CGNS/SIDS proposal for extensions

Authors: Meilin Yu and Z. J. Wang, the University of Kansas

Contact: zjw@ku.edu

## Implementation of cubic elements for unstructured zones

The proposal is an extension of the element structure definition (*Elements_t*) for unstructured zones. We propose to add cubic elements to *ElementType*. This extension can enhance the performance of CGNS in handling high-order ($\geq 2$) elements which are widely used in high-order CFD simulations. Moreover, this extension can further strengthen the capability of CGNS on accurate presentation of complex geometry, especially when a coarse grid is provided.

The new cubic element types proposed are summarized as follows.

| Dimensionality of the Element | Shape | Cubic Interpolation |
|---|---|---|
| 1-D | Line | BAR_4 |
| 2-D | Triangle | TRI_9, TRI_10 |
| | Quadrangle | QUAD_12, QUAD_16 |
| 3-D | Tetrahedron | TETRA_16, TETRA_20 |
| | Pyramid | PYRA_21, PYRA_29, PYRA_30 |
| | Pentahedron | PENTA_24, PENTA_38, PENTA_40 |
| | Hexahedron | HEXA_32, HEXA_56, HEXA_64 |

The element numbering convention is specified as follows.

Note that 'N' stands for grid point identification number. 'E' indicates edge identification number. A negative edge means that the edge is used in its reverse direction. 'F' stands for face identification number.
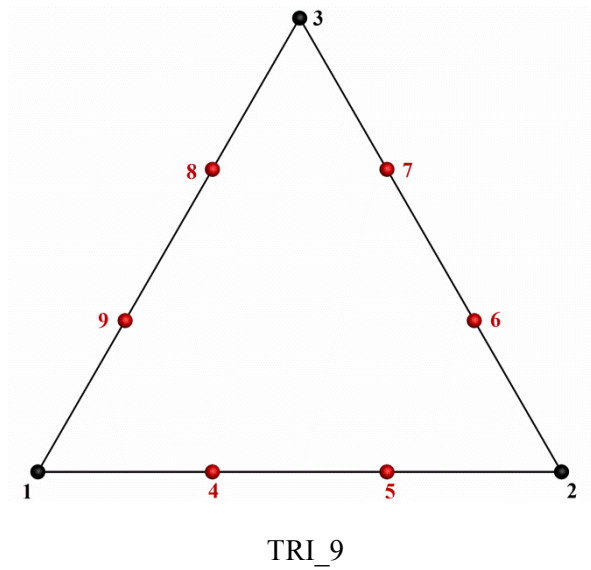
**1. 1-D Element**



BAR_4

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N3,N4 |

## 2. 2-D (Surface) Element

*2.1 Triangular Element*



TRI_9

<u>*Edge Definition*</u>

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N4,N5 |
| E2 | N2,N3 | N6,N7 |
| E3 | N3,N1 | N8,N9 |

<u>*Face Definition*</u>

| Oriented edges | Corner nodes | Mid-edge nodes | Oriented edges |
|---|---|---|---|
| F1 | N1,N2,N3 | N4,N5,N6,N7,N8,N9 | E1,E2,E3 |
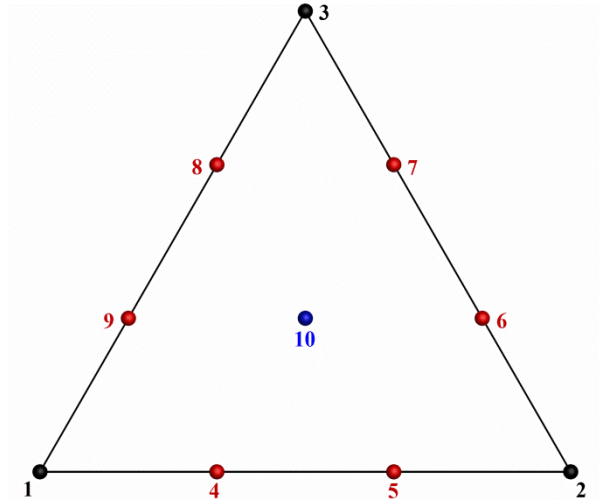
TRI_10

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N4,N5 |
| E2 | N2,N3 | N6,N7 |
| E3 | N3,N1 | N8,N9 |

*Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Mid-face nodes | Oriented edges |
|---|---|---|---|---|
| F1 | N1,N2,N3 | N4,N5,N6,N7,N8,N9 | N10 | E1,E2,E3 |

