
University of
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COMP-2032
Introduction to
Image Processing
Lecture 4
Nonlinear Filters & Thresholding

Learning Outcomes

IDENTIFY

1. Median Filtering
2. Anisotropic Diffusion
3. Bilateral Filtering
4. What is Thresholding
5. Adaptive Thresholding

Part 1

Nonlinear Filters

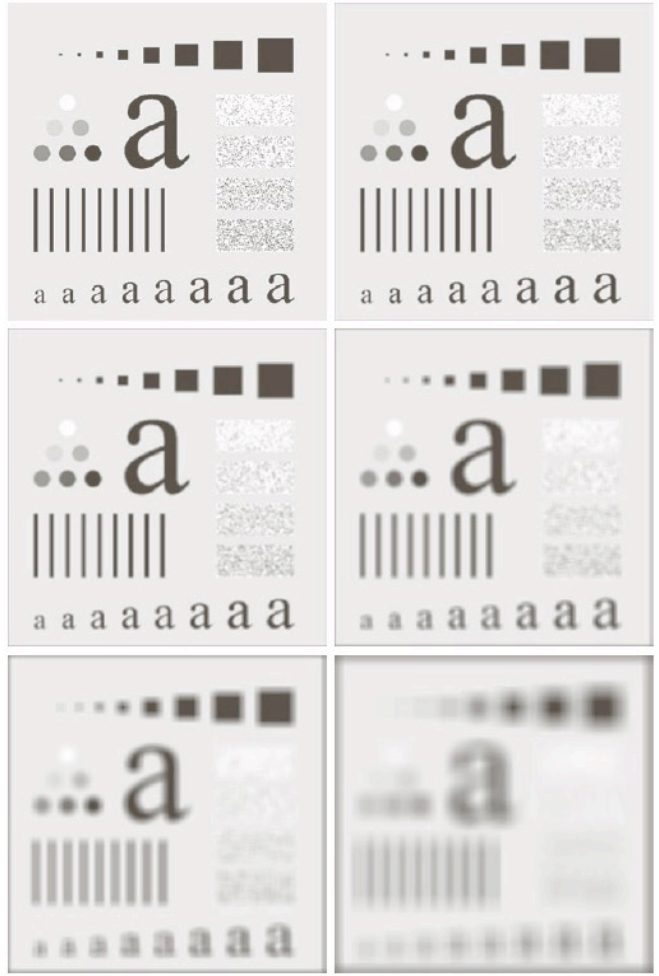
Non-Linear Filters

- Convolution with a mask of weights compute a linear function of a set of pixel values
- Many operations can be implemented this way, but not all:

- *Median filtering*
- *Anisotropic diffusion/Bilateral filtering*

Linear filters smooth sharp image changes, **nonlinear filters** tend to preserve or even enhance them

Difference



Gaussian Smoothing

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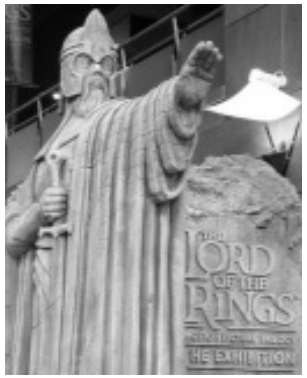
Median Filtering

Salt and Pepper Noise

Sometimes sensors either fail to respond or saturate in error

- A false saturation gives a white spot in the image (salt)
- A failed response gives a black spot in the image (pepper)
- Sometimes called **speckle noise**



1%

10%

20%

An image with varying amounts of salt and pepper noise added

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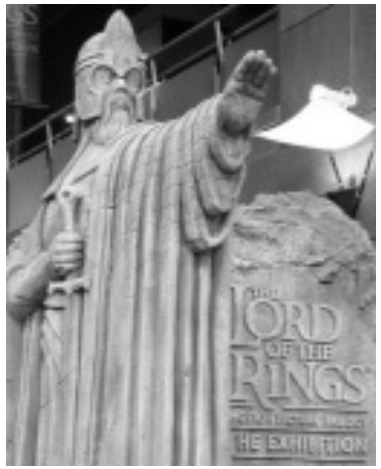
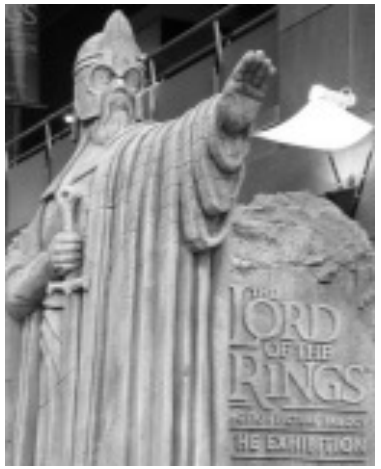
Reducing Salt and Pepper Noise



