ME 165
Basic Mechanical Engineering

Lecture 03

Rigid Bodies –Equivalent System of Forces

Sadia Tasnim
Lecturer
Department of Mechanical Engineering, BUET

Introduction

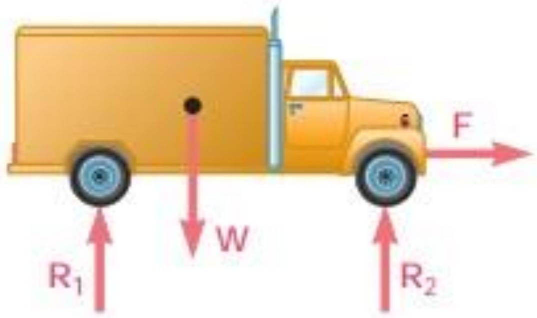
- Treatment of a body as a single particle is not always possible. In general, the size of the body and the specific points of application of the forces must be considered.

- Most bodies in elementary mechanics are assumed to be rigid, i.e., the actual deformations are small and do not affect the conditions of equilibrium or motion of the body.

- Current chapter describes the effect of forces exerted on a rigid body and how to replace a given system of forces with a simpler equivalent system.
  - moment of a force about a point
  - moment of a force about an axis
  - moment due to a couple

External and Internal Forces

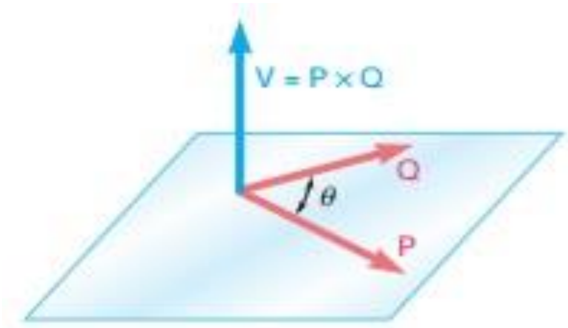
- Forces acting on rigid bodies are divided into two groups:
  - **External forces**
  - **Internal forces**



- **External forces** represent the action of other bodies on the rigid body under consideration. They are entirely responsible for the external behavior of the rigid body. They will either cause it to move or ensure that it remains at rest.

- **Internal forces** are the forces which hold together the particles forming the rigid body. If the rigid body is structurally composed of several parts, the forces holding the component parts together are also defined as internal forces.

Vector Product of Two Vectors



(a)

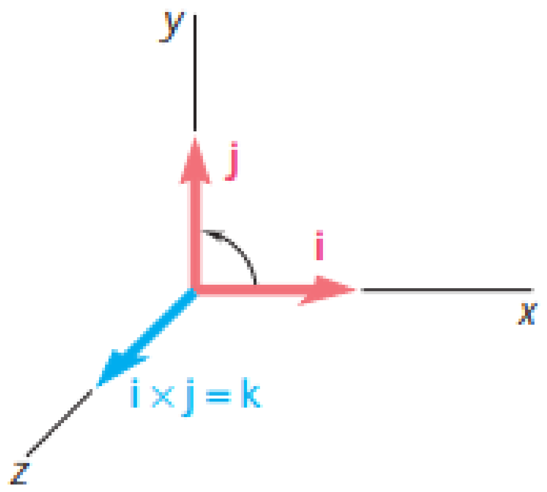
(b)

- The vector product of two vectors **P** and **Q** is defined as the vector ***V*** which satisfies the following conditions:

1. The line of action of ***V*** is perpendicular to the plane containing **P** and **Q** (Fig. a)
2. The magnitude of ***V*** is: $V = PQ \sin\theta$
3. The direction of ***V*** is obtained from the *right-hand rule*. (Fig. *b*).
4. Vector products:
   - are not commutative, $\mathbf{Q} \times \mathrm{P} \neq \mathrm{P} \times \mathrm{Q} = -(\mathrm{P} \times \mathrm{Q})$
   - are distributive, $\mathrm{P} \times \; \boldsymbol{Q_1} + \boldsymbol{Q_2} \; = \mathrm{P} \times \boldsymbol{Q_1} + \mathrm{P} \times \boldsymbol{Q_2}$
   - are not associative, $\mathrm{P} \times \mathbf{Q} \; \times \boldsymbol{S} \neq \mathrm{P} \times (\mathbf{Q} \times \boldsymbol{S})$

Vector Product Expressed in terms of Rectangular Component

$$\mathbf{i} \times \mathbf{i} = 0 \qquad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \qquad \mathbf{k} \times \mathbf{i} = \mathbf{j}$$
$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \qquad \mathbf{j} \times \mathbf{j} = 0 \qquad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$$
$$\mathbf{i} \times \mathbf{k} = -\mathbf{j} \qquad \mathbf{j} \times \mathbf{k} = \mathbf{i} \qquad \mathbf{k} \times \mathbf{k} = 0$$

