5: a: $\begin{cases} f(t,x) = x^{-2} = x' \\ x(0)=0 \end{cases}$ $\qquad \dfrac{dx(t)}{dt} = \dfrac{1}{x^2(t)}$

$\Rightarrow x(t) = (3t)^{\frac{1}{3}}$ 满足 $x(0)=0$，为一个解、

b: $\begin{cases} x' = 1+x^2 \\ x(0)=0 \end{cases}$ $\qquad \dfrac{dx(t)}{dt} = 1+x^2(t) \Rightarrow x(t) = \tan t$.

c: $\begin{cases} x' = \dfrac{1}{\sin x+\cos x} \\ x(0) = 0 \end{cases}$ $\qquad \dfrac{dx}{dt} = \dfrac{1}{\sin x+\cos x} \Rightarrow dt = (\sin x+\cos x)dx$

$\Rightarrow t = -\cos x+\sin x + C$ 代入 $x(0)=0 \Rightarrow C=1$

则 $t = \sin x - \cos x + 1 = \sqrt{2}\sin(x-\frac{\pi}{4})+1$

求得 $x = \arcsin(\frac{\sqrt{2}}{2}(t-1)) + \frac{\pi}{4}$, $t \in (1-\sqrt{2}, 1+\sqrt{2})$

12： $\begin{cases} f(x,t) = 1 + x + x^2\cos t \\ x(0) = 0 \end{cases}$ $t_0 = 0$，$x_0 = 0$

在 $R = \{(t,x): |t| \le \frac{1}{3}, |x| \le \beta\}$ 内

$|f(x,t)| \le 1 + \beta + \beta^2$，则 $\frac{\beta}{1+\beta+\beta^2} \geq \frac{1}{3} \Rightarrow \beta = 1, M = 3$

存在性定理知 在 $|t| \le \frac{1}{3}$ 上有解。

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8.2.2: $\begin{cases} x' = \sqrt{x} \\ x(0) = 0 \end{cases}$

代入 $x(t) = \frac{t^2}{4}$ 知 $x(0) = 0$，$x' = \frac{t}{2} = \sqrt{x}$ 符合方程

故 $x(t) = \frac{t^2}{4}$ 为解.

一阶泰勒级数：$x(t+h) = x(t) + hf(t,x)$

$= x(t) + h\sqrt{x}$ （数值解） $\longrightarrow$ 考虑 $x(t) \equiv 0$，亦为一个解、

代入 $x = \frac{t^2}{4}$ 发现因为忽略了后面的 $\frac{h^2}{2!}x''(t) + \frac{h^3}{3!}x'''(t) \cdots$ 等项 因此与 $\frac{t^2}{4}$ 不同

不成立；

8.2.4: $\begin{cases} x' = x^2 + xe^t \\ x(0) = 1 \end{cases}$

$x'' = 2xx' + x'e^t + xe^t$

$x''' = 2(x')^2 + 2xx'' + x''e^t + x'e^t + xe^t + x'e^t$

代入 $x(0+0.01) = x(0) + 0.01x'(0) + \frac{(0.01)^2}{2!}x''(0) + \frac{(0.01)^3}{3!}x'''(0)$

求得 $x(0.01) = 1.0204$ (1.020351)