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COMP-2032

Introduction to Image Processing

Lecture 3

Linear Filters

Learning Outcomes

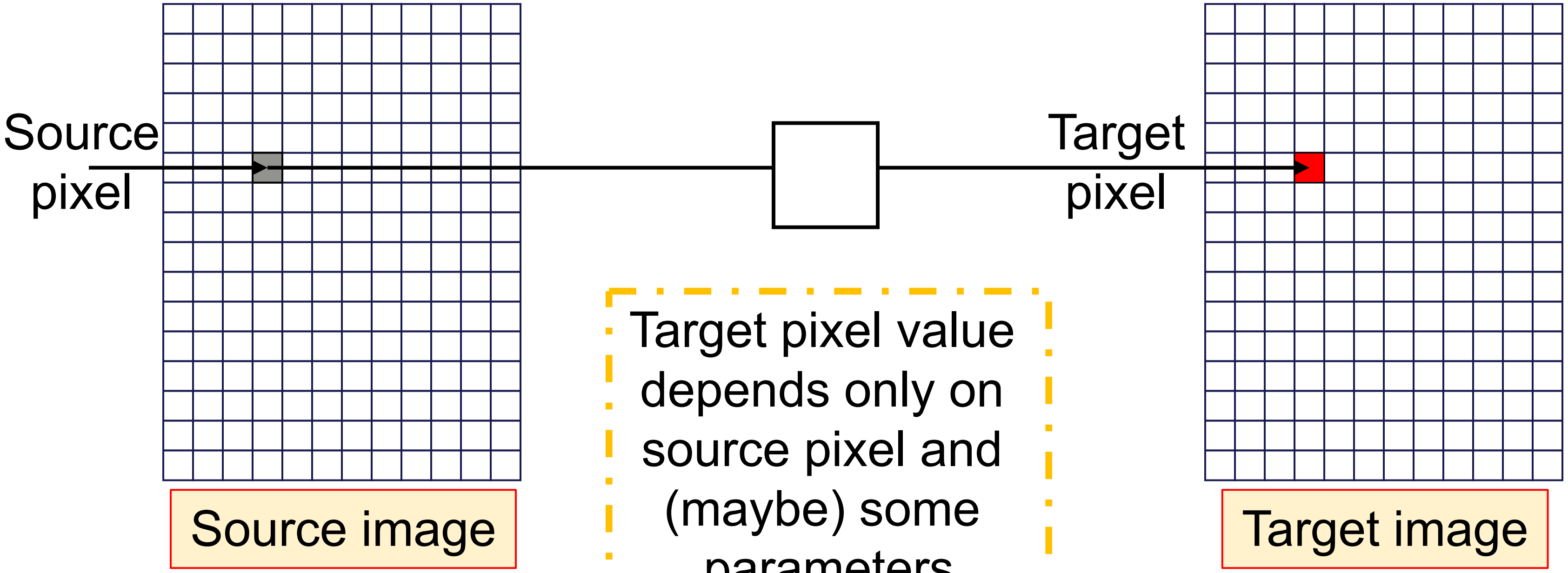
IDENTIFY

1. Convolution, Mean Filtering and Noise
2. Gaussian Filtering

Convolution, Mean Filtering & Noise

Intensity Transform

A Quick Walk DOWN Memory Lane



Target pixel value depends only on source pixel and (maybe) some parameters

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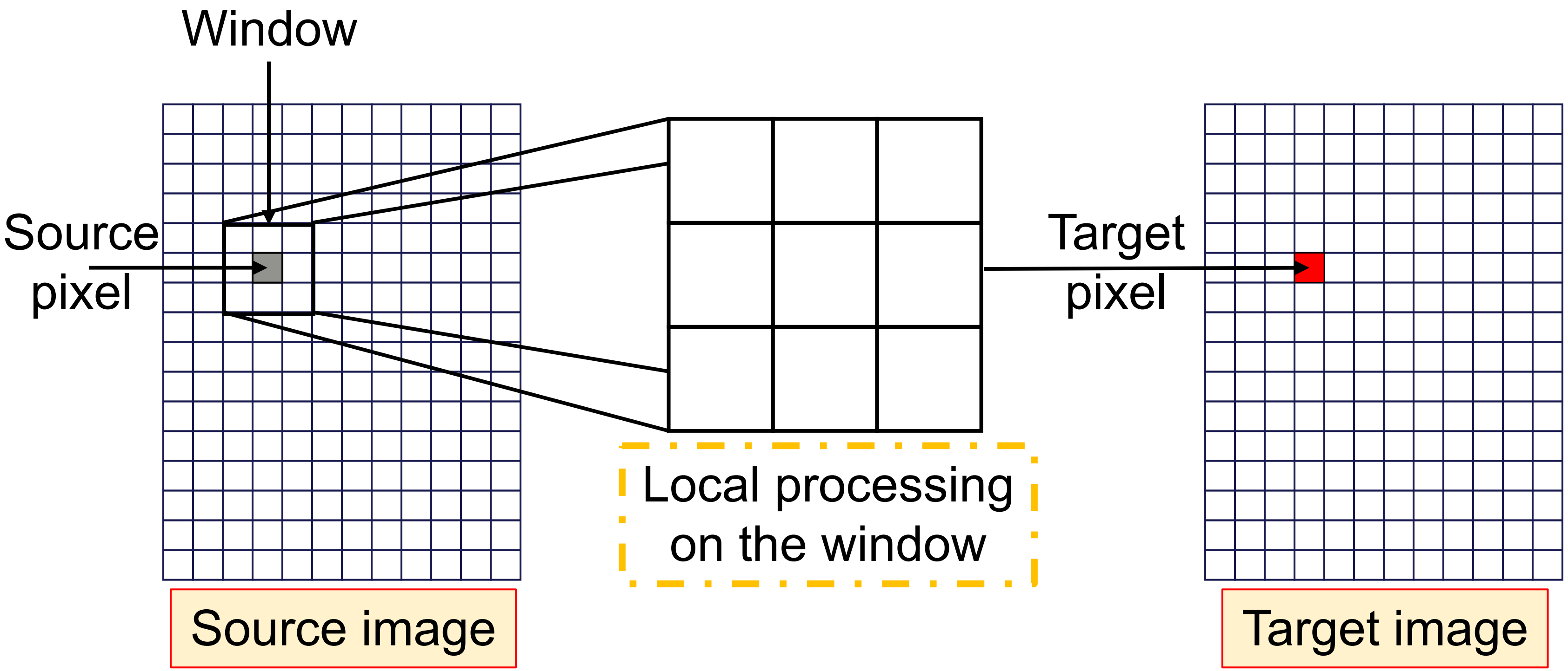
Spatial Filtering



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Why?

?

Intensity Transforms read and affect only a single pixel, their power is limited

Processes restricted to a small, compact area have access to more information but are still likely to consider a single object, surface, illumination pattern, etc.

