2017~2018 学年秋冬学期《计算理论》期末试卷回忆版

一、(24%) 判断正误

(1) If A and $A \cup B$ are both regular, then B is also regular.

(2) If A is regular and B is non-regular, then AB is non-regular.

(3) $\{xcy | x, y \in \{a, b\}^*, |x| \leq y \leq 3|x|\}$ is context-free.

(4) Every regular language can be generated by a context-free grammar.

(5) If A is recursive and $B \subseteq A$, then B is also recursive.

(6) Every recursively enumerable language is an infinite language.

(7) There exists a computable function which is not a primitive recursive function.

(8) If A, B and $\overline{A \cup B}$ are all recursively enumerable, and $A \cap B = \emptyset$, then both A and B are decidable.

(9) $\{\text{“}M\text{ ” “}w\text{ ” } | \text{TM } M \text{ accepts string } w \text{ in less than } 2018 \text{ steps}\}$ is recursive.

(10) $\{\text{“}M\text{ ” } | \text{TM } M \text{ accepts exactly } 2018 \text{ strings}\}$ is recursively enumerable but not recursive.

(11) Let $H_e = \{\text{“}M\text{ ” } | \text{TM } M \text{ halts on string } e\}$. If $H_e \leq \bar{L}$, then L is recursively enumerable but not recursive.

(12) A language is recursive if and only if it is Turing-enumerable.

二、(18%) 判断以下语言是不是正则的，并说明理由。

(1) $\{wtw | w, t \in \{a, b\}^+\}$

(2) $\{wtw | w, t \in \{a, b\}^*\}$

三、(20%) 写出生成下面这个 context-free language 的 CFG 和接受它的 PDA。

$$\{ww^R ca^m b^n | w \in \{a, b\}^*, m \neq n\}$$

四、(10%) 写出接受下面这个语言的图灵机。

$$\{uvcww^R | u, v, w \in \{a, b\}^*, |u| = 2|v|\}$$

五、(10%) 证明下面的函数是 primitive recursive function。

$$\phi_k(v_1, v_2, \ldots, v_k) = \max(v_1, v_2, \ldots, v_k)$$
$$k \geq 2, k \in \mathbb{N}$$

六、(18%) 判断下面的语言是 recursive, recursively enumerable but not recursive 还是 non-recursively enumerable 并证明。不能直接使用 Rice 定理。

(1) $\{\text{“}M\text{ ”} \mid M \text{ is a TM and } L(M) \text{ is uncountable } \}$

(2) $\{\text{“}M\text{ ”} \mid \text{TM } M \text{ accepts at least two strings with different lengths}\}$