# Automatic control III - Homework assignment 1

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## 1 Problem I

### a)

The system has the following minors:

$$
\begin{aligned}
(1\times 1):&\quad \frac{13}{17s+1},\ \frac{-19}{21s+1},\ \frac{4}{15s+1},\ \frac{7}{12s+1},\ \frac{-19}{15s+1},\ \frac{5}{12s+1}\\
(2\times 2):&\quad \frac{-19(71s+3)(21s+2)}{(21s+1)(17s+1)(15s+1)(12s+1)},\ \frac{499s+37}{(17s+1)(15s+1)(12s+1)},\\
&\quad \frac{-19(117s^s+18s+1)}{(21s+1)(15s+1)^2(12s+1)}.
\end{aligned}
$$

Least common denominator is $(21s+1)(17s+1)(15s+1)^2(12s+1)$ and the poles become $\{\frac{-1}{21}, \frac{-1}{17}, \frac{-1}{15}, \frac{-1}{15}, \frac{-1}{12}\}$.

To determine the zeros, normalize the maximal minors s.t. the pole polynomial is in the denominator:

$$
\begin{aligned}
(2\times 2):&\quad \frac{-19(71s+3)(21s+2)(15s+1)}{(21s+1)(17s+1)(15s+1)^2(12s+1)},\ \frac{(499s+37)(21s+1)(15s+1)}{(21s+1)(17s+1)(15s+1)^2(12s+1)},\\
&\quad \frac{-19(117s^s+18s+1)(17s+1)}{(21s+1)(17s+1)(15s+1)^2(12s+1)}.
\end{aligned}
$$

Where there is no common divisor, which means that there is no zero for this system.

### b)

According to Figure 1 the largest gain of all individual elements is $\approx 25$. The gain for the MIMO system is presented in Figure 2. What is presented in the plot is

$$0 \le \underline{\sigma}(G) \le \frac{|Gx|}{|x|} \le \overline{\sigma}(G), \tag{1}$$

so $\overline{\sigma}(G)$ is the gain of the system. The largest gain there is $\approx 30$, which is larger than the largest gain of the individual elements. This is to be expected since the output signals are linear combination of the input signals scaled according to the different elements in the transfer matrix.

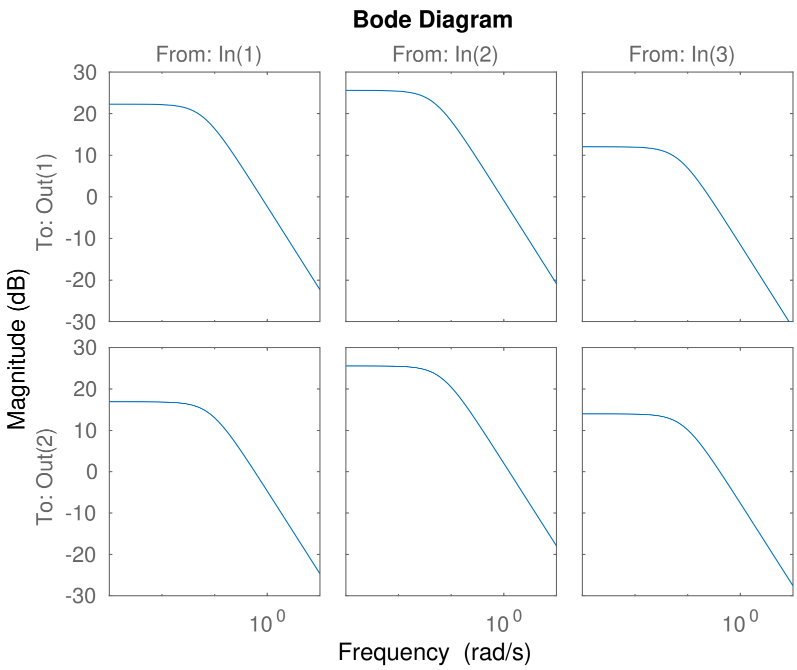


Figure 1: The gain of all individual elements of G(s) using the command bodemag.

