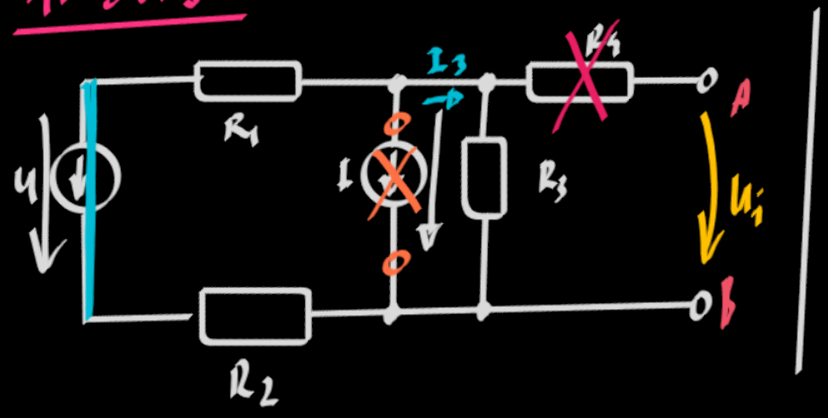
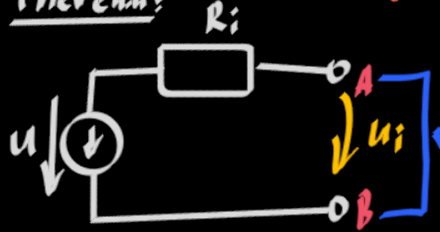
Poslední cvika ZEOA

1. SUS:



Uiu: $U_{iu} = U \frac{R_3}{R_1+R_2+R_3}$

Thevenin:



UiI:

$R_{12} = R_1 + R_2$

$I_3 = -I \frac{R_{12}}{R_{12}+R_3}$

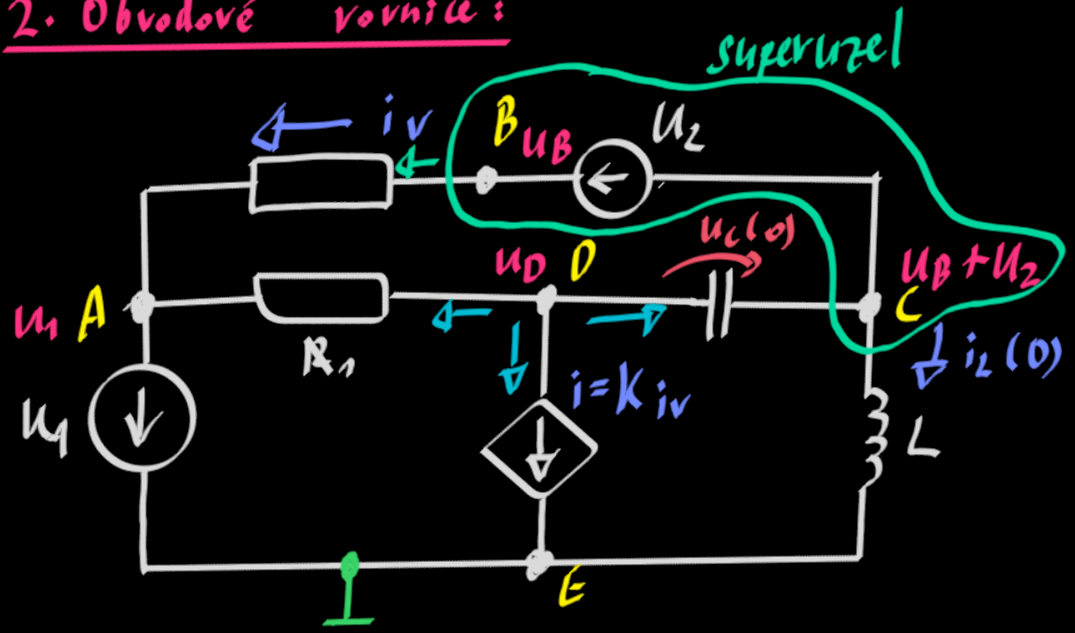
$U_{iI} = I_3 \cdot R_3 = -I \frac{R_{12}}{R_{12}+R_3} \cdot R_3$

Superpozice: $U_i = U_{iI} + U_{iU}$

$$U_i = U \frac{R_3}{R_1+R_2+R_3} - I \cdot \frac{R_1+R_2}{R_1+R_2+R_3} \cdot R_3$$

$R_i = \frac{U_i}{I_k}$ = nebo ignoruju zdroje = $\frac{(R_1+R_2)R_3}{R_1+R_2+R_3} + R_4$

2. Obvodové rovnice:



Součástky: 7

Uzly: 5 ✓

MUN: 2 rovnice

5 − 1referenc − 2zdroje napětí = 2

NSP: 2 = 7 − 4 dvojice uzlů − 1 zdroj proudu = 2

MUN: $i_L = \frac{1}{L}\int_0^t u_L(\tau)\,d\tau + i_L(0)$

$i_C = C \frac{du_C}{dt}$

MUN:

D: $\frac{u_D - u_1}{R_1} + i + C\,\frac{d(u_D - u_B + u_2)}{dt} = 0$

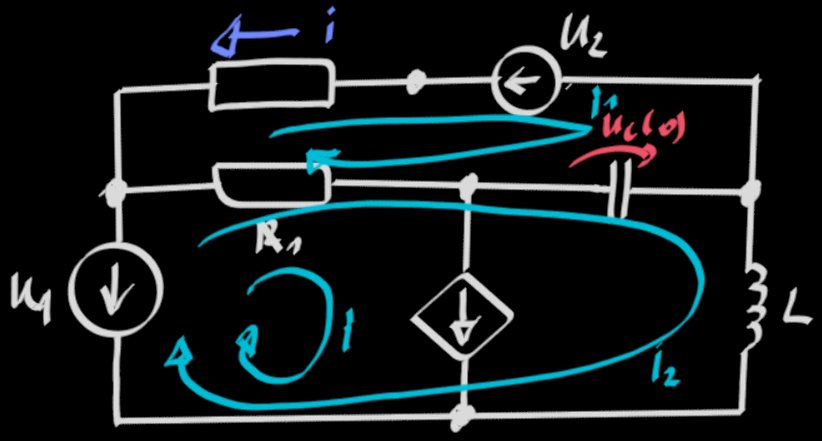
B+C: $\frac{u_B - u_1}{R_2} + C\,\frac{d(u_B + u_2 - u_D)}{dt} + \frac{1}{L}\int_0^t (u_B + u_2) - 0\; d\tau + i_L(0) = 0$

$i = k \cdot i_V = k \cdot \frac{u_B - u_1}{R_2}$

MSP:

$u_C = \frac{1}{C}\int_0^t i_C(\tau)\, d\tau + u_C(0)$

$u_L = L\,\frac{di_L}{dt}$

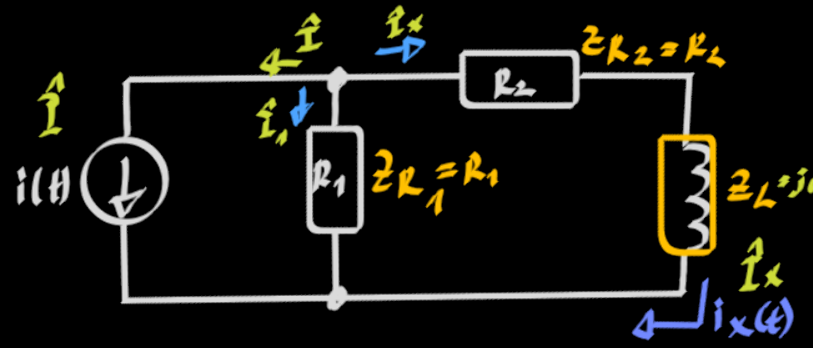


i_1: $R_2 \cdot i_1 - u_2 + \frac{1}{C}\int_0^t i_1 - i_2\; d\tau - u_C(0) + R_1 \cdot (i_1 - i_2 - i) = 0$

i_2: $-u_1 + R_1\,(i_2 + i - i_1) + \frac{1}{C}\int_0^t (i_2 - i_1)\, d\tau + u_C(0) + L\,\frac{di_2}{dt} = 0$

$i = k \cdot i_V = \underline{\underline{k \cdot (-i_1)}}$

3. HUS:



$i(t) = 10 \sin(100\pi t + \frac{\pi}{3})$ mA $\Rightarrow \hat{I} = 10\, e^{\pi/3\, j}$ mA

