Linear programming examples

A company is involved in the production of two items (X and Y). The resources need to produce X and Y are twofold, namely machine time for automatic processing and craftsman time for hand finishing. The table below gives the number of minutes required for each item:
Machine time Craftsman time Item X 13 20
Y 19 29
The company has 40 hours of machine time available in the next working week but only 35 hours of craftsman time. Machine time is costed at £10 per hour worked and craftsman time is costed at £2 per hour worked. Both machine and craftsman idle times incur no costs. The revenue received for each item produced (all production is sold) is £20 for X and £30 for Y. The company has a specific contract to produce 10 items of X per week for a particular customer.

Formulate the problem of deciding how much to produce per week as a linear program.
Solve this linear program graphically.
Solution Let

x be the number of items of X
y be the number of items of Y

then the LP is:
maximize

20x + 30y - 10(machine time worked) - 2(craftsman time worked) subject to:
13x + 19y <= 40(60) machine time
20x + 29y <= 35(60) craftsman time
x >= 10 contract
x,y >= 0

so that the objective function becomes maximize
20x + 30y - 10(13x + 19y)/60 - 2(20x + 29y)/60

i.e.

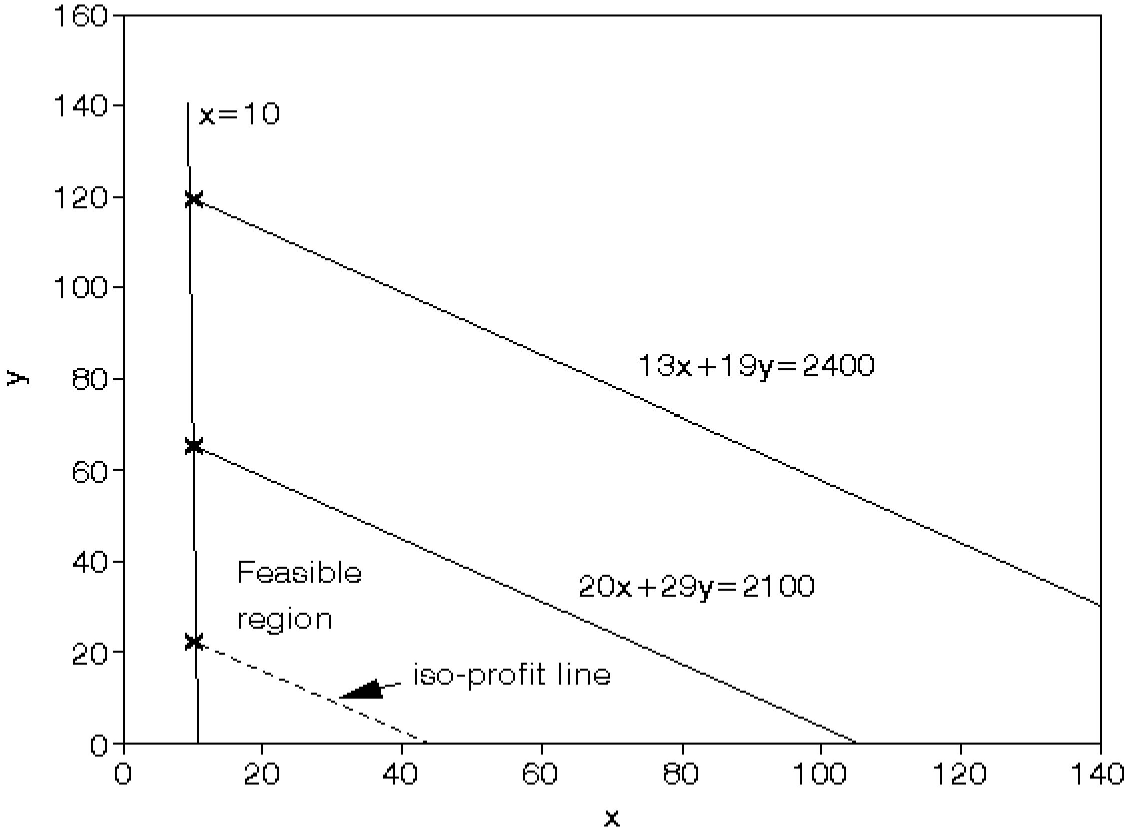
$$\max_{x,y} \; 17.1667x + 25.8667y$$

subject to:

13x + 19y <= 2400
20x + 29y <= 2100
x >= 10
x,y >= 0

It is plain from the diagram below that the maximum occurs at the intersection of x=10 and 20x + 29y <= 2100

Solving simultaneously, rather than by reading values off the graph, we have that x=10 and y=65.52 with the value of the objective function being £1866.5



x=10
13x+19y=2400
20x+29y=2100
Feasible region
iso-profit line
y
x

A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutesprocessing time on machine B. Each unit of Y that is produced requires 24 minutesprocessing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B isforecast to be 35 hours.

