

スゴスク

sgossk Series

1st and Only

sgossk

Wednesday, November 9, 2022



指示

1. どうも！きのこくんです～♡(≧▽≦*)o
Hi! I'm Shroomie-kun～♡(≧▽≦*)o
2. 【sgossk】へようこそ！
Welcome to sgossk!
3. このコンテストは【berfoer】の仲間です。
This contest is the companion to berfoer.
4. 【sgossk】テストの出題量は15問で、制限時間は75分です。楽しみにしててくださいね！
You will have 75 minutes to complete 15 questions. Please look forward to it!
5. 筆記用具や消しゴム、下書きの紙などの使用は許可されています。
You may use writing utensils, paper, and an eraser.
6. 私が主催するコンテストに参加してくれてありがとうございます！
Thanks for taking my contest!

The sgossk Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

1 Solving Problems

Each problem is worth 7 points for a correct answer, 0 points for an incorrect answer, and 2 points for a blank answer.

1. What is the value of $1001 \cdot 1002$?
2. The perimeter of a square is 56, and the perimeter of a hexagon is 35. If Katherine glues together a side of the square and a side of the hexagon with equal length so that they exactly line up, and the shapes do not overlap, she will form a new shape. What is the perimeter of this shape?
3. Three composite integers, exactly two of which are equal, sum to 42. What is the smallest possible product of these integers?
4. When Chirashizushi was 10 years old, Okonomiyaki was 18 years old, and Oyakodon was 30 years old. In how many years will the ages of Chirashizushi, Okonomiyaki, and Oyakodon form a geometric sequence in that order?
5. Ryan uses an automatic calculator to add the positive integers. After 5 seconds, the calculator adds 1, then after 5 more, it adds 2, and so forth. At a random time in the first 250 seconds, Ryan drops the calculator, and it skips any numbers that it is supposed to add for the next 7 seconds. After 250 seconds, the calculator stops adding. What is the probability that the calculator ends up at 1200? Express your answer as a common fraction.
6. Suppose that a_1, a_2, \dots is an arithmetic sequence such that the sum of a_i over all i that divide 108 is equal to 936, and the sum of a_i over all i that divide 225 is 1281. What is a_{100} ?
7. In the land of Inflatopia, a coin that is n years old is worth $n!$ pennies. Tony has 10 coins, each of which is from a different year from 2012 to 2021. If the current year is 2022, how many subsets of the coins (possibly empty) have a value, in pennies, that is a multiple of 9?
8. How many non-negative integers n at most 10^{10} satisfy

$$(S(n))^4 - 4(S(n))^3 - (S(n))^2 + 4(S(n)) = 0,$$

where $S(n)$ denotes the sum of the digits of n ?

2 Writing Problems

We were in the middle of writing the solutions document to a mock contest, but then we lost all the problem statements! Could you help guess the problem statements using the information given? It would help us out a ton!

Each “problem” will be worth 7 points. They will be scored as follows:

Plausibility (4 points):

Obviously, your problem statement will not look exactly like our problem (unless you’re secretly part of our committee), but if your problem does not match up with the solution, we will have to deduct points in this category. Note that this has little to do with how much the problem matches what we have; this is about how much the problem fits the solution.

Style & Mechanics (3 points):

A very important part of a problem is how it reads. This means that your problem should not have stylistic issues or have significant grammatical errors.

9. **Answer (9):** Let the first term of the sequence be n , and let the common difference of the sequence be r . Then, we have that $10n + 45r = 45$ and $5n + 10r = -90$, so $r = 9$.

10. **Answer (7):** Note that

$$|4 - n| \geq 3 \implies n \geq 7 \quad \text{or} \quad n \leq 1.$$

Hence, we can use geometric probability: the probability that $n \geq 7$ is $\frac{3}{10}$ and the probability that $n \leq 1$ is $\frac{1}{10}$, so the probability that $|4 - n| \geq 3$ is $\frac{3}{10} + \frac{1}{10} = \frac{2}{5}$ and our answer is $2 + 5 = 7$.

11. **Answer (21):** Note that for n to be divisible by 225, it must be divisible by 25 and 9. We find that $n \equiv 1 + 2 + 3 + \cdots + 100 \equiv 5050 \equiv 1 \pmod{9}$. Thus, for the resulting number to be divisible by 9, we must remove a 1. Regardless of what 1 we remove, the number will also be divisible by 25, as it will end with 00. Hence, we can remove any of the 21 1’s in the number for the resulting number to be divisible by 225.

12. **Answer (11):** Let $x = \frac{1}{a}$, let $y = \frac{1}{b}$, and let $z = \frac{1}{c}$. By Titu's Favorite Factoring Trick,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx).$$

Hence,

$$8 - 3xyz = 0(x^2 + y^2 + z^2 - xy - yz - zx),$$

so $xyz = \frac{8}{3}$ and $abc = \frac{1}{x} \cdot \frac{1}{y} \cdot \frac{1}{z} = \frac{1}{xyz} = \frac{3}{8}$, for an answer of $3 + 8 = 11$.

13. **Answer (85):** Note that the given expression is equal to

$$\frac{a}{7^1} + \frac{b}{7^2} + \frac{a}{7^3} + \frac{b}{7^4} + \cdots = \frac{7a + b}{7^2 - 1} = \frac{19}{48}$$

by the infinite geometric series formula. Hence, $\underline{a} \underline{b}_{\text{seven}} = 19 \implies (a, b) = (2, 5)$.

Similarly, $0.\underline{b} \underline{a} \underline{b} \underline{a} \dots_{\text{seven}}$ is equal to

$$\frac{b}{7^1} + \frac{a}{7^2} + \frac{b}{7^3} + \frac{a}{7^4} + \cdots = \frac{7b + a}{7^2 - 1},$$

so hence $0.\underline{b} \underline{a} \underline{b} \underline{a} \dots_{\text{seven}} = \frac{7 \cdot 5 + 2}{48} = \frac{37}{48}$, for an answer of $37 + 48 = 85$.

14. **Answer (40):** Let s be Kirby's speed (in miles per hour), and let c be the speed of the current. Then $s + c = \frac{60}{30} = 2$ and $s - c = \frac{60}{60} = 1$, so $2s = 3 \implies s = \frac{3}{2}$. Thus, it takes $60 \cdot \frac{2}{3} = 40$ minutes for Kirby to swim 1 mile relative to the water.

15. **Answer (5):** We get that $y = ax^2 + 20x + (4a - 1)$ has exactly one root, so hence $\sqrt{20^2 - 4 \cdot a \cdot (4a - 1)} = 0 \implies -16a^2 + 4a + 400 = 0$, so the sum of the possible values of a is $\frac{-4}{-16} = \frac{1}{4}$ by Vieta's. Consequently, our answer is $1 + 4 = 5$.



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DO NOT OPEN UNTIL WEDNESDAY, November 9, 2022

****Administration on an earlier date will disqualify your results.****

- All the information needed to administer this exam is not contained in the non-existent sgossk Teacher's Manual. PLEASE READ THE MANUAL BEFORE WEDNESDAY, NOVEMBER 9, 2022.
- Send **pog** a private message submitting your answers to the sgossk. AoPS is the only way to submit your answers.
- The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

For more information about the sgossk and our other competitions, please visit [Nowhere](#).

Questions and comments about this competition should be sent to:

pog.

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