$R_1 = R_2 = 1\,k\Omega$

$L = 1{,}59$ H

$Z_L = jwL = j(100\pi) \cdot 1{,}59$

$$\hat{I}_x = \frac{Z_{R_1}}{Z_{R_1} + Z_{R_2} + Z_L} \cdot -\hat{I}(t) = \frac{1000}{2000 + jwL} \cdot (-10) e^{\frac{\pi}{3} j} = \frac{1000}{2000 + j(100\pi)1{,}59} \cdot (-10) e^{j\pi/3} =$$

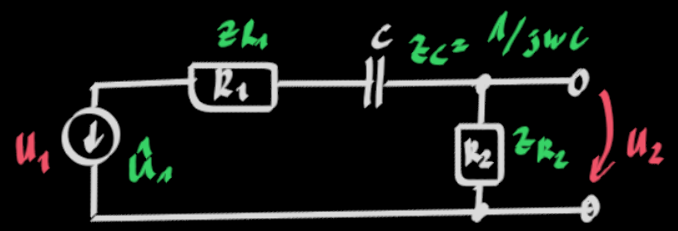
$$= \frac{1000}{2000 + 500j} \cdot -10 e^{j\pi/3} = (-10) e^{j\pi/3} \frac{1000}{\sqrt{2000^2 + 500^2}\, e^{j \arctg\left(\frac{500}{2000}\right)}} =$$

$= \dots = -0{,}00048\, e^{j\, 0{,}8}$ A $\Rightarrow (-1) = e^{\pm j\pi} \Rightarrow 0{,}00048\, e^{-j\, 2{,}34}$ A

2 zdroje?	Superpozice
Thevenin?	Obecný zdroj $\hat{U}_i$ a obecná impedance $\hat{Z}_i$

4. Frek. charakteristika:

Bodeho apmx:



$R_1 = 9\,k\Omega$
$R_2 = 1\,k\Omega$
$C = 1\,\mu F$

$$\hat{U}_2 = \hat{U}_1 \cdot \frac{Z_{R_2}}{Z_{R_2} + Z_{R_1} + Z_C} = \hat{U}_1 \cdot \frac{R_2}{R_1 + R_2 + \frac{1}{j\omega C}}$$

přenos

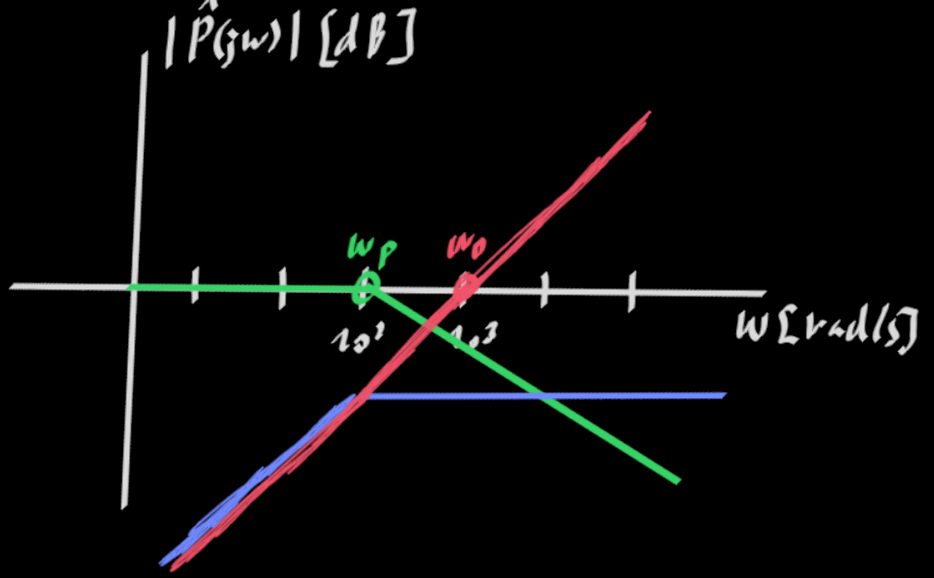
$$\hat{P}(j\omega) = \frac{\hat{U}_2}{\hat{U}_1} = \dots = \frac{R_2}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{j\omega C R_2}{j\omega C (R_1 + R_2) + 1} =$$

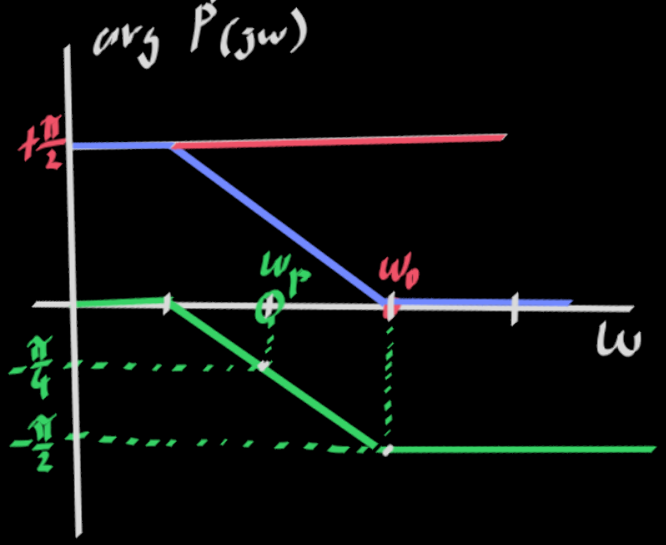
vadí čárky !

$$= \frac{j\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_p}}$$

$$\omega_0 = \frac{1}{CR_2} = 1000\ rad/s$$

$$\omega_p = \frac{1}{C(R_1 + R_2)} = 100\ rad/s$$



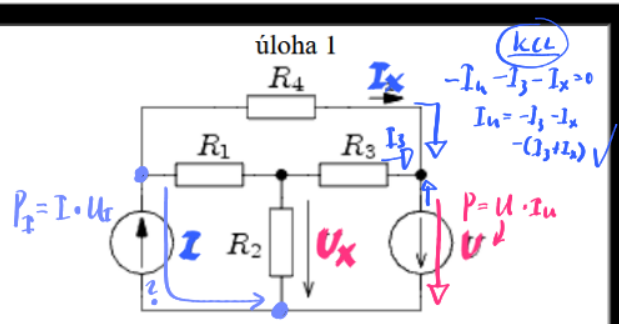


Příprava na zkoušku z moodle → SUS

Obvody podle obrázků v tabulce jsou napájeny zdrojem stejnosměrného napětí U a zdrojem stejnosměrného proudu I.

A. Určete napětí U_X a výkon P dodávaný obvodu zdrojem napětí U.
B. Určete proud I_X a výkon P dodávaný obvodu zdrojem napětí U.
C. Určete napětí U_X a výkon P dodávaný obvodu zdrojem proudu I.
D. Určete proud I_X a výkon P dodávaný obvodu zdrojem proudu I.

úloha 1

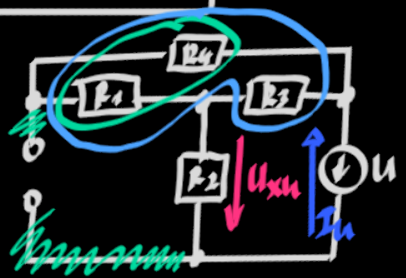


A. $R_1 = 1\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, $R_4 = 4\,\text{k}\Omega$, $U = 5\,\text{V}$, $I = 10\,\text{mA}$

KCL: $-I_4 - I_3 - I_x = 0$, $I_u = -I_3 - I_x$, $-(I_3 + I_x)$ ✓

$U_x = U_{xI} + U_{xu}$

U_{xu}:

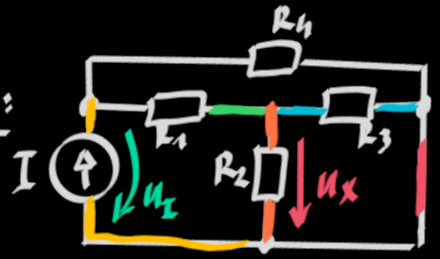


$R_1 + R_4 = R_{14} = 5000\,\Omega$

$R_{134} = R_{14} \parallel R_3 = \frac{(R_1 + R_4) R_3}{R_1 + R_4 + R_3} = 1875\,\Omega$

Dělič napětí: $U_{xu} = U \frac{R_2}{R_{134} + R_2} = 5 \frac{2000}{3875} = 2{,}581\,\text{V}$

U_{xI}:



$R_{23} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = 1200\,\Omega$

$R_{123} = R_1 + R_{23} = 2200\,\Omega$

Dělič proudu: $I_{R_{123}} = I \frac{R_4}{R_4 + R_{123}} = 6{,}45\,\text{mA} = 0{,}00645\,\text{A}$

