

**MATHS QUEST** 7  
for the Australian  
Curriculum





# MATHS QUEST



## for the Australian Curriculum

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### SUPPORT MATERIAL

KYLIE BOUCHER

**jacaranda plus**

First published 2011 by  
John Wiley & Sons Australia, Ltd  
42 McDougall Street, Milton, Qld 4064

Typeset in 10/12pt Times LT

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National Library of Australia  
Cataloguing-in-Publication data

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Author: Catherine Smith . . . [et al.]  
Title: Maths Quest 7 for the Australian Curriculum  
Publisher: Milton, Qld: John Wiley & Sons Australia, 2011.  
ISBN: 978 1 74216 281 2 (pbk)  
978 1 74246 232 5 (ebook)  
978 1 74216 283 6 (teacher edition:pbk.)  
978 1 74246 234 9 (teacher edition:ebook.)  
Notes: Includes index.  
Target audience: For secondary school age.  
Subjects: Mathematics — Textbooks.  
Other authors/  
contributors: Smith, Catherine.  
Dewey number: 510

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Cartography by MAPgraphics Pty Ltd Brisbane and the Wiley Art Studio  
Illustrated by Aptara and the Wiley Art Studio

Typeset in India by Aptara

Printed in Singapore by  
Craft Print International Ltd

10 9 8 7 6 5 4 3 2 1

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# Introduction

Australian Mathematics education is entering a historic phase. A new curriculum offers new opportunities to engage future generations of students in the exciting and challenging world of Mathematics.

The Australian Mathematics Curriculum provides students with essential mathematical skills and knowledge through the content strands of *Number and algebra*, *Measurement and geometry* and *Statistics and probability*. The Curriculum focuses on students becoming proficient in mathematical understanding, fluency, reasoning and problem solving.

*Maths Quest 7 for the Australian Curriculum* is specifically written and designed to meet the requirements and aspirations of the Australian Mathematics Curriculum.

This resource contains:

- a student textbook with accompanying eBookPLUS
- a teacher edition with accompanying eGuidePLUS.

## Student textbook

*Full colour* is used throughout to produce clearer graphs and headings, to provide bright, stimulating photos and to make navigation through the text easier.

*Are you ready?* sections at the start of each chapter provide introductory questions to establish students' current levels of understanding. Each question is supported by a SkillsHEET that explains the concept involved and provides extra practice if needed.

The first chapter, *Numeracy*, consists of six sets of 30 questions. Three sets are calculator-allowed questions and three sets are non-calculator questions. These questions are designed to promote the use of mathematics in real life.

Clear, concise *theory sections* contain *worked examples* and *highlighted important text* and *remember boxes*.

Icons appear for the eBookPLUS to indicate that interactivities and eLessons are available online to help with the teaching and learning of particular concepts.

*Worked examples* in a Think/Write format provide clear explanation of key steps and suggest presentation of solutions.

*Exercises* contain many carefully graded skills and application problems, including multiple-choice questions. Cross-references to relevant worked examples appear with the first 'matching' question throughout the exercises.

Each chapter concludes with a *summary* and *chapter review* exercise containing examination-style questions (multiple-choice, short-answer and extended-response), which help consolidate students' learning of new concepts.

A *glossary* is provided to enhance students' mathematical literacy.

There are two problem-solving chapters designed to encourage students to apply their mathematical skills in non-routine situations.

## Student website — eBookPLUS

The accompanying eBookPLUS contains the entire student textbook in HTML plus additional exercises. Students may use the eBookPLUS on laptops, school or home computers, and cut and paste material for revision, assignments or the creation of notes for exams.

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*Hungry brain* activities provide engaging, whole-class activities to introduce each chapter.

*Test yourself* tests are also available. Answers are provided for students to receive instant feedback.

*Word searches* and *crosswords* are available for each chapter.

*ProjectPLUS* activities (two, which are introduced in the text) provide students with the opportunity to work collaboratively and creatively, online, on a mathematics project.

## Teacher website — eGuidePLUS

The accompanying eGuidePLUS contains everything in the eBookPLUS and more. Two tests per chapter, fully worked solutions to *WorkSHEETS*, the work program and other curriculum advice in editable Word format are provided. Maths Quest is a rich collection of teaching and learning resources within one package.

*Maths Quest 7 for the Australian Curriculum* provides ample material, such as exercises, problem solving questions, projects, worksheets and technology files, from which teachers can assess their students.

