Stat, fast

(I already said that)

November 15, 2016

0 Synopsis

Come ready to be put on the spot. You'll be less put on the spot if you know the following.

1 Probs

- [Permut/Combin]ation
- Choose
- Random Variables
- Discrete Distributions
- Continuous Distributions
- Joint Distributions
- $\mathrm{E}[aX], \mathrm{Var}[X], \mathrm{E}[XY + Z]$ and $\mathrm{Var}[aX + bY]$
- Covariance Vs Correlation Vs Causation

  Vs Independent Vs Mutually Exclusive
- Bayes' Theorem
- Standard Error ↓

2 Inf(erence)

- MLE
- CLT
- Bootstrapping Vs Bayes

  Vs Confidence Intervals Vs Nonparametric (NP) estimation Vs NP tests ↓

3 Testing

- $\emptyset$ Vs H_a
- p Vs α Vs Type II
- Multiple testing correction via Bonferroni

  [Vs False Discovery Rate (FDR)]
- (A/B) t-test
- χ^2-test
- $X + Y \sim$?
- Bayes ↓

4 Multi-armed Bandit

- Ready, go

5 Bayes

We are monitoring credit card (cc) purchases for fraud by testing a measure of "unusual purchases".

Let π denote the overall fraud rate (for which we may have some prior belief),

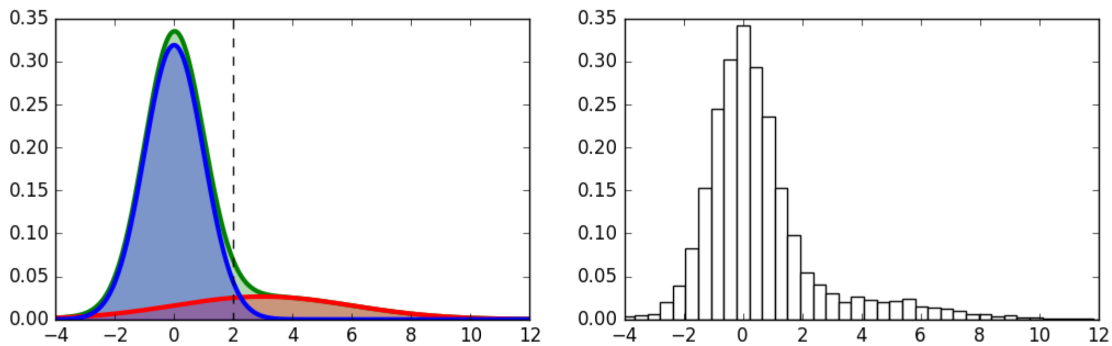
$$\begin{cases} f_i = 1: & \text{if there is in fact fraud for cc } i \text{, and} \\ f_i = 0: & \text{if not,} \end{cases}$$

and t_i be our "unusual purchases" measure which will depend on f_i. That is,

$$\begin{aligned} d(\pi) &\propto \pi^{\alpha-1}(1-\pi)^{\beta-1} \\ p(f_i|\pi) &= \pi^{f_i}(1-\pi)^{(1-f_i)} \\ t_i|f_i &\sim d_{f_i}(t_i) \end{aligned}$$

so that (upon marginalizing out the latent fraud indicator f_i) the measures t_i are generated from a mixture distribution of fraudulent d_1 and genuine d_0 cc purchases, i.e.,

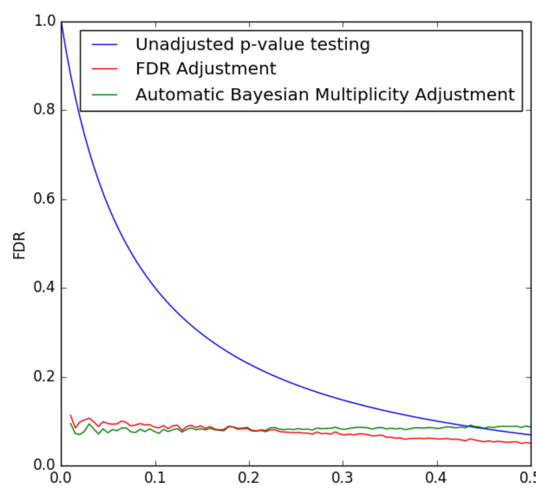
$$d(t_i|\pi) = (1-\pi)d_0(t_i) + \pi d_1(t_i)$$



