DeMathCounts

2022 Chapter Competition

Sprint Round Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the lefthand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

The problems and solutions for this competition were prepared by the DMC Editorial Board under the direction of:

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1. $_____ Dwayne spent $22.00 at the movies on soda and popcorn. If soda costs three times as much as popcorn, how much did Dwayne spend on popcorn?

2. _____ What is the value of $2022 \cdot 100 + 2022$?

3. _____ Katherine multiplies the number $\diamondsuit$ by 7 and then subtracts 2 from the result, while Taiki subtracts 2 from $\diamondsuit$ and then multiplies the resulting number by 7. What is the positive difference between Katherine and Taiki's final numbers?

4. _____ Vikram wrote an expression on the board, but he accidentally smudged one of the digits with his hand.

   $$19\blacksquare \cdot 9$$

   What is the result when the minimum possible value of Vikram's expression is subtracted from the maximum possible value of Vikram's expression?

5. _____ There are two numbers with a square of 15. What is their product?

6. _____ The first five terms of an arithmetic sequence are 3, a, b, c, and 17. What is the value of $a + b + c$?

7. _____ units Two of the sides of an isosceles triangle have lengths 10 units and 12 units. What is the sum of the possible perimeters of the triangle?

8. _____ In the equations below, what is the value of e?

$$a + b + c + d + e = 11$$
$$a + b + c + d + f = 12$$
$$a + b + c + d + e + f = 15$$

9. _____ in^2 Alice and Barbara both draw a circle. If Alice draws a circle with an area of 2 in^2, and Barbara draws twice as much as Alice, what is the area of Barbara's circle?

10. _____ If $\sqrt{x} + \sqrt{y} = 7$ and $\sqrt{x} - \sqrt{y} = 4$, what is the value of $x - y$?

11. $ _____ Evan has twice the amount of nickels as he has quarters. If the value of his nickels and quarters combined is $7.00, how much money are his nickels worth?

12. _____ Katie has a bag containing blue, pink, and red marbles. The ratio of blue to pink marbles is 5 : 7, and the ratio of pink to red marbles is 18 : 11. If the ratio of blue to red marbles is a : b, what is the value of $a + b$?

13. _____ days Every day, if Katherine remembers to water her bamboo plant, it will get 10 inches taller, but if she forgets to water it, it will get 5 inches shorter. Over a seven-day period, Katherine's bamboo plant grew from 25 inches tall to 50 inches tall. On how many days during the seven-day period did Katherine forget to water her bamboo plant?

14. _____ Hanami has 3 consecutive integers. If the product of two of these integers is 224, what is the product of the possible values of the third number?

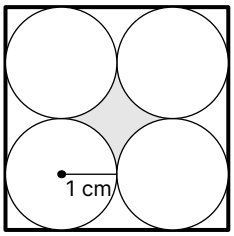
15. _____ If x and y are numbers such that $2^{12} = x^{2022}$ and $x^{1011} = 16^{y}$, what is the value of y? Express your answer as a common fraction.

16. _____ Two numbers are randomly chosen, without replacement, from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability that their product is prime? Express your answer as a common fraction.

17. _____ in Let $ABCD$ be a square, and let P be a point on side $\overline{AB}$. If the area of $\triangle BCP$ is 25 in^2 and the area of quadrilateral $APCD$ is 200 in^2, what is the length of $\overline{BP}$? Express your answer as a common fraction.

18. _____ AM At 6:00 AM, Manuel started running a marathon at 6 miles per hour at a trail in Honolulu, Hawaii. Then, Parkes started running a marathon at 8 miles per hour at the same trail. If Parkes met Manuel 2 hours after Parkes started running, at what time did the two meet?

19. _____ There are four congruent circles, each with a radius of 1 centimeter, that are internally tangent to a square and externally tangent to each other, as shown. What is the area of the shaded region? Express your answer in terms of π.



20. positive divisors If k is a real number such that

$$k\left(\tfrac{1}{2} \cdot \tfrac{1}{3} \cdot \tfrac{1}{4}\right)^{(5 \cdot 6)} = 1,$$

how many positive divisors does k have?

21. _____ Bill rolls a standard six-sided die 4 times. What is the probability that he gets any number twice in a row? Express your answer as a common fraction.

22. _____ The grid below has been filled with positive integers in such a way that the sum of the numbers in each row is the same. However, each number in the grid has been expressed in a different base, from base-two to base-ten. What is the result when the number covered by ★ is expressed in base-ten?

★	27	30
97	86	15
201	63	442

23. _____ If

$$\sqrt{4038 \cdot 2021 + 4040 \cdot 2020 + k} = 4046,$$

what is the value of k?

24. _____ values For how many integer values of n is a triangle with side lengths 7, 9, and n obtuse?

25. _____ The sequence a_n is defined for all integers n. If

$$(n-1) \cdot a_{n-2} = a_n - 3$$

for all odd $n \geq 1$, what is the value of a_5?

26. _____ units^2 The midpoints of the sides of a triangle are $(1, 0)$, $(3, 0)$, and $(2, 5)$. What is the area of this triangle?

27. _____ If p, q, and r are the roots of the polynomial

$$x^3 + 20x^2 + 21x + 22,$$

what is the value of $(p-1)(q-1)(r-1)$?

28. _____ If m and n are positive integers such that $\sqrt[m]{5}\sqrt[n]{25} = \sqrt[5]{5}$, what is the sum of the possible values of n?

29. products Ashley chooses a nonempty subset of $\{1, 2, 3, 4, 5, 6\}$ and finds the product of its elements. How many distinct products can she get from this process?

30. ordered triples How many ordered triples of positive integers (a, b, c) are there such that $a \cdot b \cdot c = 13500$?