Welcome to Automatic Control III

Lecture 1 – Introduction and linear systems theory

UPPSALA
UNIVERSITET

Alexander Medvedev
Division of Systems and Control
Department of Information Technology
Uppsala University.

Email: alexander.medvedev@it.uu.se

Original slides by Thomas Schön

What is automatic control all about?

"Automatic control is the art of getting things to behave as you want."

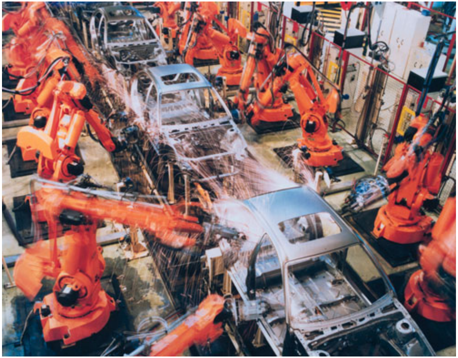
Example – automotive

Almost all abbreviations in the car sales brochures hide control systems.

- ABS (Anti-lock Braking System, controlling the brake force)
- ESC (Electronic Stability Control, controlling course stability)
- ACE (Active Cornering Enhancement, controlling shock absorbers in curves)
- TCS (Traction Control System, controlling road grip)
- ACC (Adaptive Cruise Control, controlling speed/distance)
- ANC (Active Noise Control, controlling (suppressing) sound)
- ...

Example – industrial robots

A robot arm is relatively weak and oscillates strongly when moving.

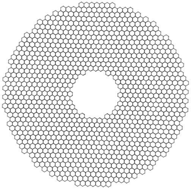


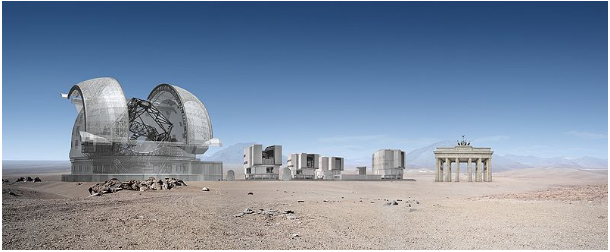
Control is necessary to achieve **speed**, **accuracy**, and **safety**.

Example – extremely large telescopes

We have reached the **technological limits** of how **large mirrors** can be made.

Large telescopes are built with lots of **small mirrors** which are then **continuously controlled** so that the image is in focus (adaptive optics).



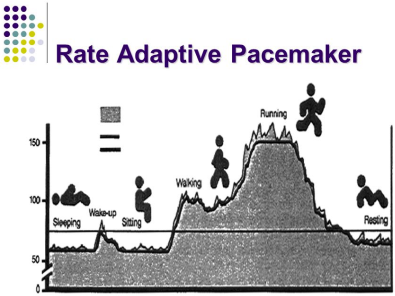


Example – mobile phones

- Controlling the signal strength between the mobile phone and the base station
- Traffic control

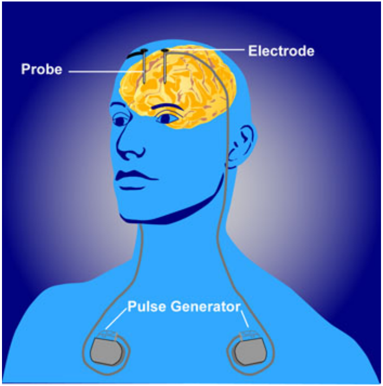
Example – medical technology

- **adaptive pacemaker**
- deep brain stimulation
- artificial pancreas
- closed-loop anaesthesia



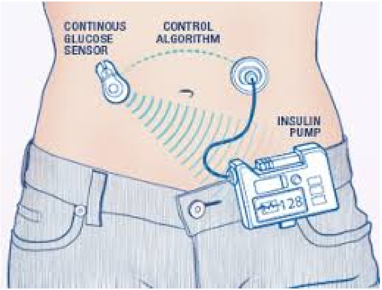
Example – medical technology

- adaptive pacemaker
- **deep brain stimulation**
- artificial pancreas
- closed-loop anaesthesia



