MA541 HW1

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1 Problem 1: Probability

1.1 Problem A

Let X = Number of points at the end of the game = B*rolls of 6 - A*rolls of 2

Then E(X) = E(B*rolls of 6) - A*(rolls of 2) =

E(B*rolls of 6) - E(A * rolls of 2) =

B*E(rolls of 6) - A*E(rolls of 2)

For each roll, the chance of rolling a 1 is 1/6, and the chance of either a 2 or a 6 is also 1/6 each so without derivation, E(rolls of 6) = E(rolls of 2)

Let E(Y) = number of expected rolls of a specific number n $\neq$ 1 until a 1 is rolled

Rolling a 1 on our first try has chance 1/6

Rolling a 1 on our second try has chance 1/6 * 5/6

Rolling a 1 on our third try has chance 1/6 * 5/6 * 5/6

However we need the expected value of rolls of n, not rolls until 1.

So consider this,

If we roll a 1 on the first try, then we get 0

If we roll n on the first try, then we get an additional 1 to our total and roll again (start over)

If we roll m $\neq$ n, 1 then we get no additional value and roll again (start over)

So E(Y) = 1/6 * 0 + 1/6 * (E+1) + 4/6 * E

Solving,

E = 1/6 * E + 1/6 + 4/6 * E

1/6 * E = 1/6

E = 1

Therefore, E(X) = B*1 - A*1 (Since E(Y) is independent of our choice of n, and E(Y) represents our E(rolls of n))

The expected number of points at the end is B - A

1.2 Problem B

Bayes rule is $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

Let A be whether the restaurant we chose was Chipotle

Let B be whether the review we chose was positive

Thus P(A|B) = the probability that the restaurant we chose was Chipotle given the nature of the review was positive

P(B|A) = the probability that the review we chose was positive given the restaurant was Chipotle

P(A) = the overall probability of restaurant Chipotlie

P(B) = the overall probability of review grade positive

We are given that Chipotle gets 200 reviews, with 120 positive and 80 negative

We are given that Five Guys gets 100 reviews, with 40 positive and 60 negative

We are given that P(A) = 0.5

P(B) = positive reviews / all reviews = (120+40)/(200+100) = 160/300 = 16/30 = 8/15

P(B|A) = 120/200 = 12/20 = 3/5

Therefore, P(A|B) = $\frac{\frac{3}{5} * \frac{1}{2}}{\frac{8}{15}} = \frac{3}{10} * \frac{15}{8} = \frac{3}{2} * \frac{3}{8} = \frac{9}{16}$

1.3 Problem C

We are trying to maximize L(p) = $p^4(1-p)^3$

log(L(p)) = $log(p^4(1-p)^3) = log(p^4) + log((1-p)^3) = 4log(p) + 3log(1-p)$

$\frac{dlogL}{dp} = \frac{4}{p} - \frac{3}{1-p}$

We find the critical points of log(L) by equating the derivative with 0. These critical points are the local minima and maxima of log(L). Since log is monotonously increasing, these are the same critical points as L.

$\frac{4}{p} - \frac{3}{1-p} = 0$

$\frac{4}{p} = \frac{3}{1-p}$

$3p = 4 - 4p$

$7p = 4$

$p = 4/7$

To confirm this is a maxima, let's take the second derivative and confirm it is negative at p=4/7

$\frac{d^2logL}{dp^2} = \frac{-4}{p^2} - \frac{3}{(1-p)^2}$

Without calculation, this is obviously negative, and thus p=4/7 is the value of p that maximizes L

Intuitively, this p is the most likely value of p for a weighted coin that would result in the given sequence of flips