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COMP-2032

Introduction to Image Processing

Lecture 10

Image Compression

Learning Outcomes

IDENTIFY

1. Redundancy
2. Huffman Coding
3. Psychovisual Redundancy - GIF
4. A Compression System - JPEG

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Redundancy

Image Compression

Individual image(s)

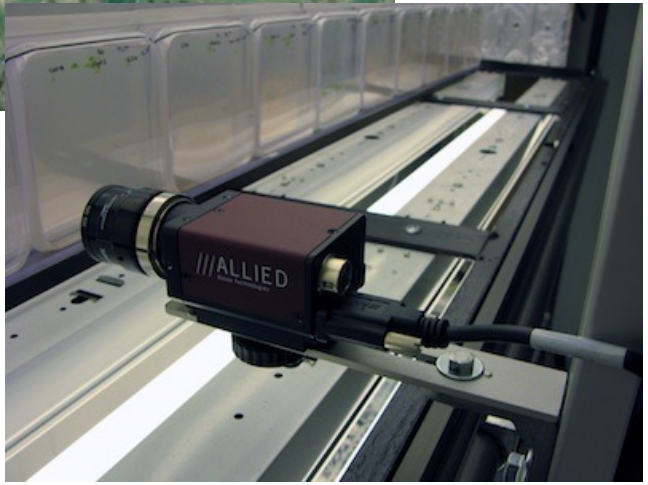
Can be large

Are easy to acquire, collections increases rapidly

In some applications, images are gathered automatically

Luckily, image data is redundant in several ways

- Coding redundancy
- Spatial redundancy
- Psychovisual redundancy



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Coding Redundancy

The grey level histogram of an image gives the probability (frequency) of occurrence of grey level r_k

$$p(r_k) = \frac{n_k}{n} \qquad k = 0,2,\ldots,L-1$$

If the number of bits used to represent each value of r_k is $l(r_k)$, the average number of bits required to represent a pixel is

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)p(r_k)$$

To code an MxN image requires MNL_{avg} bits

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Coding Redundancy

If an m-bit natural binary code is used to represent grey level then

- All pixels take the same amount of space,
- $P(r_k)$ value sum to 1, so:

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)p(r_k) = \sum_{k=0}^{L-1} mp(r_k) = m$$

- And an image occupies MN_m bits

But some pixel values are more common than others…

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Variable Length Encoding

Assigning fewer bits to the more probable grey levels than to less probable ones can achieve data compression, e.g.:

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

Build a codebook, replace 'true' pixel values with code

Lossless

The process can be reversed by inverting the codebook

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Spatial Redundancy

- Sometimes called *Interpixel Redundancy*
- Neighbouring pixels often have similar values
- Compression based on spatial redundancy involves some element of pixel grouping or transformation
- Simplest is **Run-Length Encoding**

• Maps the pixels along each scan line into a sequence of pairs $(g_1, r_1), (g_2, r_2), \ldots,$
• Where g_j is the ith grey level, r_j is the run length of ith run

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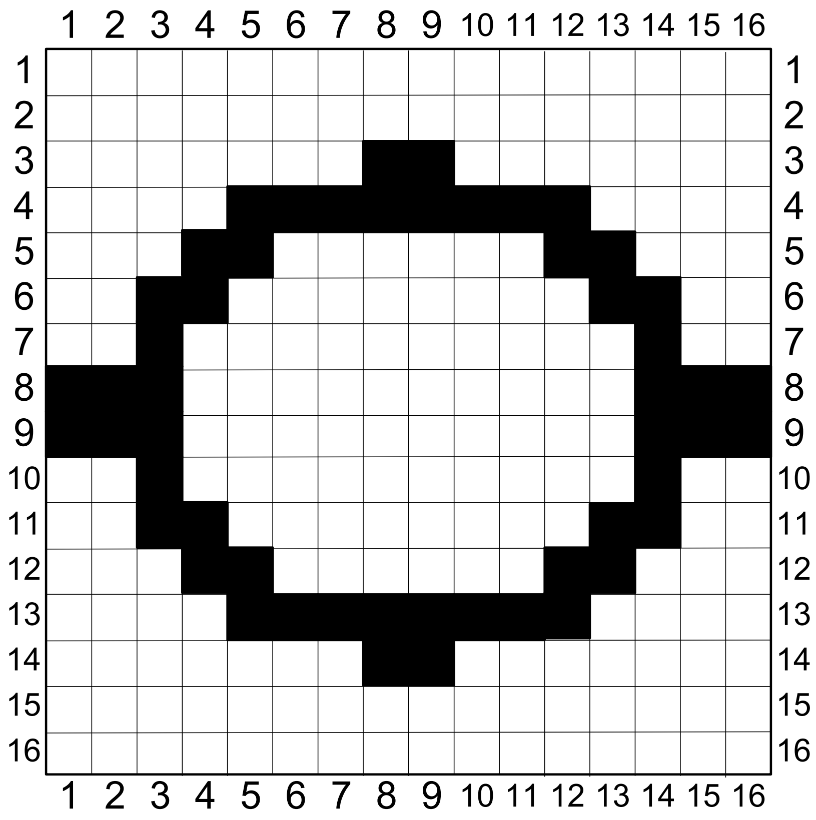
A Binary Example

Row 1: (0, 16)
Row 2: (0, 16)
Row 3: (0, 7) (1, 2) (0, 7)
Row 4: (0, 4), (1, 8) (0, 4)
Row 5: (0, 3) (1, 2) (0, 6) (1, 3) (0, 2)
Row 6: (0,2) (1, 2) (0,8) (1, 2) (0, 2)
Row 7: (0, 2) (1,1) (0, 10) (1,1) (0, 2)
Row 8: (1, 3) (0, 10) (1,3)
Row 9: (1, 3) (0, 10) (1, 3)
Row 10: (0,2) (1, 1) (0,10) (1, 1) (0, 2)
Row 11: (0, 2) (1, 2) (0, 8) (1, 2) (0, 2)
Row 12: (0, 3) (1, 2) (0, 6) (1, 3) (0, 2)
Row 13: (0, 4) (1,8) (0, 4)
Row 14: (0, 7) (1, 2) (0, 7)
Row 15: (0, 16)
Row 16: (0, 16)

encode

decode



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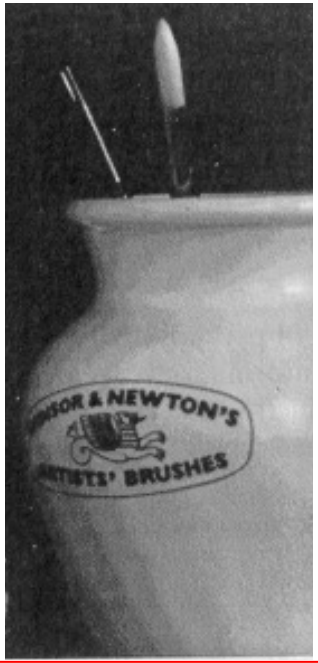
Psychovisual Redundancy

