

MATH / CS 11 Q3 - Open questions

Kunal Shrivastav

TOTAL POINTS

10 / 10

QUESTION 1

Question 3 4 pts

1.1 A: Graph 2 / 2

✓ - 0 pts Correct

- 1 pts your graph includes the line $3x+1$ instead of just integer points on it/ progress towards solution

- 2 pts No progress towards solution

1.2 B: Injectivity 2 / 2

✓ - 0 pts Correct

- 1 pts Not proving objectivity and using examples, horizontal line method

- 1.5 pts Just mentioning the definition

- 2 pts No progress towards the solutio

- 0.5 pts Minor mistakes

- 1 pts Click here to replace this description.

- 1 pts Not real numbers/left as definition

- 2 pts :(

2.3 C: Preimage of 0 2 / 2

✓ - 0 pts Solvable!

$$\begin{array}{c} x-y = 0 \\ x=y \end{array}$$

So the preimage of 0 is

$$f^{-1}(0) = \{ (k,k) \mid k \in \mathbb{Z} \}$$

Being nice. A lot of answers here have wrong notation. Like undefined y 's, defining sets as contained in \mathbb{Z} instead of \mathbb{Z}^2

- 1 pts Left as definition/not whole set

- 2 pts :(/all real numbers

QUESTION 2

Question 4 6 pts

2.1 A: $f(k,0)$ and $f(k,k)$ 2 / 2

✓ - 0 pts $f(k,0) = k-0 = k$

$f(k,k) = k-k = 0$

2.2 B: Image of f 2 / 2

✓ - 0 pts \mathbb{Z} ($(k,0)$ maps to k for any integer k)

Name: Kunal Shrivastav

Quiz 3 - V2

Math/CS 11

Wednesday, May 24, 2023

The quiz is out of 12 points. It is to be solved individually. You are not allowed to use any materials, notes, or technology to solve it.

Multiple choice

1. (1 point) Consider the function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f(n) = 1$. Which of the following is true?

- ☒ (a) It is surjective, but not injective.
- ☐ (b) It is injective, but not surjective.
- ☐ (c) It is both injective and surjective.
- ☐ (d) It is neither injective nor surjective.

$$f(0) = 1$$

$$1 \rightarrow 1$$

$$2 \rightarrow 1$$

$$3 \rightarrow 1$$

$$4 \rightarrow 1$$

2. (1 point) Consider the function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ such that $f(n) = n + 1$. Which of the following is true?

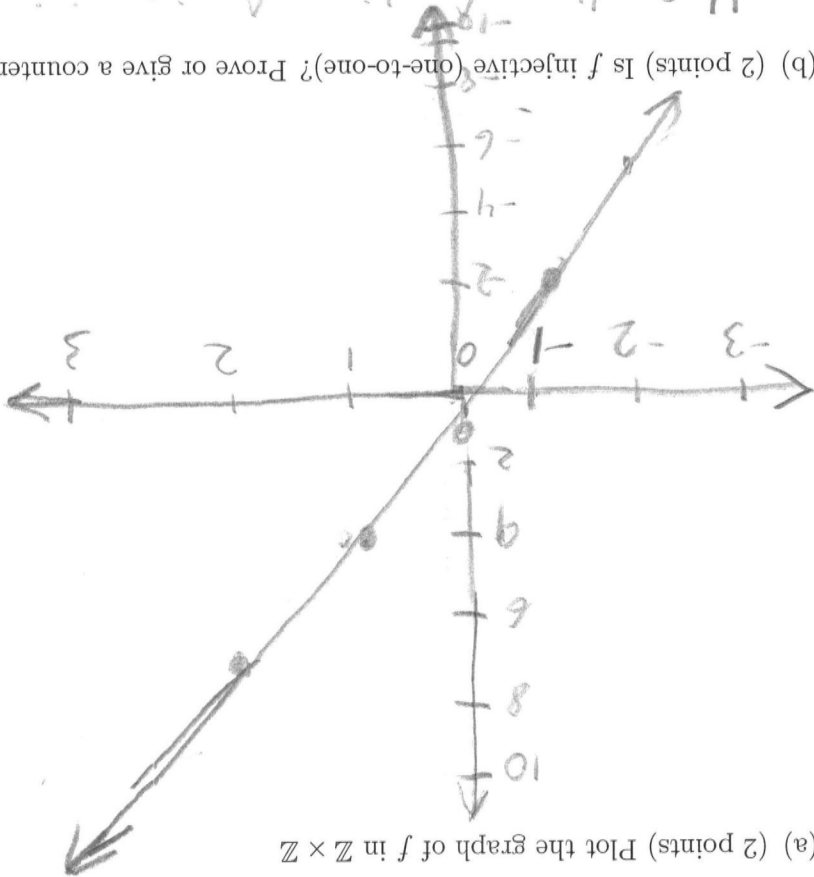
- ☐ (a) It is surjective, but not injective.
- ☐ (b) It is injective, but not surjective.
- ☒ (c) It is both injective and surjective.
- ☐ (d) It is neither injective nor surjective.

Name: Kunal Shrivastava

Open questions

3. (4 points) Consider the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = 3x + 1$.

(a) (2 points) Plot the graph of f in $\mathbb{Z} \times \mathbb{Z}$



$$\begin{aligned} f(0) &= 1 \\ f(1) &= 4 \\ f(2) &= 7 \\ f(3) &= 10 \end{aligned}$$

(b) (2 points) Is f injective (one-to-one)? Prove or give a counterexample.

Yes, the function f is injective because each input has a unique output. So if

Proof: $f(x_1) = f(x_2)$

this means that $x_1 = x_2$

$$\begin{aligned} f(x_1) &= 3x_1 + 1 \\ f(x_2) &= 3x_2 + 1 \\ \Rightarrow 0 &= 3x_1 - 3x_2 \end{aligned}$$

$$\begin{aligned} 3x_2 &= 3x_1 \\ x_2 &= x_1 \end{aligned}$$

The function is injective

Name: Kunal Shrivastav

4. (6 points) Consider the function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ such that $f(x, y) = x - y$.

(a) (2 point) Fix an integer $k \in \mathbb{Z}$. Compute $f(k, 0)$ and $f(k, k)$.

$$f(k, 0) = k - 0 = \boxed{k}$$

$$f(k, k) = k - k = \boxed{0}$$

(b) (2 point) What is the image of f ?

$$\text{Im}(f) = \{\mathbb{Z}\}$$

which means that the image is all integers because for any integer given for x minus any integer y , the result will be an integer.

(c) (2 points) What is the preimage $f^{-1}(0)$ of 0?

$$\text{Preimage } f^{-1}(0) =$$

$$0 = x - y$$

$$\Rightarrow \boxed{x = y}$$

$$\text{Preimage}_{f^{-1}(0)} = \{k \in \mathbb{Z} \mid k = x = y\}$$