Macroeconomic Policy

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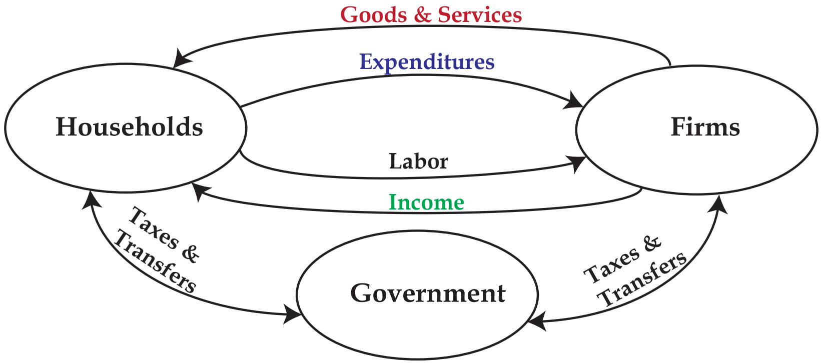
Lecture 5: The Firm and The Government

Intro

- Typical macro model:



- In previous weeks, we focused on modelling The Household.
- Today we look at:

  The Firm and **The Government**.

Roadmap

1. **The Firm**
    - Technology/Production Functions
    - Firm's problem
    - Labour Demand + Investment Demand
2. **The Government**
3. **Equilibrium** ➜ *Real Business Cycle Model* (next Lecture...)

Readings

- **The Household**
  - Part 1: Williamson Ch. 4, p. 119-142 (6E)
  - Part 2: Williamson Ch. 9, p. 328-354 + Ch. 11, p. 401-408
- **The Firm**
  - Williamson Ch. 11, p. 409-417 (6E)
  - (Optional: Williamson Ch. 4, p. 142-155 → static model)
- **The Government**
  - Williamson Ch. 11
- **Equilibrium➜** *Real Business Cycle Model*
  - Williamson Ch. 11

The Firm

The Firm

- In our model, a firm is an economic agent which transforms inputs (capital and labour) into output (consumption good).

- Similarly to the consumer's case, we assume that:

  1. Firms live for *two* periods
  2. Firms are rational
  3. Firms are identical ➜ focus on a representative firm

- Goal of the firm: Maximise the present value of its profits.

Production Functions

- The representative firm produces output today (Y) and output tomorrow (Y').

- It operates the following technology:

$$Y = zF(K, N^d) \quad \text{➜ Production function today}$$

$$Y' = z' H(K') \quad \text{➜ Production function tomorrow}$$

where:

  - K (K') is capital today (tomorrow)
  - N^d is labour employed/demanded today
  - z (z') is total factor productivity today (tomorrow)

Production Functions

☞ $Y = z\,F(K, N^d)$ is produced using capital *and labour*, while $Y' = z'\,H(K')$ is produced by *only* using capital.

- Why this assumption?
  1. Consistent with absence of labour supply from the Household in the second period.
  2. It will simplify the analysis of the "full" model significantly.
  3. All the results we will draw from the "full" model would still hold if we added labour in the second period!

Production Function Today: Properties

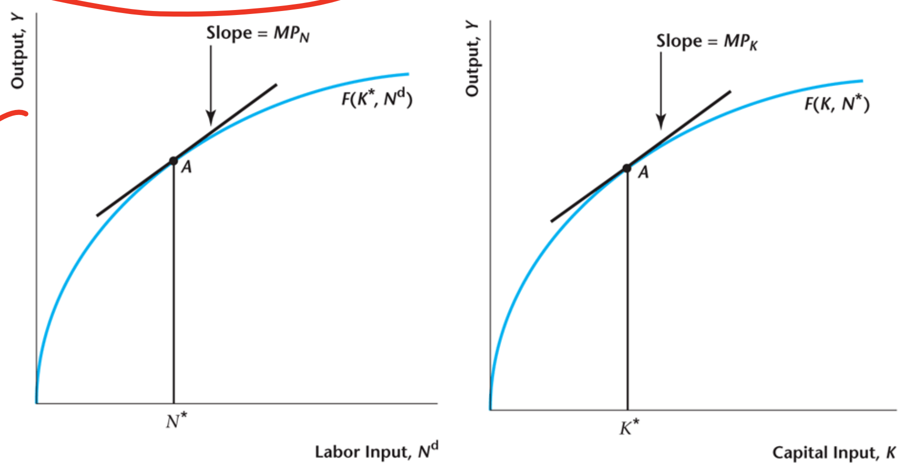
1. F is **increasing** in K and N^d:

$$F_K(K, N^d) > 0, \quad F_N(K, N^d) > 0$$

i.e. the **marginal products** of capital and labour are **positive**.