Original

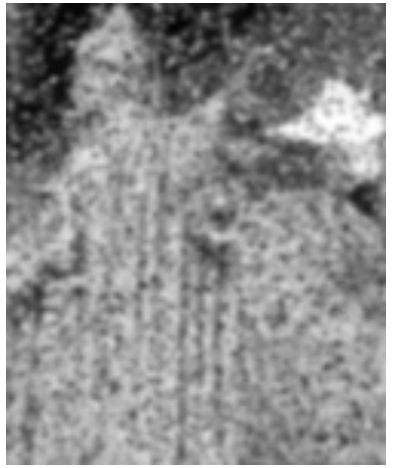
Salt & Pepper



Original

Salt & Pepper

Mean Filtering



Gaussian Filtering

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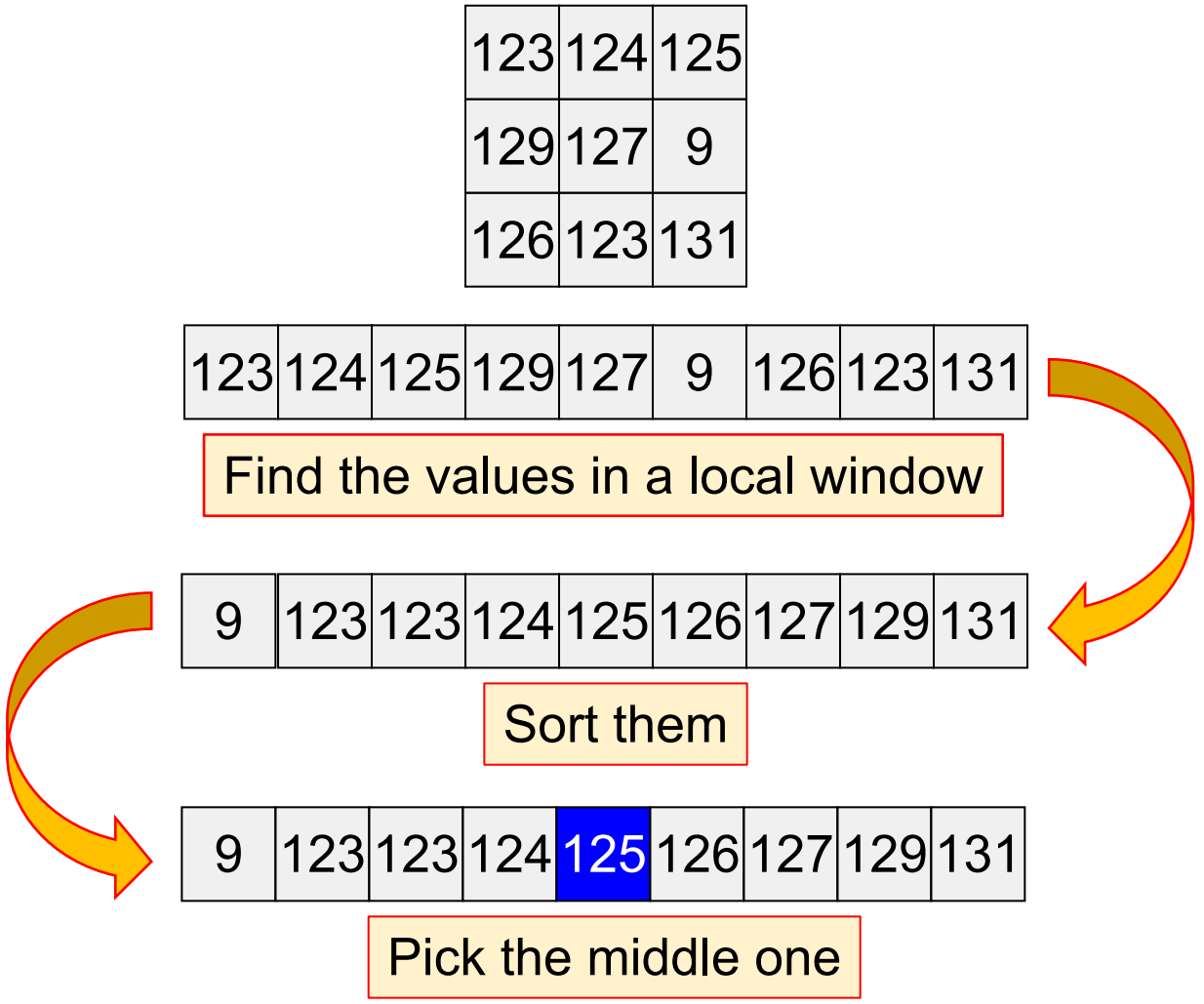
The Median Filter

Alternative: Median Filter

- Statistically the median is the middle value in a set
- Each pixel is set to the median value in a local window
- Result is a real pixel value, not a combination
- Noise pixels are outliers
- Noise would have to affect >1/2 the pixels to appear in the output



A mean filter would give 113

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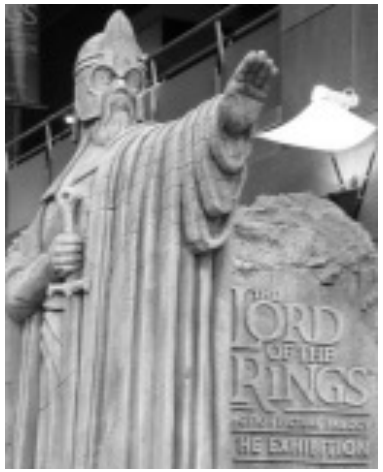
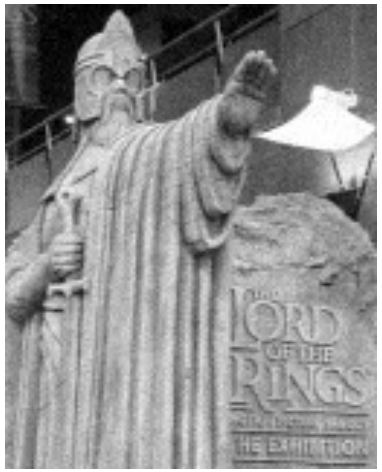
The Median Filter



Original

Salt & Pepper



Gaussian

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Anisotropic Diffusion

Anisotropic Diffusion

- Median filtering is good given small regions of speckle noise, less good if edges are important
- There exist explicit edge-preserving smoothing ops

Diffusion

- Spreading out
- Mean and Gaussian filters can be seen as diffusion processes

Anisotropic

Not the same in all directions

Basic IDEA

- Mean and Gaussian filters make each pixel more like its neighbours
- Anisotropic diffusion makes each pixel more like those neighbours **that it is already similar to**

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Anisotropic Diffusion

We have a similar function, *s(p,q)*

- *s(p,q)* has values in the range from 0 to 1
- If the pixels *p* and *q* are similar then *s(p,q)* is close to 1
- If the pixel *p* and *q* are different then *s(p,q)* is close to 0

We use *s(p,q)* to compute a weighted average of pixel values

- The new value at a pixel *p*, is based on all its neighbours, *q*

$$p' = \frac{\sum q \times s(p,q)}{\sum s(p,q)}$$

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The Similarity Function

- The smoothing function, *s(p,q)* needs to be found
- If *d* is the difference between *p* and *q* and *D* is the maximum possible difference we can use:

$$\frac{D-d}{D}$$

Other functions often used include:

$$s(p,q) = e^{\left(\frac{p-q}{K}\right)^2}$$

$$s(p,q) = \frac{1}{1+\left(\frac{p-q}{K}\right)^2}$$

K determines the amount of smoothing

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Anisotropic Diffusion

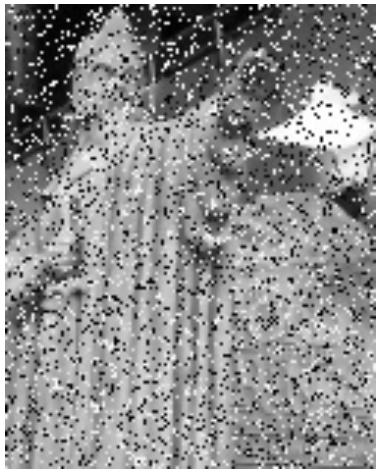
The examples here used the similarity function:

$$s(p,q) = \frac{1}{1 + \left(\frac{p - q}{K} \right)^2}$$

With $K = 25$

Salt & Pepper

Gaussian



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Anisotropic Diffusion

A higher value of **K** gives greater smoothing, but **edges are still (quite) sharp**

K = 5 | K = 25 | K = 50 | K = 100

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Anisotropic Diffusion

We can apply the filter repeatedly to give greater smoothing

Original | 1 Iteration | 2 Iteration | 3 Iteration

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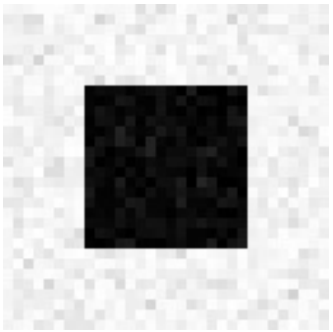
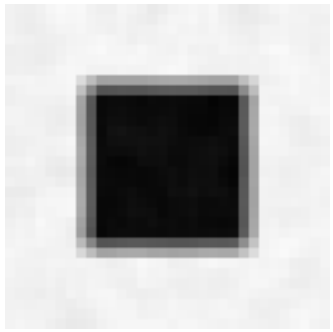
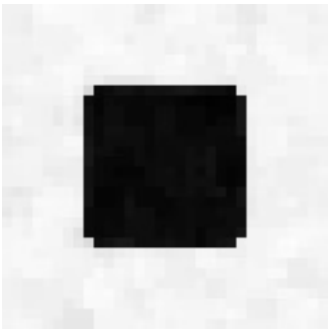
Reducing Noise near Edges