*2.2 Quadrilateral Element*
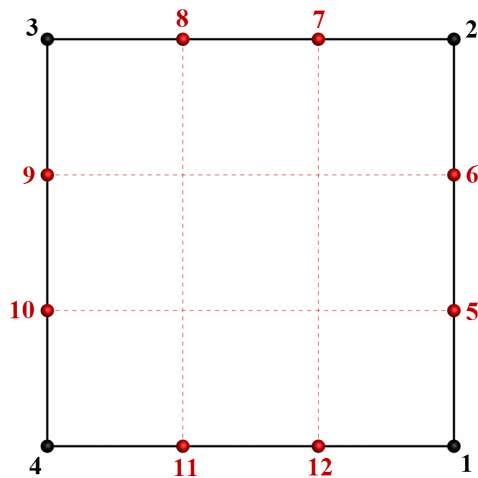


QUAD_12

## Edge Definition

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N5,N6 |
| E2 | N2,N3 | N7,N8 |
| E3 | N3,N4 | N9,N10 |
| E4 | N4,N1 | N11,N12 |

## Face Definition

| Oriented edges | Corner nodes | Mid-edge nodes | Oriented edges |
|---|---|---|---|
| F1 | N1,N2,N3,N4 | N5,N6,N7,N8,N9, N10,N11,N12 | E1,E2,E3,E4 |
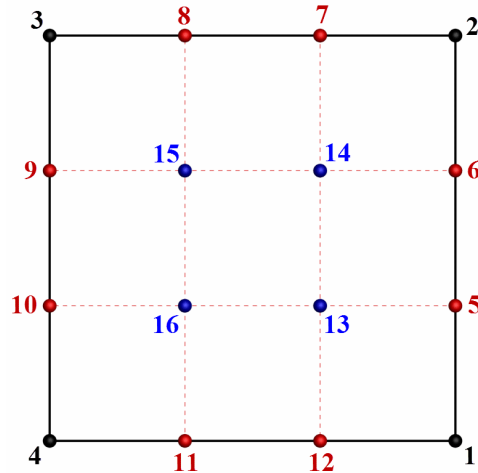
QUAD_16

## Edge Definition

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N5,N6 |
| E2 | N2,N3 | N7,N8 |
| E3 | N3,N4 | N9,N10 |
| E4 | N4,N1 | N11,N12 |

## Face Definition

| Oriented edges | Corner nodes | Mid-edge nodes | Mid-face nodes | Oriented edges |
|---|---|---|---|---|
| F1 | N1,N2,N3,N4 | N5,N6,N7,N8,N9, N10,N11,N12 | N13,N14,N15, N16 | E1,E2,E3 |

# 3. 3-D (Volume) Element

*3.1 Tetrahedral Element*
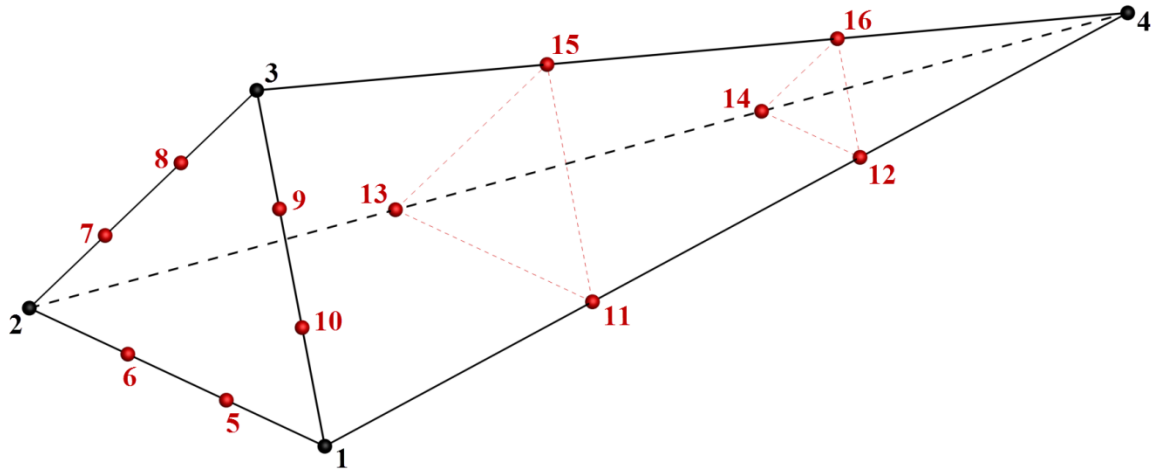


TETRA_16

## Edge Definition

| Oriented edges | Corner nodes | Mid-nodes |
|:---:|:---:|:---:|
| E1 | N1,N2 | N5,N6 |
| E2 | N2,N3 | N7,N8 |
| E3 | N3,N1 | N9,N10 |
| E4 | N1,N4 | N11,N12 |
| E5 | N2,N4 | N13,N14 |
| E6 | N3,N4 | N15,N16 |

