Algorithms Associated with Factorization Machines

Yanyu Liang
Xin Lu
Xupeng Tong

Carnegie Mellon University

{yanyul,xlu2,xtong}@andrew.cmu.edu

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Overview

Factorization Machines

- Factorization machines [1] models $y \in \mathbb{R}$ given $x \in \mathbb{R}^p$ using the following expression:

$$\hat{y} = w_0 + w^T x + \sum_{i,j} (V_i x_i)^T (V_j x_j)$$

$$= w_0 + w^T x + x^T V^T V x$$

, where V_i is $k \times 1$ vector.

- FMs is widely used in recommendation system, since it implicitly regularizes the model complexity by setting a k which is much smaller than p.

- $\sum_{i,j}(V_i x_i)^T (V_j x_j)$ makes FMs nonconvex, and researcher has proposed convex FMs [2] by replacing low rank constraint by trace norm (replace $V^T V$ with W).
- Also, by replacing one V in $V^T V$ with U, [3] proposed generalized FMs with an online learning algorithm

Our Goal

- Two block coordinate descent to solve original convexFMs
- Propose ADMM method to sovle convexFMs with element-wise l_1 constraint
- Analyze performance of generalized FMs performance

ConvexFMs

- By introducing trace norm to FMs and substitude $V^T V$ with W, the problem becomes convex.
- The formulation of convexFMs is:

$$\min_{w,W} \sum_{i=1}^{n} \underbrace{(y_i - w^T x_i - x_i^T W x_i)^2}_{f(w,W)}$$
$$+ \lambda_1 \|W\|_{\mathsf{tr}} + \lambda_2 \|w\|_2^2$$

Two Block Coordinate Descent

- convexFMs can be consider as two sub-problem:

$$\min_{w \in \mathbb{R}^d} \sum_{i=1}^{n} 1/2(y_i - w^T x_i - \pi_i)^2 + \alpha/2\|w\|_2^2,$$

where $\pi_i = \langle W, x_i x_i^T \rangle$, and

$$\min_{W \in \mathbb{S}^{d*d}} \sum_{i=1}^{n} 1/2(y_i - w^T x_i - x_i^T W x_i)^2 + \beta\|W\|_{\text{tr}}$$

- Alternatively perform gradient descent and proximal gradient descent on these problems respectively.

Proximal Gradient Descent

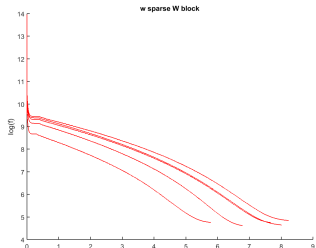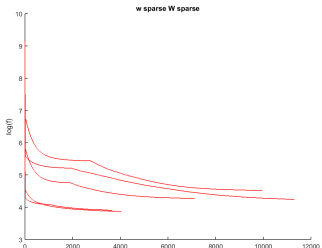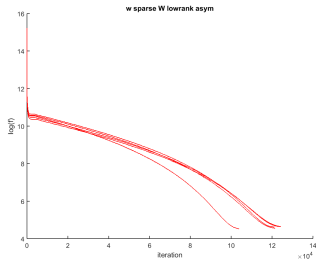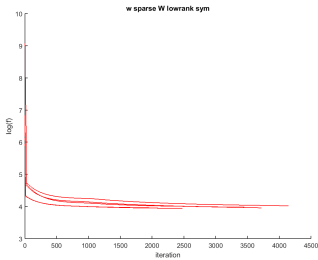- Define proximal operator using matrix soft-thresholding:

$$prox_t(W^+) = S_{\beta t}(W^+) = U\Sigma_{\beta t}V^T$$

  where $W = U\Sigma V^T$ is a SVD, and $\Sigma_{\beta t}$ is diagonal matrix with $(\Sigma_{\beta t})_{ii} = \max\{\Sigma_{ii} - \beta t, 0\}$.

- Perform backtracking line search then update using $prox_t(W - t\nabla g(W))$, where
  $\nabla g(W) = -\sum_{i=1}^{n}(y_i - w^T x_i - x_i^T W x_i)x_i x_i^T$.

Two Block Coordinate Descent Results

Convergence rate: W sym>W sparse>W asym≈W block

ConvexFMs with l_1 constraint

- l_1 penalty is widely used and it is potential helpful in many applications beyond recommendation systems.
- Consider the regression problem in convexFMs with l_1 penalty:

$$\min_{w_0, w, W} \sum_{i=1}^{n} \underbrace{(y_i - w_0 - w^T x_i - x_i^T W x_i)^2}_{f(w_0, w, W)}$$
$$+ \lambda_1 \|W\|_{\mathrm{tr}} + \lambda_2 \|W\|_1 + \lambda_3 \|w\|_2^2$$

ADMM Formulation

- By introducing axilluary variable U, it can be fit into ADMM framework and the augmented Lagrangian is:

$$\mathcal{L}(w_0, w, W) = f(w_0, w, W) + \lambda_1 \|U\|_{\text{tr}} + \lambda_2 \|W\|_1 + \lambda_3 \|w\|_2^2$$
$$+ \langle W - U, u \rangle + m\|W - U\|_2^2$$

