KMMC 2

Official Solutions

Karate Masters Mathematics Competitions 2

1st Annual

KMJJIME

Saturday, January 22, 2022

This official solutions booklet gives at least one solution for each problem on this year's competition and shows that all problems can be solved without the use of a calculator. When more than one solution is provided, this is done to illustrate a significant contrast in methods. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

People are encouraged to share copies of the problem booklet and official solutions with their friends and acquaintances to make the KMMC 2 Committee feel happy. All problems should be credited to the KMMC 2 (for example, "2022 KMMC 2 A, Problem #21"). The publication, reproduction, or communication of the competition's problems or solutions for revenue-generating purposes is illegal.

Questions and complaints about this competition should be sent by private message to

DeToasty3, **HrishiP**, **pandabearcat**, and **pog**.

The problems and solutions for this KMJJIME were prepared by the KMJJIME Editorial Board under the direction of

DeToasty3, HrishiP, pandabearcat, & pog

Answer Key:

1. (30)	2. (23)	3. (08)	4. (05)	5. (83)
6. (15)	7. (36)	8. (59)	9. (55)	10. (60)
11. (25)	12. (22)	13. (12)	14. (72)	15. (67)

Problem 1:

(pandabearcat) Ayaka has one-third the amount of money that Karate has, and Judo has one-fourth the amount of money that Ayaka has. If Karate has 72 dollars, find the sum of the amounts of money that Ayaka and Judo have, in dollars.

Answer (30):

We get that Ayaka has $72 \times \frac{1}{3} = 24$ dollars and Judo has $24 \times \frac{1}{4} = 6$ dollars, for an answer of $24 + 6 = \boxed{30}$ dollars. ■

Problem 2:

(pandabearcat & DeToasty3) Seven years ago, Saya's age was half her brother's age. Saya is currently 15 years old. Find how many years old Saya's brother currently is.

Answer (23):

Since Saya is currently 15 years old, seven years ago, she was $15 - 7 = 8$ years old. Hence, seven years ago, Saya's brother was $8 \times 2 = 16$ years old, so he is currently $16 + 7 = \boxed{23}$ years old. ■

Problem 3:

(pandabearcat) For vacation, Karate wants to go to 20 total states in either the East or the West. He also wants to go to four more states in the East than in the West. Find the number of states in the West that Karate wants to go to.

Answer (08):

Let the number of states Karate visited in the West be $\square$. Then the number of states Karate visited in the East is $\square + 4$, so

$$\square + (\square + 4) = 20.$$

Expanding, $2 \times \square + 4 = 20$, so $2 \times \square = 20 - 4 = 16$. Thus, $\square = \frac{16}{2} = \boxed{08}$. ■

Problem 4:

(DeToasty3) Judo makes a 17-inch straw by gluing some 3-inch straws and 4-inch straws together. Find how many total straws (3-inch and 4-inch) Judo used.

Answer (05):

If Judo used less than 5 straws, then the resulting straw would be at most $4 \times 4 = 16$ inches long, which is not long enough to make a 17-inch straw. However, if Judo used more than 5 straws, then the resulting straw would be at least $3 \times 6 = 18$ inches long, which is too long for a 17-inch straw. Hence, Judo used $\boxed{05}$ straws. (He can make a 17-inch straw if he uses three 3-inch straws and two 4-inch straws.) ■

Problem 5:

(pog) Aki is hiking in a straight line. He passes the first trail marker after hiking for 3 total miles and passes the second trail marker after hiking for 11 total miles. If the spaces from one trail marker to the next are equal in length, find how many total miles Aki will have hiked once he passes the 11th trail marker.

