NB-NLP
Naive Bayes for Natural Language Processing

Schwartz

April 24, 2017

How do I love thee? Let me count the Bayes

Types of Bayes

- Empirical Bayes
- **Naive Bayes**
- Full Bayes
- Variational Bayes
- Nonparametric Bayes

Types of priors

- Conjugate prior
- Jeffrey's prior
- Improper prior
- (Un)Informative prior
- Objective prior
- Uniform prior

Types of Markov Chain Monte Carlo (MCMC)

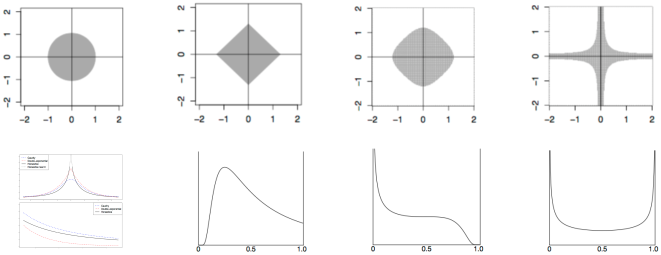
Closed form solutions for posterior distributions are rarely available...

- Gibbs Sampler (cycling through full conditional distributions)
- Metropolis-Hastings (using unnormalized posterior proportionality)
- NUTS: No U-turn sampler (universal probabilistic programming)

Types of Bayesian regularization priors

- Normal-Normal conjugate prior: *ridge regression/regularization*
- Laplace prior: *lasso regularization*
- Cauchy prior: *some other kind of regularization*
- Horseshoe prior: *some other other form of regularization*
  The manuscript presenting the "Horseshoe" prior is entitled "Shrink Globally, Act Locally: Sparse Bayesian Regularization and Prediction"

Gausian Laplace Cauchy Horseshoe



Tails

Implied shrinkage prior profiles from none (0) to total (1) shrinkage

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3. Understand how "Naive Bayes" comes to the rescue
4. Understand what Naive Bayes classification is & how it works
5. Understand how Naive Bayes can be applied to NLP problems
6. Know that Naive Bayes is super undemanding computationally

Conditional versus joint models

- Conditional/Predictive/Discriminative
  ("outcome given features")

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  So we want to model $\boldsymbol{X}_i|Y_i$... in order to get $Y_i|\boldsymbol{X}_i$

  For categorical $Y_i \in \{k : k = 1, 2, \cdots K\}$

$$\begin{aligned} f(Y_i, \boldsymbol{X}_i) &= \sum_{k=1}^{K} \Pr(Y_i = k) f(\boldsymbol{X}_i|Y_i = k) \\ &= \sum_{k=1}^{K} \pi_k f_k(\boldsymbol{X}_i) \end{aligned}$$

  so what we need is $\implies \quad f(\boldsymbol{X}_i|Y_i = k) \equiv f_k(\boldsymbol{X}_i)$

Multivariate Normal (MVN) and Multinomial (MN)

$$\left[\begin{array}{cccc}
X_{11} & X_{12} & \cdots & X_{1q} \\
X_{21} & X_{22} & \cdots & X_{2q} \\
\vdots & \vdots & \ddots & \vdots \\
\hline
X_{i1} & X_{i2} & \cdots & X_{iq} \\
\hline
\vdots & \vdots & \ddots & \vdots \\
X_{n1} & X_{n2} & \cdots & X_{nq}
\end{array}\right]$$

$$\boldsymbol{X}_i = (X_{i1}, X_{i2}, \cdots, X_{iq})^T \sim MVN(\boldsymbol{\mu}_{q\times 1}, \Sigma_{q\times q})$$

$$(2\pi)^{-\frac{k}{2}} |\Sigma|^{-\frac{1}{2}} e^{-\frac{1}{2}(\boldsymbol{X}_i - \boldsymbol{\mu})^T \Sigma^{-1} (\boldsymbol{X}_i - \boldsymbol{\mu})}$$

$$\boldsymbol{X}_i = (X_{i1}, X_{i2}, \cdots, X_{iq})^T \sim MN(\boldsymbol{p}_{q\times 1}, n_i)$$

