A Robust Approach to Image Enhancement Based on Fuzzy Logic

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***Abstract*—In this paper, we propose a robust approach to image enhancement based on fuzzy logic that addresses the seemingly conflicting goals of image enhancement: i) removing impulse noise, ii) smoothing out nonimpulse noise, and iii) enhancing (or preserving) edges and certain other salient structures. We derive three different filters for each of the above three tasks using the weighted (or fuzzy) least squares (LS) method, and define the criteria for selecting each of the three filters. The criteria are based on the local context, and they constitute the antecedent clauses of the fuzzy rules. The overall result of the fuzzy rule-based system is the combination of the results of the individual filters, where each result contributes to the degree that the corresponding antecedent clause is satisfied. This approach gives us a powerful and flexible image enhancement paradigm. Results of the proposed method on several types of images are compared with those of other standard techniques.**

I. Introduction

IMAGE enhancement is an important step in many image processing applications. The type of image enhancement algorithm to be used depends on the objective to be achieved by the enhancement process as well as the particular application. The problem of image enhancement can be stated as that of filtering out impulse noise, smoothing out nonimpulse noise, and enhancing edges or certain other salient structures in the input image. Noise smoothing and edge enhancement are inherently conflicting processes, since smoothing a region might destroy an edge and sharpening edges might lead to unnecessary noise. A plethora of techniques for this problem have been proposed in the literature [1], [2].

Noise filtering can be viewed as replacing the gray-level value of every pixel in the image with a new value depending on the local context. Ideally, the filtering algorithm should vary from pixel to pixel based on the local context. For example, if the local region is relatively smooth, then the new value of the pixel may be a type of average of the local values. On the other hand, if the local region contains edge or impulse-noise pixels, a different type of filtering should be used. However, it is extremely hard, if not impossible, to set the conditions under which a certain filter should be selected, since the local conditions can be evaluated only vaguely in some portions of an image. Therefore, a filtering system needs to be capable of performing reasoning with vague and uncertain information.

In this paper, we propose to incorporate robust adaptive filtering into a fuzzy rule-based system [3] that smooths noise while preserving edges and image details. We derive three different filters for these purposes using the weighted (or fuzzy) least squares (LS) method, which is a robust estimator [4]. Each filter is applied when certain conditions are satisfied. That is, each filter and its conditions constitute a production rule. However, due to the uncertainty and incompleteness of information contained in an image, there might be several conflicting production rules whose preconditions are satisfied by the local image. There are many sophisticated control strategies to solve this problem in traditional systems [5]–[7]. However, fuzzy rule-based systems solve this problem trivially and efficiently by combining the consequents of the rules [3]. In other words, in a fuzzy rule-based system, we make "soft" decisions based on each condition, aggregate the decisions made, and finally make a decision based on the aggregation. This approach is consistent with the principle of least commitment proposed by Marr [8]. To summarize, we propose an approach in which the selection criteria for the filters constitute the antecedent clauses of the fuzzy rules, and the corresponding filters constitute the consequent clauses of the fuzzy rules. Since the rules are fuzzy, at each pixel, each of the antecedent clauses may be satisfied, albeit to a different degree. Thus, all filters are applied, and the overall result of the enhancement is computed as the weighted combination of the results of the individual enhancement filters. The weight associated with the result of a particular filter is proportional to the degree to which the selection criteria for that filter are satisfied in the given situation. As will be shown in Section V, this gives us a smoother result compared to a method that makes hard decisions at each pixel and applies only one particular filter.

The organization of the rest of this paper is as follows. In Section II, we review adaptive filtering methods, robust estimators for image enhancement, and fuzzy approaches. In Section III, we briefly introduce a fuzzy rule-based system and describe our fuzzy rule-based approach to image enhancement. In Section IV, we present three different filters based on the fuzzy least squares method. In Section V, we describe three fuzzy rule-based filtering systems that are consistent with the architecture discussed in Section III. In Section VI, we present experimental results and comparisons with other standard methods. In Section VII, we provide a discussion of the approach as well as conclusions.

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II. Related Previous Work

One approach to smoothing noise while preserving edges is adaptive filtering [2], [9], [10]. The basic idea of adaptive filtering is to apply a filter whose parameters (weights) are adaptively determined on the basis of local context. In general, adaptive filtering can be viewed as weighted average filtering. Let $I(X_1), \cdots, I(X_N)$ be the gray levels of pixels $X_1, \cdots, X_N$, respectively, in a given window. In weighted average filtering, the gray level of the center pixel X_i is replaced by

$$I(X_i) = \frac{\sum_{j=1}^{N} w_{ij} I(X_j)}{\sum_{j=1}^{N} w_{ij}} \tag{1}$$

where w_{ij} is the weight associated with a neighboring pixel X_j. The performance of this approach depends on the weight function w_{ij}, and several ways to compute the weights are suggested in the literature [10]–[13]. Approaches of this type can smooth noisy regions while preserving conspicuous edges, but they do not preserve image details very well. The magnitude of the gradients has also been used for the determination of the weights [10], [14]. Saint-Marc *et al.* [10] proposed a weight function that decreases as the magnitude of the gradient at each pixel increases. They used the exponential function with a parameter k for this purpose, and they showed that this filter is an implementation of the anisotropic diffusion proposed by Perona and Malik [15]. This method is affected by impulse noise, and the behavior of this model is very sensitive to the choice of the parameter k [16]. Li and Chen [17] proposed a time-varying parameter $K(t)$ to overcome this problem. Other improvements of anisotropic diffusion method can be found in [18], [19].

Another class of image enhancement techniques uses robust estimation theory. In this approach, an image model is assumed, and the model parameters are estimated in a robust way. The gray level of the center pixel is obtained from the estimated parameters. The median filter is the simplest method of this class. Bovik *et al.* [28] generalized median filtering using a linear combination of order statistics (L filter). The median filter often fails to smooth out nonimpulse noise and it does not preserve fine details such as lines. Restrepo and Bovik proposed adaptive trimmed mean filters to improve performance on nonimpulse noise [34]. This approach can be viewed as an extension of the α-trimmed means [31], [35]. Several other algorithms have also been proposed to improve the capability of preserving fine details [29], [30]. Estimators from robust statistics such as M-estimators [42] have been used for image restoration [31]–[33]. However, the M-estimator is not highly robust in that it breaks down when a fraction $1/p$ of the points are outliers, where p is the number of parameters in the image model to be estimated [4]. Koivunen [36] employed the least trimmed squares (LTS) estimation technique [4] to select one of zero-, first-, and second-order polynomial image models at each pixel. As pointed out in [36], this approach yields good results, but is computationally intensive.

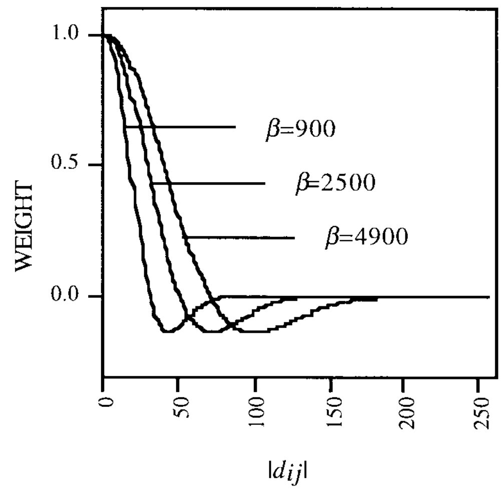


Fig. 1. Plot of the weight function for different values of scale in Filter A.

Recently, fuzzy set theory has gained popularity in pattern recognition and computer vision applications [20], [21]. Researchers in this area have made attempts to design fuzzy filters for signal/image processing with promising results [22]–[25]. In the fuzzy rule-based approach to image processing, we can incorporate human intuition [27] (heuristic rules expressed in linguistic terms) which is highly nonlinear by nature and difficult to represent by traditional mathematical modeling. Moreover, this approach also allows us to combine heuristic rules with traditional methods, which leads to a more flexible design paradigm. For example, Yang and Toh [26] used heuristic fuzzy rules in order to improve the performance of the traditional multilevel median filter. Russo proposed fuzzy rule-based operators for smoothing, sharpening, and edge detection [22], [23], [37], and used heuristic knowledge to build the rules for each of the operations. In [22], Russo and Ramponi proposed the use of operators with differing window sizes (5 × 5 or 7 × 7) where the size depended on the uniformity of the local region. The degree of uniformity of the region was obtained by using another set of fuzzy rules [38]. The two outputs from the 5 × 5 and 7 × 7 window operators were linearly combined based on the output given by the uniformity detection rule. Since the fuzzy rules in this approach were constructed based on region contours, this approach can potentially perform smoothing while preserving edges.