and we are interested in

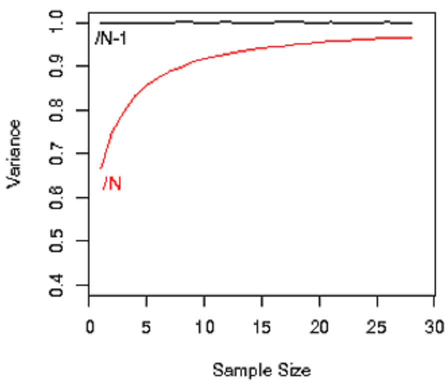
$$\begin{aligned} & p(f_i = 1|t_1, \cdots t_n, f_1, \cdots f_{i-1}, f_{i+1}, \cdots f_n) \\ \propto & \prod d_{f_i}(t_i) p(f_i|\pi) \cdot d(\pi) \\ \propto & \, d_1(t_i) \int \pi^{1+\sum\limits_{j\neq i} f_j + \alpha - 1} (1-\pi)^{\sum\limits_{j\neq i}(1-f_j)+\beta-1} d\pi \\ \approx & \, d_1(t_i)\hat{\pi} \end{aligned}$$

$$\begin{aligned} & p(f_i = 0|t_1, \cdots t_n, f_1, \cdots f_{i-1}, f_{i+1}, \cdots f_n) \\ \propto & \prod d_{f_i}(t_i) p(f_i|\pi) \cdot d(\pi) \\ \propto & \, d_0(t_i) \int \pi^{\sum\limits_{j\neq i} f_j + \alpha - 1} (1-\pi)^{1+\sum\limits_{j\neq i}(1-f_j)+\beta-1} d\pi \\ \approx & \, d_0(t_i)(1-\hat{\pi}) \end{aligned}$$



`False Discovery Rate (FDR), see: statsmodels.sandbox.stats.multicomp.fdrcorrection0`

Appendix: n-1?



Dividing by $(n-1)$ rather than n results in an unbiased estimator of σ^2

$$
\begin{aligned}
&E\left[\sum_{i=1}^{n}\left(x_i^2 - \frac{1}{n}\sum_{j=1}^{n} x_j\right)^2\right] \\
&= E\left[\sum_{i=1}^{n}\left(x_i^2 - \frac{2x_i}{n}\sum_{j=1}^{n} x_j + \left(\frac{1}{n}\sum_{j=1}^{n} x_j\right)^2\right)\right] \\
&= E\left[\sum_{i=1}^{n} x_i^2 - {\color{blue}\frac{2}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j} + {\color{red}\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j}\right] \\
&= E\left[\sum_{i=1}^{n} x_i^2 - {\color{blue}\frac{2}{n}\sum_{i=1}^{n} x_i^2 - \frac{2}{n}\sum_{j\neq i} x_i x_j} + {\color{red}\frac{1}{n}\sum_{i=1}^{n} x_i^2 + \frac{1}{n}\sum_{j\neq i} x_i x_j}\right]
\end{aligned}
$$

$$
\begin{aligned}
= E\left[\sum_{i=1}^{n} x_i^2 - {\color{blue}\frac{2}{n}\sum_{i=1}^{n} x_i^2} + {\color{red}\frac{1}{n}\sum_{i=1}^{n} x_i^2} - {\color{blue}\frac{2}{n}\sum_{j\neq i} x_i x_j} + {\color{red}\frac{1}{n}\sum_{j\neq i} x_i x_j}\right] &= E\left[\frac{n-1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\sum_{j\neq i} x_i x_j\right] \\
&= \frac{n-1}{n}\sum_{i=1}^{n} E\left[x_i^2\right] - \frac{1}{n}\sum_{j\neq i} E\left[x_i x_j\right] \\
&= \frac{n-1}{n}\sum_{i=1}^{n}(\sigma^2+\mu^2) - \frac{1}{n}\sum_{j\neq i}\mu^2 \quad (\textit{why?}) \\
&= (n-1)(\sigma^2+\mu^2) - \frac{n^2-n}{n}\mu^2 = {\color{purple}(n-1)\sigma^2}
\end{aligned}
$$

Appendix: uncorrelated $\overset{\Longrightarrow}{\underset{\Longleftarrow}{?}}$ independent *hint*