



## 2022 State Competition

### Target Round Problems 1-2

Name \_\_\_\_\_

#### **DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO SO.**

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the lefthand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

The problems and solutions for this competition were prepared by the DMC Editorial Board under the direction of:

bronzetruck2016, cj13609517288, DankBasher619, dc495, DeToasty3,  
firebolt360, HrishiP, john0512, NH14, nikenissan, pandabearcat,  
PhunsukhWangdu, pog, RedFlame2112, smartatmath, stayhomedomath,  
treemath, vsamc, YBSuburbanTea, & yusufsheikh2207.

1. \_\_\_\_\_ Ayaka, Brian, and Taiki's ages form an arithmetic sequence, in that order. Six years ago, Brian's age was a quarter of what it is now. Two years ago, Taiki's age was a third of what it is now. What is the product of Ayaka, Brian, and Taiki's ages now?

2. \_\_\_\_\_ squares How many squares have vertices at  $(0, 0)$  and  $(4, 0)$ ?



## 2022 Chapter Competition

### Target Round Problems 3-4

Name \_\_\_\_\_

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3. \_\_\_\_\_ inches      The average height of Harry, Jerry, Kerry, Mary, and Terry is 60 inches. If the average height of Harry, Jerry, and Kerry is 58 inches, while the average height of Kerry, Mary, and Terry is 54 inches, what is the average height of Harry, Jerry, Mary, and Terry?

4. \_\_\_\_\_ numbers      How many numbers  $\underline{a} \underline{b} \underline{c}$ , where  $a$ ,  $b$ , and  $c$  are digits and  $a$  is nonzero, are there such that the number  $\underline{a} \underline{b} \underline{c}_7$  is divisible by 2 and the number  $\underline{a} \underline{b} \underline{c}_6$  is divisible by 5?



## 2022 Chapter Competition

### Target Round Problems 5–6

Name \_\_\_\_\_

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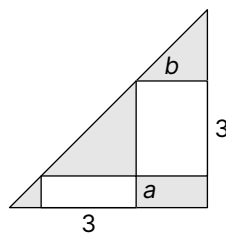
This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The third pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the lefthand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

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5. \_\_\_\_\_ miles     Analise drives a red car which costs \$48,312 and travels 20 miles per gallon of gasoline, while Henry drives a white car which costs \$41,797 and travels 12 miles per gallon of gasoline. If gasoline costs \$5.30 per gallon, what is the smallest possible value of  $n$  such that the total cost (car and fuel) for Analise when travelling  $n$  miles is less than the total cost for Henry when travelling  $n$  miles? Express your answer to the nearest hundredth.

6. \_\_\_\_\_     An isosceles right triangle passes through the vertices of two rectangles that have heights of 3 inches and widths of  $a$  inches and  $b$  inches, as shown. If the area of the shaded region is  $2022 \text{ in}^2$ , what is  $a + b$ ? Express your answer to the nearest hundredth.





## 2022 Chapter Competition

### Target Round Problems 7-8

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7. \_\_\_\_\_ A number is called *decent* if it can be written as  $2^a \cdot 3^b$ , where  $a$  and  $b$  are positive integers less than 10. What is the probability that the product of two different, randomly chosen decent numbers is a perfect cube? Express your answer as a common fraction.

8. \_\_\_\_\_ The first four terms of a geometric sequence are  $a$ ,  $b$ ,  $c$ , and  $d$ . If

$$a + 3b + 3c + d = 128 \quad \text{and} \quad 27a + 27b + 9c + d = 250,$$

then  $d = \frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. What is the value of  $mn$ ?