Portfolio analysis

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王大龍目前持有下列股票，其權重如下

彰銀	0.15
亞太電	0.15
歐買尬	0.25
士電	0.35
遠東新	0.1

1.請您計算目前投資組合的期望報酬及報酬標準差
2.請您畫出這五個資產所能構建而成的效率前緣，並標出目前投資組合的位置
3.目前投資組合是一個有效率的組合嗎？
4.如不是，要如何改進？是以三個資產來組成投資組合較好？還是四個資產？還是五個資產？
5.如果只挑三支股票，應該選哪三檔?

1402	1503	2739	2801	3682	3687	
遠東新	士電	寒舍	彰銀	亞太電	歐買尬	
0.00796	0.006538	-0.00647	0.006761	-0.00638	0.016249	
	遠東新	士電	寒舍	彰銀	亞太電	歐買尬
遠東新	0.390567	0.106275	0.087565	0.128384	0.204276	0.223919
士電	0.106275	0.161483	0.058867	0.074734	0.054558	0.225703
寒舍	0.087565	0.058867	0.463461	0.040812	0.17405	0.080734
彰銀	0.128384	0.074734	0.040812	0.179529	0.07829	0.04791
亞太電	0.204276	0.054558	0.17405	0.07829	1.001035	0.262097
歐買尬	0.223919	0.225703	0.080734	0.04791	0.262097	2.26329

Two Assets

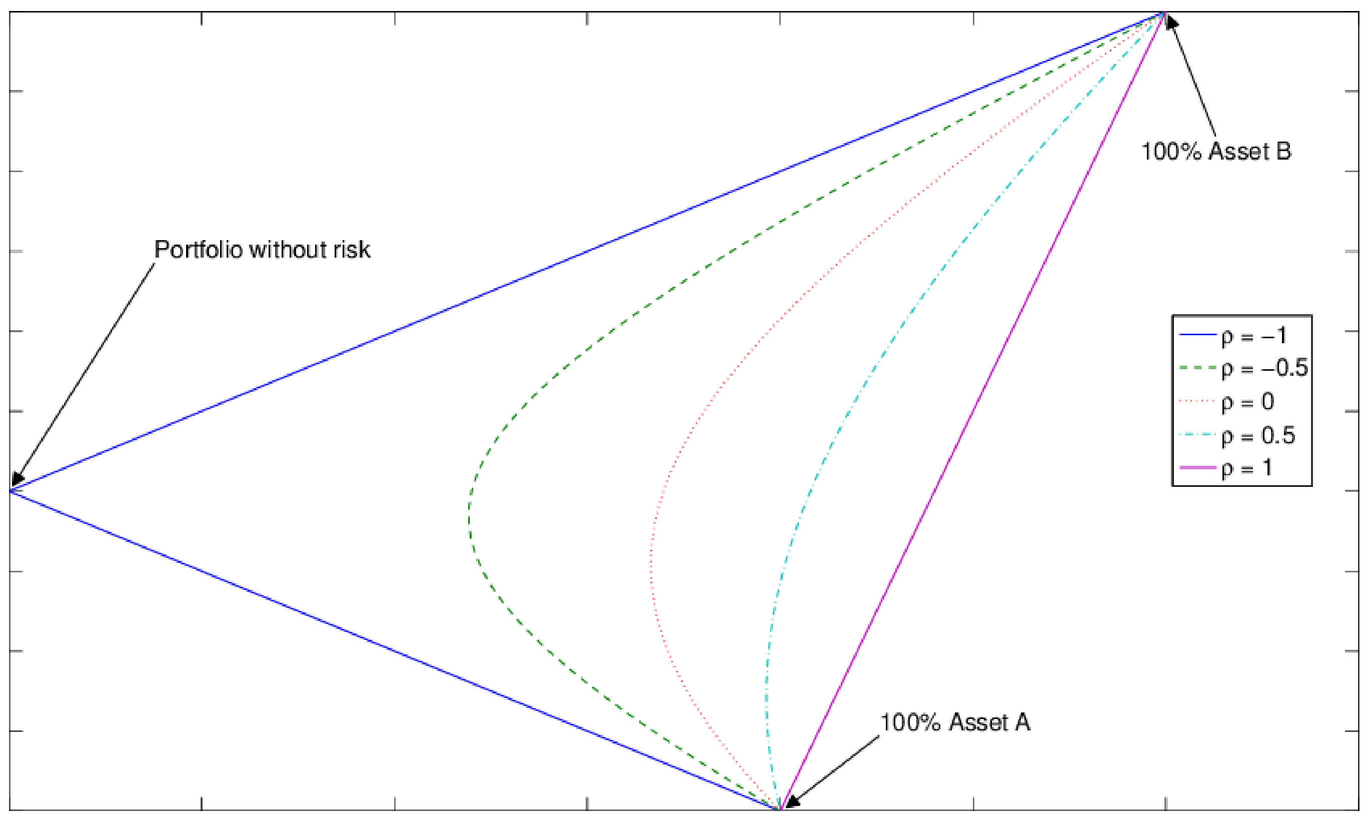
$r_A \qquad \sigma_A$

$r_B \qquad \sigma_B$

$$r_p = w_A r_A + w_B r_B$$

$$E(r_p) = w_A E(r_A) + w_B E(r_B)$$

$$Var(r_p) = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + \underbrace{2 w_A w_B Cov(r_A, r_B)}_{+ 2 w_A w_B \rho \sigma_A \sigma_B}$$