For now on, let

$$MP_N = zF_N(K, N^d), \quad MP_K = zF_K(K, N^d)$$



$$Y = zF(K, N^d)$$

$$\frac{\partial Y}{\partial K} = zF_K(K, N^d), \left[F_K(K, N^d) = \frac{\partial F}{\partial K} \right]$$

MARGINAL PRODUCT OF CAPITAL

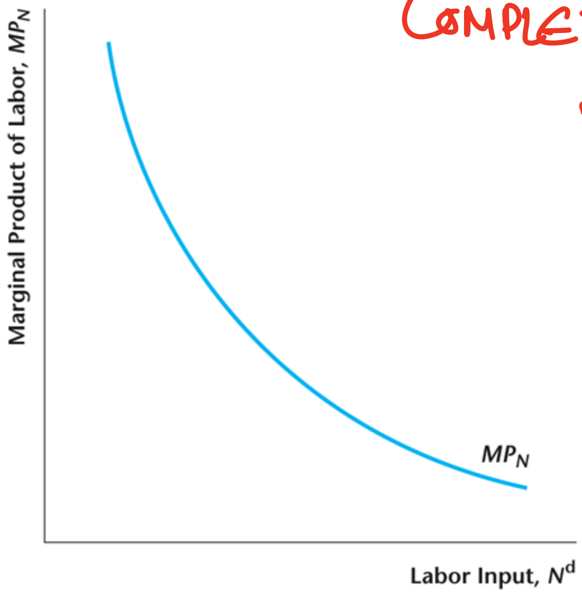
$$MP_K = zF_K(K, N^d)$$

$$MP_N = zF_N(K, N^d)$$

2. Marginal Products are **decreasing**:
   - MP_N decreases as N^d increases
   - MP_K decreases as K increases

(EXAMPLE: EVERY ADDITIONAL MEMBER IN A STUDY GROUP CONTRIBUTES MARGINALLY LESS TO COMPLETING AN ASSIGNMENT.)

Example:

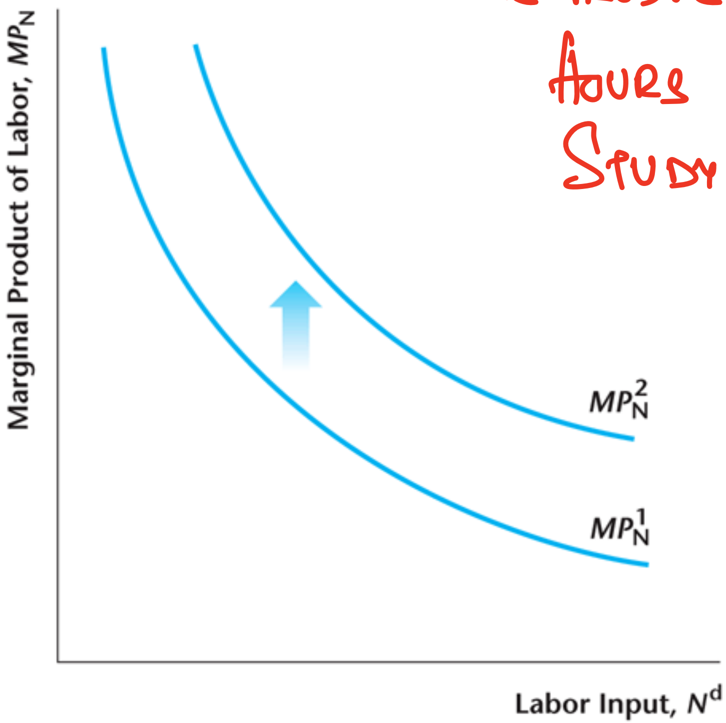


Production Function Today: Properties

3. MP_N **increases with** K and MP_K **increases with** N^d

Example: Increasing K...