III. Proposed Fuzzy Logic Approach to Image Enhancement

In this section, we discuss the general architecture of the proposed fuzzy rule-based image processing system. A typical rule in a traditional rule based system may be described as

Rule k: If (Conditions) then (Action-k or F_k).

Let us suppose that we have a set $\boldsymbol{F}$ whose elements are actions F_k, and each element F_k performs a basic filtering operation. If the set of actions $\boldsymbol{F}$ is sufficient to perform all

TABLE I
RMS ERRORS FOR FILTERS A, B, AND C ON THE NOISY LENA IMAGE

Filter	Filter A (5×5)	Filter B (5×5)	Filter C (5×5)
RMS error	14.71	12.44	11.05

TABLE II
RMS ERRORS FOR FILTERING SYSTEMS $R1$, $R2$, $R3$, AND CRISP $R3$ ON THE NOISY LENA IMAGE

Filtering system	$R1$ (5×5)	$R2$ (5×5)	$R3$ (5×5)	Crisp $R3$(5×5)
RMS error	6.40	7.89	6.94	7.93

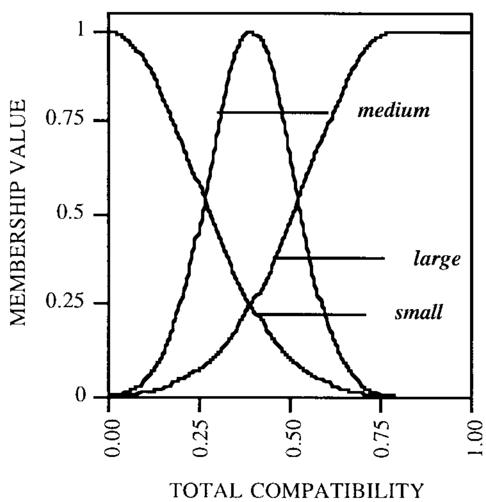


Fig. 2. Membership function for *small*, *medium*, and *large*.

image processing tasks of interest, we need only set proper conditions under which each action should be taken in a given application. However, in designing an ill-defined and complex system such as an image processing system, it is almost impossible to find a set $\boldsymbol{F}$ that would be effective for all applications. Moreover, it is hard to define the precise (crisp) conditions under which each of the filters should be applied. Therefore, a fuzzy set theoretic approach is a good solution to deal with these problems.

In the proposed rule-based system we adopt the if–then–else rule paradigm as in [23]. In general, we can write a rule base consisting of $M+1$ rules with multiple input variables as follows:

Rule 1: If X_{11} is $A_{11} \oplus \cdots \oplus X_{1i}$ is A_{1i} $\oplus \cdots \oplus X_{1N_1}$ is A_{1N_1}, then F_1

Rule k: If X_{k1} is $A_{k1} \oplus \cdots \oplus X_{ki}$ is A_{ki} $\oplus \cdots \oplus X_{kN_k}$ is A_{kN_k}, then F_k

Rule M: If X_{M1} is $A_{M1} \oplus \cdots \oplus X_{Mi}$ is A_{Mi} $\oplus \cdots \oplus X_{MN_M}$ is A_{MN_M}, then F_M

Rule $M+1$: else F_{M+1}.

In the above, A_{ki} is the linguistic label associated with the ith input variable X_{ki} in the kth rule, F_k is the desired action in the kth rule, N_k is the number of input variables in the kth rule, and $\oplus$ is the aggregation operator [39]. The action F_k can be an elementary filter or a set of rules. If an action F_k is a fuzzy set, then the rule becomes an ordinary input-output relation as in a control problem. However, in many image processing tasks, it is difficult to obtain input–output pairs from real image data. For example, suppose we build a filter for removing impulse noise. Then, we can define the set of conditions that describe a noise pixel, and this set of conditions will form the antecedent clause of an if–then rule. The desired output depends not only on the gray-level value of the noise pixel, but also on the gray-level patterns of its neighborhood. Therefore, we need a rule for each possible gray-level pattern in the neighborhood. This approach (which is similar to the one used by Russo and Ramponi [23]) could require very large number of if–then rules. To overcome this problem, we choose a more flexible rule model in which the consequent in an if–then rule is a desired type of filtering. The desired type of filtering can be a fuzzy set as in the Zadeh model [40], a linear function as in the Sugeno model [41], a set of fuzzy rules, or a general function such as a weighted average filter.

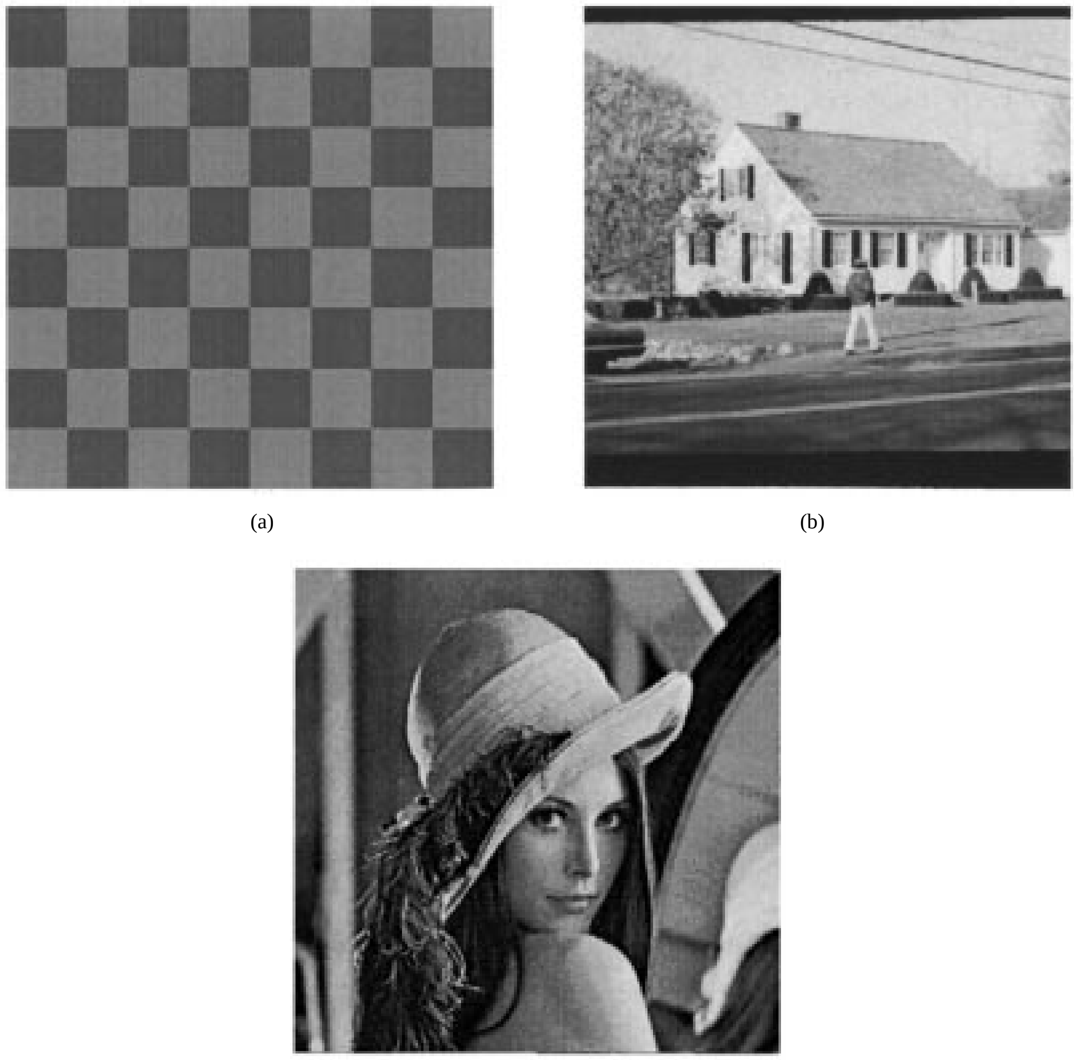
If we denote the degree of satisfaction of the antecedents in the kth rule by c_k and the input fuzzy set and the linguistic label associated with the ith input variable in the kth rule by A'_{ki} and A_{ki}, respectively, then the degree of satisfaction c_k of the antecedent in the kth rule can be written as

$$c_k = (A'_{k1} \circ A_{k1}) \oplus \cdots \oplus (A'_{kN_k} \circ A_{kN_k})$$
$$\text{for } k = 1, \cdots, M \text{ and } c_{M+1} = 1 - \max_k (c_k).$$

