likelihoods,likelihood=EM_function(data,cov
    ,k,500)
#Plotting the log-likelihood
plt.plot(log_likelihoods)
likelihood = np.array(likelihood)
#Finding the closest cluster for each pixel
predictions = np.argmax(likelihood, axis=0)
data['cluster']=predictions
data['mean_R']=np.zeros(data.shape[0])
data['mean_G']=np.zeros(data.shape[0])
data['mean_B']=np.zeros(data.shape[0])
for i in range(data.shape[0]):
    data.at[i,'mean_R']=means[data.at[i,'cluster']][0]
    data.at[i,'mean_G']=means[data.at[i,'cluster']][1]
    data.at[i,'mean_B']=means[data.at[i,'cluster']][2]
for i in range(k):
    data['weight_c'+str(i)]=b[i]
```

We ran the code for 500 iterations. The convergence of log likelihood values for 10 clusters is as shown below.



The values converge around 200 iterations.

The following are the images obtained by replacing each pixel with the mean of its cluster center.

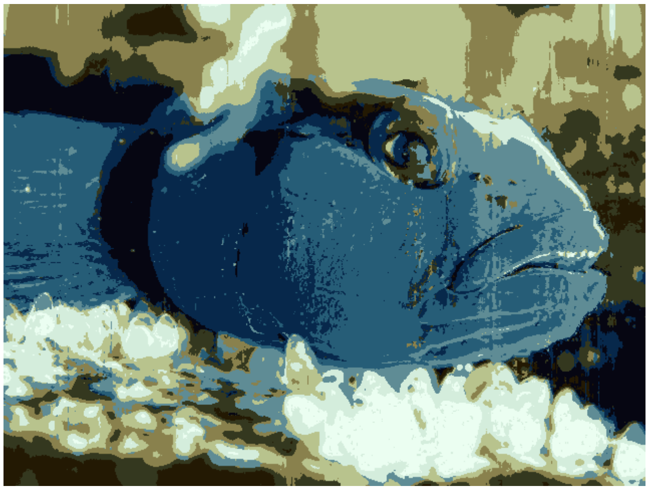


Figure 14: 10 Clusters

Figure 15: 20 Clusters

Figure 16: 50 Clusters

Following are the new set of weight maps.

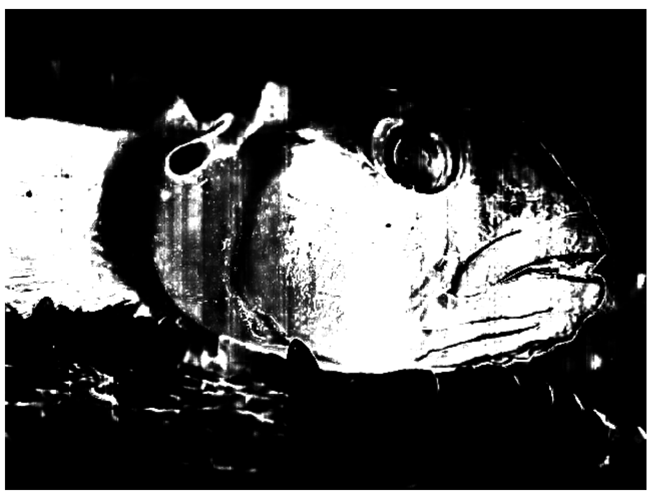


Figure 17: Cluster 1

Figure 18: Cluster 2

Figure 19: Cluster 3

Figure 20: Cluster 4

Figure 21: Cluster 5

Figure 22: Cluster 6

Figure 23: Cluster 7

Figure 24: Cluster 8

Figure 25: Cluster 9

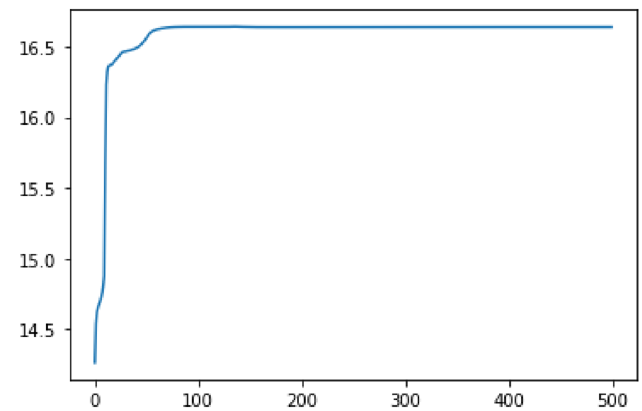
Figure 26: Cluster 10

1.4 Part d

We estimated the covariance of pixel values by assuming that pixels are normally distributed and clustered them using EM. The code for this is as given below.