IF YOU BUY A LAPTOP, THE PRODUCTIVITY OF THE HOURS YOU ARE STUDYING INCREASE.



Production Function Today: Properties

4. F exhibits **constant returns to scale**:

   For any constant $x > 0$,

   $$zF(xK, xN^d) = xzF(K, N^d),$$

   i.e. "double the size, double the output"

- This property is key to study a representative firm.
- How to define *increasing* and *decreasing* returns to scale?

IRS : $zF(xK, xN^d) > xzF(K, N^d)$

IF I DOUBLE THE SIZE OF MY FIRM, I AM GOING TO GET MORE THAN JUST DOUBLING THE SIZE OF MY OUTPUT.

$zF(\alpha k, \alpha N^d) = \alpha z F(k, N^d)$

INCREASE INPUT BY SOME FACTOR $\rightarrow$ OUTPUT INCREASES BY THE SAME FACTOR.

$\alpha z F(k, N^d) \rightarrow$ PRODUCTION CARRIED OUT BY MANY SMALL FIRM

$zF(\alpha k, \alpha N^d) \rightarrow$ PRODUCTION BY ONE FIRM OPERATING ON A LARGE SCALE.

Production Function Today: Properties

- An example of a technology that satisfies properties #1-#4 is the **Cobb-Douglas** production function:

$$Y = z\,K^{\alpha}\,N^{1-\alpha}$$

where

  - $\alpha \in (0,1)$ is called the "capital share"
  - $(1-\alpha)$ is called the "labour share"

Production Function Tomorrow

- Recall that in the second period the firm operates:

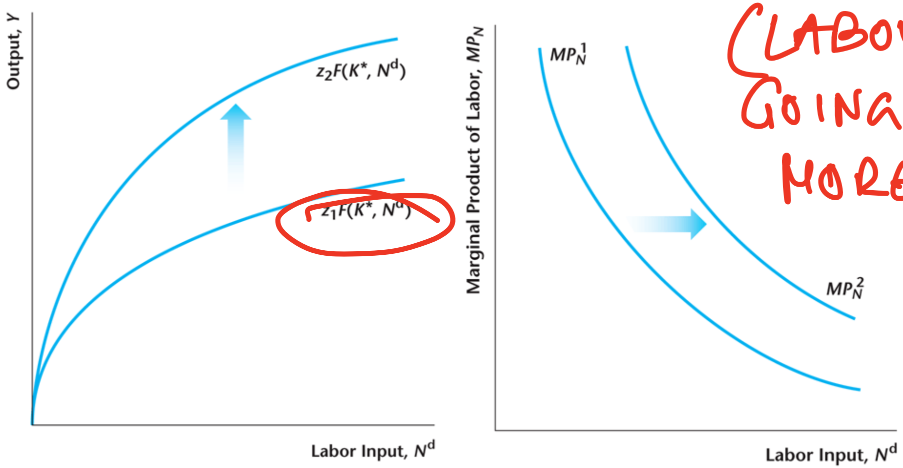
$$Y' = z' H(K')$$

- The marginal product of K' is $MP_{K'} = z' H'(K')$
- Properties:
    1. $MP_{K'}$ is positive
    2. $MP_{K'}$ is decreasing in K'
- Example:

$$Y' = z' (K')^{\beta}$$

where $\beta \in (0, 1)$.

- TFP parameters z and z' capture the degree of sophistication in the production process at each point in time.
  - Increases in z makes both K and N^d more productive.
  - Increases in z' make K' more productive.
- Example: An increase in z



$z\uparrow$, $MP_N\uparrow$
(LABOUR IS GOING TO PRODUCE MORE AT THE MARGIN)

Changing in TFP

☞ As we will see, changes in TFP are critical to understand business cycles.

- Sources of TFP changes?
  1. Technological innovations

     e.g. assembly line

  2. Weather

     IF RAINFALL IS HIGH PRODUCTIVITY IN AGRICULTURE INCREASES, DECREASES FOR CONSTRUCTION SECTOR)

     e.g. agriculture vs. construction

  3. Government regulations (not taxes or spending!)

     e.g. employment regulations (FIRMS MIGHT NOT BE ABLE TO HIRE AS MUCH AS THEY WOULD LIKE TO)

The Firm's Decisions

- To recap:

$$Y = zF(K, N^d) \text{ ➜ Production function today}$$

$$Y' = z'H(K') \text{ ➜ Production function tomorrow}$$

- Inputs are (N^d, K, K').