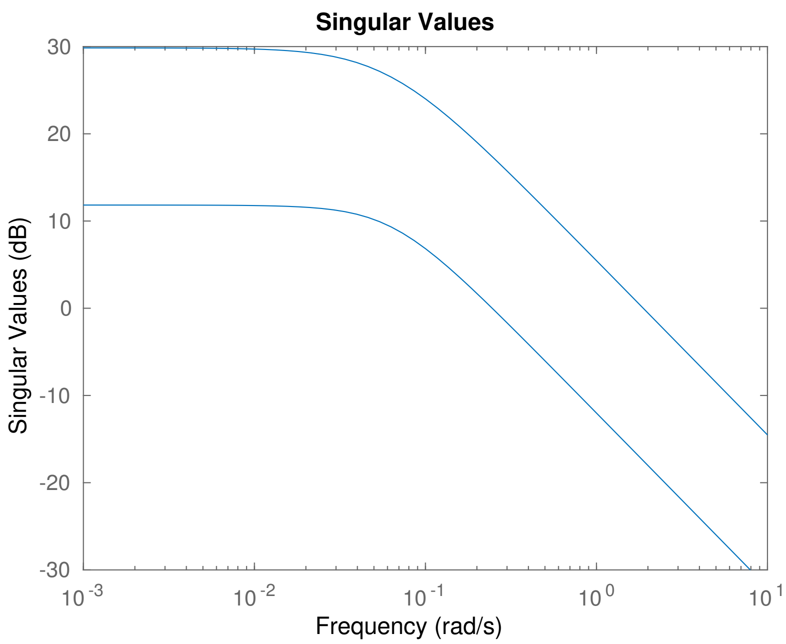


Figure 2: The gain of the system G(s) using the command sigma.

## 2 Problem II

No, the system $G(s) = \frac{1}{s(s+3)}$ is unstable and therefore we can not use the small gain theorem.

## 3 Problem III

We have the following system:

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} \frac{5s+0.025}{s^2+0.1s+0.002} & \frac{10^{-2}}{s^2+0.1s+0.002} \\ \frac{10^{-2}}{s^2+0.1s+0.002} & \frac{5s+0.025}{s^2+0.1s+0.002} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$

### a)

First we look at the RGA matrix for our system at $\omega = 0$. This was done in Matlab and the code can be seen in the appendix.

$$RGA(0) = \begin{pmatrix} 1.1905 & -0.1905 \\ -0.1905 & 1.1905 \end{pmatrix}$$

We can see that both signals affects both outputs, even if the diagonal elements are larger than the others, we can't neglect the influence of $u_1$ on $T_2$ and $u_2$ on $T_1$, so we can't use decentralized control, we have to use decoupling.
We will make the variable exchange

$$y = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \tag{2}$$

$$\tilde{y} = W_2 y \tag{3}$$

$$\tilde{u} = W_1^{-1} u \tag{4}$$

which will make

$$\tilde{G}(s) = W_2(s)G(s)W_1(s) \tag{5}$$

as diagonal as possible. With the new control- and output signals it would be possible to make a decentralized regulator

$$\tilde{u} = -F_y^{diag}\tilde{y} \tag{6}$$

Where $F_y^{diag}$ is a decentralized (PID) regulator

$$F_y^{diag} = \begin{pmatrix} F_{PID}^1(s) & 0 \\ 0 & F_{PID}^2(s) \end{pmatrix}, \tag{7}$$

we do not tune the PID parameters here.

Expressed in the original variables we then get the regulator

$$u = -W_1 F_y^{diag} W_2 y \tag{8}$$

We will achieve approximately diagonal behavior by choosing $W_1 = G^{-1}(0)\ W_2 = \mathbb{1}$ where

$$G(0) = \begin{pmatrix} 12.5 & 5 \\ 5 & 12.5 \end{pmatrix}.$$

This leads to

$$\tilde{G}(s) = \begin{pmatrix} \frac{0.476s+0.001999}{s^2+0.1s+0.002} & X \\ X & \frac{0.476s+0.001999}{s^2+0.1s+0.002} \end{pmatrix}$$

with the final regulator

$$F_y(s) = W_1 F_y^{diag}(s) \tag{9}$$

A block diagram for the system is presented in Figure 3.