Some grey level and colour differences are imperceptible; goal is to compress without noticeable change to the image

256 grey levels



16 grey levels

16 grey levels

A simple method:

Add a small random number to each pixel before quantization

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Evaluating Compression

- **Fidelity Criteria**: success is judged by comparing original and compressed version
- Some measures are objective, e.g., root mean square error (e_{rms}) and signal to noise ratio (SNR)
- Let f(x,y) be the input image, f'(x,y) be reconstructed input image from compressed bit stream, then

$$e_{rms} = \left(\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f'(x,y) - f(x,y) \right)^2 \right)^{1/2}$$

$$SNR = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f'(x,y) \right)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f'(x,y) - f(x,y) \right)^2}$$

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Fidelity Criteria

e_{rms} = **6.93**	e_{rms} = 6.78
SNR_{rm} = 10.25	SNR_{rm} = **10.39**

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Fidelity Criteria

- E_{rms} and SNR are convenient objective measures
- Most decompressed images are viewed by human beings
- Subjective evaluation of compressed image quality by human observers are often more appropriate

Rating scale of Television Allocations Study Organisation.

(Frendendall and Behrend)

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

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Image Compression Systems

- Transform input data in a way that facilitates reduction of interpixel redundancies
- Reversible

Mapper

- Transform input data in a way that facilitates reduction of psychovisual redundancies
- Not reversible

Quantiser

- Assigns the shortest code to the most frequently occurring output values
- Reversible

Symbol coder

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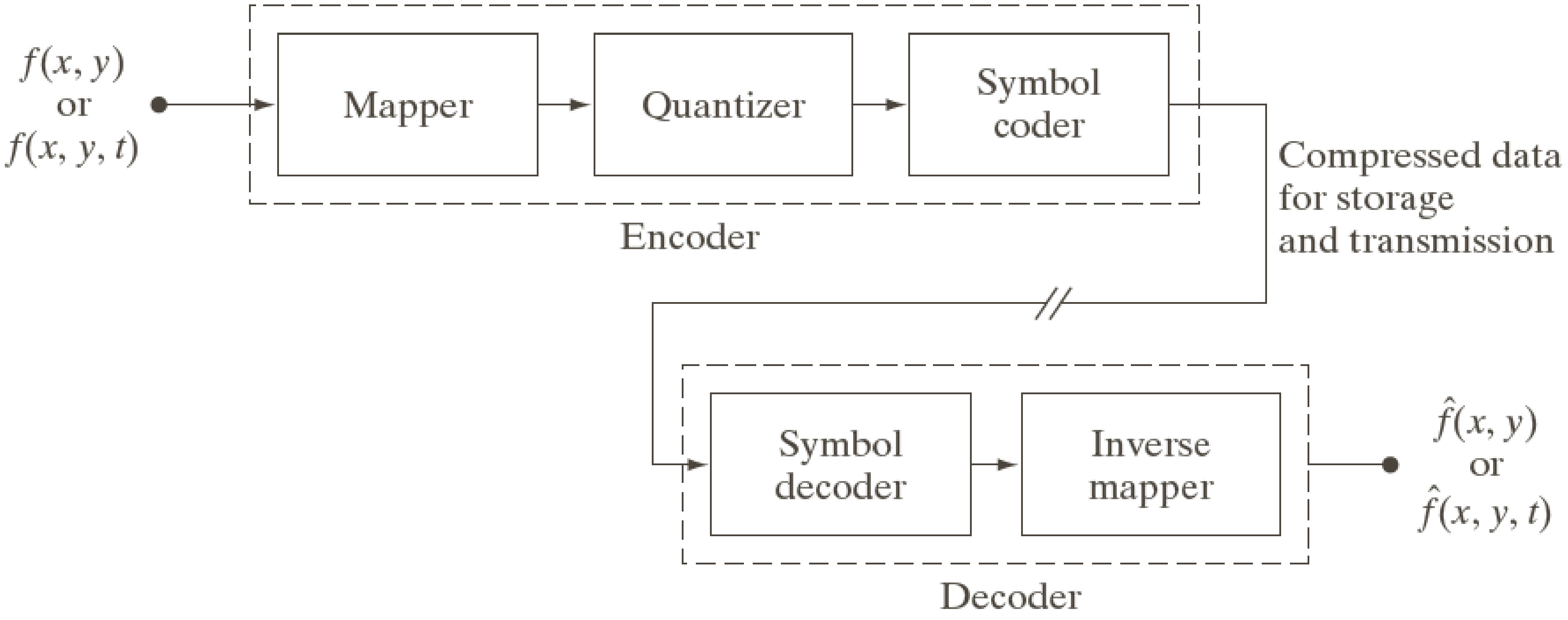
Image Compression Systems



Functional block diagram of a general image compression system

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Huffman Coding

Exploiting Coding Redundancy

Try to maintain a high level of information in compressed images

- These methods are derived from information theory
- Not limited to images, are applicable to any digital information

- Speak of symbols instead of pixel values and sources instead of images
- Exploit nonuniform probabilities of symbols (nonuniform histogram) and use a variable-length code

Evaluation requires a measure of the information content of a source:

Entropy

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Entropy

- **The idea**: associate information with probability
- A random event *E* with probability *P(E)* contains:

$$I(E) = log(\frac{1}{P(E)}) = -log(P(E))$$

units of information

Suppose that grey level values are generated by a random variable, then r_k contains:

$$I(r_k) = -log(P(r_k))$$

units of information

Note I(E) = 0 when P(E) = 1

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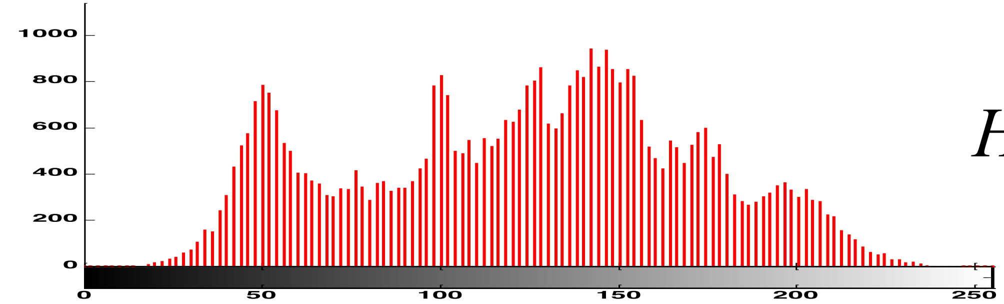
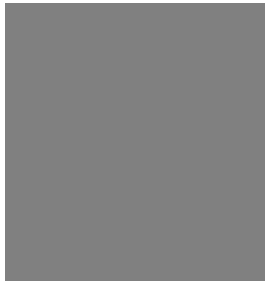
Entropy