```
rvalues=rgb_normalized_list[0].values.flatten()
gvalues=rgb_normalized_list[1].values.flatten()
bvalues=rgb_normalized_list[2].values.flatten()
list_of_tuples = list(zip(rvalues, gvalues, bvalues))
data=pd.DataFrame(list_of_tuples, columns = ['R', 'G', 'B'])
#Number of clusters
k=50
#Initializing the covariance matrix
cov = []
for i in range(k):
    cov.append(np.array(data.cov()))
cov = np.array(cov)
means,cov,weights,b,log_likelihoods,likelihood=EM_function(data,cov
    ,k,500)
#Plotting the log-likelihood
plt.plot(log_likelihoods)
likelihood = np.array(likelihood)
#Finding the closest cluster for each pixel
predictions = np.argmax(likelihood, axis=0)
data['cluster']=predictions
data['mean_R']=np.zeros(data.shape[0])
data['mean_G']=np.zeros(data.shape[0])
data['mean_B']=np.zeros(data.shape[0])
for i in range(data.shape[0]):
    data.at[i,'mean_R']=means[data.at[i,'cluster']][0]
    data.at[i,'mean_G']=means[data.at[i,'cluster']][1]
    data.at[i,'mean_B']=means[data.at[i,'cluster']][2]
for i in range(k):
    data['weight_c'+str(i)]=b[i]
```

We ran the code for 500 iterations. The convergence of log likelihood values for 10 clusters is as shown below.



The values converge around 100 iterations.

The following are the images obtained by replacing each pixel with the mean of its cluster center.

Figure 27: 10 Clusters

Figure 28: 20 Clusters

Figure 29: 50 Clusters

Following are the new set of weight maps.

Figure 30: Cluster 1

Figure 31: Cluster 2

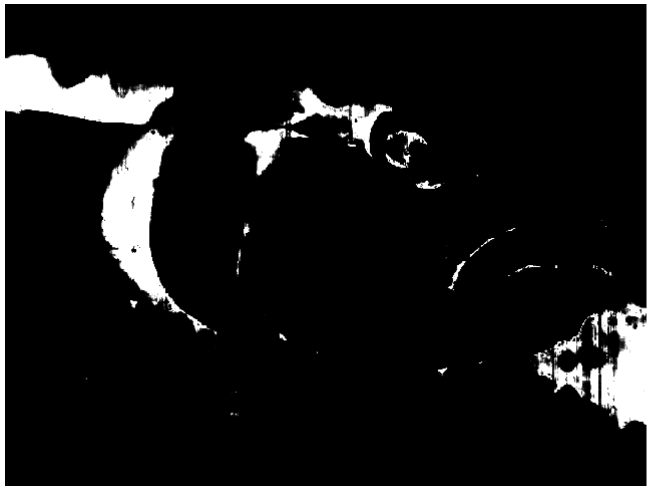


Figure 32: Cluster 3

Figure 33: Cluster 4

Figure 34: Cluster 5

Figure 35: Cluster 6

Figure 36: Cluster 7

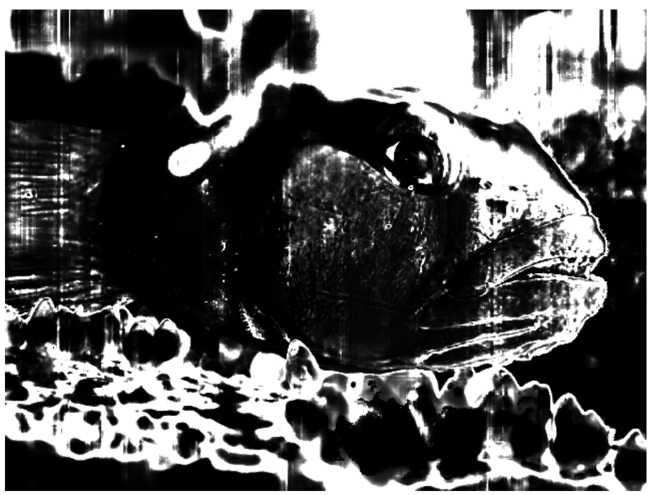


Figure 37: Cluster 8

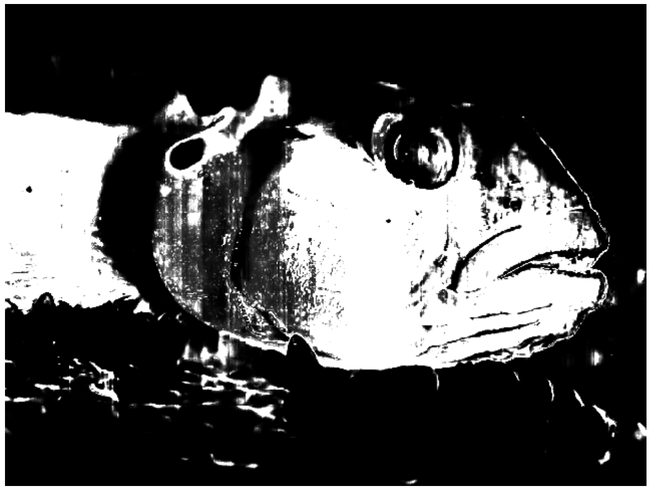


Figure 38: Cluster 9

Figure 39: Cluster 10