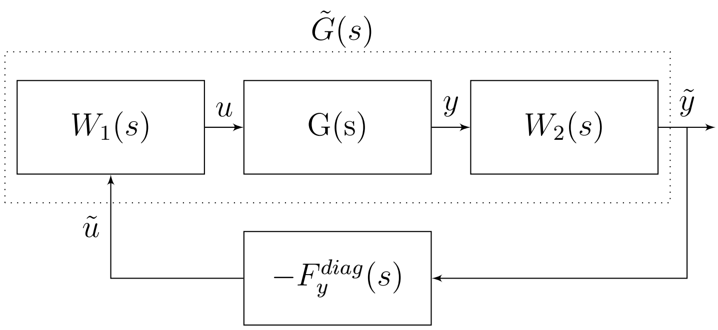


Figure 3: Block diagram for the system

## b)

The transfer function is

$$G(s) = \begin{pmatrix} \frac{5s+0.025}{s^2+0.1s+0.002} & \frac{10^{-2}}{s^2+0.1s+0.002} \\ \frac{10^{-2}}{s^2+0.1s+0.002} & \frac{5s+0.025}{s^2+0.1s+0.002} \end{pmatrix}.$$

Writing this into matlab and inverting G shows us that the degree of the denominator polynomials is 2 while the degree of the numerator polynomials is 3. Hence we must add a factor of $\frac{1}{\lambda s+1}$ to form

$$Q(s) = \frac{1}{\lambda s + 1} G^{-1}(s) \tag{10}$$

To check that the static gain is $\mathbb{1}$, we calculate $G_c(s) = G(s)Q(s)$ and used *evalfr(Gc, 0)* and got $\mathbb{1}$.

We wanted poles near to the original system so we began testing with $\lambda = \frac{1}{0.005} = 200$, i.e. the zeros of G. This did not fulfill the required rise time since it was too fast. This required us to increase $\lambda$ successively. With $\lambda = 260$ we obtain the rise time ~560 seconds.

With simulation, it can be observed that the rise time is in between 8-12 minutes.
The simulations is presented in Figure 5 and 6 and the Simulink diagram is presented in Figure 4. For the code that determined the transfer functions G and Q sent to simulink, se appendix 1.

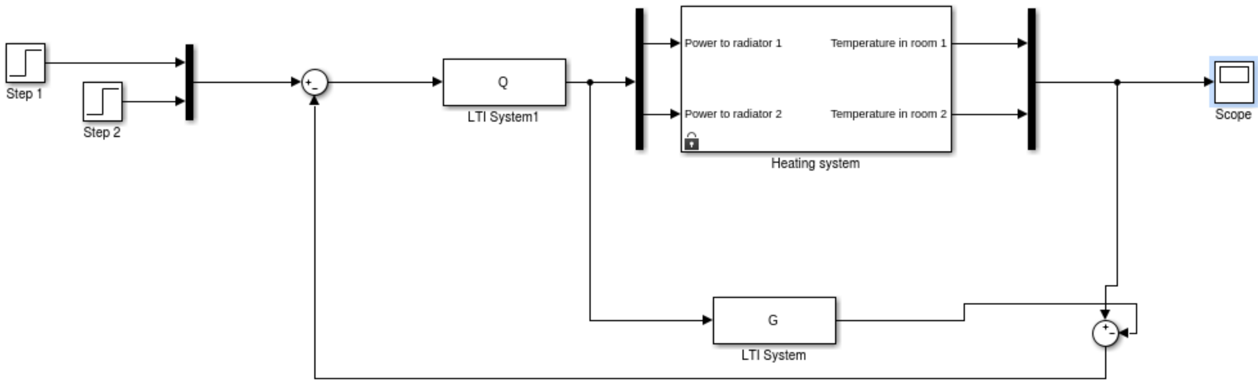


Figure 4: Simulink diagram.