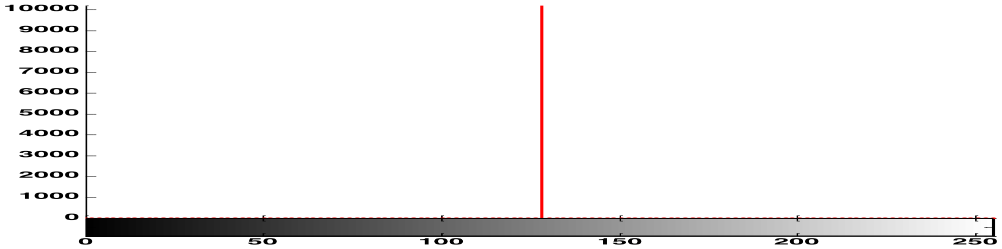
Entropy is the average information content of an image, a measure of histogram dispersion



$$H = -\sum_{k=0}^{L-1} p(r_k)\log_2 p(r_k)$$

entropy = 7.4635





entropy = 0

Can't compress to less than H bits/pixel without losing information

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Huffman Coding

- Compute probabilities of each symbol by ***histogramming*** the source
- Process probabilities to pre-compute codebook: code(i)

Codebook is static (fixed)

- Encode source symbol-by-symbol:

symbol(i) -> code (i)

- Transmit coded signal and codebook
- The need to pre-process (histogram) the source before encoding begins is a disadvantage

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Huffman Coding

Builds a binary tree in which symbols to be coded are nodes

- Create a list of nodes, one per for symbol, sorted in order of symbol frequency (or probability)
- REPEAT (until only one node left)
  - Pick the two nodes with the lowest frequencies/probabilities and create a parent of them
  - **Randomly** assign the codes 0, 1 to the two new branches of the tree and delete the children from the list
  - Assign the sum of the children's probabilities to their parent and insert it in the list

Algorithm

Path from root to node gives code for corresponding symbol

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Huffman Coding

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

New parent is higher up list (0.2) than a_1 (0.1)

Create parent node of a_3 and a_5

Create parent of new node and a_4

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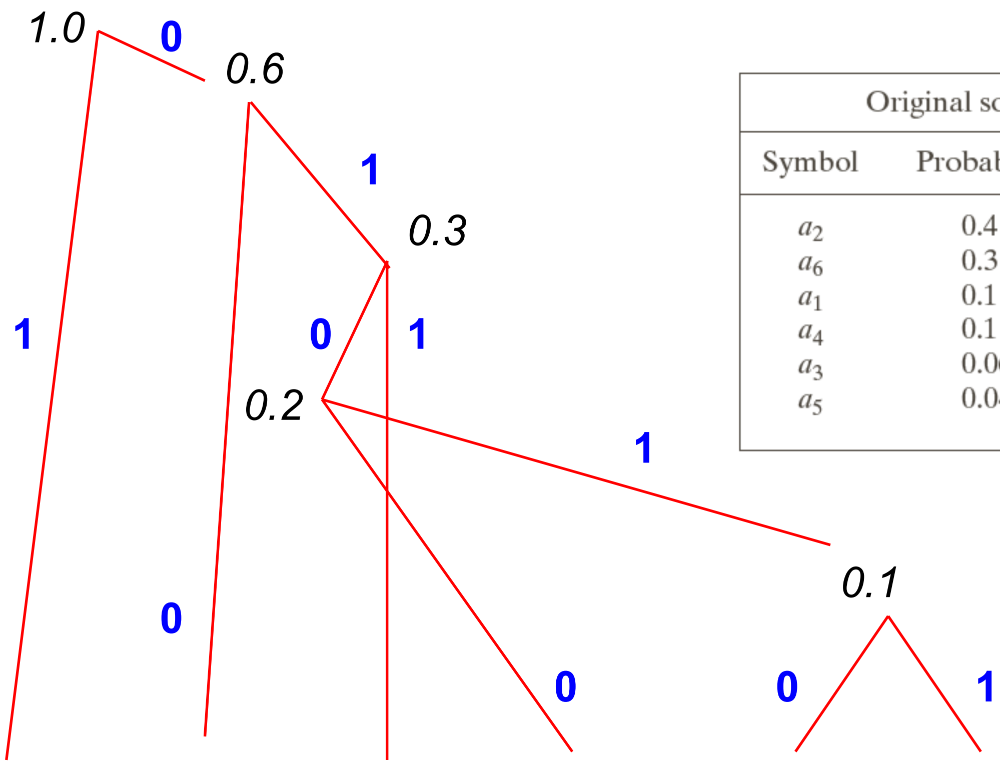
Huffman Coding



Prob$_i$	0.4	0.3	0.1	0.1	0.06	0.04
Symbol$_i$	a_2	a_6	a_1	a_4	a_3	a_5
Code$_i$	**1**	**00**	**011**	**0100**	**01010**	**01011**

Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

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Huffman Coding

- The algorithm systematically places nodes representing high probability symbols further up the tree: *their paths (and so codes) are shorter*
- No code is prefix to any other – don't need to mark boundaries between nodes
  - e.g., 01101010 must be a_1a_3

In this example

- Average length of the code is 2.2 bits/symbol
- The entropy of the source is 2.14 bits/symbol

In general

- Break image into small (e.g., 8 x 8) blocks
- Each block is a symbol to be encoded

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Break

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Psychovisual Redundancy GIF

Using Psychovisual Redundancy

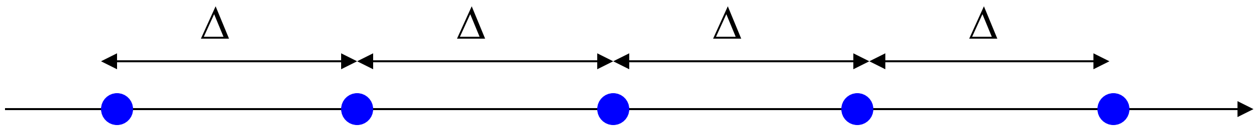
- Represent areas of grey level/colour space with fewer bits
- Lossy: cannot be inverted
- Find the best tradeoff between

