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Automatic Control III

Lecture 4 – Controller structures and control design

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Contents – lecture 4

1. Summary of lecture 3
2. Which control design methods do we have?
3. Who should control what? Relative Gain Array (RGA)
    a) The pairing problem
    b) Decentralized control
    c) Decoupled control
4. Internal Model Control (IMC)

Summary of lecture 3 (I/II)

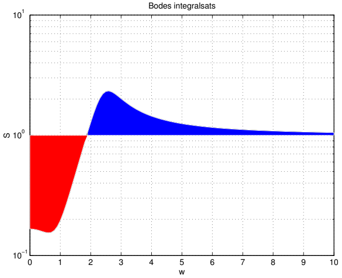
Bode's relationship provides an upper bound on the phase, which depends on the derivative (slope) of the amplitude curve.

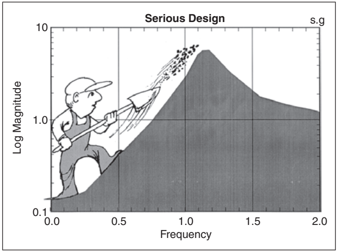
Hence, Bode's relationship provides a fundamental limit by revealing a certain **coupling** between the amplitude decay and the phase.

Summary of lecture 3 (II/II)

For stable systems we derived **Bode's integral theorem** stating that

$$\int_0^\infty \log |S(i\omega)| d\omega = 0.$$





For unstable systems (with M poles $\{p_i\}_{i=1}^M$ in the RHP):
$\int_0^\infty \log |S(i\omega)| d\omega = \pi \sum_{i=1}^M Re(p_i)$.

The most successful controller ever: PID

- Boulton och Watt 1788: speed control of steam engines, mechanical implementation.
- Hydraulic and pneumatic implementation: late 1800s
- Electronic: 1930s.
- Computer: 1950s.
- "PID-on-a-chip": 1990s.
- Applications: All.

Steam engine: http://www.youtube.com/watch?v=Hqqc6gfVrtU
PID-on-a-chip: http://www.youtube.com/watch?v=9eMWG3fwiEU

PID

- Assumes one input signal and one output signal
- If you have several input and output signals you must pair them in two and two
- Interpretation in Bode plots: lead and lag
- Tuning using intuition and experience results in not more than mediocre performance (except in very simple cases).
- Tuning using systematic analysis (poles, zeros, S, T, ...) can give controllers with very good performance.
- The first systematic approach (using poles): Maxwell 1868.

Maxwell, J.C. **On Governors**, *Proceedings of the Royal Society*, no. 100 (1868).

When PID is not enough

(Typically means that the system is multivariable and/or nonlinear)

- Internal model control (IMC)
- Minimization of quadratic criteria: LQ, LQG.
- Model predictive control (MPC) – constraints
- Systematic shaping of transfer functions: H_2, H_∞.
- Nonlinear controllers

The foundation of all modern control methods is to make use of **models**.

MIMO – who should control what?

1. Tall system (more output than input signals):

$$G = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

   All output signals cannot be controlled perfectly – prioritize.

2. Fat system (more input signals than output signals)

$$G = \begin{bmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

   How should the actuation be distributed among the control signals?

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Several input and output signals – interaction

If there are many input and output signals we can make the control design much easier by breaking down the system in sub-systems with **little interaction** between each other.

The **relative gain array (RGA)** is a way of measuring the level of cross coupling or interaction in a system.

Bristol, E. On a new measure of interaction for multivariable process control. *IEEE Transactions on Automatic Control*, 11(1), pp. 133–134, 1966.

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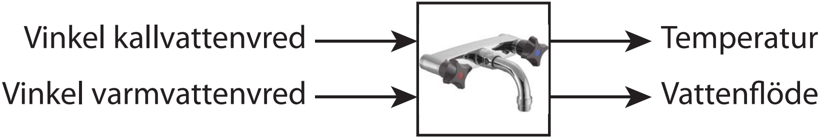
Interaction/cross coupling (I/II)

- Two-handle mixer, a system with a strong cross coupling
    - Several input signals (significantly) affect an output signal.
    - Several output signals are (significantly) affected by an input signal.



