
University of
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COMP-2032
Introduction to
Image Processing
Lecture 9
Hough Transform & Frequency Domain

Learning Outcomes

IDENTIFY

1. How to find lines?
2. The Hough Transform
3. Other parameter spaces
4. What is frequency domain?
5. The frequency domain processing

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Finding Lines:
Template Matching

Why Lines?

Geometric primitives make important vision tasks easier

E.g., matching or classifying images

- Some geometric objects are invariant under projection from the (3D) world to (2D) image
- Straight lines in the world create straight lines on the image

A straight line in the image is evidence for a straight line in the world

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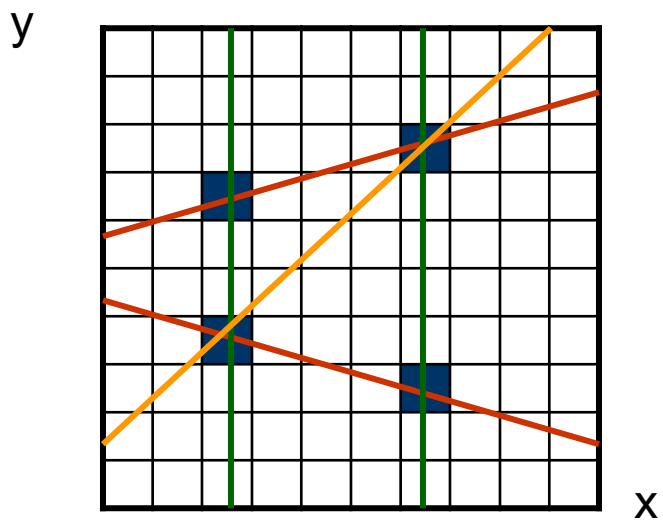
The Problem

- Suppose straight lines are important
- Edge detection provides a set of points (x_i, y_i) which are likely to lie on those lines
- But how many lines are there and what are their parameters?



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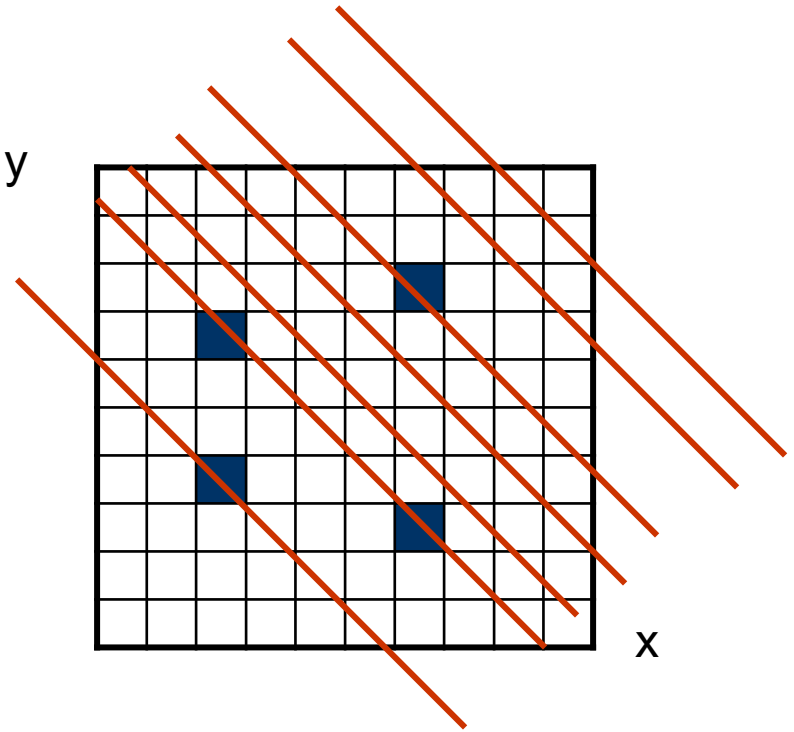
Template Matching

- One solution is to take a straight line and match it to all possible image positions and orientations
- Compute a measure of fit to the edge data

E.g., count how many edges are under each possible line

Incredibly expensive!



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Template Matching vs. Hough

Classic template matching takes a line, lies it on the image data and asks:

- *Does it fit here?*
- *Here?*
- *Here?*
- *How about here?*
- *Here then?*
- ……..

The Hough takes each data item (edge) and asks:

What lines could pass through this?

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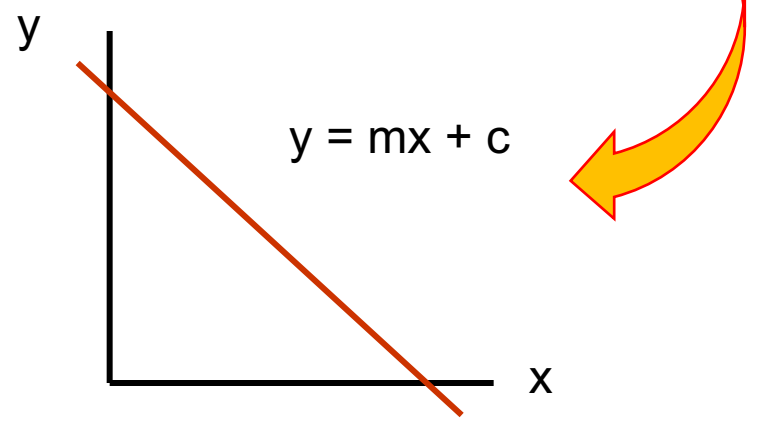
The Hough Transform

Line Parameters

The standard equation of a line is

$$y = mx + c$$



- If we know the line parameters *m, c* - we can vary *x*, compute *y* and draw the line
- This line represents the set of (*x,y*) pairs that satisfy the above equation

But we don't know the line parameters – we just have some data points (x_i, y_i)

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Parameter Space

But if a data point (x_i, y_i) lies on a line then that line's parameters m, c must satisfy

$$y_i = mx_i + c$$

So we can represent all possible lines through (x_i, y_i) by the set of (m,c) pairs that satisfy this equation

Rearranging the equation gives

$$C = -mx_i + y_i$$

This also describes a straight line, but as x_i and y_i are known and m and c are unknown, that line is in m, c space – a parameter space

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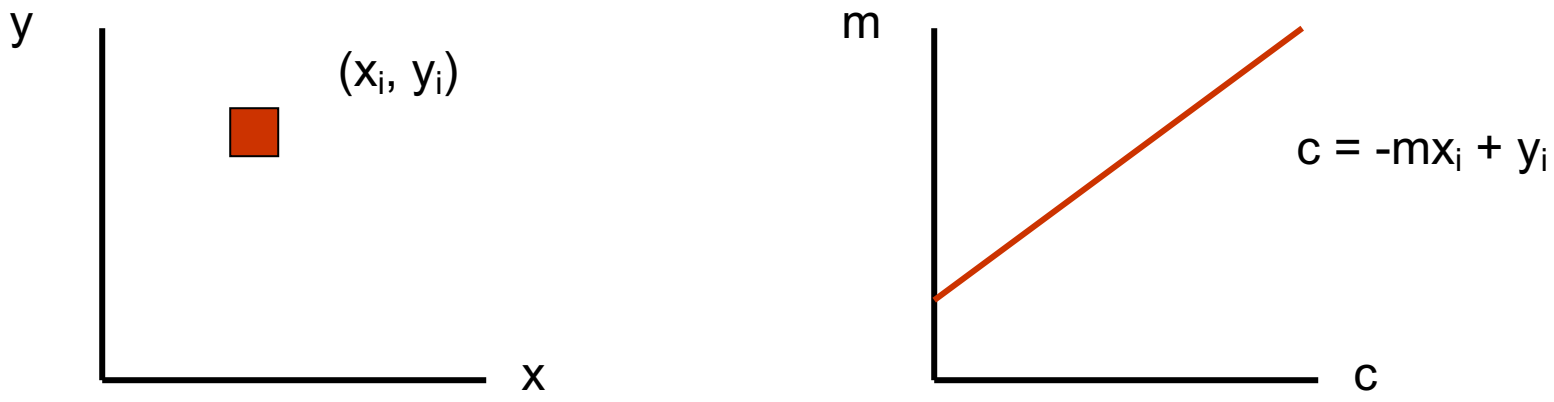
Parameter Space



This straight line in *m*, *c* space represents all the values of (*m, c*) that satisfy $y_i = mx_i + c$, and so the parameters of all the lines that could pass through (x_i, y_i)

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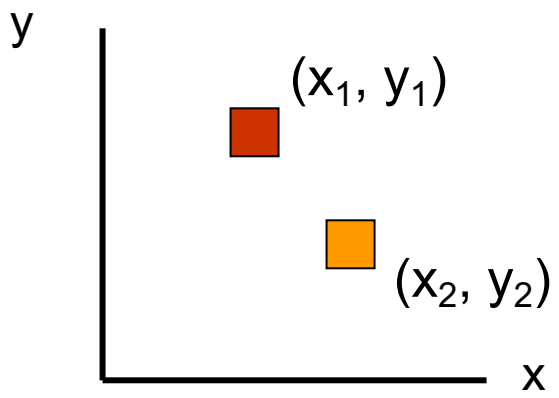
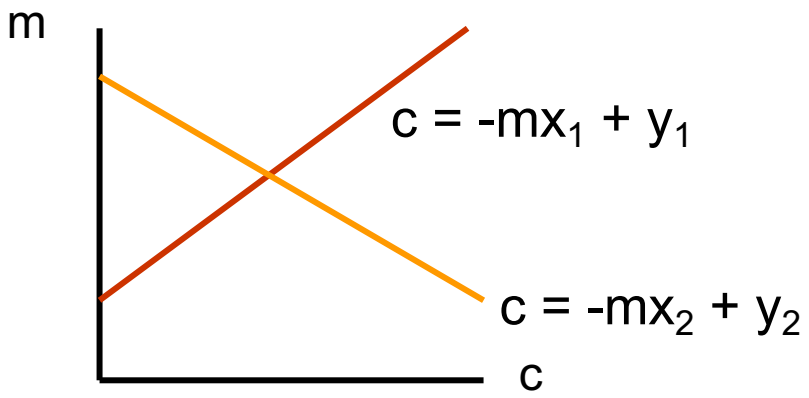
Parameter Space

Suppose there are 2 edges in our image (x_1, y_1) and (x_2, y_2)





They will each generate a line in (m,c) space representing the set of lines that could pass through them

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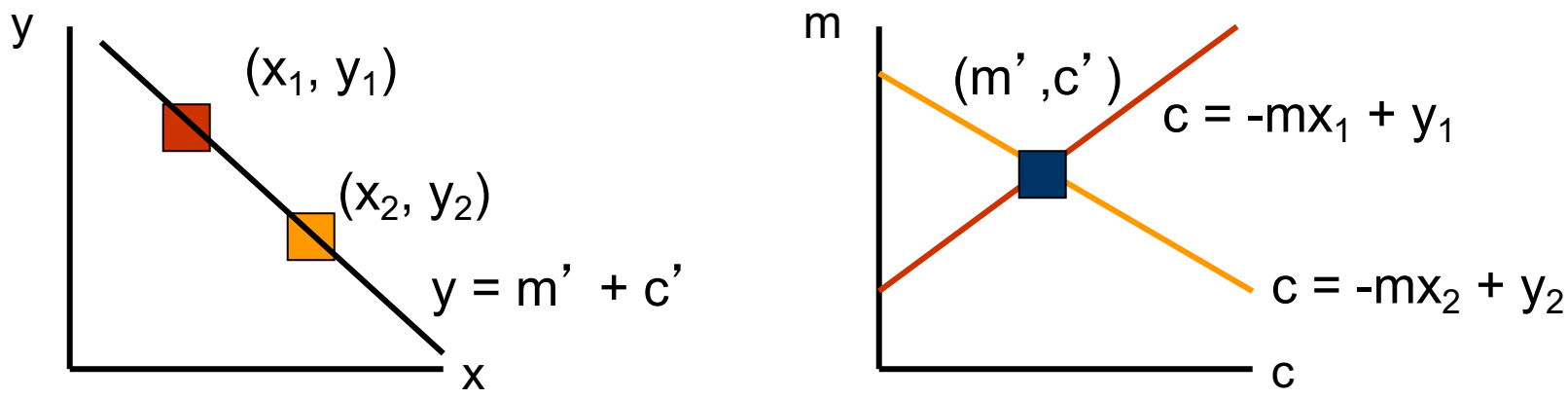
Parameter Space

