

## Homework assignment on Capital Structure: Miller's model

With corporate tax, Modigliani and Miller propose the value of the leveraged firm, L, is equal to the value of unleveraged firm, U, plus the tax shield.

$$V_L = V_U + \tau_c D ,$$

Where,  $\tau_c$  is the corporate tax rate.

In his 1977 paper, Miller considers the trade-off between the tax advantage of debt at the corporate level and the tax disadvantage at the personal level and examines the implications of this trade-off for optimal capital structure.

$$V_L = V_U + \left[ 1 - \frac{(1-\tau_c)(1-\tau_E)}{(1-\tau_D)} \right] D ,$$

The personal tax rate on interest income is  $\tau_D$ , and the personal tax rate on equity, is  $\tau_E$ .

You can derive this equation based on no arbitrage strategy that

*“Sell your holdings in firm U and buy instead a fraction  $\alpha$  of firm L's equity, and a fraction  $\beta \equiv \alpha(1-\tau_c)(1-\tau_E)/(1-\tau_D)$  of firm L's debt”*

Several remarks on Miller's models can be addressed.

Case 1,  $\tau_D = \tau_E = \tau_c = 0$

If there are no taxes at all so that  $\tau_D = \tau_E = \tau_c = 0$ , then  $V_L = V_U$ , which is exactly the result of M&M without tax.

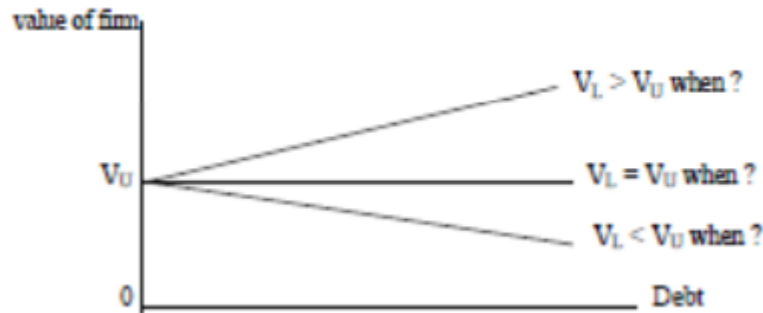
Case 2,  $\tau_D = \tau_E$

If personal taxes on debt and on equity are the same, the introduction of personal taxes does not affect the valuation formula, then  $V_L = V_U + \tau_c D$ , which is exactly the result of M&M with tax.

Case 3, if  $(1 - \tau_c)(1 - \tau_E) = (1 - \tau_D)$ , then  $V_L = V_U$ . To understand this result, note that  $(1 - \tau_D)$  is the after tax interest income on every dollar of debt, while  $(1 - \tau_c)(1 - \tau_E)$  is the after-tax income from dividends or and capital gains. If  $(1 - \tau_c)(1 - \tau_E) = (1 - \tau_D)$ , then the after-tax incomes from debt and equity are the same, so investors should be indifferent to the firm's capital structure meaning that the firm will have nothing to gain by using one type of securities rather than another.

Case 4, if  $(1 - \tau_D) > (1 - \tau_c)(1 - \tau_E)$ , then  $V_L > V_U$ . Debt is preferred to equity.

Case 5, if  $(1 - \tau_D) < (1 - \tau_c)(1 - \tau_E)$ , then  $V_L < V_U$ . All-equity may be preferred.



$V_u$	200000										
$\tau_c$		0-30% @ 1%									
$\tau_D$		0-30% @ 1%									
$\tau_E$		0-30% @ 1%									
D	0	20000	40000	60000	80000	100000	120000	140000	160000	180000	200000
$V_L$											

Assuming  $V_u=200000$ ,  $\tau_D$ ,  $\tau_E$ , and  $\tau_c$  are adjusted at 0-30% @ 1% (by using Scroll in EXCEL), we can get  $V_L$  for corresponding debt level.

Please complete the above table.

By adjusting the scrolls of  $\tau_D$ ,  $\tau_E$ , and  $\tau_c$ , you will be able to illustrate the above cases of Miller's proposition. Please illustrate these results.