	Undergraduate Course
DISCRETE MATHEMATICS	College of Computer Science
HOMEWORK 12 SOL	**ZHEJIANG UNIVERSITY**
	Fall-Winter, 2013

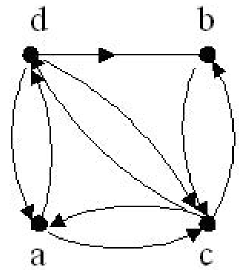
HOMEWORK 12

P618

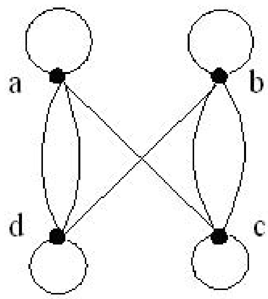
11. Draw a graph with the given adjacency matrix.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Solution:



15. Represent the given graph using an adjacency matrix.



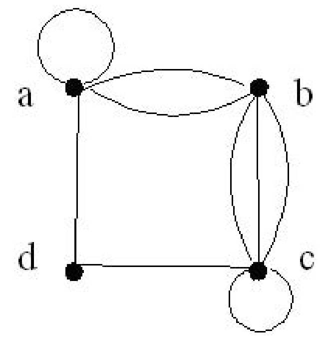
Solution: Order the vertices as a, b, c, d.

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

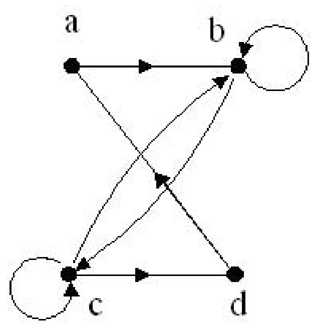
17. Draw an undirected graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solution:



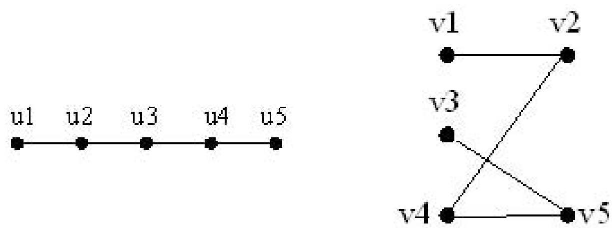
19. Find the adjacency matrix of the given directed multigraph.



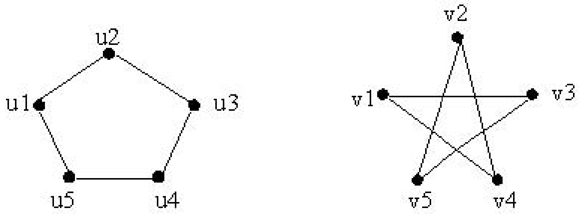
Solution: Order the vertices as a, b, c, d.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

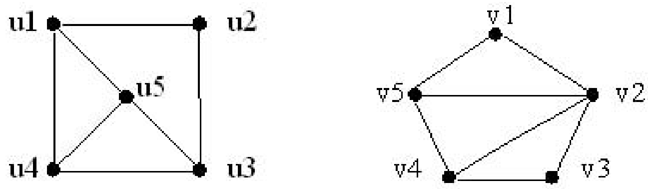
34-37. Determine whether the given pair of graphs is isomorphic.
34.



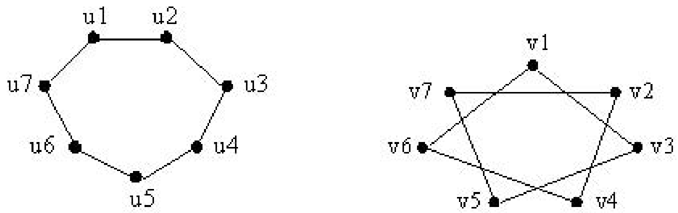
35.



36.



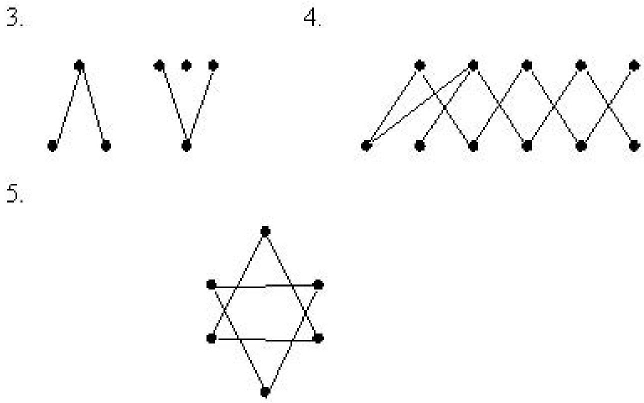
37.



Solution: The pair of graphs in Exercises 34, 35 and 37 is isomorphic. The pair of graphs in Exercise 36 is not isomorphic.

P 629

6. How many connected components does each of the graphs in Exercises 3-5 have ? For each graph find each of its connected components.