$U_{R_{123}} = R_{123} \cdot I_{123} = 14{,}2\,\text{V}$ / $U_{R_{23}} = U_{R_{123}} \frac{R_{23}}{R_1 + R_{23}} = 7{,}745\,\text{V}$ → $U_x = 10{,}326\,\text{V}$ ✓ (A)

- $P_I = U_I \cdot I$, $U_I = U_{R_1} + U_x = \underline{17{,}255\,V}$

  $I_1 = I - I_x = 6{,}935\,mA$, $U_{R_1} = R_1 \cdot I_1 = \underline{6{,}935\,V}$

$\Big\} P_I = I \cdot U_I = 0{,}01 \cdot 17{,}255 = \underline{0{,}1725\,W}$ ✓

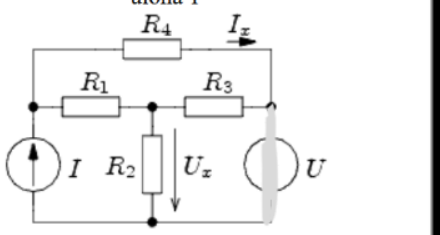
- $P_U = I_U \cdot U$, $I_U = -(I_x + I_3)$, $I_3 = \frac{U_x - U}{R_3} = \frac{10{,}32 - 5}{3000} = \underline{1{,}77\,mA}$

$I_U = -(3{,}065 + 1{,}77) = \underline{-4{,}835\,mA}$ $\Big\} P_U = I_U \cdot U = -0{,}004835 \cdot 5 = \underline{-0{,}024\,W}$

P_I a P_U

B

úloha 1



A. $R_1 = 1\,k\Omega$, $R_2 = 2\,k\Omega$, $R_3 = 3\,k\Omega$, $R_4 = 4\,k\Omega$, $U = 5\,V$, $I = 10\,mA$

$$I_x = I_{xU} + I_{xI}$$

$R_{23} = R_2 \parallel R_3 = \underline{1200\,\Omega}$

$R_{123} = \underline{2200\,\Omega}$

I_{xI}: $I_{R_4} = I \frac{R_{123}}{R_4 + R_{123}} = 0{,}01 \frac{2200}{6200} = \underline{3{,}548\,mA}$

I_{xU}:

$R_{14} = 5\,k\Omega$

$R_{134} = 1875\,\Omega$

$U_{R_{134}} = U \frac{R_{134}}{R_2 + R_{134}} = \underline{2{,}419\,V}$

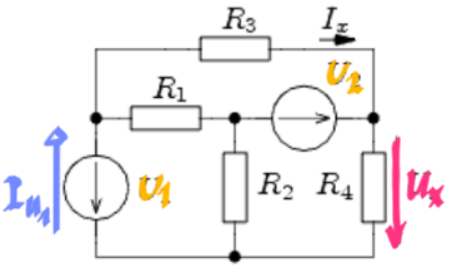
$I_{R_{134}} = \frac{U_{134}}{R_{134}} = \underline{1{,}29\,mA}$

$I_{R_4} = -I_{R_{134}} \frac{R_3}{R_{14} + R_3} = -4{,}938 \cdot 10^{-4}\,A =$

$= \underline{-0{,}4838\,mA}$

$I_x = I_{xI} + I_{xU} = 3{,}548 - 0{,}4838 = \underline{3{,}0642\,mA}$

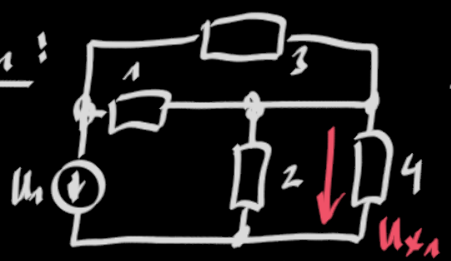
úloha 1



$R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $R_3 = 3\,\mathrm{k\Omega}$, $R_4 = 4\,\mathrm{k\Omega}$,
$U_1 = 5\,\mathrm{V}$, $U_2 = 10\,\mathrm{V}$

$U_x = U_{x1} + U_{x2}$

U_{x1}:



$R_{13} = R_1 \| R_3 = 750\,\Omega$

$R_{24} = R_2 \| R_4 = 1333\,\Omega$

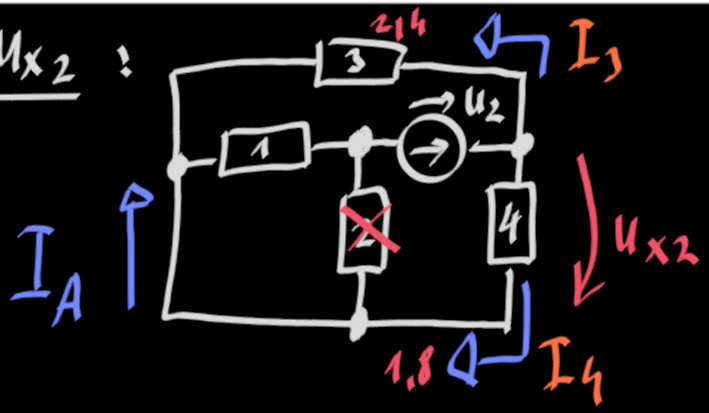
$U_{R24} = U_1 \dfrac{R_{24}}{R_{24}+R_{13}} = 3{,}2\,\mathrm{V}$

$I_1 = \dfrac{U_1}{R_{tot}} = 2{,}4\,\mathrm{mA}$

$U_x = -4\,\mathrm{V}$ ✓

E. Obvody podle obrázků v tabulce jsou napájeny zdroji stejnosměrného napětí U_1 a U_2. Určete napětí U_x a výkon dodávaný obvodu zdrojem U_1.

U_{x2}:



$R_{12} = R_1 \| R_2 = 666\,\Omega$

$R_{34} = R_3 \| R_4 = 1714\,\Omega$

$U_{x2} = U_2 \dfrac{R_{34}}{R_{34}+R_{12}} = -7{,}2\,\mathrm{V}$

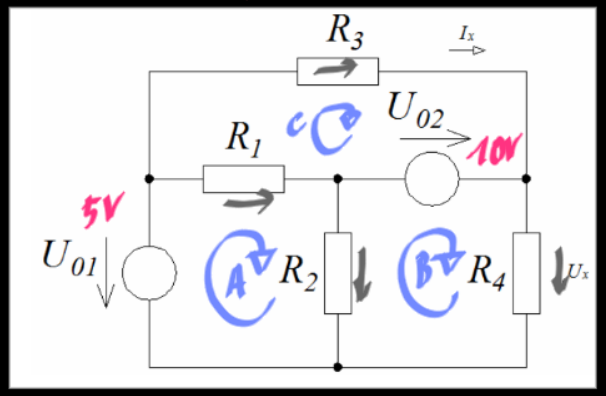
$I_{u2} = \dfrac{U_2}{R_{tot}} = 4{,}2\,\mathrm{mA}$

$P_{u1} = U_1 \cdot I_{u1}$, $I_{u1} = I_A + I_1$, $I_1 = 2{,}4\,\mathrm{mA}$, $I_A = ?$, $I_4 = I \dfrac{R_3}{R_3+R_4} = 1{,}8\,\mathrm{mA}$

$I_3 = 2{,}4\,\mathrm{mA}$

$I_{u1} = 2\,\mathrm{mA}$ má vyjít	2 0

BAPO1 way:



MSP: $r = s - z_i = 3 - 0 = 3$

I_A: $-U_{01} + U_1 + U_2 = 0$
I_B: $-U_2 + U_{02} + U_x = 0$
I_C: $U_3 - U_{02} - U_1 = 0$

I_A: $-U_{01} + R_1(I_A - I_C) + R_2(I_A - I_B) = 0$
I_B: $-R_2(I_A - I_B) + U_{02} + R_4 I_B = 0$
I_C: $R_3 I_C - U_{02} - R_1(I_A - I_C) = 0$

$I_A(R_1 + R_2) - I_B(R_2) - I_C(R_1) = U_{01}$

$-I_A(R_2) + I_B(R_2 + R_4) = -U_{02}$

$-I_A(R_1) + I_C(R_1 + R_3) = U_{02}$

$3I_A - 2I_B - 1I_C = 0{,}005$

$-2I_A + 6I_B = -0{,}01 \Rightarrow I_B = (2I_A - 0{,}01)/6 \longrightarrow = (0{,}004 - 0{,}010)/6 = \underline{-0{,}001\,A}$

$-1I_A + 4I_C = 0{,}01 \Rightarrow I_C = (I_A + 0{,}01)/4 \longrightarrow = (0{,}002 + 0{,}01)/4 = \underline{0{,}003\,1}$

$$3I_A - \frac{(2I_A - 0{,}01)}{3} - \frac{(I_A + 0{,}01)}{4} = 0{,}005 \quad |\cdot 12$$