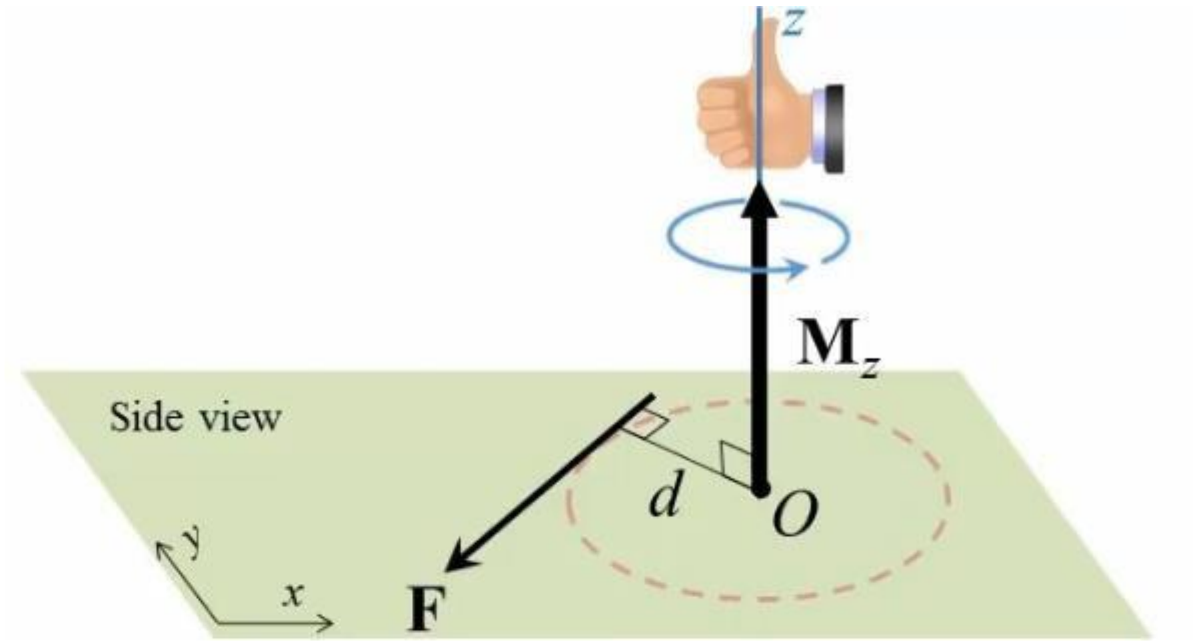


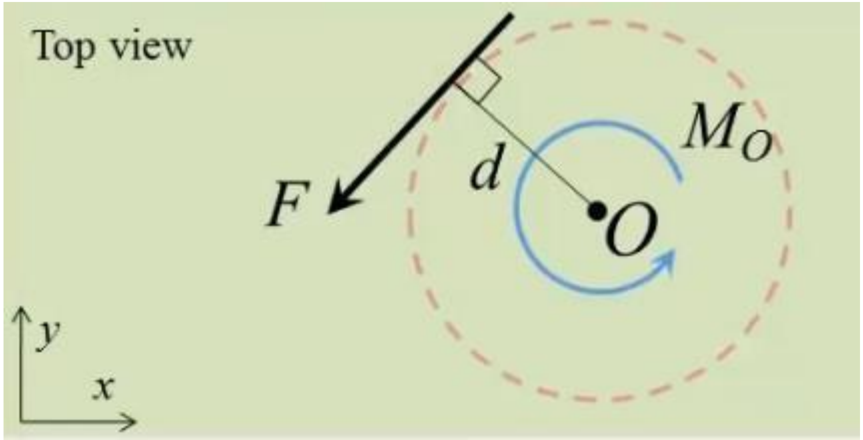
$$\begin{aligned} \boldsymbol{V} &= \boldsymbol{P} \times \boldsymbol{Q} = (P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}) \times (Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}) \\ &= (P_yQ_z - P_zQ_y)\boldsymbol{i} + (P_zQ_x - P_xQ_z)\boldsymbol{j} + (P_xQ_y - P_yQ_x)\boldsymbol{k} \end{aligned}$$