Maximal compression ←→ minimal distortion

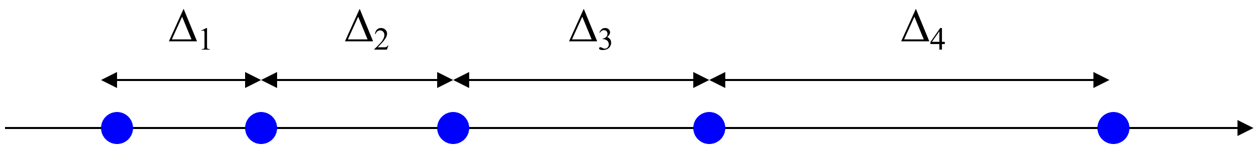
Quantisation is widely used

Scalar Quantisation (i.e., quantising scalar values)

Uniform scalar quantisation:



Non-uniform scalar quantisation:



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Vector Quantisation

Palettised image (gif)

- Map vector values (R,G,B) onto scalar values
- Multiple vectors map to each scalar

Vector quantisation

True colour R,G,B
8 bits each
1677216 possible colours

gif
8 bits per pixel
256 possible colours

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Vector Quantisation

(r, g, b)

For each pixel in the original image

Find the closest colour in the Colour Table

r_0	g_0	b_0
r_1	g_1	b_1
⋮	⋮	⋮
r_{255}	g_{255}	b_{255}

Colour Table

Record the index of that colour (for storage or transmission)

To reconstruct the image, place the indexed colour from the Colour Table at the corresponding spatial location

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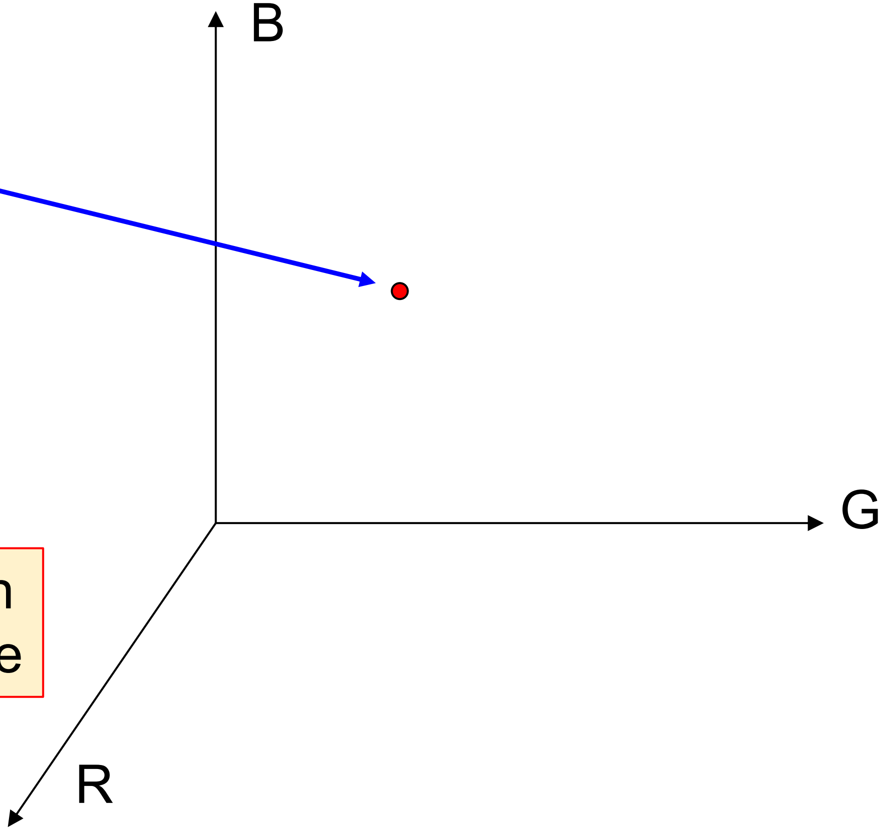
Building a Palette

(r, g, b)



A pixel corresponds to a point in the 3 dimensional R, G, B space

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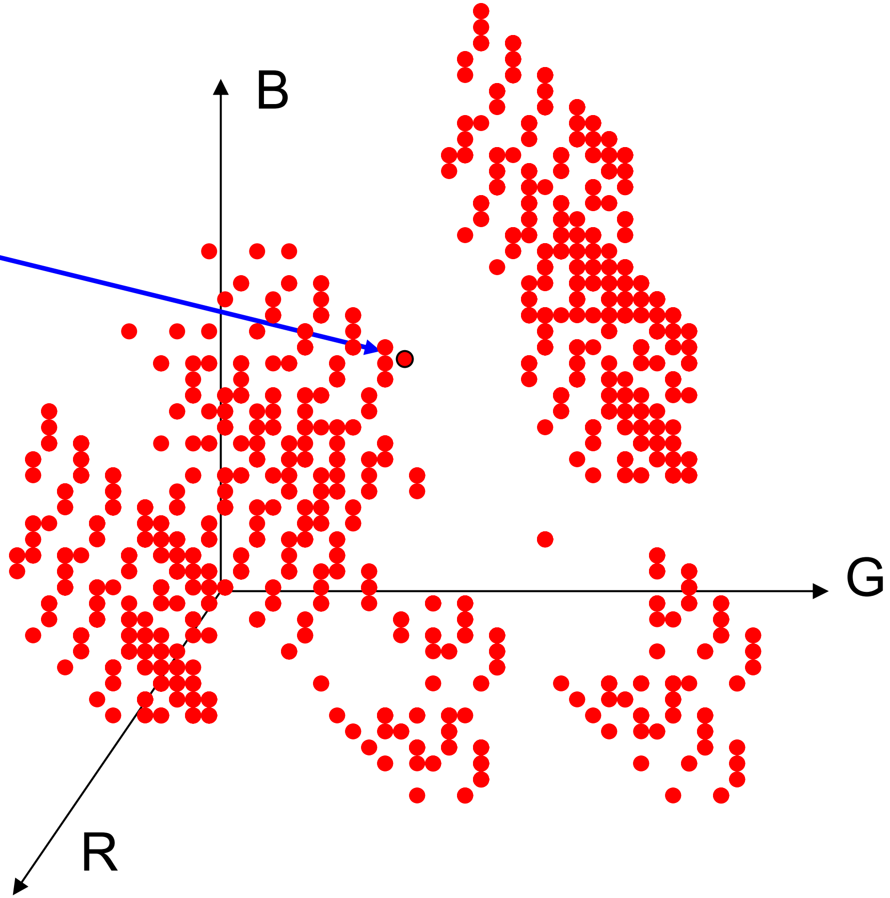
Building a Palette