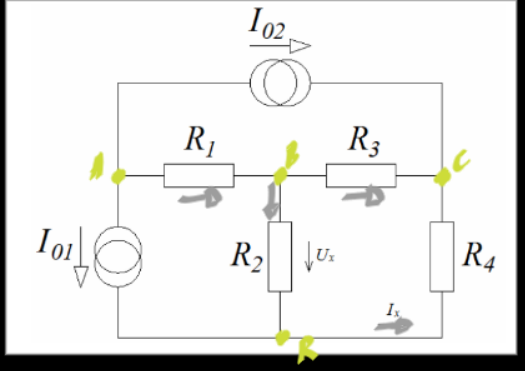
$36I_A - 8I_A + 0{,}04 - 3I_A - 0{,}03 = 0{,}06$

$25I_A = 0{,}050 \quad |:25$

$I_A = \underline{\underline{0{,}002\,A}}$

$I_x = I_C = \underline{3\,mA}$

$U_x = R_4 \cdot I_B = 4000 \cdot (-0{,}001) = \underline{\underline{-4\,V}}$



HUN: $r = u - 2u - 1 = 4 - 0 - 1 = 3$

A: $I_{01} + I_{02} + I_1 = 0$

B: $-I_1 + I_2 + I_3 = 0$

C: $-I_{02} - I_3 - I_x = 0$

A: $I_{01} + I_{02} + \frac{(U_A - U_B)}{R_1} = 0$

B: $-\frac{(U_A - U_B)}{R_1} + \frac{U_B}{R_2} + \frac{(U_B - U_C)}{R_3} = 0$

C: $-I_{02} - \frac{(U_B - U_C)}{R_3} - \left(-\frac{U_C}{R_4}\right) = 0$

A: $U_A - U_B = -R_1(I_{01} + I_{02})$

B: $U_A\left(-\frac{1}{R_1}\right) + U_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + U_C\left(-\frac{1}{R_3}\right) = 0$

C: $U_B\left(-\frac{1}{R_3}\right) + U_C\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = I_{02}$

A: $U_A - U_B = -15 \Rightarrow U_A = -15 + U_B$

B: $U_A\left(-\frac{1}{1000}\right) + U_B\left(\frac{11}{6000}\right) + U_C\left(-\frac{1}{3000}\right) = 0 \mid \cdot 6000$

$-6U_A + 11U_B - 2U_C = 0$

C: $U_B\left(-\frac{1}{3000}\right) + U_C\left(\frac{4+3}{12\,000}\right) = 0{,}010 \mid \cdot 12\,000$

$-4U_B + 7U_C = 120 \Rightarrow U_C = \frac{1}{7}(4U_B + 120)$

$-6(-15 + U_B) + 11U_B - \frac{2}{7}(4U_B + 120) = 0$

$90 - 6U_B + 11U_B - \frac{8}{7}U_B - \frac{240}{7} = 0$

$5U_B - \frac{8}{7}U_B = -\frac{630}{7} + \frac{240}{7}$

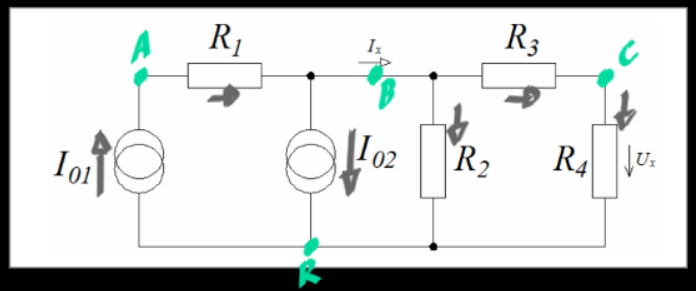
$\frac{27}{7}U_B = -\frac{390}{7}$

$U_B = -14{,}4\ V$

$U_A = -0{,}556\ V$

$U_C = 8{,}889\ V$

✓



MUN: $r = u - z_4 - 1 = 4 - 0 - 1 = 3$

A: $-I_{01} + G_1 U_1 = 0$

B: $-G_1 U_1 + I_{02} + G_2 U_2 + G_3 U_3 = 0$

C: $-G_3 U_3 + G_4 U_x = 0$

A: $-I_{01} + G_1 (U_A - U_B) = 0$

B: $-G_1 (U_A - U_B) + I_{02} + G_2 (U_B) + G_3 (U_B - U_C) = -I_{02}$

C: $-G_3 (U_B - U_C) + G_4 (U_C) = 0$

A: $U_A - U_B = I_{01} \cdot R_1$

B: $U_A (-G_1) + U_B (G_1 + G_2 + G_3) + U_C (-G_3) = -I_{02} \mid \cdot 1000$

C: $U_B (-G_3) + U_C (G_3 + G_4) = 0 \mid \cdot 1000$

A: $U_A - U_B = 5 \Rightarrow U_A = U_B + 5$

B: $-U_A + \frac{11}{6} U_B - \frac{1}{3} U_C = -I_{02}$

C: $-\frac{1}{3} U_B + \frac{7}{12} U_C = 0 \Rightarrow U_C = \left(\frac{1}{3} U_B\right) \cdot \frac{12}{7} = \frac{4}{7} U_B$

$-U_B - 5 + \frac{11}{6} U_B - \frac{4}{21} U_B = -I_{02}$

$U_B \left(\frac{-126 + 231 - 24}{126}\right) = -5$

$\frac{81}{126} U_B = -5$

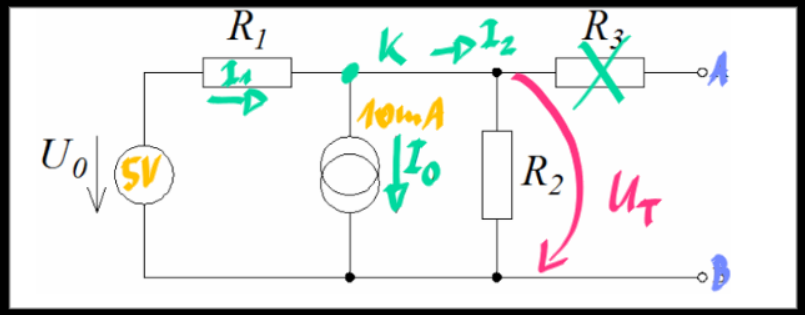
$U_B = -7{,}778\ V$

$U_x = U_C = -4{,}444\ V$

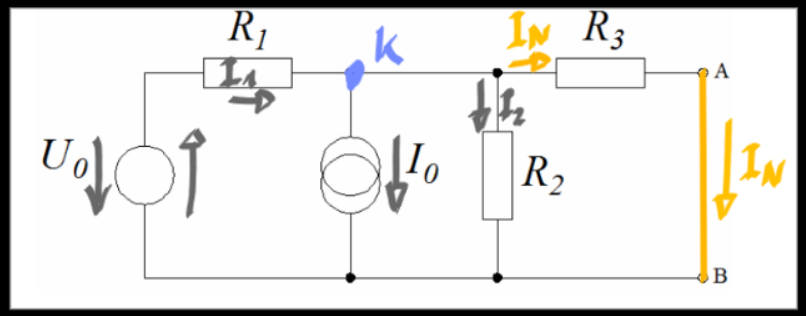
$U_A = -2{,}778\ V$

$I_x = \frac{U_A - U_B}{R_1} = \ldots = 5\ mA$

Věty o náhradních zdrojích



Norton: I_N = zkrat mezi A-B, R_N



Thevenin: U_T = napětí naprázdno mezi A-B

I. KCL: $-I_1 + I_0 + I_2 = 0$

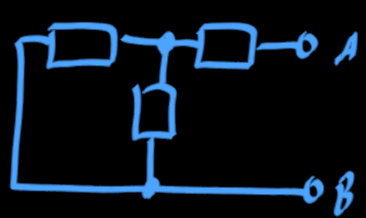
$-G_1(U_0 - U_T) + I_0 + G_2 U_T = 0$

$U_T(G_1 + G_2) = G_1 U_0 - I_0$

$1{,}5\,U_T = 5 - 10$

$U_T = -3{,}333\,V$

R_T:



$R_T = 3667\,\Omega$

$I_3 = I_N$

$-I_1 + I_0 + I_2 + I_3 = 0$

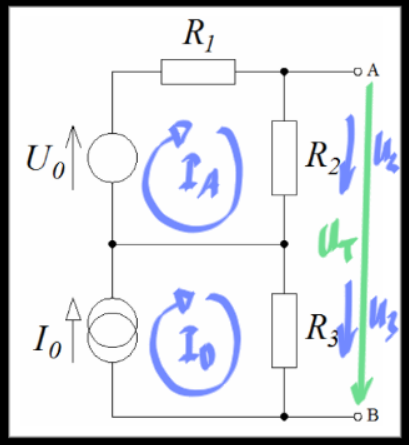
$-G_1(U_0 - U_K) + I_0 + G_2(U_K) + G_3(U_K) = 0$