We can then compute the output from the fuzzy rule-based system for an input vector as

$$Y = f[(c_1 \circ F_1), \cdots, (c_{M+1} \circ F_{M+1})]. \tag{2}$$

The function f is the defuzzification function, and the symbol $\circ$ denotes the composition operator [3]. If we select the multiplication for the composition operator, the weighted average for the defuzzification function, and the filter output for the consequent, then the output Y from the fuzzy rule-



(a) (b) (c)

Fig. 3. Original images of (a) chess board, (b) outdoor scene, (c) Lena.

based system for image enhancement is

$$Y = \frac{\sum_{k=1}^{M+1} c_k F_k}{\sum_{k=1}^{M+1} c_k}. \tag{3}$$

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IV. Robust Filter Design and Fuzzy Least Squares

A. Robust Estimator

The classical LS method consists of minimizing the sum of the squared residuals (errors). Due to its computational efficiency, the LS method has been widely used. It is, however, known that the LS method can completely break down even in the presence of a single outlier [42]. One way to achieve robustness is to replace the squared residual r_j^2 (corresponding to the jth observation) by a function of the residual $\rho(r_j)$. Huber [42] proposed a more sophisticated method which uses the constraint that ρ is a symmetric function with a unique minimum at zero. The objective function to be minimized is

$$J = \sum_{j=1}^{N} \rho\left(\frac{r_j}{\sigma}\right)$$

where σ is a scale parameter that reflects the dispersion of the data set. This is the so called M-estimator. Minimization of the above objective function involves the derivative of $\rho(\,)$, and several functions for the derivative of ρ have been proposed [47]. Rousseeuw [43] replaced the "sum" in the LS method by the median and proposed the least median of squares (LMS) estimator, given by

$$\underset{\theta}{\text{minimize}}\ \underset{j}{\text{median}}\ r_j^2$$

where θ is the parameter to be estimated. It can be shown that the LMS estimator has a 50% breakdown point (i.e., it can tolerate up to 50% noise and outlier points), but it also has a low efficiency [43]. The weighted least squares (WLS) method

Fig. 4. (a) Noisy Lena image. (b) Result of Filter A. (c) Result of Filter B. (d) Result of Filter C.

is also a robust estimator [4] that has been used in computer vision [44], [45]. Its objective function is given by

$$J = \sum_{j=1}^{N} w_j r_j^2.$$

In [44] and [45], the inverse of the squared residual r_j^2 was used as the weight w_j. It can be shown that the WLS is a particular formulation of the M-estimator [48]. The WLS is also related to the possibilistic clustering method (PCM) [46], [48], which is designed for the case of multiple clusters. In the case of a single cluster, the membership value (weight) in the PCM is given by

$$w_j = \left[1 + \left(\frac{r_j^2}{\eta}\right)\right]^{1/m-1}$$

where η is an estimated scale parameter and $m > 1$. Through this relation, we can view the weight function in WLS as the membership function in fuzzy set theory. The robust estimate, therefore, represents the cluster center (prototype) and the membership value (weight) of a point is determined by its distance from the prototype. During the minimization process, a sample that is far from the prototype, i.e., an outlier, will be treated less importantly and vice versa. We use the weighted (or equivalently fuzzy or possibilistic) LS method for the design of the image smoothing filter, and estimate the scale in a robust way.

B. Filter Design

We can view the noise smoothing problem as the estimation of the prototype for a given set of pixels in a spatial window. That is, filtering is a process of replacing the gray level of a pixel by a prototypical value such that the differences between the prototype and its neighbors are minimized in some sense. In fact, the weighted average filtering is optimal with respect to the following objective function:

$$J = \sum_{j=1}^{N} w_{ij}[I(X_i) - I(X_j)]^2 \tag{4}$$

where X_i and X_j are the center pixel and its neighbors, respectively. If we differentiate the objective function J with respect to $I(X_i)$, we will obtain (1) as the update equation for

(a) (b) (c) (d)

Fig. 5. Restored images of Fig. 4(a) based on Filtering systems (a) $R1$; (b) $R2$; (c) $R3$; and (d) crisp version of $R3$.

the center pixel X_i. In general, the weight function is defined on the basis of the local gray-level distribution.

We regard the weight function w_{ij} as a membership function μ_{ij} that represents the degree of compatibility of a neighboring pixel X_j with respect to X_i. The membership function μ_{ij} is any decreasing function of the scaled residual d_{ij}^2/β_{i_i} such that

$$\mu\left(\frac{d_{ij}^2}{\beta_i}\right): I(X_j) \rightarrow [0, 1], \text{ where } d_{ij}^2 = [I(X_i) - I(X_j)]^2. \tag{5}$$

The parameter β_i represents a robust estimate of scale, and it can be determined on the basis of the variations in pixel intensities in a given spatial window. It is to be noted that $\mu_{ij} \neq \mu_{ji}$ unless $\beta_i = \beta_j$.

Suppose that the center pixel is regarded as the prototype of its neighbors in a given spatial window. Then, the objective function to be minimized can be written as follows:

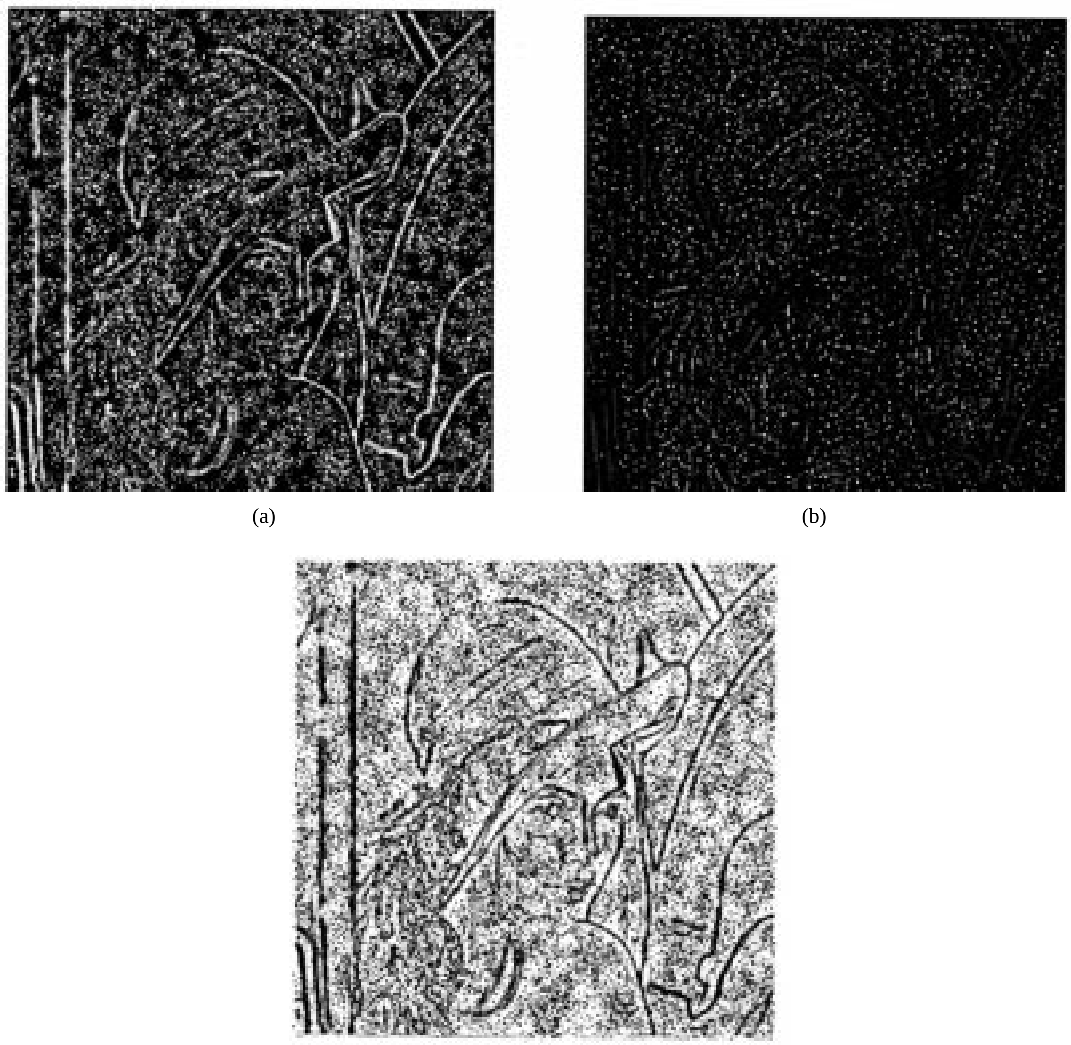
$$J = \sum_{j=1}^{N} \mu_{ij} d_{ij}^2. \tag{6}$$

