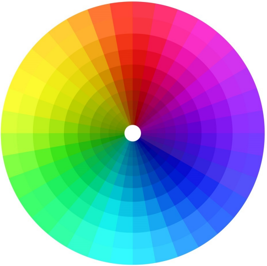
Nonparametric methods: Improved periodogram methods



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Summary from last lecture

Practical non-parametric methods

Periodogram:

$$\hat{\phi}_p(\omega) = \frac{1}{N}\left|\sum_{t=1}^{N} y(t)e^{-i\omega t}\right|^2$$

Correlogram:

$$\hat{\phi}_c(\omega) = \sum_{k=-(N-1)}^{N-1} \hat{r}(k)e^{-i\omega k}, \qquad \hat{r}(k) = \frac{1}{N}\sum_{t=k+1}^{N} y(t)y^*(t-k)$$

- Periodogram and Correlogram
- Asymptotically Unbiased
- High variance (inconsistent)
- Resolution $\omega \approxeq 2\pi/N$

Today: How can the periodogram be improved?

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What is the problem with the periodogram?

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- Smaller M should give less variance. But at what cost?
- More generally, we could give different weights to different lags $|k|$. Small $|k|$ higher weight.

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- The Blackman-Tukey PSD estimate:

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- Rectangular window: $w(k) = 1$ for $|k| < M \Rightarrow$ truncated sum.
- Generally: $w(0) = 1$, $w(-k) = w(k)$.

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Alternative interpertation

- Let $\hat{r}(k) = 0$ for $|k| \geq N$ and $w(k) = 0$ for $|k| \geq M$.

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- Possible interpretation:

$$\xrightarrow{\hat{\phi}_p(\omega)} \boxed{W} \xrightarrow{\hat{\phi}_{\mathrm{BT}}(\omega)}$$

Analysis

Analysis of the Blackman-Tukey method

$$\hat{\phi}_{\mathrm{BT}}(\omega) = \sum_{k=-(M-1)}^{M-1} w(k)\hat{r}(k)e^{-i\omega k} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\hat{\phi}_p(\zeta)W(\omega-\zeta)d\zeta.$$

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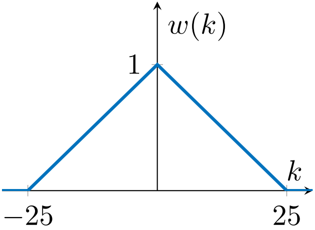
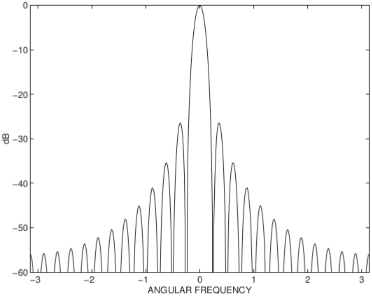
- "Smoothed periodogram".
- Reduces variance. Smaller M gives smaller variance.
- Slightly increases bias. Smaller M gives higher bias, and worse resolution.
- If $W(\omega) \geq 0$, then $\hat{\phi}_{\mathrm{BT}}(\omega) \geq 0$.

Example: The Bartlett (triangular) window

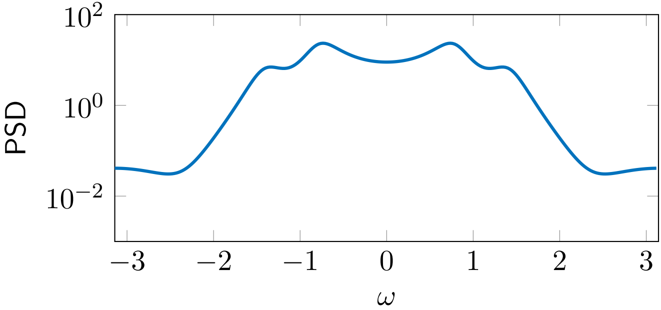




The triangular window with $M = 25$.
Main lobe 3db: $2\pi/M$.

Example

$$\hat{\phi}_{\mathrm{BT}}(\omega) = \sum_{k=-(M-1)}^{M-1} w(k)\hat{r}(k)e^{-i\omega k}.$$



True spectrum estimated with $N = 256$ samples.

