Uppsala University
Department of Information Technology
Division of Systems and Control
September 3, 2019

Automatic control III

Homework assignment 3
2019

Deadline (for this assignment):
Wednesday October 16, 23:59

All homework assignments are compulsory and form an important part of the examination.

The assignments are to be solved in groups of up to 4 students (sign up on studentportalen). All group members should understand and be able to explain the entire solution of each problem. There will be an oral examination (October 22), based on all three assignments. Each group member will be given approximately 5 minutes to present the solution of one (by the examiner) arbitrarily chosen problem from one of the three assignments.

Instructions on how to hand in the assignments can be found on the course homepage. The solution should be handed in as one pdf. It should include a clearly presented and well motivated answer to all questions, with satisfactory equation typesetting, equation numbering, complete sentences, etc. All figures should have correct labels, a relevant figure text and be referenced in the text. If you use MATLAB (or some other language) for plotting or computations, you must *include your code* with your solution. However, a full report is not required, so no introduction, abstract, etc is needed, just solutions to the problems.

Before handing in your solutions, make sure (for each exercise) that

(I) You have answered all questions

(II) Your answers are reasonable

Solutions not fulfilling (I) and (II) will be rejected.

Problem I   Analysis of nonlinear feedback systems

The Van der Pol oscillator[1] is a well-studied and widely used example of a second order system with a limit cycle. The system is governed by the differential equation

$$\frac{d^2y}{dt^2} - \mu(1-y^2)\frac{dy}{dt} + y = 0, \tag{1}$$

where $\mu > 0$. Introduce the state variables $x_1 = y$ and $x_2 = \dot{y}$. The system (1) is then equivalent with the state space representation

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \mu(1-x_1^2)x_2 - x_1. \end{aligned}$$

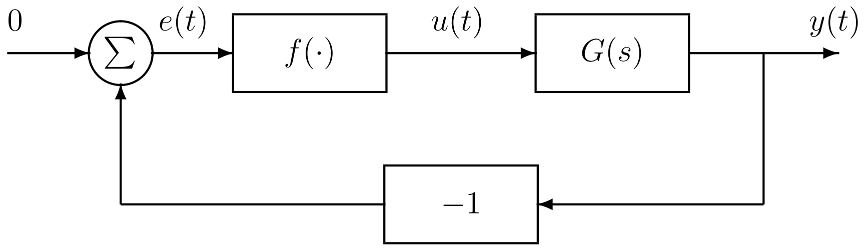
(a) Perform a phase plane analysis of the Van der Pol oscillator, i.e. find all stationary points and determine their type (focus, saddle etc) for all $\mu > 0$.

(b) Let $u = -y^3$. Show that the system

$$\frac{d^2y}{dt^2} - \mu\frac{dy}{dt} + y = \frac{\mu}{3} \cdot \frac{du}{dt} \tag{2}$$

is equivalent with (1) for this particular choice of u.

(c) The system (2), with $u = -y^3$, can be represented as the feedback system in the block diagram below.



What is $G(s)$ and $f(\cdot)$ in this particular case?

(d) Determine the sector and the circle in the circle criterion corresponding to this particular $f(\cdot)$. Also show that the circle criterion is not fulfilled in this case.

(e) Show that the describing function method indicates a limit cycle for the Van der Pol oscillator. Determine the amplitude C and the frequency ω indicated by the describing function for the cases $\mu = 0.1$, $\mu = 1.0$ and $\mu = 4.0$ respectively.

(f) Compare the theoretical values on ω and C obtained in (e) with simulations in Simulink. Comment on and try to explain any differences between your theoretical values and those obtained from simulations. When simulating, use a dirac impulse as reference signal and make sure you simulate for a long enough time.

(g) Use MATLAB to plot the phase portrait for $\mu = 0.1$, $\mu = 1.0$ and $\mu = 4.0$ respectively. A suitable range is $x_1, x_2 \in [-5, 5]$. Make sure to include both *arrows* indicating the direction of the tracks across the whole plane as well as some *example solutions* for different initial values (both close to and far out from the equilibrium point). (Hint: The functions **quiver** and **ode45** may be useful. Normalize your input to quiver so that all arrows are of similar size.)

Does the solution close to the equilibrium point behave as you expect? Compare with your result in (a). What happens further out from the equilibrium? Try to relate this behaviour to your results and conclusions in (f).

[1] van der Pol, B. (1920): "A theory of the amplitude of free and forced triode vibrations", in *Radio Review*, 1: 701–710.

Problem II Feedback design for nonlinear systems

A DC motor

$$\Theta(s) = \frac{1}{s(s+1)} U(s)$$

is used as an actuator in a position servo. The input u is the voltage over the motor, and θ is the angle of the motor axis. A gear box is used to transform the rotational motion to linear motion. Due to an imperfection in the manufacture there is a backlash in the gear box.

Thus the linear position is $y = f(\theta)$, where f represents a backlash with $H = D = 0.02$, and its associated describing function (for $C \geq 0.02$) is

$$\mathrm{Re}\, Y_f(C) = \frac{1}{\pi}\left[\frac{\pi}{2} + \arcsin\left(1 - \frac{0.04}{C}\right) + 2\left(1 - \frac{0.04}{C}\right)\sqrt{\frac{0.02}{C}\left(1 - \frac{0.02}{C}\right)}\right],$$

$$\mathrm{Im}\, Y_f(C) = -\frac{0.08}{\pi C}\left(1 - \frac{0.02}{C}\right).$$

Assume that proportional control is used, i.e. $U(s) = K(R(s) - Y(s))$.

(a) Present a block diagram describing the system.

(b) How large values of the gain K (give at least one decimal) can be used if a limit cycle is to be avoided according to the describing function method?

(c) Compare your result in (b) with simulations in Simulink (use a dirac impulse as reference signal). If the results do not agree, try to explain why. Include at least one example of a K which gives a limit cycle and one that doesn't to support your claims.

Problem III Optimal control

Design a feedback control $u(t)$ for the system

$$\dot{x}(t) = x(t) + u(t) + 1 \tag{3}$$

with $x(0) = 0$, minimizing the criterion

$$\int_0^T \left(x(t) + u^2(t)\right) dt \tag{4}$$

for a given value of $T > 2$. Also plot your $u(t)$ and the evolution of $x(t)$ for this input. Using the plots, give an intuitive explanation of why this is the solution to the given problem.

Problem IV Course overview

For all concepts included in the course,

- poles and zeros of MIMO systems
- internal stability
- performance limitations
- equilibrium points
- phase planes
- Lyapunov functions
- describing functions
- the circle criterion
- RGA
- IMC
- $\mathcal{H}_2$
- $\mathcal{H}_\infty$

write one sentence summarizing the problem addressed by that concept, as the following example:

> *"Lead/lag-design is a method for designing feedback control for linear time-invariant SISO systems."*

Clearly state what the **purpose** of the concept is (analysis or control design), and **what kind of system** (linear/nonlinear, SISO/MIMO, etc.) it is relevant for.