Automatic Control III, Homework 3

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Problem 1, H_∞ control and basic limitations

a)

A phase plane analysis of the Van der Pol oscillator includes finding all stationary points and determining their type. First, we find the stationary points by looking at for which state variables the derivative of the states yields zero. For the system

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) = x_2 \\ \dot{x}_2 &= f_2(x_1, x_2) = \mu(1 - x_1^2)x_2 - x_1 \end{aligned} \tag{1}$$

We get

$$\begin{aligned} \dot{x}_1 &= 0 \rightarrow x_2 = 0 \\ \dot{x}_2 &= 0 \rightarrow x_1 = 0 \end{aligned} \tag{2}$$

which corresponds to the stationary point of the system. We now investigate the jacobian of this state space at this stationary point. We get

$$\begin{aligned} \frac{\partial f_1}{\partial x_1} &= 0 \\ \frac{\partial f_1}{\partial x_2} &= 1 \\ \frac{\partial f_2}{\partial x_1} &= -2_2 x_1 - 1|_{x_2=0, x_1=0} = -1 \\ \frac{\partial f_2}{\partial x_2} &= \mu(1 - x_1^2)|_{x_1=0} = \mu \end{aligned} \tag{3}$$

which yields the linearized state space matrix A as the jacobian. Solving the eigenvalues of this matrix gives the types of the phase plane. We get the eigenvalues by solving

$$det(\lambda I - A) = 0 \iff \lambda^2 - \mu\lambda + 1 = 0 \tag{4}$$

The solution to this equation will be the following:

$$\lambda = \frac{\mu}{2} \pm \sqrt{(\frac{\mu}{2})^2 - 1} \tag{5}$$

Analysis of this second order polynomial, using the PQ-formula enlightens us with the following

$$\begin{aligned} &\mu = 2 \rightarrow \lambda = 1 \pm \sqrt{1-1} = 1 \\ &\iff \lambda \in \mathbb{R} \quad \lambda_1 = \lambda_2 > 0 \quad \rightarrow unstable \quad single \quad tangential \quad node \quad or \quad star \quad node \\ &\mu > 2 \rightarrow \lambda_1 \neq \lambda_2, \quad \lambda \in \mathbb{R}, \quad \lambda_1, \lambda_2 > 0 \rightarrow unstable \quad double \quad tangential \quad node \\ &\mu < 2 \rightarrow \lambda_1 \neq \lambda_2 \quad \lambda \in \mathbb{C} \rightarrow unstable \quad focus \end{aligned} \tag{6}$$

For $\mu = 2$ the solution could both be star node or a single tangential node. The number of linearly independent eigenvectors must be investigated to reach a conclusion. $\mu = 2$ corresponds to $\lambda = 1$ which gives

$$(A - \lambda I)\begin{bmatrix} \vec{v_1} \\ \vec{v_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -\lambda & 1 \\ -1 & 2-\lambda \end{bmatrix}\begin{bmatrix} \vec{v_1} \\ \vec{v_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \vec{v_1} \\ \vec{v_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{7}$$

where $\vec{v_1}$ and $\vec{v_2}$ are the corresponding eigenvectors. There is only one linearly independent eigenvector since the rows are dependent on each other. Thus, the node is single tangential.

b)

$$\frac{d^2y}{dt^2} - \mu * \frac{dy}{dt} + y = \frac{\mu}{3} * \frac{du}{dt} \tag{8}$$

With $u = -y^3$ the equation becomes

$$\frac{d^2y}{dt^2} - \mu * \frac{dy}{dt} + y = -3y^2\frac{dy}{dt} \rightarrow \tag{9}$$

$$\frac{d^2y}{dt^2} - \mu\frac{dy}{dt} + y + 3y^2\frac{dy}{dt} = 0 \rightarrow \frac{d^2y}{dt^2} - \mu(1-y^2)\frac{dy}{dt} + y = 0 \tag{10}$$

which shows that the systems are equivalent for this particular choice of $+u$.

c)

By taking the Laplace transformation of the differential equation the equation becomes

$$s^2Y - \mu sY + Y = \frac{\mu}{3}sU \iff \frac{Y}{U} = G = \frac{\frac{\mu s}{3}}{s^2 - \mu s + 1} \tag{11}$$

where the latter is the transfer function G. Making an high level analysis of the non-linearity yields the following. The output of the non-linearity is given as $u = -y^3$ where as the input is fed back as $-y$. Hence $f(e) = e^3$ where e is the input to the non linearity.

d)

When analyzing equation 11 we can see that, for small frequencies, the transfer function will be small and that $G(s)$ will be independent of μ in this case. For big frequencies, the quadratic term in the numerator will take over and $G(s)$ will be independent of μ in this case aswell. For frequencies around 1 we will see that $G(s)$ will be around a constant $-\frac{1}{3}$. From this analysis we see that $G(s)$ will be independent of μ. When plotting the Nyquist diagram for $G(s)$ in MatLab we can see that this analysis holds and that the Nyquist diagram is approximately independent of μ. Atleast when μ lies between 0 and 4.

To analyze the circle criterion we want to enclose the nonlinear behaviour ($f(e) = e^3$) between two straight lines. The lines for this corresponds to $k_1 = 0$ and $k_2 = \infty$. This in turn, corresponds to an infinitely large circle, covering the whole left complex plane when trying to plot a circle that intersect the points $-\frac{1}{k_1}$ and $-\frac{1}{k_2}$.

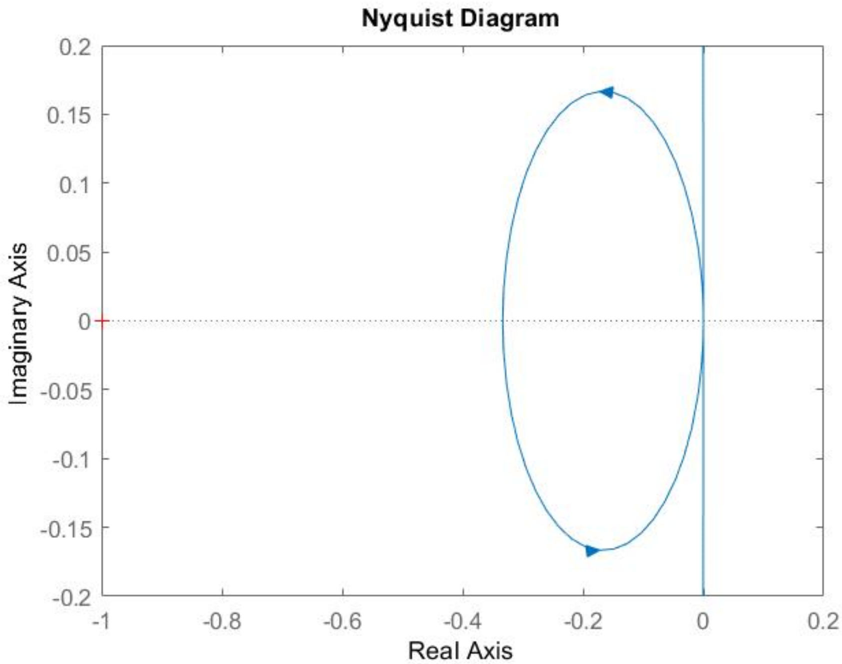

Figure 1: *Nyquist plot and the circle span by k1 and k2. μ is 0.01.*

Figure 1 shows that the Nyquist curve is enclosed by the circle spanned by $-\frac{1}{k_1}$ and $-\frac{1}{k_2}$, so that the circle criterion is not fulfilled. This means we can't guarantee a stable system, for all μ (between 0 and 4).