The gray level of the center pixel is replaced by the new value obtained by minimizing the objective function. Differentiating the objective function in (6) with respect to the gray level of the center pixel X_i, we obtain the following weighted average filter:

$$I(X_i) = \frac{\sum_{j=1}^{N} (\mu_{ij} + d_{ij}^2 \mu'_{ij}) I(X_j)}{\sum_{j=1}^{N} (\mu_{ij} + d_{ij}^2 \mu'_{ij})} \tag{7}$$

where $\mu'_{ij} = \partial\mu_{ij}/\partial(d_{ij}^2)$. Note that the term $d_{ij}^2 \mu'_{ij}$ is negative, since μ is a decreasing function. This means that if $|d_{ij}^2 \mu'_{ij}| > \mu_{ij}$, the neighboring pixel X_j can have a negative weight. The negative weight has an enhancing effect on edges and fine details. We will discuss this behavior in more detail for particular choices of the scale parameter β_i and the membership function μ in the next section.

Note that the objective function (6) is based on the assumption that a given center pixel is a prototype of its neighboring



(a) (b)

(c)

Fig. 6. Weights of filters used in Filtering system $R3$: (a) Filter A; (b) Filter B; and (c) Filter C.

pixels. Therefore, if the given center pixel itself is a noise pixel, the objective function (6) is not valid and consequently the filter (7) will not work well. Hence, we need to construct a different objective function for a different filtering scheme such as smoothing. If a given center pixel is a noise pixel, then we would like to update the gray level of the center pixel in such a way that a new value maximizes the degrees of membership to which its neighbors represent the center pixel. That is, we would like to maximize

$$\mu_{1i} \text{ and } \cdots \text{ and } \mu_{ji} \text{ and } \cdots \text{ and } \mu_{Ni}.$$

One can build different objective functions according to the interpretation of the "and" connective. Here we present two extreme methods: multiplication and averaging (See [49] and [50] for a discussion on fuzzy connectives.) Choosing multiplication, we have the following objective function:

$$J = \prod_{j=1}^{N} \mu_{ji} \tag{8}$$

or

$$\begin{aligned} J_1 &= \ln J \\ &= \sum_{j=1}^{N} \ln \mu_{ji}. \end{aligned} \tag{9}$$

Setting the first derivative of J_1 with respect to $I(X_i)$ to zero, we obtain

$$I(X_i) = \frac{\displaystyle\sum_{j=1}^{N} \frac{\mu'_{ji}}{\mu_{ji}} I(X_j)}{\displaystyle\sum_{j=1}^{N} \frac{\mu'_{ji}}{\mu_{ji}}} \tag{10}$$

where $\mu'_{ji} = \partial\mu_{ji}/\partial(d_{ji}^2)$. If we choose the averaging operator as the "and" connective, the objective function becomes

$$J = \frac{1}{N} \sum_{j=1}^{N} \mu_{ji} \tag{11}$$

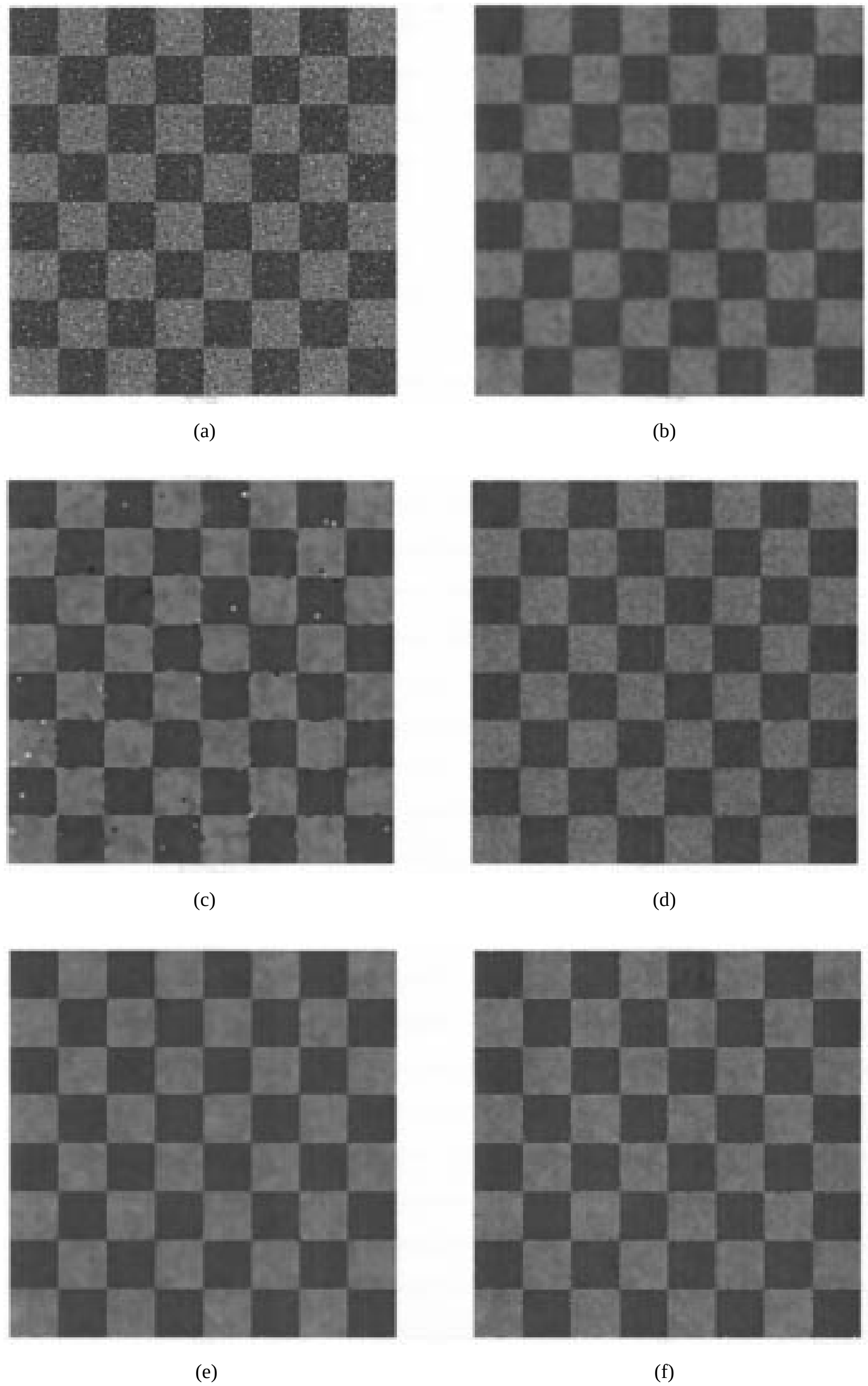


Fig. 7. (a) Noisy chess board image. Results of (b) the median filter, (c) the Saint-Marc filter, and (d) Filtering systems $R1$, (e) $R2$, and (f) $R3$.

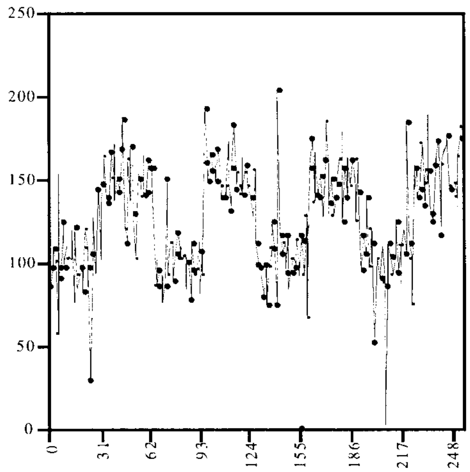
and the corresponding filter is

$$I(X_i) = \frac{\sum_{j=1}^{N} \mu'_{ji} I(X_j)}{\sum_{j=1}^{N} \mu'_{ji}}. \tag{12}$$

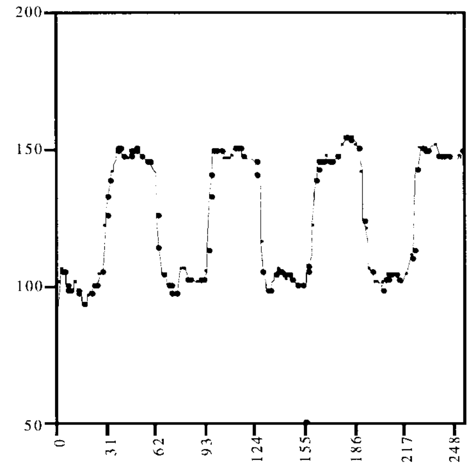
We will discuss the properties of the two filters, (10) and (12), for particular choices of the scale parameter β_i and the membership function μ in the following section.