$U_K(G_1 + G_2 + G_3) = G_1 U_0 - I_0$

$\frac{11}{6} U_K = -5$

$U_K = -2{,}73\,V \Rightarrow I_N = \frac{U_K}{R_3} = -0{,}91\,mA$

R_T



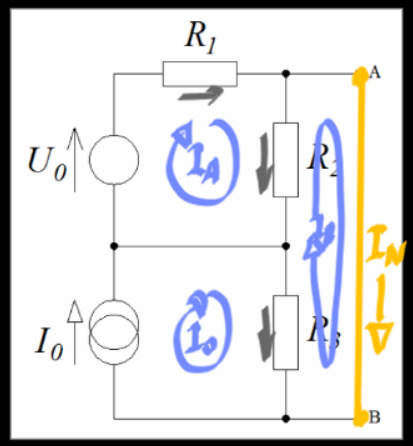
Thevenin: $R_T = (R_1 \parallel R_2) + R_3 = 3667\Omega$

I_A: $U_0 + R_1 \cdot I_A + R_2 I_A = 0$

$I_A = \frac{-U_0}{R_1+R_2}$ $\quad U_2 = -5{,}33V$

$I_A = -1{,}667\,mA$

$U_3 = R_3 \cdot I_0 = 30V$ $\quad U_T = 26{,}67$



Norton:

I_A: $U_0 + U_1 + U_2 = 0$

$U_0 + R_1 I_A + R_2(I_A - I_B) = 0$

I_B: $-U_2 - U_3 = 0$

$-R_2(I_A - I_B) - R_3(I_0 - I_B) = 0$

$I_A(R_1+R_2) - I_B(R_2) = -U_0$

$3I_A - 2I_B = -0{,}005$

$-I_A(R_2) + I_B(R_2+R_3) = R_3 I_0$

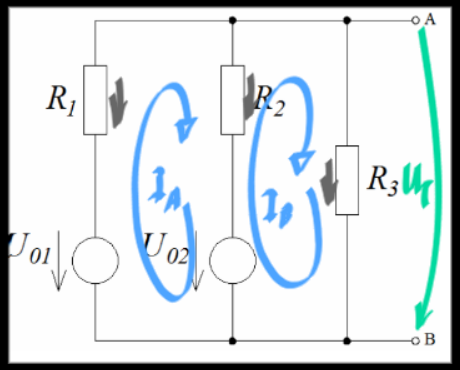
$-2I_A + 5I_B = 0{,}103 \Rightarrow I_B = \frac{1}{5}(0{,}103 + 2I_A)$

$3I_A - \frac{2}{5}(0{,}103 + 2I_A) = -0{,}005$

$3I_A - 0{,}0412 - \frac{4}{5}I_A = -0{,}005$

$\frac{11}{5}I_A = 0{,}0037$

$I_A = 3{,}18\,mA$ / $I_B = \ldots = 7{,}27\,mA$ ✓



Thevenin:

I_A: $-U_{01} - R_1 I_A + R_2 (I_A - I_B) + U_{02} = 0$

$I_A (+R_1 + R_2) + I_B (-R_2) = U_{01} - U_{02} \mid :1000$

$3 I_A - 2 I_B = -0{,}005$ ✓

I_B: $-U_{02} - R_2 (I_A - I_B) + R_3 I_B = 0$

$I_A (-R_2) + I_B (+R_2 + R_3) = U_{02} \mid :1000$

$-2 I_A + 5 I_B = 0{,}01$ ✓

$\rightarrow I_A = -\frac{1}{2}(0{,}01 - 5 I_B)$

$-\frac{3}{2}(0{,}01 - 5 I_B) - 2 I_B = -0{,}005$

$-0{,}015 + \frac{11}{2} I_B = -0{,}005$

$I_B = 1{,}82\,mA$ ✓ $\Rightarrow U_T = R_3 I_B = 5{,}45V$ ✓

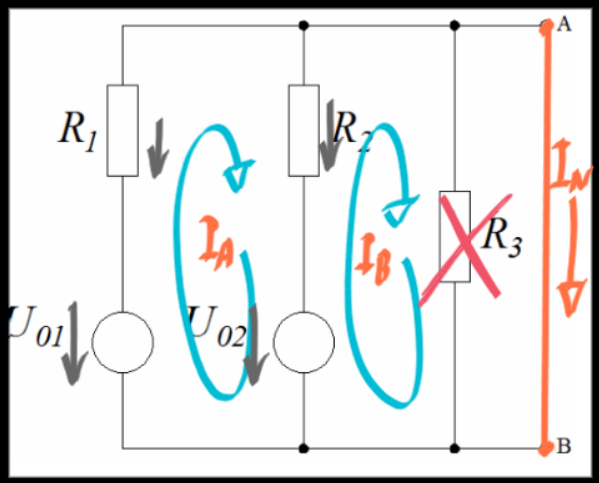
$R_T = R_1 \parallel R_2 \parallel R_3 = 545{,}45\,\Omega$ ✓

Zátěž:

$I_T = \frac{U_T}{R_T + R_4} = 3{,}53\,mA$

$U_4 = R_4 \cdot I_T = 3{,}53V$ ✓

$P_4 = U_4 \cdot I = 0{,}012\,W$



Norton:

$U_1 = -R_1 I_A$

$U_2 = R_2 (I_A - I_B)$

I_A: $-U_{01} - U_1 + U_2 + U_{02} = 0$

$R_1 I_A + R_2 (I_A - I_B) = U_{01} - U_{02}$

$3 I_A - 2 I_B = -0{,}005$

$\rightarrow 3 I_B - 0{,}015 - 2 I_B = -0{,}006$

I_B: $-U_{02} - U_2 = 0$

$-R_2 (I_A - I_B) = U_{02}$

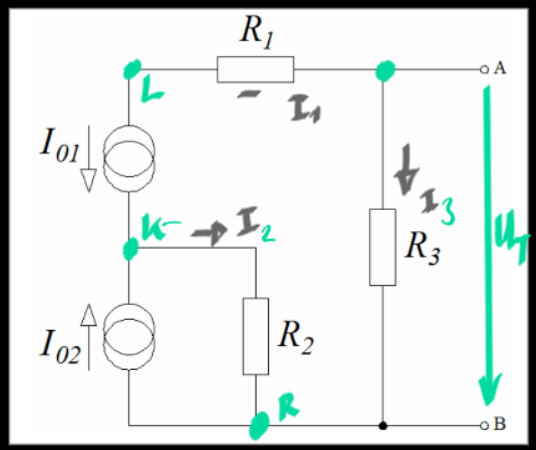
$I_A - I_B = -0{,}005$

$\Rightarrow I_A = I_B - 0{,}005$

$I_B = 0{,}01\,A$ ✓

$I_A = 0{,}005\,A$ ✓

$\Rightarrow I_N = I_B$, $R_N = R_T = 545{,}45\,\Omega$



Thevenin:

$r = d - z_u = 3 - 0 = 3$

MhN:

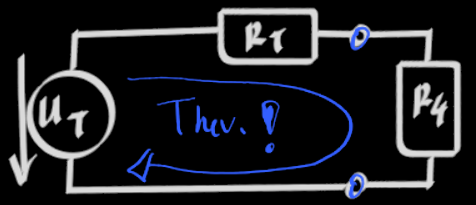
K: $-I_{01} - I_{02} + I_2 = 0 \Rightarrow I_2 = I_{01} + I_{02} = 15\,mA$

L: $I_{01} + I_1 = 0 \Rightarrow I_1 = -I_{01} = -5\,mA$

M: $-I_1 + I_3 = 0 \Rightarrow I_3 = -I_{01} = -5\,mA$

$U_T = U_3 = R_3 \cdot I_3 = -15\,V$

$R_T = R_3 = 3000\,\Omega$



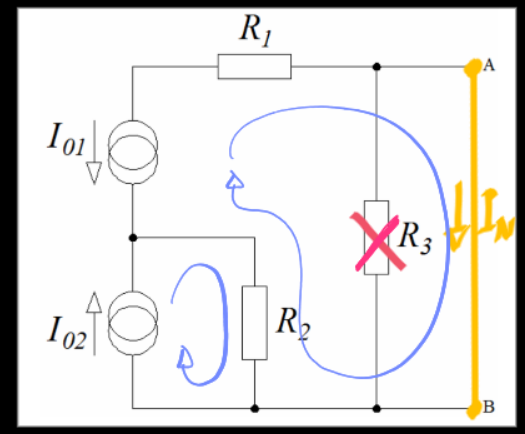
R_4, $U_4 = ?$

$I = \frac{U_T}{R_T + R_4} = -3{,}75\,mA$ (Ohm. z.)