(r, g, b)



Map all pixels into R,G,B space,
Clouds of pixels are formed

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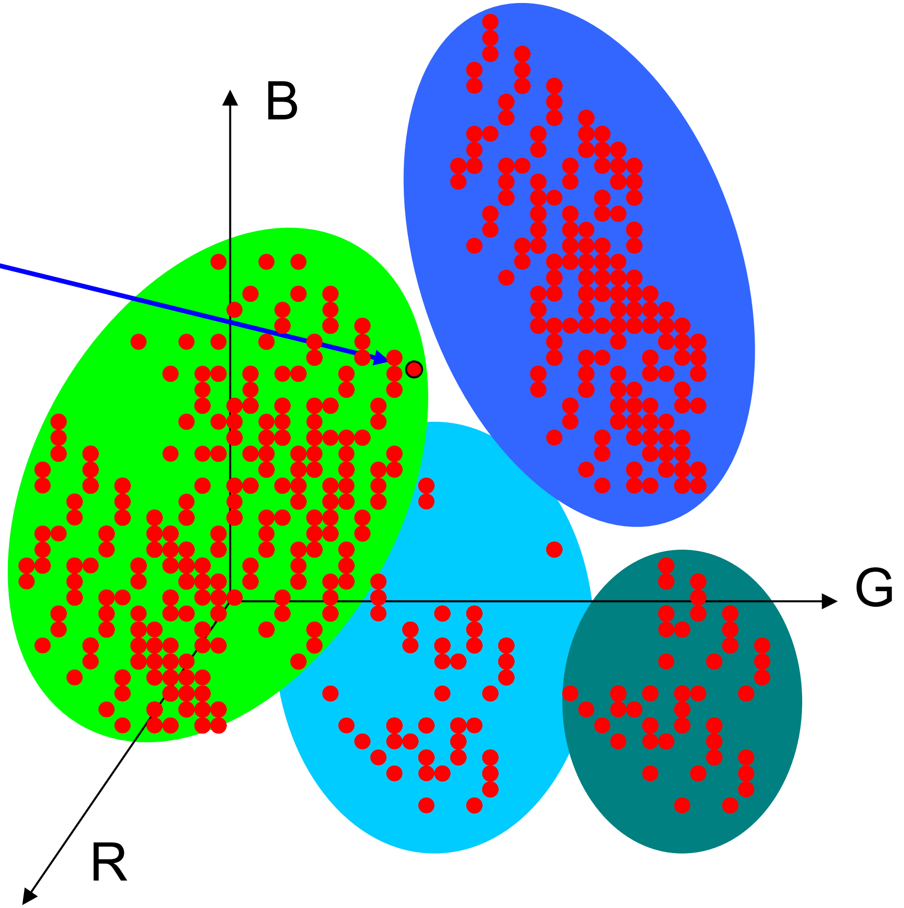
Building a Palette



Group pixels that are close to each other, and replace them by one single colour

Close to each other means of a similar colour

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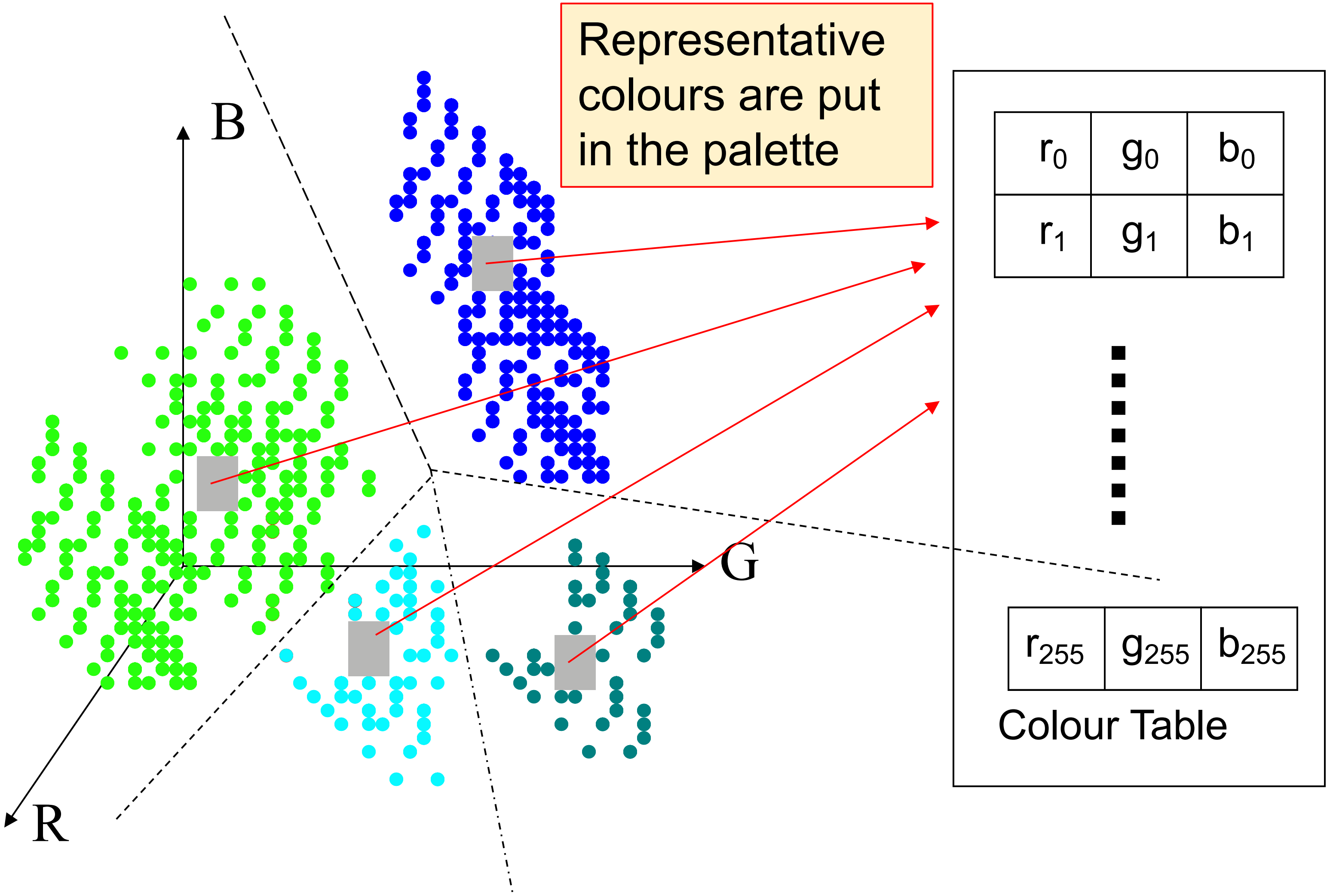
Building a Palette