100% Asset B
Portfolio without risk
ρ = −1
ρ = −0.5
ρ = 0
ρ = 0.5
ρ = 1
100% Asset A

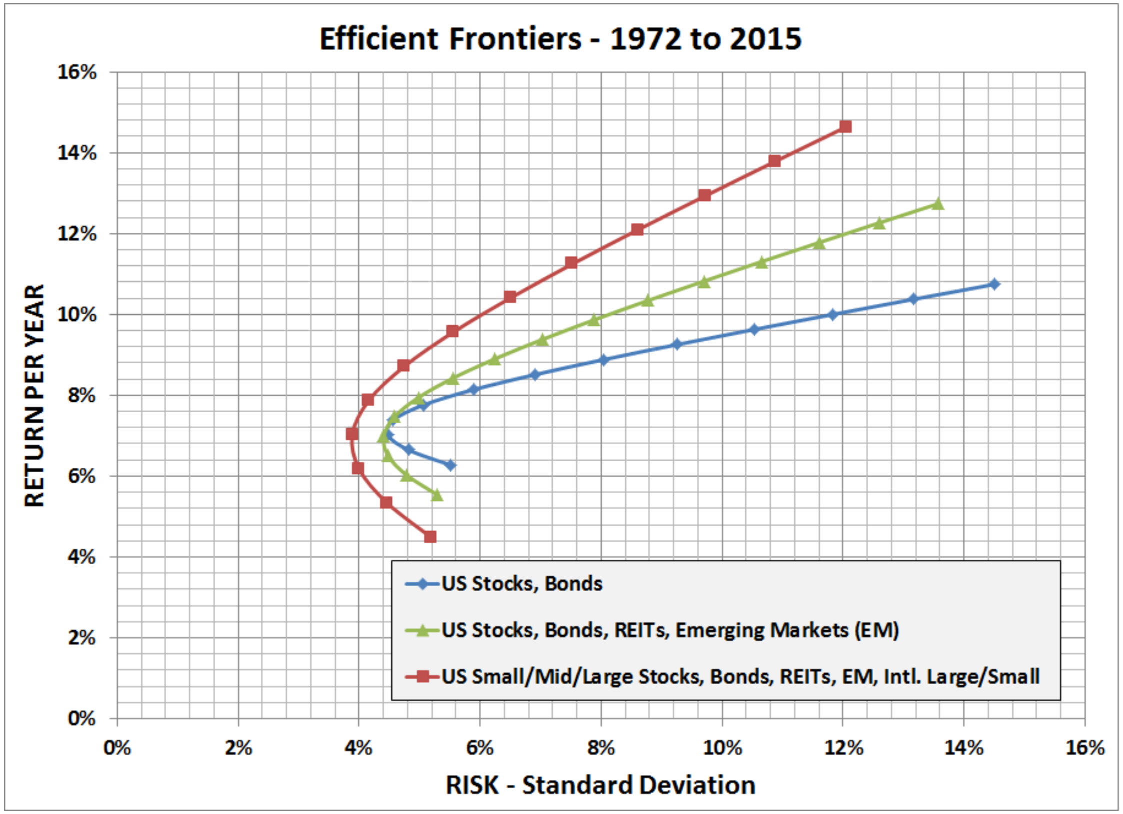
Three assets

$$r_p = w_1 r_1 + w_2 r_2 + w_3 r_3 \quad , \quad w_1 + w_2 + w_3 = 1$$

$$\Rightarrow E(r_p) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3)$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2 w_1 w_2 \sigma_{12} + 2 w_1 w_3 \sigma_{13} + 2 w_2 w_3 \sigma_{23}$$

More assets



Efficient Frontiers - 1972 to 2015
RETURN PER YEAR
16%
14%
12%
10%
8%
6%
4%
2%
0%
0%
2%
4%
6%
8%
10%
12%
14%
16%
RISK - Standard Deviation
US Stocks, Bonds
US Stocks, Bonds, REITs, Emerging Markets (EM)
US Small/Mid/Large Stocks, Bonds, REITs, EM, Intl. Large/Small