$$\boldsymbol{V} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

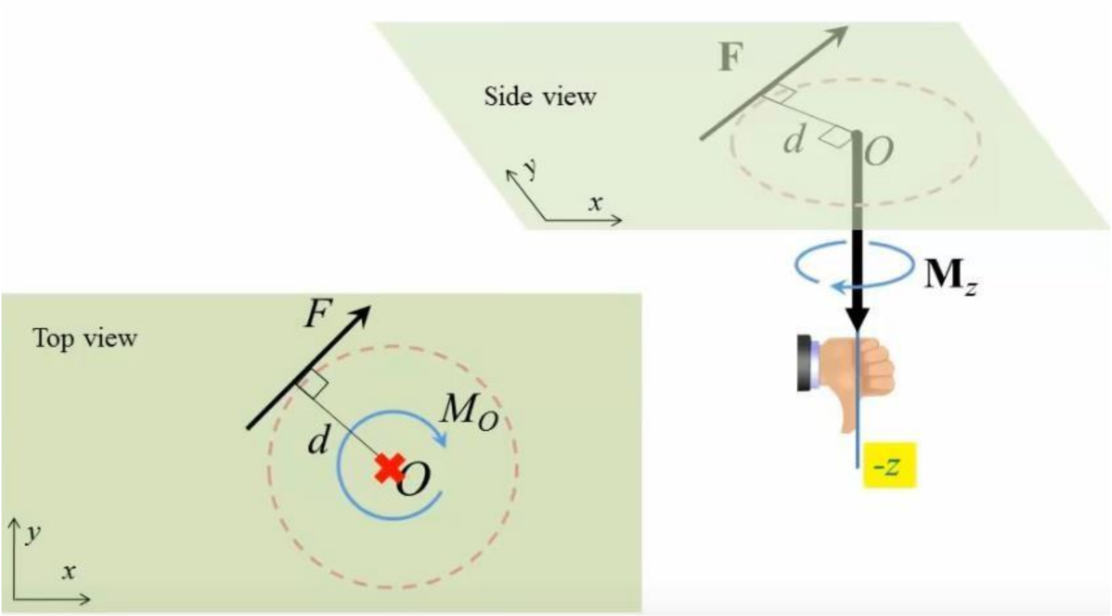
Moment of Force

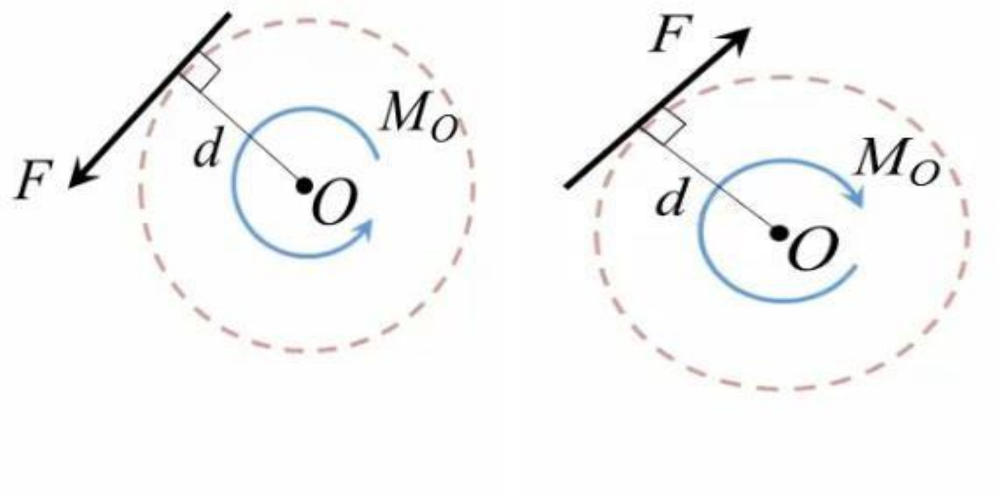
- Moment is a physical quantity that describes the rotational effect (or rotational tendency) about an axis produced by a force.
  Just like force, moment is also a **vector**.



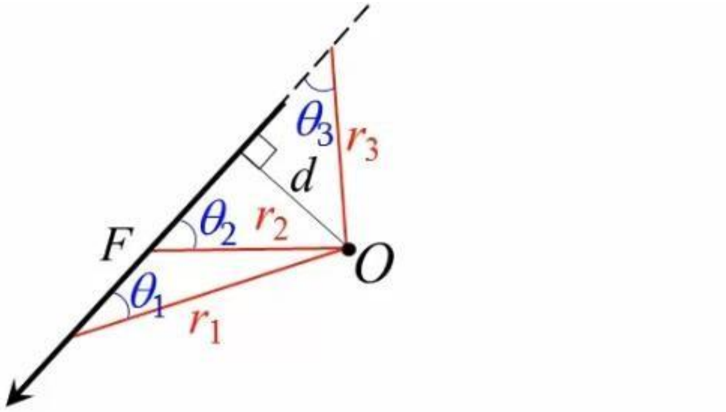


Moment of Force About a Point

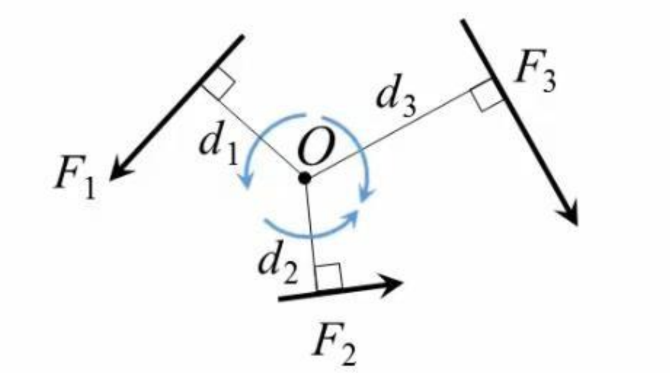




$$M_O = Fd \qquad M_O = -Fd$$

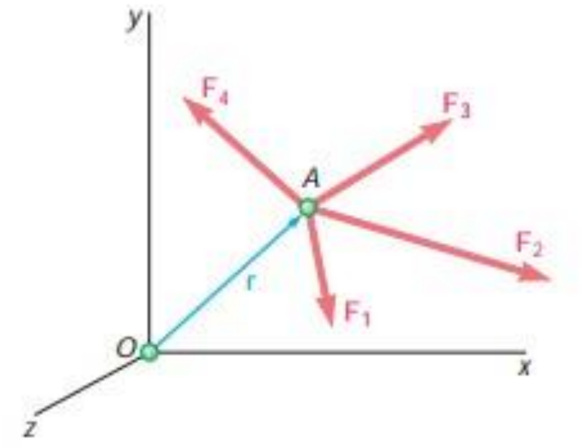


$$M_O = Fd = F \cdot r_1 \cdot \sin\theta_1 = F \cdot r_2 \cdot \sin\theta_2 = F \cdot r_3 \cdot \sin\theta_3$$



$$(M_R)_O = \sum Fd = F_1 d_1 + F_2 d_2 - F_3 d_3$$

Varignon's Theorem



- If several forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, . . . are applied at the same point $\boldsymbol{A}$ and if we denote by **r** the position vector of $\boldsymbol{A}$, then from the theorem we found that

$$\boldsymbol{r} \times (\boldsymbol{F_1} + \boldsymbol{F_2} + \cdots) = \boldsymbol{r} \times \boldsymbol{F_1} + \boldsymbol{r} \times \boldsymbol{F_2} + \cdots$$