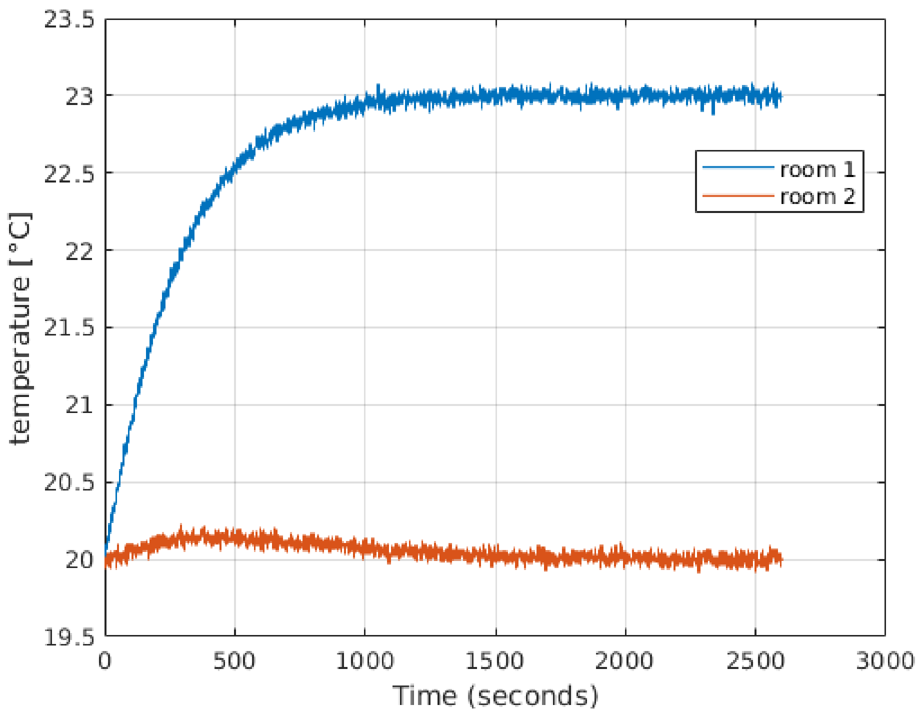


Figure 5: Simulation of the rise time when stepping u1 from 20 to 23 and holding u2 constant 20.

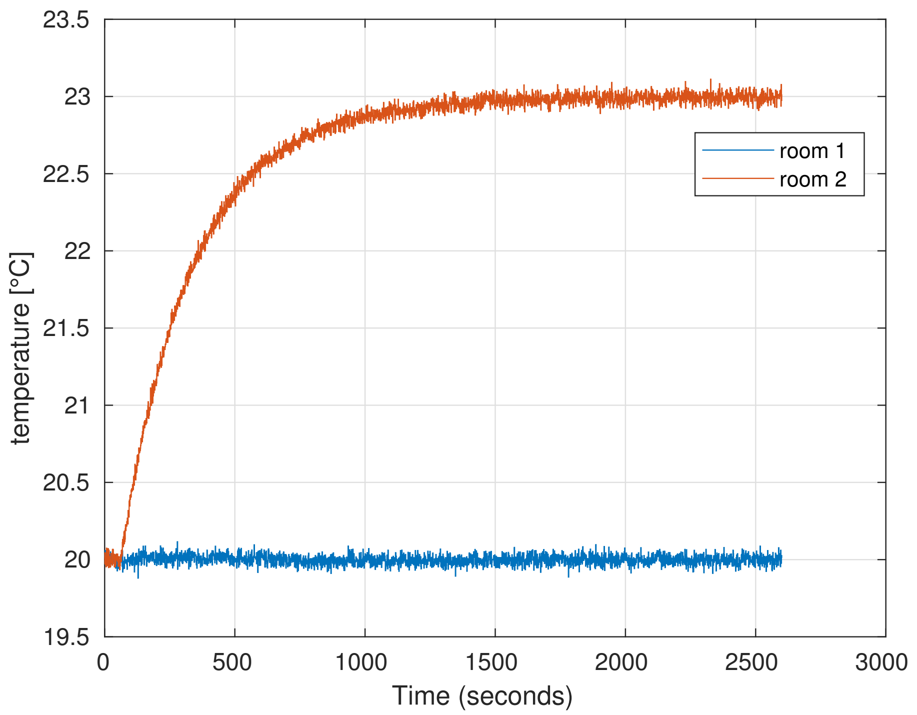


Figure 6: Simulation of the rise time when stepping u2 from 20 to 23 and holding u1 constant 20.

# A Appendix

```
%%%%%%RGA
G0 = (1/0.002)*[0.025 (10^-2) ; (10^-2) 0.025]
G0.*pinv(G0.')
G = G0
RGAmat = G.*pinv(G).'

%%%%%%%%%%%%%%%% 2b
G0 = (1/0.002)*[0.025 (10^-2) ; (10^-2) 0.025]
G0.*pinv(G0.')
G = G0
RGAmat = G.*pinv(G).'
lambda = 280;

G = [tf([5 0.025],[1 0.1 0.002]) tf([10^(-2)],[1 0.1 0.002]); tf([10^(-2)]

pole(G);
tzero(G);

Q = tf(1,[lambda 1])*inv(G);

Gc = G*Q;

step(Gc)
```