Many assets

$$\sum_{i=1}^{N} w_i = 1$$

$$r_p = \sum_{i=1}^{N} w_i r_i$$

$$\Rightarrow E(r_p) = \sum_{i=1}^{N} w_i \, E(r_i)$$

$$Var(r_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$= \sum_{i=1}^{N} \left(\frac{1}{N}\right)^2 \sigma_i^2 + \sum_{i\neq j}^{N}\sum \left(\frac{1}{N}\right)^2 \sigma_{ij}$$

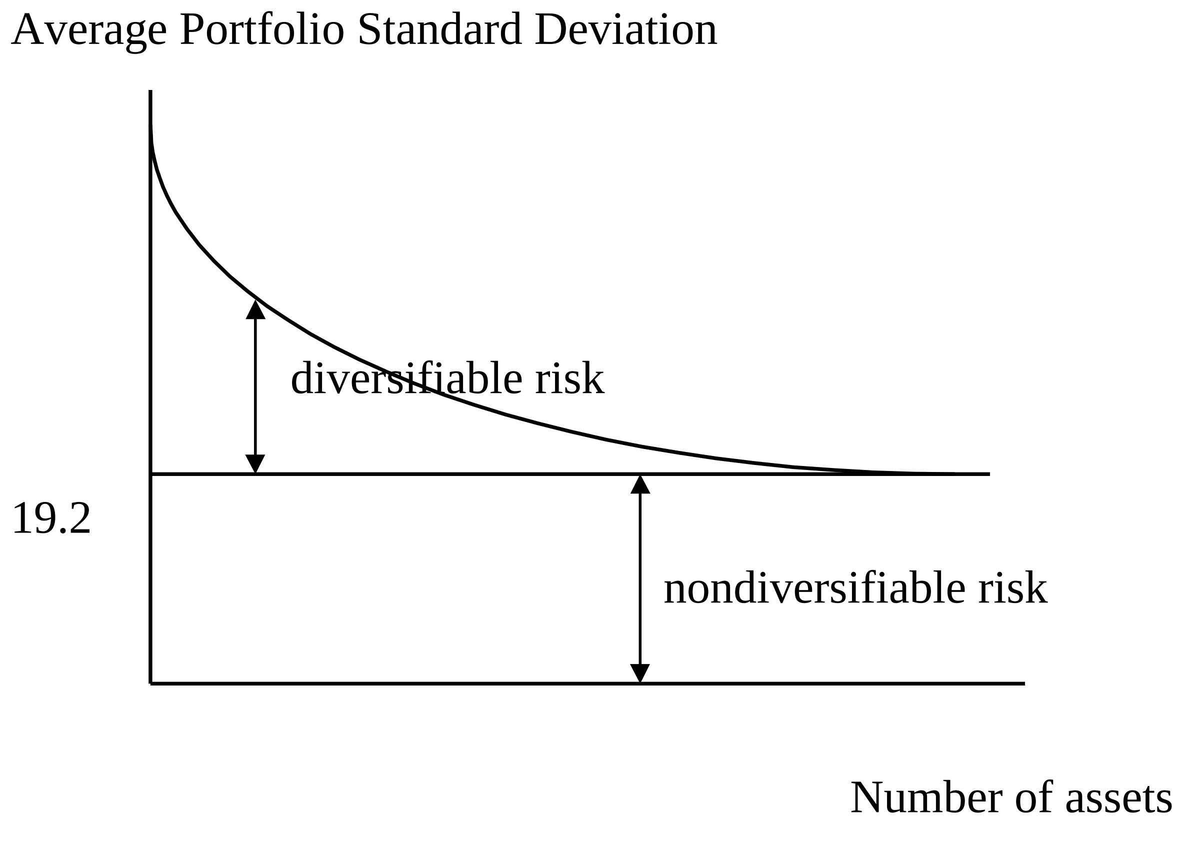
$$= \frac{1}{N}\left(\frac{1}{N}\sum_{i=1}^{N} \sigma_i^2\right) + \frac{N(N-1)}{N^2}\left(\frac{1}{N(N-1)}\sum_{i\neq j}^{N}\sum \sigma_{ij}\right)$$

$$= \frac{1}{N}(\text{平均變異數}) + \frac{N(N-1)}{N^2}(\text{平均共變數})$$

When $N \to \infty$

$$= 0 + (\text{平均共變數})$$

Diversification



Total risk = non-systematic risk + systemtic risk

diversifiable

firm-specific

Non-diversifiable

market risk

(Beta risk)