Example – medical technology

- adaptive pacemaker
- deep brain stimulation
- **artificial pancreas**
- closed-loop anaesthesia



UPPSALA UNIVERSITET

Example – medical technology

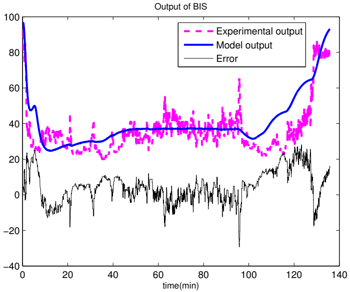
- adaptive pacemaker
- deep brain stimulation
- artificial pancreas
- **closed-loop anaesthesia bispectral index sensor (BIS)**

Example – medical technology

- adaptive pacemaker
- deep brain stimulation
- artificial pancreas
- **closed-loop anaesthesia bispectral index sensor (BIS)**

Example – medical technology

- adaptive pacemaker
- deep brain stimulation
- artificial pancreas
- **closed-loop anaesthesia bispectral index sensor (BIS)**



Example – Aerospace

- Stabilization
- Cruise control
- Altitude control
- Navigation
- Weapon systems
- Quadrotors (www.youtube.com/watch?v=3CR5y8qZf0Y)

Used almost everywhere

1. Improves already existing technical systems and enables the creation of new ones
2. Central area for many high-tech companies
3. Rewarding area filled with many fun applications.
4. Mathematics intensive

To summarize it is fair to say that Automatic Control is used almost everywhere, but it is often hidden.

Automatic control is sometimes refereed to as the **"hidden technology"**.

Used almost everywhere

1. Improves already existing technical systems and enables the creation of new ones
2. Central area for many high-tech companies
3. Rewarding area filled with many fun applications.
4. Mathematics intensive

To summarize it is fair to say that Automatic Control is used almost everywhere, but it is often hidden.

Automatic control is sometimes refereed to as the **"hidden technology"**.

Aim of the course

What is new in this course?

1. More on multivariable systems
2. Systematic design methods
3. Fundamental bounds on control performance
4. Nonlinear dynamics
5. Optimal control

Course structure and practicalities (I/IV)

- People:
  - Lecturer and examiner: Alexander Medvedev, `www.it.uu.se/katalog/almed173`
  - Problem solving sessions: Anna Wigren, `http://www.it.uu.se/katalog/annwi999`
  - Homework assignments: Anna Wigren `http://www.it.uu.se/katalog/annwi999`
- 10 lectures (theory and examples)
- 8 problem solving sessions (solve problems, **discuss and ask questions**)
- 1 computer lab (2h)
- 3 homework assignments
- **Feel free to ask questions!**

Course structure and practicalities (II/IV)

Date	Time	Event	Instructor
3/09	10:15	F1	AM
4/09	13:15	F2	AM
9/09	15:15	Lab	AW
10/09	10:15	F3	AM
12/09	10:15	L1	AW
12/09	13:15	F4	AM
16/09	13:15	F5	AM
18/09	8:15	L2	AW
19/09	13:15	F6	AM
20/09	15:15	L3	AW
23/09	15:15	F7	AM
25/09	13:15	F8	AM
26/09	13:15	L4	AW
30/09	10:15	L5	AW
01/10	10:15	F9	AM
01/10	23:59	HW 1, deadline	AW
02/10	10:15	L6	AW
07/10	13:15	F10	AM
07/10	23:59	HW 2, deadline	AW
09/10	13:15	L7	AW
14/10	8:15	L8	AW
16/10		exam sign-up deadline	
16/10	23:59	HW 3, deadline	AW
22/10	TBD	oral exam	AM+AW
30/10	8-13	written exam	AM+AW

Course structure and practicalities (III/IV)

Examination (passing the course (grade 3)):

- Three compulsory homework assignments:
  - Homeworks 1-3 are solved in groups of up to four students. Each group hands in one solution in the form of a written report.
- An individual oral exam must be passed. The oral exam consists of presenting the solution to one work assignment in 5 min. The assignment to present is selected arbitrarily by the instructors.

Examination (for grades 4 and 5):

- Besides fulfilling the requirements for grade 3 you have to take an additional written exam (you can bring the textbook and your hand-written notes with you).