## About eBookPLUS

This book features eBookPLUS: an electronic version of the entire textbook and supporting multimedia resources. It is available for you online at the JacarandaPLUS website ([www.jacplus.com.au](http://www.jacplus.com.au)).

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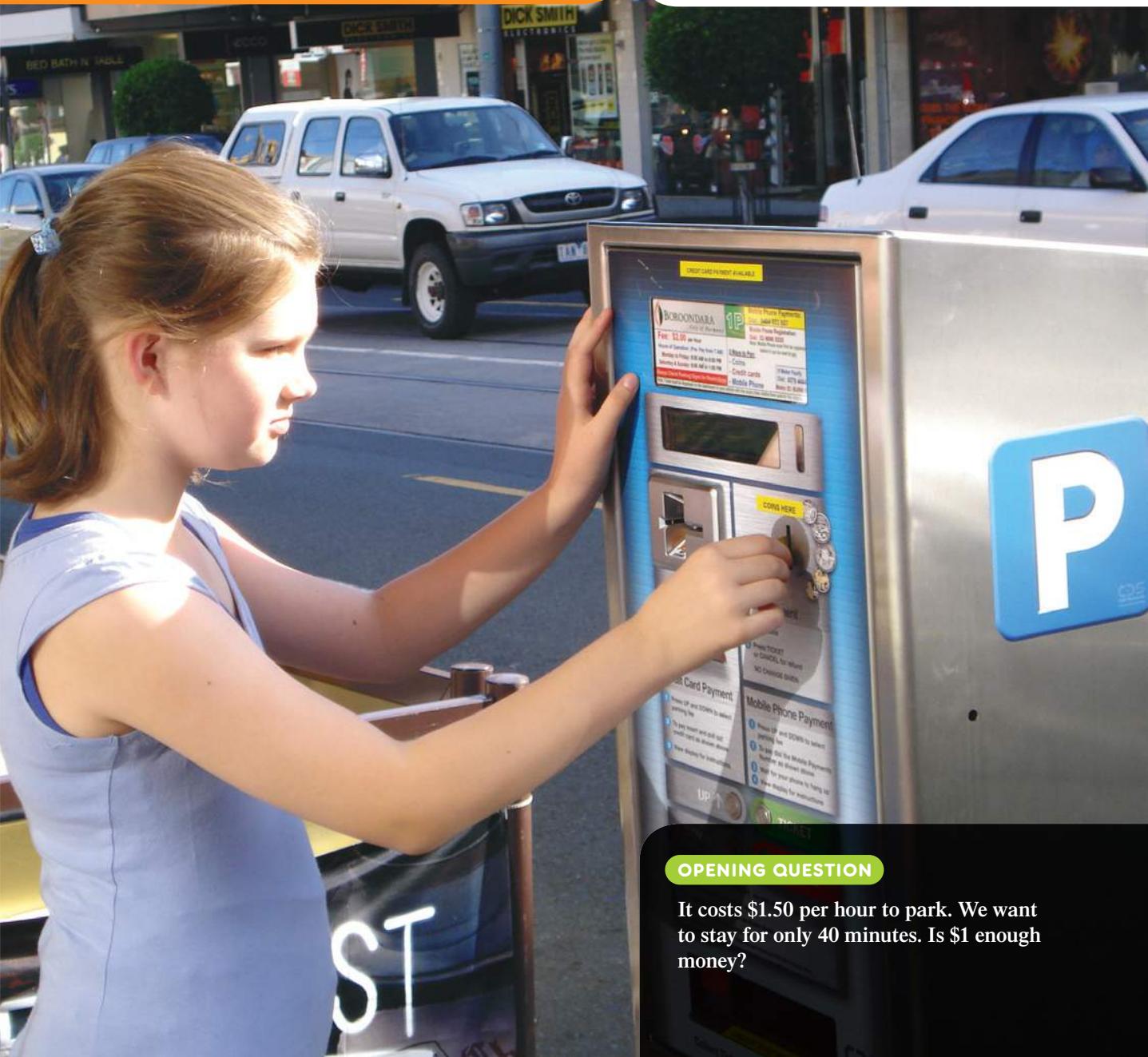
The contributions of the following authors are also acknowledged: Brett Barber, Stephen Broderick, Robert Cahn, Steve Craven, Caroline Denney, Andrea Dineen, John Dowsey, Rodney Ebbage, Chris Evangelou, Dennis Fitzgerald, Ray Hawkins, Stephen Heames, Linda Johnson, Robert Johnson, Carolyn Mews, Mario Panaccio, Poppy Pantelidis, David Phillips, Tony Priddle, Colin Shnier, David Tynan, Jill Vincent, Don Wagstaff, Jenny Watson.