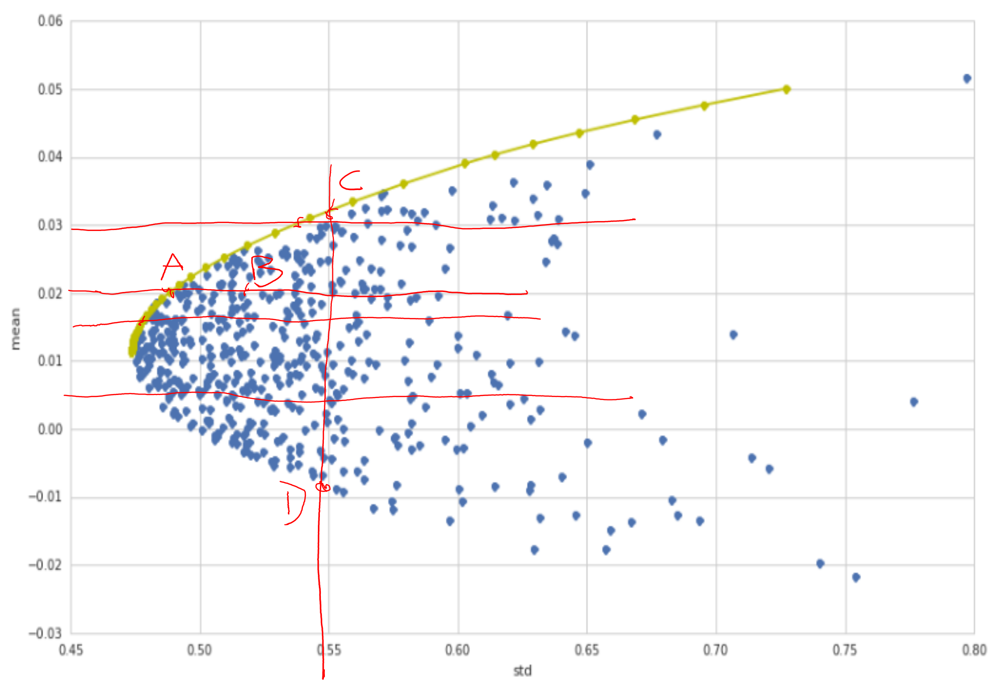
Optimization

$$\min_{w_i} \sigma_p^2$$

$$\text{s.t.} \quad E(r_p) = \bar{r}$$

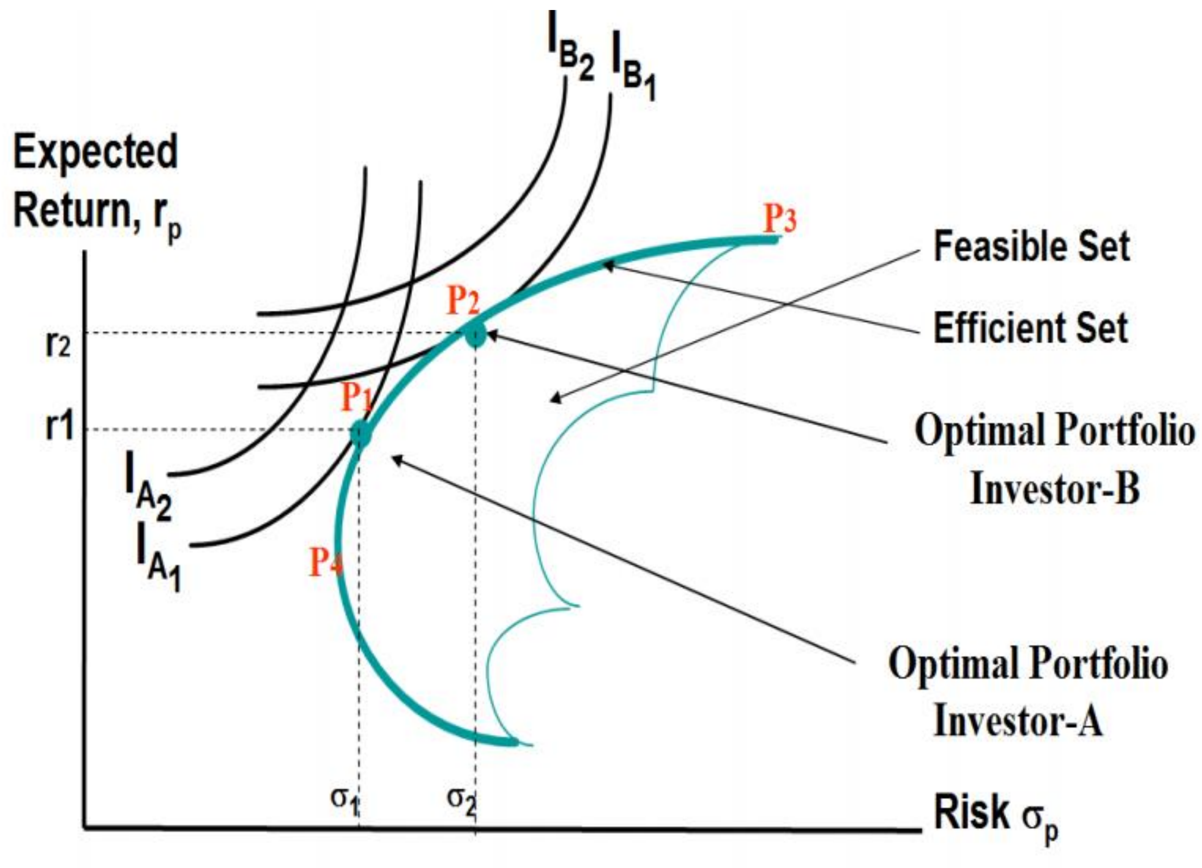
$$\sum_{i=1}^{N} w_i = 1$$

A >> B ,
C >> D