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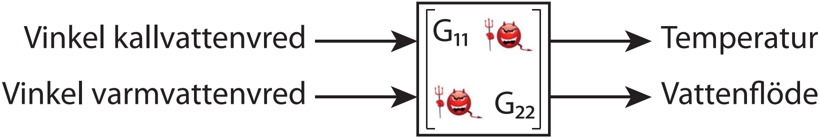

Interaction/cross coupling (II/II)

- Mixer with one handle, a system with a “nice” cross coupling.
  - Each input signal affects (almost) only one output signal.
  - Each output is affected (almost) only by one input signal.

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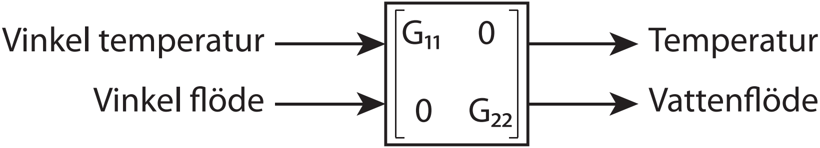

Decentralized control

Idea (decentralized control): Build a controller for a MIMO system where **one** output signal controls **one** input signal.

The result is a set of "single variable loops"

$$u_i = F_r^i r_j - F_y^i y_j,$$

where the individual controllers are all independent ("they do not know of each other").

- F_y is a quadratic transfer matrix. If the number of input and output signals is different some of them are simply discarded.
- The less cross couplings there are, the better this strategy works
- We want to pair the input and output signals that have the strongest connections, the **pairing problem**.
- How do we determine the couplings between the various input and output signals?

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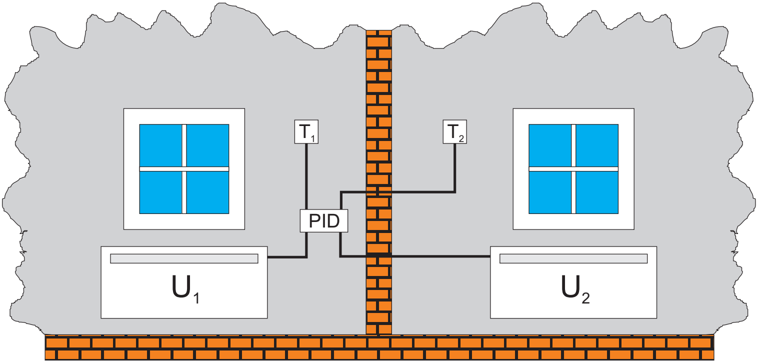
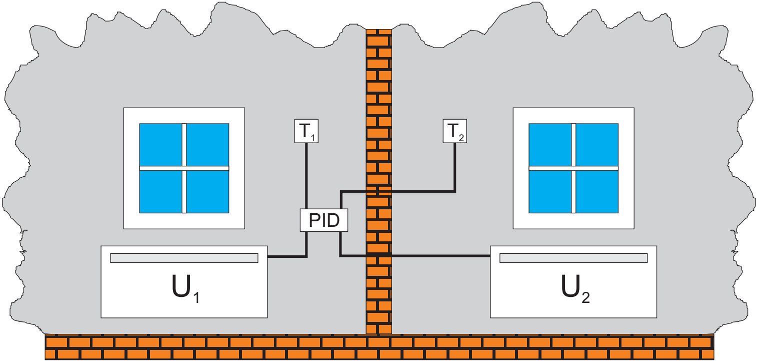
Temperature control



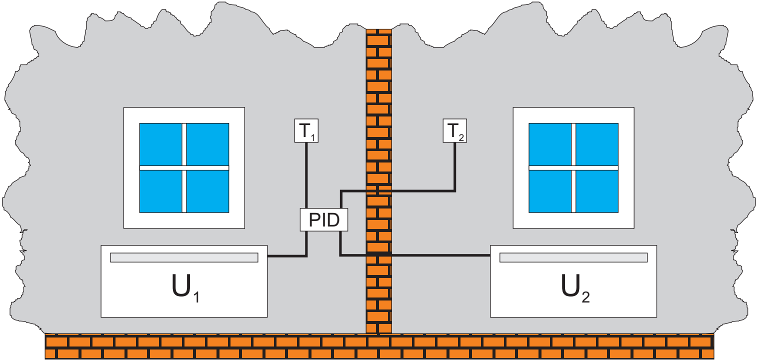
- Two rooms with a separating inner wall.
- The temperatures T_1, T_2 are states, which are both measured.
- Both rooms can be both heated and cooled by U_1 and U_2.