## Face Definition

| Oriented edges | Corner nodes | Mid-edge nodes | Oriented edges |
|:---:|:---:|:---:|:---:|
| F1 | N1,N3,N2 | N10,N9,N8,N7,N6,N5 | -E3,-E2,-E1 |
| F2 | N1,N2,N4 | N5,N6,N13,N14,N12,N11 | E1,E5,-E4 |
| F3 | N2,N3,N4 | N7,N8,N15,N16,N14,N13 | E2,E6,-E5 |
| F4 | N3,N1,N4 | N9,N10,N11,N12,N16,N15 | E3,E4,-E6 |
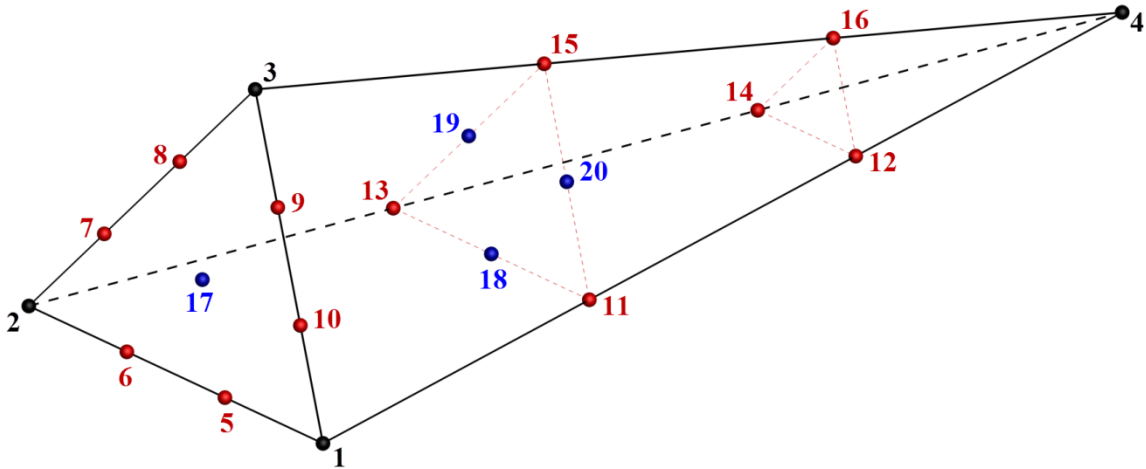
TETRA_20

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N5,N6 |
| E2 | N2,N3 | N7,N8 |
| E3 | N3,N1 | N9,N10 |
| E4 | N1,N4 | N11,N12 |
| E5 | N2,N4 | N13,N14 |
| E6 | N3,N4 | N15,N16 |

*Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Mid-face nodes | Oriented edges |
|---|---|---|---|---|
| F1 | N1,N3,N2 | N10,N9,N8,N7,N6,N5 | N17 | -E3,-E2,-E1 |
| F2 | N1,N2,N4 | N5,N6,N13,N14,N12,N11 | N18 | E1,E5,-E4 |
| F3 | N2,N3,N4 | N7,N8,N15,N16,N14,N13 | N19 | E2,E6,-E5 |
| F4 | N3,N1,N4 | N9,N10,N11,N12,N16,N15 | N20 | E3,E4,-E6 |