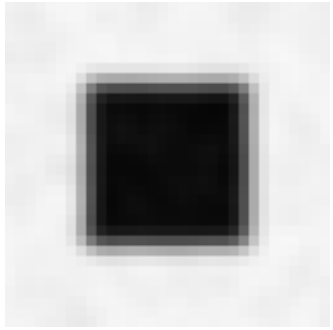
Noise Image



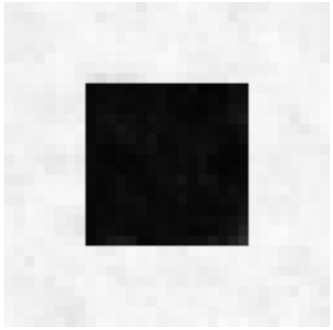
Mean



Median



Gaussian



Anisotropic Diffusion

Anisotropic diffusion is to mean filtering as ???? Is to Gaussian filtering…

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Bilateral Filtering

Bilateral Filtering

- Anisotropic Diffusion is related to mean filtering
- If the similarity function is always 1 we get a mean filter

$$p' = \frac{\sum q \times s(p,q)}{\sum s(p,q)}$$

Sums pixel values in a region

Counts pixel values in a region

Bilateral filters modify Gaussian smoothing in a similar way

- One Gaussian weights pixels that are near the source
- Another Gaussian weights pixels that have similar intensity to the source pixel

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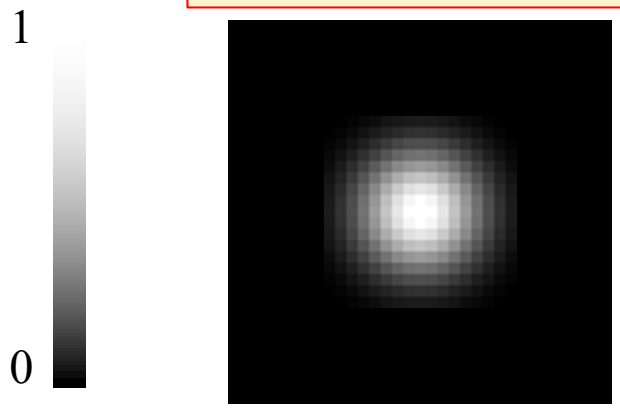
Gaussian Smoothing Again

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}\left(\| \mathbf{p} - \mathbf{q} \|\right) I_{\mathbf{q}}$$

normalized Gaussian function



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Bilateral Filtering

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}}-I_{\mathbf{q}}|\right) I_{\mathbf{q}}$$

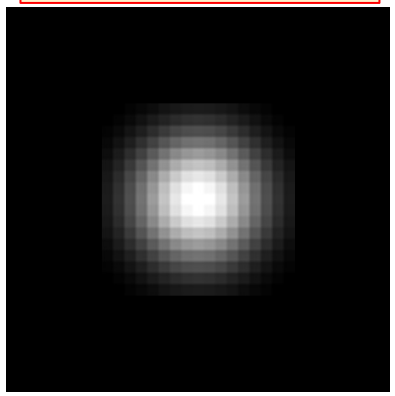
new

not new

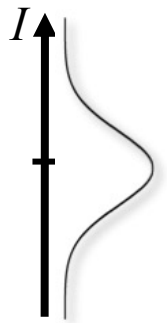
new

normalization factor

space weight



range weight



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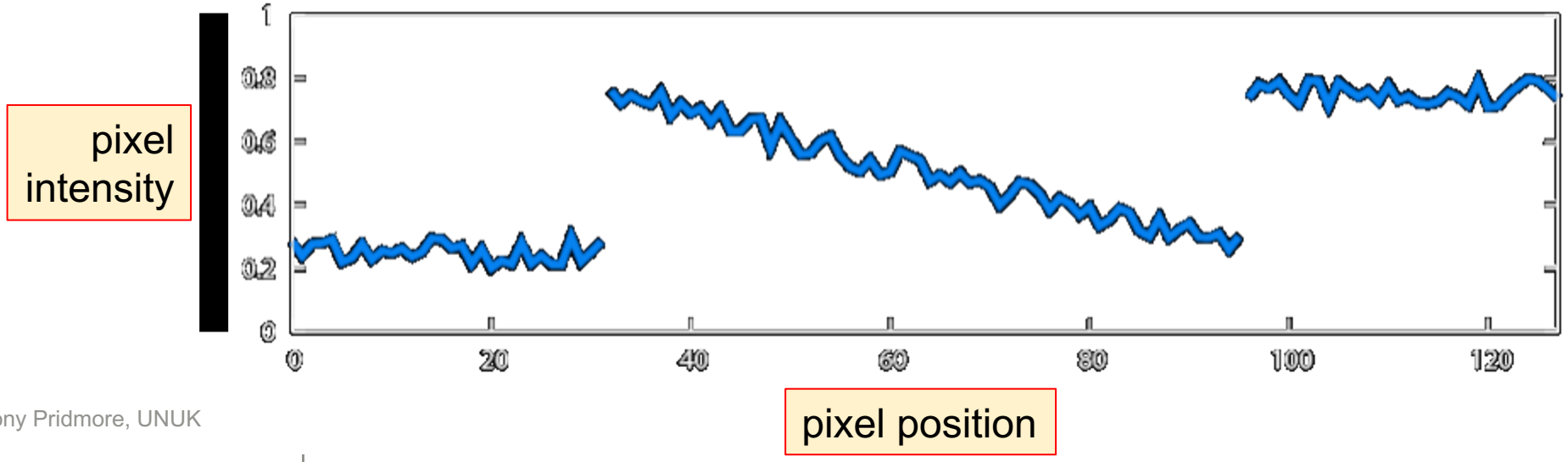
A One Dimensional Example

1D image = line of pixels

Better visualised as a plot



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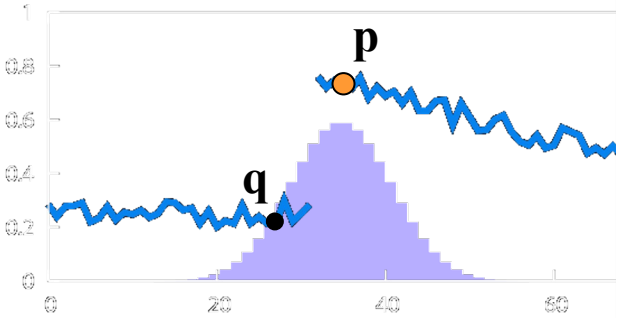
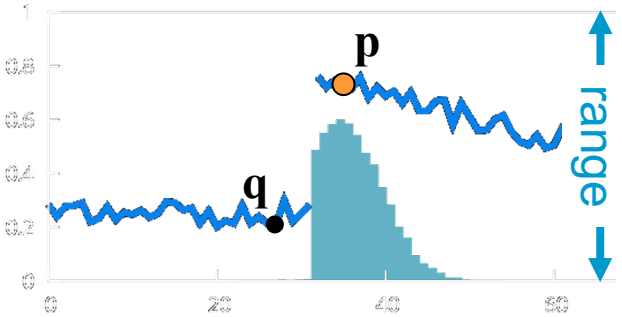
Gaussian & Bilateral Smoothing