Course structure and practicalities (IV/IV)

Problem solving sessions

- 8 sessions
- Solve problems, **discuss and ask questions**
- Exercise manual available as wiki at **http://regtek3.it.uu.se**.
- The exercise manual is also available as pdf in the student portal

Outline – entire course

1. **Lecture 1:** Introduction and linear multivariable systems
2. **Lecture 2–5:** Multivariable linear control theory
   a) systems theory, closed-loop system
   b) Basic limitations
   c) Controller structures and control design
   d) $\mathcal{H}_2$ and $\mathcal{H}_\infty$ loop shaping
3. **Lecture 6–8:** Nonlinear control theory
   a) Linearization and phase portraits
   b) Lyapunov theory and the circle criterion
   c) Describing functions
4. **Lecture 9:** Optimal control
5. **Lecture 10:** Summary and repetition

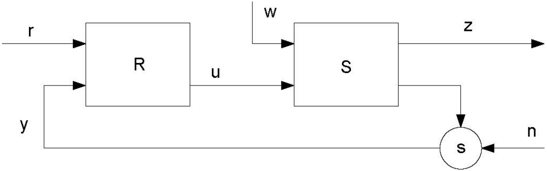
UPPSALA UNIVERSITET

Outline lecture 1

1. Introduction
2. What is automatic control?
3. Course administration
4. Systems theory – linear multivariable systems
   a) Signal size
   b) Gain
   d) The frequency function and singular values
   e) Stability and the small gain theorem

The control problem

z: Controlled signal
r: Reference signal
w: Process disturbance
n: Measurement error
u: Control signal
y: Measurement signal



R - controller, S - controlled system, s - sensor

We want $r - z$ to be small (i.e. z should follow r nicely) despite w, n and model uncertainties. At the same time we want u to take on reasonable values.

- SISO - single-input single-output system
- MIMO - multi-input multi-output system

Multivariable vs. scalar

The following concepts generalize straightforwardly from SISO to multivariable systems:

- The weight function is now a matrix
- The transfer function is now a matrix
- Stability
- State space formulation and the solution of the state equation
- Controllability and observability

The following requires some thought:

- Gain and plotting the gain
- Poles and zeros
- Turning $G(s)$ into a state space description

Multivariable vs. scalar

The following concepts generalize straightforwardly from SISO to multivariable systems:

- The weight function is now a matrix
- The transfer function is now a matrix
- Stability
- State space formulation and the solution of the state equation
- Controllability and observability

The following requires some thought:

- Gain and plotting the gain
- Poles and zeros
- Turning $G(s)$ into a state space description

Gain

For a linear transformation $y = Ax$, the operator norm ("the gain") is **defined** as how much larger the norm of y can (maximally) become compared to the norm of x:

$$|A| = \sup_{x \neq 0} \frac{|y|}{|x|} = \sup_{x \neq 0} \frac{|Ax|}{|x|}$$

You can think of $|A|$ as the "gain" of the transformation A.

For a dynamical system $\mathcal{S}$ ($y = \mathcal{S}(u)$) we can similarly **define** the gain of the system as

$$\|\mathcal{S}\| = \sup_{u} \frac{\|y\|_2}{\|u\|_2} = \sup_{u} \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

"Choose u such that the quotient between the size of the output signal and the input signal is maximized".

Gain

For a linear transformation $y = Ax$, the operator norm ("the gain") is **defined** as how much larger the norm of y can (maximally) become compared to the norm of x:

$$|A| = \sup_{x \neq 0} \frac{|y|}{|x|} = \sup_{x \neq 0} \frac{|Ax|}{|x|}$$

You can think of $|A|$ as the "gain" of the transformation A.

For a dynamical system $\mathcal{S}$ ($y = \mathcal{S}(u)$) we can similarly **define** the gain of the system as

$$\|\mathcal{S}\| = \sup_{u} \frac{\|y\|_2}{\|u\|_2} = \sup_{u} \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

"Choose u such that the quotient between the size of the output signal and the input signal is maximized".