*3.2 Pyramid Element*
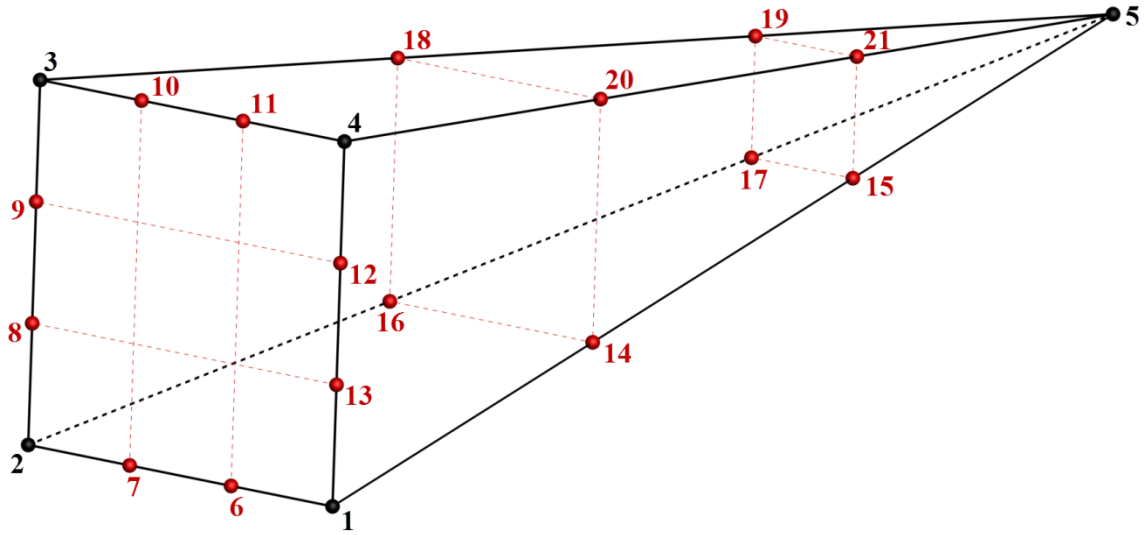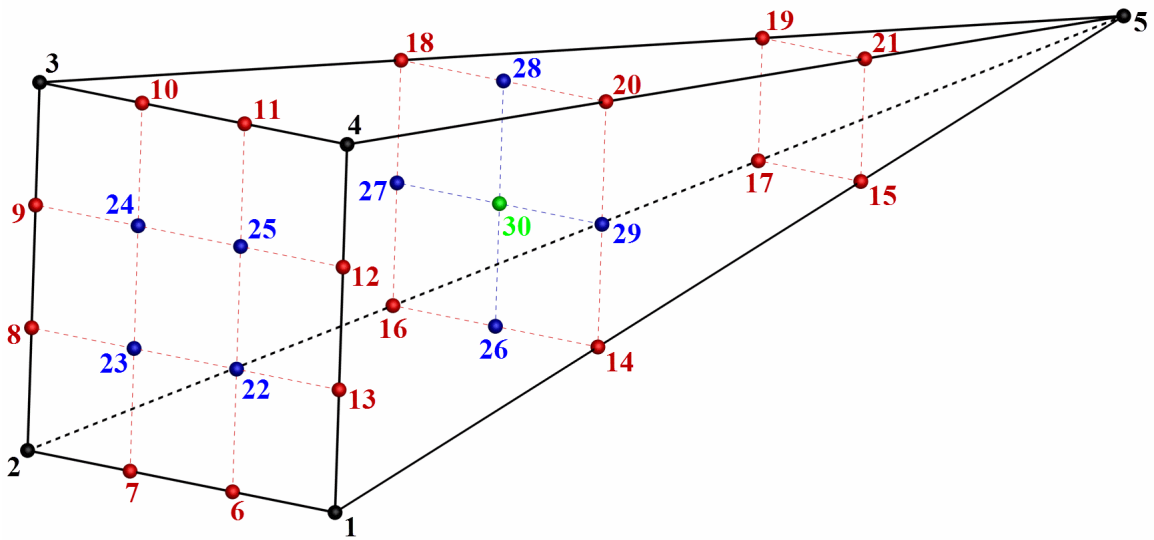
PYRA_21

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N6,N7 |
| E2 | N2,N3 | N8,N9 |
| E3 | N3,N4 | N10,N11 |
| E4 | N4,N1 | N12,N13 |
| E5 | N1,N5 | N14,N15 |
| E6 | N2,N5 | N16,N17 |
| E7 | N3,N5 | N18,N19 |
| E8 | N4,N5 | N20,N21 |

*Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Oriented edges |
|---|---|---|---|
| F1 | N1,N4,N3,N2 | N13,N12,N11,N10,N9,N8,N7,N6 | -E4,-E3,-E2,-E1 |
| F2 | N1,N2,N5 | N6,N7,N16,N17,N15,N14 | E1,E6,-E5 |
| F3 | N2,N3,N5 | N8,N9,N18,N19,N17,N16 | E2,E7,-E6 |
| F4 | N3,N4,N5 | N10,N11,N20,N21,N19,N18 | E3,E8,-E7 |
| F5 | N4,N1,N5 | N12,N13,N14,N15,N21,N20 | E4,E5,-E8 |
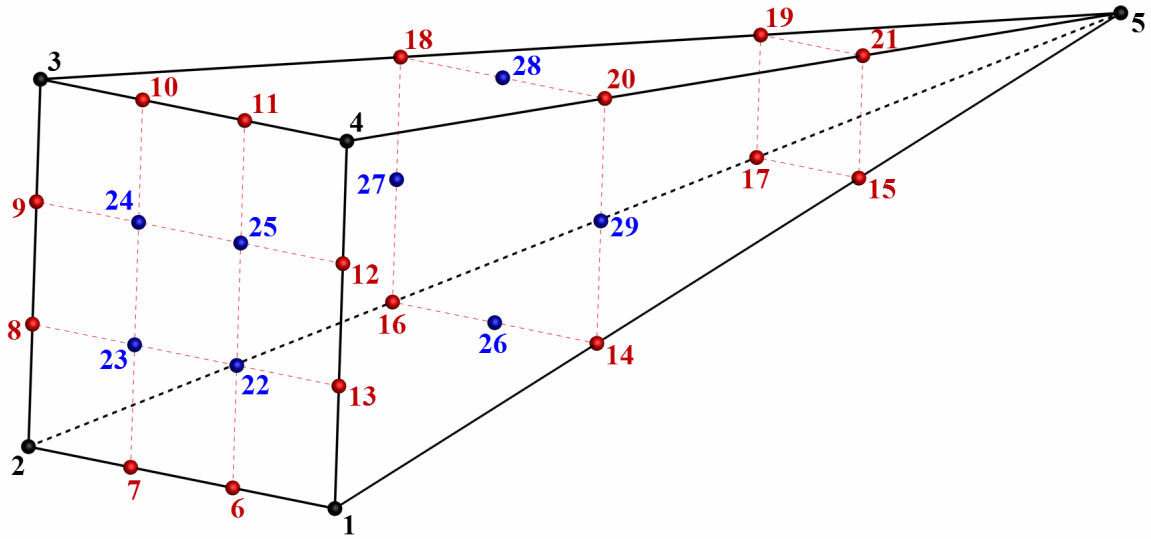
PYRA_29



PYRA_30

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|:---:|:---:|:---:|
| E1 | N1,N2 | N6,N7 |
| E2 | N2,N3 | N8,N9 |
| E3 | N3,N4 | N10,N11 |
| E4 | N4,N1 | N12,N13 |
| E5 | N1,N5 | N14,N15 |
| E6 | N2,N5 | N16,N17 |
| E7 | N3,N5 | N18,N19 |
| E8 | N4,N5 | N20,N21 |

| Oriented edges | Corner nodes | Mid-edge nodes | Mid-face nodes | Oriented edges |
|:---:|:---:|:---:|:---:|:---:|
| F1 | N1,N4,N3,N2 | N13,N12,N11,N10,N9, N8,N7,N6 | N22,N25,N24, N23 | -E4,-E3,-E2,-E1 |
| F2 | N1,N2,N5 | N6,N7,N16,N17,N15,N14 | N26 | E1,E6,-E5 |
| F3 | N2,N3,N5 | N8,N9,N18,N19,N17,N16 | N27 | E2,E7,-E6 |
| F4 | N3,N4,N5 | N10,N11,N20,N21,N19,N18 | N28 | E3,E8,-E7 |
| F5 | N4,N1,N5 | N12,N13,N14,N15,N21,N20 | N29 | E4,E5,-E8 |

