EOSC 454 / 556B Assignment 1: Introduction to inversion

DUE: February 6, 2024

Overview

In this assignment, you will work with the problem of estimating interval velocity from RMS velocity. You are welcome to use the notebook that we wrote together in class as a starting point, but please be mindful that you should rename variables to be more descriptive and that the setup you will be asked to use will be different from the exercise that we completed in class.

Recall that the RMS velocity is related to the interval velocity by

$$V_{rms}^2(t_j) = \frac{1}{t_j}\int_0^{t_{max}} v_{int}^2(u)du \tag{1}$$

and that the analytic inverse is given by

$$v_{int} = V_{rms}(t)\left(1 + \frac{2tV'_{rms}(t)}{V_{rms}(t)}\right)^{1/2} \tag{2}$$

Instructions

Please submit your assignment on canvas. In your submission, please include:

- A write-up: this can be completed in word, LaTeX, Jupyter or similar. Please submit a PDF. For graduate students, I recommend that you work with LaTeX to build familiarity with it. For figure, please include axes labels, a legend (where appropriate), and a figure caption.
- Jupyter notebook / code: please submit a pdf of the notebook you wrote along with either (a) a zip file including the notebook and any additional python files you wrote, or (b) a link to a GitHub repository with your code

You are welcome to work with your classmates (and encouraged to do so!). You each must hand in your own work. Please indicate who you collaborated with in your report.

Q1. In this question, we will set up the analytic solutions for the interval velocity and the RMS velocity. In this example, we will work with an interval velocity that has the form

$$v_{int} = v_0 + \alpha \sin(\omega t) + \beta t \tag{3}$$

where $\omega = 2\pi f$ (with f being the frequency), α is a scalar value and β is a scalar value.

a. Create a function in python that computes v_{int} given values of v_0, α, β, ω and t. Plot the interval velocity using $v_0 = 2000$m/s, $\alpha = 50$m/s, $\beta = 20$m/s^2 and with $0s \leq t \leq 2s$. You can discretize the time axis using 200 points. Don't forget to include axes labels (with units).

b. Describe how each of the parameters v_0, α, β, and ω influence the character of the interval velocity. Labeled plots to illustrate the impact of each are encouraged.

c. Next, set create a function for the analytic solution of V_{rms}. This is a challenging integral. You are welcome to use Wolfram Alpha, Sympy, or any of your other favourite tools to compute the integral. Create two plots: one using the values from part (a) and the second using those same values, but setting the value for $\beta = 0$ (as a hint, you should get the same plot as what we obtained in class when working through this together when $\beta = 0$)

d. Finally, create a function to estimate the interval velocity v_{int} from the RMS velocity. This function should take two vectors: the time and the RMS velocity. It should use a finite difference approach to compute the derivative in equation 2.

Q2. Next, we will explore the implications when we have a finite amount of data.

a. Downsample your data by taking every n-th data point. You can choose what n is, and please be sure to document your choice. Plot the RMS velocity and downsampled values that will be your RMS data.

b. Next, linearly interpolate your downsampled V_{rms} data onto the original time grid and use this to estimate the interval velocity. Plot your results.

c. Now use a cubic spline interpolation and estimate the interval velocity.

d. Choose 3 different decimation levels and plot the V_{rms} data and corresponding recovered v_{int} models using both the linear and cubic spline interpolation. Comment on how the recovered model changes with decimation level.

Q3. In this question, we will explore the impacts when there are noise in the data.

a. Choose a decimation level where the solution was reasonably well recovered when using cubic spline interpolation. Now add Gaussian random noise. Explore using different levels of noise (e.g. with standard deviations of 0.5 m/s, 1 m/s, 1.5m/s). Generate plots that show the noisy data and recovered interval velocity for 3 different noise levels. Comment on how the recovered model changes with noise level. Which regions of the model are most / least affected?

b. Next, we will simplify the model. Set $\alpha = 0$, and explore different levels of random noise. What happens if you add correlated noise (e.g. of the form $cos(\omega t)$)? Include some figures to show your results.

Q4. **[Bonus]** Derive the analytic solution for v_{int} as shown in equation 2 from the expression for the RMS velocity (equation 1).