Answer (83):

Note that, each subsequent trail marker, Aki will have hiked for 8 more miles than the last one. Hence, on the third marker, Aki will hike $11 + 8$ miles, on the fourth marker, Aki will hike $11 + 8 + 8$ miles. We see that this pattern will continue, so Aki will have walked $11 + 8 \times 9 = \boxed{83}$ miles by the time he passes his 11th trail marker. ■

Problem 6:

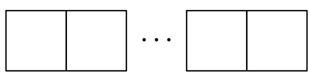
(DeToasty3) Karate has four different whole numbers, where three of the numbers are 10, 7, and 12, and the other number is unknown. If Karate can split his four numbers into two pairs so that the sums of the two numbers in both pairs are equal, find the greatest possible value of the unknown number.

Answer (15):

Let the unknown number be $\triangle$. Note that the unknown number will be larger if the sum of the other two numbers is as large as possible. Thus, the other numbers must be 10 and 12 in some order, so $\triangle + 7 = 22$. Thus, the greatest possible value of the unknown number is $22 - 7 = \boxed{15}$.

Problem 7:

(pog) Karate glues together ten identical squares by their sides so that they line up exactly, and they form a rectangle with a perimeter of 132 inches, as shown in the picture. Find the area of one of the squares, in square inches.



Answer (36):

Let the side length of one square be $\square$. The rectangle has a width of $10 \times \square$ and a height of $\square$, so its perimeter is $2(10 \times \square + \square) = 22 \times \square$. Hence,

$$22 \times \square = 132,$$

so $\square = \frac{132}{22} = 6$. Thus, one of the squares has an area of $6 \times 6 = \boxed{36}$.

Problem 8:

(pandabearcat & pog) Karate currently has 84 pennies. Karate can trade five of his pennies for a nickel. After making some number of trades, Karate ends up with 64 coins (pennies and nickels). Find how many of these 64 coins are pennies.

Answer (59):

Note that, for each trade Karate makes, he gains 1 coin and loses 5 coins, for a net loss of 4 coins. Thus, if $\ominus$ is the number of trades that Karate made,

$84 - 4 \times \ominus = 64$, so $4 \times \ominus = 20$. Hence, Karate made $\ominus = \frac{20}{4} = 5$ trades, so 5 of his coins are nickels and the other $64 - 5 = \boxed{59}$ coins are pennies. ■

Problem 9:

(pandabearcat) Karate lies on his bed at 9:50 PM and falls asleep at 10:10 PM. He then sleeps for 25 minutes, and then he wakes up and reads a book for 15 minutes. Right after reading, Karate falls asleep again until he wakes up at 11:20 PM. Find how many minutes Karate was asleep from 9:50 PM to 11:20 PM.

Answer (55):

After Karate's first sleep for 25 minutes, it is 10:35 PM, and after Karate reads a book for 15 minutes, it is 10:50 PM. Hence, Karate's second sleep is from 10:50 PM to 11:20 PM, so it is 30 minutes long. Hence, Karate was asleep for a total of $25 + 30 = \boxed{55}$ minutes. ■

Problem 10:

(DeToasty3) Karate writes all of the whole numbers from 1 to 100 (including 1 and 100) on a sheet of paper. Then, Karate erases all numbers that are multiples of 3 from the paper. Then, Karate erases all numbers whose tens digit is equal to 2 from the paper. Find how many whole numbers are left on the paper.

Answer (60):

The multiples of 3 from 1 to 100 are $3, 6, 9, \ldots, 99$. This list corresponds to the list $1, 2, 3, \ldots, 33$, so Karate erases 33 multiples of 3 from the paper. Now, when Karate erases all numbers whose tens digits are equal to 2, note that 21, 24, and 27 were already erased since they are multiples of 3, so Karate only erases $10 - 3 = 7$ more numbers. Hence, Karate erased a total of $33 + 7 = 40$ numbers, so there are $100 - 40 = \boxed{60}$ numbers left on the paper. ■

Problem 11:

(DeToasty3 & pog) Aki and Hanami each have a whole number from 1 to 20 (including 1 and 20). They only know that at least one of their numbers is odd. Aki says, "My number is less than 7, but it is greater than 4." Hanami says, "Then, I know that my number is at least three times as large as yours." Aki says, "Then, I know your number." Find the sum of Aki and Hanami's numbers.