$$\frac{n_i!}{\prod_{j=1}^{p} X_{ij}!} \prod_{j=1}^{q} p_j^{X_{ij}}$$

Multivariate Models

- $\boldsymbol{X}_i \sim MVN(\boldsymbol{\mu}_{q\times 1}, \Sigma_{q\times q})$
  i.e.,

$$\boldsymbol{X}_i \equiv \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_q \end{bmatrix} \sim MVN\left(\begin{bmatrix} \mu_{X_1} \\ \mu_{X_2} \\ \vdots \\ \mu_{X_q} \end{bmatrix}, \begin{bmatrix} \sigma^2_{X_1} & \sigma_{X_1X_2} & \cdots & \sigma_{X_1X_q} \\ \sigma_{X_2X_1} & \sigma^2_{X_2} & \cdots & \sigma_{X_2X_q} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{X_qX_1} & \sigma_{X_qX_2} & \cdots & \sigma^2_{X_q} \end{bmatrix} \right)$$

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- What is n? And how do we estimate $\mu_{X_j}, \sigma^2_{X_j}, \sigma_{X_jX_{j'}}$?

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- What does this matrix specify? And why use it?

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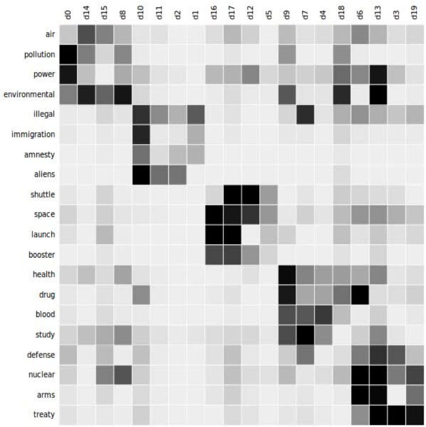

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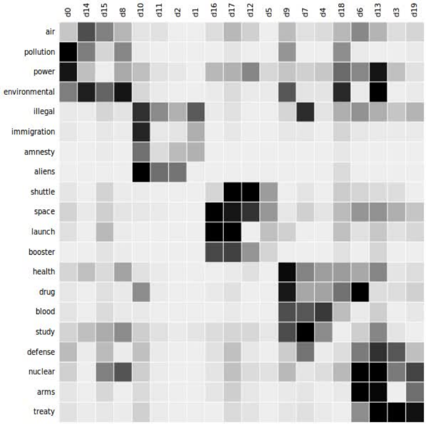
$$\begin{aligned} & f(Y_i, \boldsymbol{X}_i) \\ = & \sum_{k=1}^{K} \Pr(Y_i = k) f(\boldsymbol{X}_i | Y_i = k) \\ = & \sum_{k=1}^{K} \pi_k f_k(\boldsymbol{X}_i) \end{aligned}$$



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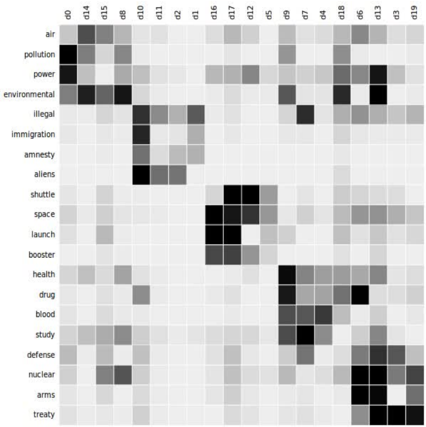

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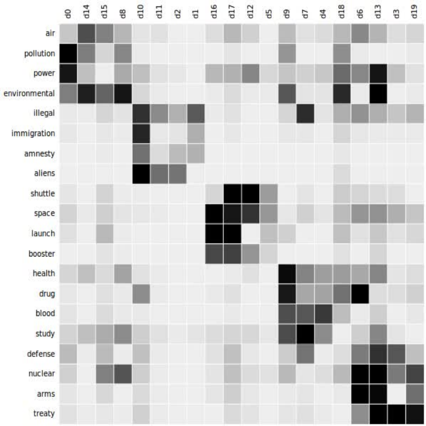

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Why is this even called *Bayes*? Why is this called *Naive*?