## 3.3 Pentahedral Element
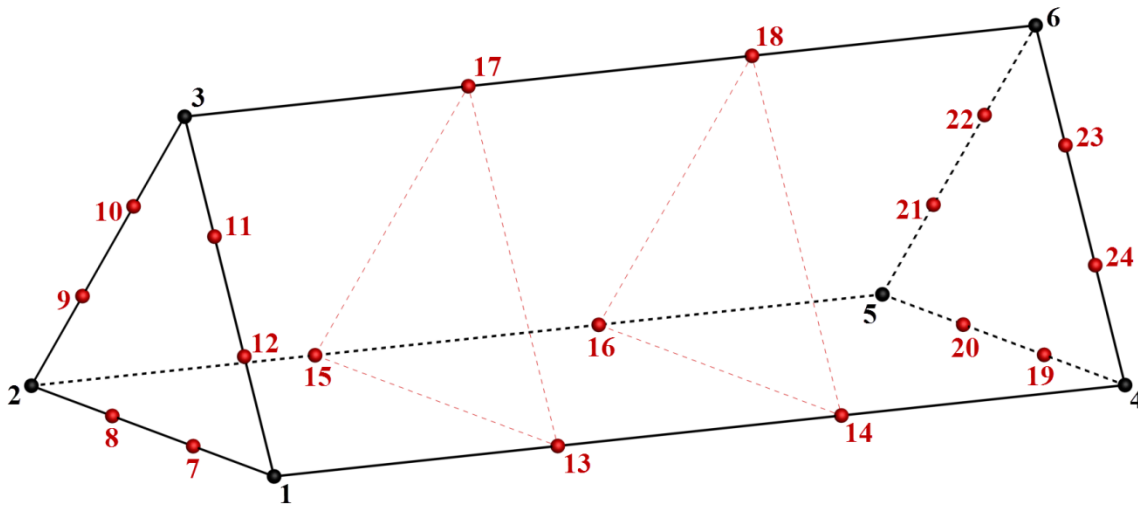


PENTA_24

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|:---:|:---:|:---:|
| E1 | N1,N2 | N7,N8 |
| E2 | N2,N3 | N9,N10 |
| E3 | N3,N1 | N11,N12 |
| E4 | N1,N4 | N13,N14 |
| E5 | N2,N5 | N15,N16 |
| E6 | N3,N6 | N17,N18 |
| E7 | N4,N5 | N19,N20 |
| E8 | N5,N6 | N21,N22 |
| E9 | N6,N4 | N23,N24 |

*Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Oriented edges |
|:---:|:---:|:---:|:---:|
| F1 | N1,N2,N5,N4 | N7,N8,N15,N16,N20,N19,N14,N13 | E1,E5,-E7,-E4 |
| F2 | N2,N3,N6,N5 | N9,N10,N17,N18,N22,N21,N16,N15 | E2,E6,-E8,-E5 |

| F3 | N3,N1,N4,N6 | N11,N12,N13,N14,N24,N23,N18,N17 | E3,E4,-E9,-E6 |
| F4 | N1,N3,N2 | N12,N11,N10,N9,N8,N7 | -E3,-E2,-E1 |
| F5 | N4,N5,N6 | N19,N20,N21,N22,N23,N24 | E7,E8,E9 |
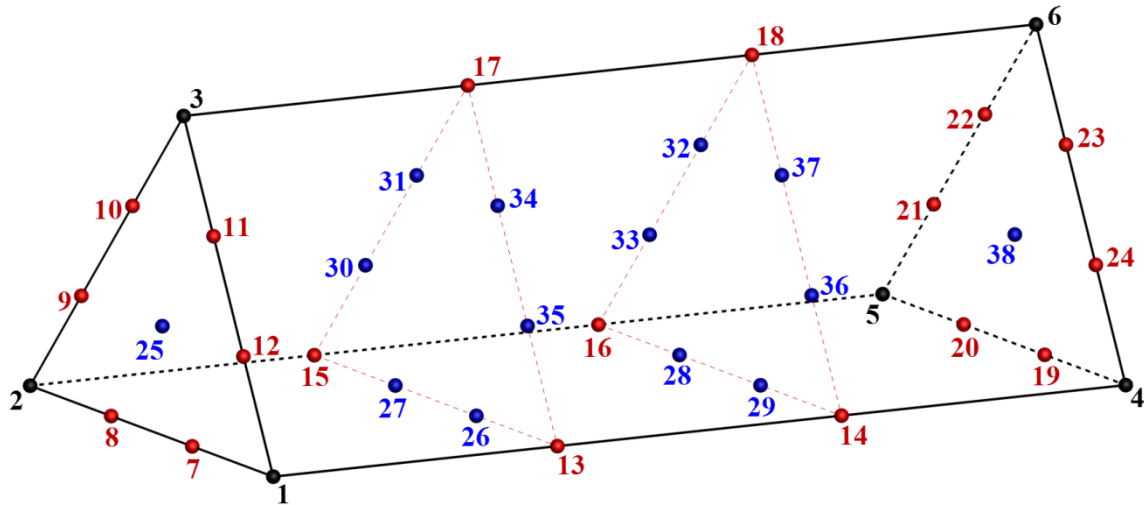