The intersection of our 2 lines is the intersection of 2 sets of line parameters



The point of intersection therefore gives the parameters of a line through both (x_1, y_1) and (x_2, y_2)

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Parameter Space

Taking this one step further

- All pixels which lie on a line is (x,y) space are represented by lines which all pass through a single point in (m,c) space
- The point through which they all pass gives the values of m' and c' in the equation of the line: $y = m'x + c'$

To detect lines all we need to do is transform each edge point into m,c space and look for places where lots of lines intersect

This is what Hough Transform do

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The Hough Transform

1. Quantise (m,c) space into a two-dimensional array A for appropriate steps of m and c
2. Initialise all elements of $A(m,c)$ *to zero*
3. For each pixel (x_i, y_i) which lies on some edge in the image, we add 1 to all elements of $A(m, c)$ whose indices m and c satisfy $y_i = mx_i + c$
4. Search for elements of $A(m, c)$ which have large values – each one found corresponds to a line in the original image

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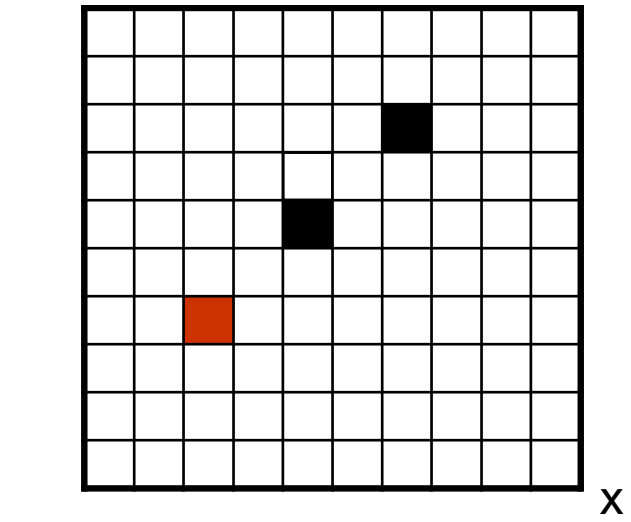
The Hough Transform



m

m'

c

c'

0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	1
0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	1	0	1	0	0
1	1	1	1	1	1	3	1	1	1
0	0	0	0	0	1	0	1	0	0
0	0	0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0

There is one line: $y = m'x + c'$

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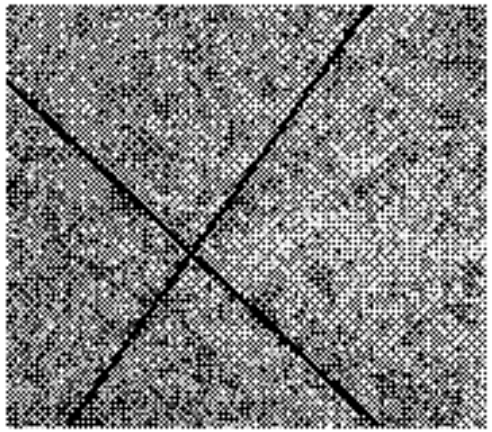
A (More) Real Example

Image

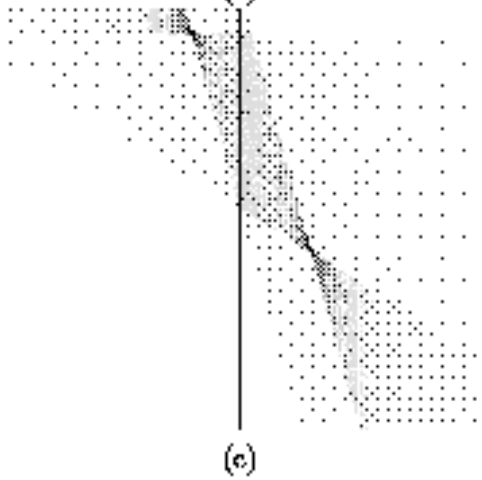


(a)

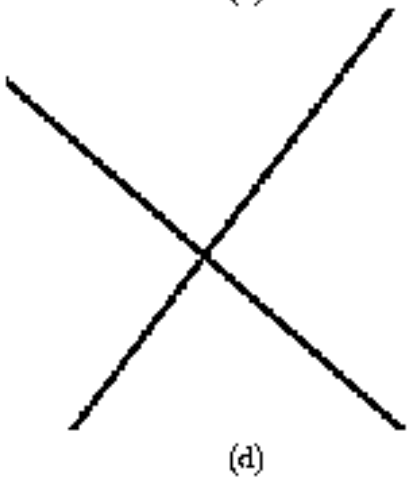
(b)

Edges

Accumulators



(c)



(d)

Result

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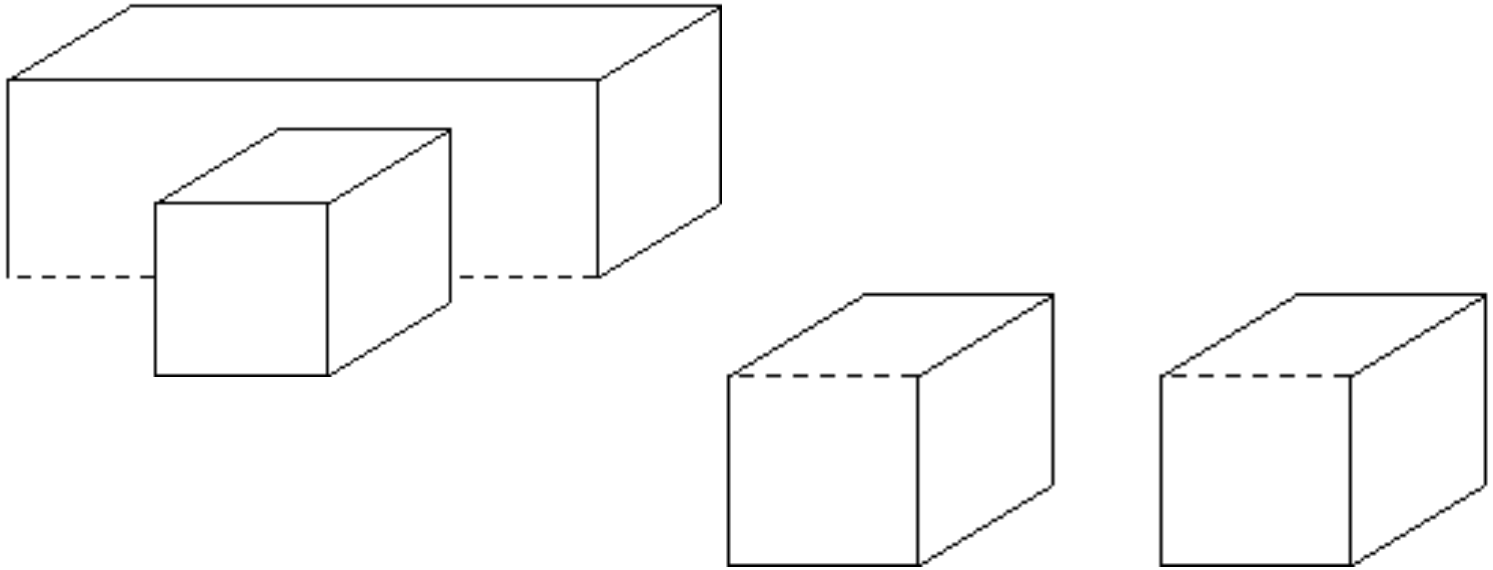
Line Parameters

The Hough Transform only supplies the parameters of the lines it detects; this can be an advantage or a disadvantage



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Other Parameter Spaces

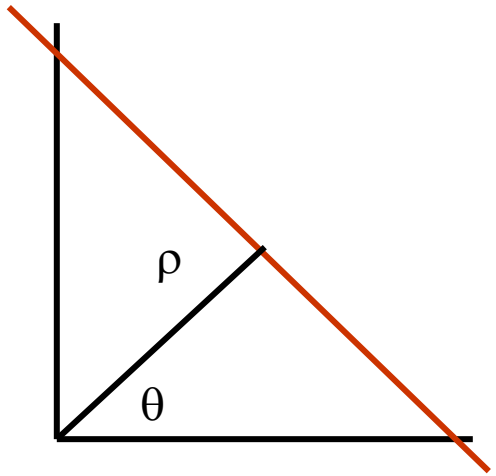
Straight Lines AGAIN

Another equation for a straight line

$$\rho = x.\cos(\theta) + y.\sin(\theta)$$

For an n x n image

- ρ is in range [0, 2n]
- θ is in range [0, 2π]



Each x_i, y_i generates a sinusoid in ρ, θ space

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A ρ, θ Example

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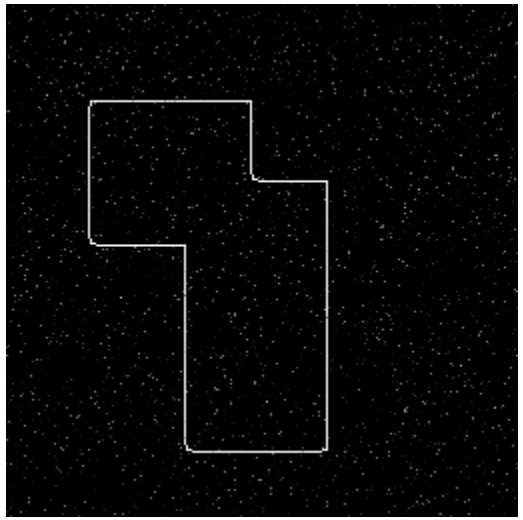
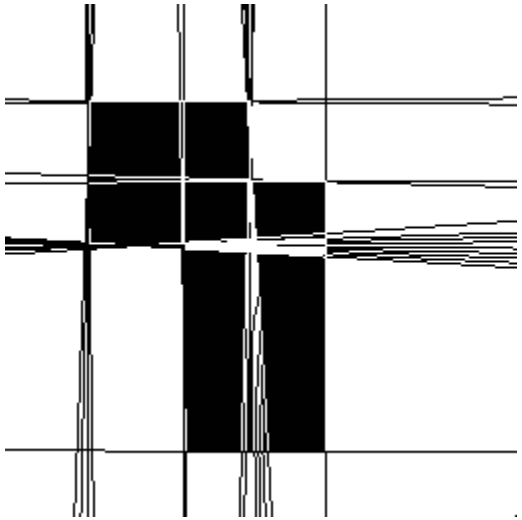
A ρ, θ Example





A key question is what happens to peaks in parameter space when noise or occlusion arise?

