Uppsala University
Department of Information Technology
Division of Systems and Control
September 3, 2019

Automatic control III

Homework assignment 1
2019

Deadline (for this assignment):
Tuesday October 1, 23:59

All homework assignments are compulsory and form an important part of the examination.

The assignments are to be solved in groups of up to 4 students (sign up on studentportalen). All group members should understand and be able to explain the entire solution of each problem. There will be an oral examination (October 22), based on all three assignments. Each group member will be given approximately 5 minutes to present the solution of one (by the examiner) arbitrarily chosen problem from one of the three assignments.

Assignment 2 and **assignment 3** (not in this document) are to be solved in the same groups.

Instructions on how to hand in the assignments can be found on the course homepage. The solution should be handed in as one pdf. It should include a clearly presented and well motivated answer to all questions, with satisfactory equation typesetting, equation numbering, complete sentences, etc. All figures should have correct labels, a relevant figure text and be referenced in the text. If you use MATLAB (or some other language) for plotting or computations, you must *include your code* with your solution. However, a full report is not required, so no introduction, abstract, etc is needed, just solutions to the problems.

Before handing in your solutions, make sure (for each exercise) that

(I) You have answered all questions

(II) Your answers are reasonable

Solutions not fulfilling (I) and (II) will be rejected.

Problem I Distillation column

An approximate model of a binary distillation column[1] is

$$Y(s) = G(s)U(s), \tag{1}$$

$$\text{with} \quad Y(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}, \quad U(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \end{bmatrix}, \quad \text{and} \tag{2}$$

$$G(s) = \begin{bmatrix} \dfrac{13}{17s+1} & \dfrac{-19}{21s+1} & \dfrac{4}{15s+1} \\ \\ \dfrac{7}{12s+1} & \dfrac{-19}{15s+1} & \dfrac{5}{12s+1} \end{bmatrix}. \tag{3}$$

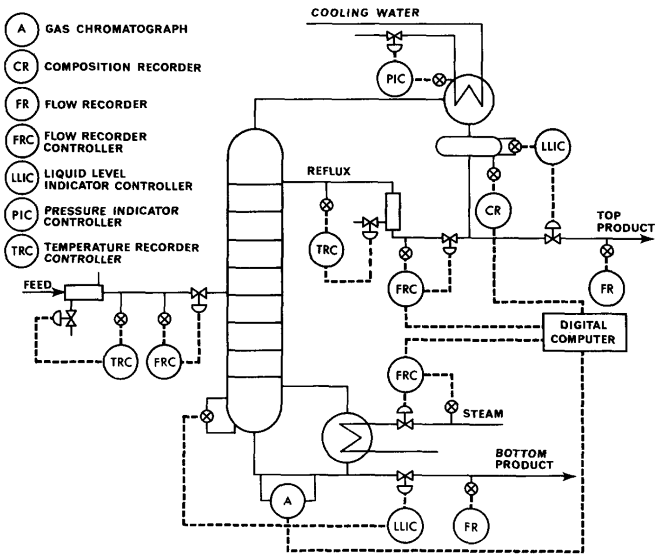
The inputs are u_1 reflux rate, u_2 steam rate and u_3 feed rate, whereas the outputs are y_1 distillate purity and y_2 bottoms purity.

(a) Calculate the poles and zeros (with multiplicity) for the transfer function $G(s)$, both analytically (by hand) and using MATLAB. Is there a difference? Why?

(b) What is the largest gain of all individual elements of $G(s)$?
What is the gain of the (MIMO) system $G(s)$?

Does any of the individual gains reach the MIMO gain? Why/why not?

(Hint: The MATLAB commands **bodemag** and **sigma** might be useful.)