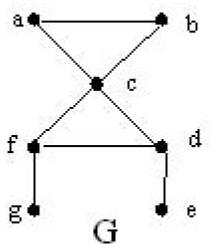
Solution: The graph in Exercises 3 has three connected components.
The graph in Exercises 4 has one connected component.
The graph in Exercises 5 has two connected components

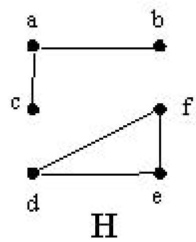
26. Show that a connected graph with n vertices has at least $n-1$ edges.
Solution: The graph is connected, so the degree of each vertices is 2 at least.(except the start and the end vertex). And the degree of the start and the end is 1 at least. Since the total degree is at least $2(n-2)+2=2n-2$. Therefore, a connected graph with n vertices has at least $n-1$ edges.

27. Let $G=(V,E)$ be a simple graph. Let R be the relation on V consisting of pairs of vertices (u,v) such that there is a path from u to v or such that $u=v$. Show that R is an equivalence relation.
Solution:
a) R is reflexive by definition.
b) Assume the $(u,v)\in R$; then there is a path from u to v. Then $(v,u)\in R$ since there is a path from v to u, namely, the path from u to v traversed backward. Hence, R is symmetric.
c) Assume $(u,v)\in R$ and $(v,w)\in R$; then there are paths from u to v and from v to w. Putting these two paths together gives a path from u to w, Hence $(u,w)\in R$. It follows that R is transitive. Hence, R is an equivalence relation on V.

34. Show that a vertex c in the connected simple graph G is a cut vertex if and only if there are vertices u and v, both different from c, such that every path between u and v passes through c.
Solution: If c is a cut vertex, then the removal of c and all edges incident with it produces a subgraph that is not connected. Hence, every path between u and v passes through c.
If every path between u and v passes through c, then the removal of c and all edges incident with it produces a subgraph with more connected components.
Hence, c is a cut vertex.

52. Use Exercise 52 to show that the graph G in Figure 2 is connected whereas the graph H in the figure is not connected.



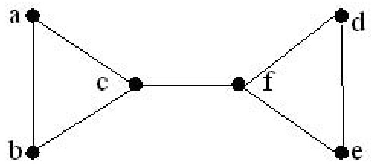


Solution: The graph G is connected if and only if every off-diagonal entry of $A+A^2+\cdots+A^{n-1}$ is positive where A is the adjacency matrix of G. Let G be a graph with adjacency matrix A and H be a graph with adjacency matrix B. Since every (i,j) entry of A^6 does not equal to 0, the graph G is connected. Since some (i,j) entry of B^5 equals to 0, the graph H is not connected.

P 645

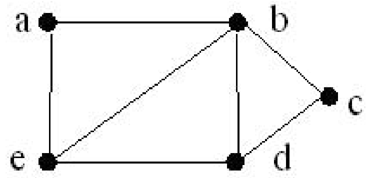
Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

30.



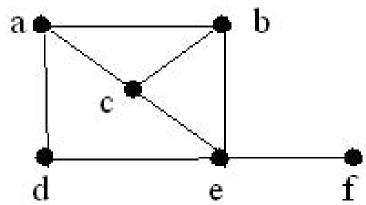
Solution: No Hamilton circuit exists, because every edge in the graph is incident to a vertex of degree 2 and must be in the circuit. And the degrees of the vertices a, b, d, e are all 2, every edge incident with these vertices must be part of any Hamilton circuit. Since the edge $\{e, f\}$ can be removed from consideration, it is easy to see there is no Hamilton circuit.

31.



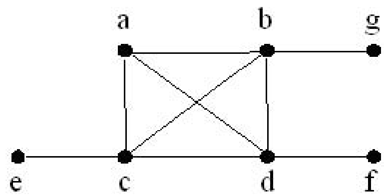
Solution: a, b, c, d, e a is a Hamilton circuit.

32.



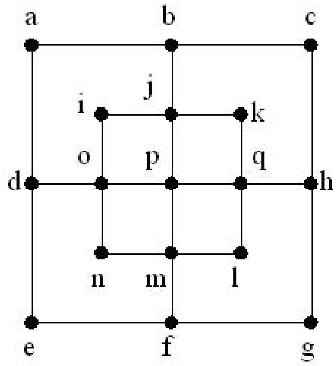
Solution: No Hamilton circuit exists, because once a purported circuit has reached f, it would have nowhere to go.

33.



Solution: No Hamilton circuit exists, because once a purported circuit has reached e, f or g, it would have nowhere to go.

34.



Solution: No Hamilton circuit exists, because every edge in the graph is incident to a vertex of degree 2 and must be in the circuit. And the degrees of the vertices a, c, e, g, i, k, n and l are all 2, the edges incident with these vertices must be part of any Hamilton circuit. We can remove the edges $\{b, j\}, \{d, q\}, \{f, m\}, \{h, o\}, \{o, p\}, \{j, p\}, \{q, p\}$ and $\{m, p\}$ from consideration. It is easy to see that no Hamilton circuit exists.