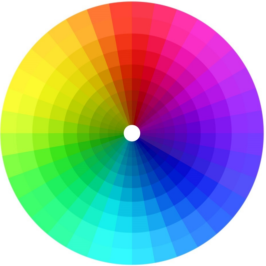
Spectral Processing of Signals



Per Mattsson

Systems and Control
Department of Information Technology
Uppsala University

2019-09-03

Short survey

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per.mattsson@it.uu.se

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- Comfortable in the frequency domain?
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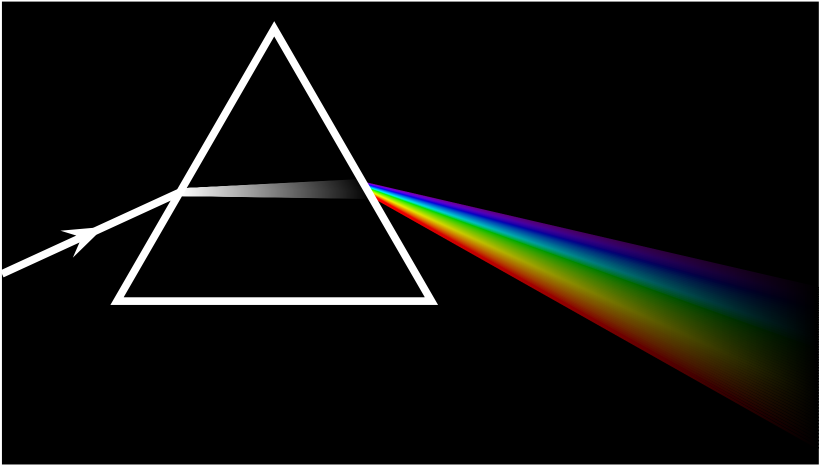
Basic courses to remember

Transform Methods (Fourier), Linear Algebra, Probability Theory/Statistics, Signals and Systems, Scientific Computing

Motivation

Visible light analogy

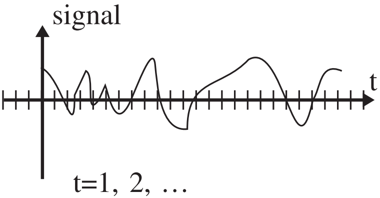
- Splitting (white) light into (all) the colors of the rainbow
- Splitting a signal into its spectral components, and quantifying them

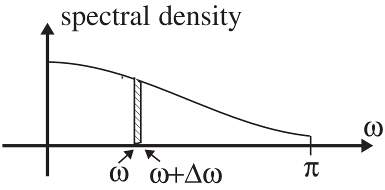


Spectral Estimation

Informal definition

From a finite record of a stationary data sequence (signal), estimate how the total signal power is distributed over frequency, or more practically, over a set of narrow frequency intervals (bins).





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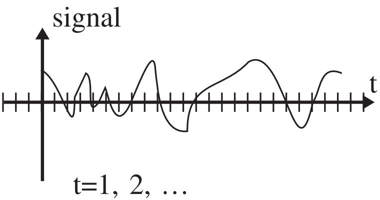

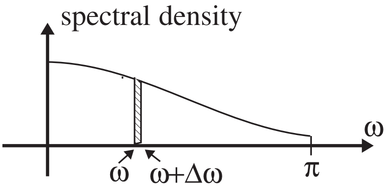

- Assumption: signal properties (spectral content) ≈ constant

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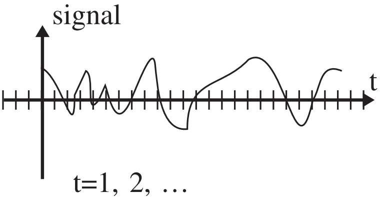

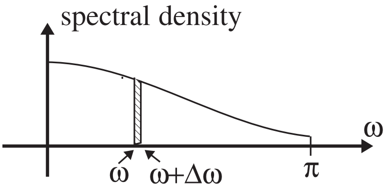

- Assumption: signal properties (spectral content) ≈ constant
- Question: Which frequencies contribute to the total signal?