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# 1

# Numeracy



- 1A Set 1A
- 1B Set 1B
- 1C Set 1C
- 1D Set 1D
- 1E Set 1E
- 1F Set 1F

## WHAT DO YOU KNOW?

- 1 List what you know about numeracy.  
Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of numeracy.

## OPENING QUESTION

It costs \$1.50 per hour to park. We want to stay for only 40 minutes. Is \$1 enough money?

In this chapter there are six sets of questions. These questions provide the opportunity for students to use their numeracy skills in everyday situations.

## 1A Set 1A

### Calculator allowed

- 1 Our feet produce approximately 100 litres of sweat a year.  
Approximately how many millilitres of sweat do your feet produce each day?  
**A** 270 mL      **B** 27 mL  
**C** 37 mL      **D** 2.7 mL
- 2 The brain grows at an amazing rate during development. At times during development 250 000 neurones (nerve cells) are added every minute. If neurones multiplied at the given rate for an hour, how many neurones would be added?  
**A** 1 500 000      **B** 15 000 000  
**C** 6 000 000      **D** 600 000
- 3 Nerve impulses to and from the brain travel as fast as 274 metres every second. Which is the closest estimate of the number of metres a nerve impulse travels in an hour?  
**A** 970 000      **B** 980 000      **C** 990 000      **D** 100 000
- 4 On average, human beings blink their eyes 6205 000 times each year. How many times do you blink in a day?
- 5 How much timber would be left if eleven 1.75-m lengths are cut from a 22-m piece of timber?  
**A** 3.25      **B** 2.25  
**C** 2.35      **D** 2.75
- 6 On three tests, a student scored a total of 255 marks.  
If the student improved her performance on each successive test by 4 marks, what was the mark for each test?  
**A** 80, 84, 88      **B** 81, 85, 89      **C** 78, 86, 91      **D** 79, 83, 87
- 7 A student scored 42 out of 50 for a Maths test. What percentage did she obtain for the Maths test?  
**A** 82%      **B** 84%      **C** 75%      **D** 16%



- 8** In a notebook, an author writes a short story of 4000 words. He can type 125 words in three minutes. Keeping the same rate, how long would it take him to type the story?
- A 32 minutes      B  $41\frac{2}{3}$  minutes      C 96 minutes      D  $10\frac{2}{3}$  minutes
- 9** In your soccer team,  $\frac{1}{3}$  of the players ride a bike to practice, 25% walk to practice and the remaining 15 players are driven by their parents. How many players do you have in your team?
- A 42      B 36      C 30      D 21
- 10** Jonathan is saving his money for a two-week surfing trip to Hawaii. He has estimated the costs of his adventure to be:
- i Airfare return: \$2000
  - ii Accommodation: \$1400
  - iii Food: \$500
  - iv Extra costs: \$400
- If Jonathan can earn \$80 per day laying bathroom tiles, how many full days of work does he need to complete in order to save the money for the trip?
- 11** A bottle containing 1 litre of lemonade was poured into big glasses of volume 120 mL and small glasses of volume 80 mL. How much lemonade was left in the bottle after 5 big glasses and 4 small glasses of lemonade had been poured out?

- A 620 mL      B 378 mL      C 280 mL      D 80 mL



- 12** There are 184 students who need to be seated for a dinner in a hall. If all the tables need to be full, select whether the school should use:
- A 18 tables      B 19 tables      C 23 tables      D 31 tables

- 13** The table below shows the cost of road tax for motor cars.

Engine capacity	Road tax
1001 to 1600 cc	75 cents per cc
1601 to 2000 cc	85 cents per cc
2001 to 2600 cc	95 cents per cc

The road tax on a motor car of engine capacity 2400 cc is:

- A 95 cents  
B \$950  
C \$228  
D \$2280



- 14 A waiter works 40 hours in order to earn \$640. Assuming the same rate of pay per hour, calculate how much she will earn in 15 hours.

- 15 There is space in a multi-storey car park for 15 rows of 20 cars on each of the 8 floors. How many spaces are left if 2124 cars have already parked?