Gain

For a linear transformation $y = Ax$, the operator norm ("the gain") is **defined** as how much larger the norm of y can (maximally) become compared to the norm of x:

$$|A| = \sup_{x \neq 0} \frac{|y|}{|x|} = \sup_{x \neq 0} \frac{|Ax|}{|x|}$$

You can think of $|A|$ as the "gain" of the transformation A.

For a dynamical system $\mathcal{S}$ ($y = \mathcal{S}(u)$) we can similarly **define** the gain of the system as

$$\|\mathcal{S}\| = \sup_{u} \frac{\|y\|_2}{\|u\|_2} = \sup_{u} \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

"Choose u such that the quotient between the size of the output signal and the input signal is maximized".

Gain

For a linear transformation $y = Ax$, the operator norm ("the gain") is **defined** as how much larger the norm of y can (maximally) become compared to the norm of x:

$$|A| = \sup_{x \neq 0} \frac{|y|}{|x|} = \sup_{x \neq 0} \frac{|Ax|}{|x|}$$

You can think of $|A|$ as the "gain" of the transformation A.

For a dynamical system $\mathcal{S}$ ($y = \mathcal{S}(u)$) we can similarly **define** the gain of the system as

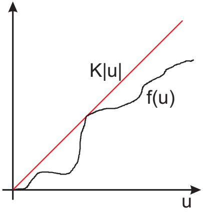
$$\|\mathcal{S}\| = \sup_{u} \frac{\|y\|_2}{\|u\|_2} = \sup_{u} \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

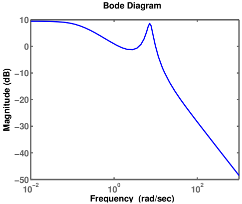
"Choose u such that the quotient between the size of the output signal and the input signal is maximized".

Computing the gain

As we have seen it is fairly easy to compute the gain for:

1. Static nonlinearity
   - The gain varies with the amplitude.
   - The gain does not vary with the frequency.
2. Linear dynamical systems (scalar)
   - The gain does not vary with the amplitude.
   - The gain varies with the frequency.





How big is $G(i\omega)$ if $G(i\omega)$ is a matrix?

With m input and p output signals, defining the gain of a system becomes more complicated.

$$G(i\omega) = \begin{bmatrix} G_{11}(i\omega) & G_{12}(i\omega) & \dots & G_{1m}(i\omega) \\ \vdots & \vdots & & \vdots \\ G_{p1}(i\omega) & G_{p2}(i\omega) & \dots & G_{pm}(i\omega) \end{bmatrix}$$

How to define the gain of a matrix?

Computing singular values – SVD

The singular values are computed using the **singular value decomposition (SVD)**, which states that every matrix $A \in \mathrm{R}^{n\times m}$ can be written as

$$A = U\Sigma V^*,$$

where $U \in \mathrm{R}^{n\times n}$ and $V \in \mathrm{R}^{m\times m}$ are unitary ($UU^* = I$) matrices and $\Sigma \in \mathrm{R}^{n\times m}$ with the singular values on the diagonal and zeros elsewhere,

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_n \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

In MATLAB: `[U,Sigma,V] = svd(A)`.

Connection, singular values and eigenvalues

The SVD (applicable to any $n \times m$ matrix) is more general than the eigenvalue (applicable to certain square matrices) decomposition, but the two are connected.

Let $A = U\Sigma V^*$. Hence,

$$AA^* = U\Sigma \underbrace{V^*V}_{=I} \Sigma^* U^* = U \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 & \mathbf{0} \\ 0 & \sigma_2^2 & \cdots & 0 & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_n^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} U^*,$$

which is the eigenvalue decomposition of AA^*. Hence, σ_i^2 are the non-zero eigenvalues of AA^*.

How big is $G(i\omega)$ if $G(i\omega)$ is a matrix? (revisited)

We can now answer this question.

- $|G(i\omega)|$ = the largest singular value of $G(i\omega)$
- $|Y(i\omega)| \leq |G(i\omega)||U(i\omega)|$
- $\|G\|_\infty$ = the largest singular value of $G(i\omega)$ for any ω.
- $\|Y\|_2 \leq \|G\|_\infty \|U\|_2$

Plotting the singular values of $G(i\omega)$ as a function of ω corresponds to the plot of the amplitude curve for SISO systems.