e)

The describing function for $f(e) = e^3$ is given as:

$$Y_f(s) = \frac{3}{4}C^2 \tag{12}$$

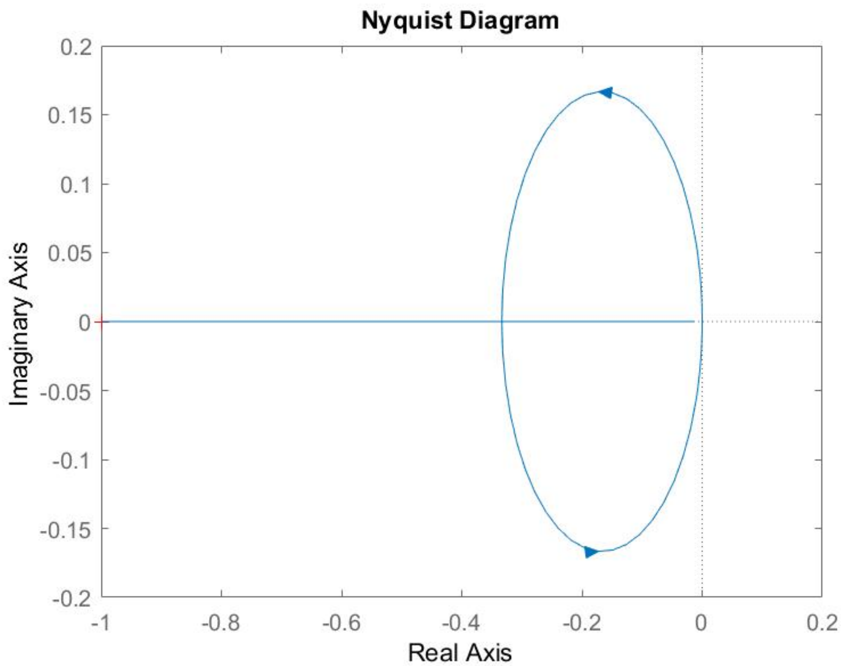

Figure 2: *Nyquist diagram for $G(s)$ where $\mu = 1$, together with $-\frac{1}{Y_f(s)}$ for* $0 < C < \infty$

In Figure 2 we see that $-\frac{1}{Y_f(s)}$ will cross the Nyquist diagram. $-\frac{1}{Y_f(s)}$ will be 0 for $C = 0$ and become more negative when C is getting larger. This means $-\frac{1}{Y_f(s)}$ will cross the Nyquist diagram and end outside the circle, resulting in a stable limit cycle. We can see that $-\frac{1}{Y_f(s)}$ will cross the Nyquist diagram on the real axis at $\frac{-1}{3}$ which gives a solution for C

$$-\frac{1}{Y_f} = -\frac{1}{3} \rightarrow \frac{4}{3C^2} = \frac{1}{3} \rightarrow C = 2. \tag{13}$$

Thus a solution for ω can be obtained

$$G = \frac{\frac{\mu s}{3}}{s^2 - \mu s + 1} = \frac{\frac{1s}{3}}{s^2 - s + 1} = \frac{1}{3} \rightarrow w = 1. \tag{14}$$

f)

To test our theory, simulations in smulink was made:

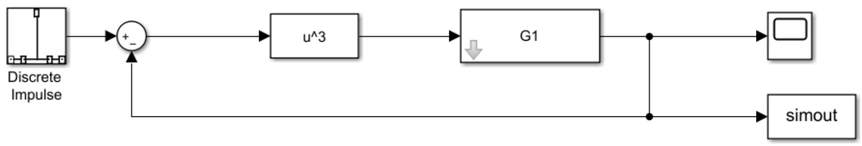

Figure 3: *Block diagram in simulink.*

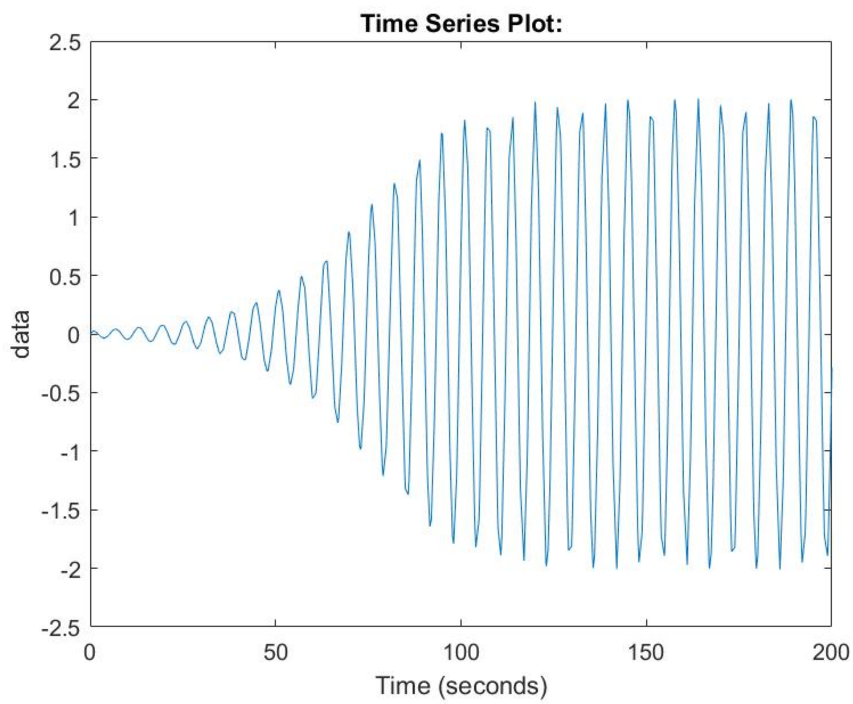

Figure 4: *Simulation of the system when* $\mu = 0.1$

In Figure 4 we see the that a stable limit cycle is created when $\mu = 0.1$. The period time is 6.28 seconds and corresponds to a angle frequency $\omega \approx 1$.