- 16 Your friend is training for a triathlon. On one training day she swam 1500 m, rode a bike 47 000 m and ran 14 km.  
How many kilometres were completed?

- 17 \$5320 was raised by a group of students to give to 38 refugees. How much does each refugee receive?

- 18 A plane leaves Melbourne at 0935 and arrives in Osaka at 2007 (Melbourne time). How long did the trip take?

9 : 35      20 : 07

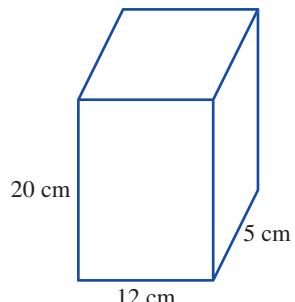
- A 9 hours 42 minutes  
B 10 hours 72 minutes  
C 9 hours 28 minutes  
D 10 hours 32 minutes

- 19 During the day, a patient had her temperature measured as 37.3 °C, 38 °C, 37.8 °C and 38.5 °C. Her average temperature was:

- A 37.8 °C      B 37.9 °C      C 38 °C      D 38.2 °C

- 20 Boxes for packaging are in the shape of cuboid, as shown. A company offers a promotion of an extra 20% free. What is the new volume?

- A 1200 cm<sup>3</sup>  
B 240 cm<sup>3</sup>  
C 1440 cm<sup>3</sup>  
D 81.4 cm<sup>3</sup>



- 21 A couple bought a laptop with an interest free loan of \$2400. They paid a deposit of \$800, and paid off the remainder over the next 5 months. How much were the monthly repayments?

- A \$480  
B \$320  
C \$160  
D \$200



- 22** A restaurant served 315 meals. One-third of the people ate a two-course meal while the remainder ate a three-course meal. How many courses did the chef prepare?  
**A** 840      **B** 210      **C** 630      **D** 1575
- 23** The following is the exchange rate as given by Travelex.

	A\$1
US\$	0.92
€	0.58

You have \$1500 in Australian currency and you want to change \$500 to US currency and the remainder to Euros. How much will you receive of each currency?

- 24** At the Corio supermarket, shopping trolleys are 1 metre long. You work collecting them and stacking them into an area. Each shopping trolley that is added to the stack adds on 25 cm. How long would the line be if you stacked 20 trolleys?  
**A** 6 m      **B** 5.75 m      **C** 60 m      **D** 57.5 m



- 25** At another supermarket, shopping trolleys are also 1 metre long. Your friend works collecting them and stacking them into an area. Each shopping trolley in the stack adds on 0.25 m. How many trolleys could be stacked in a single line of 10 metres?

**A** 37      **B** 36      **C** 35      **D** 34

- 26** There are 128 Year 7 students. Three-quarters of them own a pet. A survey shows that 37 students have cats,  $\frac{1}{3}$  of the students who own a pet have dogs and the remaining students have horses. How many have horses?

- 27** How many 55-cent stamps can you buy for \$10?

**A** 2      **B** 18      **C** 11      **D** 19

- 28** In 1969 the first 12-sided 50-c coin (as shown) came into existence. It weighs 15.55 g. Coins are usually stored in bags of 400 coins for transport.

How much would the bag weigh (in kilograms)?



- 29** Clare buys a fashion magazine for \$10.25 and chocolates for \$3.99. She pays with a \$20 note. How much change does she receive? (Assume rounding to the nearest 5c.)

**A** \$6.75      **B** \$5.80      **C** \$5.76      **D** \$5.75

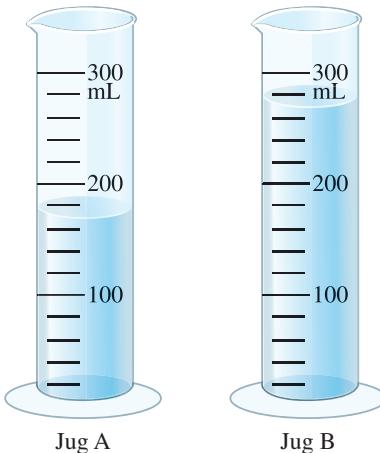
- 30** What number would you insert into the following expression to get an answer of 19?

$$5 \times 3 + \underline{\quad} \div 6$$

# 1B Set 1B

## Non-calculator

- 1 The figure 2010415 is closest to:
- A 2 000 000      B 2 010 000      C 2 020 000      D 2 100 000?
- 2 Jennifer's softball match will take 90 minutes from start to finish, including breaks. Her team must finish by 12.20 pm. What is the latest time the match can start?
- 3 Jug A and Jug B contain different amounts of water. How much *more* water is in Jug B?