Gaussian blur



Bilateral filter



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|) I_{\mathbf{q}}$$

space

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|) I_{\mathbf{q}}$$

space range

normalization

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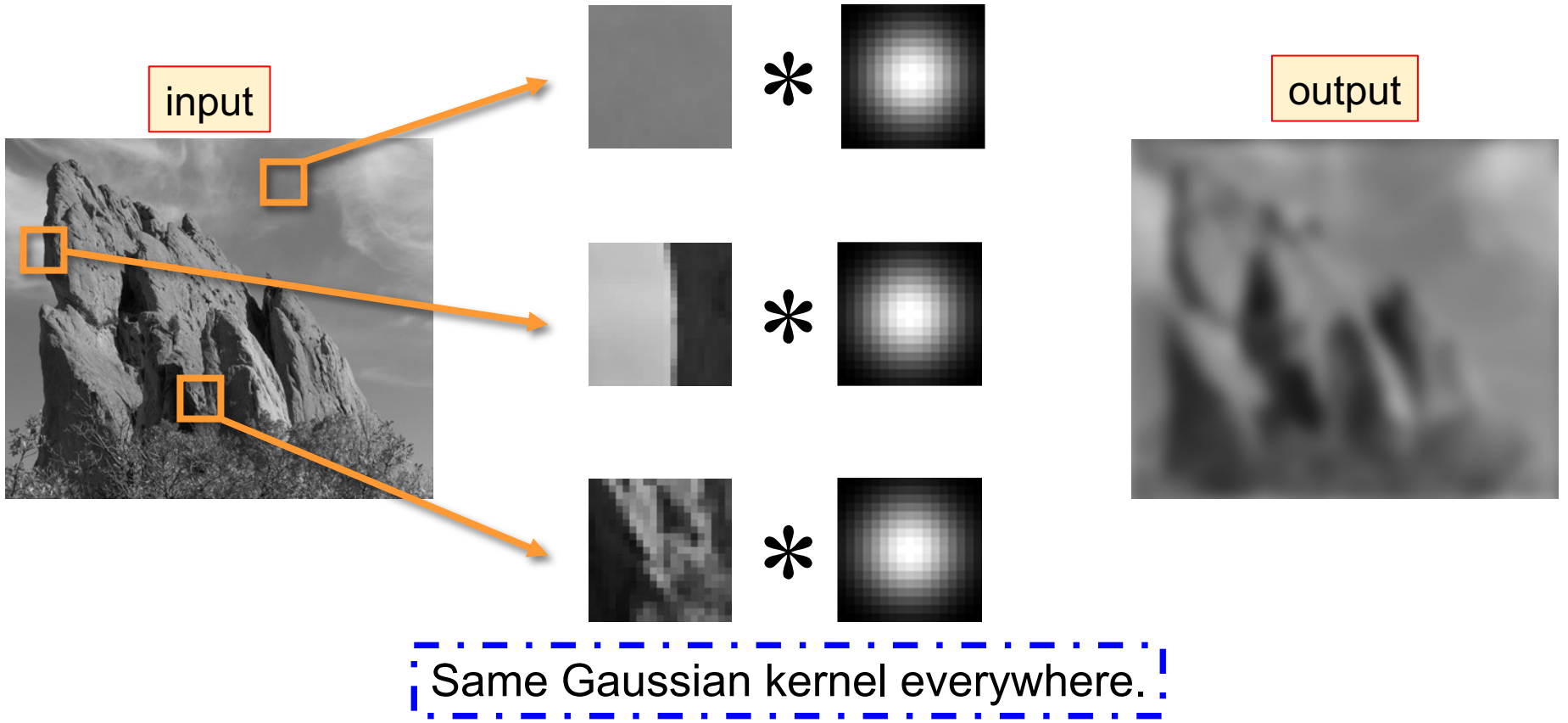
In 2D: Gaussian Smoothing



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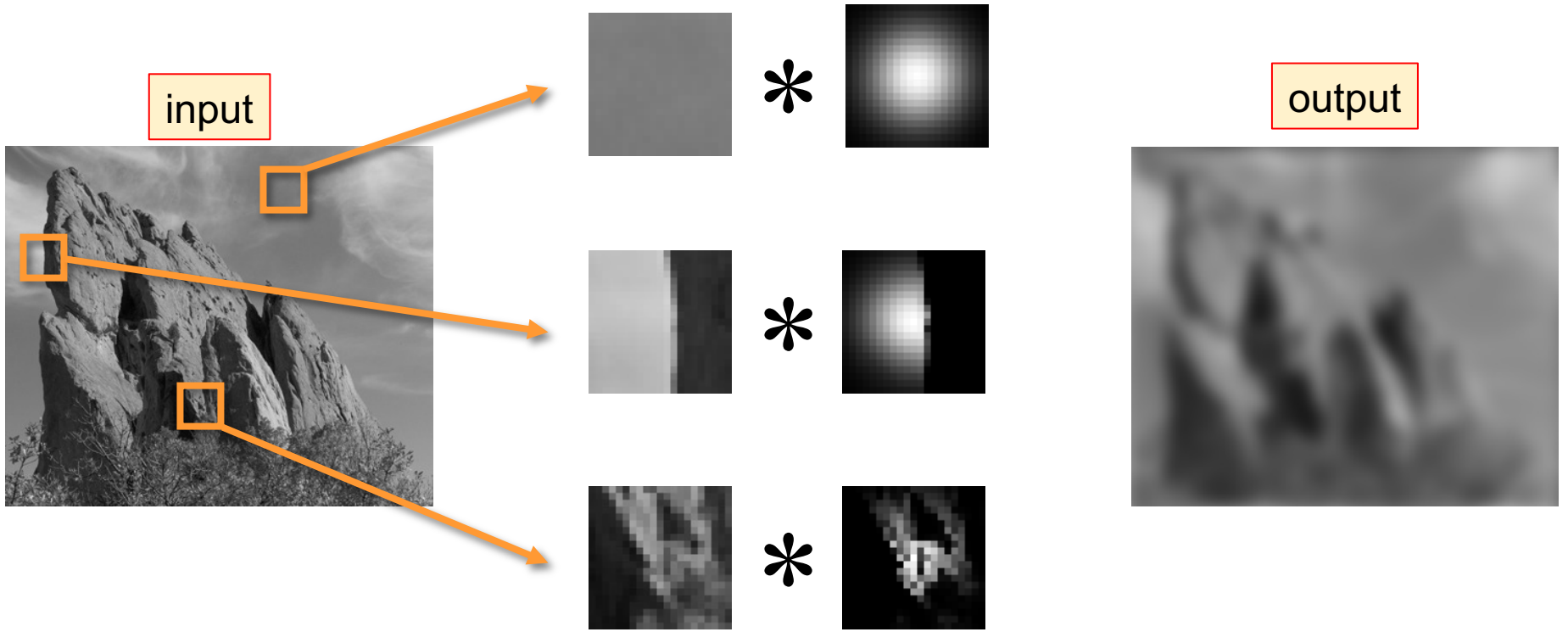
In 2D: Bilateral Filtering



The kernel shape depends on the image content.