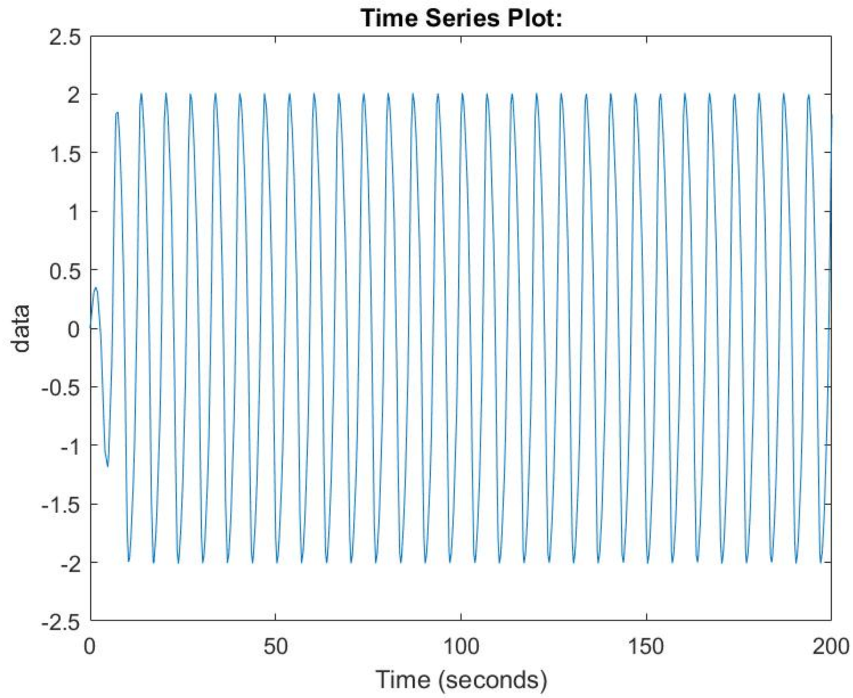

Figure 5: *Simulation of the system when* $\mu = 1$

In Figure 5 we see the that a stable limit cycle is created when $\mu = 1$. The period time is 6.67 seconds and corresponds to a angle frequency $\omega \approx 1$.

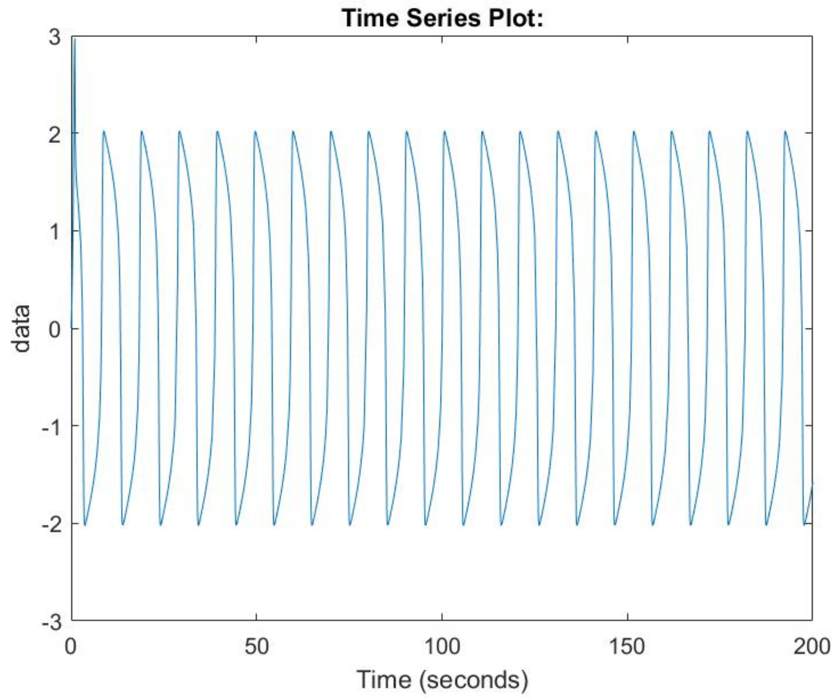

Figure 6: *Simulation of the system when* $\mu = 4$

In Figure 6 we see the that a stable limit cycle is created when $\mu = 4$. The period time is 10.2 seconds and corresponds to a angle frequency $\omega \approx 0.62$. This result contradicts the previous conclusions that the angle frequency ω should be ≈ 1. This could be explained by plotting the corresponding bode plots.

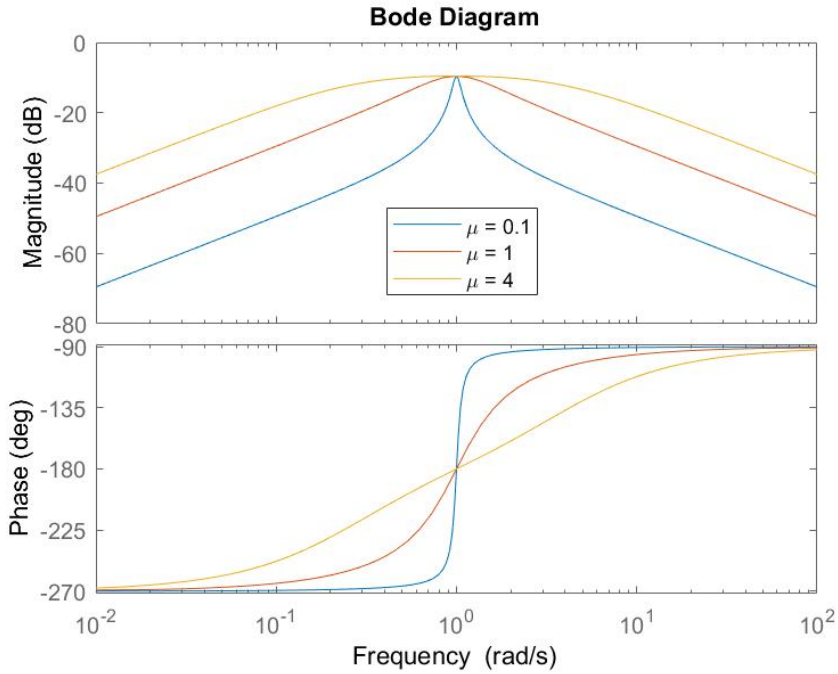

Figure 7: *Bode plots for the different transfer functions.*

In Figure 7 we see the bode plots for the different transfer functions corresponding to the different μ. When $\mu = 4$ the pass band will be wider compared to the other pass bands. This means that several frequencies will affect our limit cycle and $\omega = 1.6$ makes sense.

g)