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Building a Palette

Many clustering algorithms exists

- Supervised
- unsupervised

- We know how many clusters we need: *one per palette entry*
- We need clusters that are spread across the colour space
- A unsupervised method…

K-Means Clustering

- Start with estimates of the mean of each cluster ☞ $\mu_1, \mu_2, \ldots, \mu_k$
- Assign each point, p, to the cluster where ☞ $|p - \mu_i|$ is smallest
- Recompute the means
- Repeat until no changes are made to the clusters

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A Compression System
JPEG

Spatial Redundancy

- Run-length encoding needs adjacent pixels to be **equal**
- Pixels are more often **highly correlated** (dependent)
  - Not equal, but can predict the image pixels to be coded from those already coded

- Each pixel value (except at the boundaries) is predicted based on its neighbours (e.g., linear combination) to get a predicted image
- The difference between the original and predicted images yields a differential or residual image <u>with a reduced set of values</u>
- The differential image is encoded using Huffman coding, or similar

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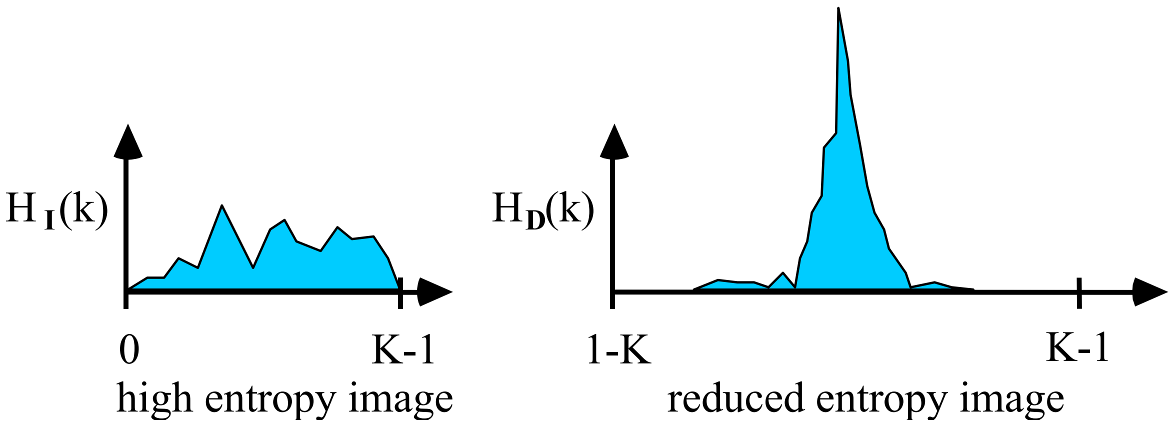
Differential Pulse-code Modulation

Code the difference between adjacent pixels

Original pixels:
82, 83, 86, 88, 56, 55, 56, 60, 58, 55, 50, ……

DPCM:
82, 1, 3, 2, -32, -1, 1, 4, -2, -3, -5, ……

- Prediction is that the next pixel value – current one
- Need the first value to provide a point of reference
- Invertible (lossless) and lower entropy



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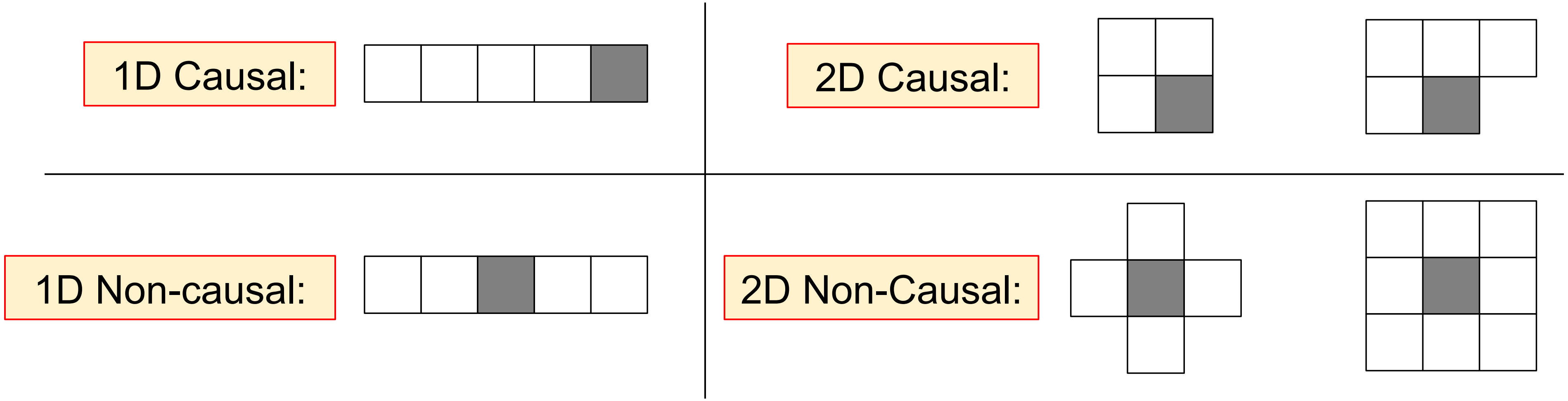
Predictive Coding

- Higher order pattern prediction
- Use both 1D and 2D patterns (*to predict shaded pixel*)



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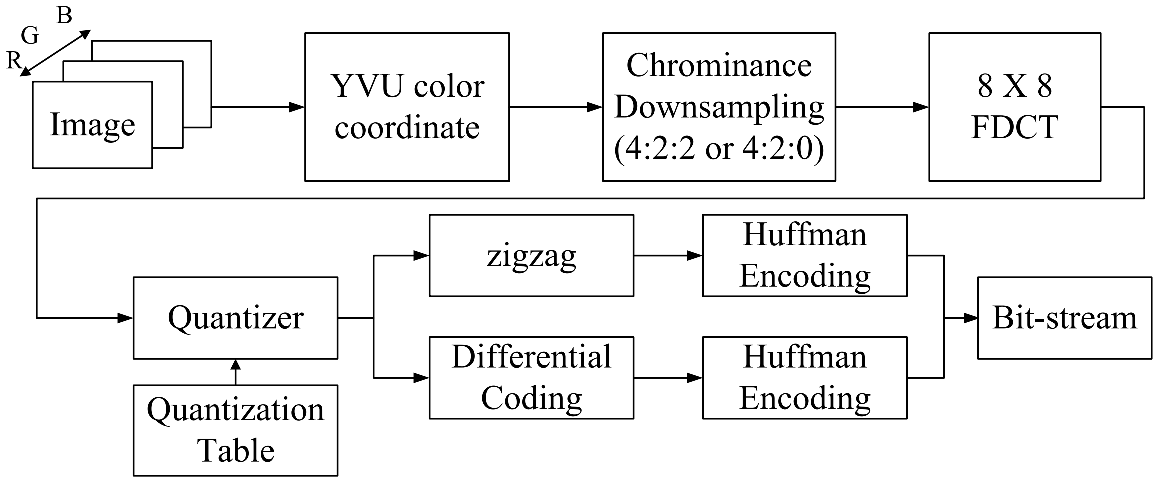
A Complete System: JPEG

