MATH / CS 11 Q2 - Open questions

Kunal Shrivastav

TOTAL POINTS

9 / 9

QUESTION 1

1 Question 4: Induction 5 / 5

✓ **+ 5 pts** *Full correct argument*

+ 4 pts Correct proof but minor mistake (e.g. not concluding the argument, assuming the claim is true for all $$k$$, etc.)

+ 3 pts Correct setup but didn't prove induction step

+ 2 pts Everything correct up to stating the induction hypothesis

+ 1 pts Little progress

+ 0 pts No progress towards solution

+ 3 pts Using induction correctly without mentioning the conclusion and hypothesis correctly

QUESTION 2

2 Question 5: De Morgan Law 4 / 4

✓ **+ 1 pts** *$$ \overline{ A \cap B} = \{x \in \mathbb{U} | \lnot(p(x) \land q(x))\} $$*

✓ **+ 1 pts** *$$\overline A \cup \overline B = \{ x \in \mathbb{U} | \lnot p(x) \lor \lnot q(x)\}$$*

✓ **+ 2 pts** *How the statements are shown to be equivalent has no single correct answer, what I will demonstrate is equivalence through truth tables. $$\begin{array}{|c|c||c|c|c|c|c|} \hline P & Q & P \land Q & \lnot (P \land Q) & \lnot P \lor \lnot Q & \lnot P & \lnot Q \\ \hline T & T & T & F &F & F & F \\ \hline T & F & F & T & T & F & T \\ \hline F & T & F & T & T & T & F \\ \hline F & F & F & T & T & T & T \\ \hline\end{array} $$*

Since the column for $$\lnot (P \land Q)$$ and $$\lnot P \lor \lnot Q$$ are the same, they are equivalent.

+ 0 pts :(

Name: Kunal Shrivastav

Quiz 2 - V2

Math/CS 11

Wednesday, May 10, 2023

The quiz is out of 14 points. It is to be solved individually. You are not allowed to use any materials, notes, or technology to solve it.

Multiple choice

1. (1 point) True or false. We have $\emptyset \subseteq S$ and $S \subseteq S$ for every set S.

   (a) True

   (b) False

2. (1 point) Which of the following statements is correct:

   (a) A set is an ordered list of elements without repetitions. ×

   (b) A set is an ordered list of elements where we keep track of repetitions.

   (c) A set is an unordered list of elements without repetitions. ×

   (d) A set is an unordered list of elements where we keep track of repetitions. ×

3. (1 point) Let $\mathbb{Z}$ denote the set of all integers. Consider the sets

$$R = \{x \in \mathbb{Z} \,|\, x \text{ is a multiple of } 2\}, \qquad S = \{x \in \mathbb{Z} \,|\, x \text{ is a multiple of } 6\}.$$

   Which of the following is correct: 2, 4, 6, 8, 10, 12, 6, 12, 18,

   (a) $R = S$ (this means both sets are contained in each other.)

   (b) None of the other options are correct.

   (c) $R \subseteq S$

   (d) $S \subseteq R$

Name: ______________________

Open questions

4. (5 points) Use (weak) induction to prove that the formula

$$0 + 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

is true for all $n \geq 0$.

$P(n) = \frac{n \cdot (n-1)}{2}$ is T for all $n \geq 0$

Base Case: $P(0) \equiv 0 = \frac{0(0+1)}{2} = 0$ ✓

Inductive Step:

$P(n+1)$:

$\underbrace{0 + 1 + 2 + 3 + \ldots + n}_{n(n+1)/2} + (n+1) \stackrel{?}{=} \frac{(n+1)(n+2)}{2}$

Assume starting statements is true

$\frac{n(n+1)}{2} + (n+1) \stackrel{?}{=} \frac{(n+1)(n+2)}{2}$

$= \frac{n^2+n}{2} + \frac{2n+2}{2}$

factor it or expand.

$= \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2}$

or $\frac{n^2+3n+2}{2} = \frac{n^2+3n+2}{2}$ ✓

This is equal to so the formula is true for all $n \geq 0$

Name:

5. (4 points) Let $p(x), q(x)$ be predicates with x in a universe $\mathbb{U}$. Consider sets

$$A = \{x \in \mathbb{U} \,|\, p(x)\} \qquad \text{and} \qquad B = \{x \in \mathbb{U} \,|\, q(x)\}.$$

(a) (1 point) Write the definition of $\overline{A \cap B}$ as a set in terms of $p(x)$ and $q(x)$.

$$\boxed{\overline{A \cap B} = \{x \in \mathbb{U} \,|\, \neg p(x) \vee \neg q(x)\}}$$

sign flips according to OR (DML)

because -

$A \cap B = \{x \in \mathbb{U} \,|\, p(x) \wedge q(x)\}$

$\Rightarrow \overline{A \cap B} = \{x \in \mathbb{U} \,|\, \neg(p(x) \wedge q(x))\}$ ← flip/negate everything!

(b) (1 point) Write the definition of $\overline{A} \cup \overline{B}$ as a set in terms of $p(x)$ and $q(x)$.

$$\boxed{\overline{A} \cup \overline{B} = \{x \in \mathbb{U} \,|\, \neg p(x) \vee \neg q(x)\}}$$

because

$\overline{A} = \{x \in \mathbb{U} \,|\, \neg p(x)\}$

$\overline{B} = \{x \in \mathbb{U} \,|\, \neg q(x)\}$

union = or since we are finding the either part of the set not in between/common

(c) (2 points) Prove the De Morgan Law for sets $\overline{A \cap B} = \overline{A} \cup \overline{B}$ using the previous parts.

For $(A \cap B)^c$:

$A \cap B = \{x \in \mathbb{U} \,|\, p(x) \wedge q(x)\}$

$\Rightarrow \boxed{\overline{A \cap B} = \{x \in \mathbb{U} \,|\, \neg p(x) \vee \neg q(x)\}}$

sign flips to OR (DML)

For $(A)^c \cup (B)^c$:

$A \cup B = \{x \in \mathbb{U} \,|\, p(x) \vee q(x)\}$

$\Rightarrow \boxed{\overline{A} \cup \overline{B} = \{x \in \mathbb{U} \,|\, \neg p(x) \vee q(x)\}}$

Equal!

$\Rightarrow \boxed{\overline{A \cap B} = \overline{A} \cup \overline{B}}$