- 4 A table of numbers is shown below. What is the probability of choosing a number from the table that is divisible by 7?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

- A  $\frac{1}{25}$       B  $\frac{2}{25}$       C  $\frac{3}{25}$       D  $\frac{4}{25}$
- 5 Human blood travels 675 780 km per week on its journey through the body. How far does the blood travel to the nearest thousand km in one week?

- A 670 000      B 700 000      C 675 000      D 676 000

- 6 The average human heart will beat three thousand million times in its lifetime.

This can be represented as

- A  $3 \times 10^6$       B  $3 \times 10^7$   
C  $3 \times 10^8$       D  $3 \times 10^9$

- 7 A chef made ten apple pies. For one apple pie he used  $\frac{7}{12}$  of a tablespoon of cinnamon and  $\frac{5}{12}$  of a tablespoon of nutmeg. How much more cinnamon than nutmeg was used for all the apple pies?

- A  $\frac{1}{6}$       B  $\frac{5}{6}$       C  $\frac{5}{3}$       D  $1\frac{1}{3}$

- 8 Hue leaves home and rides on a bike 4 km west, then 2 km north, followed by 4 km east and finally 5 km south. Which direction and how far must Hue ride to get back home?

- A South 3 km      B South 1 km      C North 3 km      D North 1 km



- 9 Peta has 36 small buttons and  $n$  large buttons. Which expression shows how many buttons Peta has altogether?

A  $n - 36$       B  $36 - n$   
C 36      D  $n + 36$

- 10 Of the 48 children in the park,  $\frac{3}{4}$  are wearing a hat. How many children are not wearing a hat?

A 12      B 36  
C 1      D 48



- 11 Three quarters of the students on the athletics team do high jump. Of the students who do the high jump, one-half also do the long jump. What fraction of the athletics team does the long jump?

A  $\frac{1}{2}$       B  $\frac{3}{8}$       C  $\frac{2}{3}$

D  $\frac{3}{4}$

- 12 The Ambefield aquifer pumps 15 million litres per day from its bore field. If it is to be shared among the 200 000 residents, how many litres would each person receive per day?

A 750 L  
B 75 L  
C 30 L  
D 7.5 L



- 13 The Ambefield aquifer is estimated to hold 50 billion litres. Assuming no addition or depletion of water, approximately how many days could water be pumped if 15 million litres are extracted each day?

A 34  
B 340  
C 3400  
D 34 000

- 14 A boy borrowed two-fifths of his friend's CD collection. If he borrowed 20 CDs, how many were in the collection?

A 50  
B 70  
C 8  
D 24



- 15 On a map, the distance between Seaspray and Unanda is represented by 10.3 cm. What is the actual distance if 1 cm represents 7 km?

A 17.3 km      B 103 km      C 72.1 km

D 70 km

- 16 Seven cubic metres can be written as:

A  $(7 \text{ m})^3$       B  $7 \text{ m}^3$       C 21 m      D 343 m

- 17 At the local Sunday market, you have sold  $\frac{2}{3}$  of your mangoes. You started with 24 boxes, each containing 20 mangoes. How many mangoes are left?

A 320      B 160      C 16      D 32

- 18 Six thousand people attended a tennis match. If  $\frac{3}{10}$  of them entered with a free pass, how many people paid for a ticket?

A 1800      B 600      C 2000      D 4200

- 19 The difference between the volumes of water in two tanks is 436 L. If the smaller tank has a volume of 7844 L, what is the volume of water in the larger tank?