PENTA_38
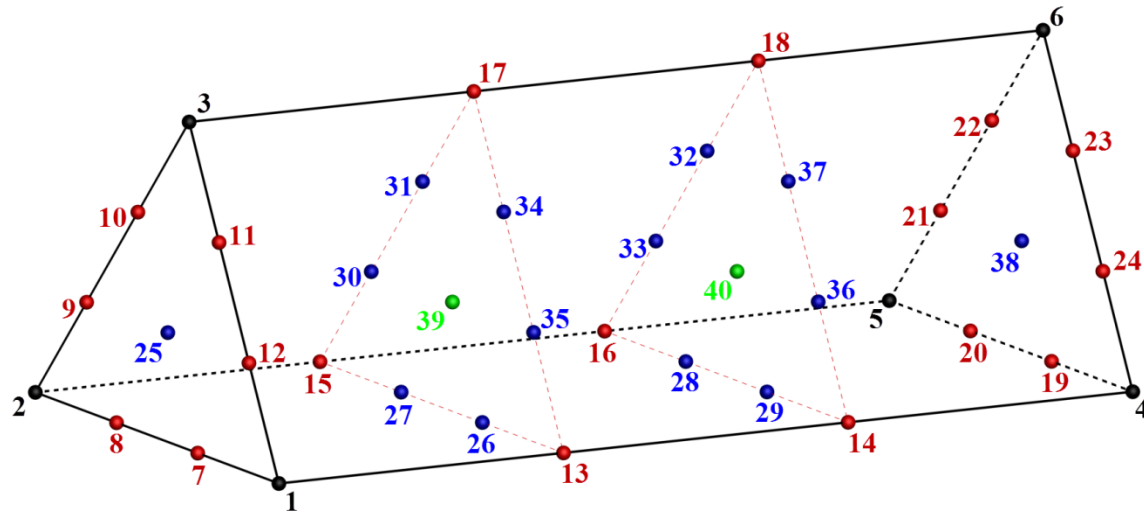


PENTA_40

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
| --- | --- | --- |
| E1 | N1,N2 | N7,N8 |
| E2 | N2,N3 | N9,N10 |
| E3 | N3,N1 | N11,N12 |
| E4 | N1,N4 | N13,N14 |
| E5 | N2,N5 | N15,N16 |
| E6 | N3,N6 | N17,N18 |

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E7 | N4,N5 | N19,N20 |
| E8 | N5,N6 | N21,N22 |
| E9 | N6,N4 | N23,N24 |

*Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Mid-face nodes | Oriented edges |
|---|---|---|---|---|
| F1 | N1,N2,N5,N4 | N7,N8,N15,N16,N20, N19,N14,N13 | N26,N27,N28, N29 | E1,E5,-E7,-E4 |
| F2 | N2,N3,N6,N5 | N9,N10,N17,N18,N22, N21,N16,N15 | N30,N31,N32, N33 | E2,E6,-E8,-E5 |
| F3 | N3,N1,N4,N6 | N11,N12,N13,N14,N24, N23,N18,N17 | N34,N35,N36, N37 | E3,E4,-E9,-E6 |
| F4 | N1,N3,N2 | N12,N11,N10,N9,N8,N7 | N25 | -E3,-E2,-E1 |
| F5 | N4,N5,N6 | N19,N20,N21,N22,N23,N24 | N38 | E7,E8,E9 |

*3.4 Hexahedral Element*



HEXA_32

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N9,N10 |
| E2 | N2,N3 | N11,N12 |
| E3 | N3,N4 | N13,N14 |
| E4 | N4,N1 | N15,N16 |

| | | |
|---|---|---|
| E5 | N1,N5 | N17,N18 |
| E6 | N2,N6 | N19,N20 |
| E7 | N3,N7 | N21,N22 |
| E8 | N4,N8 | N23,N24 |
| E9 | N5,N6 | N25,N26 |
| E10 | N6,N7 | N27,N28 |
| E11 | N7,N8 | N29,N30 |
| E12 | N8,N5 | N31,N32 |