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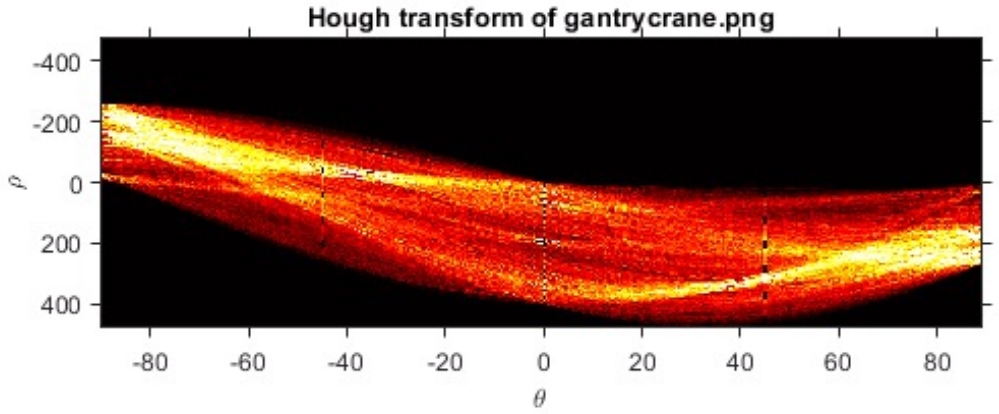
The Hough Transform in Matlab



Like many implementations, Matlab's Hough allows you to specify which part of the parameter space you're interested in…

https://uk.mathworks.com/help/images/ref/hough.html

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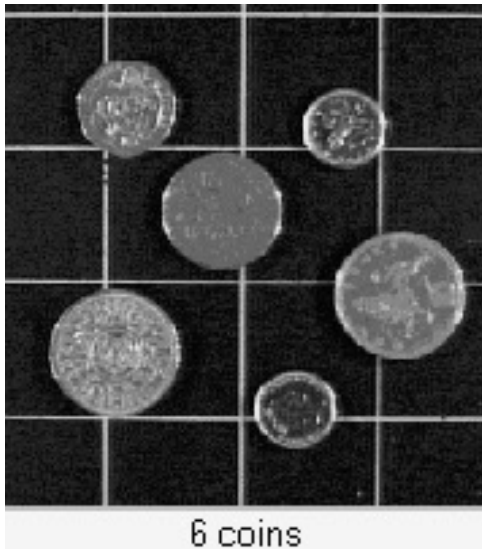
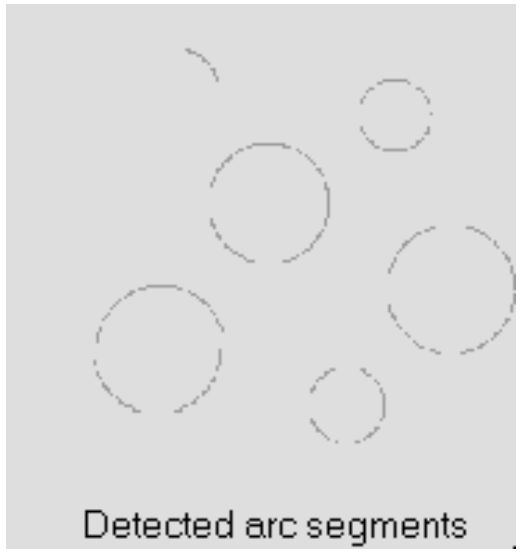
Circles

Parametric equation is $r^2 = (x-a)^2 + (y-b)^2$

- Full problem ⟶ cones in (*a, b, r*) space
- Known r ⟶ circles in (*a, b*) space



6 coins



Detected arc segments

How about ellipses?

The Hough can in principle be applied to any parameterizable curve, though its not always practical

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Break

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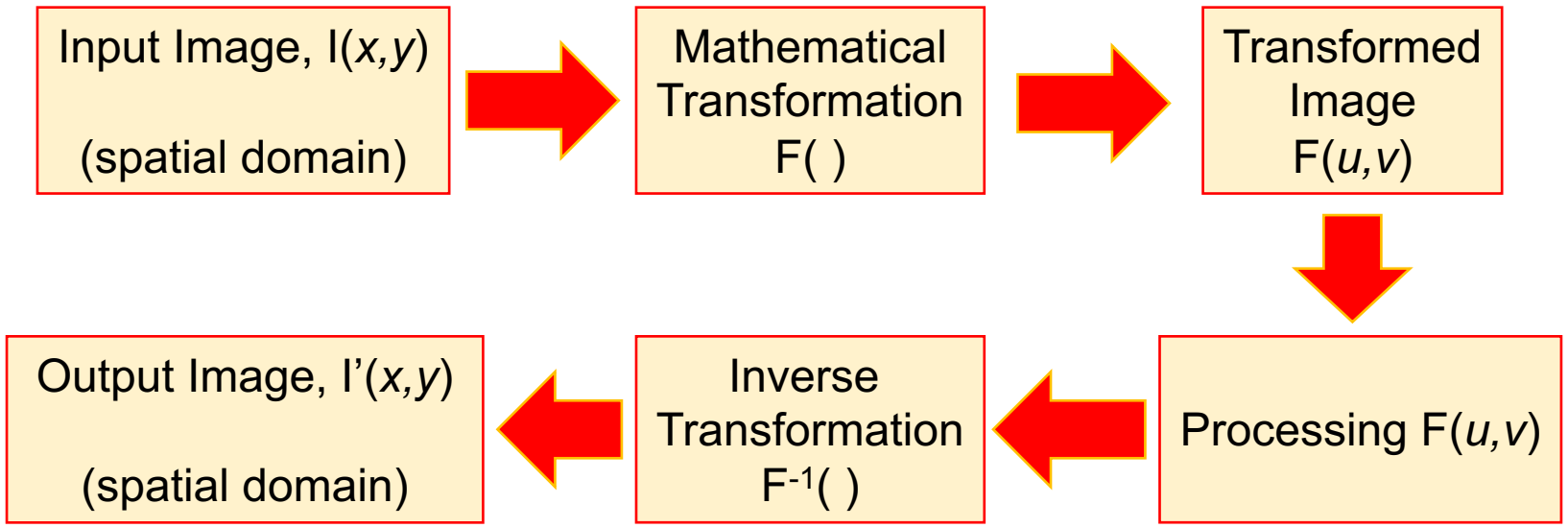
Frequency Domain

An Alternative Representation

Image Transform: The Idea



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The Fourier Transform

- There are many different transformations, the Fourier Transform (FT) or its fast implementation (FFT) is the best known
- COMP 2032 will treat FFT as a black box and will **NOT** consider the underlying mathematics in detail
- Focus will be on the key idea, the frequency space representation, and how it can be used

Any given function (*an image is a 2D function*) can be approximated by a weighted sum of ***basis functions***, e.g., sines and cosines

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The Fourier Transform

The basic idea (Fourier 1807)

These 4 functions

… can be combined to form this one

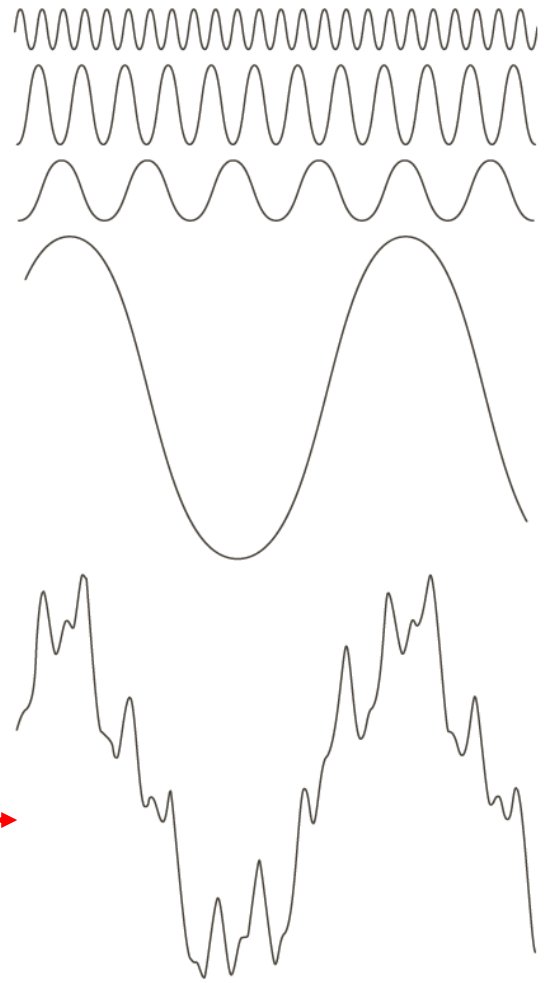

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Image Transforms

Suppose we just use cosines:

$$f(t) = \sum_i F_i \cos(u_i t)$$

- The weight, F_i, indicates the importance of $cos(u_i t)$
- u_i represents the frequency of the cosine signal
- A larger u_i means $cos(u_i t)$ changes faster → higher frequency component of $f(t)$
- A smaller u_i means $cos(u_i t)$ changes slower → lower frequency component of $f(t)$

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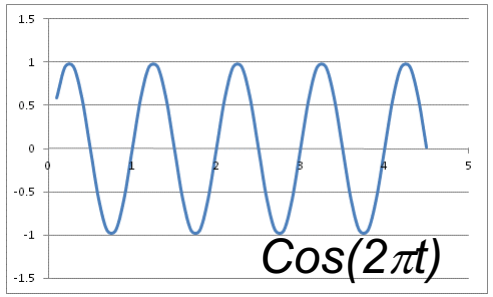
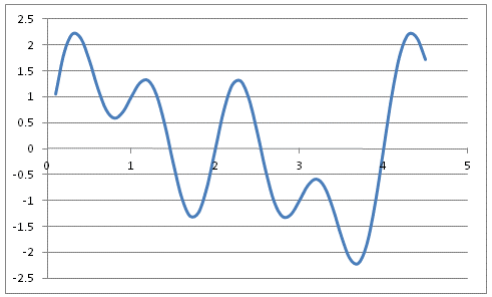
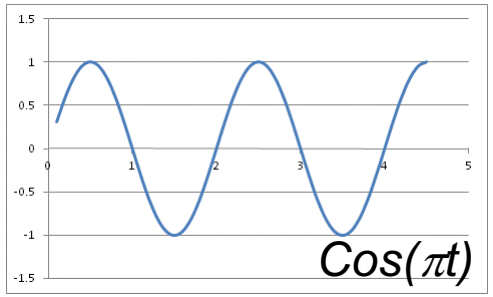
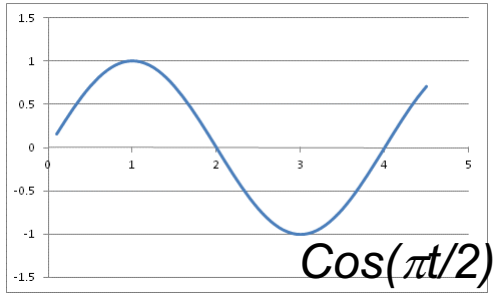
Image Transforms