- So, *the moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point O.*

Rectangular Components of the Moment of Force

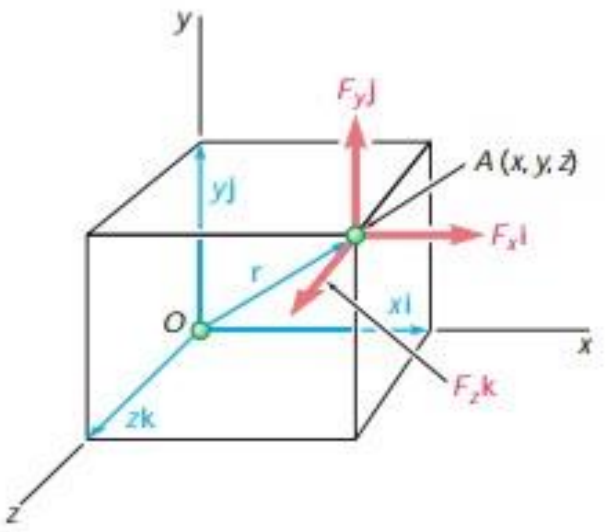
$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

So now,

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

where,

$$M_x = yF_z - zF_y$$
$$M_y = zF_x - xF_z$$
$$M_z = xF_y - yF_x$$



- We can also write Mo in the form of determinant like:

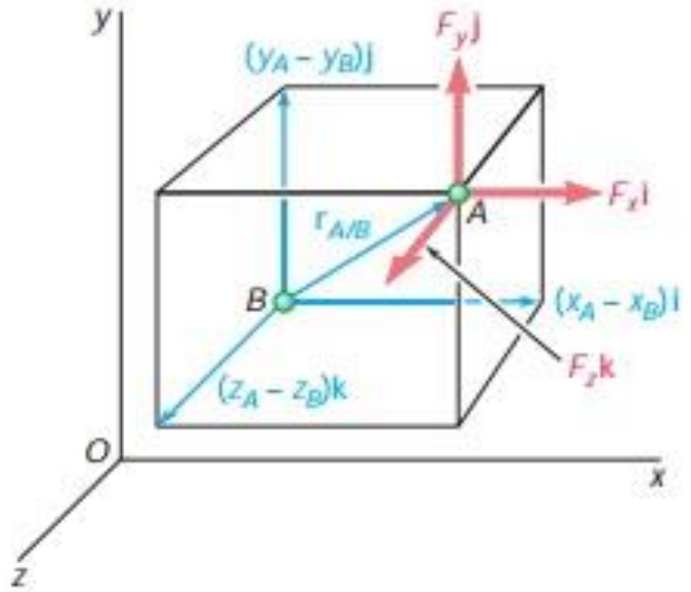
$$\boldsymbol{M_o} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

Rectangular Components of the Moment of Force

- General Case:

The moment $\mathbf{M}_B$ about an arbitrary point B of a force $\mathbf{F}$ applied at A

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$



In this case, **r** is replaced by $\mathbf{r}_{A/B}$

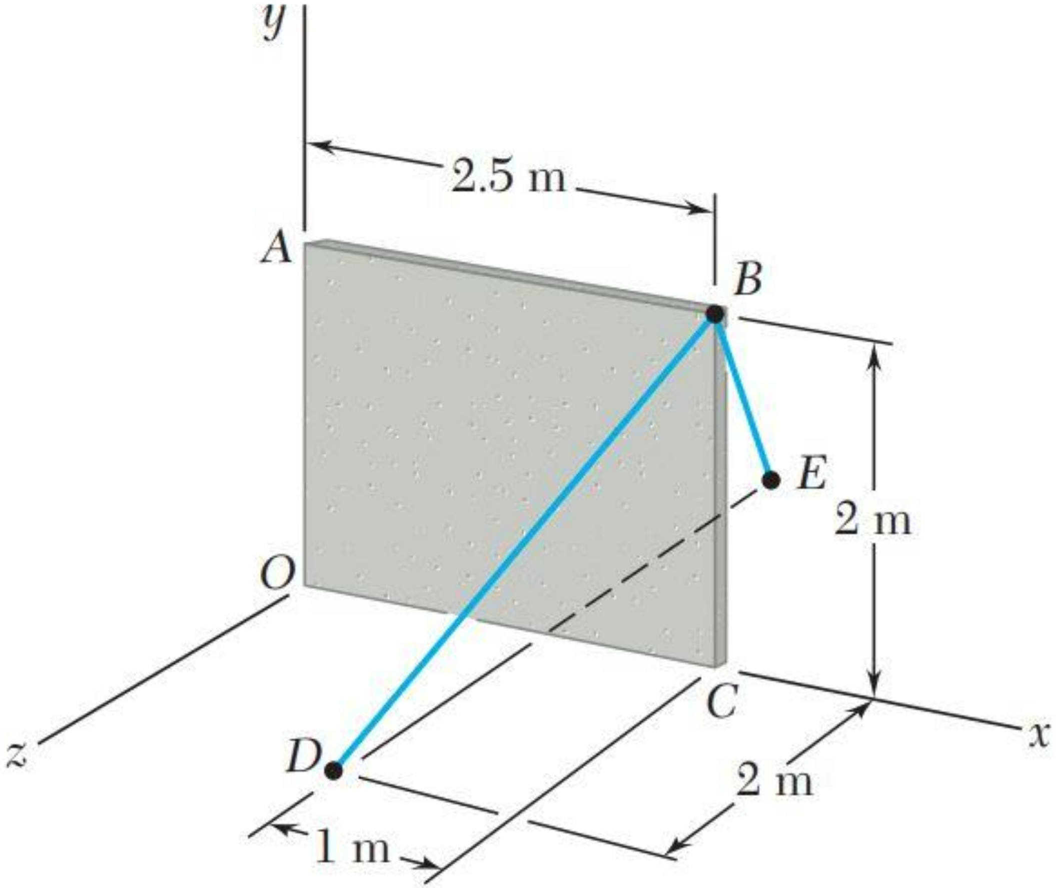
This vector is the *position vector of A relative to B.*

$$\boldsymbol{M_B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$

where, $x_{A/B} = x_A - x_B \qquad y_{A/B} = y_A - y_B \qquad z_{A/B} = z_A - z_B$