Answer (25):

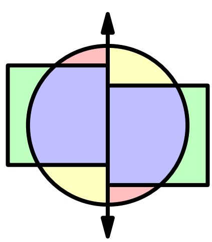
Since Aki's number is less than 7 but greater than 4, it is either 5 or 6. Thus, if Hanami knows her number must be at least three times Aki's number, it must be at least $3 \times 6 = 18$, so Hanami's number can be 18, 19, or 20.

First, if Aki's number is 5, then he cannot find out Hanami's number, as it can be any of 18, 19, or 20. Thus, Aki's number cannot be 5. Next, if Aki's number is 6, then he can find out Hanami's number, as he knows that at least one of their numbers is odd, so since his number is not odd, he knows that Hanami's number must be odd, so it must be 19.

Consequently, Aki's number is 6 and Hanami's number is 19, for an answer of $6 + 19 = \boxed{25}$. ■

Problem 12:

(DeToasty3) Aki has two squares and a circle, where the area of the circle is twice the area of one square. He cuts the circle in half with a line and glues the squares on opposite sides of the line. In the picture, regions with the same color are identical. If the sum of the areas of 1 red, 1 yellow, 2 green, and 3 blue regions is $90\frac{3}{4}$ square inches, find the perimeter of one square, in inches.



Answer (22):

Since the area of the circle is twice the area of one square, the area of half of a circle is equal to the area of a square. Aki's squares are comprised of 1 green and 1 blue region, while each half of Aki's circle is comprised of 1 red, 1 blue, and 1 yellow region. Thus, the sum of the areas of the red and yellow regions is the same as the area of a green region.

Hence, the given sum is the same as the sum of the areas of 3 green and 3 blue regions, which we are given is equal to $90\frac{3}{4}$ inches. This is the same as the area of three of Aki's squares, so the area of one of Aki's squares is

$$\frac{90+\frac{3}{4}}{3} = 30 + \frac{3}{12} = \frac{363}{12} = \frac{121}{4}.$$

Note that since $121 = 11 \times 11$ and $4 = 2 \times 2$, we get that $\frac{11}{2} \times \frac{11}{2} = \frac{121}{4}$, so the side length of one square is $\frac{11}{2}$, and the perimeter of one square is equal to $\frac{11}{2} \times 4 = \frac{44}{2} = \boxed{22}$. ■

Problem 13:

(DeToasty3 & pog) Karate chooses two different whole numbers and writes their sum, their positive difference, and their product on a board. If there is exactly one even number on the board, and that number is equal to 72, find the sum of all possible values of the smaller of Karate's two numbers.

Answer (12):

Let $\square$ and $\triangle$ be the two numbers, where $\triangle$ is the smaller number. Note that if $\square + \triangle$ is even, then so is $\square - \triangle$, and if $\square - \triangle$ is even, then so is $\square + \triangle$. However, we are given that exactly one even number is on the board, so $\square + \triangle$ and $\square - \triangle$ are both odd, and the even number on the board is $\square \times \triangle$, which we are given is equal to 72.

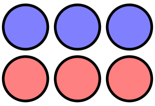
Since $\square + \triangle$ is odd, we wish to find values of $\square$ and $\triangle$ such that one of them is even and the other one is odd. Looking at the numbers that multiply to 72, we get that the possible values of $\square$ and $\triangle$ are

$$72 \times 1, 24 \times 3, \quad \text{and} \quad 9 \times 8,$$

so the sum of all possible values of $\triangle$ is $1 + 3 + 8 = \boxed{12}$. ■

Problem 14:

(DeToasty3 & pog) Karate, Judo, Naruto, Haruka, Ayaka, and Saya each stand on one of the six circles in the picture so that each circle has exactly one person standing on it. Find how many ways they can stand so that Karate and Judo both stand on blue circles, and Naruto and Saya stand on circles with different colors.