$f(t) = Cos(\pi t/2)+Cos(\pi t)+Cos(2\pi t)$

$F_1 = 1 \quad u_1 = \pi/2$
$F_2 = 1 \quad u_2 = \pi$
$F_3 = 1 \quad u_3 = 2\pi$

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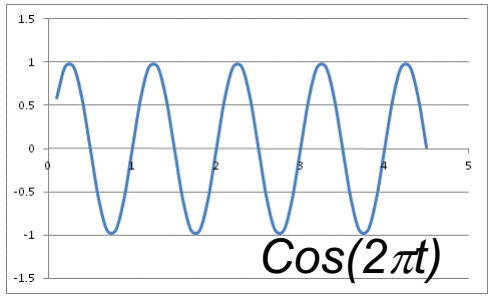
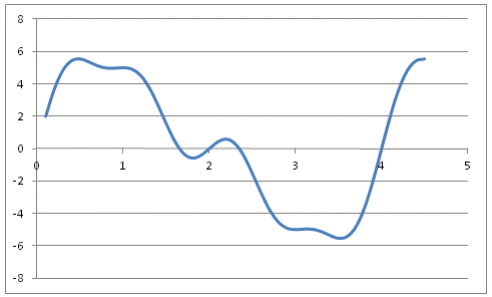
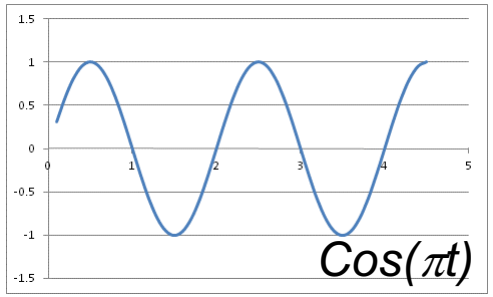
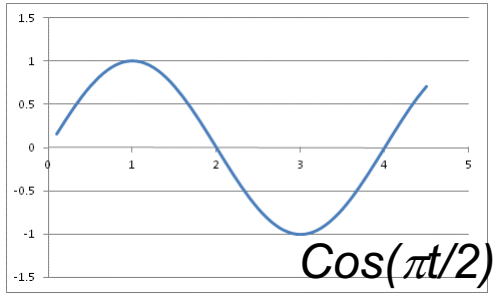
Image Transforms









$f(t) = 5*Cos(\pi t/2)+2*Cos(\pi t)+Cos(2\pi t)$

$F_1 = 5 \quad u_1 = \pi/2$

$F_2 = 2 \quad u_2 = \pi$

$F_3 = 1 \quad u_3 = 2\pi$

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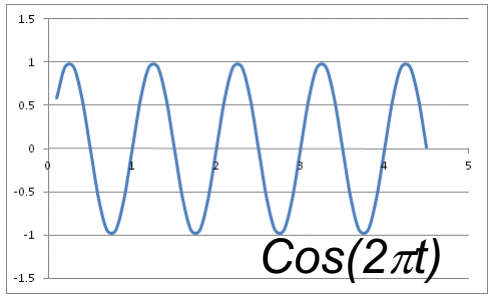
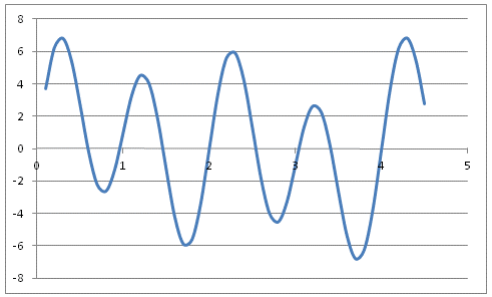
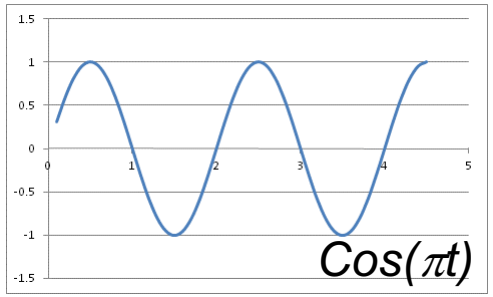
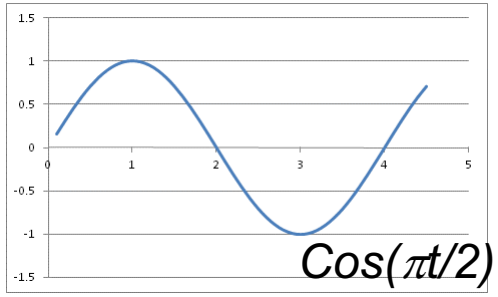
Image Transforms









$f(t) = Cos(\pi t/2)+2*Cos(\pi t)+5*Cos(2\pi t)$

$F_1 = 1 \quad u_1 = \pi/2$
$F_2 = 2 \quad u_2 = \pi$
$F_3 = 5 \quad u_3 = 2\pi$

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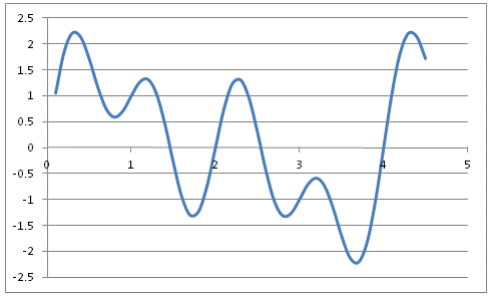
Image Transforms



$F_1 = 1 \quad u_1 = \pi/2$
$F_2 = 1 \quad u_2 = \pi$
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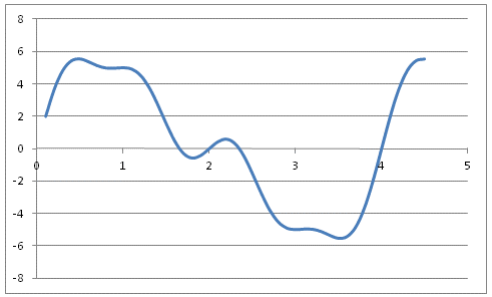
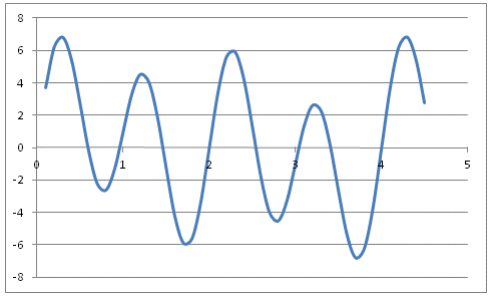
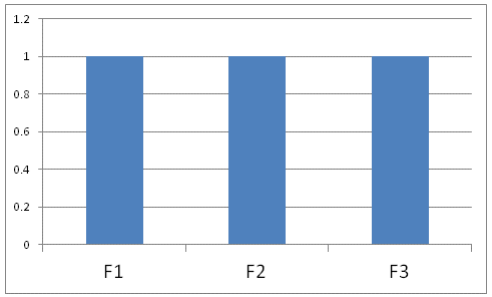


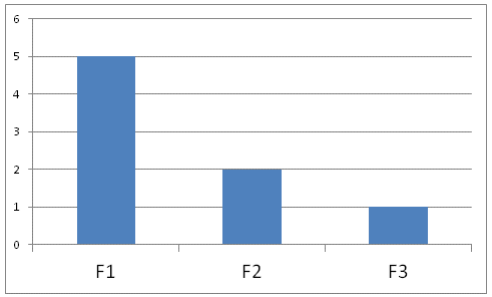
$F_1 = 5 \quad u_1 = \pi/2$
$F_2 = 2 \quad u_2 = \pi$
$F_3 = 1 \quad u_3 = 2\pi$

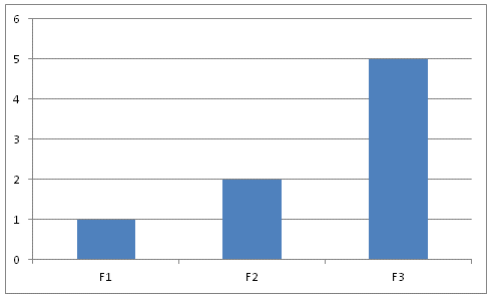


$F_1 = 1 \quad u_1 = \pi/2$
$F_2 = 2 \quad u_2 = \pi$
$F_3 = 5 \quad u_3 = 2\pi$

$$f(t) = F_1 * Cos(\pi t/2) + F_3 * Cos(\pi t) + F_3 * Cos(2\pi t)$$







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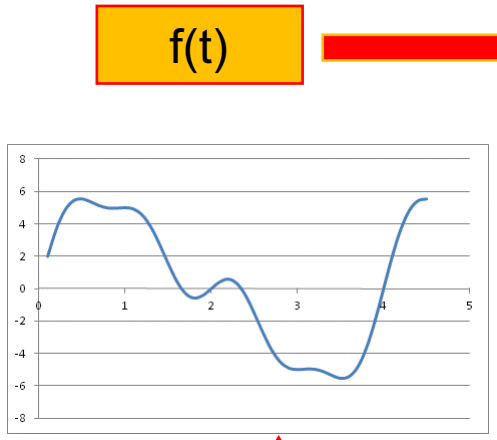
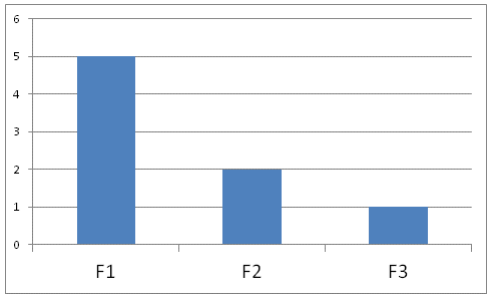
The Fourier Transform

Simplified, informal version:





f(t) → FFT → F(u)

$$f(t) = \sum_i F_i \cos(u_i t)$$

Input signal (image)

basis functions: actually sines and cosines

Power spectrum or frequency distribution

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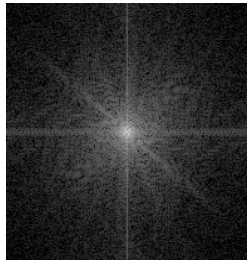
Amplitude vs Phase

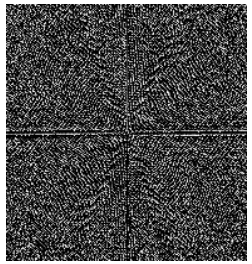
A = “Aron”

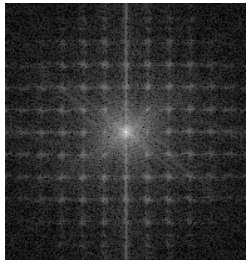
FA = fft2(A)

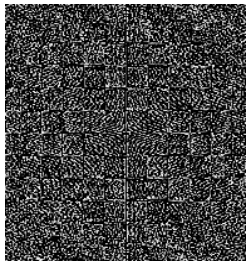
log(abs(FA))

angle(FA)









P = “Phyllis”

FA = fft2(P)

log(abs(FP))

angle(FP)

Image processing focuses on the power spectrum, the amplitude

To accurately represent a signal (image), basis functions need to be shifted (and rotated)

- **Amplitude**: relative prominence of basis function
- **Phase**: relative displacement of basis function