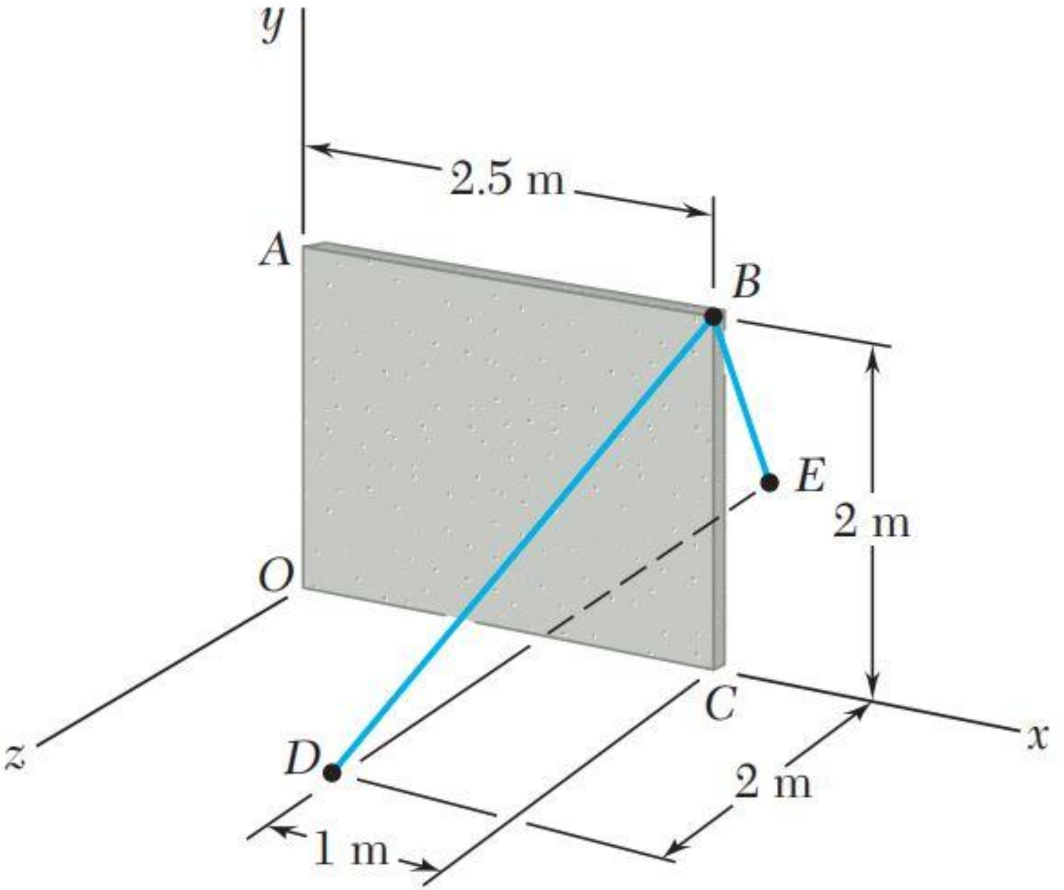
❑ Problem:

A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about point O of the force exerted by the cable at B.



Rectangular Components of the Moment of Force

❑ **Solution:**



Ans: M = 1200i-1500j-900k N.m

Moment of Force About a Given Axis

Let, OL be an arbitrary axis through the origin O.

Moment of* F *about O as the vector product of **r** and **F**:

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

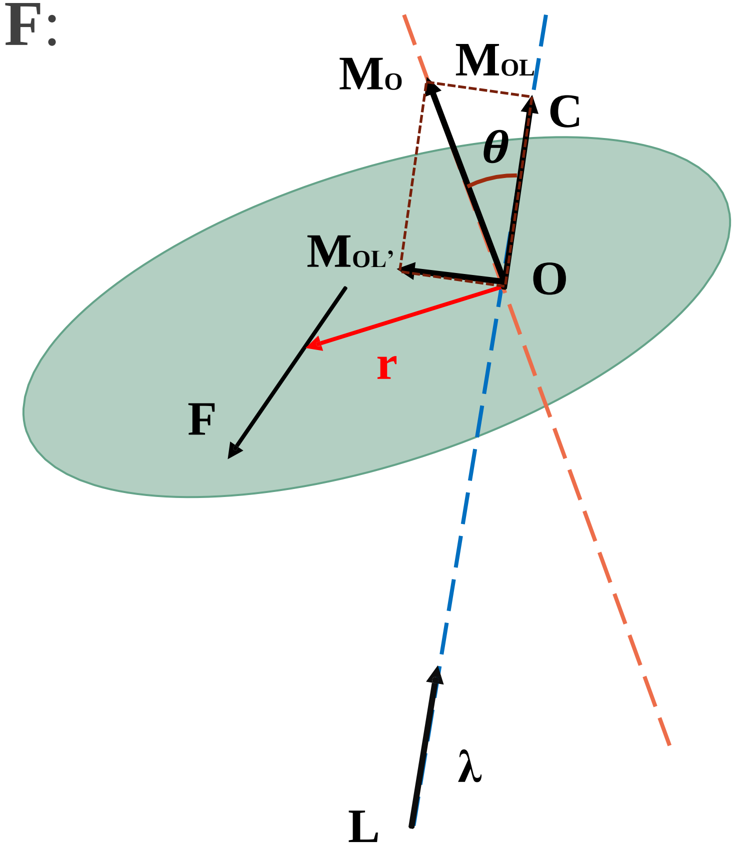
So, the moment M_{OL} of **F** about OL as the projection OC of the moment $\mathbf{M_O}$ onto the axis OL.