For multivariable systems the actual gain depends on the **direction** of the input vector $U(i\omega)$.

How big is $G(i\omega)$ if $G(i\omega)$ is a matrix? (revisited)

We can now answer this question.

- $|G(i\omega)| =$ the largest singular value of $G(i\omega)$
- $|Y(i\omega)| \leq |G(i\omega)||U(i\omega)|$
- $\|G\|_\infty =$ the largest singular value of $G(i\omega)$ for any ω.
- $\|Y\|_2 \leq \|G\|_\infty \|U\|_2$

Plotting the singular values of $G(i\omega)$ as a function of ω corresponds to the plot of the amplitude curve for SISO systems.

For multivariable systems the actual gain depends on the **direction** of the input vector $U(i\omega)$.

How big is $G(i\omega)$ if $G(i\omega)$ is a matrix? (revisited)

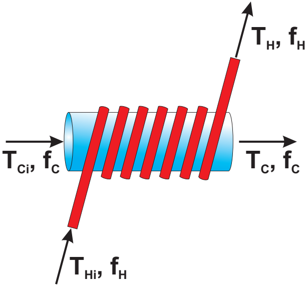
We can now answer this question.

- $|G(i\omega)| =$ the largest singular value of $G(i\omega)$
- $|Y(i\omega)| \leq |G(i\omega)||U(i\omega)|$
- $\|G\|_\infty =$ the largest singular value of $G(i\omega)$ for any ω.
- $\|Y\|_2 \leq \|G\|_\infty \|U\|_2$

Plotting the singular values of $G(i\omega)$ as a function of ω corresponds to the plot of the amplitude curve for SISO systems.

For multivariable systems the actual gain depends on the **direction** of the input vector $U(i\omega)$.

Example – a heat exchanger (I/III)



f_C, f_H flows,
V_C, V_H volumes,
T_C, T_H temperatures.

Model

$$V_C \frac{dT_C}{dt} = f_C(T_{C_i} - T_C) + \beta(T_H - T_C),$$
$$V_H \frac{dT_H}{dt} = f_H(T_{H_i} - T_H) - \beta(T_H - T_C).$$

Example – a heat exchanger (II/III)

Assume (for simplicity): $f_{\mathsf{C}} = f_{\mathsf{H}} = f$ is constant.

Inputs: $u_1 = T_{\mathsf{C}_i}$ and $u_2 = T_{\mathsf{H}_i}$.

States: $x_1 = T_{\mathsf{C}}$ and $x_2 = T_{\mathsf{H}}$.

Using the following numerical values $f = 0.01$ (m^3/min), $\beta = 0.2$ (m^3/min) and $V_{\mathsf{H}} = V_{\mathsf{C}} = 1$ (m^3), results in

$$\dot{x} = \begin{pmatrix} -0.21 & 0.2 \\ 0.2 & -0.21 \end{pmatrix} x + \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix} u,$$

$$y = x.$$

UPPSALA UNIVERSITET

Example – a heat exchanger (II/III)

Assume (for simplicity): $f_{\mathsf{C}} = f_{\mathsf{H}} = f$ is constant.

Inputs: $u_1 = T_{\mathsf{C}_i}$ and $u_2 = T_{\mathsf{H}_i}$.

States: $x_1 = T_{\mathsf{C}}$ and $x_2 = T_{\mathsf{H}}$.

Using the following numerical values $f = 0.01$ (m^3/min), $\beta = 0.2$ (m^3/min) and $V_{\mathsf{H}} = V_{\mathsf{C}} = 1$ (m^3), results in

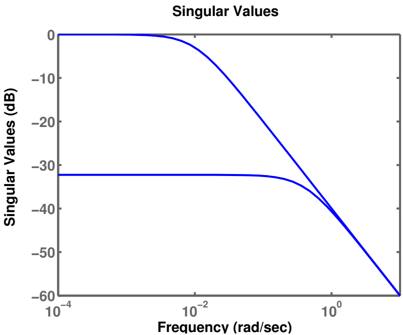
$$\dot{x} = \begin{pmatrix} -0.21 & 0.2 \\ 0.2 & -0.21 \end{pmatrix} x + \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix} u,$$
$$y = x.$$