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Break

Part 2

Thresholding

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What is Thresholding?

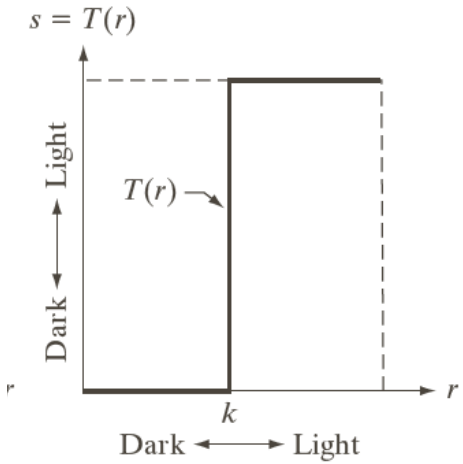
Binary Images

Many image processing operations make a decision:

Is this the colour/object/edge I am interested in?

The result is a binary image

- Pixels can have only two values: 0 or 1
- Binary images also need noise removal, enhancement, etc.



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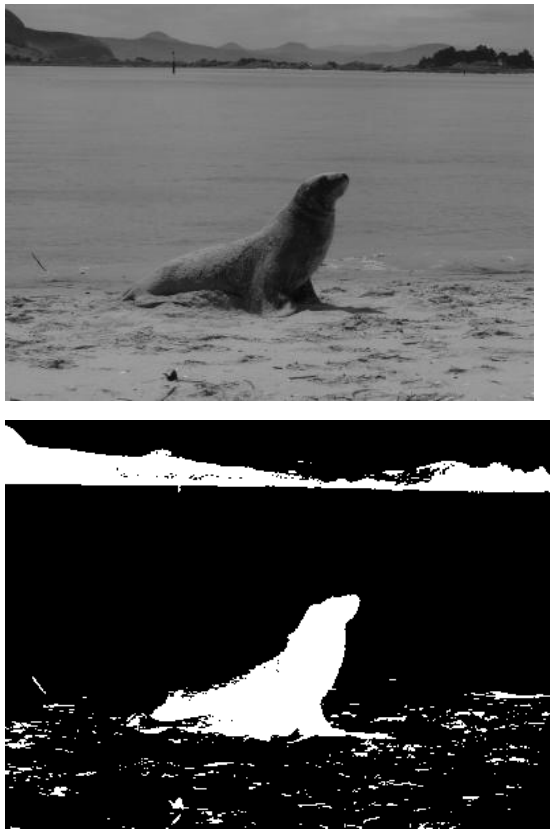
Binarisation: Thresholding

- A dark object on a light background in a grey-level image
- Choose a threshold value, T
- Consider each pixel in turn
  - If the brightness at a pixel is less than T, that pixel is object
  - Otherwise, it is part of the background

Basic idea extends to colour; define sets of colour values that correspond to objects



Threshold, *T = 96*

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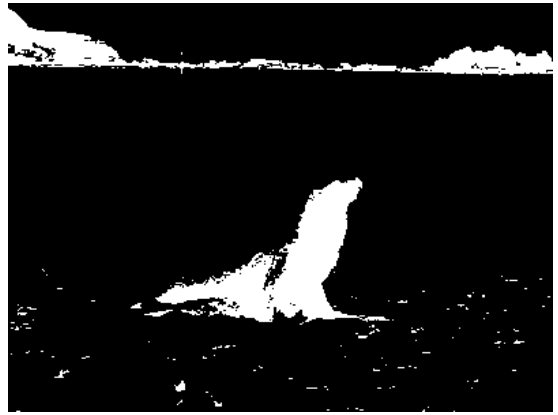
Too Simple?

The value of the threshold is very important

- If it is too high, background pixels will be classified as foreground
- If it is too low, object pixels will be considered background

Assumes there are exactly two regions, with no overlap in their brightness – *is that true*?

T = 128



T = 64

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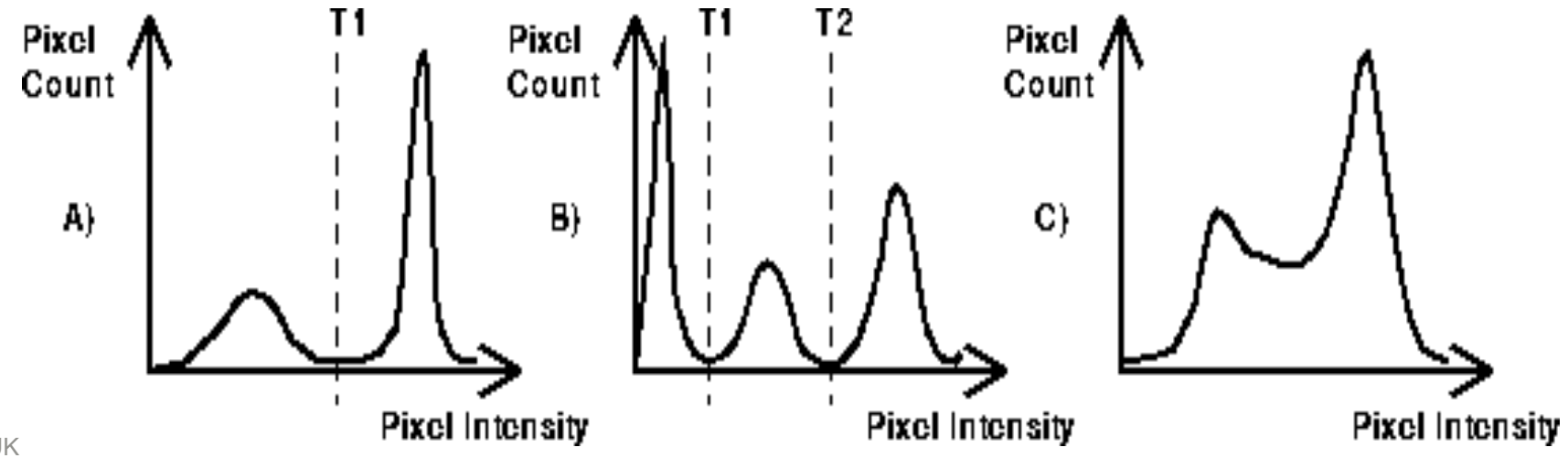
Adaptive Thresholds

- If the user chooses *t* for each of a set of images, there is no guarantee the results will be consistent
- Automatic methods choose a threshold based on image properties: histogram are commonly used