Images are spatially organised data structures, many important attributes vary slowly across the image

- ***Object identity***
- ***Viewed surface orientation, colour, etc***
- ***Illumination***

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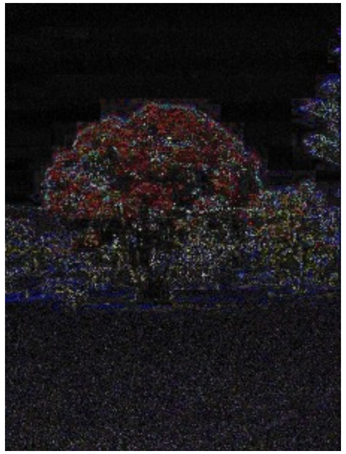
Image Noise

Noise Small errors in image values

- Imperfect sensors introduce noise
- Image compression methods are lossy: repeated coding & decoding adds noise

Original

JPEG



Difference
(Enhanced)

Noise is often modelled as additive:

Recorded value = true value + random noise value

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Gaussian Noise

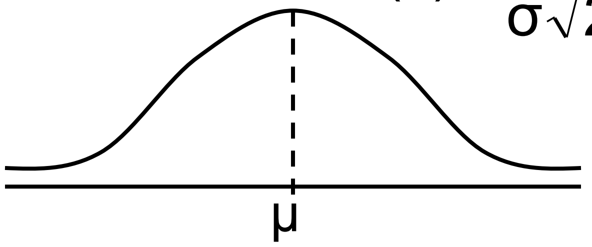
Sensors often give a measurement a little off the true value