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Notice how probability of a word is independent of previous words

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Tricks

Laplace Smoothing:

$$\hat{p}_{kX_j} = \frac{\#(\text{times } X_j \text{ appears in Class } k) + \alpha}{\#(\text{words in Class } k) + \alpha \times |\text{Vocab}|}$$

Exponentiating the sum of log probabilities:

$$\begin{aligned}
&\Pr(Y_i = k|\boldsymbol{X}_i) \\
&= \frac{\pi_k \hat{p}_{kX_1}^{X_{i1}} \cdot \hat{p}_{kX_2}^{X_{i2}} \cdots \hat{p}_{kX_q}^{X_{ip}}}{\sum_{k'=1}^{K} \pi_{k'} \hat{p}_{k'X_1}^{X_{i1}} \cdot \hat{p}_{k'X_2}^{X_{i2}} \cdots \hat{p}_{k'X_q}^{X_{ip}}} = \frac{1}{\sum_{k'=1}^{K} \frac{\pi_{k'} \hat{p}_{k'X_1}^{X_{i1}} \cdot \hat{p}_{k'X_2}^{X_{i2}} \cdots \hat{p}_{k'X_q}^{X_{ip}}}{\pi_k \hat{p}_{kX_1}^{X_{i1}} \cdot \hat{p}_{kX_2}^{X_{i2}} \cdots \hat{p}_{kX_q}^{X_{ip}}}} \\
&= \frac{1}{\sum_{k'=1}^{K} \frac{\pi_{k'}}{\pi_k} exp\left(\sum_{j=1}^{q} log(\hat{p}_{k'X_j}^{X_{ij}}) - \sum_{j=1}^{q} log(\hat{p}_{kX_j}^{X_{ij}})\right)}
\end{aligned}$$

What is the assumption on the covariance matrix doing?

Back to the MVN...

$$
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&\propto \pi_k f_k(\boldsymbol{X}_i) \\
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\qquad
\begin{bmatrix}
\hat{\sigma}^2_{kX_1} & 0 & \cdots & 0 \\
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\vdots & \vdots & \ddots & \vdots \\
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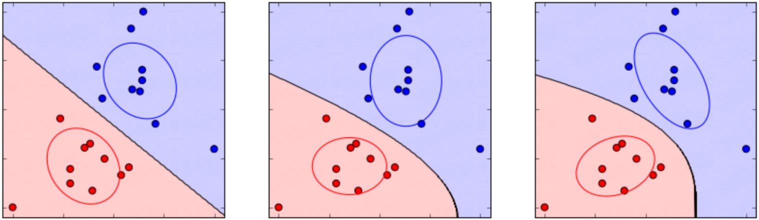
\end{bmatrix}
$$



What is the assumption on the covariance matrix doing?

Back to the MVN...

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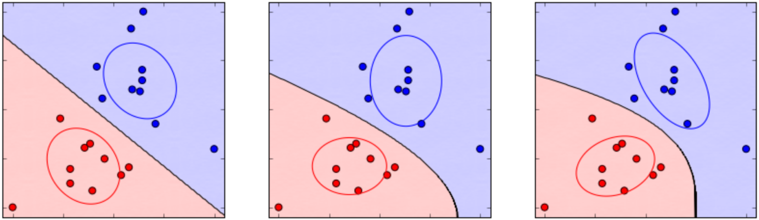

Linear Discriminant Analysis (LDA) Naive Bayes Quadratic Discriminant Analysis (QDA)

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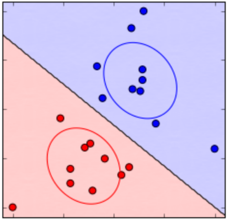

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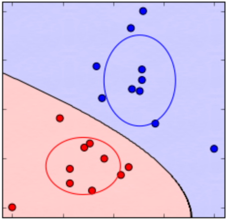
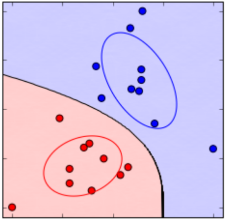


Naive Bayes



Quadratic Discriminant Analysis (QDA)

$\Uparrow\Uparrow\Uparrow\Uparrow\Uparrow$

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