# CS513 HW1: Probability

Aidan Fischer - 10447681

I pledge my honor that I have abided by the Stevens Honor System.

## 1)

$P(J) = .2$

$P(S) = .3$

$P(J \cap S) = .08$

$P(J \cup S) = P(J) + P(S) - P(J \cap S) = .2 + .3 - .08 = .42$

### a)

$P(J|S) = \frac{P(J \cap S)}{P(S)} = \frac{.08}{.3} = \frac{4}{15}$

### b)

$P(J) = P(J|S)P(S) + P(J|\bar{S})P(\bar{S})$

$.2 = \frac{4}{15} * .3 + P(J|\bar{S}) * .7$

$.2 = .08 + P(J|\bar{S}) * .7$

$.12 = P(J|\bar{S}) * .7$

$P(J|\bar{S}) = \frac{12}{70}$

### c)

$P(Both|One) = \frac{P(Both \cap One)}{P(One)} = \frac{P(Both)}{P(One)} = \frac{.08}{.42} = \frac{4}{21}$

## 2)

$P(H) = .8$

$P(S) = .9$

$P(S \cup H) = P(S) + P(H) - P(H \cap S) = .91$

$1.7 - P(H \cap S) = 0.91 => P(H \cap S) = .79$

### a)

$P(H - S) = P(H) - P(H \cap S) = .8 - .79 = .01$

### b)

$P(S - H) = P(S) - P(H \cap S) = .9 - .79 = .11$

### c)

$P(\bar{S} \cap \bar{H}) = 1 - P(S \cup H) = 1 - .91 = .09$

## 3)

Independent if $P(J \cap S) = P(J)P(S)$

Since $.08 \neq .2 * .3 = .06$, the events are not independent.

## 4)

Individual rolls are independent events.

### a)

The probability that the second die shows 5 is $\frac{1}{6}$

The probability that the sum is 6 is the sum of the probabilities of the dice rolls that add to 6

Since the rolls are independent, we can just take ordered pairs of dice rolls as the independent events, with each possible pair having a probability of $\frac{1}{36}$

There are 5 pairs that add to 6 (1,5),(2,4),(3,3),(4,2),(5,1) so the probabilty the sum is $\frac{5}{36}$. Since there is only one pair that has the second dice show 5, the chance of both events being true is $\frac{1}{36}$.

Since $\frac{5}{36} * \frac{1}{6} \neq \frac{1}{36}$, the events are not independent.

### b)

By similar reasoning, take the pairs that add to 7 - (1,6),(2,5),(3,4),(4,3),(5,2),(6,2). There are 6, so the chance the sum is 7 is $\frac{1}{6}$. The chance that the first die is 5 is $\frac{1}{6}$. The chance that both happen is once again $\frac{1}{36}$.

Since $\frac{1}{6} * \frac{1}{6} = \frac{1}{36}$, the events are independent.

## 5)

$P(CTX) = .6$

$P(CAK) = .3$

$P(CNJ) = .1$

$P(OTX) = .3$

$P(OAK) = .2$

$P(ONJ) = .1$

### a)

$P(Oil) = P(CTX)P(OTX) + P(CAK)P(OAK) + P(CNJ)P(ONJ) = .6*.3 + .3*.2 + .1*.1 = .18 + .06 + .01 = .25$

### b)

$P(CTX|Oil) = \frac{P(CTX \cap Oil)}{P(Oil)} = \frac{.3*.6}{.25} = .6 * 1.2 = .72$

# 6)

## a)

$\frac{1490}{2201}$

## b)

$\frac{325}{2201}$

## c)

$\frac{203}{711}$

## d)

$P(Survived) = \frac{711}{2201}$

$P(First) = \frac{325}{2201}$

$P(FirstandSurvived) = \frac{203}{2201}$

$\frac{711}{2201} * \frac{325}{2201} \neq \frac{203}{2201}$, so the events are not independent.

## e)

$P(FirstClass \cap Child|Survived) = \frac{6}{203}$

## f)

$P(Adult|Survived) = \frac{654}{711}$

## g)

$P(Adult|Survived) = \frac{654}{711}$

$P(Child|Survived) = \frac{57}{711}$

$P(FirstClass|Survived = \frac{203}{711}$

$P(Adult|Survived)P(FirstClass|Survived) \neq P(Adult \cap FirstClass)$ So they aren't independent.

# 7)

```
AIGenerated=c(970,30,1000)
HumanGenerated=c(70,930,1000)
Total=c(1040,960,2000)
confmatrix=data.frame(AIGenerated,HumanGenerated,Total,row.names=c("Predicted AI","Predicted Human","To
print(confmatrix)
```

```
##                 AIGenerated HumanGenerated Total
## Predicted AI            970             70  1040
## Predicted Human          30            930   960
## Total                  1000           1000  2000
```

Accuracy $= \frac{TP+TN}{TP+FP+FN+TN} = \frac{1900}{2000} = .95$

Precision $= \frac{TP}{TP+FP} = \frac{970}{1040} \approx .93$

Recall $= \frac{TP}{TP+FN} = \frac{970}{1000} = .97$

F1 $= \frac{2*Precision*Recall}{Precision+Recall} = \frac{2*\frac{970}{1040}*.97}{\frac{970}{1040}+.97} \approx .95$