A06

Sunday, 20. October 2019 15:37

a) $g: \hat{y} = ax^2 + bx + c$

$$\sum (\hat{y}_i - y_i)^2 = 0 \rightarrow \text{MIN}$$

$$\sum (ax_i^2 + bx_i + c - y_i)^2 = 0$$

$$\frac{\partial f}{\partial a} = 0 \wedge \frac{\partial f}{\partial b} = 0 \wedge \frac{\partial f}{\partial c} = 0$$

$$\text{I} : 2\sum (a \cdot x_i^2 + b \cdot x_i + c - y_i) \cdot x_i^2 = 0 \quad | \cdot \tfrac{1}{2}$$
$$\text{II} : 2\sum (a \cdot x_i^2 + b \cdot x_i + c - y_i) \cdot x_i = 0 \quad | \cdot \tfrac{1}{2}$$
$$\text{III} : 2\sum (a \cdot x_i^2 + b \cdot x_i + c - y_i) \cdot 1 = 0 \quad | \cdot \tfrac{1}{2}$$

$$\text{I} \quad \sum a \cdot x_i^4 + b \cdot x_i^3 + c \cdot x_i^2 - y_i x_i^2 = 0$$
$$\text{II} \quad \sum a \cdot x_i^3 + b \cdot x_i^2 + c \cdot x_i - y_i x_i = 0$$
$$\text{III} \quad \sum a \cdot x_i^2 + b \cdot x_i + c\sum 1 - y_i = 0$$

Matrix Notation

$$\text{I} \wedge \text{II} \wedge \text{III} \begin{pmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum x_i^2 y_i \\ \sum x_i y_i \\ \sum y_i \end{pmatrix}$$

b) $g^*: \hat{z} = a \cdot x + b \cdot y + c$

$$(\hat{z} - z_i)^2 \rightarrow \text{MIN}$$

$$(a.x + b.y + c - z_i)^2$$

$$\frac{\partial f}{\partial a} = 0 \wedge \frac{\partial f}{\partial b} = 0 \wedge \frac{\partial f}{\partial c} = 0$$

$$\text{I}: \quad 2 \cdot \bar{\Sigma}(a.x + b.y + c - z_i)\, x = 0 \qquad 1:2$$
$$\text{II}: \quad 2 \cdot \bar{\Sigma}(a.x + b.y + c - z_i)\, y = 0 \qquad 1:2$$
$$\text{III}: \quad 2 \cdot \bar{\Sigma}(a.x + b.y + c - z_i)\, 1 = 0 \qquad 1:2$$

$$\text{I}: \quad \bar{\Sigma}\, a.x^2 + b.y + cx - z_i x = 0$$
$$\text{II}: \quad \bar{\Sigma}\, a.xy + b.y^2 + c.y - z_i.y = 0$$
$$\text{III}: \quad \bar{\Sigma}\, a.x + b.y + c.\bar{\Sigma}\, 1 - z_i = 0$$

Matrix Notation

$$\text{I},\text{II},\text{III} \begin{pmatrix} \bar{\Sigma} x_i^2 & \bar{\Sigma} y_i & \bar{\Sigma} x_i \\ \bar{\Sigma} x_i y_i & \bar{\Sigma} y_i^2 & \bar{\Sigma} y_i \\ \bar{\Sigma} x_i & \bar{\Sigma} y_i & n \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \bar{\Sigma} z_i x_i \\ \bar{\Sigma} z_i y_i \\ \bar{\Sigma} z_i \end{pmatrix}$$