Spectral Processing of Signals
Homework 1: Periodogram Methods

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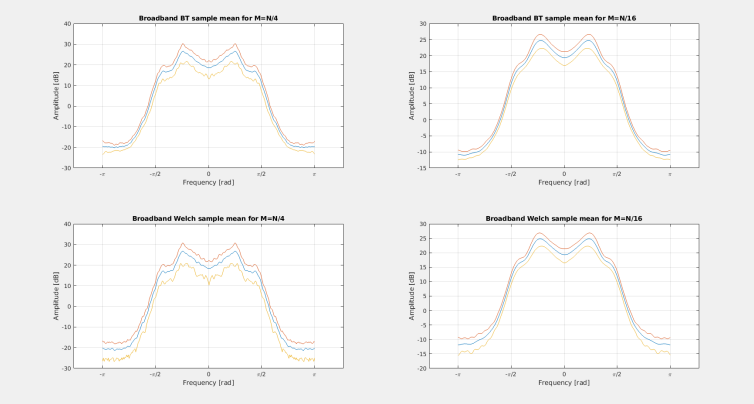

Figure 1: Blackman-Tukey method using Bartlett window and Welch method using a rectangular window with size M=N/4 and M=N/16 for a broadband arma-process. Errorbars are shown with one standard deviation away from the mean.

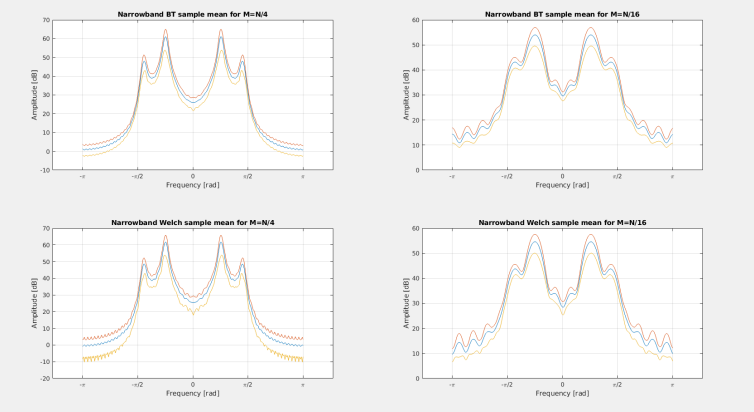

Figure 2: Blackman-Tukey method using Bartlett window and Welch method using a rectangular window with size M=N/4 and M=N/16 for a narrowband arma-process. Errorbars are shown with one standard deviation away from the mean.

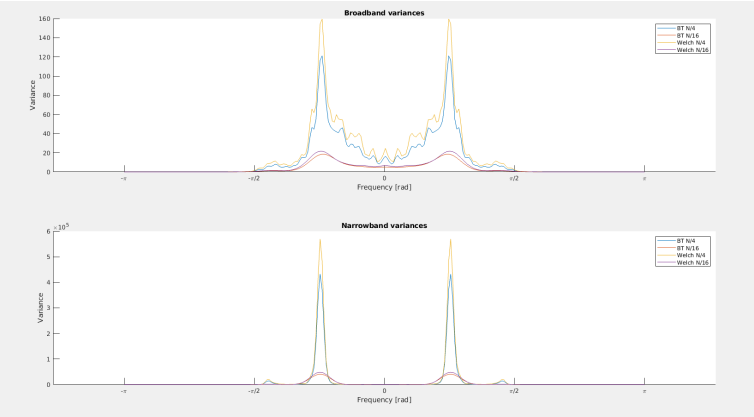

Figure 3: Variances for both methods, BT using a Bartlett window and Welch using a rectangular window, and different window sizes (M=N/4 and M=N/16) and for broadband and narrowband

At first glance, one can notice that the graphs are plotted on the interval $[-\pi, \pi]$ and the power spectral density (PSD) estimation is shown in a dB-scale. Choosing a larger window size, M=N/4=64 compared to M=N/16=16, gives higher peaks for both broadband and narrowband. For the smaller window size the variance is clearly reduced whilst the bias is slightly increased for both methods and and band types. This confirms the theory given in lecture 3, hence we should get worse resolution on this smaller window size as well. A smaller window size also gives a smoother power spectrum estimation. Comparing the two methods, the Welch method have slighly higher variance on all sets and window sizes.