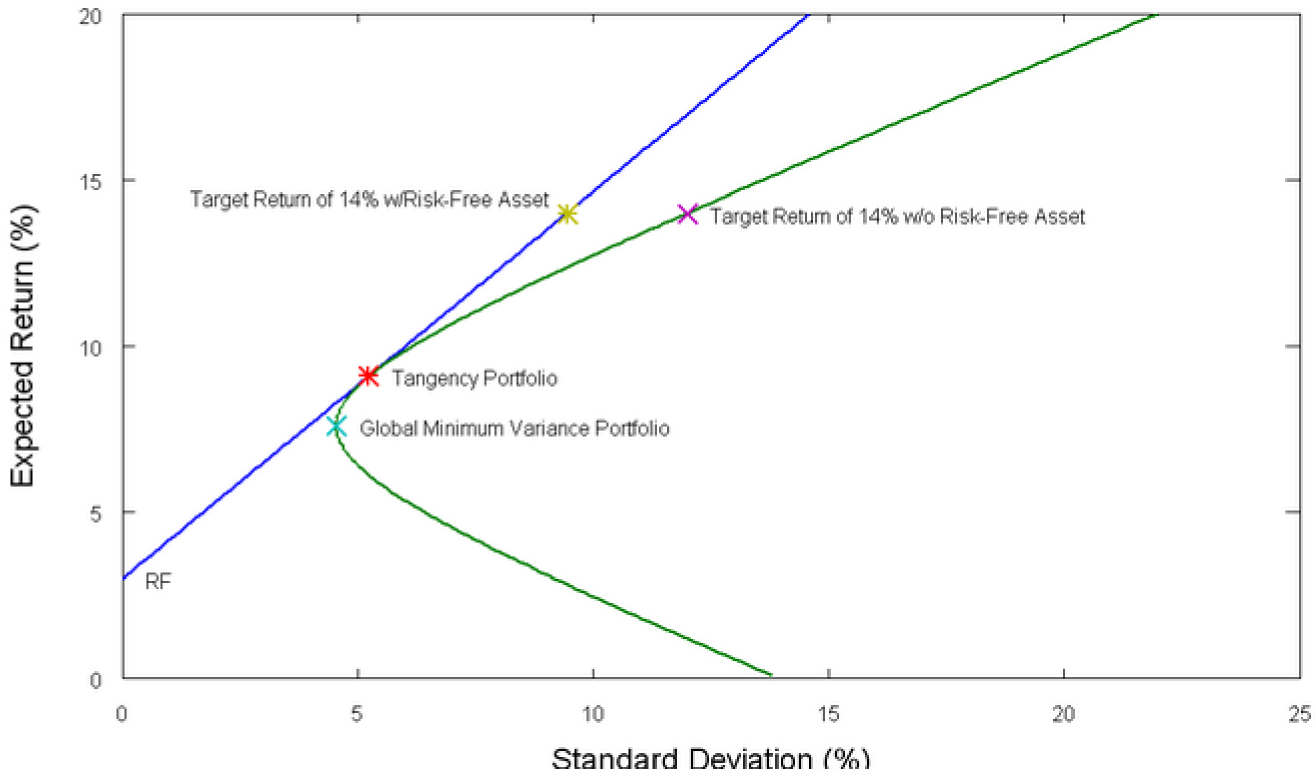


0.06
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
mean
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
std
A
B
C
D

Portfolio selection



With risk free asset
Capital Allocation Line (CAL)