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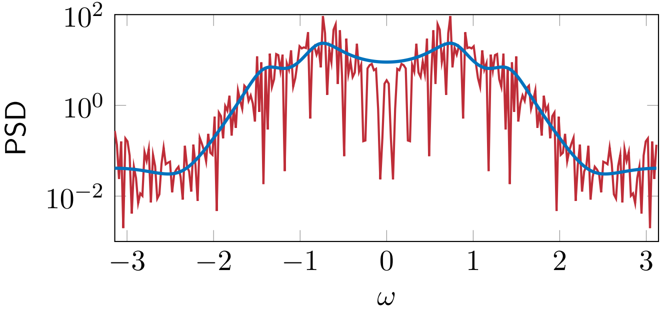

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Periodogram $M = N = 256$ and $w(k) = 1$

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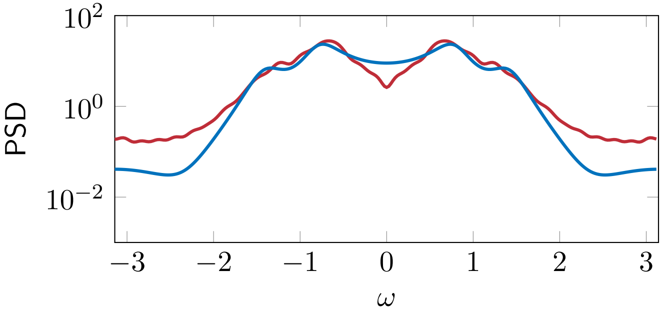

True spectrum estimated with $N = 256$ samples.
Blackman-Tukey with $M = 32$.

Example

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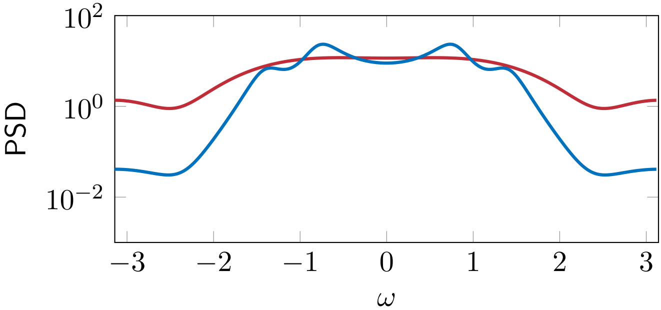


True spectrum estimated with $N = 256$ samples.
Blackman-Tukey with $M = 4$.

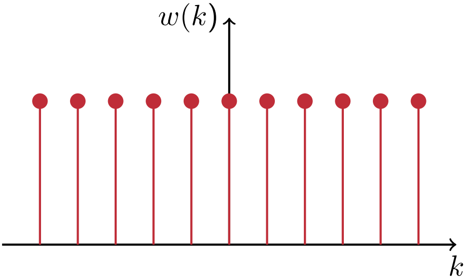
Time-Bandwidth product

For the typical window functions, we can define

$$\text{Equivalent time width:}\quad N_e = \frac{\sum_{k=-(M-1)}^{M-1} w(k)}{w(0)}$$

$$\text{Equivalent bandwidth:}\quad \beta_e = \frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} W(\omega)\mathrm{d}\omega}{W(0)}$$

Example: Rectangular window



$$N_e = \frac{\sum_{k=-(M-1)}^{M-1} w(k)}{w(0)} = 2M - 1$$

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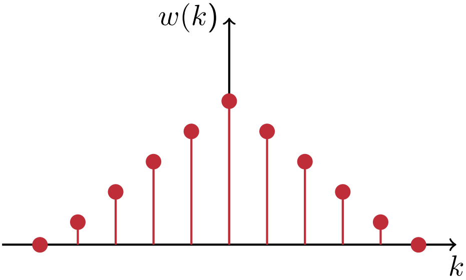

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By the DTFT:

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- No "correct" way to choose M, depends on application. But important to know how choice of M affects estimate.
- Rule of thumb: $M \leq N/10$.

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Fixed window functions

TABLE 2.1: Some Common Windows and their Properties

The windows satisfy $w(k) \equiv 0$ for $|k| \geq M$, and $w(k) = w(-k)$; the defining equations below are valid for $0 \leq k \leq (M-1)$.