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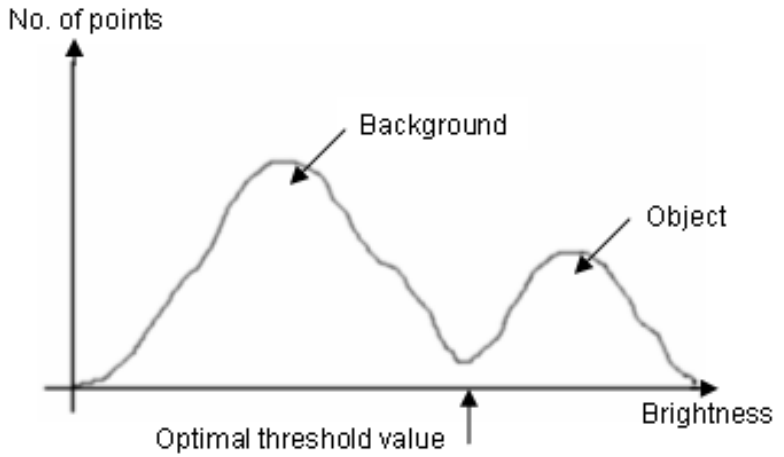
Adaptive Thresholding

Otsu Thresholding

- Assumes histograms are *bimodal*; two regions can be separated by one threshold
- Think of the histrogram as made of two normal (Gaussian) distributions, described by their means and deviations

If a threshold is wrong, it will include histogram bins from peak A in peak B

- Peak A's deviation will be too small
- Peak B's deviation will be too big
- Peak A's size (area) will be too small
- Peak B's size will be too big



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Otsu Thresholding

- Find the threshold which minimises a weighted sum of the variations of the two regions that threshold produce
- Weights are the areas of the histogram assigned to each region

$$\sigma_w^2(t) = q_1(t)\sigma_1^2(t) + q_2(t)\sigma_2^2(t)$$

$$q_1(t) = \sum_{i=1}^{t} P(i) \qquad q_2(t) = \sum_{i=t+1}^{I} P(i)$$

This is small when the two regions are **both physically small and have low deviations**

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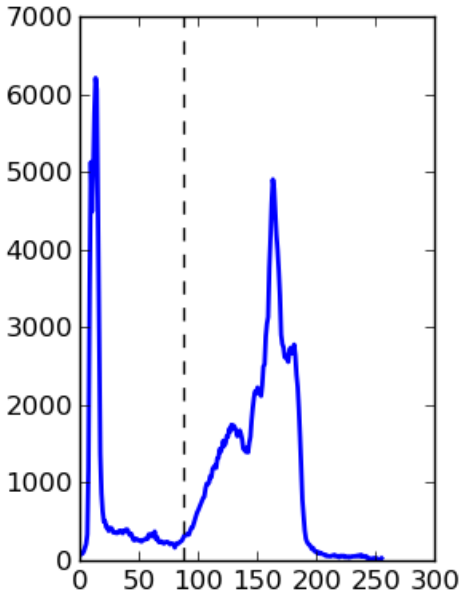
Otsu Thresholding

- Consider all possible threshold values (0 – 255)
- Compute weighted sum
- Pick *t* with the smallest value

Algorithm

A recursive version exists that is very efficient



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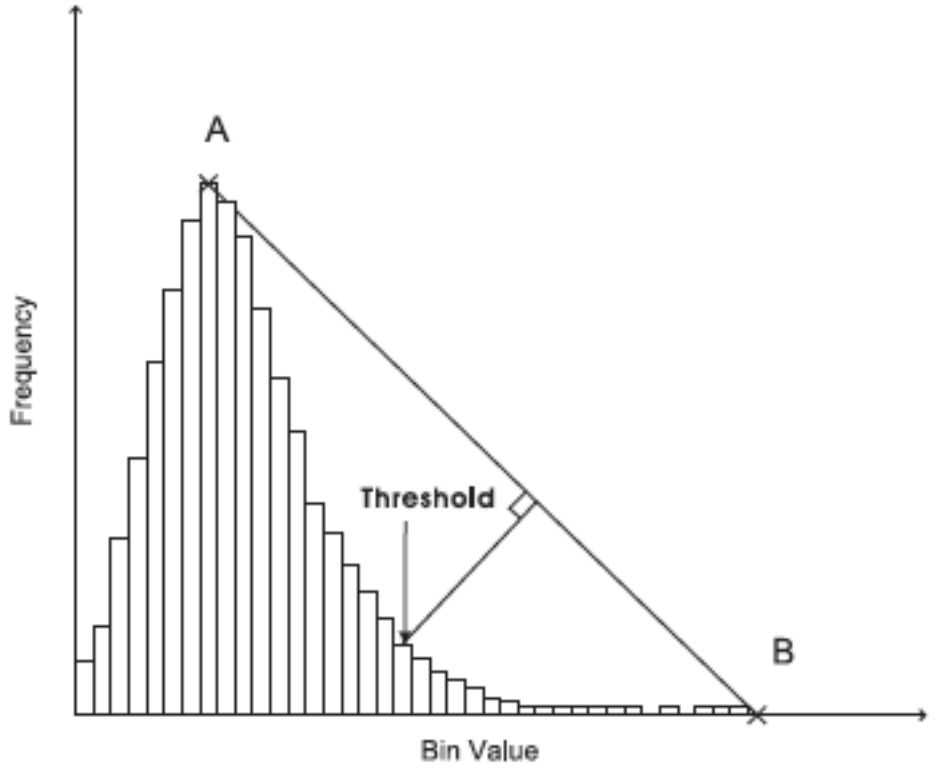
Unimodal Thresholding

Many histograms are not bimodal, there is often only one peak e.g., text is mainly white, with a small amount of black

- Finds the peak
- Draws a line from there to the top of the furthest bin
- Finds the top of the bin that is furthest from this line; that bin value is the threshold

Rosin's unimodal method



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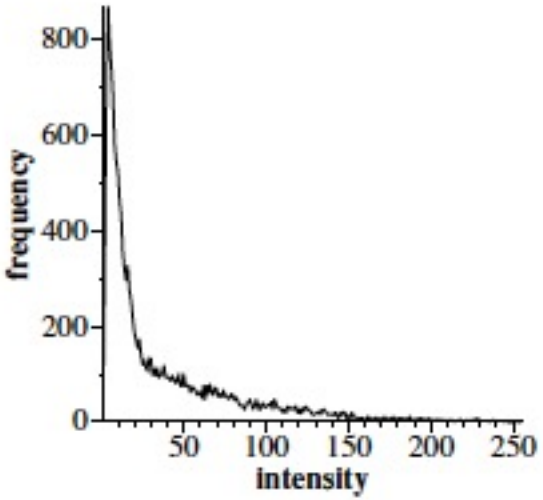
Unimodal Thresholding

Can be applied to any suitable image, e.g., intensity gradients

Edge of magnitudes (**inverted for display**)