Sharpe ratio

$$\frac{R_p - R_f}{\sigma_p}$$

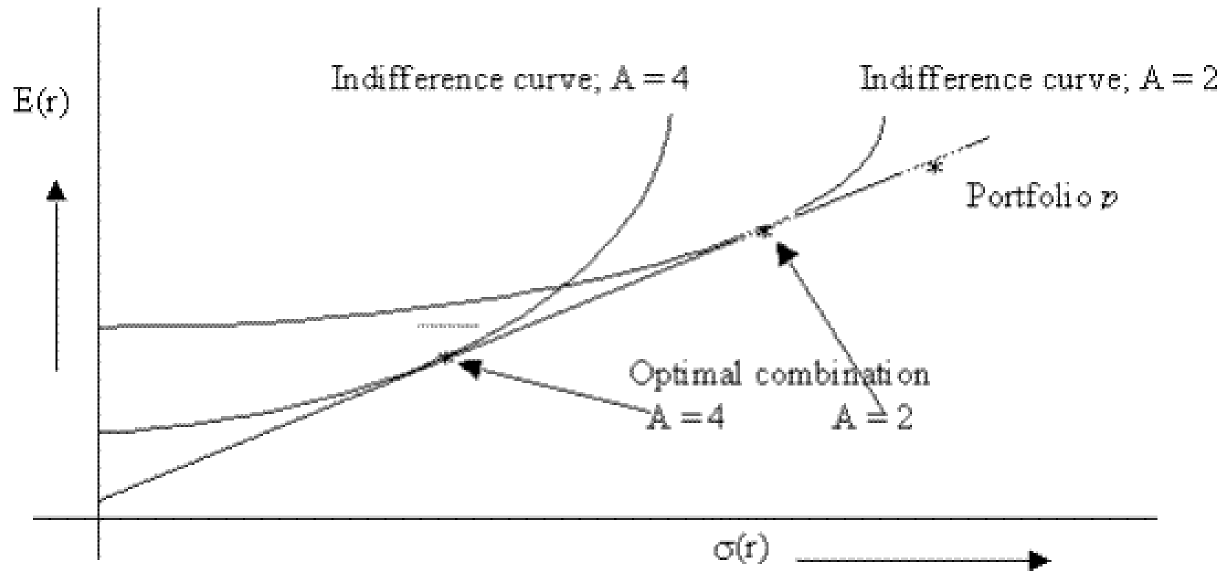
Where:

R_p = Portfolio Return

R_f = Risk-Free Rate (3-month Treasury Rate is standard)

σ_p = Portfolio Risk, aka Standard Deviation of Returns

Portfolio selection with risk free asset



5.5 Portfolio Asset Allocation: Expected Return and Risk

Expected Return of the Complete Portfolio

$$E(r_C) = y \times E(r_p) + (1 - y) \times r_f$$

where $E(r_C) =$ Expected Return of the complete portfolio

$E(r_p) =$ Expected Return of the risky portfolio

$r_f =$ Return of the risk free asset

$y =$ Percentage assets in the risky portfolio

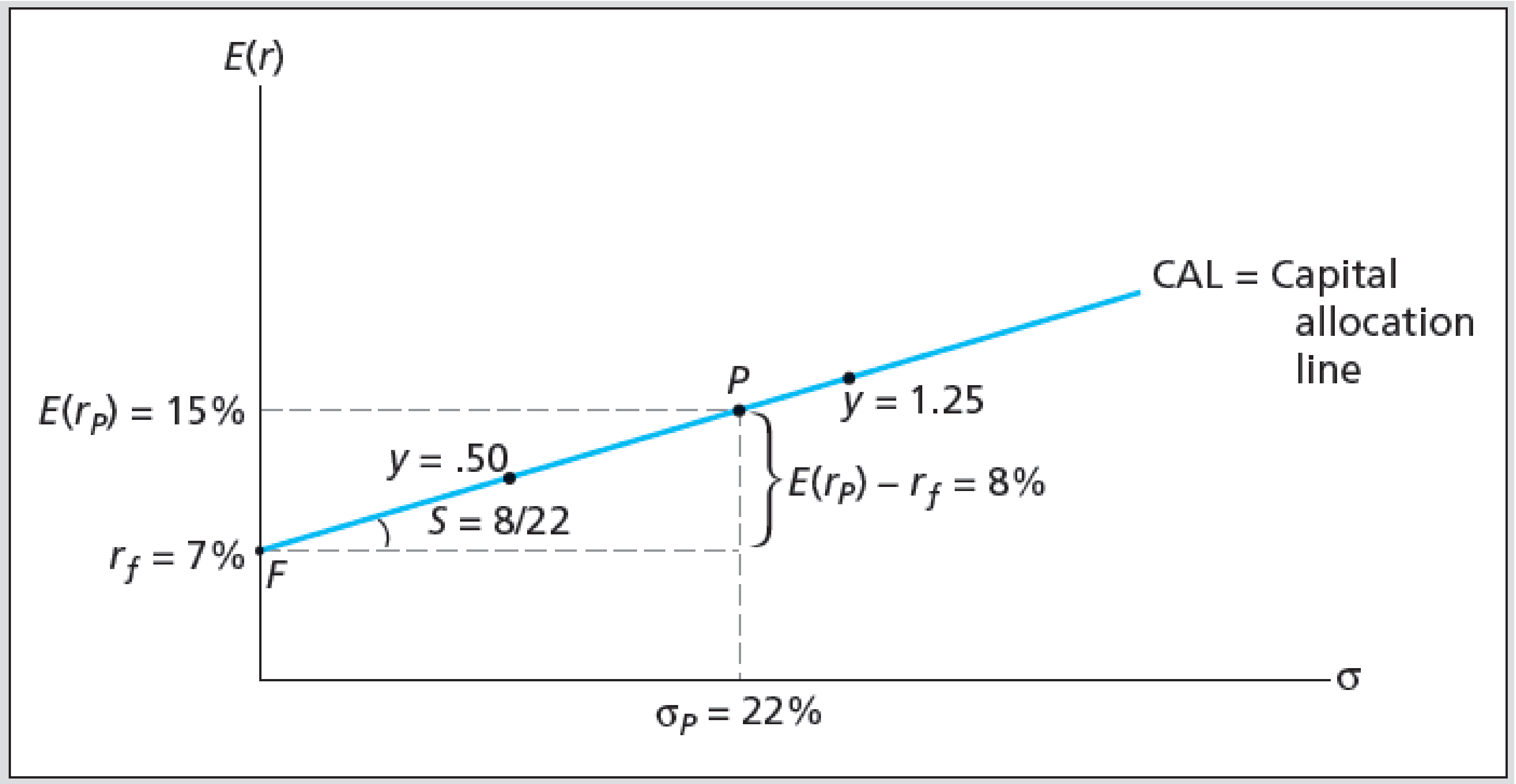
Standard Deviation of the Complete Portfolio