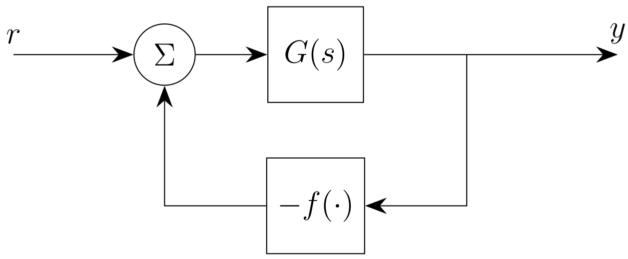


A schematic diagram of a binary distillation column

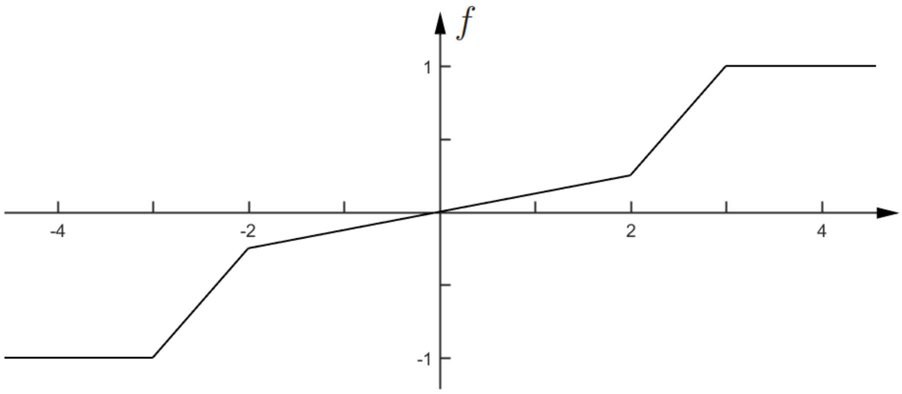
[1] R. K. Wood and M. W. Berry, *Terminal composition control of a binary distillation column*, Chemical Engineering Science, 1973, vol 28, pp. 1707-1717.

Problem II Small gain theorem

A nonlinear feedback system is described by the block diagram

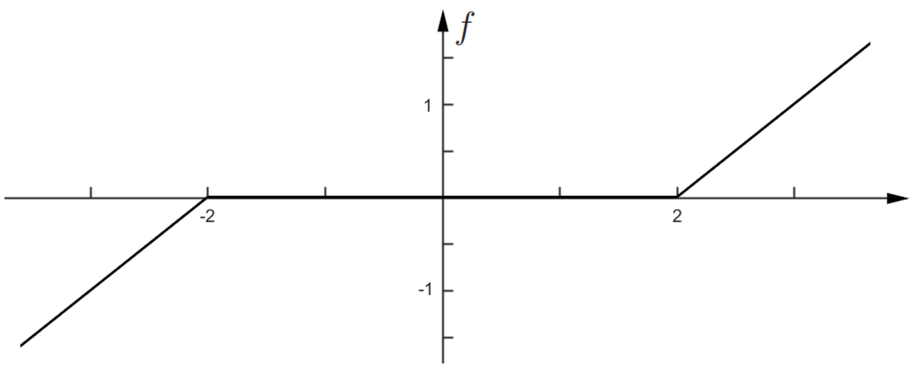


(a) The linear dynamical system $G(s)$ is given by $G(s) = \frac{s+3}{(s+1)(s+2)}$ and f is a static nonlinearity defined as



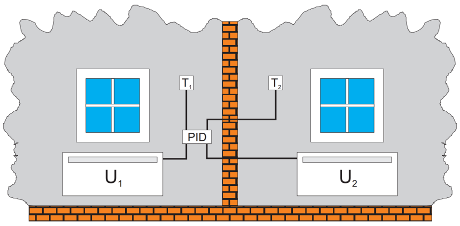
Using the small gain theorem, can stability of the closed loop system be guaranteed?

(b) For another nonlinear feedback system the linear part is given by $G(s) = \frac{1}{s(s+3)}$ and the static nonlinearity f is



Can stability be guaranteed according to the small gain theorem in this case?

Problem III RGA and IMC for a heating system



You are assigned the task to design a control system for the heating of two rooms, according to the figure above. The temperatures T_1 and T_2 are measured, and the control signals are the radiator powers U_1 and U_2. U_1 and U_2 are assumed to take both negative and positive values (i.e., they can both heat and cool). Your colleague has done some system identification, and presents the following model to you:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \dfrac{5s+0.025}{s^2+0.1s+0.002} & \dfrac{10^{-2}}{s^2+0.1s+0.002} \\[2ex] \dfrac{10^{-2}}{s^2+0.1s+0.002} & \dfrac{5s+0.025}{s^2+0.1s+0.002} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{4}$$

(a) For complexity reasons, you are asked to first investigate a solution based on static decoupling and regular SISO PID-controllers. However, the heating in one room affects the other, so a straightforward one-loop-at-a-time approach may not be successful.

Derive the RGA matrix of the system to determine if a *decentralized controller* can be used. Which coupling of the signals does the RGA matrix suggest? Also explain how *static decoupling* can be applied to this problem and present a block diagram which clearly illustrates the decoupled system (you do *not* have to find parameters for the controller).

