DISCRETE MATHEMATICS	Undergraduate Course
HOMEWORK 6 SOL	College of Computer Science
	Zhejiang University
	Fall-Winter 2014

HOMEWORK 6

P361-362

26. Thirteen people on the softball team show up for a game.

a) How many ways are there to choose 10 players to take the field?

b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

c) Of the 13 people who show up, 3 are women. How many ways are there to choose 10 players to take the field if at least one of the players must be a woman?

Solution :

a) $C(13, 10) = 286$.

b) $P(13, 10) = 13!/3! = 1,037,836,800$.

c) There is only one way to choose the 10 players without choosing a woman, since there are exactly 10 men. Therefore there are $286 - 1 = 285$ ways to choose the players if at least one of them must be a woman.

28. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

Solution : $C(40, 17)$

32. How many strings of six lowercase letters from the English alphabet contain

a) the letter a?

b) the letters a and b?

c) the letters a and b in consecutive positions with a preceding b, with all the letters distinct?

d) the letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?

Solution :

a) $26^6 - 25^6$.

b) $26^6 - (25^6 + 25^6) + 24^6$.

c) $5P(24, 4)$.

d) $C(6,2)P(24,4)$.

P369

11. Give a formula for the coefficient of x^k in the expansion of $(x^2 - 1/x)^{100}$, where k is integer.

Solution : By the binomial theorem, the typical term in this expansion is $C(100, j)x^{100-j}(1/x)^j$, which simplifies to $C(100, j)x^{100-2j}$. As j runs from 0 to 100, the exponent runs from 100 down to -100 in decrements of 2. If we let k denote the exponent, then solving $k = 100-2j$ for j we obtain $j = (100 - k)/2$. Thus the values of k for which x^k appears in the expansion are -100, -98, $\cdots$, -2, 0, 2, 4, $\cdots$, 100. and for such values of k the coefficient is $C(100, (100 - k)/2)$.

24. Show that if p is a prime and k is an integer such that $1 \leq k \leq p-1$, then p divides $C(p, k)$.

Solution : We note that

$$C(p, k) = \frac{p!}{k!(p-k)!}$$

Clearly p divides the numerator. On the other hand, p can not divide the denominator, since the prime factorization of these factorials contains only numbers less than p. Therefore the factor p does not cancel when this fraction is reduced to the lowest terms (i.e. to a whole number), so p divides $C(p, k)$.

P379

6. How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?

Solution : $C(3+5-1, 5) = C(7, 5) = C(7, 2) = 21$

10. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a) a dozen croissants?

b) three dozen croissants?

c) two dozen croissants with at least two of each kind?

d) two dozen croissants with no more than two broccoli croissants?

e) two dozen croissants with at least five chocolate croissants and at least three almond croissants?

f) two dozen croissants with at least one plain croissants, at least two cherry croissants, at least three chocolate croissants, at least one almond croissant, at least two apple croissants, and no more than three broccoli croissant?

Solution : a) $C(6+12-1, 12) = C(17, 12) = 6188$

b) $C(6+36-1, 36) = C(41, 36) = 749,398$

c) $C(6+12-1, 12) = C(17, 12) = 6188$

d) We first compute the number of ways to violate the restriction, by choosing at least three broccoli croissants: $C(6+21-1, 21) = C(26, 21) = 65,780$.

Since there are $C(6+24-1, 24) = C(29, 24) = 118,755$ ways to pick 24 croissants without any restriction, there must be 118,755-65,780=52,975 ways to choose two dozen croissants with at least five chocolate croissants and at least three almond croissants.
e) $C(6+16-1, 16) = 20,349$.
f) First let us include all the lower bound restrictions. If we choose the required 9 croissants, then there are $24-9=15$ left to choose, and if there are no restrictions on the broccoli croissant then there would be $C(6+15-1, 15) = 15,504$ ways to make the selections. If addition we were to violate the broccoli croissant restriction by choose at least 4 broccoli croissants, there would be $C(6+11-1, 11) = 4,368$ choice. Therefore the number of ways to make the selection without violating the restriction is $15504 - 4368 = 11,136$.

14. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 17$$

where x_1, x_2, x_3, x_4 are nonnegative integers?

$Solution:$ $C(17+4-1, 17) = C(20, 17) = C(20, 3) = 1,140$.

16. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$$

where $x_i, i = 1, 2, 3, 4, 5, 6,$ is a nonnegative integer such that
a) $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$?
b) $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 > 5$ and $x_6 \geq 6$?
c) $x_1 \leq 5$?
d) $x_1 < 8$ and $x_2 > 8$?

$Solution:$ a) $C(17+6-1, 17) = C(22, 17) = 26,334$.
b) $C(7+6-1, 7) = C(12, 7) = 792$.
c) $C(6+29-1, 29) - C(6+23-1, 23) = C(34, 5) - C(28, 5)$.
d) The number of solutions with $x_2 \geq 9$ but without the restriction on x_1 is $C(6+20-1, 20) = C(25, 20) = 53,130$.
The number of solution violating the additional restriction by having $x_1 \geq 8$ is $C(6+12-1, 12) = C(17, 12) = 6,188$. Therefore the answer is $53,130 - 6,188 = 46,942$.

32. How many different strings can be made from the letter in $AARDVARK$, using all the letters, if all three As must be consecutive?

$Solution:$ We can treat the 3 consecutive As as one letter. Thus we have 6 letter, of which 2 are the same, the answer is $6!/2! = 360$.

42. In bridge, the 52 cards of a standard deck are dealt to four players. How many different ways are there to deal bridge hands to four players?

Solution : $52!/(13!)^4$.