$$M_{OL} = M_O . \cos\theta$$

Here, **λ** is the unit vector along OL

$$M_{OL} = \boldsymbol{\lambda} . \mathbf{M_O} = \boldsymbol{\lambda} . (\mathbf{r} \times \mathbf{F})$$

$$\boldsymbol{M_{OL}} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

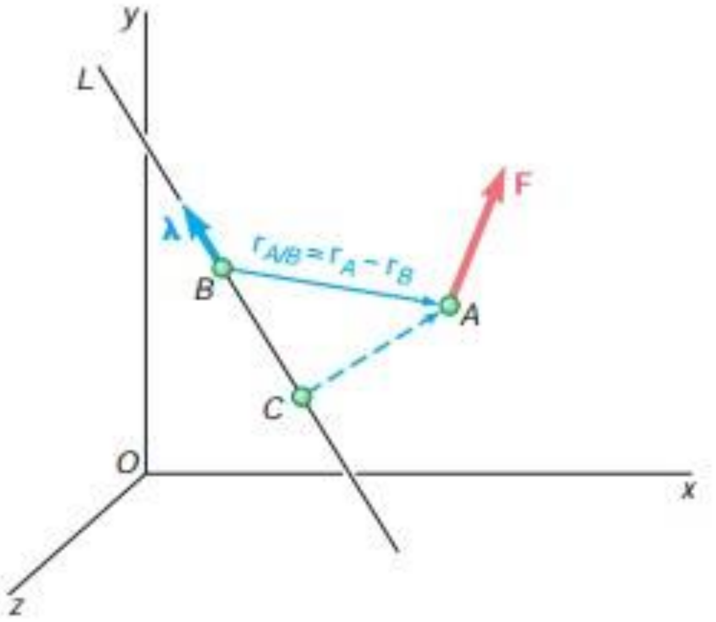


Moment of Force About a Given Axis

- The moment of a force **F** applied at *A* about an axis which does not pass through the origin is obtained by choosing an arbitrary point *B* on the axis and determining the projection on the axis *BL* of the moment **M***B* of **F** about *B*.

- $M_{BL} = \boldsymbol{\lambda} \cdot \mathbf{M}_B = \boldsymbol{\lambda} \cdot (\mathbf{r}_{A/B} \times \mathbf{F})$