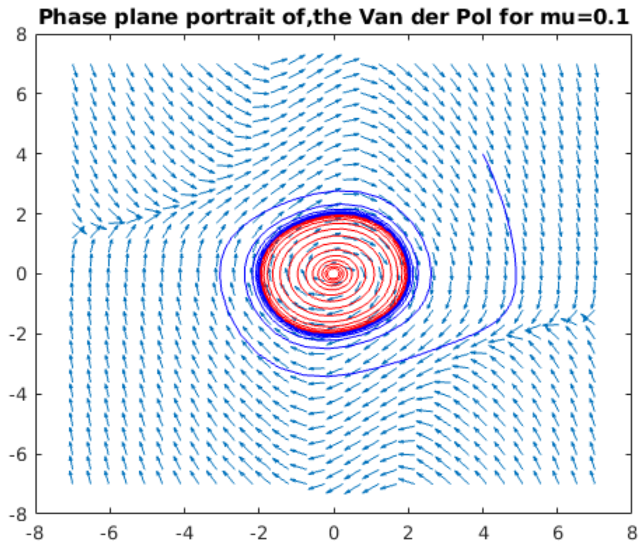

Figure 8: *Phase plane portrait for the Van der Pol equations with* $\mu = 0.1$.

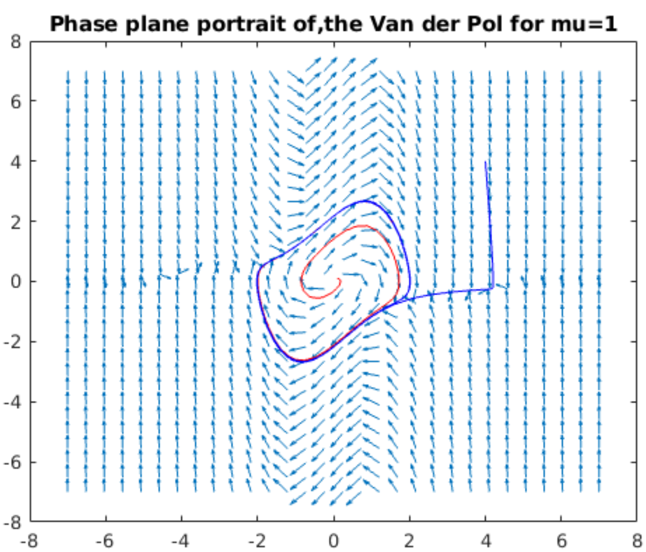

Figure 9: *Phase plane portrait for the Van der Pol equations with* $\mu = 1$.

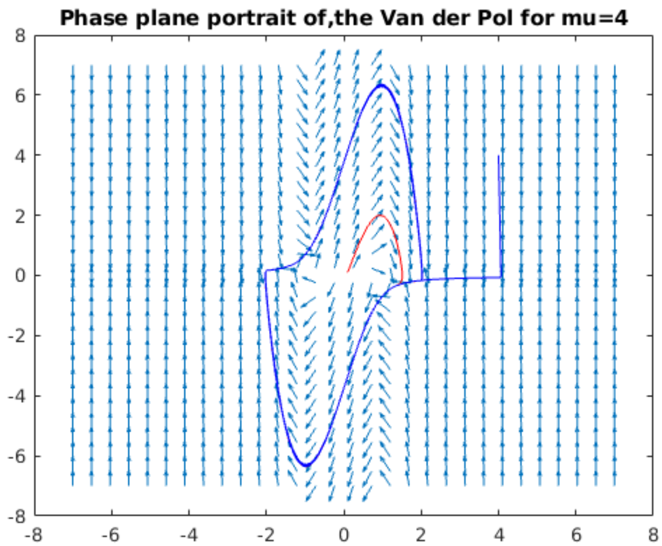

Figure 10: *Phase plane portrait for the Van der Pol equations with* $\mu = 4$.

In Figure 8, Figure 9 and Figure 10 we see the phase plane portraits for the different choices of $\mu = 0.1, \mu = 1, \mu = 4$. Coherently with the previous analysis all these choices of μ yields stable limit cycles. For higher values of μ, the solutions faster converge to the oscillation, which we can see in the phase portraits as well. Two solutions using Matlabs built in solver *ode45* are shown in each phase plane. The blue curve have initial guess at (4,4) and the red curve has an initial guess at (0.1,0.1).

In Figure 6 one can notice that the frequency is slightly lower than the other choices of μ. In the phase planes we can see that for the same choice of μ. This feels intuitive since we step all solutions in *ode45* with the same step length, hence a larger curve takes longer time for one cycle compared to a smaller cycle, hence a lower frequency for a larger choice of μ. In question a) we studied the linearised case and found that the system has an unstable focus point for all $\mu < 2$. This corresponds well to what we see in Figure 8 and Figure 9 when looking close to the equilibria. Looking at Figure 10 we see that close to the equilibria the phase plane behaves as the linearised case when $\mu > 2$.

Furthermore, the result is consistent to that of 1a. When $\mu = 0.1$ and $\mu = 1$ ($\mu < 2$)the phase plane should look like an unstable focus which it does. The phase plane spirals outwards from the equilibrium which it should for an unstable focus. Similarly, for $\mu = 4$ near the equilibrium the solution looks analog to that of an unstable double tangential node which can be seen in course book, figure 13.3.

Problem 2, Equilibria and stability

a)

Here we have a DC motor:

$$\Theta(s) = \frac{1}{s(s+1)}U(s) \tag{15}$$

where u is the voltage and Θ is the angle of the motor axis. A gearbox is used to transform the rotational movement (Θ) to a linear movement $y = f(\Theta)$. Due to imperfections in the gearbox, f will represent a backlash with $D = H = 0.02$.

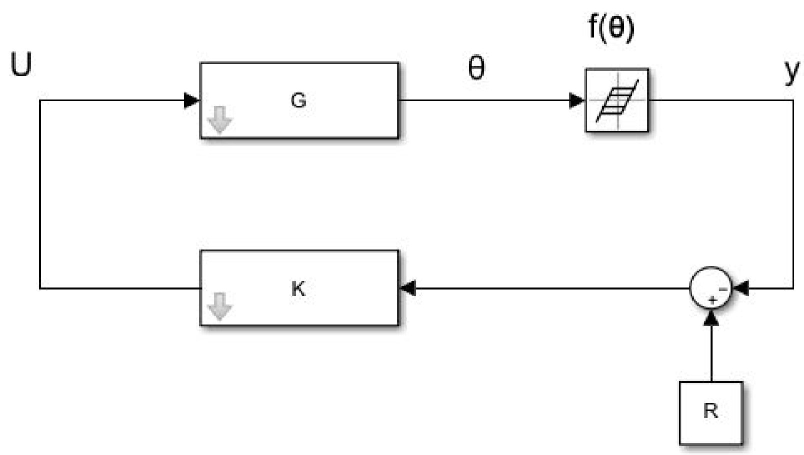

Figure 11: *Block diagram of the problem*

Figure 12 shows a block diagram that the describes our system. The associated describing function for f is:

$$ReY_f(C) = \frac{1}{\pi}\left[\frac{\pi}{2} + arcsin(1 - \frac{0.04}{C}) + 2(1 - \frac{0.04}{C})\sqrt{\frac{0.02}{C}(1 - \frac{0.02}{C})}\right] \tag{16}$$

$$ImY_f(C) = -\frac{0.08}{\pi C}(1 - \frac{0.02}{C}) \tag{17}$$

for $C \geq 0.02$.
A further assumption is that proportional control is used, $U(s) = K(R(s) - Y(s))$. To analyze this, the proportional K is merged with the transfer function $\frac{1}{s(s+1)}$ to create:

$$G(s) = \frac{K}{s(s+1)}. \tag{18}$$

The Nyquist diagram for the new transfer function $G(s)$ will be plotted together with $-\frac{1}{Y_f(s)}$.

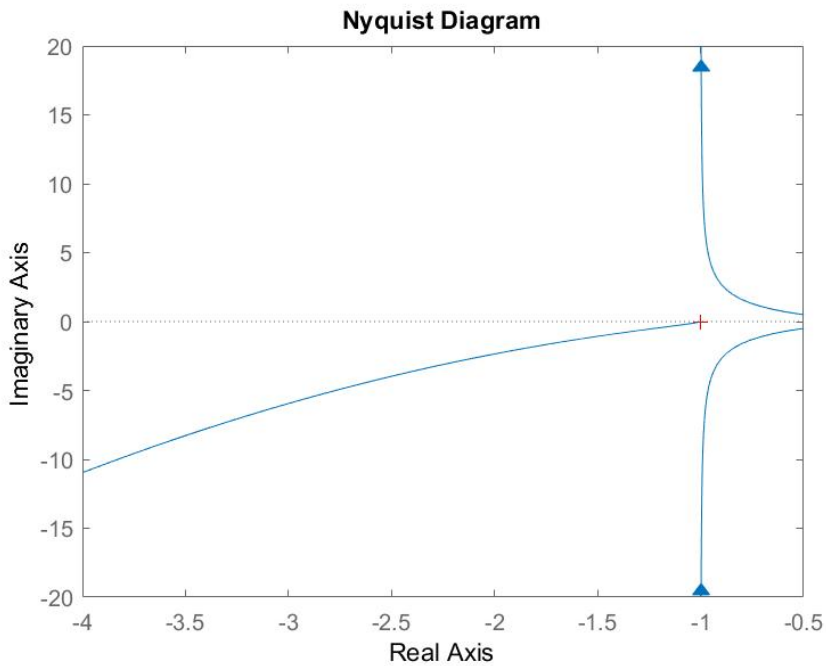

Figure 12: *Nyquist diagram for $G(s)$ where $K = 1$, together with $-\frac{1}{Y_f(s)}$ for* $0.02 < C < \infty$

Figure 12 shows that the describing function will not cross the Nyquist diagram for $G(s)$ meaning that a limit cycle is avoided.

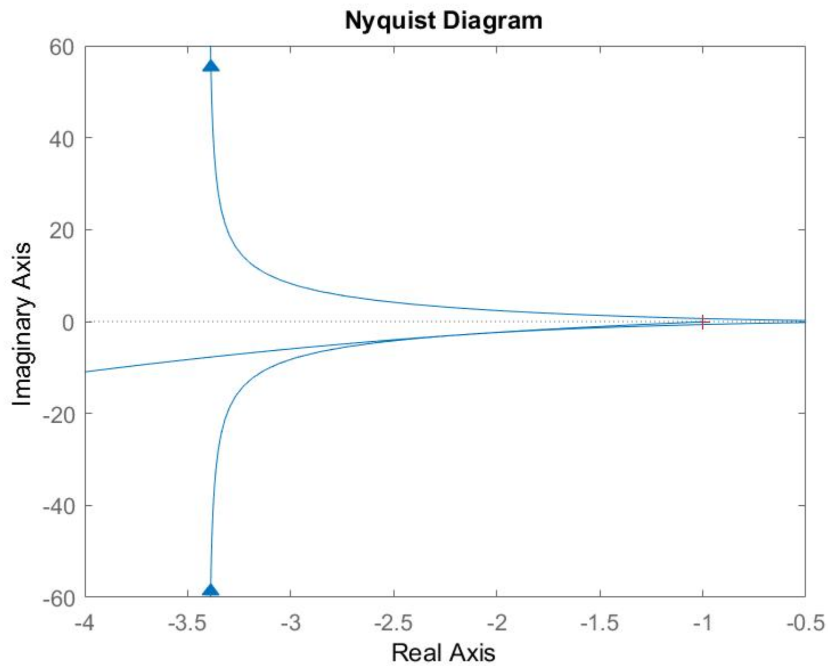

Figure 13: *Nyquist diagram for $G(s)$ where $K = 3.4$, together with $-\frac{1}{Y_f(s)}$ for* $0.02 < C < \infty$

Figure 13 shows that the describing function will (barely) not cross the Nyquist diagram for $G(s)$ meaning that a limit cycle is avoided.

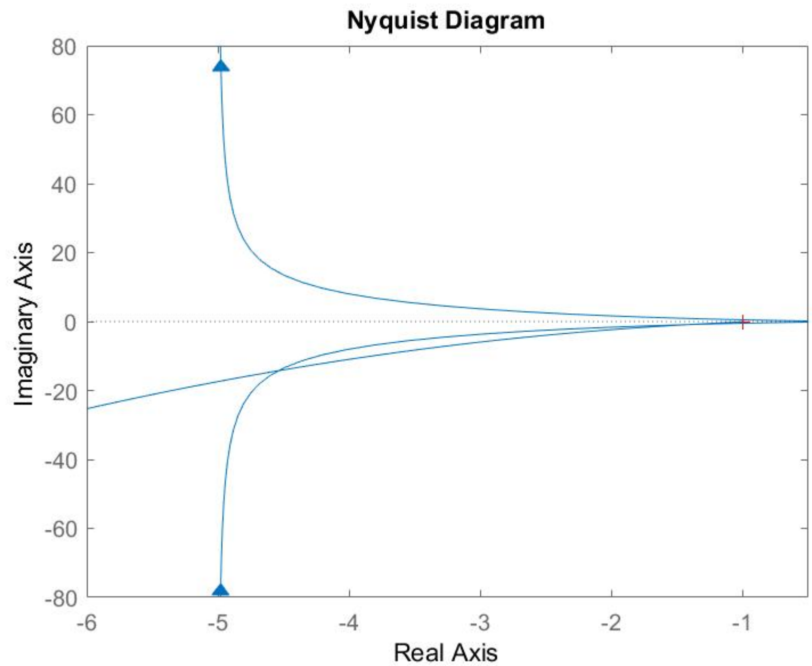

Figure 14: *Nyquist diagram for* $G(s)$ *where* $K = 5$*, together with* $-\frac{1}{Y_f(s)}$ *for* $0.02 < C < \infty$

Figure 14 shows that the describing function will cross the Nyquist diagram for $G(s)$ meaning that a limit cycle is created. The describing function will come from the left, cross the Nyquist curve, then cross the curve again, landing outside the Nyquist curve. This will result in a stable limit cycle.