- On **average** they give the right value
- They tend to give values near the right value rather than far from it

We model this with a **Gaussian**

- Mean (μ) = 0
- Variance (σ^2) indicates how much noise there is

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$



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Gaussian Noise

The level of noise is related to the Gaussian parameter, σ

σ = 1

σ = 10

σ = 20

Image with varying degrees of Gaussian noise added

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Noise Reduction

If you have multiple images, taking the mean value of each pixel will reduce noise

- Noise is randomly added to each value
- Mean value added is 0
- If you average a large set of estimates of the same pixel, the random noise values will cancel out

42 43 44 41 40 42 42 44 40

42

Given only a single image, averaging over a local region has a similar effect

42	43	44
41	40	42
42	44	40

42

Ideally, we would choose the region to only include pixels that should have the same value

We need a spatial filter…

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Spatial Filtering: Convolution

Many filters follow a similar pattern – multiplying each image value by a corresponding filter entry, and summing the results

$F_{(-1,-1)}$	$F_{(0,-1)}$	$F_{(+1,-1)}$
$F_{(-1,0)}$	$F_{(0,0)}$	$F_{(+1,0)}$
$F_{(-1,+1)}$	$F_{(0,+1)}$	$F_{(+1,+1)}$

Filter Window

$P_{(x-1,y-1)}$	$P_{(x,y-1)}$	$P_{(x+1,y-1)}$
$P_{(x-1,y)}$	$P_{(x,y)}$	$P_{(x+1,y)}$
$P_{(x-1,y+1)}$	$P_{(x,y+1)}$	$P_{(x+1,y+1)}$

Picture Window

$F_{(-1,-1)} \times P_{(x-1,y-1)}$

$+ F_{(0,-1)} \times P_{(x,y-1)}$

$+ F_{(+1,-1)} \times P_{(x+1,y-1)}$

$+ F_{(-1,0)} \times P_{(x-1,y)}$

$+ \ldots$

$+ F_{(+1,+1)} \times P_{(x+1,y+1)}$

Result

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Filtering

More generally, with a filter with radius *r*

- $p_{x,y}$ is the original image value at (x,y)
- $p'_{x,y}$ is the new image value at (x,y)

$$p'(x,y) = \sum_{dx=-r}^{+r} \sum_{dy=-r}^{+r} f_{dx,dy} \times p_{x+dx,y+dy}$$

Many, though not all, filters work this way, e.g. **the mean filter:**

3 × 3 mean filter

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

5 × 5 mean filter

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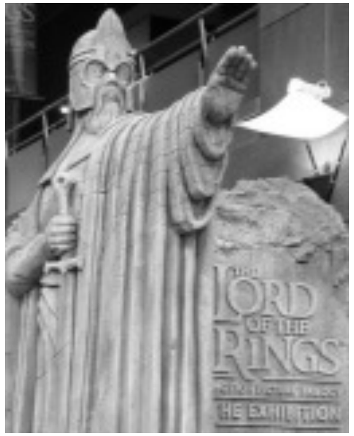
The Mean Filter



Original

Gaussian

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Key Points

Spatial filters operate on local image regions

Many can be formulated as convolution with a suitable mask

REMEMBER

Noise reduction via mean filtering is a classic example

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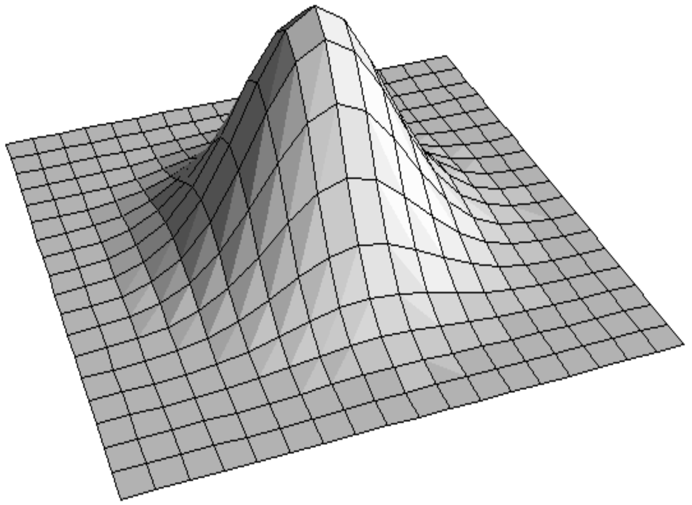
Break