$$\sigma_C = y \times \sigma_p$$

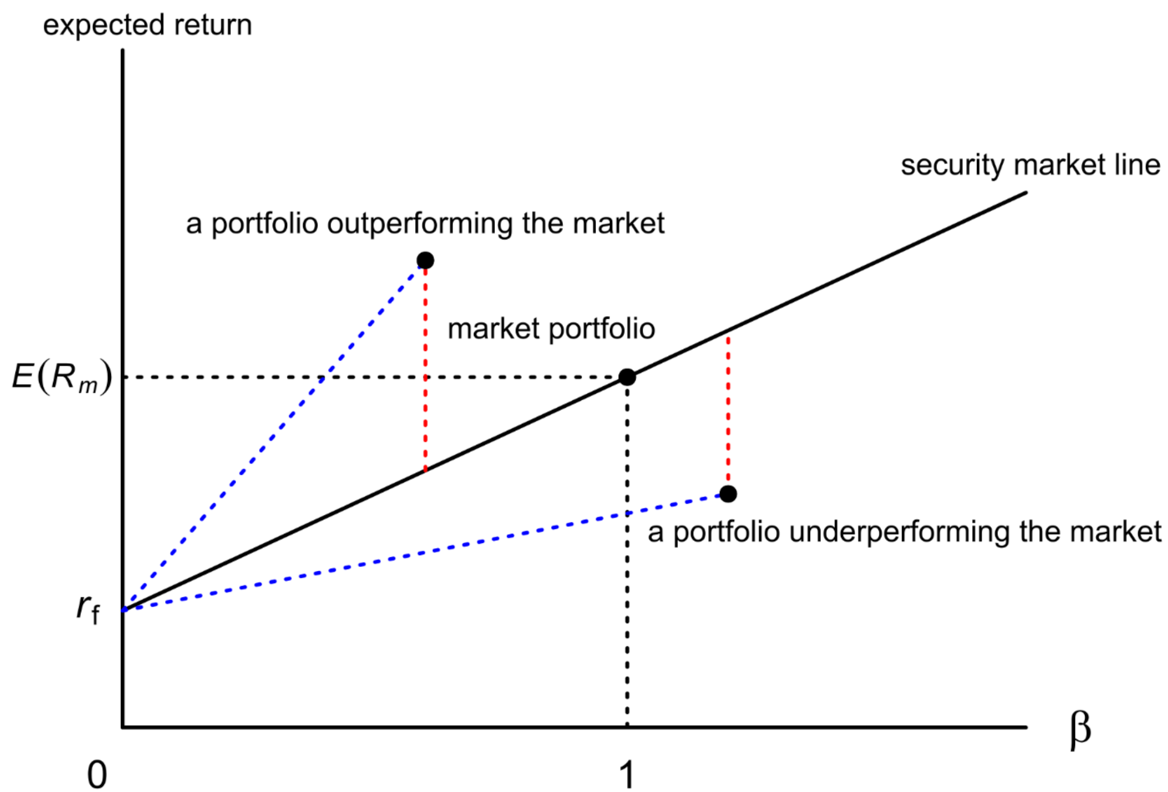
where $\sigma_C =$ Standard deviation of the complete portfolio