C. Membership Function

In theory, the membership function can be any decreasing function of d_{ij}^2/β_i. If we assume that the membership function is exponentially decreasing with respect to d_{ij}^2/β_i, and that



(a)



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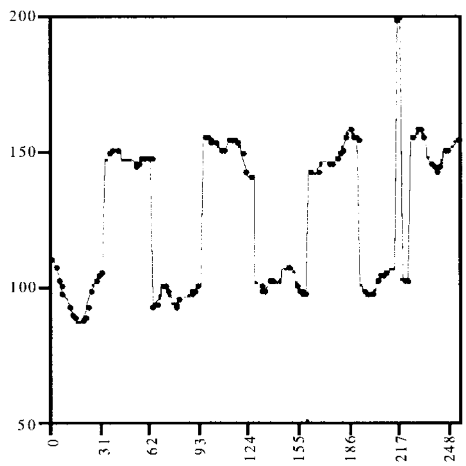

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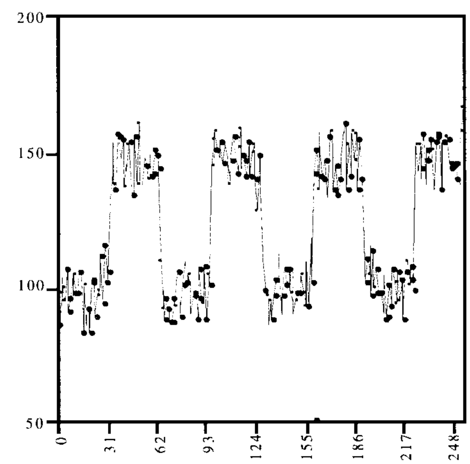
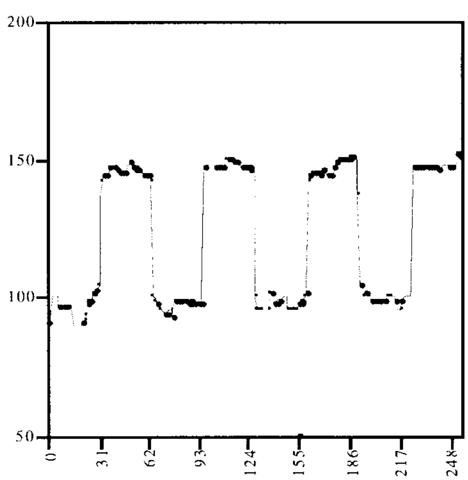
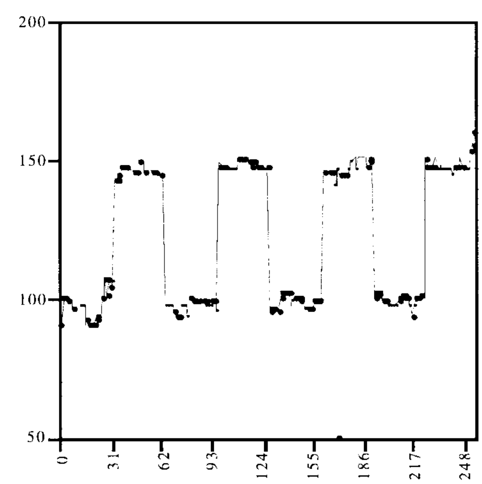


(d)



(e)



(f)

Fig. 8. (a) Intensity profile of a row of the noisy chess board image. Profile of noisy chess board image after applying (b) the median filter, (c) the Saint-Marc filter, (d) filtering system $R1$, (e) $R2$, and (f) $R3$.

the membership value depends on the spatial distances as well as the gray-level differences between the center pixel and its neighbors, the resulting membership function is

$$\mu_{ij} = \mu\left(\frac{d_{ij}^2}{\beta}\right) = \omega_{ij}\,\exp\left(\frac{-d_{ij}^2}{\beta_i}\right) \tag{13}$$

where

$$\omega_{ij} = \exp\left\{\frac{-\|X_i - X_j\|_2^2}{\alpha}\right\}.$$

The new parameter ω_{ij} considers the spatial distances, and does not affect the derivations of the filters given in the previous section. It can be ignored for small window sizes such as 3 $\times$ 3 and 5 $\times$ 5. Thus, we have the following filters from (7), (10), and (12):

$$\text{Filter } A\text{:}\quad I(X_i) = \frac{\sum_{j=1}^{N} \mu_{ij}\, 1 - \left(\frac{d_{ij}^2}{\beta_i}\right) I(X_j)}{\sum_{j=1}^{N} \mu_{ij}\, 1 - \left(\frac{d_{ij}^2}{\beta_i}\right)} \tag{14}$$

$$\text{Filter } B\text{:}\quad I(X_i) = \frac{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j} I(X_j)}{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j}} \tag{15}$$

$$\text{Filter } C\text{:}\quad I(X_i) = \frac{\sum_{j=1}^{N} \frac{\mu_{ji}}{\beta_j} I(X_j)}{\sum_{j=1}^{N} \frac{\mu_{ji}}{\beta_j}}. \tag{16}$$

Since β_i is a scale estimate, it should reflect the variance (dispersion) of the gray-level differences between the center pixel and its neighboring pixels. We can simply take the mean of d_{ij}^2 in the neighborhood as β_i. That is

$$\beta_i = \frac{1}{N-1} \sum_{j=1}^{N} d_{ij}^2. \tag{17}$$

It may appear that this value for the scale parameter is sensitive to outliers (impulse noise). However, since the maximum value of d_{ij}^2 is limited, and the weights [i.e., multipliers of $I(X_j)$ in (14)–(16)] associated with impulse noise pixels tend to be small, the overall outputs from filters are not affected much by impulse noise. The scale estimate for β_i can be also defined in a robust way. For example, initial estimate can be calculated [4] as

$$s = 1.4926\left(1 + \frac{5}{N-1}\right)\sqrt{\operatorname*{med}_j d_{ij}^2},$$

The final scale estimate is then obtained as

$$\beta_i = \sqrt{\frac{\sum_{j=1}^{N} w_j d_{ij}^2}{\sum_{j=1}^{N} w_j - 1}} \tag{18}$$

where $w_j = 1$ if $|d_{ij}/s| \le 2.5$, otherwise zero. This estimate has a 50% breakdown point [4]. We now briefly discuss the Filters A, B, and C to provide more insight into their operations.

Filter A: We illustrate the weight in Filter A for various values of scale β in Fig. 1. The weight is zero at $d_{ij}^2 = \beta_i$, and the weight has the maximum 1 and the minimum $-e^{-2}$ at $d_{ij}^2 = 0$ and $d_{ij}^2 = 2\beta_i$, respectively. The weight approaches zero as d_{ij}^2 goes to infinity. Therefore, Filter A does not consider neighboring pixels with very large distances or distances near β_i. If d_{ij}^2 is smaller than β_i, then the weight for a pixel X_j will be positive. If d_{ij}^2 is larger than β_i, the weight for a pixel X_j will be negative. If the intensity $I(X_j)$ of a pixel X_j is more similar to $I(X_i)$ than those of other neighboring pixels, then d_{ij}^2 will be smaller than β_i, and therefore, the weight for the pixel X_j will be larger than those for other neighboring pixels. The meaning of the negative weight can be understood more clearly in the context of an edge region. Suppose that a center pixel lies on an object boundary. The d_{ij}^2 values will be smaller than β_i for pixels on one side of the edge, and will be larger than β_i for pixels on the other side. The neighbors from the other side, therefore, will have negative weights. Hence, after applying the Filter A, the updated $I(X_i)$ will be further away from the pixels that have negative weights. Thus, the negative weight has a sharpening effect, and consequently we have an adaptive filter with smoothing and sharpening properties.