A set of methods with a common *baseline* system

- Discrete Cosine Transform
- Quantisation
- Variable length encoding

A JPEG-compatible product must only include support for the baseline



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JPEG Compression

Conversion RGB to YUV is optional, but common

Y is grey level, U,V are colour (Lecture 2)

Y is kept at full resolution, U and V are downsampled

Y

U

V

Human vision is more sensitive to grey-level variations than colours

The rest of the process is applied (separately) to each field

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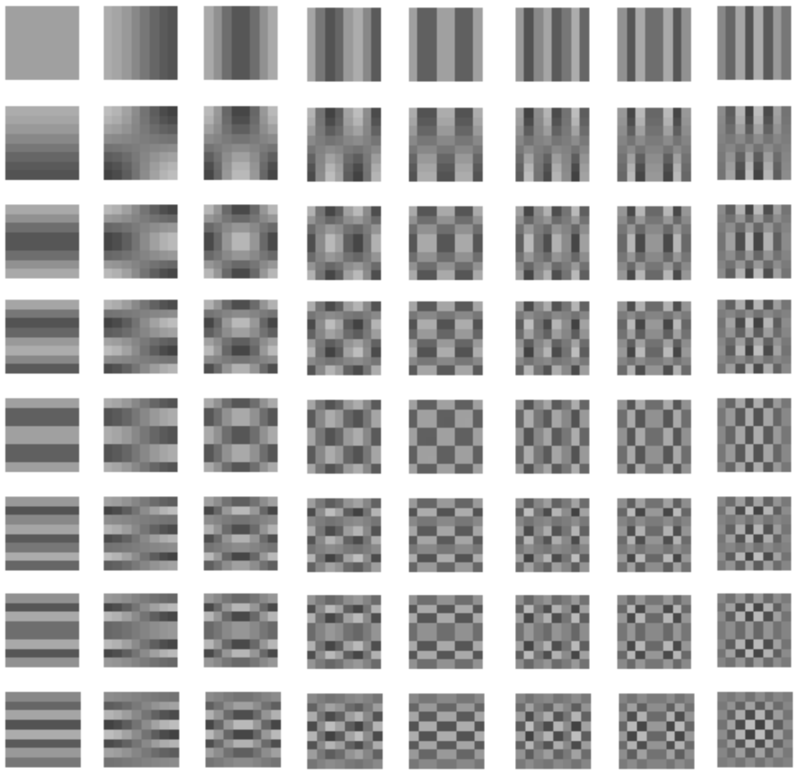
Image Transforms

- Like FFT, basis functions are different
- Top left is DC level
- All other are ”AC”
- Frequency of basis functions increases with distance from origin
- Basis patterns vary in 2D



$$X(u,v) = \frac{4C(u)C(v)}{N^2} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x(m,n) \cos\left(\frac{(2m+1)u\pi}{2N}\right) \cos\left(\frac{(2n+1)v\pi}{2N}\right)$$

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JPEG Compression

- The image is broken in 8 x 8 pixel blocks, which are processed sequentially, top left to bottom right
- First subtract half maximum intensity value so values are distributed about 0

$$
\begin{bmatrix}
52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\
63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\
62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\
63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\
67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\
79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\
85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\
87 & 79 & 69 & 68 & 65 & 76 & 78 & 94
\end{bmatrix}
\quad
\begin{bmatrix}
-76 & -73 & -67 & -62 & -58 & -67 & -64 & -55 \\
-65 & -69 & -73 & -38 & -19 & -43 & -59 & -56 \\
-66 & -69 & -60 & -15 & 16 & -24 & -62 & -55 \\
-65 & -70 & -57 & -6 & 26 & -22 & -58 & -59 \\
-61 & -67 & -60 & -24 & -2 & -40 & -60 & -58 \\
-49 & -63 & -68 & -58 & -51 & -60 & -70 & -53 \\
-43 & -57 & -64 & -69 & -73 & -67 & -63 & -45 \\
-41 & -49 & -59 & -60 & -63 & -52 & -50 & -34
\end{bmatrix}
$$

Image block

Normalised image block

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JPEG Compression

$$\begin{bmatrix} -76 & -73 & -67 & -62 & -58 & -67 & -64 & -55 \\ -65 & -69 & -73 & -38 & -19 & -43 & -59 & -56 \\ -66 & -69 & -60 & -15 & 16 & -24 & -62 & -55 \\ -65 & -70 & -57 & -6 & 26 & -22 & -58 & -59 \\ -61 & -67 & -60 & -24 & -2 & -40 & -60 & -58 \\ -49 & -63 & -68 & -58 & -51 & -60 & -70 & -53 \\ -43 & -57 & -64 & -69 & -73 & -67 & -63 & -45 \\ -41 & -49 & -59 & -60 & -63 & -52 & -50 & -34 \end{bmatrix}$$

Normalised image block

2D-DCT

high frequency

DC component

high frequency

$$\begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.13 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.88 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix}$$

DCT block

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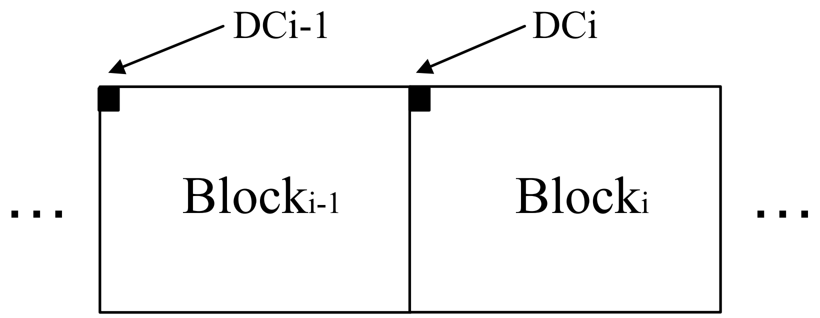
JPEG Compression

