浙江大学 2007-2008 学年 秋冬 季学期

研究生《计算理论》课程期末考试试卷

开课学院：计算机学院 考试形式：闭卷，允许带 ________ 入场

考试时间: 2007 年 1 月15日，所需时间:120 分钟，任课教师: ________

考生姓名：________________ 学号：________________ 专业：________________

题序	1	2	3	4	5	6	7	总分
得分								
评卷人								

Zhejiang University
Theory of Computation, Fall-Winter 2007
Final Exam

1. **(16%)** Suppose there are four languages A, B, C and D. Each of these languages may or may not be recursively enumerable. However, we know the following about them:

   i. There is a reduction from A to B

   ii. There is a reduction from B to C

   iii. There is a reduction from D to C

   Below are eight statements indicate whether each is:

   - CERTAIN to be true: regardless of what problems$A$, B, C and D are
   - MAYBE true: depending on what A, B, C and D are
   - NEVER true: regardless of what A, B, C and D are

   (a) (   ) A is recursively enumerable but not recursive, and C is recursive.

   (b) (   ) A is not recursive and D is not recursively enumerable.

   (c) (   ) The complement of A is not recursively enumerable, but the complement of B is recursively enumerable.

   (d) (   ) The complement of B is not recursive, but the complement of C is recursive.

   (e) (   ) If A is recursive, then the complement of B is recursive.

   (f) (   ) If C is recursive, then the complement of D is recursive.

   (g) (   ) If C is recursively enumerable, then the union of B and D is recursively enumerable.

   (h) (   ) If C is recursively enumerable, then the intersection of B and D is recursively enumerable.

2. **(14%)** Suppose there are four languages A, B, C and D. Each of these languages may or may not be in the class $\mathcal{NP}$. However, we know the following about them:

   i. There is a polynomial-time reduction from A to B
   ii. There is a polynomial-time reduction from B to C
   iii. There is a polynomial-time reduction from D to C

   Below are seven statements Indicate whether each is:

   • CERTAIN to be true, regardless of what problems A, B, C and D are and regardless of the resolution of unknown relationships among complexity classes of "which is $\mathcal{P} = \mathcal{NP}$" is one example.

   • MAYBE true, depending on what A, B, C and D are and/or depending on the resolution of unknown relationships such as $\mathcal{P} = \mathcal{NP}$?

   • NEVER true, regardless of what A, B, C and D are and regardless of the resolution of unknown relationships such as $\mathcal{P} = \mathcal{NP}$?

   (a) (   ) If A is $\mathcal{NP}$-complete then C is $\mathcal{NP}$-complete.

   (b) (   ) A is $\mathcal{NP}$-complete and C is in $\mathcal{P}$.

   (c) (   ) B is $\mathcal{NP}$-complete and D is in $\mathcal{P}$.

   (d) (   ) If A is $\mathcal{NP}$-complete and B is in $\mathcal{NP}$ then B is $\mathcal{NP}$-complete.

   (e) (   ) If C is $\mathcal{NP}$-complete then D is in $\mathcal{NP}$.

   (f) (   ) C is in $\mathcal{P}$ and the complement of D is not in $\mathcal{P}$.

   (g) (   ) B is not in $\mathcal{P}$ and A is not in $\mathcal{NP}$.

3. **(12%)** Consider the binary operator $\circ$ on languages as follows: given two languages L_1 and L_2 over Σ, $L_1 \circ L_2$ consists of words of the form uv such that $u \in L_1$, $v \in L_2$ and $|u| = |v|$.

   (a) Prove that if L_1 and L_2 are regular languages, then $L_1 \circ L_2$ is context-free.

   (b) Give a counter-example to disprove that if L_1 is a regular language and L_2 is a context-free language, then $L_1 \circ L_2$ is context-free.

4. **(20%)**

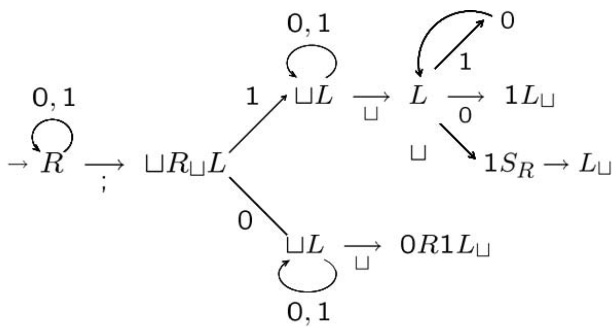
   (a) Give a context-free grammar for the language

   $$L_3 = \{a^m b^m c a^{2n} b^{2n} \mid m, n \in \mathbb{N}\}.$$

   (b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_3.

5. **(10%)** Show that the following language
   $\{$"M_1" "M_2" $\mid M_1, M_2$ are Turing machines and both M_1 and M_2 halt on blank tape$\}$
   is recursively enumerable. An informal description suffices.

6. **(16%)**Let the following Turing machine M compute function $f(x, y)$, where x and y are represented by binary strings respectively and separated with the symbol ";", i.e. the initial configuration in form of $\triangleright\underline{\sqcup}x; y$.



   (a) Describe the key configurations when M started from the configuration $\triangleright\underline{\sqcup}10111; 10$.

   (b) Try to give the function $f(x, y)$ computed by Turing Machine M.

7. **(12%)** The **non-tautology, NT** problem is defined as follows: given a Boolean expression E, does there exist a truth-assignment for the variables of E that makes E false.

   (a) Prove NT is in $\mathcal{NP}$.

   (b) Describe a polynomial-time reduction from SAT to NT and show that NT problem is $\mathcal{NP}$-complete.