The demand for X in the current week is forecast to be 75 units and for Y is forecast to be95 units. Company policy is to maximize the combined sum of the units of X and the unitsof Y in stock at the end of the week.

- Formulate the problem of deciding how much of each product to make in thecurrent week as a linear program.
- Solve this linear program graphically.

Solution

Let

- x be the number of units of X produced in the current week
- y be the number of units of Y produced in the current week

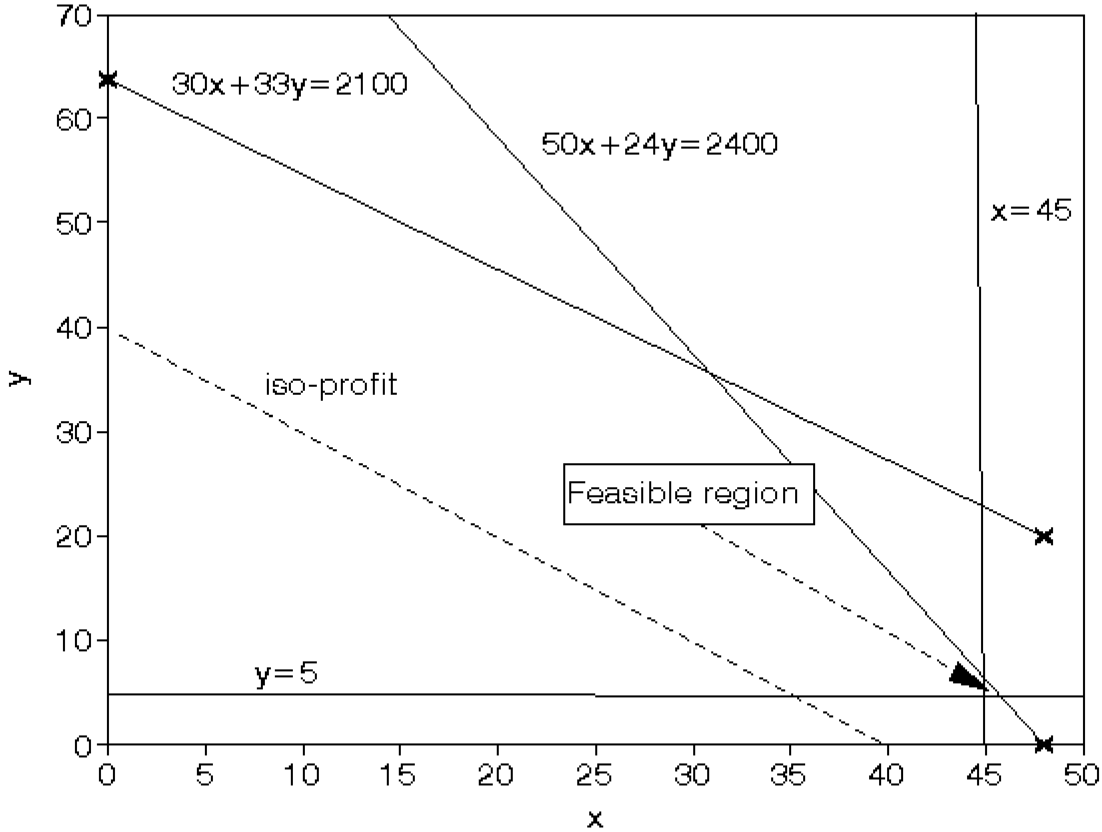
then the constraints are:

- 50x + 24y <= 40(60) machine A time
- 30x + 33y <= 35(60) machine B time
- x >= 75 - 30
- i.e. x >= 45 so production of X >= demand (75) - initial stock (30), which ensures wemeet demand
- y >= 95 - 90
- i.e. y >= 5 so production of Y >= demand (95) - initial stock (90), which ensures wemeet demand

The objective is: maximize (x+30-75) + (y+90-95) = (x+y-50)
i.e. to maximize the number of units left in stock at the end of the week

It is plain from the diagram below that the maximum occurs at the intersection of x=45 and50x + 24y = 2400

Solving simultaneously, rather than by reading values off the graph, we have that x=45 andy=6.25 with the value of the objective function being 1.25



$$\underset{x,y}{\text{Max}} \quad x + y - 50$$

st.

$50x + 24y <= 40 * 60 \quad (1)$

$30x + 33y <= 35 * 60 \quad (2)$

$x >= 75 - 30 \quad (3)$

$y >= 95 - 90 \quad (4)$