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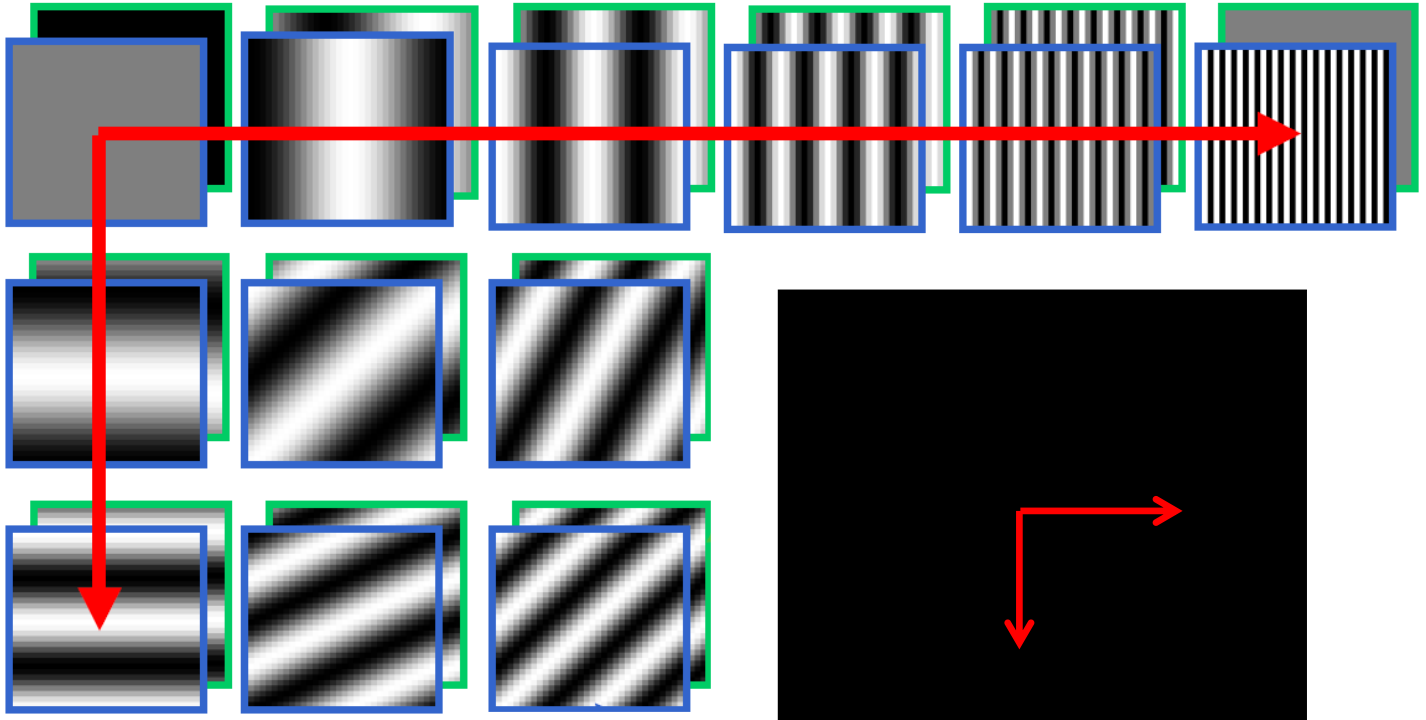
The Power Spectrum in 2D



The principle is the same, but the *basis functions* are oriented

- Distance from centre reflects frequency of corresponding basis function
- Direction from centre reflects orientation of corresponding basis function
- Origin records lowest possible frequency – flat, proportional to mean intensity

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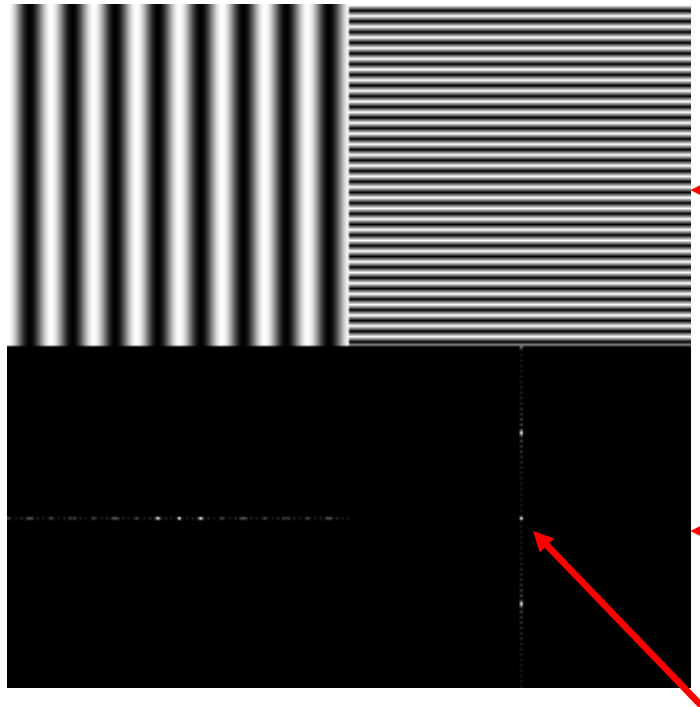
The Power Spectrum in 2D



Two images

Their power spectra

Origin at centre

Two responses because basis functions are even: f(-x) = f(x)

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Frequency Domain

Translation does not change the spectrum

Rotation does

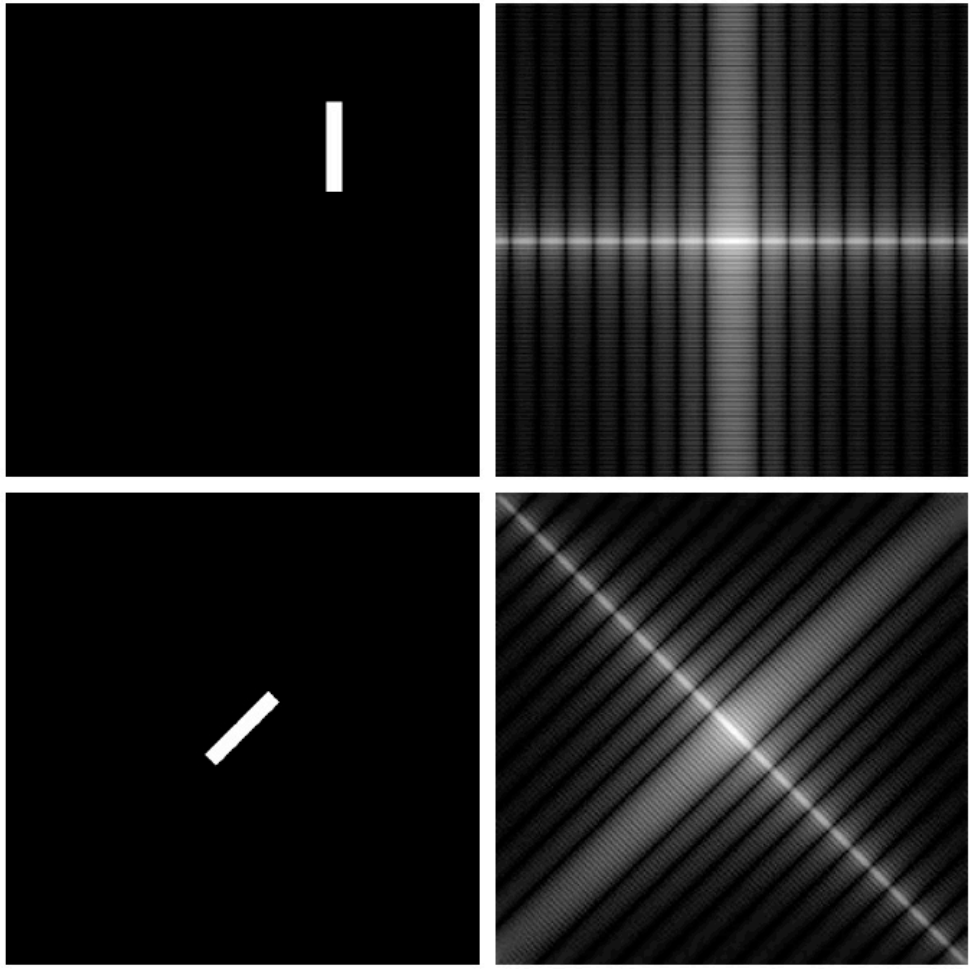

Image | Spectrum

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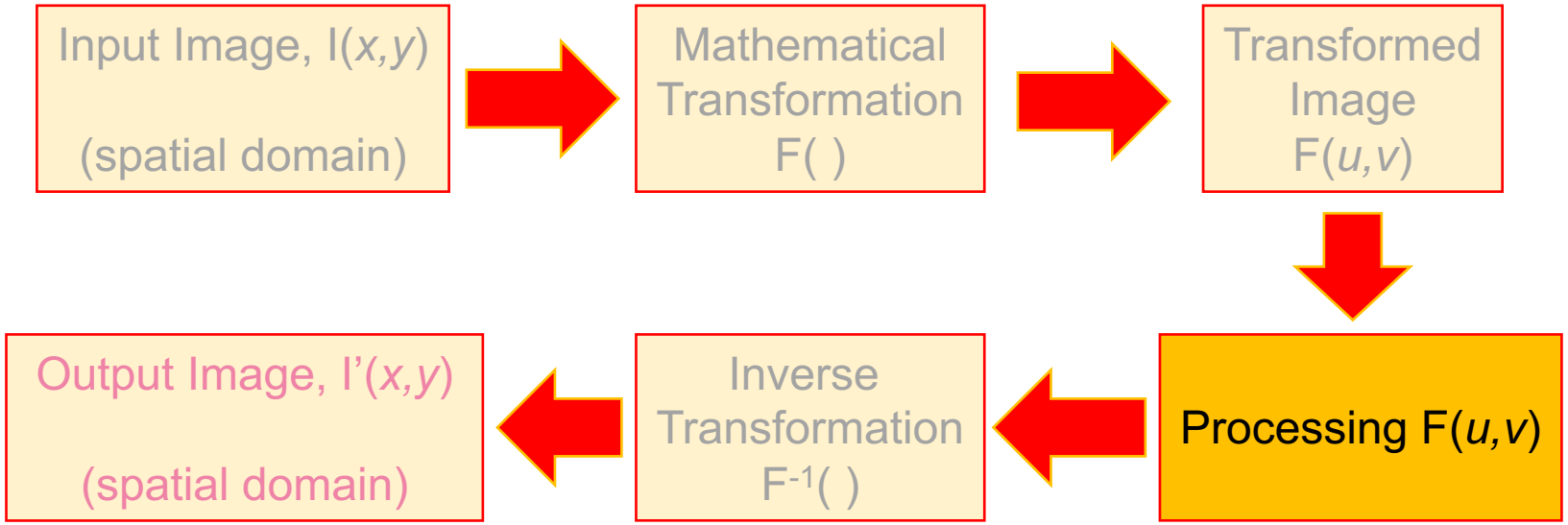
University of Nottingham
UK | CHINA | MALAYSIA

Frequency Domain Processing

Frequency Domain Processing



- Smoothing = low pass filtering = attenuate high frequency components
- Sharpening = high pass filtering = attenuate low frequency components
- Band limiting = set all components to 0 outside a given frequency range