- How are these inputs determined?

The Firm's Decisions

- The firm hires labour N^d in the labour market.
- The firm owns K, and takes it as fixed.
  - i.e., capital is fixed in the short-run.
  - Captures the idea that building new plant and equipment takes considerable time!

The Firm's Decisions

- K' is *not* fixed! The firm **invests** today to accumulate K' for tomorrow.
  - How exactly?
    - Denote the firm's investment today by I.
    - Then K' is given by:

$$K' = (1 - d)K + I$$

MORE INVESTMENT TODAY + STOCK OF UNDEPRECIATED CAPITAL TODAY.

where $0 < d < 1$ is the depreciation rate.

- This equation is a.k.a. the *law of motion for capital.*
  - LHS: Capital tomorrow
  - RHS: Undepreciated Capital today + Investment

The Firm's Problem

- The firm chooses (N^d, K', I) to maximise the present value of profits:

$$V = \pi + \frac{\pi'}{1+r}$$

where

REV

- Profits today are ➜ $\pi = Y - wN - I$ COSTS (LABOUR COST + INVESTMENT COST)
- Profits tomorrow are ➜ $\pi' = Y' + \underbrace{(1-d)K'}_{\text{liquidation value}}$

The firm owns the capital and liquidates it in the last period. *Why?*

$(1-d)k'$ represents liquidation value of capital.

THE FIRM OWNS THE CAPITAL AND LIQUIDATES IT IN THE LAST PERIOD.

- THE FIRM ONLY LIVES FOR TWO PERIODS.
- SO, THERE IS NO NEED FOR CAPITAL AFTER THAT.

The Firm's Problem

- Formally, the firm solves:

$$\max_{N^d, K', I} \overbrace{zF(K, N^d) - wN^d - I}^{=\pi} + \frac{1}{1+r}\underbrace{[z' H(K') + (1-d)K']}_{=\pi'}$$

subject to

$$K' = (1-d)K + I$$

- How do we solve this problem?

The Firm's Problem

- First-order conditions for N^d and K' give:

TAKING DERIVATIVE W.R.T. N^d.

$$z F_N(K, N^d) = w \tag{1}$$

TAKING DERIVATIVE W.R.T. K'.

$$z' H'(K') - d = r \tag{2}$$

- Equations (1) and (2) pin down the two optimal decisions for the firm:
  1. **Labour demand** $N^d(w)$
  2. Investment demand $K'(r)$ (or $I(r)$)

Labour Demand

$zf_N(K, N^d) = w$

- Equation (1) can be written as:

$MP_N = w$

$$MP_N = w$$

REVENUE GENERATED BY HIRING AN ADDITIONAL WORKER.

COST OF HIRING AN ADDITIONAL WORKER.

- Intuition:
  - Suppose $MP_N > w$
    ➜ By hiring an additional worker, ΔRevenues $>$ ΔCosts
  - Suppose $MP_N < w$
    ➜ By hiring an additional worker, ΔRevenues $<$ ΔCosts

$MP_N > w$

REV. GENERATED BY HIRING AN ADDITIONAL WORKER > COST OF HIRING AN ADDITIONAL WORKER

$\hookrightarrow$ INCREASE HIRING ($N^d \uparrow$)

$\hookrightarrow MP_N \downarrow$

$\rightarrow MP_N = w$,

Labour Demand

- The labour demand curve depicts how much labour the firm demands for different levels of w:

$$N^d(w)$$

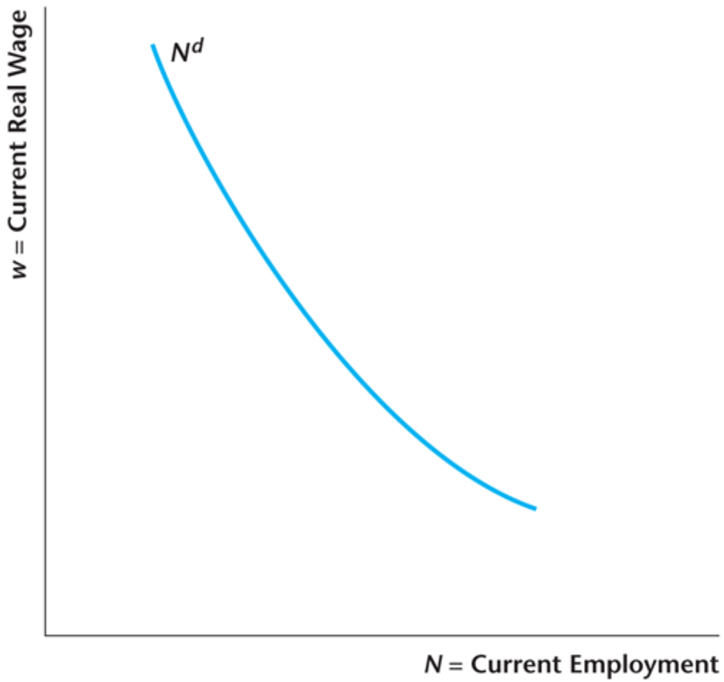
- $N^d(w)$ is *downward slopping.*
- Why? By Equation (1) previously: $MP_N = w$
  - If $\uparrow w$ ➜ $\uparrow MP_N$, so that $MP_N = w$.
  - Since MP_N decreases with N ➜ $\downarrow N$

THERE IS A NEGATIVE RELATIONSHIP BETWEEN MP_N AND N

- Intuition:
  - As w grows (falls), less (more) profitable to hire more workers.

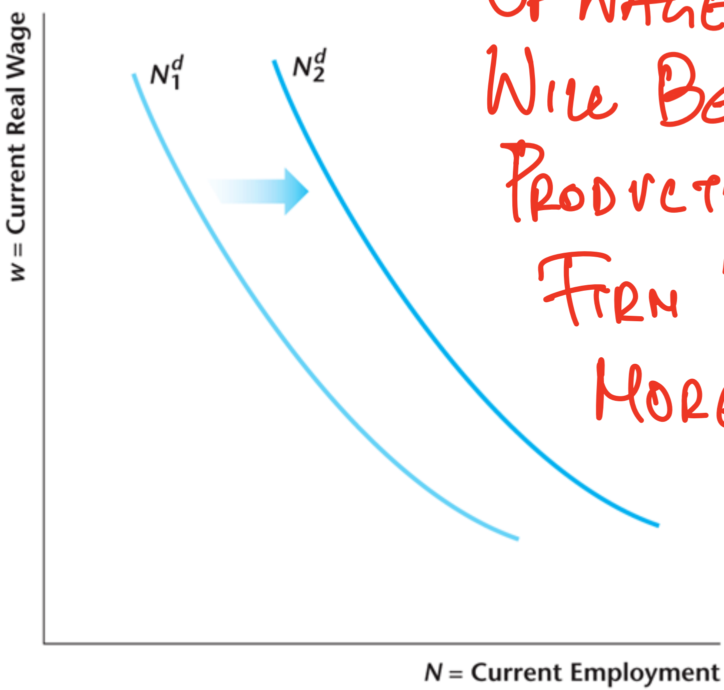
Labour Demand

- Graphically:



Labour Demand

- Increasing z or K:



At Any Given Level Of Wage, Labour Will Become More Productive. So The Firm Will Hire More Workers.

Back to the Firm's Problem

- Recall that first-order conditions for N^d and K' give:

$$z\,F_N(K, N^d) = w \tag{1}$$

$$z'\,H'(K') - d = r \tag{2}$$

- Equations (1) and (2) pin down the two optimal decisions for the firm:

  1. Labour demand $N^d(w)$
  2. **Investment demand** $K'(r)$ (or $I(r)$)

Investment Demand

- Equation (2) from first-order conditions of the firm can be written as:

$$MP_{K'} - d = r$$

$z' F'(K') - d = r$

$MP_{K'} - d = r$

- This optimality condition drives the optimal **investment decision** of the firm.

CAPITAL RETURN (NET OF DEPRECIATION) = RETURN OF BONDS

- Intuition:
  - Two assets in the model: Bonds and Capital.
  - Bond return is r.
  - Capital return (net of depreciation) is $MP_{K'} - d$.
  - Supp. $MP_{K'} - d < r$ (similar logic if "$>$" instead).
  - Then the firm would invest less in capital and more in bonds.
  - This increases $(MP_{K'} - d)$ until it equalises r.