- Then the ADMM loop is:
  1. Update w_0, w, W:

  $$w_0^k, w^k, W^k = \arg \min_{w0, w, W} f(w_0, w, W) + \lambda_2 \|W\|_1$$
  $$+ \frac{\rho}{2} \|W - U^{k-1} + u^{k-1}\|_2^2$$

  2. Update U:

  $$U^k = \arg \min_U \lambda_1 \|U\|_{\text{tr}} + \frac{\rho}{2} \|W^k - U + u^{k-1}\|_2^2$$
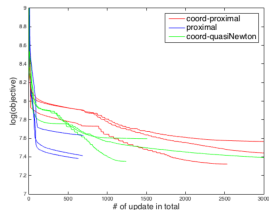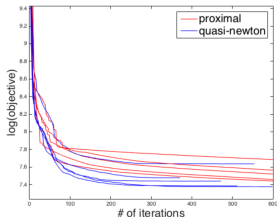
  3. Update u:

  $$u^k = u^{k-1} + W^k - U^k$$

ADMM Subproblem

- The second step can be solve exactly with proximal operator of trace norm.
- We explored two approaches to solve the first subprobelm:
  - proximal graident descent on w_0, w, W (proximal)
  - blockwise coordinate descent on w_0, w and W respectively, where we applied coordinate descent on the first block and tried proximal gradient (coor-proximal) and Quasi-Newton (coor-newton) method on the second block

ADMM Results

- Quasi-Newton method performs better than proximal graident descent in solving $\arg\min_W g(W)$ subproblem (as we expected)
- proximal gradient descent performs better and stable than blockwise coordinate descent

ADMM Results (con'd)

- ADMM converges and it achieves feasiblity along the path
- with the same number of updates (consider inner loop), ADMM outperforms sub-gradient method
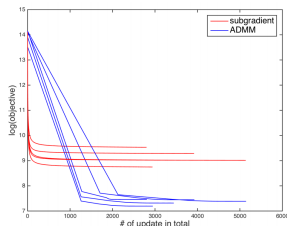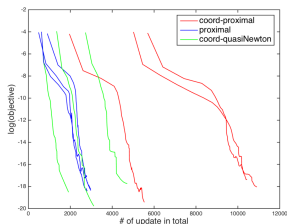
gFM proposed by [3] removes several redundant constraints compared to the original FM, while its learning ability is kept. Reforming $\hat{y}$ in gFMs as follow:

$$\hat{y} = X^T w^* + \mathcal{A}(U^T V) + \xi$$

- When $w = 0$, gFM is equal to the symmetric rank-one matrix sensing problem
- When $w \neq 0$, the gFM has an extra perturbation term $X^T w$.

# One pass algorithm solving generalized Factorization Machine

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Algorithm 1 One pass algorithm solving

Ensure: $w^{(T)}, U^{(T)}, V^{(T)}$

1: Initialize: $w^{(0)} = 0, V^{(0)} = 0. U^{(0)} = \text{SVD}\left(H_1^0 - \frac{1}{2}h_2^{(0)}I, k\right)$
2: **for** $t = 1, 2, \cdots, T$ **do**
3:     Retrieve $x^{(T)} = [x_{(t-1)n+1}, \cdots, x_{(t-1)n+n}]$. Define $\mathcal{A}(M) \triangleq \left[X_i^{(t)^T} M X_i^{(t)}\right]$
4:     $\hat{U}^{(t)} = \left(H_1^{(t-1)} - \frac{1}{2}h_2^{(t-1)}I + M^{(t-1)^T}U^{(t-1)}\right)$
5:     Orthogonalize $\hat{U}^{(t)}$ via QR decomposition: $U^{(t)} = QR(\hat{U}^{(t)})$
6:     $w^{(t)} = h_3^{(t-1)} + w^{(t-1)}$
7:     $V^t = (H_1^{(t-1)} - \frac{1}{2}h_2^{(t-1)}I + M^{(t-1)})U^{(t)}$
8: **end for**
9: **Output:** $w^{(T)}, U^{(T)}, V^{(T)}$

Result

- Fine tune the rank k in order to have a better convergence rate
- Higher learning rate might cause the algorithm unable to learn or even increased error rate
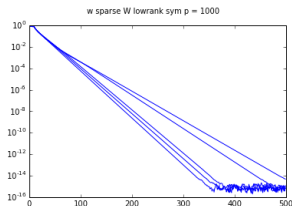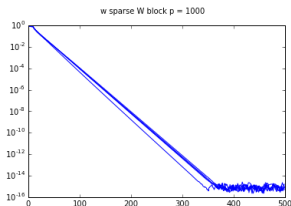
References I

S. Rendle, "Factorization machines," in 2010 IEEE International Conference on Data Mining. IEEE, 2010, pp. 995–1000.

M. Blondel, A. Fujino, and N. Ueda, "Convex factorization machines," in Joint European Conference on Machine Learning and Knowledge Discovery in Databases. Springer, 2015, pp. 19–35.

M. Lin and J. Ye, "A non-convex one-pass framework for generalized factorization machine and rank-one matrix sensing," in Advances In Neural Information Processing Systems, 2016, pp. 1633–1641.

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