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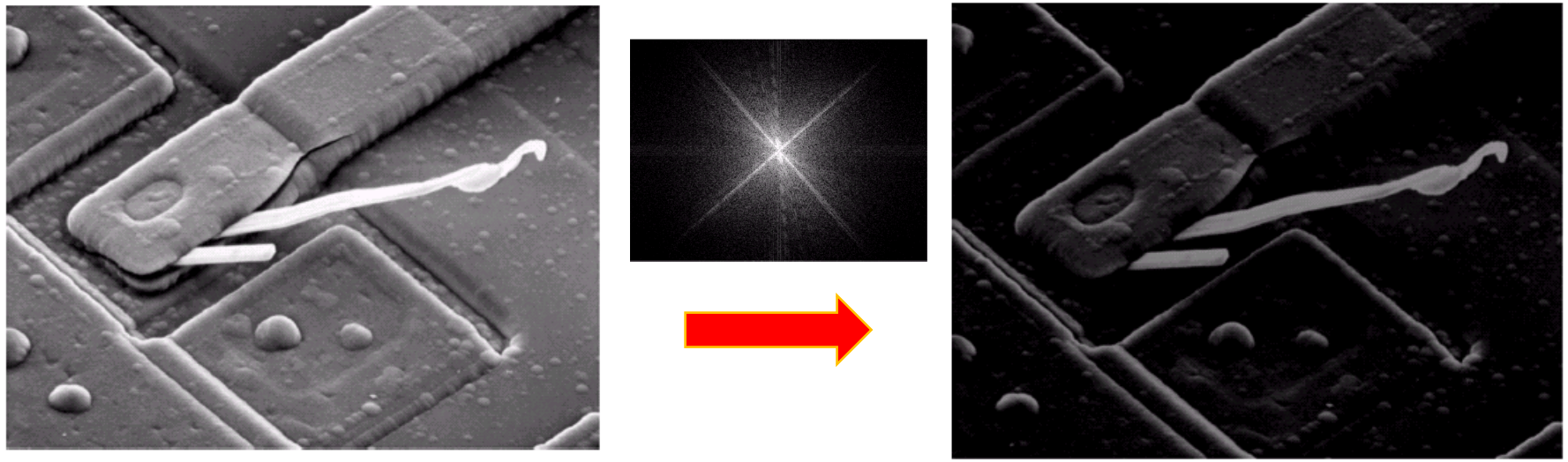
The Simplest Example

Set the origin of the power spectrum to zero, leave the rest unchanged

- Reduces average intensity of image to 0
- Negative intensities have been set to 0 in the display



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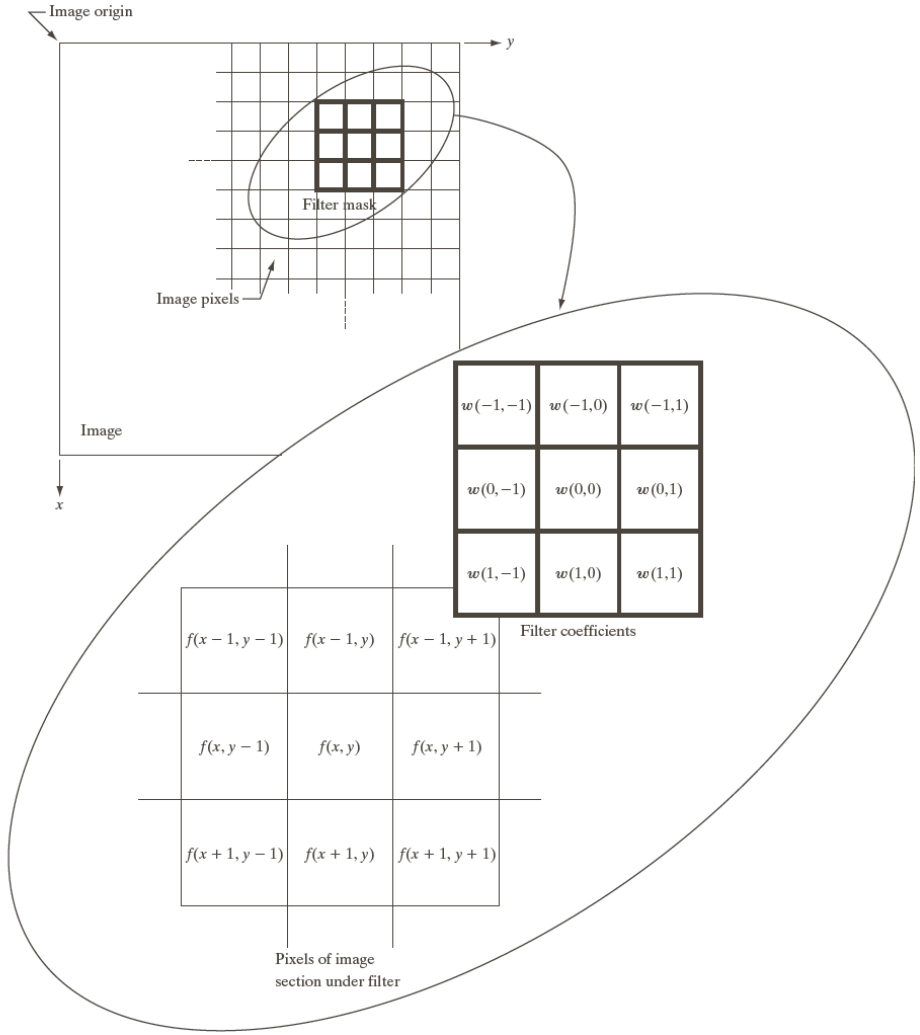
Convolution

- In spatial domain, processing is achieved via convolution (*)
- The *convolution theorem* underpins frequency domain processing

$$f(x,y) * g(x,y) \Leftrightarrow F(u,v)G(u,v)$$

Convolution in spatial domain becomes multiplication in frequency domain



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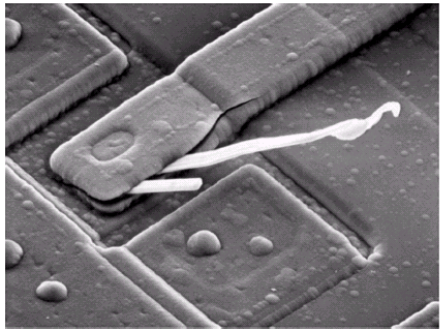
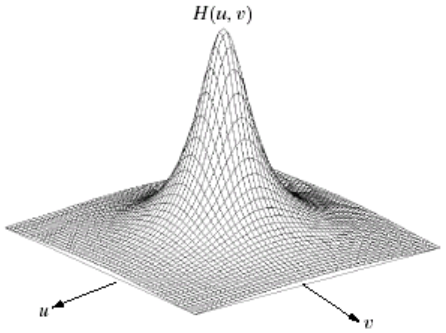
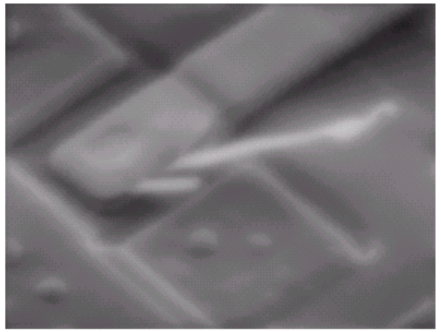
Filtering in Frequency Domain

Filtering = Multiply power spectrum by a mask designed to change the weights of selected basis functions

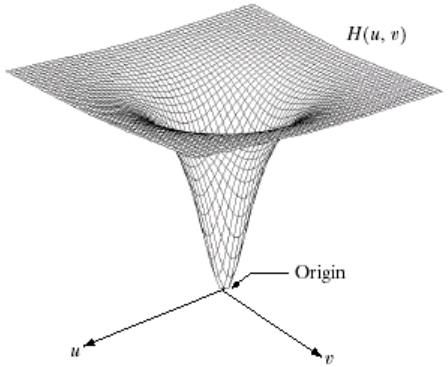


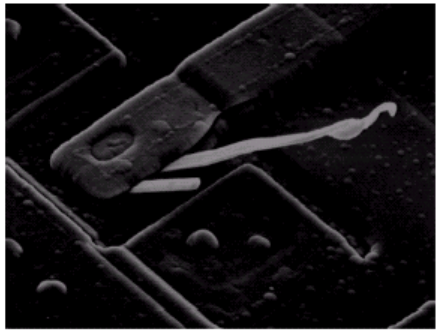
Low-pass filtering





High-pass filtering





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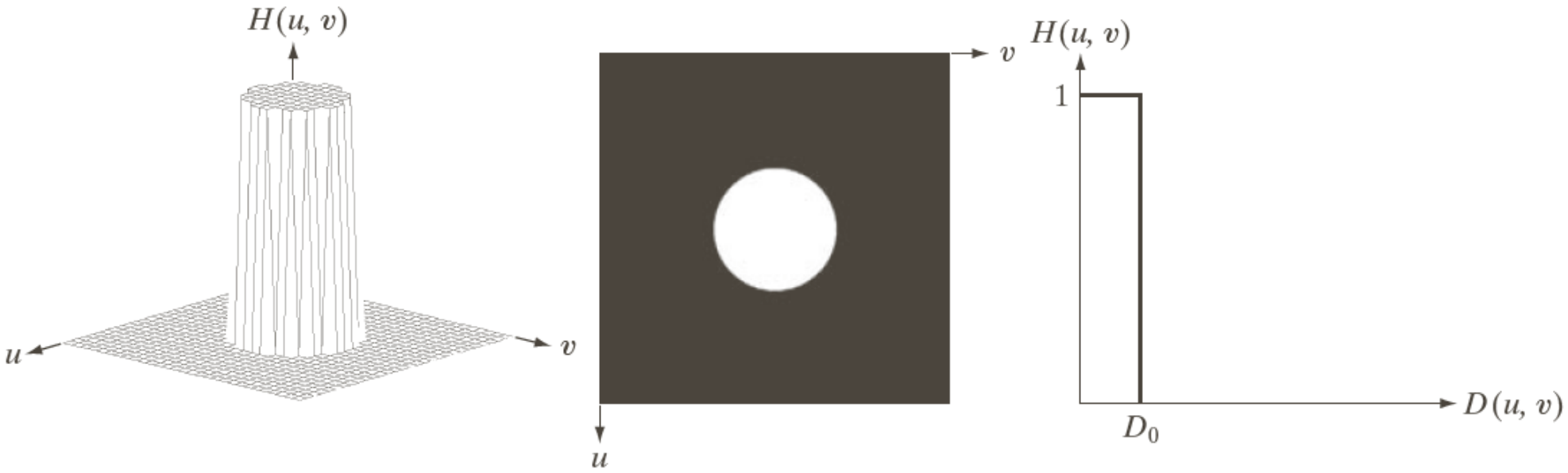
Ideal Low-pass Filtering



Perspective plot | Image view | Profile

Allows user to specify precisely which frequencies are to be kept and which removed – good if you know which are which