## *Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Oriented edges |
|---|---|---|---|
| F1 | N1,N4,N3,N2 | N16,N15,N14,N13,N12,N11,N10,N9 | -E4,-E3,-E2,-E1 |
| F2 | N1,N2,N6,N5 | N9,N10,N19,N20,N26,N25,N18,N17 | E1,E6,-E9,-E5 |
| F3 | N2,N3,N7,N6 | N11,N12,N21,N22,N28,N27,N20,N19 | E2,E7,-E10,-E6 |
| F4 | N3,N4,N8,N7 | N13,N14,N23,N24,N30,N29,N22,N21 | E3,E8,-E11,-E7 |
| F5 | N1,N5,N8,N4 | N17,N18,N32,N31,N24,N23,N15,N16 | E5,-E12,-E8,E4 |
| F6 | N5,N6,N7,N8 | N25,N26,N27,N28,N29,N30,N31,N32 | E9,E10,E11,E12 |
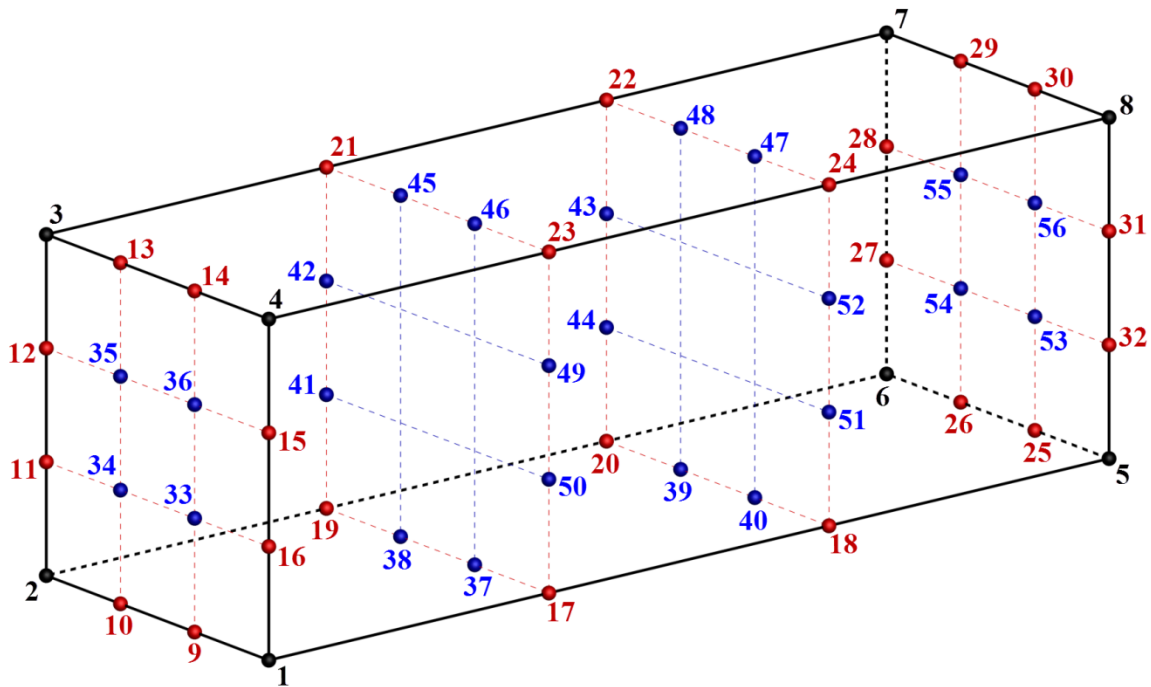


HEXA_56

HEXA_64

*Edge Definition*

| Oriented edges | Corner nodes | Mid-nodes |
|---|---|---|
| E1 | N1,N2 | N9,N10 |
| E2 | N2,N3 | N11,N12 |
| E3 | N3,N4 | N13,N14 |
| E4 | N4,N1 | N15,N16 |
| E5 | N1,N5 | N17,N18 |
| E6 | N2,N6 | N19,N20 |
| E7 | N3,N7 | N21,N22 |
| E8 | N4,N8 | N23,N24 |
| E9 | N5,N6 | N25,N26 |
| E10 | N6,N7 | N27,N28 |
| E11 | N7,N8 | N29,N30 |
| E12 | N8,N5 | N31,N32 |

*Face Definition*

| Oriented edges | Corner nodes | Mid-edge nodes | Mid-face nodes | Oriented edges |
|---|---|---|---|---|
| F1 | N1,N4,N3,N2 | N16,N15,N14,N13,N12, N11,N10,N9 | N33,N36,N35, N34 | -E4,-E3,-E2,-E1 |
| F2 | N1,N2,N6,N5 | N9,N10,N19,N20,N26, N25,N18,N17 | N37,N38,N39, N40 | E1,E6,-E9,-E5 |
| F3 | N2,N3,N7,N6 | N11,N12,N21,N22,N28, N27,N20,N19 | N41,N42,N43, N44 | E2,E7,-E10,-E6 |
| F4 | N3,N4,N8,N7 | N13,N14,N23,N24,N30, | N45,N46,N47, | E3,E8,-E11,-E7 |

| F5 | N1,N5,N8,N4 | N29,N22,N21<br>N17,N18,N32,N31,N24,<br>N23,N15,N16 | N48<br>N49,N50,N51,<br>N52 | E5,-E12,-E8,E4 |
|---|---|---|---|---|
| F6 | N5,N6,N7,N8 | N25,N26,N27,N28,N29,<br>N30,N31,N32 | N53,N54,N55,<br>N56 | E9,E10,E11,E12 |