Gaussian Filtering

Gaussian Filters

- Convolution with a mask whose weights are determined by a 2D Gaussian function
- Higher weight is given to pixels near the source pixel
- These are more likely to lie on the same object as the source pixel



$$P(x,y) = \frac{1}{\sigma^2 2\pi} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

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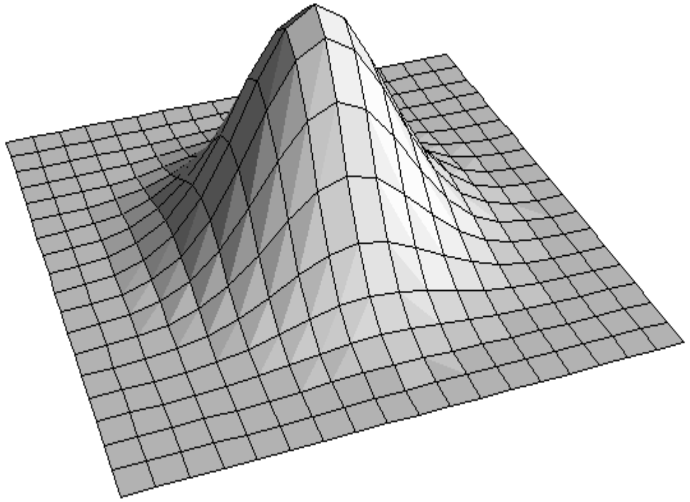
Discrete Gaussian Filters

The Gaussian

- Extends **infinitely** in all direction, but we want to process just a **local** window
- Has a volume underneath it of 1, which we want to maintain



We can approximate the Gaussian with a discrete filter

- We restrict ourselves to a square window and **sample** the Gaussian function
- We **normalise** the result so that the filter entries add to 1

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Example

Suppose we want to use 5x5 window to apply a Gaussian filter with $\sigma^2 = 1$

- The centre of the window has x = y = 0
- We sample the Gaussian at each point
- We then normalise it

$$
\begin{array}{c|ccccc|}
-2 & 0.00 & 0.01 & 0.02 & 0.01 & 0.00 \\
-1 & 0.01 & 0.06 & 0.10 & 0.06 & 0.01 \\
0 & 0.02 & 0.10 & 0.16 & 0.10 & 0.02 \\
1 & 0.01 & 0.06 & 0.10 & 0.06 & 0.01 \\
2 & 0.00 & 0.01 & 0.02 & 0.01 & 0.00 \\
\hline
 & -2 & -1 & 0 & 1 & 2
\end{array}
\times \frac{1}{0.96}
$$