Filters B and C: If a pixel X_j has a large β_j associated with it, then pixel X_j is less reliable because its neighboring pixels are significantly different from it. Filter B assigns a small weight to a neighboring pixel that has a large β_j and Filter C assigns a large weight to a similar neighboring pixel that has a low β_j. Filters B and C come from different interpretations of the "and" connective. If we select the multiplication operator, we maximize all the degrees to which its neighbors represent the new value for the center pixel. Suppose we have an impulse noise pixel. Then, we should not consider the degree to which its neighbors represent the noise pixel. A more important criterion in this case is the reliability of its neighbors. Therefore, we can conclude that Filter B is suitable for removing impulse noise. If we build the objective function using the averaging operator, then we are not forced to blindly maximize every membership value. In this case, we would be considering both the similarity and the reliability of its neighbors. Thus, the resulting filter, i.e., Filter C, assigns a small weight to a very similar neighbor if it has a large β_j, and vice versa. Therefore, we can apply Filter C to smooth a region while preserving edges.

To illustrate the solo performance of Filters A, B, and C, we present the results of applying each of the filters to a

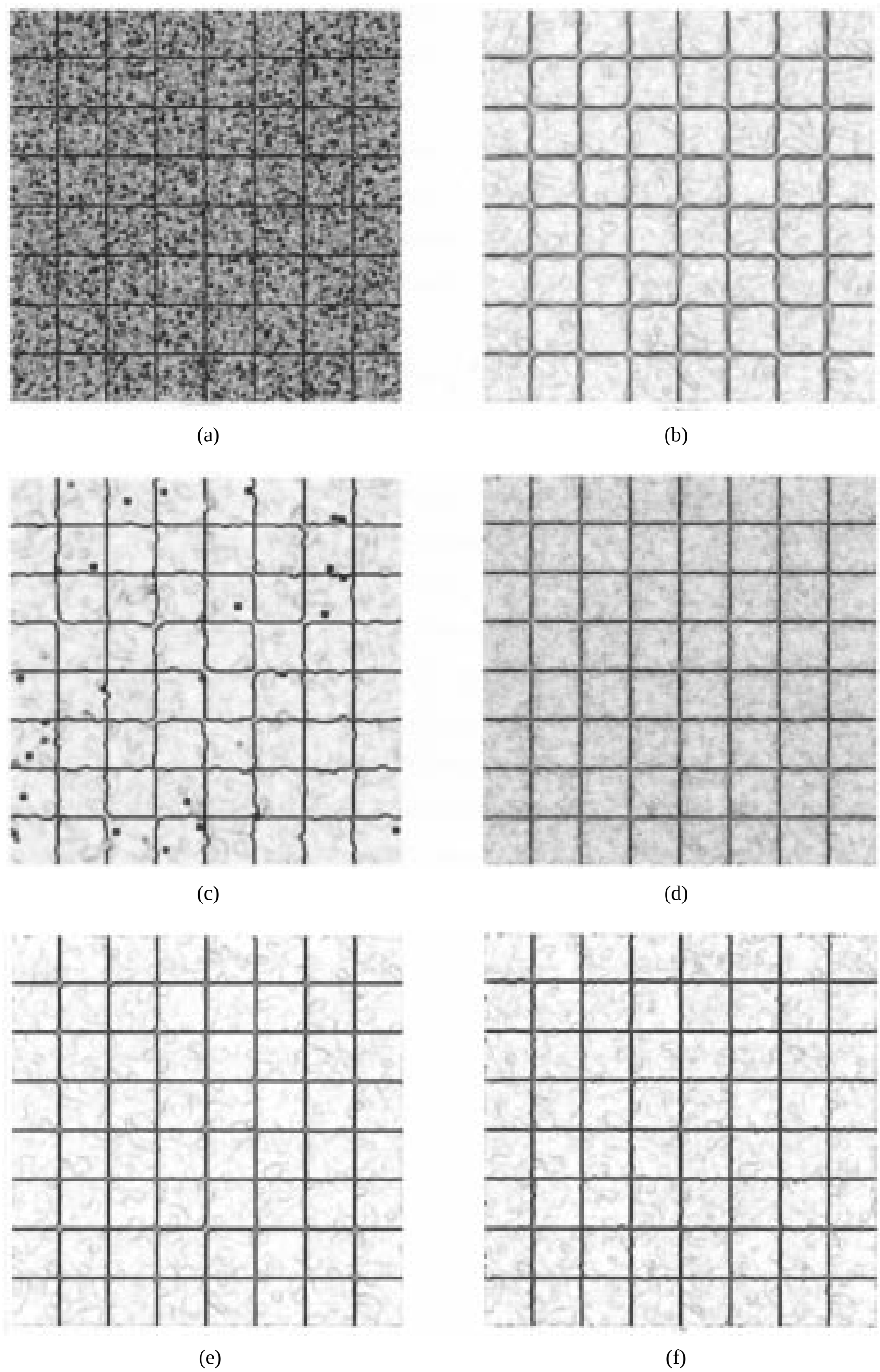
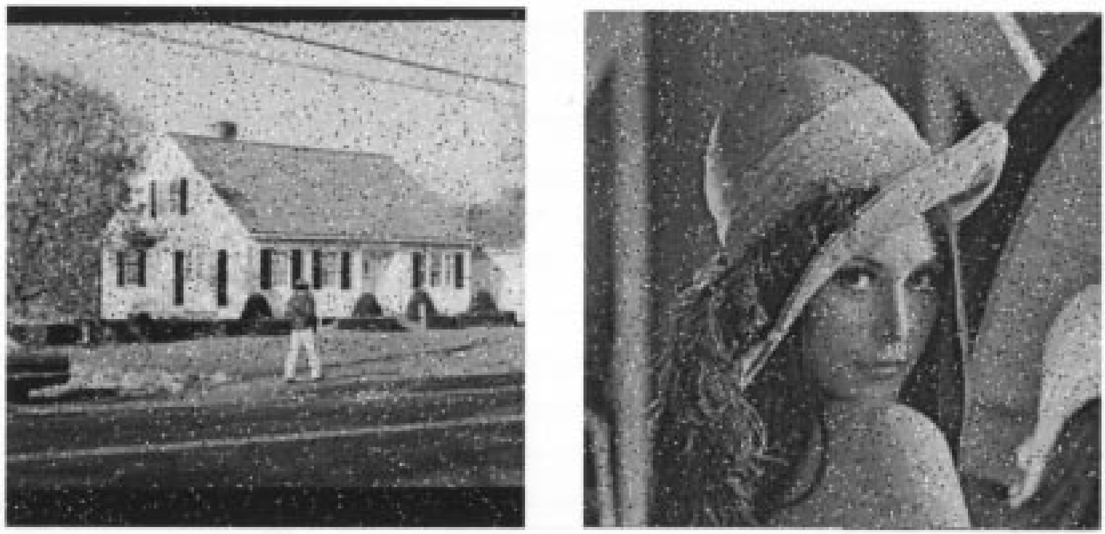


Fig. 9. (a) Sobel edge image of a noisy chess board image, and edge images obtained after applying (b) the median filter, (c) the Saint-Marc filter, and (d) filtering systems $R1$, (e) $R2$, and (f) $R3$.

noisy version of the Lena image obtained by adding a mixture consisting of 5% impulse noise and zero-mean Gaussian noise with $\sigma = 5$. We used a window of size 5×5 for all filters. Fig. 4(a) shows the noisy Lena image and Fig. 4(b)–(d) shows images obtained by applying Filters A, B, and C, respectively. We also present the root mean square (RMS) errors between the original Lena image and the filtered images in Table I. Filter A tends to sharpen edge regions and preserve image details. However, it also increases noise. Filter B smoothes out impulse noise effectively but also blurs image details and edge regions. Filter C smoothes out the Gaussian noise while preserving edges. However, some impulse noise still remains and image details are somewhat blurred. It is clear from this experiment that none of the three filters is effective everywhere in the image, and the type of filter that we need to apply at each pixel depends on the local information.

V. Fuzzy Rule-Based Filtering

From the example presented in the previous section, we see that we need to decide the conditions under which the



(a) (b)

Fig. 10. Noisy images of (a) the outdoor scene and (b) the Lena image contaminated by 10% impulse noise.

three filters should be applied. As described in Section III, these conditions constitute the antecedent clauses in the fuzzy if–then rules and the filters constitute the consequent clauses in the fuzzy rules.