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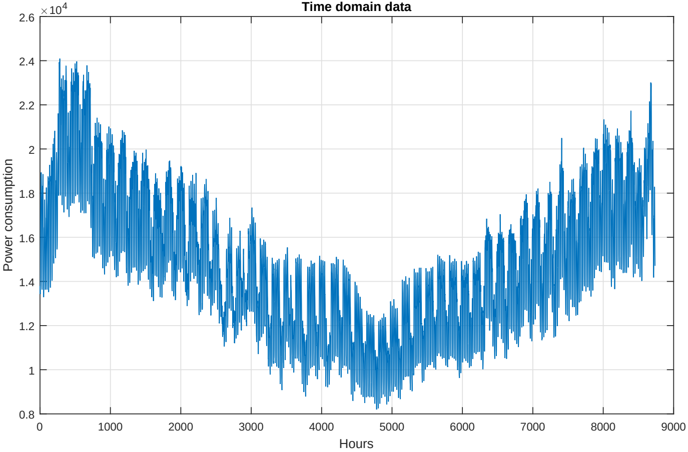
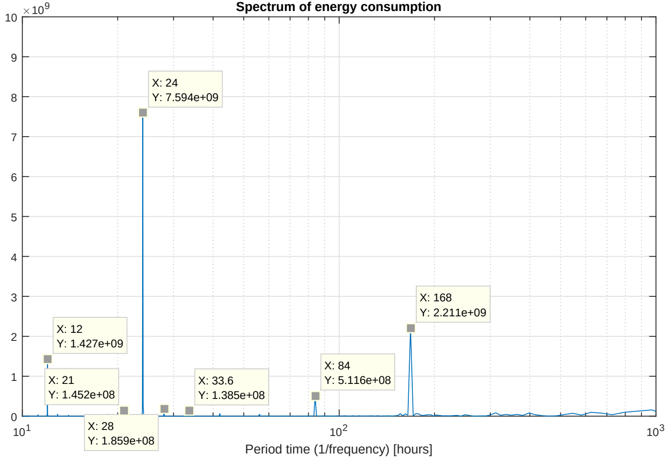
Example: Power consumption (1)

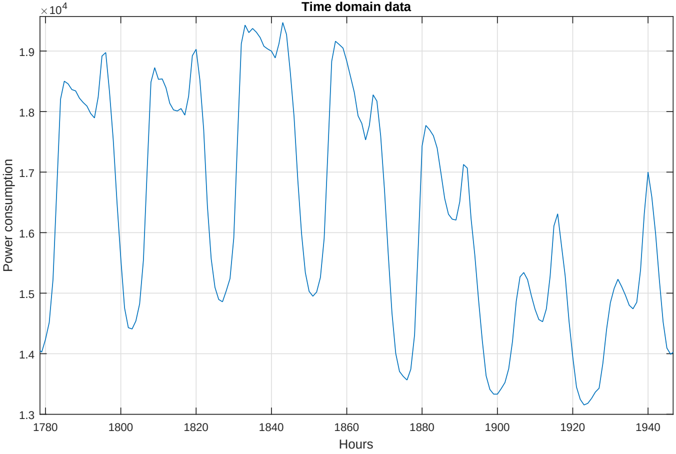


What periodicities do you expect?

Example: Power consumption (2)



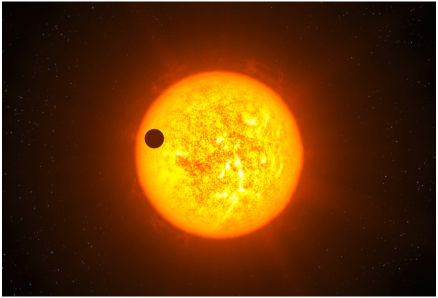
Example: Power consumption (3)

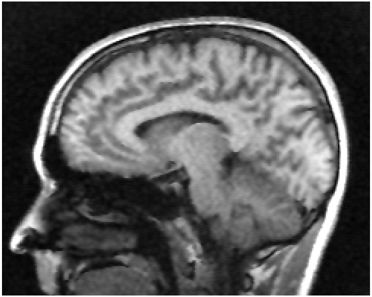


Clear vs. hidden periodicities

Applications

- Hidden periodicity finding: ecology, astronomy, climate/weather, seismology, econometrics, etc.
- Speech processing/coding and audio devices
- Medical diagnosis (EEG, ECG, MRI)
- Automatic control
- Vibration monitoring and fault detection
- Radar, Sonar
- Digital communications





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The Course

Course structure

- 10 lectures
- 2 Exercise/discussion sessions
- 4 Homeworks, 4hp (graded)
- 4 Computer labs, 1hp (mandatory)

Computer-based course

It can be helpful to have a computer at hand when studying this course, since we will focus on *using* methods for spectral analysis, on data.