Temperature control



$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{0.005s+0.0002438}{s^2+0.0975s+0.002025} & \frac{9.375e-05}{s^2+0.0975s+0.002025} \\ \frac{9.375e-05}{s^2+0.0975s+0.002025} & \frac{0.005s+0.0002438}{s^2+0.0975s+0.002025} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Temperature control



Which sensor should be used in controlling which heating/cooling source (i.e. the pairing problem)?

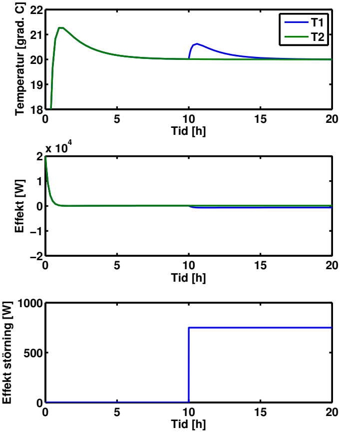
Temperature control

Decentralized PI control

- T_1 is used for U_1.
- T_2 is used for U_2.

$$F(s) = \begin{bmatrix} 1000 + \frac{500}{s} & 0 \\ 0 & 1000 + \frac{500}{s} \end{bmatrix}$$

After 10 hours 10 people enter room 1.



Temperature control

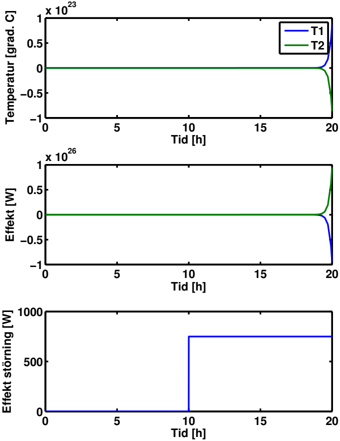
Decentralized PI control

- T_2 is used for U_1.
- T_1 is used for U_2.

$$F(s) = \begin{bmatrix} 0 & 1000 + \frac{500}{s} \\ 1000 + \frac{500}{s} & 0 \end{bmatrix}$$

After 10 hours 10 people enter room 1.

Whooops...



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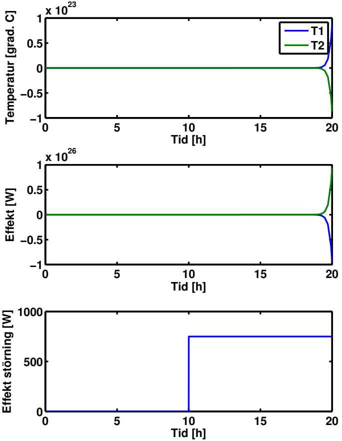
After 10 hours 10 people enter room 1.

Is there any way in which we can predict this problem "analytically"?



RGA again

- Use RGA to decide which input to pair with which output
  - $RGA(A) = A \times (A^{\dagger})^{T}$, $\times$ - element-wise matrix multiplication, $\cdot^{\dagger}$ is pseudoinverse
  - RGA in Matlab: RGA(A) = `A.*pinv(A).';` `.'` is necessary to get a transpose instead of conjugate transpose
- The two main rules in designing a decentralized controller using RGA. Pair measurement signals and control signals such that the diagonal elements in
  1. $RGA(G(i\omega_c))$ are close to 1 in the complex plane
  2. $RGA(G(0))$ are positive (if they are negative this can lead to instability)
- "Pairing" implies a change of the position of rows and columns in the RGA-matrix.

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Temperature control – RGA

Design rules from the previous slide in different words:

1. Select input-output pairs so that the diagonal elements of RGA$(i\omega_c)$ are close to 1.
2. Avoid pairing that gives negative diagonal elements of RGA$(G(0))$.

In the temperature control example:

$$\text{RGA}(G(i8)) \approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{RGA}(G(0)) = \begin{pmatrix} 1.17 & -0.17 \\ -0.17 & 1.17 \end{pmatrix}$$

The pairing where

- T_2 is used for U_1 and
- T_1 is used for U_2,

breaks both of these rules (the rows change places)!

Decoupled control

Select W_1 and W_2 to make $W_1(s)G(s)W_2(s)$ mostly diagonal.