Histogram of edge magnitudes

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Unimodal Thresholding

Rosin

Otsu

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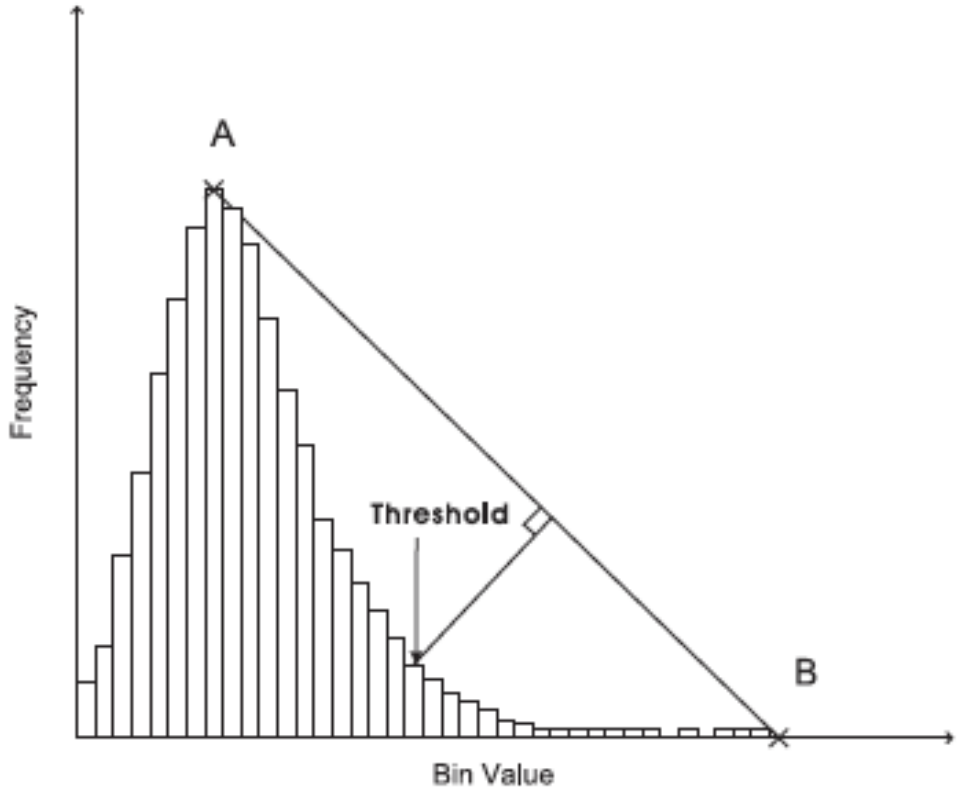
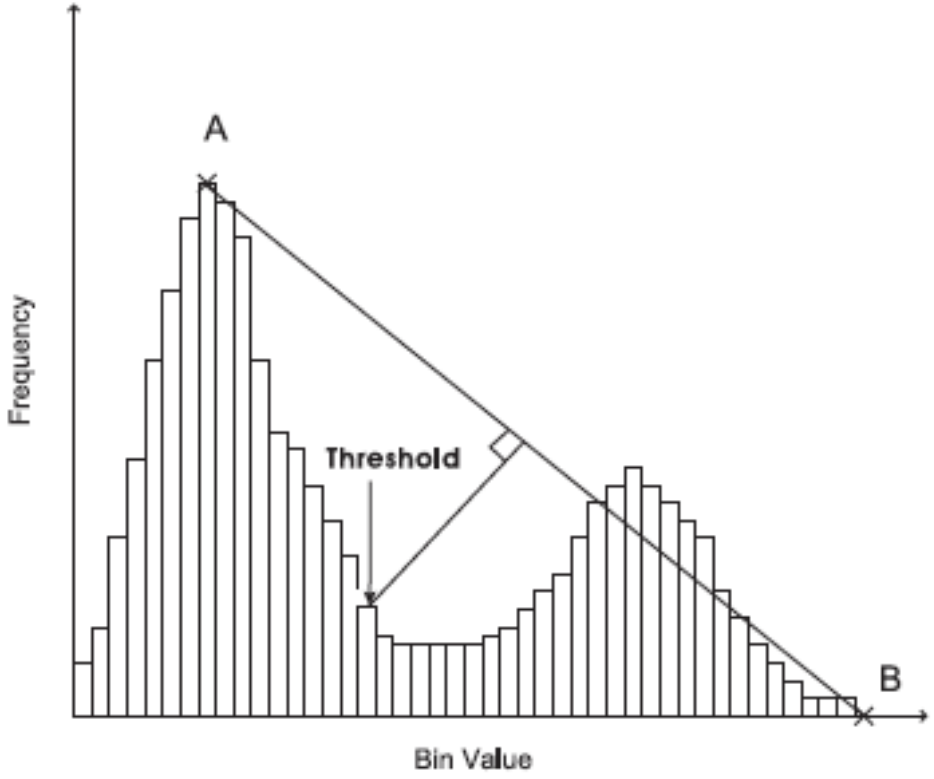
Otsu vs Rosin





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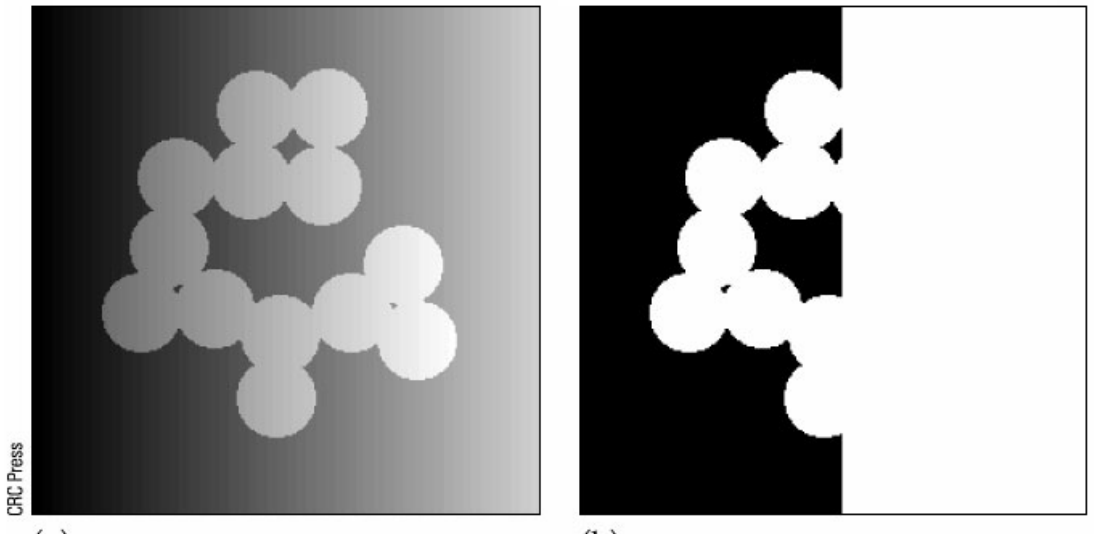
Local Adaptive Methods

- Imaging conditions and object properties can vary within a single image as well as across sets of images
- Histograms can be too complex for any method's assumption to be true