Example – a heat exchanger (III/III)

$$G(s) = \frac{0.01}{(s+0.01)(s+0.41)} \begin{pmatrix} s+0.21 & 0.2 \\ 0.2 & s+0.21 \end{pmatrix}$$

Plot the singular values in MATLAB:

```
A = [-0.21 0.2;0.2 -0.21];
B = 0.01*eye(2); C = eye(2);
D=zeros(2,2);
sigma(A,B,C,D)
```



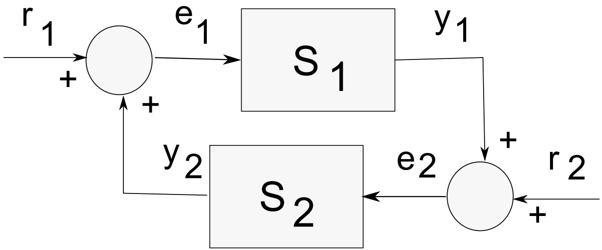
Stability

- A system is **input-output stable** if its gain is finite.
- A solution to a differential equation is stable if a small perturbation of the initial condition gives a small, possibly vanishing, effect.
- For linear time-invariant systems, stability is a system property, all solutions have the same stability properties.
- The stability theory for multivariable linear time-invariant systems is the same as in the basic course.
- In this course we will also analyze stability for nonlinear systems, namely Lyapunov theory (Chapter 12).

Stability – the small gain theorem

Two stable systems $\mathcal{S}_1$ and $\mathcal{S}_2$ which are connected according to the figure below result in a closed-loop system that is stable if

$$\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1.$$



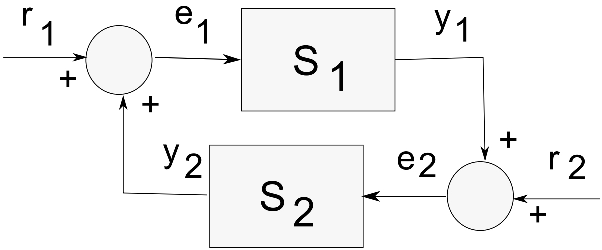
Note that the small gain theorem is valid both for linear and nonlinear systems. For linear systems the criterion is simplified to

$$\|\mathcal{S}_2\mathcal{S}_1\| < 1.$$

Stability – the small gain theorem

Two stable systems $\mathcal{S}_1$ and $\mathcal{S}_2$ which are connected according to the figure below result in a closed-loop system that is stable if

$$\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1.$$



Note that the small gain theorem is valid both for linear and nonlinear systems. For linear systems the criterion is simplified to

$$\|\mathcal{S}_2\mathcal{S}_1\| < 1.$$

Useful MATLAB commands

`ss2zp`, `zp2ss`, `tf2zp`, `zp` Transformations between different representations (state space (`ss`), transfer functions (`tf`), zeros and poles (`zp`))

`tzero` Calculation of zeros (for multivariable systems)

`lsim`, `step`, `impulse` Simulation, step and impulse responses

`pole`, `eig`, `roots` Eigenvalues and poles

`bode` Frequency function and Bode plot

`sigma` Computes the singular values of the frequency function

`obsv`, `ctrb` Observability and controllability

UPPSALA UNIVERSITET

Control engineering joke

You might be a control engineer if

- You can work miracles in Matlab, but haven't a clue how to use Excel.

UPPSALA UNIVERSITET

A few concepts to summarize lecture 1

Automatic control: "the art of getting things to behave as you want."

Singular value: The singular values of a matrix A are given by $\sigma_i = \sqrt{\lambda_i}$, where λ_i denotes the eigenvalues of A^*A.

Singular values of the frequency function: Plotting the singular values of $G(i\omega)$ (for a multivariable system) as a function of ω corresponds to the plot of the amplitude curve for SISO systems.

Input-output stability: A systems is input-output stable if its gain is finite.

Small gain theorem: Let the two systems S_1 and S_2 be connected in a feedback loop, then the closed-loop system is input-output stable if $\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1$.