$U_4 = R_4 \cdot I = -3{,}75\,V$

$P_4 = U_4 \cdot I = 14{,}06\,mW$

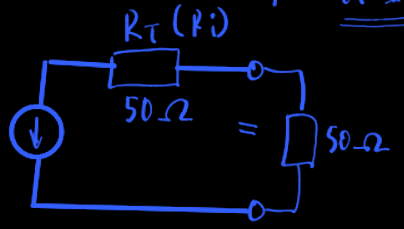
$P = U \cdot I = (R \cdot I) \cdot I = R_4 \frac{U_T}{R_T + R_4}$



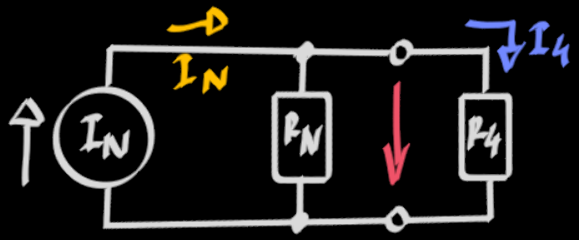
Norton:

$I_N = -I_{01} = -5\,mA$

$R_N = R_T = 3000\,\Omega$



Imp. / Odp. přizpůsobení



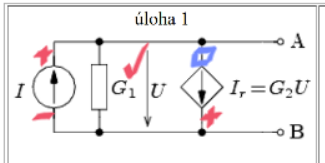
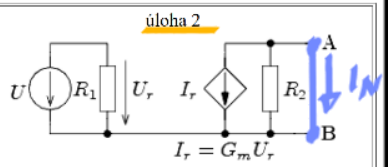
$U = \frac{R_N R_4}{R_N + R_4} \cdot I = -3{,}75\,V$

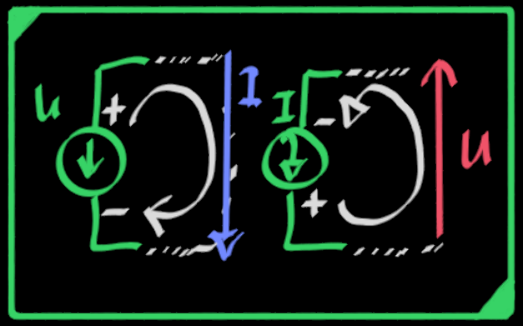
$I_4 = I \frac{R_N}{R_N + R_4} = -3{,}75\,mA$

$P_4 = U I_4 = \dots = 14{,}06\,mW$

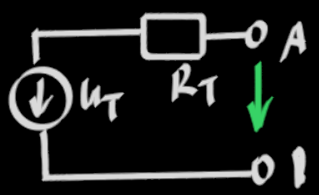
Pro obvody podle obrázků v tabulce nakreslete z hlediska výstupních svorek A a B náhradní zapojení podle

U. Théveninova teorému
V. Nortonova teorému a určete jeho parametry.

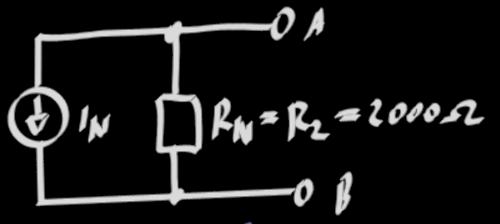
úloha 1	úloha 2
	
U $G_1 = 1\,\mathrm{mS}$, $G_2 = 2\,\mathrm{mS}$, $I = 5\,\mathrm{mA}$	U $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $G_m = 1\,\mathrm{mS}$, $U = 5\,\mathrm{V}$
výsledek	výsledek
V $G_1 = 1\,\mathrm{mS}$, $G_2 = 2\,\mathrm{mS}$, $I = 5\,\mathrm{mA}$	V $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $G_m = 1\,\mathrm{mS}$, $U = 5\,\mathrm{V}$
výsledek	výsledek



Úloha 2: $R_T = R_2 = 2000\,\Omega$



$I_r = G_m u_R = G_m \cdot u$



$I_N = -5\,\mathrm{mA}$

$u_T = ? = R_2 \cdot (-I_N) = -R_2 \cdot G_m \cdot u =$

$= -2 \cdot u = \underline{-10\,\mathrm{V}}$ ✓

Úloha 1:

$R_1 = 1\,\mathrm{k\Omega}$

$u = u_I + u_{Ir}$

$u = 5 + -2u$

$3u = 5$

$u = 5/3\,\mathrm{V}$

$u_I = R_1 \cdot I = \underline{5\,\mathrm{V}}$

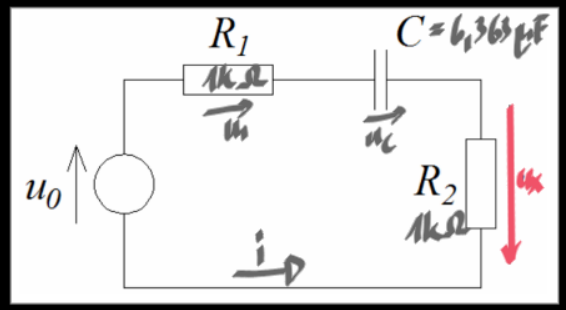
$u_{Ir} = -R_1 \cdot I_r = -R_1 \cdot \frac{u}{R_2} = \frac{u}{1000} = \frac{u}{500}$

$= -2u$

$G_2 = \frac{2}{1000} = \frac{1}{500}$) $\underline{500\,\Omega}$

HUS

Příklad č.12: $u_0(t) = u_m \sin(\omega t + \varphi)$, $\hat{U}_0 = U e^{j\varphi}$



$U_m = 100\,V$, $f = 50\,Hz$, $\varphi = \pi/3$ $\Rightarrow$ $\hat{U}_0 = 100\, e^{j\pi/3}$

$j = e^{j\pi/2}$

$-1 = e^{j\pi}$

$$\hat{I} = \frac{\hat{U}_0}{\hat{Z}} = \frac{\hat{U}_0}{R_1 + R_2 + X_C} = \frac{\hat{U}_0}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{j\omega C \hat{U}_0}{j\omega C (R_1 + R_2) + 1}$$

$$= \frac{e^{j\pi/2} \cdot 100\pi \cdot 6{,}363 \cdot 10^{-6} \cdot 100 e^{j\pi/3}}{j \cdot 100\pi \cdot 6{,}363 \cdot 10^{-6} \cdot 2000 + 1} = \frac{0{,}2\, e^{j(\frac{\pi}{2} + \frac{\pi}{3})}}{4j + 1} = \frac{0{,}2\, e^{j2{,}62}}{4{,}12\, e^{1{,}32}} = 0{,}048\, e^{j1{,}3}\ A$$

$$\hat{U}_x = -(R_2 \cdot \hat{I}) = -10^3 \cdot 0{,}048\, e^{j1{,}3} = 1000\, e^{j\pi} \cdot 0{,}048\, e^{j1{,}3} = 48\, e^{j4{,}44} = 48\, e^{-j1{,}84}\ V$$

$u_x(t) = 48 \sin(314{,}16\, t - 1{,}84)\ V$

Výkony: $S = \frac{\hat{U}_0}{\sqrt{2}} \cdot \frac{\hat{I}}{\sqrt{2}} = \frac{1}{2} \hat{U}_0 \hat{I} = 2{,}4\, e^{-0{,}25j}\ VA$

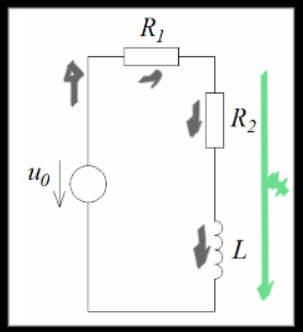
$= 2{,}4 (\cos(-0{,}25) + j \sin(0{,}25)) =$ činný

$= (2{,}35 - 0{,}6j)\ VA$

$P = 2{,}35\,W$

$Q = -0{,}6\,VAr$ jalový

Příklad č. 13



$u_0(t) = 100 \sin(100\pi t + \pi/3) = 100\, e^{j\pi/3}$

$U_{ef} = 100\,V$, $Z = R_1 + R_2 + Z_L$

$\hat{U}_x = Z_{R2L}\,\hat{I} = (X_L + R_2) \cdot \frac{\hat{U}_0}{Z} = (j\omega L + R_2) \cdot \frac{100\, e^{j\pi/3}}{2000 + j\omega L} = (1000 + 500j) \cdot \frac{100\, e^{j\pi/3}}{(2000 + 500j)} =$