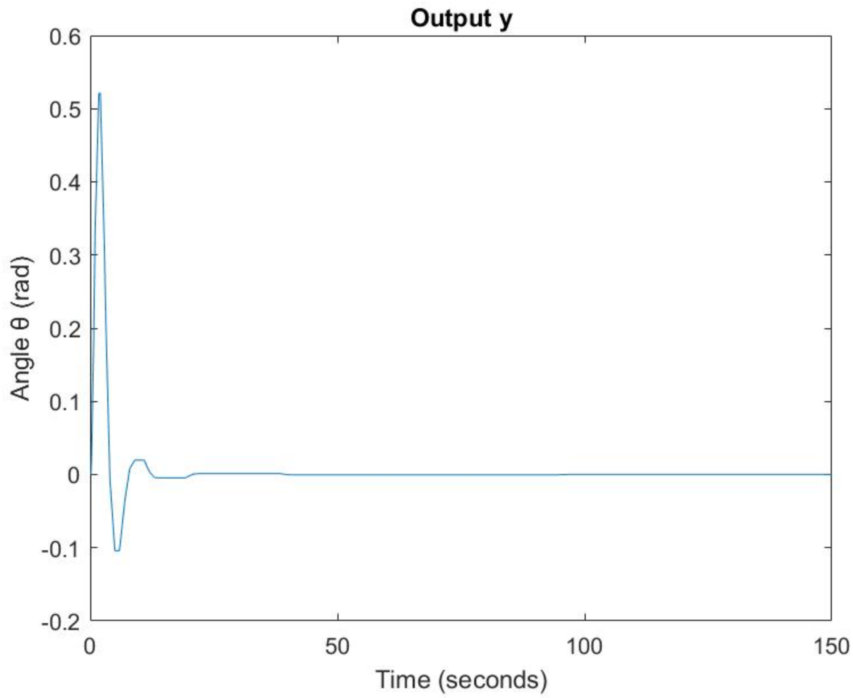

Figure 15: *The output y when K = 1*

Figure 15 shows that the output y does not result in a limit cycle when $K = 1$.

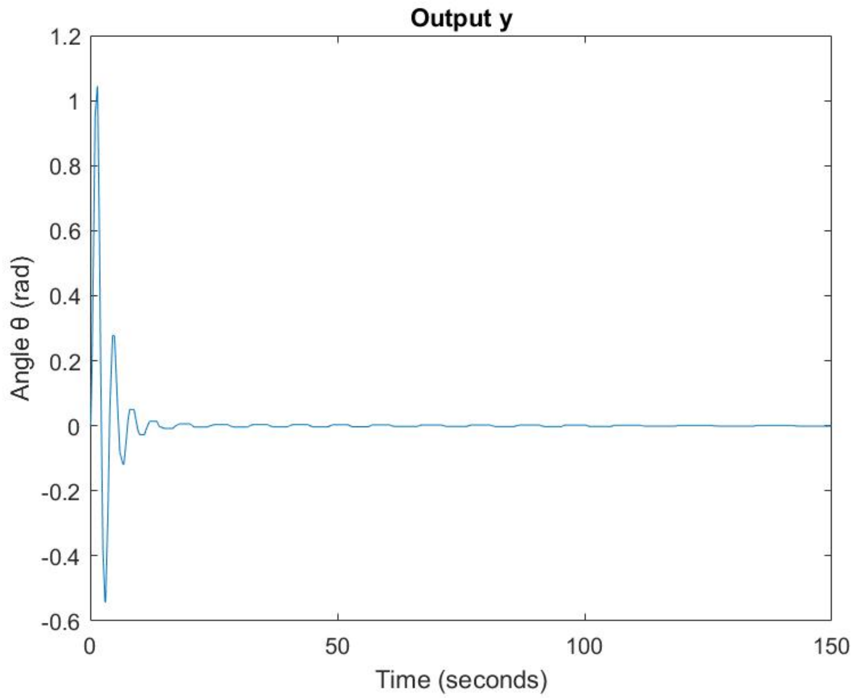

Figure 16: *The output y when $K = 3.4$*

Figure 16 shows that the output y does not result (barely) in a limit cycle when $K = 3.4$.

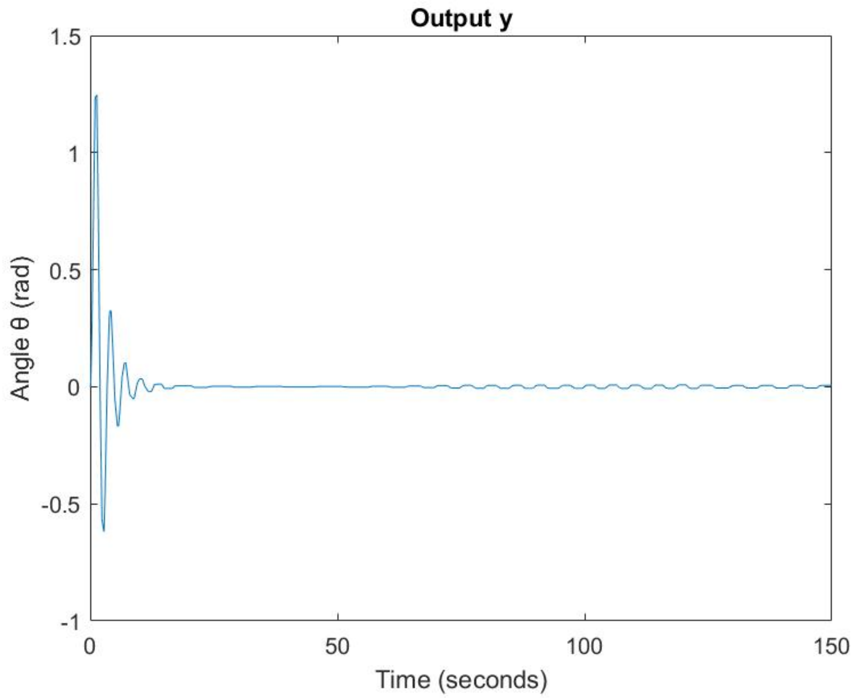

Figure 17: *The output y when $K = 5$*

Figure 15 shows that the output y results in a limit cycle when $K = 5$.