$$\boldsymbol{M_{BL}} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$



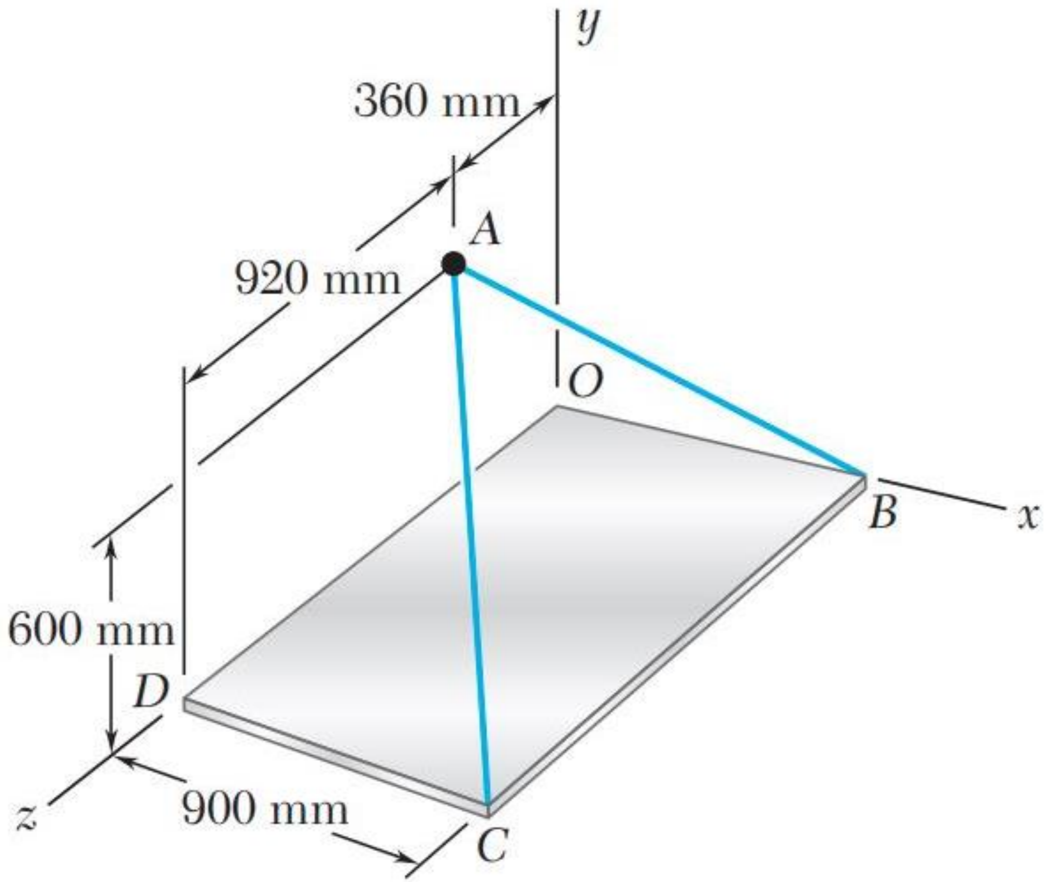
Where,

$\lambda_x, \lambda_y, \lambda_z$ = direction cosines of axis *BL*

Rectangular Components of the Moment of Force

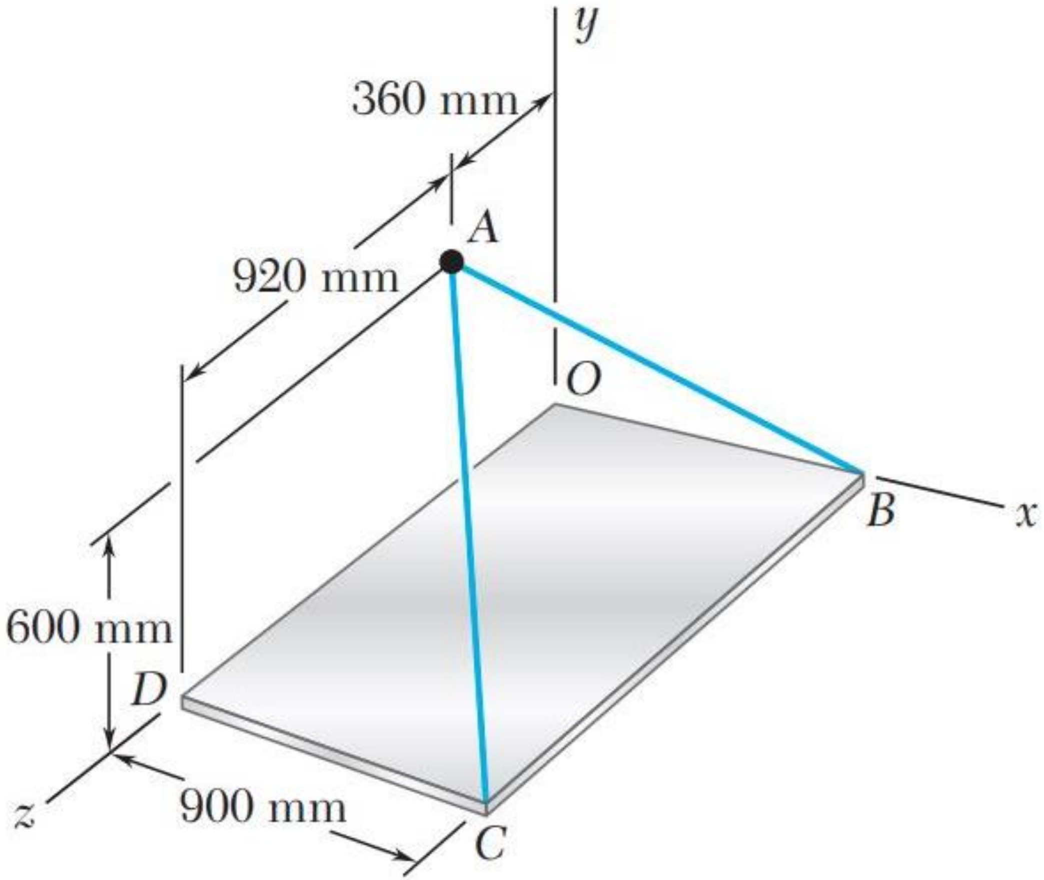
❑ Problem:

Knowing that the tension in cable AB is 570 N, determine the moment about each of the coordinate axes of the force exerted on the plate at B.