Otsu

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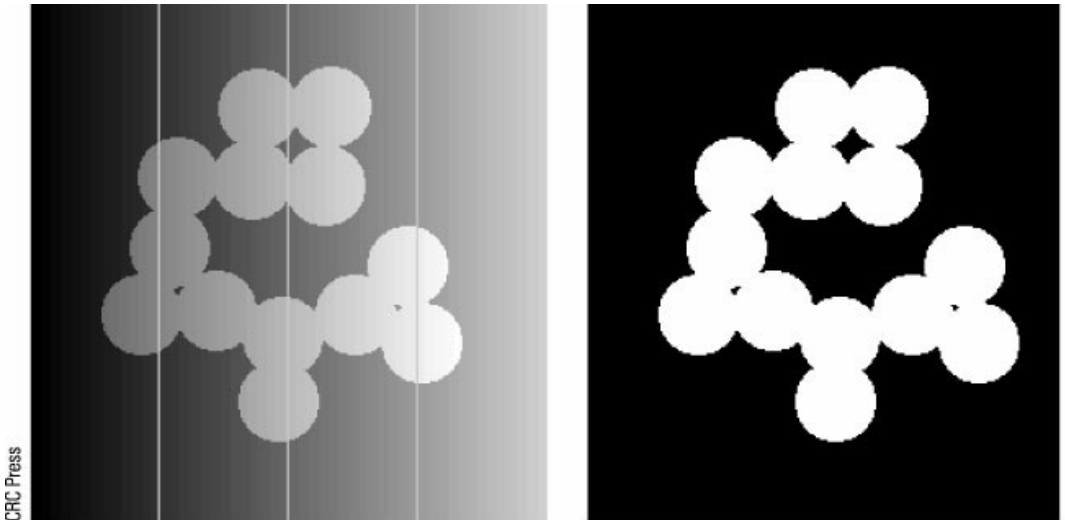
Local Adaptive Methods

- Assumptions about histograms may, however, be true for local areas of the image
- Divide image into sub-regions, apply a threshold selection method independently to each



- The histograms of each vertical strip of this image are bimodal
- Otsu can be applied to each strip

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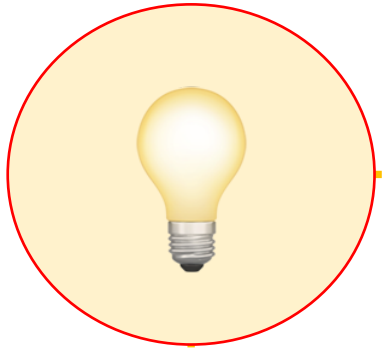
Summary



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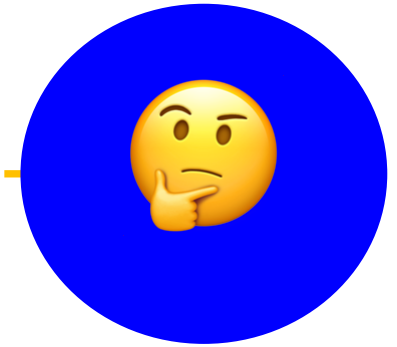


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Questions

One last thing…

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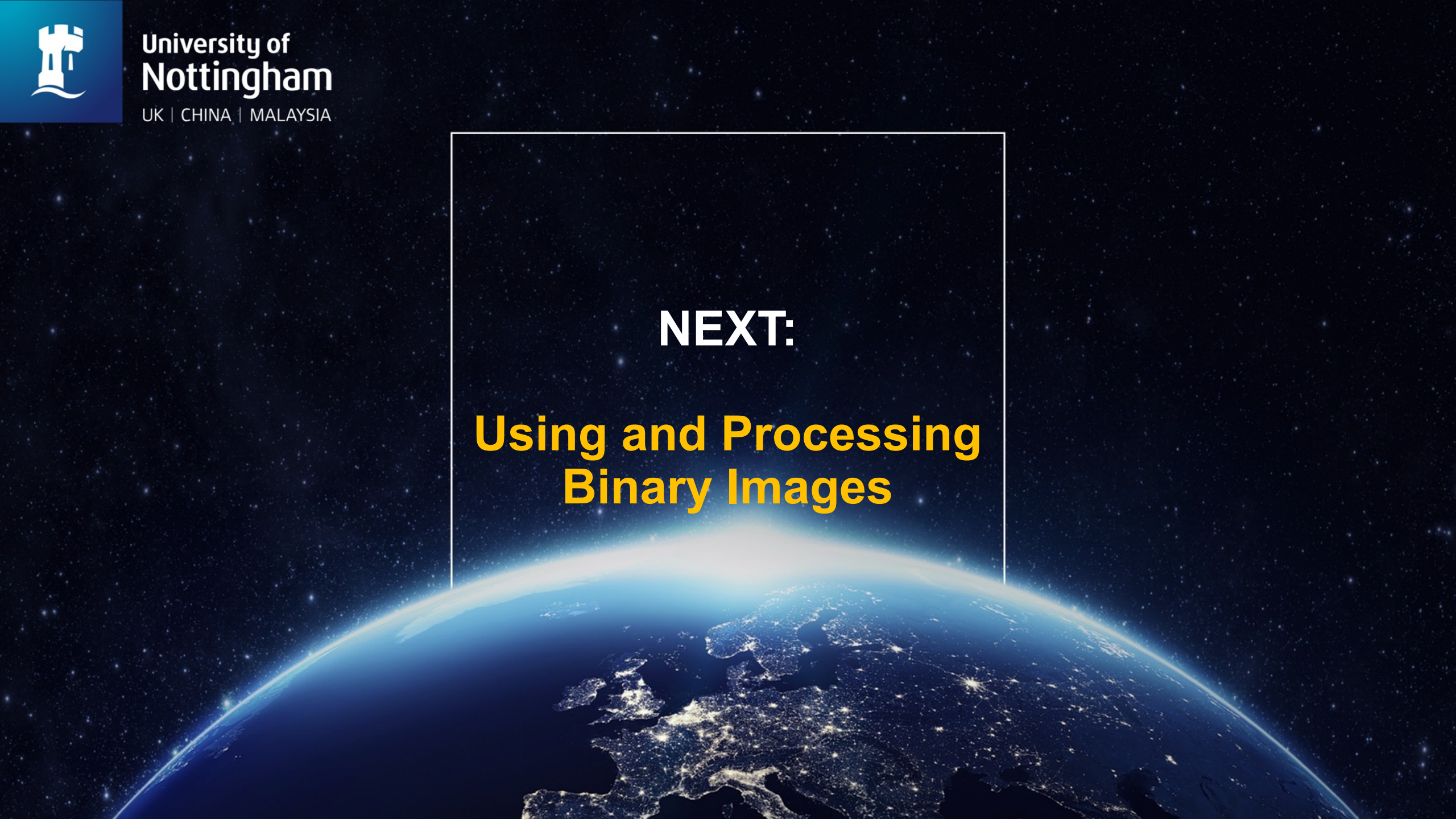
LET'S PLAY KAHOOT

Get Ready Everyone!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NEXT:
Using and Processing
Binary Images