$= 1118\, e^{j0{,}464} \cdot \frac{100\, e^{j\pi/3}}{2061\, e^{j0{,}245}} = 54{,}25\, e^{j1{,}27}\,V$ ✓

$u_x(t) = 54{,}25 \sin(100\pi t + 1{,}27)$

$\hat{I}_{ef} = \frac{\hat{U}_{ef}}{Z} = \frac{100\, e^{j\pi/3}}{2061\, e^{j0{,}245}} = 0{,}049\, e^{j0{,}8}\,A$

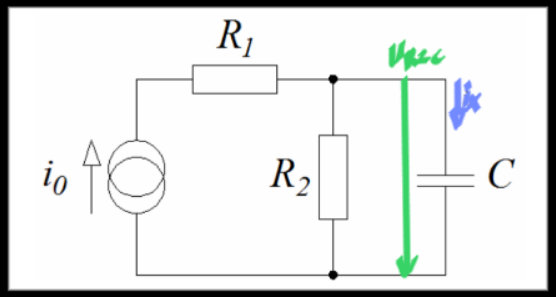
$S = U_{ef} \cdot I_{ef}^{*} = 4{,}9\, e^{j0{,}25}$

$= 4{,}9\,(\cos(0{,}25) + j \sin(0{,}25))$

$= (4{,}7 + 1{,}2j)\,VA$

$P = 4{,}7\,W$, $Q = 1{,}2\,VAr$ ✓

Příklad č. 14



$i_0(t) = I_m \sin(\omega t + \varphi) = 0{,}01 \sin(100\pi + \pi/3)$

$\hat{I}_0 = I\, e^{j\varphi} = 0{,}01\, e^{j\pi/3}$

$\hat{U}_{R2C} = \frac{R_2 X_C}{R_2 + X_C} \cdot I_0 = \frac{\frac{R_2}{j\omega C}}{R_2 + \frac{1}{j\omega C}} \cdot \hat{I}_0 = \frac{R_2}{R_2 j\omega C + 1} \cdot \hat{I}_0 = \frac{1000}{2j + 1} \cdot 0{,}01\, e^{j\pi/3} =$

$= \frac{10\, e^{j\pi/3}}{2{,}24\, e^{j1{,}11}} = 4{,}46\, e^{-j0{,}63}$

$\hat{I}_x = \frac{\hat{U}_{R2C}}{X_C} = \frac{4{,}46\, e^{-j0{,}63}}{1/j\omega C} = e^{j\pi/2} \cdot 0{,}002 \cdot 4{,}46 \cdot e^{-j0{,}63} = 0{,}009\, e^{j1{,}51}\,A$

$\hat{U} = Z \cdot \hat{I} = \left(R_1 + \frac{R_2 X_C}{R_2 + X_C}\right) \cdot \hat{I}_0 = \left(1000 + \frac{1000}{2{,}24\, e^{j1{,}11}}\right) \cdot 0{,}01\, e^{j\pi/3}$

$= (1000 + 446{,}43\, e^{-j1{,}11}) \cdot 0{,}01\, e^{j\pi/3} =$

$= [1000 + 446{,}43\,(0{,}445 - j\,0{,}896)] \cdot 0{,}01\, e^{j\pi/3} =$

$= [1198{,}7 - 400j] \cdot 0{,}01\, e^{j\pi/3} =$

$= 1263{,}7\, e^{-j0{,}32} \cdot 0{,}01\, e^{j\pi/3} = 12{,}64\, e^{j0{,}73}$

$S = \frac{1}{2}\,\hat{U} \cdot I^{*} = \frac{1}{2}(12{,}64\, e^{j0{,}73} \cdot 0{,}01\, e^{-j\pi/3}) =$

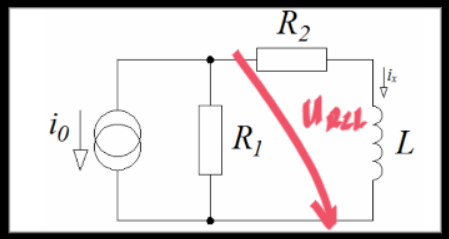
$= \frac{1}{2}[0{,}1264\,[\cos(-0{,}317) + j \sin(-0{,}317)]] =$

$= \frac{1}{2}(0{,}12 - 30{,}04) = 0{,}06 - 0{,}02j\,VA$

$P = 0{,}06\,W$

$Q = -0{,}02\,VAr$

Příklad 2.15



$i_0(t) = 0{,}01 \sin(100\pi t + \pi/3)$

$\hat{I}_0 = 0{,}01\, e^{j\pi/3}$ / $Z_{R2L} = R_2 + X_L = 1000 + j\omega L = 1000 + 500j = \underline{\underline{1118\, e^{j0{,}46}\ \Omega}}$

$Z = \dfrac{R_1 \cdot Z_{R2L}}{R_1 + Z_{R2L}} = \dfrac{1000 \cdot 1118\, e^{j0{,}46}}{1000 + 1000 + 500j} = \dfrac{1\,118\,000\, e^{j0{,}46}}{2061{,}6\, e^{j0{,}24}} = \underline{\underline{542{,}3\, e^{j0{,}22}\ \Omega}}$

$\hat{U} = Z \cdot \hat{I} = 542{,}3\, e^{j0{,}22} \cdot 0{,}01\, e^{j\pi/3} = \underline{\underline{5{,}423\, e^{j1{,}26}\ V}}$

$\hat{I}_x = \dfrac{\hat{U}}{Z_{R2L}} = \dfrac{5{,}423\, e^{j1{,}26}}{1118\, e^{j0{,}46}} = \underline{\underline{4{,}85\, e^{j0{,}8}\ mA}}$

$\underline{\underline{i_x(t) = 4{,}85 \cos(100\pi t + 0{,}8)}}$

$I_{ef} = 50\ mA$

$\hat{U}_{ef} = Z \hat{I}_{ef} = 542{,}3\, e^{j0{,}22} \cdot 0{,}05\, e^{j\pi/3} = \underline{\underline{27{,}11\, e^{j1{,}26}}}$

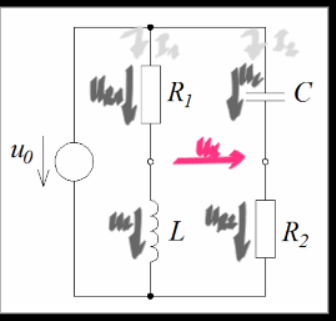
$\hat{S} = \hat{U}_{ef} \cdot I_{ef}^{*} = 27{,}11\, e^{j1{,}26} \cdot 0{,}05\, e^{-j\pi/3} = 1{,}356\, e^{j0{,}21} =$

$= 1{,}326 + 0{,}282j$

$\hookrightarrow Q = 0{,}282\ VAr$

$\hookrightarrow P = 1{,}326\ W$

Příklad č.16



$u_0(t) = 100 \sin(100\pi t + \pi/3)$

$\hat{U}_0 = 100\, e^{j\pi/3}$

$C = 6{,}363\,\mu F,\ L = 3{,}183\,H$

$$\hat{U}_x = \hat{U}_L - \hat{U}_{R_2} = X_L \hat{I}_1 - R_2 \hat{I}_2 = X_L \frac{\hat{U}_0}{R_1 + X_L} - R_2 \frac{\hat{U}_0}{R_1 + X_L} = \hat{U}_0 \left(\frac{X_L}{R_1 + X_L} - \frac{R_2}{R_1 + X_L} \right) =$$

$$= \left(\frac{j\omega L}{1000 + j\omega L} - \frac{1000}{1000 + \frac{1}{j\omega C}} \right) \hat{U}_0 = \left(\frac{1000\, e^{j\pi/2}}{1000 + 1000j} - \frac{1000 j\omega L}{1000 j\omega C + 1} \right) \hat{U}_0 =$$

