Numerical Analysis Homework2.1

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1 Introduction

对函数

$$f(x)=\frac{1}{1+25x^2},x\in[-1,1]$$

构造$Newton$插值多项式$p_L(x)$,插值节点取为:

$$1.x_i=1-\frac{2}{N}i,i=0,1,...,N$$

$$2.x_i=-cos(\frac{2i+1}{2N+2}\pi),i=0,1,...,N$$

并计算如下误差

$$max_i\{|f(y_i)-p(y_i)|,y_i=\frac{i}{50}-1,i=0,1,...,100\}$$

并且对$N=5,10,20,40$比较以上两组节点的结果，并在一张图中画出$N=20$时$f(x)$数值计算结果.

2 Method

本次实验采用MATLAB进行编程.一共有两个文件,一个是函数文件,用于计算$Newton$插值多项式,另一个是用于计算以及输出结果的。考虑使用$Newton$插值法构造插值多项式.对不同的N,根据给出的数据点,构造插值多项式来逼近。然后对所有节点遍历，求出最大误差。

3 Results

4 Discussion

观察到第一组节点的拟合效果较差。当N越大时,误差反而越大.而第二组节点的拟合效果整体就较好.当N越大时,逼近效果更好,误差较小。大概是第一组节点取值比较均匀，不能有效反映出函数的所有信息,所以会在一个局部出现较大的误差.而第二组节点取值相对随机一点,更有效的反映出函数的全部信息。

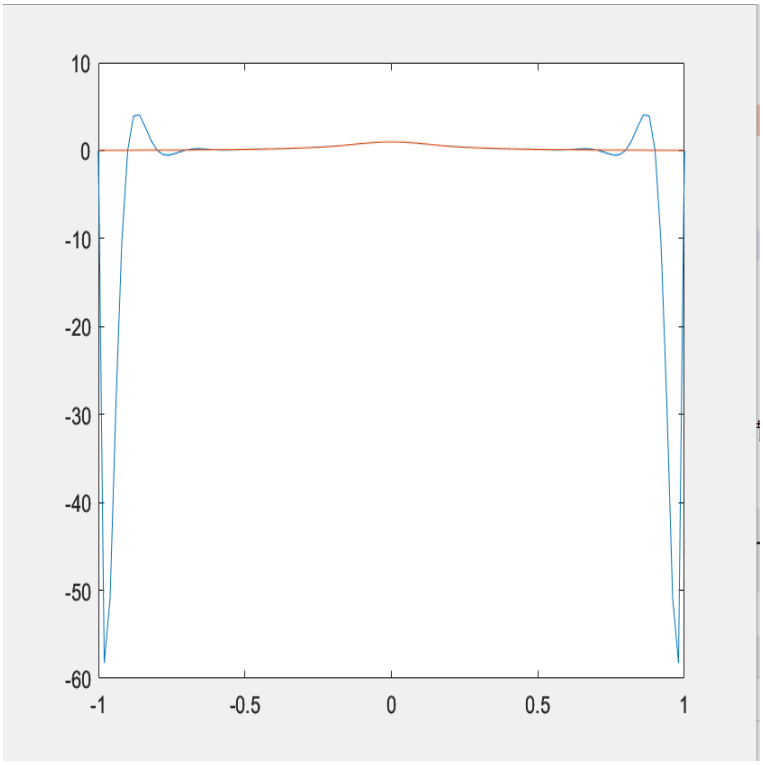

图 1: 当N=10时的图像

```
>> main
N=5
    "Max Error of grid (1) :"    "0.43269"

    "Max Error of grid (2) :"    "0.55591"

N=10
    "Max Error of grid (1) :"    "1.9156"

    "Max Error of grid (2) :"    "0.10893"

N=20
    "Max Error of grid (1) :"    "58.2781"

    "Max Error of grid (2) :"    "0.015325"

N=40
    "Max Error of grid (1) :"    "78689.0375"

    "Max Error of grid (2) :"    "0.00027386"
```