Problem 3, Optimal Control

$$\dot{x}(t) = x(t) + u(t) + 1 \tag{19}$$

where $x(t)$ is some state and u(t) is our control signal. We want to find the optimal control signal, which in turn is the control signal that minimizes

$$\int_0^T (x(t) + u^2(t))dt. \tag{20}$$

In order to find the optimal control signal we look at Equation 19 and Equation 20 and define the following two equations

$$L(x, u) = x(t) + u(t)^2 \tag{21}$$

and

$$f(x, u) = x(t) + u(t) + 1. \tag{22}$$

We go in with using theorem 18.2. The Hamilton function is defined as

$$H(x,u,\lambda) = L(x,u) + \lambda f(x,u) = x(t) + u(t)^2 + \lambda(x(t) + u(t) + 1) \quad (23)$$

and the optimal control signal u^* is found by minimizing Equation 23 with respect to u.

$$\frac{dH}{du} = 0 \rightarrow 2u^* + \lambda = 0 \rightarrow u^* = -\frac{\lambda}{2}, \quad (24)$$

u^* can now be used to decide λ and x(t). According to theorem 18.2 the time derivative of λ is equal to the x derivative of the Hamilton equation,

$$-\frac{dH}{dx} = -\lambda - 1 = \dot{\lambda} \quad (25)$$

the resulting first-order linear ordinary differential equation is

$$\dot{\lambda} = -\lambda - 1 \rightarrow \lambda = Ae^{-t} - 1. \quad (26)$$

Using the condition that $\lambda(T) = 0$ gives

$$A = e^T \quad (27)$$

where T is the final time. The expression for the optimal control signal becomes

$$u^* = -\frac{\lambda}{2} = -\frac{e^T e^{-t} - 1}{2}. \quad (28)$$

$x(t)$ can now be calculated with the use of Equation 19, Equation 28 and Equation 26,

$$\dot{x} = x + 1 - \frac{\lambda}{2} \rightarrow \dot{x} = x + 1 - \frac{e^T e^{-t} - 1}{2} \rightarrow x(t) = \frac{e^T e^{-t}}{4} + Be^t - \frac{3}{2}, \quad (29)$$

where B is the integration constant achieved by solving the differential equation. Using the condition that $x(0) = 0$ we get

$$B + \frac{e^T}{4} - \frac{3}{2} = 0 \rightarrow B = \frac{-e^T + 6}{4} \quad (30)$$

hence x(t) can be written as

$$x(t) = \frac{-e^T e^t + 6e^t + e^{T-t}}{4} - \frac{3}{2}. \quad (31)$$

We now have the expression for the optimal control signal and for the state $x(t)$.

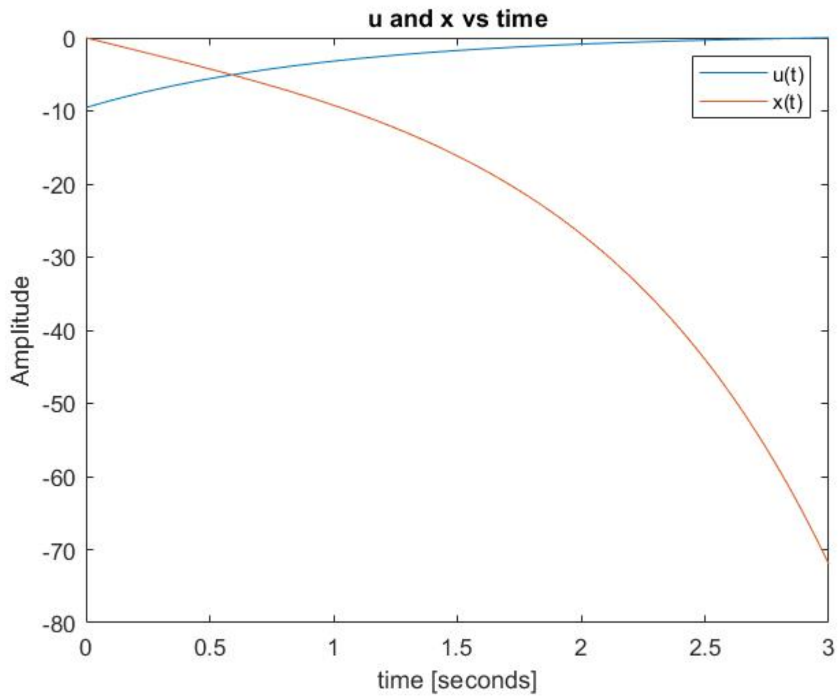

Figure 18: *Plot of how x and the optimal control input u^* varies over time.*

In Figure 18 we get an understanding about weather the criterion is minimized or not. Intuitively one would assume that in order to minimize the criterion the values of the two terms within the integral inEquation 20 should be as small as possible. The smallest values of those two terms are $u^2 = 0$ and $x = -\infty$ which is exactly what is shown in Figure 18.

Problem 4: Course overview

- Poles and zeros of MIMO systems determines at which frequencies the system is damped or amplified and stability, determined by looking at when the denominator and numerator of the minors transfer functions are zero. For both linear and non-linear systems.
- Internal stability of a closed loop system is a criterion of stability based

on the plant model G and the feedback controller F_y and is promised if all transfer functions from the process noise and input noise to input and output are stable. For linear systems, both MIMO and SISO.

- Performance limitations can be caused by the natural environment of the system, such as the amplitude or the rise time of the control signal, or by principle, such as an unstable or non-min phase system in which the sensitivity cannot become arbitrary good, or it could be caused by noise which limits the performance unless we have *a priori* information about its nature.

- Equilibrium points occurs when all the state variables of the system are constant, often used for non-linear systems where one wants to linearize around an equilibria.

- Phase planes are a way to visualize different characteristics in non-linear differential equations graphically, usually with two axis, one for each state (ODE), where one can see where different initial settings asymptotically goes. Works for linear systems.

- Lyapunov functions can be seen as a generalized distance measure to the equilibrium used when checking for stability in non-linear systems.

- In cases when non-linear systems not tend towards an equilibria, one can use the describing function (derived from the fourier expansion of the output of the static non-linearity) to determine whether there are a limit cycle or not and if the limit cycle is stable or not.

- The circle criterion is a non-linear extension of the linear Nyquist criterion, which says that the Nyquist curve of the open loop system should not intersect or be on the right side of a circle centered at the real axis with radius determined by two straight lines fully encircling the non-linearity of the system. This will guarantees stability if the criteria are fulfilled. If not, we can't guarantee stability.