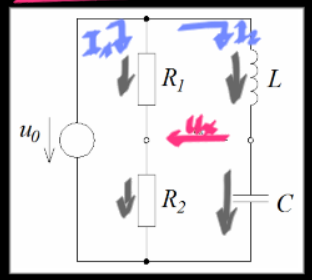
$$= \left(0{,}7071\, e^{j0{,}785} - \frac{2\, e^{j\pi/2}}{2{,}24\, e^{j1{,}11}} \right) \hat{U}_0 = \left(0{,}7071\, e^{j0{,}785} - 0{,}893\, e^{j0{,}461} \right) \hat{U}_0 =$$

$$= (0{,}5 + 0{,}5j - 0{,}8 - 0{,}4j)\, \hat{U}_0 = (-0{,}3 + 0{,}1j)\, \hat{U}_0 = 0{,}3162\, e^{j2{,}82} \cdot 100\, e^{j\pi/3} =$$

$$= 31{,}62\, e^{j3{,}87} = 31{,}62\, e^{-j2{,}42}\ V$$

$u_x(t) = 31{,}62 \sin(100\pi t - 2{,}42)\ V$

Příklad č.17



$u_0(t) = 100 \sin(100\pi t + \pi/3) = 100\, e^{j\pi/3}$

$C = 6{,}363\,\mu F,\ L = 3{,}183\,H$

$\hat{U}_x = \hat{U}_C - \hat{U}_{R_2}$

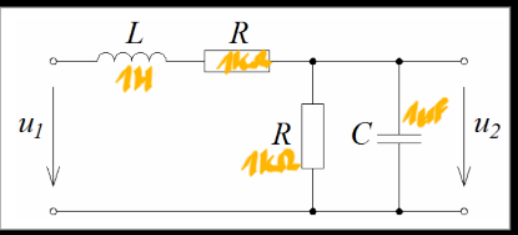
$$\hat{U}_C = X_C \cdot \hat{I}_2 = \frac{1}{j\omega C} \cdot \frac{\hat{U}}{X_L + X_C} = \hat{U} \cdot \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \hat{U} \cdot \frac{1}{-2 + 1} = \hat{U} \cdot (-1)$$

$$\hat{U}_{R_2} = Z_{R_2} \cdot \hat{I}_1 = R_2 \cdot \frac{\hat{U}}{R_1 + R_2} = \hat{U} \frac{1}{2} = \hat{U} \cdot \frac{1}{2}$$

$$\hat{U}_x = \hat{U}_C - \hat{U}_{R_2} = \hat{U} \left(-1 - \frac{1}{2} \right) =$$

$$= 100\, e^{j\pi/3} \cdot 1{,}5\, e^{j\pi} = 150\, e^{j\,1/3\pi}\ V$$

Příklad č. 18



$\hat{U}_1 = U_1 e^{j\varphi_1}$

$\varphi = -\pi/4$

Přenos: $\hat{P} = \hat{U}_2 / \hat{U}_1$

$\omega_x = ?$ $\Rightarrow \varphi$

$$\hat{U}_2 = \hat{U}_1 \frac{Z_{RC}}{Z_{RL} + Z_{RC}} = \hat{U}_1 \frac{\frac{R \cdot X_C}{R + X_C}}{X_L + R + \frac{R X_C}{R + X_C}} = \hat{U}_1 \frac{\frac{\frac{R}{j\omega C}}{R + 1/j\omega C} \cdot \frac{j\omega C}{j\omega C}}{j\omega L + R + \frac{\frac{R}{j\omega C}}{R + 1/j\omega C}} = \hat{U}_1 \frac{\frac{R}{1 + j\omega CR}}{j\omega L + R + \frac{R}{1 + j\omega CR}} = \hat{U}_1 \frac{R}{(j\omega L + R)(1 + j\omega CR) + R} =$$

$$= \hat{U}_1 \frac{R}{j\omega L - \omega^2 RLC + j\omega CR^2 + 2R} =$$

$$= \hat{U}_1 \frac{R}{j\omega (L + CR^2) + R(2 - \omega^2 LC)} \checkmark$$

$$\hat{P} = \frac{\hat{U}_2}{\hat{U}_1}, \text{ pro } \varphi = -\pi/4$$

$$\operatorname{tg} \varphi = \frac{\operatorname{Im}\{\hat{P}\}}{\operatorname{Re}\{\hat{P}\}}$$

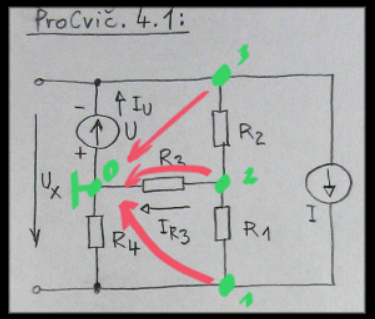
$$\hat{P} = \frac{R}{(2R - \omega^2 RLC) + j\omega(L + CR)} \cdot \frac{(2R - \omega^2 RLC) - j\omega(L + CR)}{(2R - \omega^2 RLC) - j\omega(L + CR)}$$

meh ...

Obvodové rovnice

- řízené a plovoucí zdroje (!)

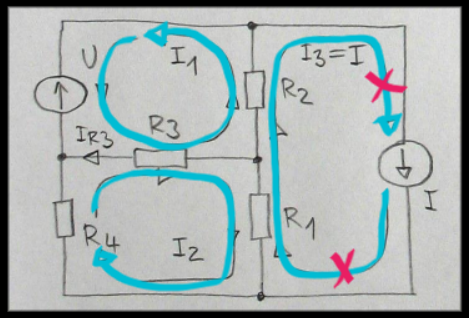
ProCvič. 4.1:



MUN: $r = uzly - 1 - z_u =$
$= 4 - 1 - 1 = 2$

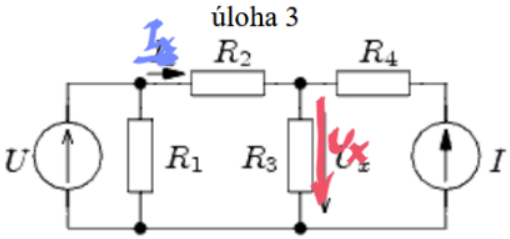
1: $\frac{U_1}{R_4} + \frac{U_1 - U_2}{R_1} - I = 0$

2: $\frac{U_2 - U_1}{R_1} + \frac{U_2}{R_3} + \frac{U_2 + U}{R_2} = 0$



MSP: $s =$ prvky − dvojice uzlů − $z_I =$
$= 6 - 3 - 1 = 2$ smyčky

úloha 3



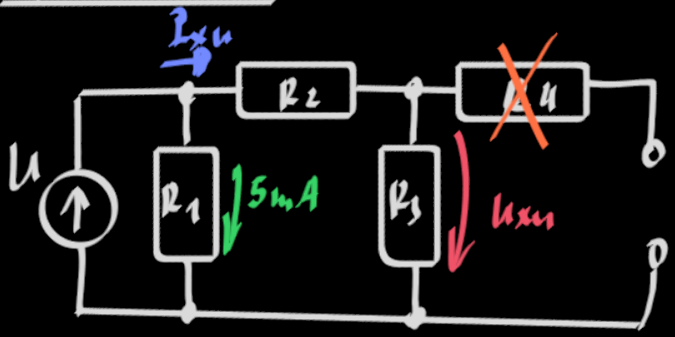
A. $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $R_3 = 3\,\mathrm{k\Omega}$, $R_4 = 4\,\mathrm{k\Omega}$, $U = 5\,\mathrm{V}$, $I = 10\,\mathrm{mA}$

A. Určete napětí U_X a výkon P dodávaný obvodu zdrojem napětí U.
B. Určete proud I_X a výkon P dodávaný obvodu zdrojem napětí U.
C. Určete napětí U_X a výkon P dodávaný obvodu zdrojem proudu I.
D. Určete proud I_X a výkon P dodávaný obvodu zdrojem proudu I.

$U_x = U_{xu} + U_{xI}$
$I_x = I_{xu} + I_{xI}$

$R_{tot} = 833\,\Omega$

U_{xu} a I_{xu}:



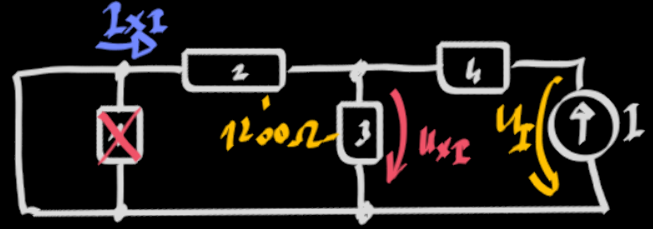
$U_{xu} = U_{R_3} = U \frac{R_3}{R_2 + R_3} = 3\,\mathrm{V}$, $I_u = U / R_{tot} = 6\,\mathrm{mA}$

$I_{xu} = I_{R_{23}} = I_u \frac{R_1}{R_1 + R_2 + R_3} = 1\,\mathrm{mA}$

$U_x = 20\,\mathrm{V}$

$I_x =$

U_{xI} a I_{xI}:



$I_{xI} = -I \cdot \frac{R_3}{R_2 + R_3} = -4{,}33\,\mathrm{mA}$

$U_{xI} = R_3 \cdot I_{R_3} = 3000 \cdot (5{,}67) = 17\,\mathrm{V}$

$U_I = I \cdot R_{tot} = 0{,}01 \cdot 5200 = 52\,\mathrm{V}$, $U_{R_{23}} = U_{xI} = U_I \frac{R_4}{5200} = 40\,\mathrm{V}$