Alternative combinations were also investigated where a good combination was found to use the Bartlett window using the Welch method. This gave smaller variance peaks compared to the latter and also smoother, less oscillating estimates for some cases. Also choosing M=N/8 as window size reduced the variance while keeping it reasonably biased. Hence a the proposed choice would be the Welch method using a Bartlett window of size M=N/8 as it seems to balance the trade-offs. Its estimates for broadband is shown in the lower left subplot in Fig.4, for narrowband in the lower left in Fig.5 and its variances in Fig.6.

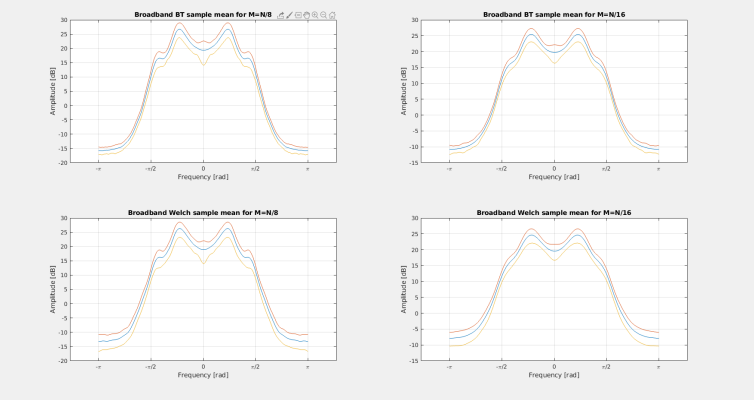

Figure 4: Blackman-Tukey method using Bartlett window and Welch method using a Bartlett window with size M=N/8 and M=N/16 for a broadband arma-process. Errorbars are shown with one standard deviation away from the mean.

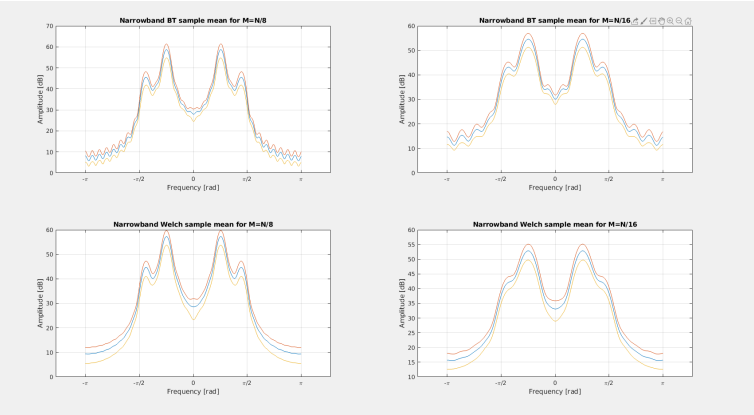

Figure 5: Blackman-Tukey method using Bartlett window and Welch method using a rectangular window with size M=N/8 and M=N/16 for a narrowband arma-process. Errorbars are shown with one standard deviation away from the mean.

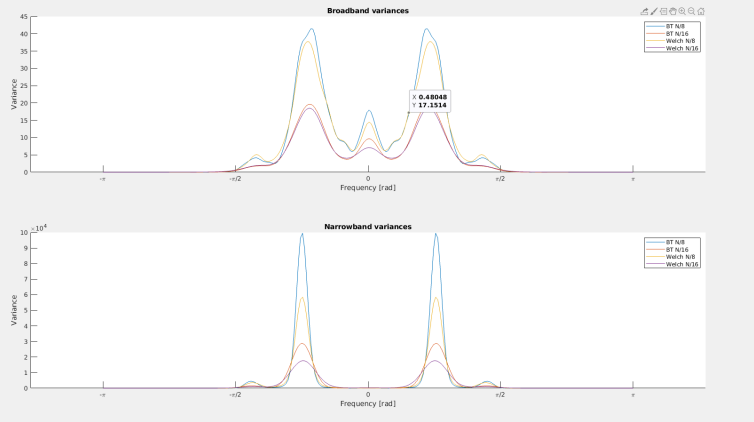

Figure 6: Variances for both methods, BT and Welch using a Bartlett window, and different window sizes (M=N/8 and M=N/16) and for broadband and narrowband