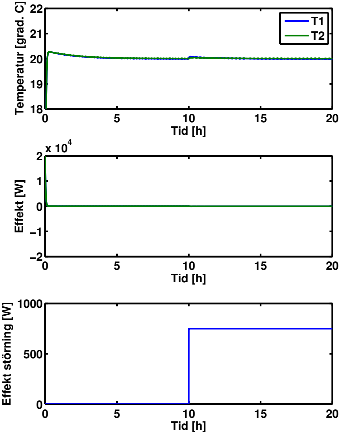
- In order to obtain a completely decoupled "virtual system" we would need s-dependent weight matrices W_1 and W_2.
- This is in general not possible, since it would lead to a complicated and/or non-proper controller.
- Instead we choose one frequency where the system becomes decoupled:
  - $\omega = 0$
  - $\omega = \omega_c$ ($G^{-1}(\omega_c)$ is often approximated to get rid of complex valued elements).
- The choice $W_1 = G^{-1}(0)$ and $W_2 = I$ results in decoupling in stationarity.
- With the right choice of W_1 and W_2 we can make the two-handle mixer behave as a one-handle mixer. Easier to control!!

Temperature control – decoupled

After 10 hours 10 people enters room 1.

Decoupled control using

$$W_1 = G^{-1}(0), \text{ and } W_2 = I.$$



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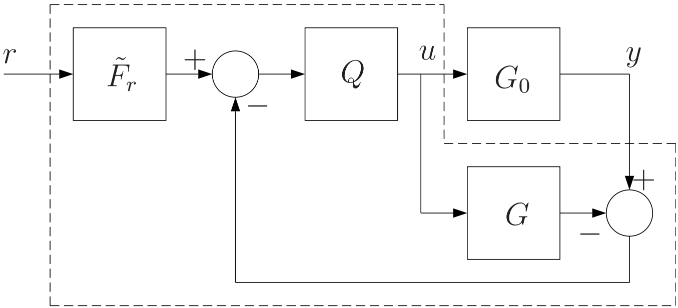
Internal Model Control (IMC)

Feedback only using the new information $y - Gu$.



Results in (if $G_0 = G$)

- $G_c = GQ\tilde{F}_r, \quad S = I - GQ, \quad T = GQ$

How do we choose Q? (I/II)

The "ideal" case $Q = G^{-1}$ would result in $S = 0, G_c = I$, but it is infeasible since $F_y = \infty$.

Hence, we have to approximate!

Some rules of thumb:

1. If G has more poles than zeros: The inverse of G cannot be realized. Use

$$Q(s) = \frac{1}{(\lambda s + 1)^n} G^{-1}(s).$$

Choose n such that $Q(s)$ is proper (# poles = # zeros).
Choose λ to adjust the bandwidth of the closed loop system.

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How do we choose Q? (II/II)

2. If $G(s)$ non-minimum phase $\Leftrightarrow$ G has an "unstable zero" and contains a factor $(-\beta s + 1), \beta > 0$. Two alternatives
   a) Omit the factor when $Q = G^{-1}$ is formed.
   b) Replace the factor $(-\beta s + 1), \beta > 0$ with $(\beta s + 1), \beta > 0$ when $Q = G^{-1}$ is formed.

3. If G contains a time delay, i.e. a factor $e^{-s\tau}$:
   a) Omit the factor when $Q = G^{-1}$ is formed.
   b) Approximate the factor using first-order Padé approximation

$$e^{-s\tau} \approx \frac{1 - s\tau/2}{1 + s\tau/2}$$

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A few concepts to summarize lecture 4

Cross couplings: The key difficulty in controlling MIMO systems is that there are cross couplings between input and output signals. If we change one input signal this affects several output signals.

Relative gain array (RGA): The relative gain array is a measure of the amount of cross couplings in a matrix
(RGA$(A) = A \times (A^{\dagger})^T$).

Decentralized control: Let every input be determined by feedback from one single output.

The pairing problem: The pairing problem is to select which input-output pairs that should be used for the feedback.

Decoupled control: Decoupled control makes use of a change of variables such that suitable pairings of measurements and control signals becomes easier to see.

Internal model control (IMC): Choose $Q \approx G^{-1}$ $(y = GQr)$ and let the new information in the form of $y - Gu$ be fed back to affect u.