$MP_{K'} - d < r$

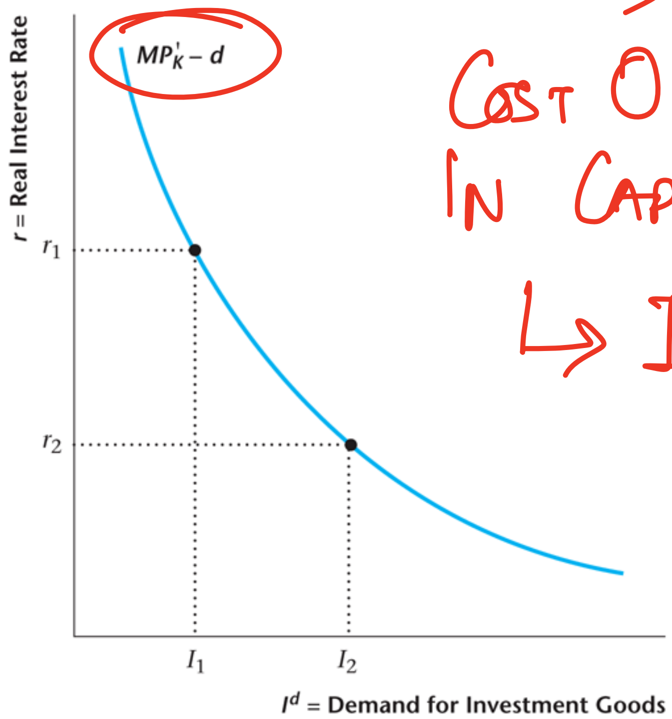
RETURN ON CAPITAL (NET OF DEPRECIATION) < RETURN ON BONDS

$\Rightarrow$ LEADS FIRMS TO REDUCE INVESTMENT IN CAPITAL AND INCREASE INVESTMENT IN BONDS

$\Rightarrow I \downarrow \rightarrow K' \downarrow = (1-d)K + I \longrightarrow MP_{K'} \uparrow$

$(MP_{K'} - d) \uparrow$

Investment Demand



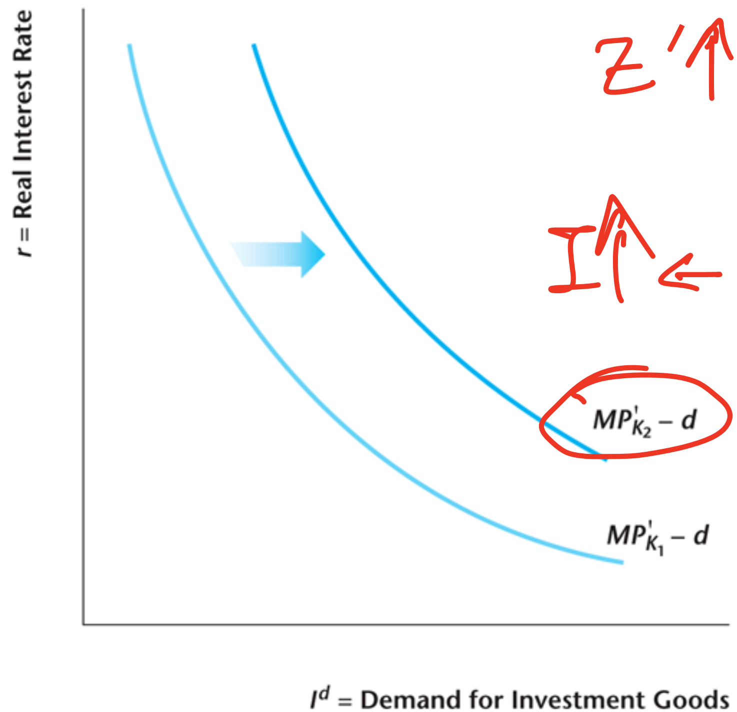
$r \uparrow \rightarrow$ OPPORTUNITY COST OF INVESTING IN CAPITAL INCREASE

$\hookrightarrow I \downarrow$

☞ <u>Important</u>: Interest rate = Opportunity cost of investing ($\downarrow r, \uparrow I$)

Investment Demand: Experiments

- Q1: Suppose $\uparrow z'$. Would the firm increase or decrease I?
- A: The firm would increase I because $MP_{K'}$ goes up for all K'.



