

# Make Up

Nolan Pofi

Prob 2:

10.12

y	$y^2 \pmod{11}$
1	1
2	4
3	9
4	5
5	3
6	3
7	5
8	9
9	4
10	1

$QR(11) = \{1,3,4,5,9\}$

x	$x^3 + x + 6 \pmod{11}$	Y
0	6	none
1	8	none
2	5	(4,7)
3	3	(5,6)
4	8	none
5	4	(2,9)
6	8	none
7	4	(2,9)
8	9	none
9	7	none
10	4	(2,9)

Points are  $\{(2,4), (2,7), (3,5), (3,6), (5,2), (5,9), (7,2), (7,9), (8,3), (8,8), (10,2), (10,9)\}$

10.13

-P = (5,-8) = (5,9)

-Q = (3,-0) = (3,0)

-R = (0,-6) = (0,11)

10.14

$$2G = (2,7) + (2,7)$$

$$m = \frac{3(2)^2+1}{2*7} = \frac{13}{14} \mod 11 = \frac{2}{3} \mod 11 = 2 * 4 \mod 11 = 8$$

$$x = 8^2 - 2 - 2 \mod 11 = 5$$

$$y = 8(2 - 5) - 7 \mod 11 = 2$$

$$2G = (5,2)$$

$$3G = (5,2) + (2,7)$$

$$m = \frac{7-2}{2-5} = \frac{5}{-3} \mod 11 = \frac{5}{8} \mod 11 = 5 * 7 \mod 11 = 2$$

$$x = 2^2 - 5 - 2 \mod 11 = 8$$

$$y = 2(5 - 8) - 2 \mod 11 = 3$$

$$3G = (8,3)$$

$$4G = (8,3) + (2,7)$$

$$m = \frac{7-3}{2-8} = \frac{4}{-6} \mod 11 = \frac{4}{5} \mod 11 = 4 * 9 \mod 11 = 3$$

$$x = 3^2 - 8 - 2 \mod 11 = 10$$

$$y = 3(8 - 10) - 3 \mod 11 = 2$$

$$4G = (10,2)$$

$$5G = (10,2) + (2,7)$$

$$m = \frac{7-2}{2-10} = \frac{5}{-8} \mod 11 = \frac{5}{3} \mod 11 = 5 * 4 \mod 11 = 9$$

$$x = 9^2 - 10 - 2 \mod 11 = 3$$

$$y = 9(10 - 3) - 2 \mod 11 = 6$$

$$5G = (3,6)$$

$$6G = (3,6) + (2,7)$$

$$m = \frac{7-6}{2-3} = \frac{1}{-1} \mod 11 = \frac{1}{10} \mod 11 = 1 * 10 \mod 11 = 10$$

$$x = 10^2 - 3 - 2 \mod 11 = 7$$

$$y = 10(3 - 7) - 6 \mod 11 = 9$$

$$6G = (7,9)$$

$$7G = (7,9) + (2,7)$$

$$m = \frac{7-9}{2-7} = \frac{-2}{-5} \mod 11 = \frac{9}{6} \mod 11 = 9 * 2 \mod 11 = 7$$

$$x = 7^2 - 7 - 2 \mod 11 = 7$$

$$y = 7(7 - 7) - 9 \mod 11 = 2$$

$$7G = (7,2)$$

$$8G = (7,2) + (2,7)$$

$$m = \frac{7-2}{2-7} = \frac{5}{-5} \mod 11 = \frac{5}{6} \mod 11 = 2 * 2 \mod 11 = 10$$

$$x = 10^2 - 3 - 2 \mod 11 = 3$$

$$y = 10(7 - 3) - 2 \mod 11 = 5$$

$$8G = (3,5)$$

$$\begin{aligned}
9G &= (3,5) + (2,7) \\
m &= \frac{7-5}{2-3} = \frac{2}{-1} \pmod{11} = \frac{5}{10} \pmod{11} = 2 * 10 \pmod{11} = 9 \\
x &= 9^2 - 3 - 2 \pmod{11} = 10 \\
y &= 9(3 - 10) - 5 \pmod{11} = 9 \\
9G &= (10,9)
\end{aligned}$$

$$\begin{aligned}
10G &= (10,9) + (2,7) \\
m &= \frac{7-9}{2-10} = \frac{-2}{-8} \pmod{11} = \frac{9}{3} \pmod{11} = 9 * 4 \pmod{11} = 3 \\
x &= 3^2 - 10 - 2 \pmod{11} = 8 \\
y &= 3(10 - 8) - 9 \pmod{11} = 8 \\
10G &= (8,8)
\end{aligned}$$

$$\begin{aligned}
11G &= (8,8) + (2,7) \\
m &= \frac{7-8}{2-8} = \frac{-1}{-6} \pmod{11} = \frac{10}{5} \pmod{11} = 10 * 9 \pmod{11} = 2 \\
x &= 2^2 - 8 - 2 \pmod{11} = 5 \\
y &= 2(8 - 5) - 8 \pmod{11} = 9 \\
11G &= (5,9)
\end{aligned}$$

$$\begin{aligned}
12G &= (5,9) + (2,7) \\
m &= \frac{7-9}{2-5} = \frac{-2}{-3} \pmod{11} = \frac{9}{8} \pmod{11} = 9 * 7 \pmod{11} = 8 \\
x &= 8^2 - 5 - 2 \pmod{11} = 2 \\
y &= 8(5 - 2) - 9 \pmod{11} = 4 \\
12G &= (2,4)
\end{aligned}$$

$$\begin{aligned}
13G &= (2,4) + (2,7) \\
m &= \frac{7-4}{2-2} = \frac{3}{0} \pmod{11} = 3/0 \\
x &= 2 + 0 \pmod{11} = 2 \\
y &= 4 + 3 \pmod{11} = 7 \\
13G &= (2,7)
\end{aligned}$$

10.15

calculations done using values from prev problem since same curve.

a).

$$\begin{aligned}
P_B &= n_B \times G \\
P_B &= 7G = (7,2)
\end{aligned}$$

b).

$$\begin{aligned}
C_m &= \{kG, P_m + kP_B\} \\
&= \{3G, (10,9) + 3(7,2)\} \\
&= \{(8,3), (10,9) + (3,5)\} \\
&= \{(8,3), (10,2)\}
\end{aligned}$$

$$\begin{aligned}
P_m &= \{kP_B - n_B(kG, P_m)\} \\
&= (10,2) - (3,5) \\
&= (10,2) + (3,-5)
\end{aligned}$$

$$\begin{aligned}
&= (10,2) + (3,6) \\
&= (10,9)
\end{aligned}$$

prob 3  
 31531, 485827, 15485863 are all prime  
 520482 is not prime and factors to (3 ,173494)