Field of applied mathematics

Signal processing, and in turn, spectral analysis, is based on mathematical results and algorithms. So we will need some math to *understand* the methods.

Course content

Activity	Content	To read
L1	The spectral estimation problem.	1.1 – 1.5
L2	The periodogram and Correlogram methods	2.1 – 2.4
L3	Improved periodogram-based methods	2.5 – 2.7.2
L4	Parametric methods for rational spectra	3.1 – 3.4, 3.7
L5	Line spectra, NLS and rational methods for	4.1 – 4.4
L6	Subspace methods for line spectra	4.5 – 4.8
L7	Order selection	C.1 – C.8
L8	Filter-bank methods	5.1 – 5.5
L9	Summary and repetition (buffer)	
L10	Applications (guest lectures)	

Prepare by looking in the book **before** the lectures! Appendix A is good for shaping up your linear algebra.

Homeworks

Discuss in pairs, hand in individual reports in studentportalen.

HW 1 Periodogram Methods, C2.22:
Refined Methods: Variance–Resolution Tradeoff.
Deadline: 2019-09-17 23:59

HW 2 Rational Parametric Methods, C3.20:
AR and ARMA Estimators applied to Measured Data.
Deadline: 2019-09-29 23:59

HW 3 Rational Parametric Methods for Line Spectra, C3.18:
AR and ARMA Estimators for Line Spectral Estimation.
Deadline: 2019-10-13 23:59

HW 4 Parametric Methods for Line Spectra, C4.14:
Line Spectral methods applied to Measured Data.
Deadline: 2019-11-03 23:59

Start as soon as you can!

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Computer labs

Mandatory! Oral presentation in the lab. (See schedule)

CL 1 Periodogram Methods, C2.19 and C2.20:
Zero Padding Effects on Periodogram Estimators and *Resolution and Leakage Properties of the Periodogram*.

CL 2 Parametric Methods for Rational Spectra, C3.17:
Comparison of AR, ARMA and Periodogram Methods for ARMA Signals.

CL 3 Parametric Methods for Line Spectra, C4.12:
Resolution Properties of Subspace Methods for Estimation of Line Spectra.

CL 4 Filter Bank Methods, C5.13:
The Capon Method.

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- Continuous/discrete (sampled) signals:
  $y(t) = y_c(tT_s), \quad t = 0, 1, 2, \ldots$

per.mattsson@it.uu.se

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- Parseval's theorem (Energy preservation)

$$\sum_{t=-\infty}^{\infty} |y(t)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |Y(\omega)|^2 \mathrm{d}\omega$$

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Deterministic signals (not too interesting in practice)

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- Finite *energy*: $\sum_{t=-\infty}^{\infty} |y(t)|^2 < \infty$

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- DTFT of the autocorrelation also gives the ESD

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per.mattsson@it.uu.se

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Linear filters

- **Transfer function:**

$$\color{red}{H(q)} = \sum_{k=0}^{\infty} h_k q^{-k}$$

where q^{-1} is the unit delay operator, $q^{-1}y(t) = y(t-1)$.

$$\xrightarrow[\phi_u(\omega)]{u(t)} \boxed{\color{red}{H(q)}} \xrightarrow[\phi_y(\omega)]{y(t)}$$

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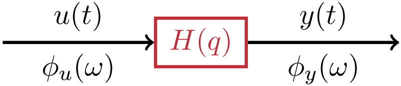

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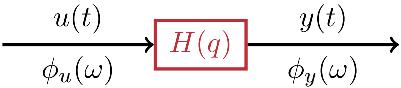

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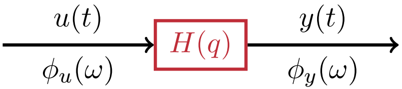