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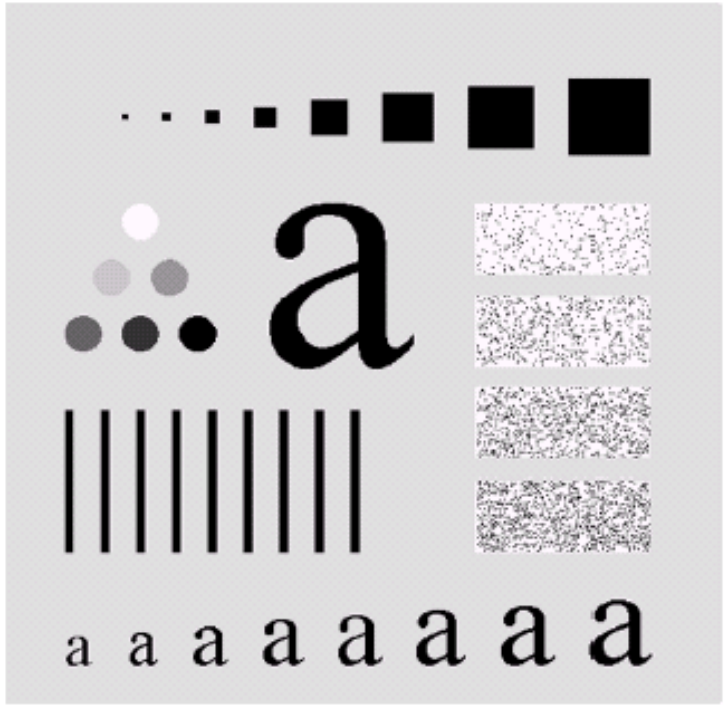
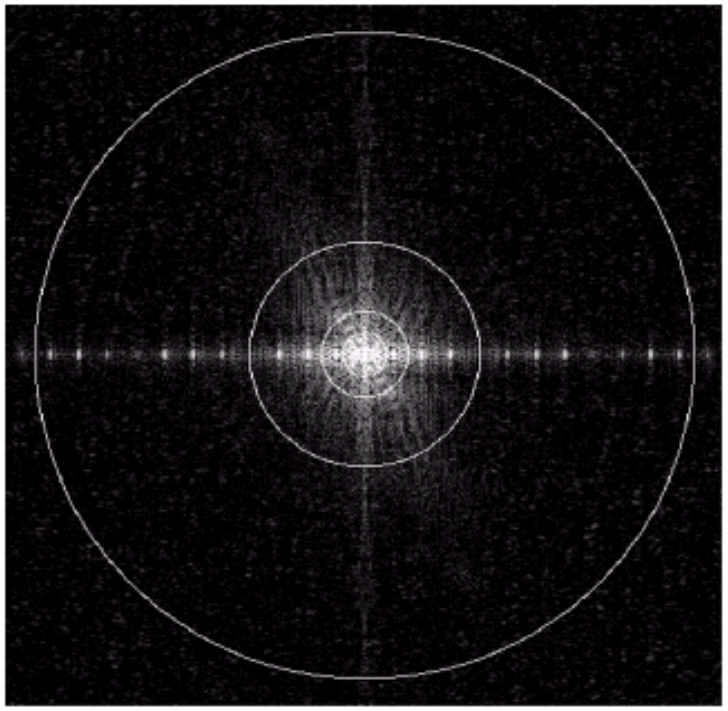
Ideal Low-pass Filtering

- Sample image and its Fourier spectrum
- Circles mark ideal cut-offs at D_0 = 5, 15, 30, 80, 230





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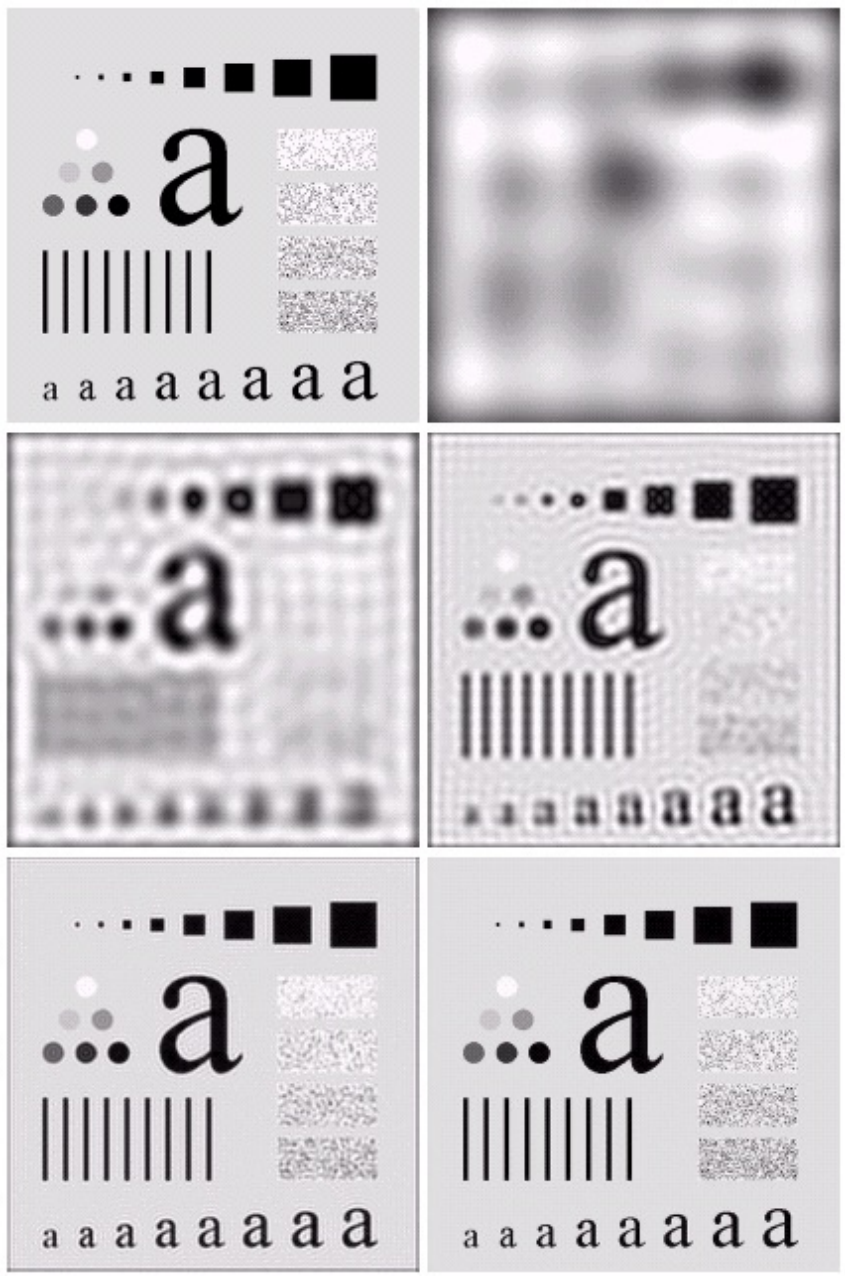
Unintended Consequences

- Ideal low-pass filtering with (*top-left to bottom-right)* D_0 = 5, 15, 30, 80, 230
- Note the *ringing* artefact caused by hard boundary between frequencies kept and removed
- Frequency of ringing effect increases with D_0

  - The “missing” components needed to make the uniform regions uniform become higher frequency as D_0 increases



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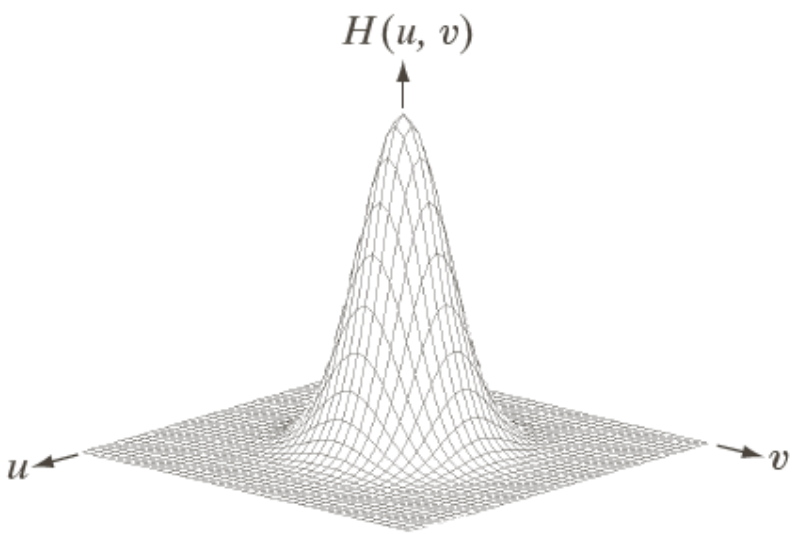
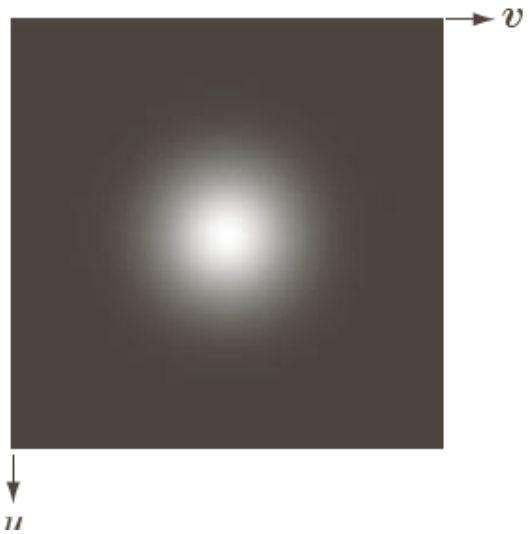
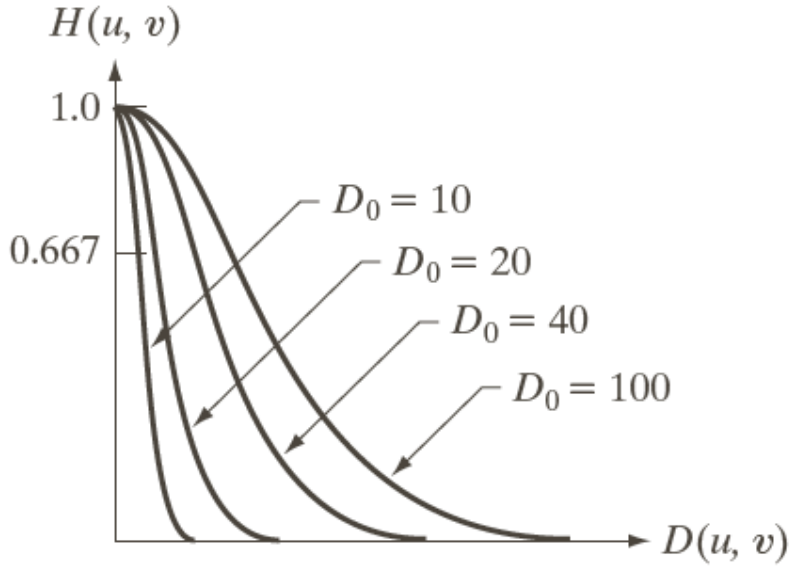
Low-pass Gaussian Filters







Perspective plot

Image view

Profile

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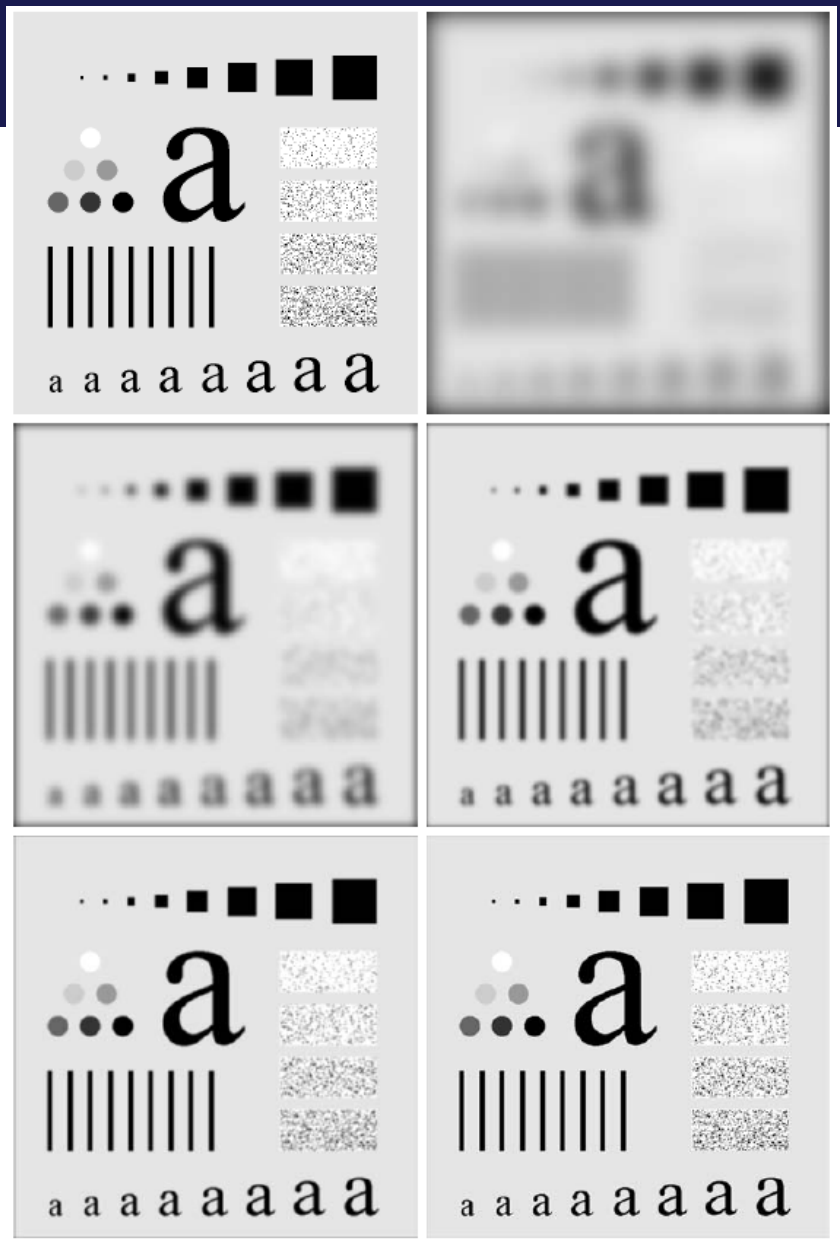
Low-pass Gaussian Filters