图 2: 输出结果

5 Computer Code

```
function y = Newton(X,Y,x)

n=length(X); m=length(x);
for t=1:m
    z=x(t); A=zeros(n,n);
    A(:,1)=Y';
    s=0.0;
    for  j=2:n
       for i=j:n
           A(i,j)=(A(i,j-1)- A(i-1,j-1))/(X(i)-X(i-j+1));
       end
    end
    C=A(n,n);
    for k=1:n
        p=1.0;
        for j=1:k-1
            p=p*(z-X(j));
        end
        s=s+A(k,k)*p;
    end
    ss(t)=s;
end
    y=ss;
end



error=zeros(101,2,4);
xi=zeros(101,1);
p1=zeros(101,1); %#ok
p2=zeros(101,1); %#ok
fi=zeros(101,1);
N=5;
x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1); %#ok
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;
```

```
for i=0:N
    x1(i+1)=1-(2/N)*i;
    y1(i+1)=1/(1+25*x1(i+1)*x1(i+1));
    x2(i+1)=-cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+25*x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/50-1;
    fi(i+1)=1/(1+25*xi(i+1)*xi(i+1));
end
p1=Newton(x1,y1,xi);
p2=Newton(x2,y2,xi);

for i=0:100
    error(i+1,1,1)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,1)=abs(p2(i+1)-fi(i+1));
end



N=10;

x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1);
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=1-(2/N)*i;
    y1(i+1)=1/(1+25*x1(i+1)*x1(i+1));
    x2(i+1)=-cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+25*x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/50-1;
    fi(i+1)=1/(1+25*xi(i+1)*xi(i+1));
end
p1=Newton(x1,y1,xi);
```

```
p2=Newton(x2,y2,xi);

for i=0:100
    error(i+1,1,2)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,2)=abs(p2(i+1)-fi(i+1));
end


N=20;

x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1);
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=1-(2/N)*i;
    y1(i+1)=1/(1+25*x1(i+1)*x1(i+1));
    x2(i+1)=-cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+25*x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/50-1;
    fi(i+1)=1/(1+25*xi(i+1)*xi(i+1));
end
p1=Newton(x1,y1,xi);
p2=Newton(x2,y2,xi);


for i=0:100
    error(i+1,1,3)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,3)=abs(p2(i+1)-fi(i+1));
end

plot(xi,p1);
hold on;
plot(xi,p2);
```

```
N=40;

x1=zeros(N+1,1);
y1=zeros(N+1,1);
f1=zeros(101,1);
x2=zeros(N+1,1);
y2=zeros(N+1,1);
t=2*N+2;

for i=0:N
    x1(i+1)=1-(2/N)*i;
    y1(i+1)=1/(1+25*x1(i+1)*x1(i+1));
    x2(i+1)=-cos(pi*((2*i+1)/t));
    y2(i+1)=1/(1+25*x2(i+1)*x2(i+1));
end

for i=0:100
    xi(i+1)=i/50-1;
    fi(i+1)=1/(1+25*xi(i+1)*xi(i+1));
end
p1=Newton(x1,y1,xi);
p2=Newton(x2,y2,xi);


for i=0:100
    error(i+1,1,4)=abs(p1(i+1)-fi(i+1));
    error(i+1,2,4)=abs(p2(i+1)-fi(i+1));
end

e=max(error);

format long
e;

disp('N=5');
disp(["Max Error of grid (1) :"  e(1,1,1)] );
disp(["Max Error of grid (2) :"  e(1,2,1)] );

disp('N=10');
disp(["Max Error of grid (1) :"  e(1,1,2)] );
disp(["Max Error of grid (2) :"  e(1,2,2)] );
```

```
disp('N=20');
disp(["Max Error of grid (1) :"  e(1,1,3)] );
disp(["Max Error of grid (2) :"  e(1,2,3)] );

disp('N=40');
disp(["Max Error of grid (1) :"  e(1,1,4)] );
disp(["Max Error of grid (2) :"  e(1,2,4)] );
```