- $y(t) = H(q)u(t) = \sum_{k=0}^{\infty} h_k u(t-k)$,

Then $\phi_y(\omega) = |H(\omega)|^2 \phi_u(\omega)$, where $H(\omega) = \sum_{k=0}^{\infty} h_k e^{-i\omega k}$

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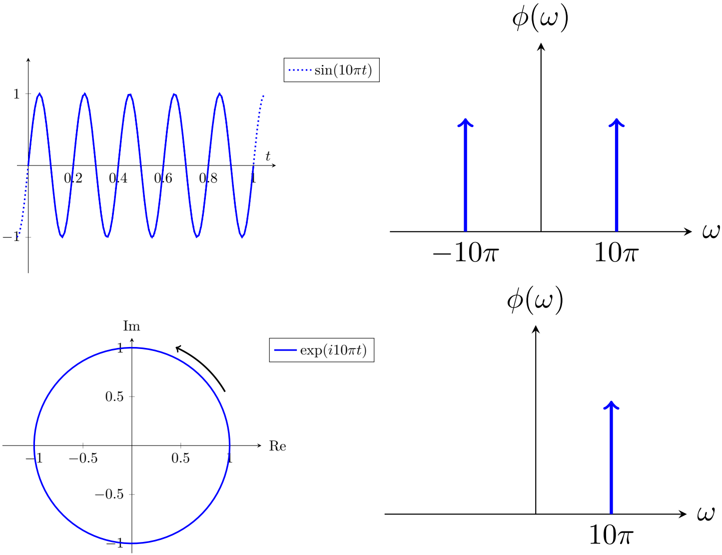
Signals and Sampling

In this course we will mainly consider discrete-time signals, but they often come from sampling of continuous-time signals.

per.mattsson@it.uu.se

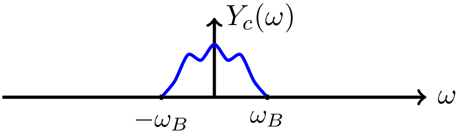
Sinusoids



$\sin(10\pi t)$
1
−1
0.2
0.4
0.6
0.8
1
t
$\phi(\omega)$
ω
-10π
10π
Im
Re
$\exp(i10\pi t)$
1
0.5
−0.5
−1
$\phi(\omega)$
ω
10π

Sampling

- Consider a continuous-time signal y_c with the spectrum



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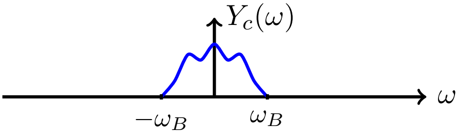

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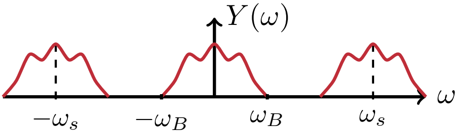

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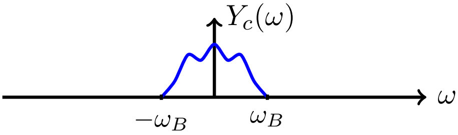

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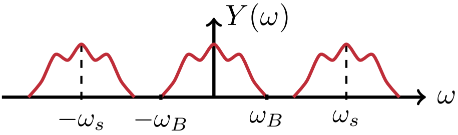

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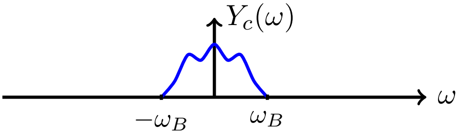

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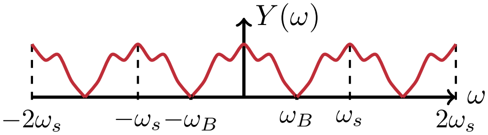

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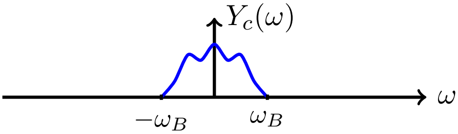

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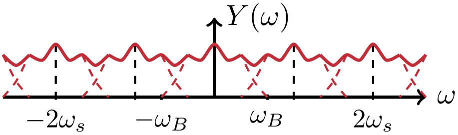