$z'\uparrow \rightarrow MP_K'\uparrow$

$I\uparrow \leftarrow (MP_K' - d)\uparrow$

Figure: Increasing z'.

Investment Demand: Experiments

- Q2: Suppose $\downarrow K$. Would the firm increase or decrease I?
- A: As $\downarrow K$, $\downarrow K' = (1-d)K + I$. Then $MP_{K'}$ increases.

As $K \downarrow \rightarrow$

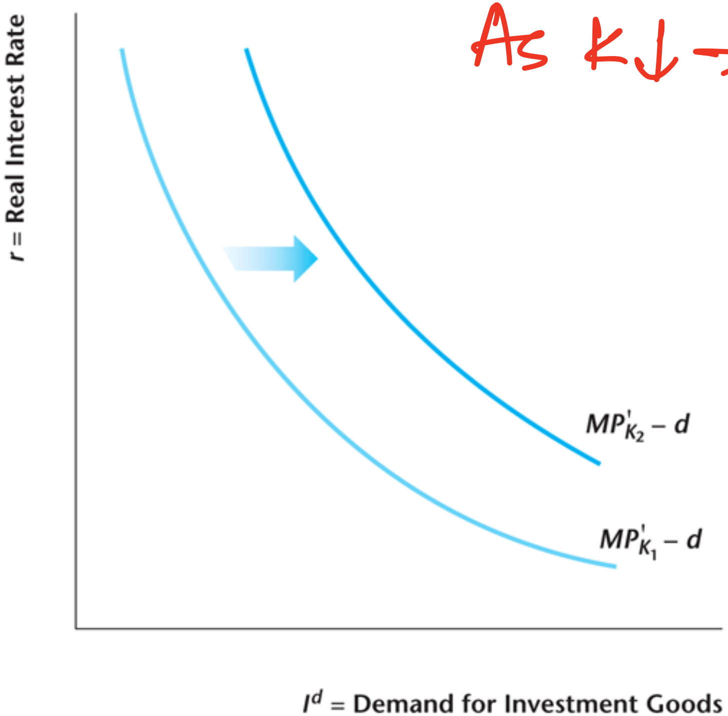


Figure: Decreasing K.

As $K \downarrow$

$$K' = (1-d)K + I$$

$$K' \downarrow \longrightarrow MP_K' \uparrow \longrightarrow (MP_K' - d) \uparrow$$

$$\downarrow$$

$$I \uparrow$$

The Government

The Government

- The government purchases goods in the amounts G today and G' tomorrow.
- Government expenditures in practice:
    - Include *public goods*, such as: national defense, education, roads, bridges,...
- Government expenditures in the model (G and G'):
    - Abstract from public good feature for simplicity.

      G and G' are just unproductive expenses taking away resources from the private sector.
    - G and G' are exogenous, i.e. determined "outside the model."

The Government

- Government expenditures are financed via lump-sum taxes and by issuing debt.

- Government's budget constraints:

$$G = T + B \quad \text{➜ Today}$$
$$G' + (1+r)B = T' \quad \text{➜ Tomorrow}$$

- Lifetime budget constraint:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

$G = T + B$

TAX REV.

BONDS

TAXES HOUSEHOLDS AND SELLS BONDS TO FINANCE GOVT. EXP.

$G' + (1+r)B = T'$

REPAYMENT OF THE BORROWING IN PERIOD 1.

$B = G - T$

$G' + (1+r)(G-T) = T'$

$$\Rightarrow G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

$G + \frac{G'}{1+r} \rightarrow$ PRESENT VALUE OF GOVT. EXP.

$T + \frac{T'}{1+r} \rightarrow$ PRESENT VALUE OF TAXES.

Taking Stock

- We have finished modelling the behaviour of the key economic actors *in isolation*:
  - The Household ➜ $N^s(w, we, r)$, $C(we, r)$
  - The Firm ➜ $N^d(w)$, $I(r)$
  - The Government
- What's next?
  - Study "consistency" in the actions of these agents

    ➜ Equilibrium model of the macroeconomy
  - Study applications/policy implications