MATH / CS 11 Final Exam (Free Response [Extra Credit])

Kunal Shrivastav

TOTAL POINTS

6 / 6

QUESTION 1

1 Induction 3 / 3

✓ **+ 3 pts** *Correct*

+ 1 pts Proving base case

+ 1 pts Stating induction hypothesis correctly and attempting induction step

+ 0 pts No progress towards solution

+ 0.5 pts Writing induction step abstractly (without progress on it)

+ 2 pts Correct induction step without base case

QUESTION 2

2 Inclusion-exclusion principle 3 / 3

+ 1 pts Correctly identifying $$A$$ and $$B$$

+ 1 pts Correctly using IEP (including intersection set) but counting elements wrong (in a consistent way)

✓ **+ 3 pts** *Correct*

+ 0 pts No progress towards solution

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Final exam - extra credit open questions

Introduction to Discrete Structures

Friday 16, June 2023

Instructions: The exam is 2 hours long. There is a total of 34 multiple-choice questions, each having a value of 1 point. There are 2 extra credit open questions below, each worth 3 points. The maximum score for the exam is capped at 34 points. No calculators, cellphones, smartwatches or books allowed.

1. (3 points) Use induction to prove that for all $n \geq 1$ we have

$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}.$$

Base Case:

$A_1 = 1$ — $P(1) = 3(1) - 2 = \frac{1(3(1)-1)}{2}$

(Since $3n-2=1$, when $n=1$) $3-2 = 2/2$ $\quad 1 = 1$ ✓

Inductive step:

$$1 + 4 + 7 + \ldots + (3n-2) + (3(n+1)-2) \stackrel{?}{=} \frac{(n+1)(3(n+1)-1)}{2}$$

(Assume this is true)

$$\Rightarrow \frac{n(3n-1)}{2} + 3n + 1 \stackrel{?}{=} \frac{(n+1)(3n+2)}{2}$$

$$\Rightarrow \frac{3n^2 - n}{2} + \frac{6n+2}{2} \stackrel{?}{=} \frac{(n+1)(3n+2)}{2}$$

$$\Rightarrow \frac{3n^2 + 5n + 2}{2} \stackrel{?}{=} \frac{(n+1)(3n+2)}{2}$$

$$\Rightarrow \frac{3n^2 + 5n + 2}{2} = \frac{3n^2 + 5n + 2}{2}$$

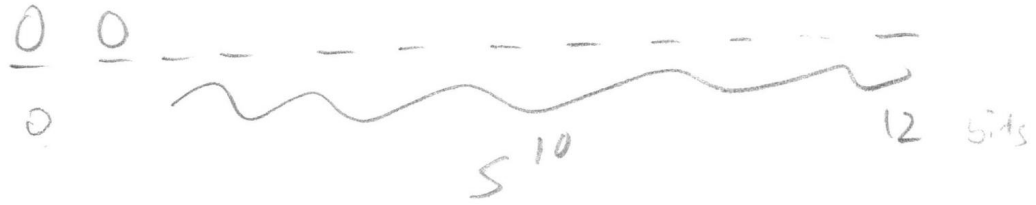
Therefore by weak induction, the statement is true for all $n \geq 1$

2. (3 points) Recall that the inclusion-exclusion principle for finite sets A and B says that

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Use this to count how many strings of length 12 begin OR end with two zeros. You can leave your answer in calculator-ready form.

$S = \{0, 1\}$

$|A|$ 

$|A \cap B|$ 

$|B|$ 

$\Rightarrow$ # of permutations

$= |S|^{10} + |S|^{10} - |S|^8$ $\quad |S| = 2$ (cardinality)

$= 2^{10} + 2^{10} - 2^8$ possible permutations

$= 2^{18}$ possible permutations