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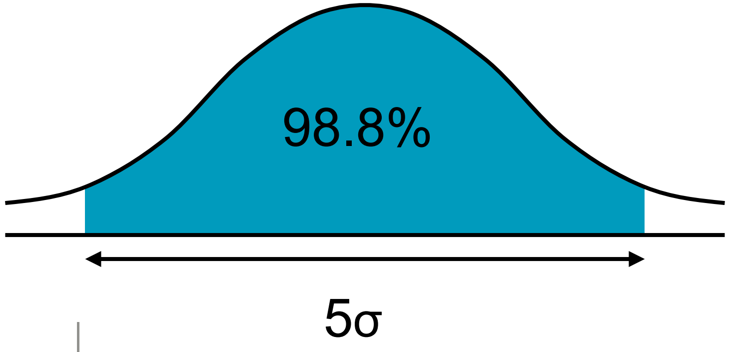
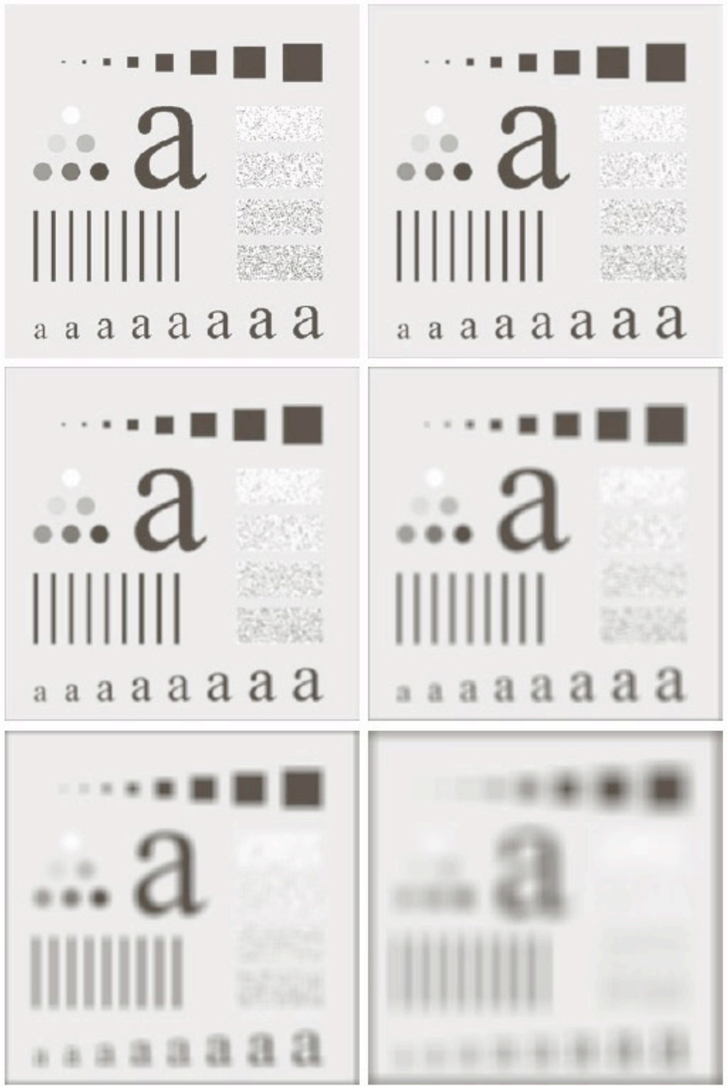
Gaussian Filters

How big should the filter window be ?

- With Gaussian filters this depends on the variance (σ^2)
- Under a Gaussian curve 98% of the area lies within 2σ of the mean
- A filter width of 5σ gives more than 98% of the values we want





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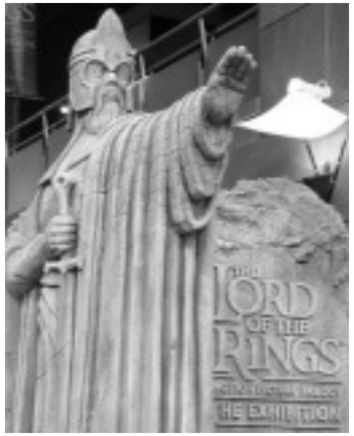
The Gaussian Filter



Original

Gaussian

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Separable Filters

The Gaussian filter is separable

- A 2D Gaussian is equivalent to two 1D Gaussians
- First you filter with a ‘horizontal’ Gaussian
- Then with a ‘vertical’ Gaussian…

0.06
0.24
0.40
0.24
0.06

0.06	0.24	0.40	0.24	0.06

$$
\begin{aligned}
P(x,y) &= \frac{1}{\sigma^2 2\pi} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \\
&= \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} e^{\frac{-y^2}{2\sigma^2}} \\
&= \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \right) \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-y^2}{2\sigma^2}} \right) \\
&= P(x) \times P(y)
\end{aligned}
$$

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Separable Filters

The separated filter is separable is more efficient

- Given an N x N image and a n x n filter we need to do $O(N^2n^2)$ operations
- Applying two n x 1 filters to a N x N image takes $O(2N^2n)$ operations

Example

- A 600 x 400 image and a 5 x 5 filter
- Applying it directly takes around 6,000,000 operations
- Using a separable filter takes around 2,400,000 – less than half as many

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Key Points

Like mean filtering, and Gaussian smoothing can be used to remove additive noise

Gaussian smoothing emphasises pixels near the source pixels, where it is more likely image properties are fixed

REMEMBER

Gaussian smoothing is separable, and so efficient

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Summary

1. Convolution, Mean Filtering and Noise
2. Gaussian Filtering

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Questions

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NEXT:

Nonlinear Filtering & Thresholding