(b) Design a multivariable IMC controller based on the model (4). You have to fulfill the following requirements:

(i) The order of the denominator polynomial should not be greater than 3 for any element in Q.

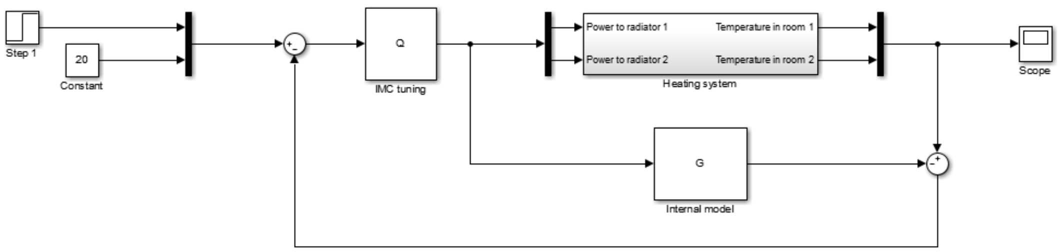
(ii) Provided that the model (4) is correct, the static gain should be I (verify by calculations!).

(iii) You should, for each of the rooms, obtain a a rise time of 10 ± 2 minutes for a step from 20 to 23, when the reference signal for the other room is kept constant at 20.

Make sure to clearly *motivate* you choice of the Q-matrix and include plots verifying (iii) for *both* rooms in your solution.

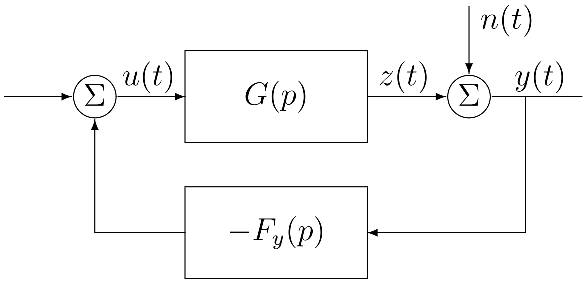
To verify (iii), a simulation model `Heating system` is provided in the Simulink file `simulink_model.m`. (The functions `fcnA.p`, `fcnB.p` and `fcnC.p` are part of the simulation model, and should be saved in your MATLAB directory for the file to work.) Note that the unit in Simulink is seconds (not minutes!), and that (4) is a simpler model than the simulation model provided in Simulink. Due to this plant-model mismatch, this will be a robustness test of the controller you have designed.

Your final Simulink diagram should look similar to this:



Problem IV $\mathcal{H}_2$ control

This assignment deals with an inverted pendulum. Your task is to design a controller F_y, according to the picture below.



Here, u denotes the control input, z is the controlled variable, n is measurement noise, y is the measured (noisy) output, and F_y is the feedback controller. As a model for the system, the transfer function (from u to z)

$$G(s) = \frac{-(0.1s + 1)}{s^2 - 1}, \tag{5}$$

is used, which captures the fact that there is one stable and one unstable pole. For designing the feedback controller, you will use continuous-time $\mathcal{H}_2$ methods. After a discussion with your colleagues about this particular application, you conclude that a good weighting of the transfer functions would be

$$W_S(s) = \frac{1}{s + 0.01}, \quad W_T(s) = 1, \quad W_u(s) = 1. \tag{6}$$

(a) Write the transfer function (5) on controllable canonical form. Determine the extended model of the system. Check if all conditions related to the matrices M, D and innovation form are fulfilled. If these conditions are not fulfilled, how should the standard recipe for determining the optimal $\mathcal{H}_2$ controller be modified?

(b) Determine the optimal $\mathcal{H}_2$ controller. (Hint: If using MATLAB, the function **care** might be useful.) What is the feedback transfer function $F_y(s)$ in this case? Does it have integral action? If the order of your controller is high you can try to reduce it using the MATLAB function **minreal**.

(c) Plot the Bode diagrams of the transfer functions $S(s)$, $T(s)$ and $G_{wu}(s)$. Do they look as you expected?

(d) For each of the transfer functions $S(s)$, $T(s)$ and $G_{wu}(s)$, simulate the response for three different sine wave inputs of frequencies $\omega = 0.01,\ 1,\ 100$ rad/s respectively. Adjust the simulation time to fit 5-10 periods in your plot. For the higher frequencies you may have to adjust the sample time to a small number if the sine wave is not smooth (double click on the sine block), try for example 10^{-4}.

Relate your results to your results in **(c)**. A Simulink model for simulating the response of $T(s)$ could look like below (and similar for $S(s)$ and $G_{wu}(s)$)