Window Name	Defining Equation	Approx. Main Lobe Width (radians)	Sidelobe Level (dB)
Rectangular	$w(k) = 1$	$2\pi/M$	-13
Bartlett	$w(k) = \frac{M-k}{M}$	$4\pi/M$	-25
Hanning	$w(k) = 0.5 + 0.5\cos\left(\frac{\pi k}{M}\right)$	$4\pi/M$	-31
Hamming	$w(k) = 0.54 + 0.46\cos\left(\frac{\pi k}{M-1}\right)$	$4\pi/M$	-41
Blackman	$w(k) = 0.42 + 0.5\cos\left(\frac{\pi k}{M-1}\right) + 0.08\cos\left(\frac{\pi k}{M-1}\right)$	$6\pi/M$	-57

Window design

Windows with design parameter:

- Chebyshev (constant peak sidelobe ripples)
- Kaiser (main lobe/sidelobe trade-off. $\gamma = 0 \implies$ rect.)

$$w(k) = \frac{I_0(\gamma\sqrt{1-[k/(M-1)]^2})}{I_0(\gamma)}, \quad |k| \leq M-1$$

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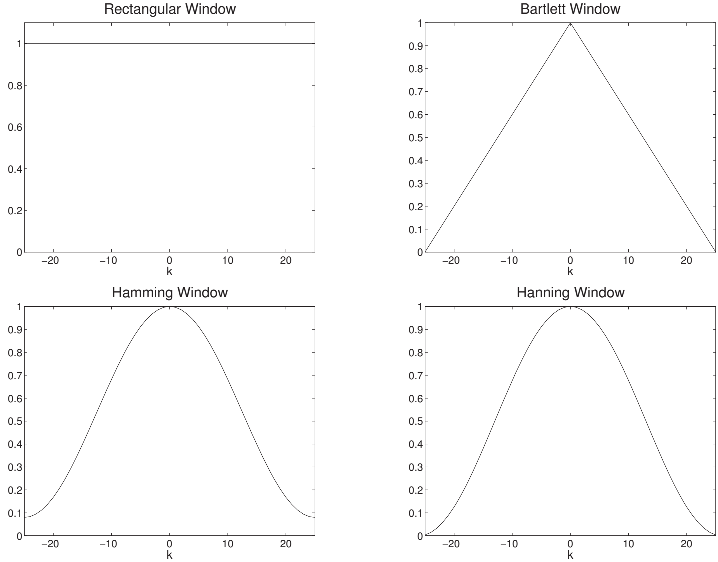
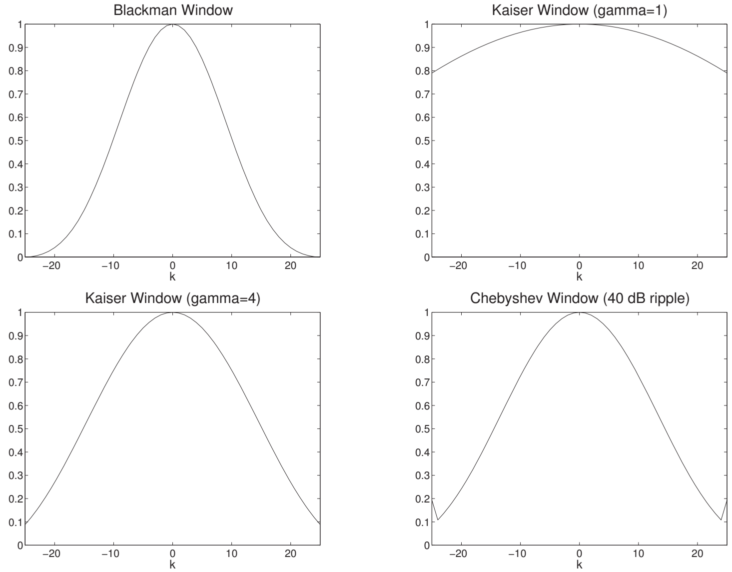
- Temporal lag windows (for the Periodogram) can also be designed
- One can relate the lag windows to temporal windows
- These windowed Periodograms have the same *average* behavior as the corresponding windowed Correlograms

Windows (1)

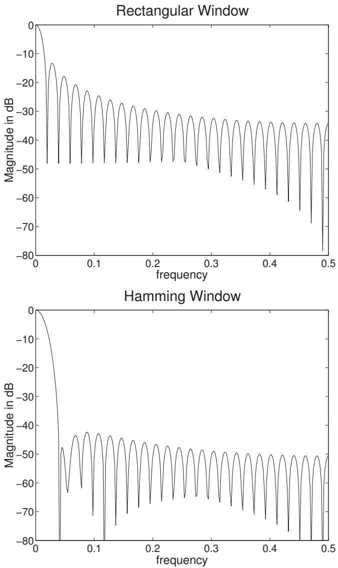

Rectangular Window
Bartlett Window
Hamming Window
Hanning Window
k
−20
−10
0
10
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Windows (2)