Answer (72):

First, note that if Naruto and Saya have to stand on circles with different colors, then one of the two must be standing on a blue circle. Also, since Karate and Judo have to stand on blue circles, the three people who are standing on blue circles must be either Karate, Judo, and Naruto or Karate, Judo, and Saya. As a result, the three people who are standing on red circles must be either Haruka, Ayaka, and Saya or Haruka, Ayaka, and Naruto.

Next, we take a look at how many ways the six people can be ordered from left to right. Note that for each row of circles, there are three choices for who goes in the leftmost circle, then there are two choices left for who goes in the middle circle, and finally, there is one choice left for who goes in the rightmost circle. For each way to put someone in the leftmost circle, there are two ways to put someone in the middle circle and one way to put someone in the rightmost circle, so there are $3 \times 2 \times 1 = 6$ ways to order the three people in each row of circles. Since there are two rows, we realize that for each of the 6 ways to order the people in the blue circles row, there are 6 ways to order the people in the red circles row. Thus, there are $6 \times 6 = 36$ ways to order the six people from left to right.

Finally, for each of the 36 ways to order the six people from left to right, there are two choices for which people stand on the blue circles, which leaves one choice for which people stand on the red circles. Thus, there are $36 \times 2 = \boxed{72}$ ways for the six people to stand on the six circles. ■

Problem 15:

(pog & DeToasty3) Karate has two whole numbers $\square$ and $\triangle$ such that

$$\square + \triangle = 2022 \quad \text{and} \quad 4 < \tfrac{\square}{\triangle} < 5.$$

Find the number of possible values of $\triangle$.

Answer (67):

Note that the equation $\square + \triangle = 2022$ tells us that $\square = 2022 - \triangle$. From here, we may plug this value into the chain of inequalities to get

$$4 < \tfrac{2022-\triangle}{\triangle} < 5.$$

Now, since $\triangle$ is a positive number, we can multiply all three sides of the inequality chain by $\triangle$ to get

$$4 \times \triangle < 2022 - \triangle < 5 \times \triangle.$$

Finally, adding $\triangle$ to all three sides of the inequality chain and combining like terms, we get

$$5 \times \triangle < 2022 < 6 \times \triangle.$$

Next, if we divide all three sides of the inequality chain by 5, we get

$$\triangle < \frac{2022}{5} < \frac{6}{5} \times \triangle.$$

This means that $\triangle$ is less than $\frac{2022}{5} = 404\frac{2}{5}$. If we instead divide all three sides of the inequality chain by 6, we get

$$\frac{5}{6} \times \triangle < \frac{2022}{6} < \triangle.$$

This means that $\triangle$ is greater than $\frac{2022}{6} = 337$. This means that $\triangle$ can be any whole number from 338 to 404 (including 338 and 404). Thus, our answer is $404 - 338 + 1 = \boxed{67}$.

OR

Note that we can add $\frac{\triangle}{\triangle} = 1$ to the inequality, as then

$$\frac{\square}{\triangle} + \frac{\triangle}{\triangle} = \frac{\square + \triangle}{\triangle} = \frac{2022}{\triangle},$$

which removes one of the unknown numbers.

Thus, we get that

$$5 < \frac{2022}{\triangle} < 6.$$

As $\triangle$ increases, $\frac{2022}{\triangle}$ decreases. Since $\frac{2022}{404} > 5$ and $\frac{2022}{403} < 5$, the smallest possible value of $\triangle$ is 403. From here, all values of $\triangle$ will work until $\frac{2022}{337} = 6$, while $\frac{2022}{338} < 6$ works.

Hence, $\triangle$ can be any whole number from 338 to 404 (including 338 and 404). If we subtract 337 from every whole number in this set, it is equivalent to finding how many numbers there are from $338 - 337 = 1$ to $404 - 337 = 67$, for an answer of $\boxed{67}$. ■