- AC and DC components are processed separately
- DC components summarise patch intensity, so should vary smoothly over patches,

Use different coding (DCPM)

$$\text{Diff}_i = \text{DC}_i - \text{DC}_{i-1}$$



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JPEG Compression

AC components are quantised

- Divide the DCT block entries by values in a quantisation table
- Different tables for luminance (Y) and chrominance (UV) blocks

$$Q_Y = \begin{pmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{pmatrix} \quad Q_C = \begin{pmatrix} 17 & 18 & 24 & 47 & 99 & 99 & 99 & 99 \\ 18 & 21 & 26 & 66 & 99 & 99 & 99 & 99 \\ 24 & 26 & 56 & 99 & 99 & 99 & 99 & 99 \\ 47 & 66 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \end{pmatrix}$$

$$X'(u,v) = Round\left(\frac{X(u,v)}{Q(u,v)}\right)$$

X(u,v): original DCT coefficient
X'(u,v): DCT coefficient after quantization
Q(u,v): quantization value

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JPEG Compression

Why quantise ?

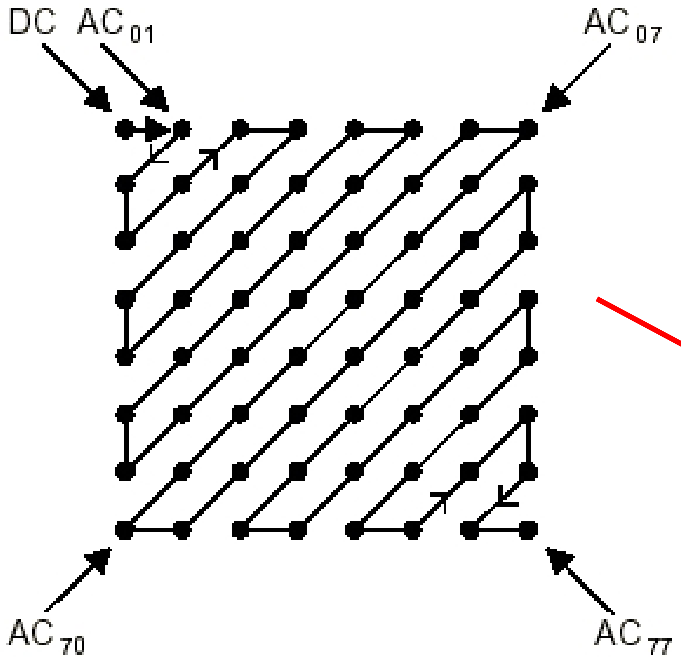
- Further compression by representing DCT coefficients with no greater precision than is necessary to achieve the desired image quality
- Generally, the “high frequency coefficients” have larger quantisation values
- Quantisation makes most coefficients zero, it makes JPEG compression efficient, but ”lossy”

$$\begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

ACK: Prof. Tony Pridmore, UNUK

JPEG Compression

Zigzag coding orders elements of quantised DCT block (roughly) on frequency



Ordering produces long sequences of zeros

Sequences of AC values can be represented using:

- Run-length coding
- Huffman coding

−26 −3 0 −3 −2 −6 2 −4 1 −3….

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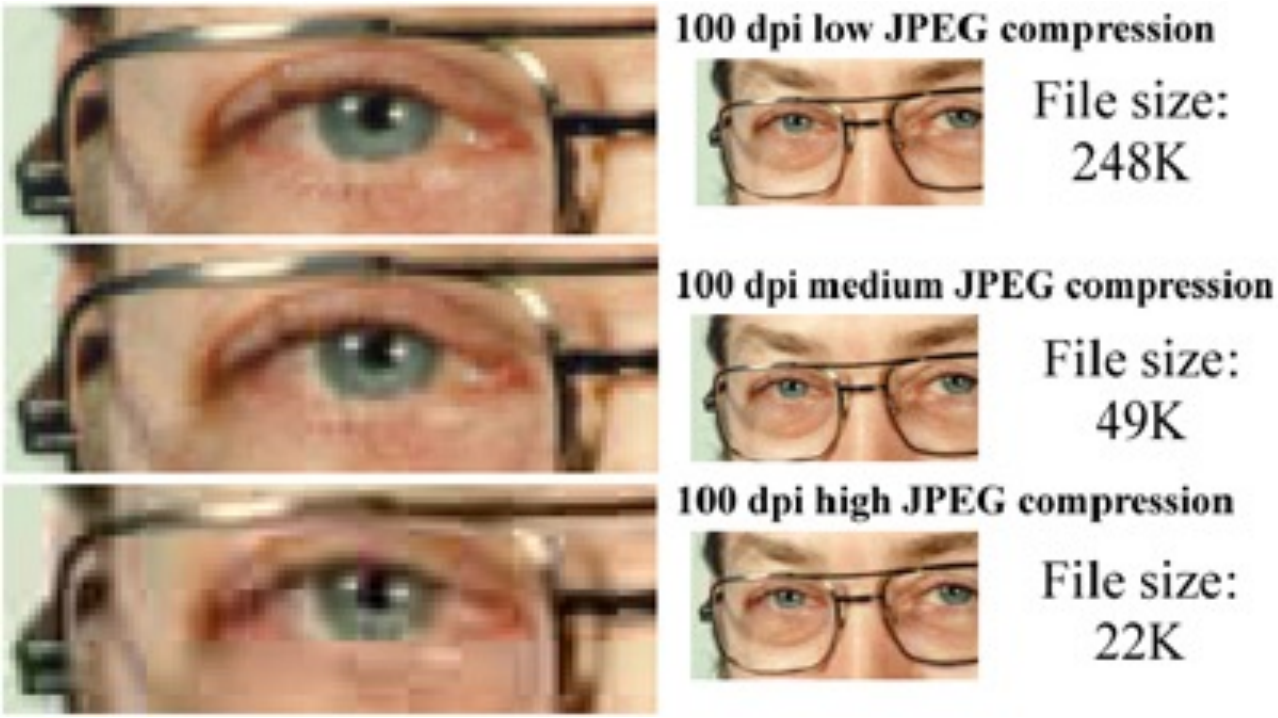
JPEG Compression

Increasing the amount of quantisation reduces file size but introduces artefacts: **blocks become visible**



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Summary

1. Redundancy
2. Huffman Coding
3. Psychovisual Redundancy - GIF
4. A Compression System - JPEG

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Questions

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NEXT:

Interactive Methods & Compositing