Gradient magnitude has been used as an input for adaptive smoothing algorithms [1], [17], [33], [34]. However, it is known to be sensitive to noise. To overcome this problem, one can smooth the image before computing the gradient. However, this leads to a "catch-22" situation: One needs a reliable edge detection algorithm to smooth the image without destroying edges, but one needs a reliable smoothing algorithm to avoid detecting spurious edges in an image. To circumvent this situation, we avoid the use of gradients, and use the following heuristic rules.

If a given center pixel is an impulse noise pixel, then the gray level of this pixel will be significantly different from its neighbors. This means that the degree to which its neighboring pixels are compatible with this noise pixel will be small. Equivalently, the total compatibility given by

$$TC = \frac{1}{K} \sum_{j=1,\, j\neq i}^{N} \mu_{ji},$$

where $K = \sum_{j=1,\, j\neq i}^{N} \omega_{ji}$, is *small* if the center pixel is an impulse noise pixel. Therefore, we would like to apply Filter B in this case. If a given center pixel lies in a smooth region where the variation in gray level is not significant, then there is a high possibility that its neighboring pixels are compatible with this center pixel. Thus, we may apply Filter C if TC is *large*. If a given center pixel lies on an object boundary, the compatibility with its neighboring pixels will be *medium* when compared with the aforementioned cases. In this case, we can apply Filter A. However, it is almost impossible to define the precise ranges for the labels *small*, *medium*, and *large*. We would rather make soft decisions about each condition, aggregate the decisions made, and make a final decision based on the aggregation. Since each filter is applied to a certain degree, the final result of a fuzzy rule-based system is smoother when compared to a method that makes crisp decisions at each pixel and applies only one particular filter.

The rule construction for image enhancement largely depends on the application domain, since the desired type of enhancement varies from application to application. Out of this consideration, we can build the various types of rule-based filtering systems for image enhancement based on the particular application. We now present three sample filtering systems.

Filtering System $R1$: If we want to remove impulse noise, the following rules involving Filter B can be used:

Rule 1: If total compatibility is $small$

$$\text{then } y_1 = \frac{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j} I(X_j)}{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j}}.$$

Rule 2: Else, $y_2 = I(X_i)$.

Filtering System $R2$: For an edge-preserving noise filter, we can use the following rules involving Filters B and C.

Rule 1: If total compatibility is $small$

$$\text{then } y_1 = \frac{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j} I(X_j)}{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j}}.$$

Rule 2: Else, $y_2 = \dfrac{\sum_{j=1}^{N} \frac{\mu_{ji}}{\beta_j} I(X_j)}{\sum_{j=1}^{N} \frac{\mu_{ji}}{\beta_j}}.$

(a) (b) (c) (d)

Fig. 11. Enhanced versions of the noisy images in Fig. 10: (a), (c) median filter; (b), (d) filtering system $R1$.

Filtering System $R3$: If we need noise filtering that preserves fine details as a preprocessing step for a general computer vision system, we can use the following rules involving Filters A, B, and C.

Rule 1: If total compatibility is $small$

$$\text{then } y_1 = \frac{\sum_{j=1}^{N} \mu_{ij} \frac{1-d_{ij}^2}{\beta_i} I(X_j)}{\sum_{j=1}^{N} \mu_{ij} \frac{1-d_{ij}^2}{\beta_i}}.$$

Rule 2: If total compatibility is $medium$

$$\text{then } y_2 = \frac{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j} I(X_j)}{\sum_{j=1,\, j\neq i}^{N} \frac{1}{\beta_j}}.$$

$$\text{Rule 3:} \quad \text{Else, } y_3 = \frac{\sum_{j=1}^{N} \frac{\mu_{ji}}{\beta_j} I(X_j)}{\sum_{j=1}^{N} \frac{\mu_{ji}}{\beta_j}}.$$

In the above filtering systems, the membership function for the linguistic labels *small*, *medium*, and *large* are defined on the domain of total compatibility TC (see Fig. 2). The final value of each of the filtering systems is computed using (3).

In order to demonstrate the advantage of fuzzy rule-based filtering methods over individual Filters A, B, and C, we show the results of applying filtering systems $R1$, $R2$, and $R3$ to the same image [Fig. 4(a)] that was used in Section IV. Fig. 5 shows the filtered images and Table II shows the RMS errors. It is clear that filtering systems $R1$, $R2$, and $R3$ outperform Filters A, B, and C. In Fig. 5(d), we also present the result of applying a crisp version of filtering system $R3$ to the image in Fig. 4(a). This result was obtained by applying only one of

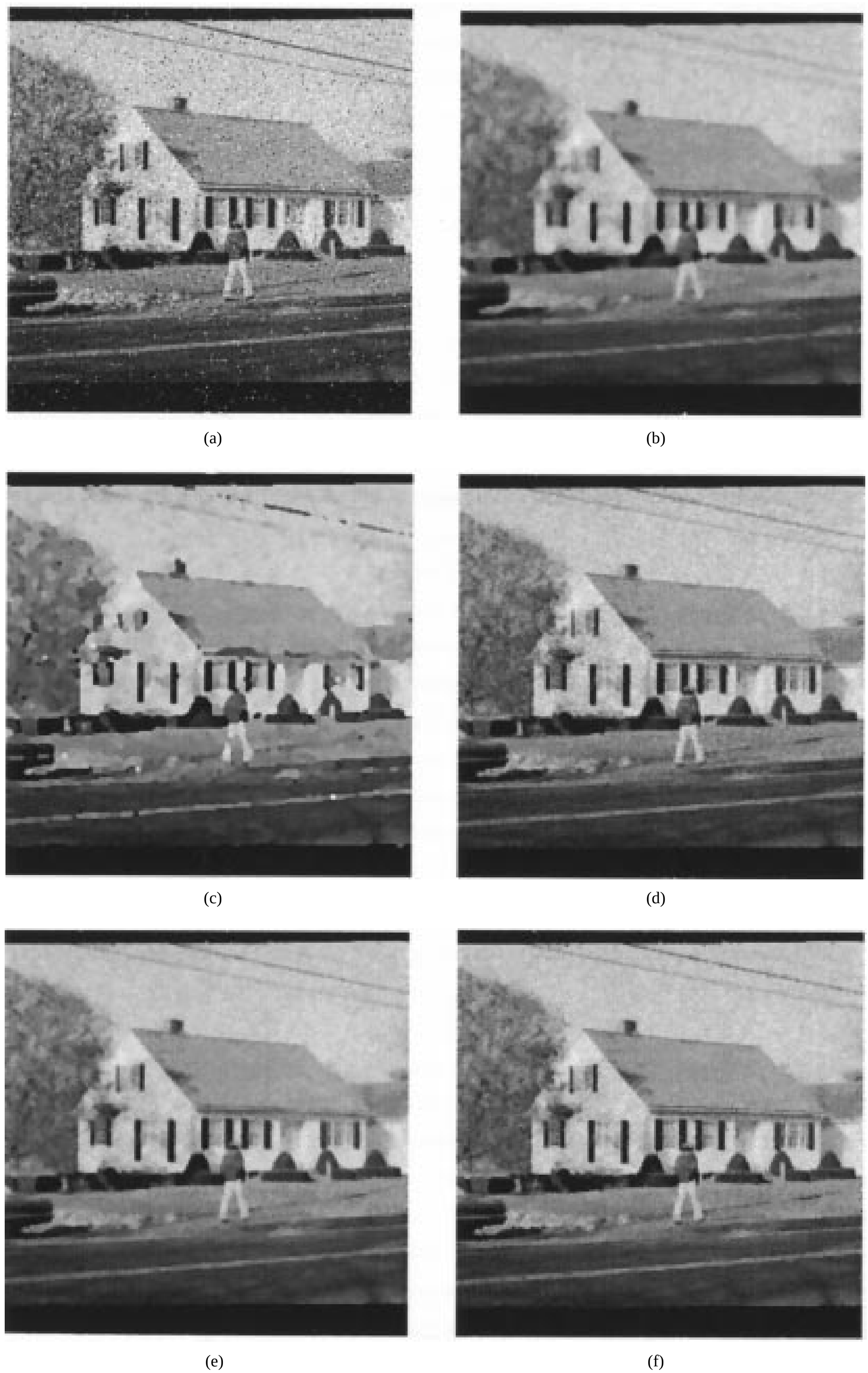


Fig. 12. (a) Outdoor scene contaminated by mixture noise. Results of (b) the median filter, (c) Saint-Marc filter, and filtering systems (d) $R1$, (e) $R2$, and (f) $R3$.

(a) (b) (c) (d) (e) (f)

Fig. 13. (a) Lena image contaminated by mixture noise. Results of (b) the median filter, (c) Saint-Marc filter, and filtering systems (d)$R1$, (e) $R2$, and (f) $R3$.