- The Relative Gain Array (RGA) matrix is a measure of interaction or cross coupling between different inputs and outputs in a MIMO system. It holds for linear systems.

- Internal Model Control (IMC) is a technique used for synthesising robust, in terms of disturbances and model mismatch, controllers for linear and nonlinear systems, where the feedback is given as the difference between the (stable) model and the real system.

- If one cannot use one (or many) simple SISO controllers, and when one wants to find an optimal controller with respect to the sensitivity function S, the complementary sensitivity function T and the transfer function from noise to input $G_w u$ as an integral of their $\mathcal{H}_2$-norm summed with an extension of the system, one does $\mathcal{H}_2$ control.
- If one wants the above, but instead wants to set limits on S, T and G_{wu}, the feedback F_y gets determined by an extension of the system and the $\mathcal{H}_\infty$-norm of S, T, G_{wu}, then one does $\mathcal{H}_\infty$ control.

Matlab Code

```
%Homework 3
close all
clear all
%% exercise 1
if true
my=[0.1 1 4];
s=tf('s');
k1=10^-3;
k2=9.99*10^3;
r=(1/k1-1/k2)/2;
w=linspace(0,2*pi,10000);
unit_circle=(cos(w)+1i*sin(w))*r-(1/k2+r);

G1=(my(1)/3*s/(s^2-my(1)*s+1));
G2=(my(2)/3*s/(s^2-my(2)*s+1));
G3=(my(3)/3*s/(s^2-my(3)*s+1));

%% calling to ode45
t=[0 100];
x1_0=1;
x2_0=1;
my=1;
[t,p1]=ode45('phasepotrait1',t,x1_0,x2_0);
[t,p2]=ode45('phasepotrait2',t,x1_0,x2_0);
plot(t,p1)
hold on
plot(t,p2)
```

```
%% Plotting

% figure(1)
% nyquist(G1)
% hold on
% plot(unit_circle)
%
% figure(2)
% nyquist(G2)
% hold on
% plot(unit_circle)
%
% figure(3)
% nyquist(G3)
% hold on
% plot(unit_circle)
end
if true
close all
clear all
T=10;
nr=100;
t=linspace(0,T,nr);
u=(1-exp(T-t))/2;
x=0.25*exp(T-t)+0.25*(6*exp(t)-exp(T+t))-3/2;


plot(t,u)
hold on
plot(t,x)
legend('u(t)','x(t)')
title('u and x vs time')
xlabel('time [seconds]')
ylabel('Amplitude')












end
```

Phase plane portraits

```
% hw3 regler3
close all
clear all

mu = [0.1 1 4];
s = tf('s');
G = (mu(3)/3)*s/(s^2 - mu(3)*s+1);

w = 0:1/100:2*pi;
unitcircle = cos(w)+1j*sin(w);
%figure(1)
%nyquist(G);

x1 = linspace(-7,7,30);
x2 = linspace(-7,7,30);
x1dot = zeros(length(x2),length(x1));
x2dot = zeros(length(x2),length(x1));

%% mu = .1 ...
    ------------------------------------------------------------------
for i=1:length(x1)
    for j=1:length(x2)
        x1dot(j,i) = x2(j);
        x2dot(j,i) = mu(1)*(1-x1(i)^2)*x2(j)-x1(i);
    end
end

% normalizing vector
x1dot = x1dot./sqrt(x1dot.^2 + x2dot.^2);
x2dot = x2dot./sqrt(x1dot.^2 + x2dot.^2);
figure(2)
quiver(x1,x2,x1dot,x2dot);
hold on
% ode45 solution with mu=1, y0=[.1,.1]
tspan = [0 1000];
y0 = [.1;.1];
[t,y] = ode45(@sysODE_01,tspan,y0);
plot(y(:,1)',y(:,2),'r'), title({'Phase plane portrait of,the ...
    Van der Pol for mu=0.1'});
hold on
% ode45 solution with mu=1, y0=[4,4]
tspan = [0 1000];
y0 = [4;4];
[t,y] = ode45(@sysODE_01,tspan,y0);
plot(y(:,1)',y(:,2),'b');
```

```
%% mu = 1 ...
    -----------------------------------------------------------------
for i=1:length(x1)
    for j=1:length(x2)
        x1dot(j,i) = x2(j);
        x2dot(j,i) = mu(2)*(1-x1(i)^2)*x2(j)-x1(i);
    end
end

% normalizing vector
x1dot = x1dot./sqrt(x1dot.^2 + x2dot.^2);
x2dot = x2dot./sqrt(x1dot.^2 + x2dot.^2);
figure(3)
quiver(x1,x2,x1dot,x2dot);
hold on
% ode45 solution with mu=1, y0=[.1,.1]
tspan = [0 1000];
y0 = [.1;.1];
[t,y] = ode45(@sysODE_1,tspan,y0);
plot(y(:,1)',y(:,2),'r'), title({'Phase plane portrait of,the ...
    Van der Pol for mu=1'});
hold on
% ode45 solution with mu=1, y0=[4,4]
tspan = [0 1000];
y0 = [4;4];
[t,y] = ode45(@sysODE_1,tspan,y0);
plot(y(:,1)',y(:,2),'b');

%% mu = 4 ...
    -----------------------------------------------------------------
for i=1:length(x1)
    for j=1:length(x2)
        x1dot(j,i) = x2(j);
        x2dot(j,i) = mu(3)*(1-x1(i)^2)*x2(j)-x1(i);
    end
end

% normalizing vector
x1dot = x1dot./sqrt(x1dot.^2 + x2dot.^2);
x2dot = x2dot./sqrt(x1dot.^2 + x2dot.^2);
figure(4)
quiver(x1,x2,x1dot,x2dot);
hold on
% ode45 solution with mu=4, y0=[.1,.1]
tspan = [0 1000];
y0 = [.1;.1];
[t,y] = ode45(@sysODE_4,tspan,y0);
plot(y(:,1)',y(:,2),'r'), title({'Phase plane portrait of,the ...
```

```
    Van der Pol for mu=4'});
hold on
% ode45 solution with mu=4, y0=[4,4]
tspan = [0 1000];
y0 = [4;4];
[t,y] = ode45(@sysODE_4,tspan,y0);
plot(y(:,1)',y(:,2),'b');
%--------------------------------------------------------------------------


function y_out = sysODE_01(t,y)
    mu = .1;
    y_out = [y(2);mu*(1-y(1)^2)*y(2)-y(1)];
end
function y_out = sysODE_1(t,y)
    mu = 1;
    y_out = [y(2);mu*(1-y(1)^2)*y(2)-y(1)];
end
function y_out = sysODE_4(t,y)
    mu = 4;
    y_out = [y(2);mu*(1-y(1)^2)*y(2)-y(1)];
end
```