Numerical Analysis Homework1.1

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1 Introduction

对函数

$$f(x) = \frac{1}{1+x^2}, x \in [-5, 5]$$

构造 $Lagrange$ 插值多项式 $p_L(x)$, 插值节点取为:

$$1.x_i = 5 - \frac{10}{N}i, i = 0, 1, ..., N$$

$$2.x_i = -5cos(\frac{2i+1}{2N+2}\pi), i = 0, 1, ..., N$$

并计算如下误差

$$max_i\{|f(y_i) - p(y_i)|, y_i = \frac{i}{10} - 5, i = 0, 1, ..., 100\}$$

并且对 $N = 5, 10, 20, 40$ 比较以上两组节点的结果，并在一张图中画出 $N = 10$ 时 $f(x)$ 数值计算结果.

2 Method

本次实验采用 MATLAB 进行编程. 一共有两个文件, 一个是函数文件, 用于计算 $Lagrange$ 插值多项式, 另一个是用于计算以及输出结果的。考虑使用 $Lagrange$ 插值法构造插值多项式. 对不同的 N, 根据给出的数据点, 构造插值多项式来逼近。然后对所有节点遍历，求出最大误差。

3 Results

4 Discussion

观察到第一组节点的拟合效果较差。当 N 越大时, 误差反而越大. 而第二组节点的拟合效果整体就较好. 当 N 越大时, 逼近效果更好, 误差较小。大概是第一组节点取值比较均匀，不能有效反映出函数的所有信息, 所以会在一个局部出现较大的误差. 而第二组节点取值相对随机一点, 更有效的反映出函数的全部信息。

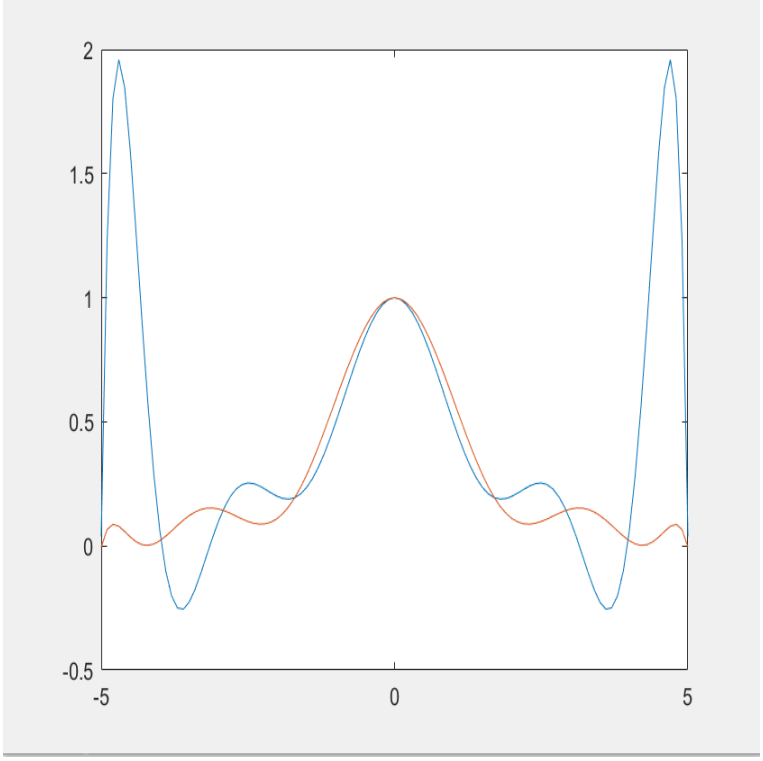

图 1: 当 N=10 时的图像

```
>> main
N=5
    "Max Error of grid (1) :"    "0.43269"

    "Max Error of grid (2) :"    "0.55591"

N=10
    "Max Error of grid (1) :"    "1.9156"

    "Max Error of grid (2) :"    "0.10893"

N=20
    "Max Error of grid (1) :"    "58.2781"

    "Max Error of grid (2) :"    "0.015325"

N=40
    "Max Error of grid (1) :"    "78689.0375"

    "Max Error of grid (2) :"    "0.00027386"
```

图 2: 输出结果

5 Computer Code

```
error=zeros(101,2,4);
xi=zeros(101,1);
p1=zeros(101,1); %#ok
p2=zeros(101,1); %#ok
fi=zeros(101,1);
N=5;
x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1); %#ok
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=5-(10/N)*i;
    y1(i+1)=1/(1+x1(i+1)*x1(i+1));
    x2(i+1)=-5*cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/10-5;
    fi(i+1)=1/(1+xi(i+1)*xi(i+1));
end
p1=Lagrange(x1,y1,xi);
p2=Lagrange(x2,y2,xi);

for i=0:100
    error(i+1,1,1)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,1)=abs(p2(i+1)-fi(i+1));
end



N=10;

x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1);
x2=zeros(N+1,1);
```

```
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=5-(10/N)*i;
    y1(i+1)=1/(1+x1(i+1)*x1(i+1));
    x2(i+1)=-5*cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/10-5;
    fi(i+1)=1/(1+xi(i+1)*xi(i+1));
end
p1=Lagrange(x1,y1,xi);
p2=Lagrange(x2,y2,xi);


for i=0:100
    error(i+1,1,2)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,2)=abs(p2(i+1)-fi(i+1));
end

plot(xi,p1);
hold on;
plot(xi,p2);


N=20;

x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1);
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=5-(10/N)*i;
    y1(i+1)=1/(1+x1(i+1)*x1(i+1));
    x2(i+1)=-5*cos(pi*((2*i+1)/t));
```

```
    y2(i+1)=1/(1+x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/10-5;
    fi(i+1)=1/(1+xi(i+1)*xi(i+1));
end
p1=Lagrange(x1,y1,xi);
p2=Lagrange(x2,y2,xi);


for i=0:100
    error(i+1,1,3)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,3)=abs(p2(i+1)-fi(i+1));
end

N=40;

x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1);
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=5-(10/N)*i;
    y1(i+1)=1/(1+x1(i+1)*x1(i+1));
    x2(i+1)=-5*cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/10-5;
    fi(i+1)=1/(1+xi(i+1)*xi(i+1));
end
p1=Lagrange(x1,y1,xi);
p2=Lagrange(x2,y2,xi);


for i=0:100
```

```
    error(i+1,1,4)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,4)=abs(p2(i+1)-fi(i+1));
end

e=max(error);

format long
e;

disp('N=5');
disp(["Max Error of grid (1) :"  e(1,1,1)] );
disp(["Max Error of grid (2) :"  e(1,2,1)] );

disp('N=10');
disp(["Max Error of grid (1) :"  e(1,1,2)] );
disp(["Max Error of grid (2) :"  e(1,2,2)] );

disp('N=20');
disp(["Max Error of grid (1) :"  e(1,1,3)] );
disp(["Max Error of grid (2) :"  e(1,2,3)] );

disp('N=40');
disp(["Max Error of grid (1) :"  e(1,1,4)] );
disp(["Max Error of grid (2) :"  e(1,2,4)] );
```

```
function z = Lagrange(x,f,x0)

n = length(x) ;

m = length(x0);

tt=zeros(m,1);

    for i = 1:m
    D = x0(i);
    y = 0.0;
    for k = 1:n

    l = 1.0;
        for j = 1:n

            if j~=k
            l = l*(D-x(j))/(x(k)-x(j));
            end
        end

        y = y + l*f(k);
    end
        tt(i)=y;

    end
    z=tt;
end
```