Low-pass Gaussian Filtering with cut-off frequencies at (*top-left to bottom-right)*
D_0 = 5, 15, 30, 80, 230



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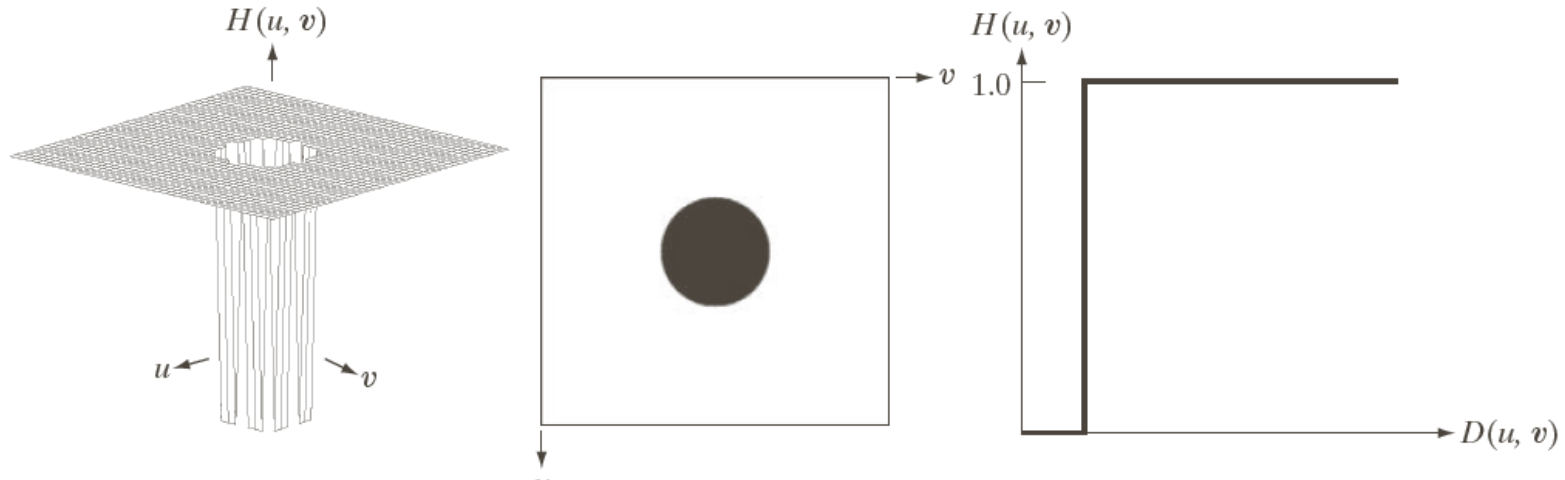
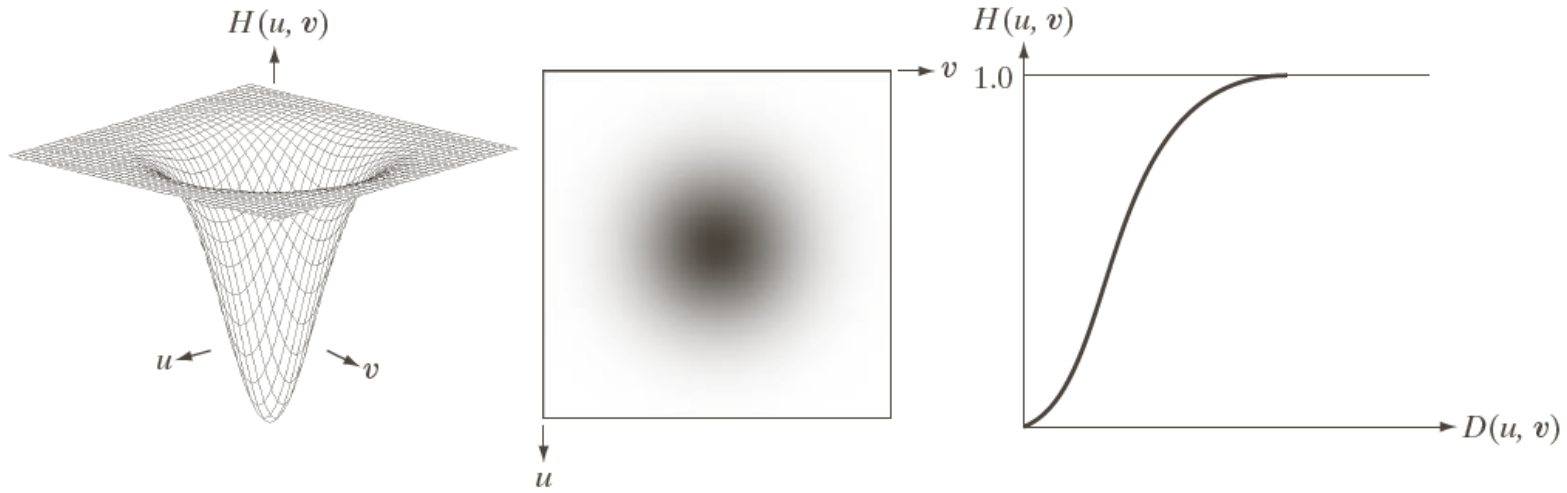
High-pass Filters

Ideal



Gaussian



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High-pass Gaussian Filters

Gaussian High-pass filtering with D_0 = 30, 60 and 160

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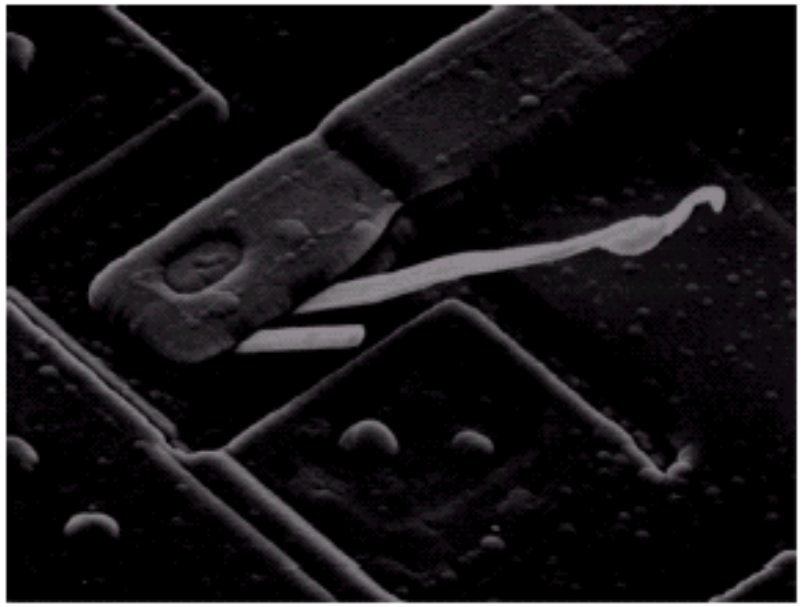
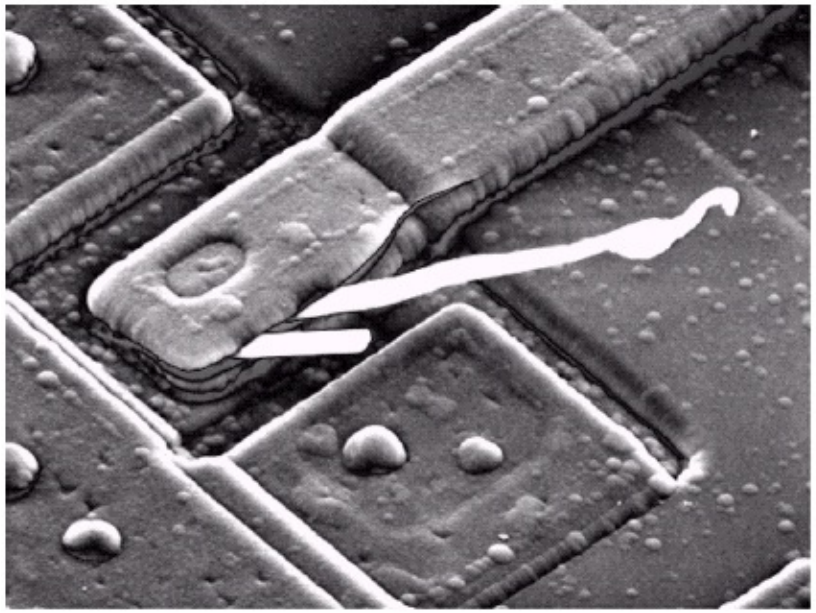
High-pass Filtering

Raw high-pass filters set origin, and so mean intensity to 0, making images dark. Add a small constant to the filter so the mean remains positive



Gaussian high pass filtered SEM image



Gaussian high pass filter plus a constant

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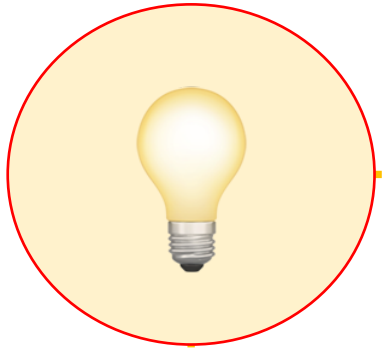
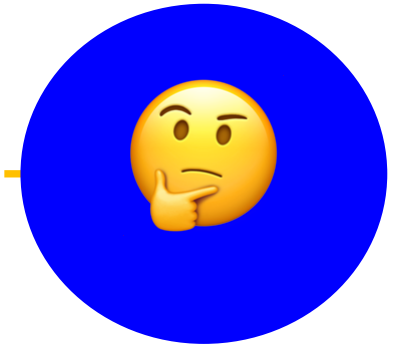
Summary



1. How to find lines?
2. The Hough Transform
3. Other parameter spaces
4. What is frequency domain?
5. The frequency domain processing

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Questions


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NEXT:
Image Compression