TABLE III
RMS ERRORS FOR DIFFERENT FILTERING METHODS ON NOISY CHESS BOARD IMAGES

Filtering method	median (5×5)	Saint-Marc (3×3)	$R1$ (5×5)	$R2$ (5×5)	$R3$ (5×5)
σ = 10, 1% black and white impulse noise	4.44	3.74	5.32	3.05	3.52
σ = 15, 5% Gaussian impulse noise	6.42	7.94	7.39	4.32	5.38
cpu time	1	0.26	1.67	2.67	3.35

TABLE IV
RMS ERRORS FOR DIFFERENT FILTERING METHODS ON REAL IMAGES WITH VARYING LEVELS OF GAUSSIAN IMPULSE NOISE

Image/ Filtering method	Lena image 5%	Lena image 10%	Outdoor Scene 5%	Outdoor Scene 10%
median (5×5)	11.25	11.43	14.84	14.97
$R1$ (5×5)	3.46	6.82	6.92	8.39

TABLE V
RMS ERRORS FOR DIFFERENT FILTERING METHODS ON REAL IMAGES WITH VARYING LEVELS OF MIXTURE NOISE

Image/ Filtering method	Lena image $\sigma = 5$	Lena image $\sigma = 10$	Lena image $\sigma = 15$	Outdoor scene $\sigma = 5$	Outdoor scene $\sigma = 10$	Outdoor scene $\sigma = 15$
median (5×5)	11.49	11.78	12.36	12.9	15.18	15.58
Saint-Marc (3×3)	15.36	15.9	16.45	18.08	18.16	18.9
$R1$ (5×5)	7.28	9.12	11.45	8.6	10.31	12.26
$R2$ (5×5)	8.22	9.0	10.11	9.87	10.71	11.68
$R3$ (5×5)	7.46	8.71	10.74	9.14	10.38	11.98

the Filters A, B, and C at each location, whichever had the largest c_k (degree of satisfaction of the antecedent clause). The RMS error of filtering system $R3$ is smaller than that of the crisp version of $R3$ (see Table II). In order to illustrate how the rule-based filters combine Filters A, B, and C in different conditions, in Fig. 6 we present the weights c_k (degrees of satisfaction of the antecedent clause) c_1, c_2, and c_3 for Filters A, B, and C in filtering system $R3$. The weights are scaled by 255 and, therefore, a brighter value indicates a larger weight. Filter A has a small weight in most of the image except in edge regions. Filter B has a large weight in locations contaminated by impulse noise. Filter C has a large weight in most regions and a small weight in edge regions.

VI. EXAMPLES

In this section, we present examples of image enhancement based on the proposed fuzzy rule-based filters. Three of the images used in the experiments are shown in Fig. 3. Fig. 3(a) is a synthesized 256 $\times$ 256 image of a chess board; Fig. 3(b) is a 256 $\times$ 256 image of an outdoor scene; and Fig. 3(c) is a 256 $\times$ 256 Lena image. All images were digitized into 256 gray levels. The contaminated images were constructed by adding zero-mean Gaussian noise and a small percentage of impulse noise to the original images. In order to make the results comparable with other robust filters, the impulse noise was generated by two different methods: replacing a certain percent of the pixels by i) black or white pixels with equal probability as in [32] and [36]; and ii) Gaussian impulse noise, as in [33]. The three different filtering systems presented in Section V, i.e., the filtering system for removing impulse noise ($R1$), the filtering system for edge preserving smoothing ($R2$), and the filtering system for detail preserving smoothing ($R3$), were applied to these contaminated images. Throughout the experiment, we used a window of size 5 $\times$ 5 for Filters A, B, and C. Using the RMS error, we quantitatively compare the results with those obtained using the median and the Saint-Marc adaptive filtering algorithm [10]. We also present a qualitative comparison.

A. Synthesized Chess Board Image

As in [36], a synthesized chess board image was used to study the characteristics of the proposed filtering systems. The gray level of the darker regions is 100, and the gray level of the brighter regions is 150. Several levels of noise were used in comparing the performance of the filtering system. Fig. 7(a) shows one of the noisy images obtained by adding zero-mean Gaussian noise ($\sigma = 15$) and 5% of Gaussian impulse noise ($\sigma = 100$). The RMS error of each filtering algorithm on images with different types and levels of noise is shown in Table III. This quantitative comparison shows that filtering system $R2$ smooths out noise better than the other filters. The CPU time of each filter normalized by that of the median filter is also shown in Table III.

The outputs from the filters are shown in Fig. 7(b)–(f). The outputs show that all the filters except the Saint-Marc filter remove impulse noise effectively. As mentioned in Section II, methods based on anisotropic diffusion cannot handle impulse noise well. We can also see that the median filter and filtering system $R1$ do not attenuate nonimpulse noise effectively. To visualize the performance of the filters in edge regions, we show the intensity profiles of filtered images in Fig. 8 and the Sobel edge images in Fig. 9. The Saint-Marc filter and filtering systems $R2$ and $R3$ enhance step edges. The filtering system $R1$ preserves edges but it does not suppress nonimpulse noise effectively. The median filter slightly blurs the edges. Filtering system $R2$ preserves the location of the edges very well, as seen by the straightness of the Sobel edges. However, filtering system $R3$ seems to preserve the corners better. Overall, filtering system $R2$ outperforms the other filters quantitatively and qualitatively in this experiment.

B. Lena Image and Outdoor Scene

The experiment in the previous section shows that filtering system $R2$ is more applicable to a piecewise continuous image if we are to smooth out noise while preserving edges. However, in images such as the outdoor scene and the Lena image, we need to preserve not only edges but also fine details while smoothing out noise. We first show the performance of filtering system $R1$ when only impulse noise is present. Then, we present the experiment results in the case of the mixture noise model.

To demonstrate how filtering system $R1$ can effectively remove impulse noise while preserving fine details, we added different percentages (5%, 10%) of Gaussian impulse noise ($\sigma = 100$) to the Lena image and the outdoor scene (see Fig. 10). The results are compared to those of the median filter. The RMS error from each case is shown in Table IV, and the outputs are shown in Fig. 11. Filtering system $R1$ removes impulse noise effectively and preserves fine details well. The results indicate that filtering system $R1$ outperforms the median filter quantitatively and qualitatively.

The images were then contaminated by the mixture of zero-mean Gaussian noise ($\sigma = 5, 10, 15$) and 5% Gaussian impulse noise ($\sigma = 100$). Figs. 12(a) and 13(a) show the noisy images with $\sigma = 10$. The RMS errors and the outputs are shown in Table V, and Fig. 12(b)–(f) and 13(b)–(f), respectively. The results indicate that filtering system $R1$ compares favorably with the others when the value of σ is small. Filtering systems $R2$ and $R3$ compare favorably with the others when the value of σ is medium and large, respectively. This result can be interpreted as follows. Filtering system $R1$ modifies the pixel value only when it is significantly different from its neighbors. Therefore, when the amount of contamination is relatively small, $R1$ is better. When an image is severely contaminated, filtering system $R2$, which consists of the two smoothing Filters, B and C, is more effective. Filtering system $R3$ is favorable when the noise level is unknown, since it outperforms other filters when the amount of contamination is moderate and works well overall.

VII. CONCLUSION

In this paper, a robust filtering method based on fuzzy logic was proposed. We have derived three different filters based on the weighted (fuzzy) LS method. Each filter is applicable in a different situation. To evaluate the situation at a particular pixel, we used the total compatibility of the neighboring pixels with the center pixel. The total compatibility is the mean value of the membership degrees to which the neighboring pixels represent a center pixel in a given window. Each filter and the corresponding condition in which it is to be applied constitute a fuzzy if–then rule in the proposed fuzzy rule-based system for image enhancement. Through experiments, we showed that this fuzzy rule-based filtering approach can provide a flexible image enhancement paradigm.

We constructed three different fuzzy rule-based filtering systems, and demonstrated the performance of these filtering using synthesized and real images. Filtering system $R1$ is most effective for removing impulse noise while preserving fine details, and outperforms the others when the noise level is small. Filtering system $R2$ outperforms the others with regard to the RMS criterion when images are severely contaminated and can be used for smoothing noise while preserving edges. Filtering system $R3$ is somewhat less effective for noise attenuation. However, in real images, it is preferable to the others when the noise level cannot be predetermined. The CPU time of the filtering systems compares very favorably with that of the median filter.

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