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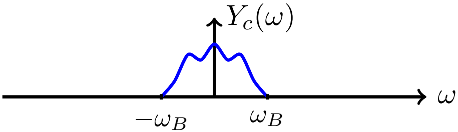

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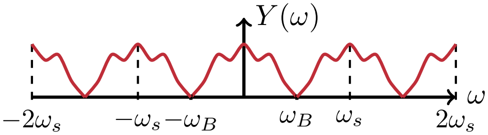

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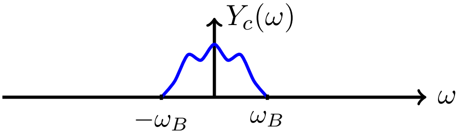

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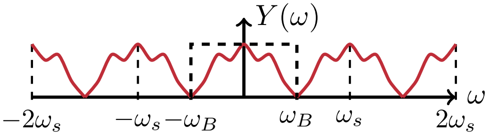

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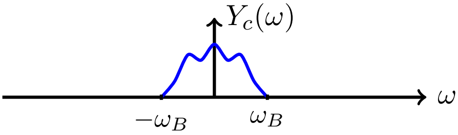

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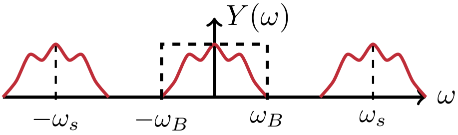

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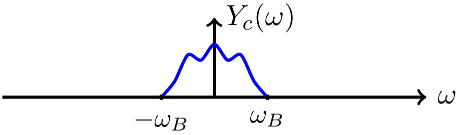

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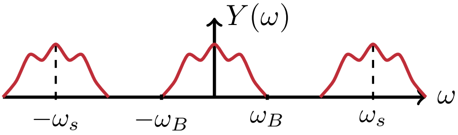

- Nyquist frequency: Reconstruction possible if,

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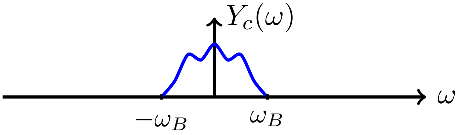

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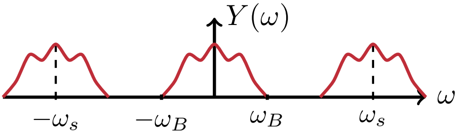

- Nyquist frequency: Reconstruction possible if,

$$\omega_s - \omega_B$$

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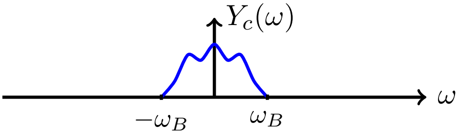

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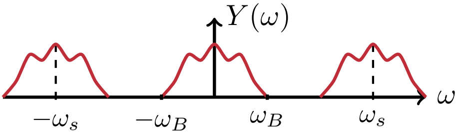

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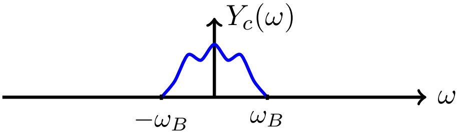

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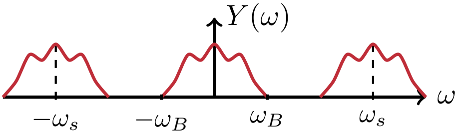

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$$\omega_s - \omega_B > \omega_B \iff \frac{\omega_s}{2} > \omega_B$$

Example: Sinusoid

- **Sampling:** $T_s = 1$ s, dvs $\omega_s = 2\pi$ rad/s.

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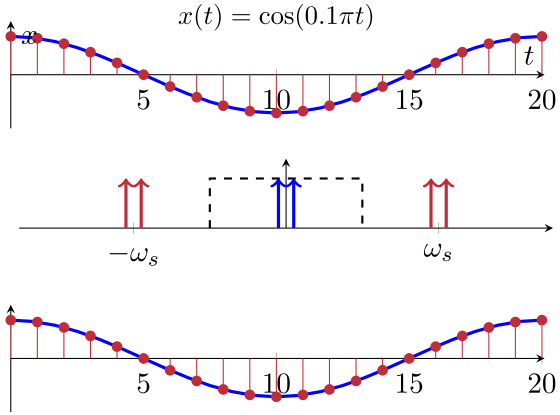
Spectrum:

Reconstructed signal:



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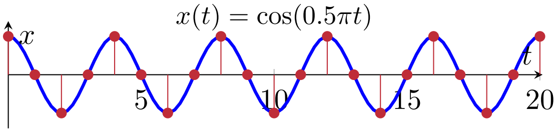

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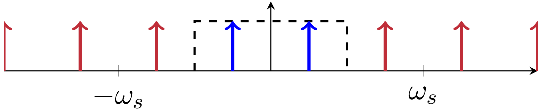
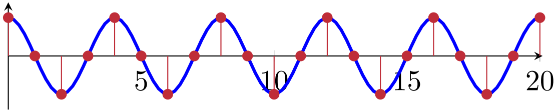


Reconstructed signal:



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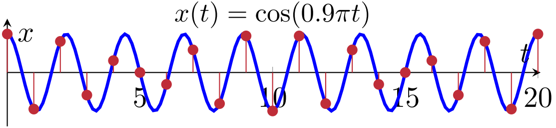

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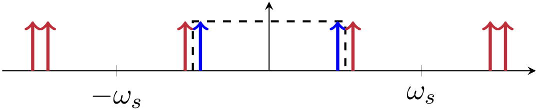

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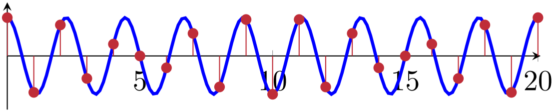

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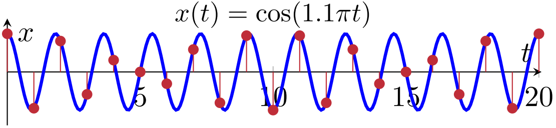

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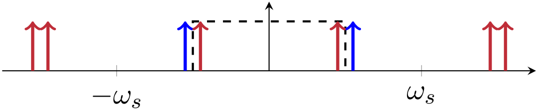

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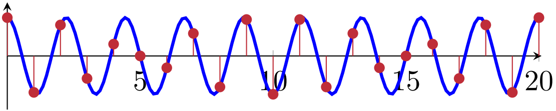

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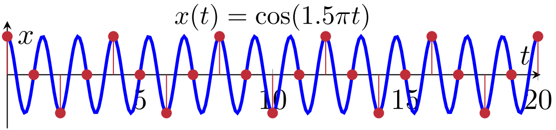

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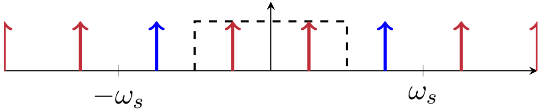

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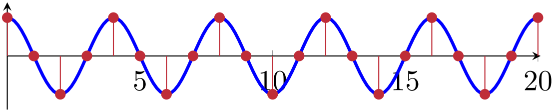

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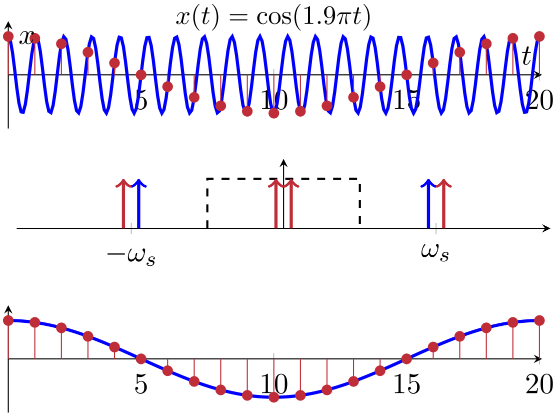

Summary

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The Spectral Estimation Problem

From a finite record $\{y(1), y(2), \ldots, y(N)\}$ of a stationary data sequence, determine an estimate $\hat{\phi}(\omega)$ of the PSD $\phi(\omega)$ for $\omega \in [-\pi,\ \pi]$ (normalized frequency).

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Next time:

- Parametric vs. non-parametric approaches to spectral analysis
- How to estimate the PSD from data
- How to estimate the ACS from data
- The Periodogram and correlogram methods (non-parametric)
- Spectral analysis in MATLAB (useful functions)