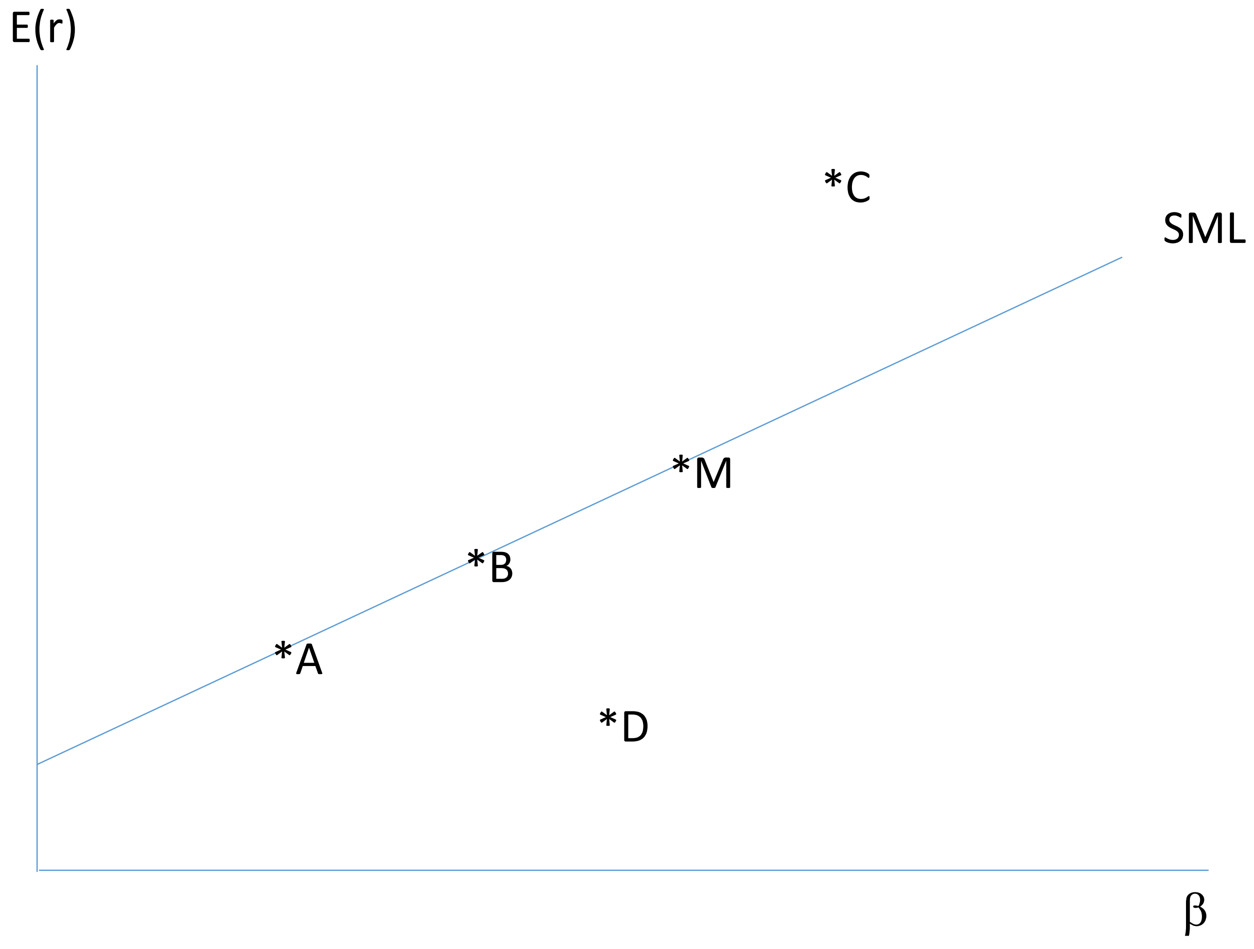
$\sigma_P =$ Standard deviation of the risky portfolio

Figure 5.6 Investment Opportunity Set



Security Market Line When only consider market risk




E(r)
*C
SML
*M
*B
*A
*D
β

Derive CAPM

$$\frac{E(r_A) - r_f}{\beta_A} = \frac{E(r_B) - r_f}{\beta_B} = \frac{E(r_m) - r_f}{\beta_m}$$

$\beta_m = 1$

$$\forall i \quad \frac{E(r_i) - r_f}{\beta_i} = E(r_m) - r_f$$

$$\Rightarrow E(r_i) = r_f + \beta_i \left(E(r_m) - r_f \right)$$

Capital Asset Pricing Model CAPM

$E(r_i) = R_f + \beta_i(E(r_m) - R_f)$

$E(r_i)$ = return required on financial asset i
R_f = risk-free rate of return
β_i = beta value for financial asset i
$E(r_m)$ = average return on the capital market