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Approximate Controllability of Fractional Delay Dynamic Inclusions with Nonlocal Control Conditions

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Abstract: We introduce a nonlocal control condition and the notion of approximate controllability for fractional order quasilinear control inclusions. Approximate controllability of a fractional control nonlocal delay quasilinear functional differential inclusion in a Hilbert space is studied. The results are obtained by using the fractional power of operators, multi-valued analysis, and Sadovskii's fixed point theorem. Main result gives an appropriate set of sufficient conditions for the considered system to be approximately controllable. As an example, a fractional partial nonlocal control functional differential inclusion is considered.

Keywords: approximate controllability; control theory; multivalued maps; fractional dynamic inclusions; fractional power; fixed points; semigroup theory.

MSC 2010: 26A33; 34A60; 34G25; 93B05.

1 Introduction

We are concerned with the fractional delay quasilinear control inclusion

$$D_t^{\alpha}[u(t)-g(t,u(\sigma(t)))] \in Au(t)+\int_0^t f(t,s,B_1\mu_1(\delta(s)))ds \tag{1}$$

subject to the nonlocal control condition

$$u(0)+h(u(t)) = B_2\mu_2(t)+u_0, \tag{2}$$

where the unknown $u(\cdot)$ takes its values in a Hilbert space H with norm $\|\cdot\|$, D_t^{α} is the Caputo fractional derivative with $0<\alpha\le 1$ and $t \in J=[0,a]$. Let A be a closed linear operator defined on a dense domain $D(A)$ in H into H that generates an analytic semigroup $Q(t)$, $t\ge 0$, of bounded linear operators on H and $u_0 \in D(A)$. We assume that $\{B_i : U \to H,\ i=1,2\}$ is a family of bounded linear operators, the control functions μ_i, $i=1,2$, belong to the space $L^2(J,U)$, a Hilbert space of admissible control functions with U as Hilbert space, and $\sigma,\delta : J \to J'$ are delay arguments, $J'=[0,t]$. It is also assumed that $g: J\times H \to H$ and $h: C(J':H) \to H$ are given abstract functions and $f: \Delta \times H \to H$ is a multi-valued map, $\Delta = \{(t,s) : 0\le s\le t\le a\}$.

2 Main results

We obtain existence and approximate controllability results for the fractional nonlocal control inclusion (1)–(2). We consider the following hypotheses:

(H$_1$) There exists a constant $p \in (0,1)$ such that the function $g(\cdot,\cdot)$ maps $[0,a] \times H_q$ into H_{p+q} and $A^p g : [0,a] \times H_q \to H_q$ satisfies a Lipschitz condition, that is, there exists a constant $L_1 > 0$ such that

$$\|A^p g(t_1,u_1) - A^p g(t_2,u_2)\|_q \le L_1 \left(|t_1 - t_2| + \|u_1 - u_2\|_q\right) \tag{3}$$

for any $0 \le t_1, t_2 \le a$, $u_1, u_2 \in H_q$.

(H$_2$) For any $u(\cdot) \in \Omega$, $u_a \in H$, we take the controls

$$\mu_1 = B_1^* T_\alpha^*(a-t)\mathscr{R}(\lambda, \Gamma_{0,1}^a)P(u(\cdot)), \quad \mu_2 = B_2^* S_\alpha^*(a)\mathscr{R}(\lambda, \Gamma_{0,2}^a)P(u(\cdot)). \tag{4}$$

Theorem 1 *If (H$_1$)–(H$_2$) are satisfied and $\lambda\mathscr{R}(\lambda,\Gamma_{0,i}^a) \to 0$ in the strong operator topology as $\lambda \to 0^+$, $i = 1,2$, then the nonlocal-control fractional delay system* (1)–(2) *is approximately controllable on J.*

Proof. We use the existence result of mild solutions, it is easy to see that F^λ has a fixed point in Ω_r for any $\lambda \in (0,1)$. This implies that there exists $\overline{u}^\lambda \in F^\lambda(\overline{u}^\lambda)$, that is, there is $\overline{v}^\lambda \in S_{f,\overline{\mu_1}^\lambda}$ such that

$$\begin{aligned}\overline{u}^\lambda(t) = S_\alpha(t)\left[B_2\overline{\mu_2}^\lambda(t) + u_0 - h\left(\overline{u}^\lambda\right) - g\left(0,\overline{u}^\lambda(\sigma(0))\right)\right] + g\left(t,\overline{u}^\lambda(\sigma(t))\right)\\ + \int_0^t (t-s)^{\alpha-1}\left\{AT_\alpha(t-s)g\left(s,\overline{u}^\lambda(\sigma(s))\right) + T_\alpha(t-s)\left[\overline{v}^\lambda(\delta(s)) + B_1\overline{\mu_1}^\lambda(s)\right]\right\}ds.\end{aligned}$$

Moreover, by the assumption that we have $\lambda\mathscr{R}(\lambda,\Gamma_{0,i}^a) \to 0$ in the strong operator topology as $\lambda \to 0^+$, $i = 1,2$, we ensure that $\|u_a - \overline{u}^\lambda(a)\| \to 0$ as $\lambda \to 0^+$. Therefore, the fractional dynamic inclusion (1)–(2) is approximately controllable on J. ∎

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