Rectangular Components of the Moment of Force

- **Solution:**

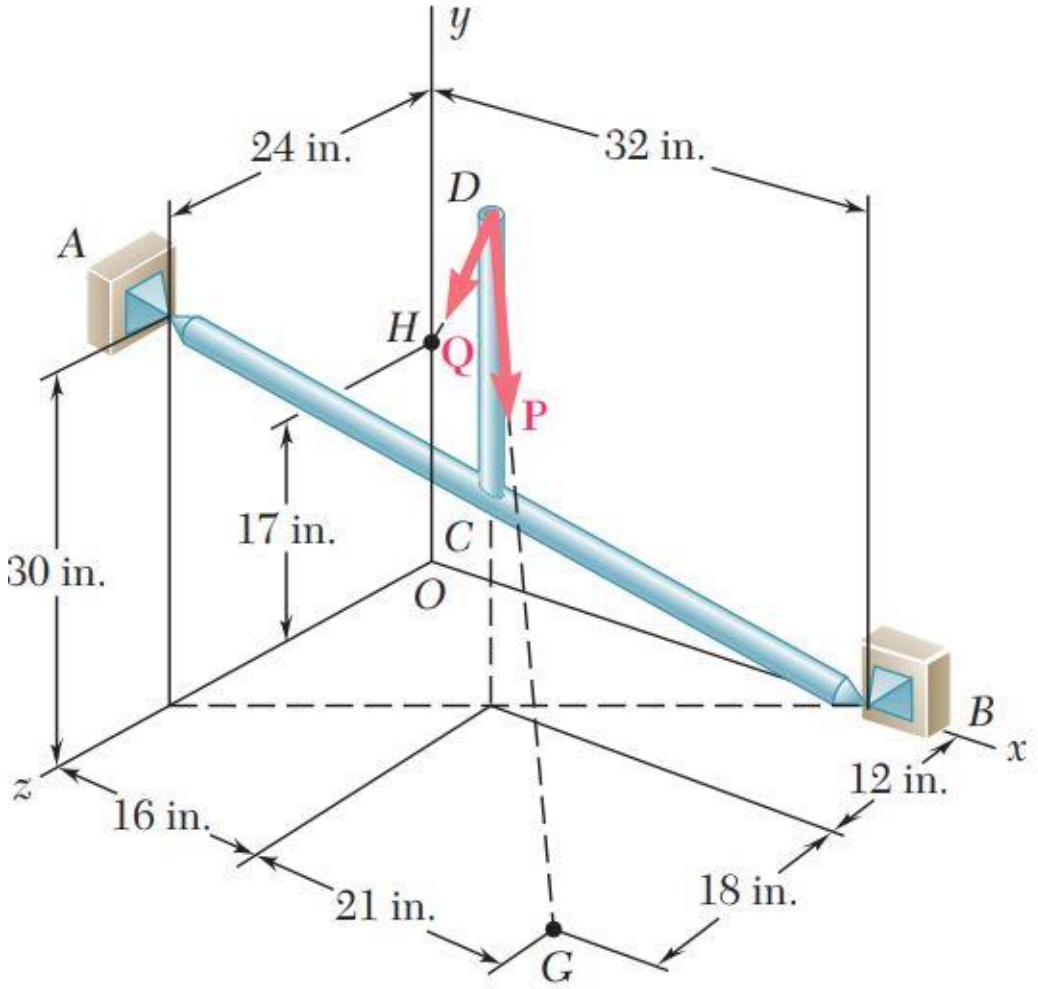


Ans: M = 0i-162j-270k N.m

Rectangular Components of the Moment of Force

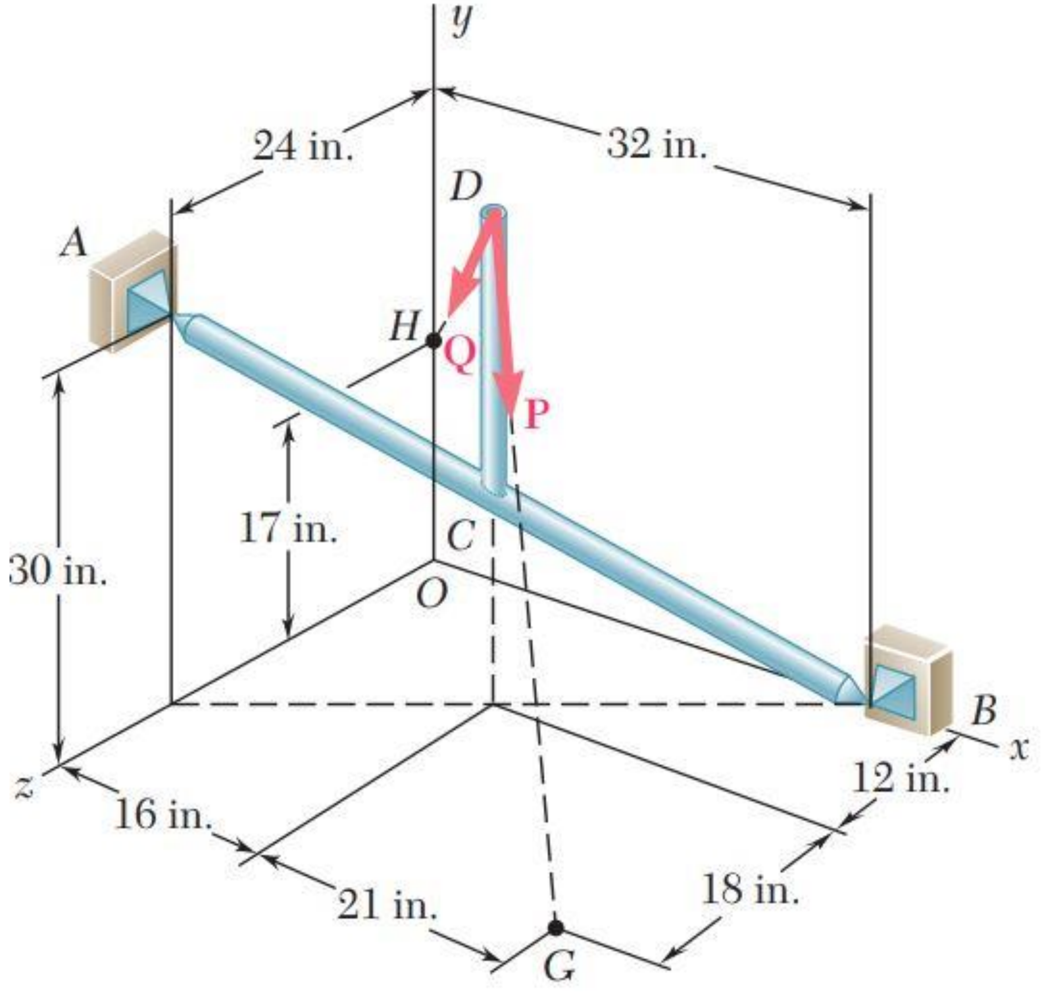
❑ Problem:

The 23-in. vertical rod CD is welded to the midpoint C of the 50-in. rod AB. Determine the moment about AB of the 235-lb force **P**.



Rectangular Components of the Moment of Force

- **Solution:**



Ans: M = 2484 lb-in

Practice: (Ex. 3.58)

Rectangular Components of the Moment of Force

❑ Problem:

A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x axis of the forces exerted by the cables on the barn at points A and D is equal to 4728 lb-ft, determine the magnitude of T_{DE} when T_{AB} =255 lb.