

MATH / CS 11 Quiz 4

Kunal Shrivastav

TOTAL POINTS

7 / 6

QUESTION 1

Question 4 2 pts

- 2 pts No progress towards solution

- 2 pts I could not read your answer

1.1 a 2 / 2

✓ - 0 pts If A and B are sets such that $|A| > |B|$, then no $f: A \rightarrow B$ is injective.

- 1 pts Incomplete answer

- 2 pts No progress towards solution.

1.2 b 1 / 0

✓ + 1 pts Correct

+ 0.5 pts Reasonable attempt

+ 0 pts No progress towards solution

QUESTION 2

Question 5 4 pts

2.1 a 2 / 2

✓ - 0 pts A map $g: A \rightarrow B$ is surjective if for all $y \in B$ there exists some $x \in A$ such that $g(x) = y$.

- 1 pts Incomplete definition

- 2 pts No progress towards solution

- 2 pts I could not read!

2.2 b 2 / 2

✓ - 0 pts Given any y in the codomain we have that $f((y+1)/2) = y$

- 1 pts Incomplete attempt

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Quiz 4 - V2

Math/CS 11

Wednesday, June 7, 2023

The quiz is out of 12 points. It is to be solved individually. You are not allowed to use any materials, notes, or technology to solve it.

Multiple choice

1. (1 point) We say that two sets A and B have the same cardinality, written as $|A| = |B|$, if
 - (a) There exists an injective map $f : A \rightarrow B$.
 - (b) There exists a surjective map $f : A \rightarrow B$.
 - ☒ (c) There exists a bijective map $f : A \rightarrow B$.
 - (d) Both sets are equal.

2. The sets \mathbb{N} and \mathbb{Z} have the same cardinality.
 - (a) True.
 - ☒ (b) False.

3. (1 point) 18 people get on an elevator. They can stop on floors 2, 3, 4, or 5. Can you guarantee that at least five people will get off the elevator on at least one of those floors?
 - (a) Yes, because you can have all 18 people get off on the same floor. ~~X~~
 - (b) Yes, because if you have a limit of four people per floor that's only $4 \times 4 = 16$ people.
 - (c) No, because you can have $1 - 1 - 1 - 14$ so three floors can have fewer than five people.
 - (d) No, because you can have all 18 people get off on the same floor.

16

$5 > 4$

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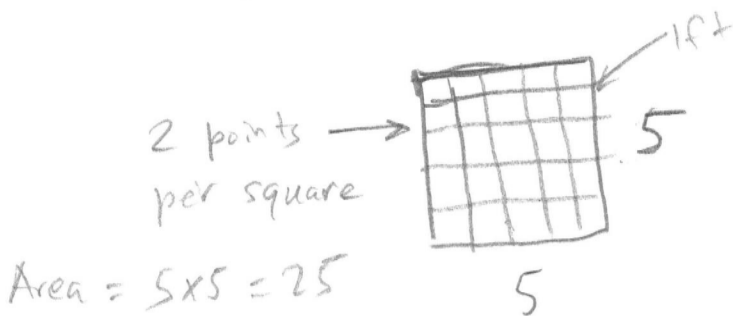
Open questions

4. (3 points)

(a) (2 points) Write the mathematical statement of the pigeonhole principle.

If $|A| > |B|$, assuming $|A|$ & $|B|$ are finite, then for $f: A \rightarrow B$, there is no injective maps.

(b) (1 points) 51 points are scattered inside a square with sides of length 5 ft. Prove that some set of 3 of these points lies inside a square with side length equal to 1 ft.



W.T.S $51 >$ points that can fit in

\Rightarrow This means you will have 24 squares with 2 points and 1 square with 3 points.

25 squares exist in the square with side length of 5. If there is 2 points in each square you will have $51 - (25 \cdot 2) = 1$ point remaining. You will need to put this last point into a tiny square with 2 points ² as no more squares are left.

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5. (4 points) Remember that \mathbb{R} is the set of real numbers, this is, of all numbers.
Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x - 1$.

(a) (2 point) Write the definition of a function $g: A \rightarrow B$ being surjective. $\{A\} = \text{domain}$

$$g: A \rightarrow B \mid \forall b \in B \exists a \in A \quad \text{or } \{B\} = \text{codomain}$$

For all elements b in set B , there exists an element a in set A that maps to it, in the function $f(x) = 2x - 1$.

(b) (2 point) Show that f is surjective.

$$\text{Let } y = f(x)$$

$$\Rightarrow y = 2x - 1$$

$$\Rightarrow \frac{y+1}{2} = x$$

plug in for x $\Rightarrow f\left(\frac{y+1}{2}\right) = 2\left(\frac{y+1}{2}\right) - 1$

$$= y + 1 - 1$$

$$= y \quad \checkmark$$

\Rightarrow The function is surjective.