Blackman Window
Kaiser Window (gamma=1)
Kaiser Window (gamma=4)
Chebyshev Window (40 dB ripple)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−20
−10
0
10
20
k

Windows (3)



Rectangular Window
Bartlett Window
Hamming Window
Hanning Window
Magnitude in dB
frequency
0
−10
−20
−30
−40
−50
−60
−70
−80
0
0.1
0.2
0.3
0.4
0.5

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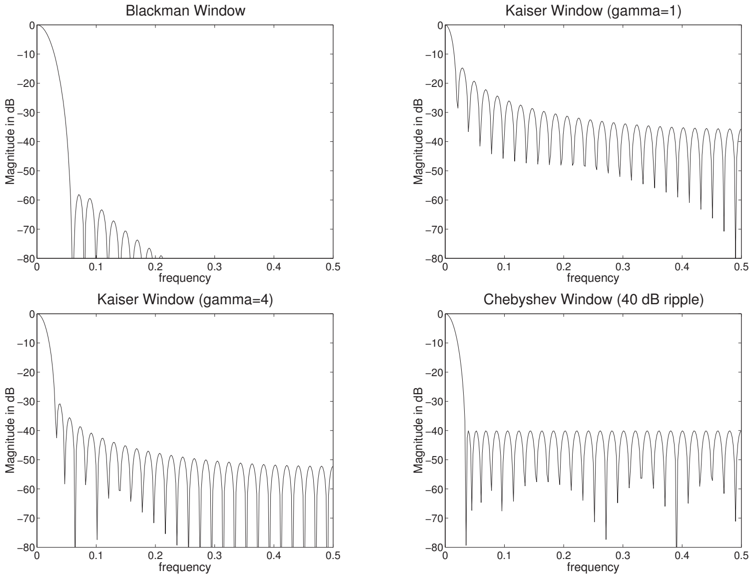
Windows (4)



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Other methods

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The Bartlett method

- Split up available sample into L = N/M subsamples
- Average the Periodograms obtained from each

That is, $y_j(t) = y((j-1)M + t),\ t = 1, \dots, M,\ j = 1, \dots, L$

$$\hat{\phi}_j(\omega) = \frac{1}{M} \left| \sum_{t=1}^{M} y_j(t) e^{-i\omega t} \right|^2$$

$$\hat{\phi}_{\mathrm{B}}(\omega) = \frac{1}{L} \sum_{j=1}^{L} \hat{\phi}_j(\omega)$$

- Reduces the variance of the estimate
- Reduces the resolution (fewer samples)
- Similar to $\hat{\phi}_{\mathrm{BT}}(\omega)$ with a rectangular window.

The Welch method

- Refines the Bartlett method in two ways:
  - Overlapping subsamples $\rightarrow$ more averaging (but correlated)
  - Windowing the data (reduce correlation)
- S subsamples, $1 - K/M$ overlap, P window power

$$y_j(t) = y((j-1)K + t), \; t = 1, \dots, M, \; j = 1, \dots, S$$

$$\hat{\phi}_j(\omega) = \frac{1}{MP} \left| \sum_{t=1}^{M} v(t) y_j(t) e^{-i\omega t} \right|^2$$

$$\hat{\phi}_{\mathrm{W}}(\omega) = \frac{1}{S} \sum_{j=1}^{S} \hat{\phi}_j(\omega)$$

- $K = M \implies$ no overlap, as in Bartletts
- Commonly used: $K = M/2$

Wrapping up

- There are many variations of the Periodogram/Correlogram
- Reduce variance at the cost of resolution (bias)
- Smoothing or averaging the spectrum
- All can essentially be expressed as Blackman-Tukey estimators
- All special cases of the Filter bank approach (Lecture 8)

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Twist

The refined methods are needed for estimating continuous PSDs, but the *unmodified* Periodogram can be shown to be a satisfactory estimator for discrete (or line) spectra corresponding to sinusoidal signals.

Summary

- How can we improve the periodogram?
- The Blackman-Tukey method
- Different window functions and design
- Time-Bandwidth product
- The Bartlett method
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Next lecture:

- Parametric Methods for Rational Spectra

Exercises:

- Exercise 1.5, 1.7, 1.10, 2.3, 2.6, 2.10

MATLAB

Useful functions:

- btse()

Try the following:

- Implement the BT estimator using the FFT (how?)

Remember:

- The MATLAB built-in functions might note be directly compatible with the provided code!
- Window functions in MATLAB are defined differently, use the definitions in the book to compute the window vectors directly!