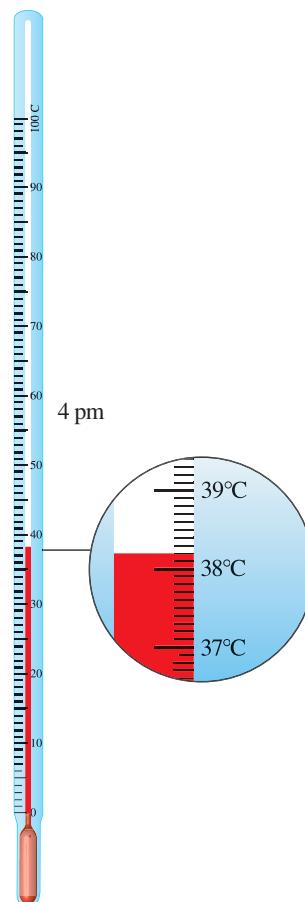
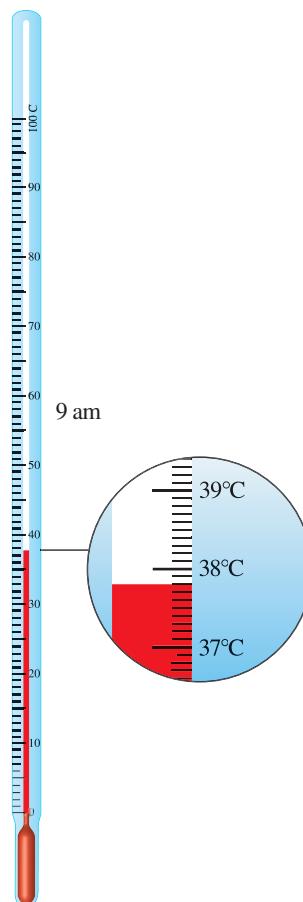
- 20** A nurse takes your temperature at 9 am in the morning and then again at 4 pm in the afternoon (see diagrams). What is the difference in the temperature?

A  $0.3^\circ\text{C}$       B  $0.1^\circ\text{C}$   
C  $0.4^\circ\text{C}$       D  $1.1^\circ\text{C}$

- 21** It takes Lyng 15 minutes to walk to school. If school begins at 9.03 am, what time must she leave home so that she can get to school just as school begins?

- 22** There are 42 apples in a box. If you picked up 28 apples, what fraction of apples would be left in the box?

A One-quarter  
B One-third  
C One-half  
D Three-quarters



- 23** It takes Shannon exactly three hours to walk 18 km. How long would it take Shannon to walk (assuming the same speed) 30 km?

- 24** A piece of rope had 160 cm cut from the end. The rope that remained measured 75 metres. How long was the original piece of rope?

- 25** Jarrod goes to sleep at 2130 and wakes at 6.00 am. His brother goes to sleep at 2215 and sleeps for the same length of time that Jarrod did. What time does he wake up?

- 26** A student works in a shop each day from 14 January to 24 January, inclusive. The student is paid \$40 for each day. How much does he earn in total?

- 27** You can walk 100 m per minute and can run 200 m per minute. How far can you travel if you walk for 20 minutes and run for 15 minutes?

A  $20 \times 100 + 15 \times 200$   
B  $20 \times 200 - 15 \times 200$   
C  $15 \times 100 + 20 \times 200$   
D  $20 \times 200 + 15 \times 100$

- 28** A hard-disc recorder was set to begin taping a show at 1835. It ended at 2015. How long was the show?

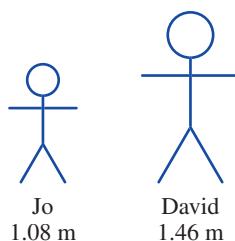
A 110 min      B 100 min      C 85 min      D 80 min

- 29** What is the difference between Jo's height and David's height?

A 0.32 m      B 0.38 m      C 0.42 m      D 0.48 m

- 30** A sketchbook contains 100 pieces of paper and is 14 mm thick. What is the thickness of one sheet of paper?

A 1.4 mm      B 10 mm      C 0.14 mm      D 7.14 mm



# 1C Set 1C

## Calculator allowed

- 1 A school has 434 DVDs in its collection. It has been decided to store them in boxes that hold 32 DVDs each. How many boxes are needed?

A 13

B 13.5

C 13.6

D 14

- 2 A cardboard box weighs 244 grams. It contains 48 smaller boxes, each weighing 127 grams. Which of the following is the closest to the total weight of the large box and its contents?

A 6.3 kg

B 6.31 kg

C 6.309 kg

D 6.4 kg

- 3 A truck is carrying 12 boxes each weighing 250 kg and 20 pipes each weighing 17 kg. How many excess kilograms is the truck carrying if it has a weight limit of 2.8 tonnes?

A 2737

B 310

C 540

D 217



- 4 The following is the exchange rate given by Travelex.

A\$1 = US\$0.92

A\$1 = €0.58

Jinny has just had a holiday and has brought back US\$100 and €150.

If she exchanges this into Australian dollars with the above rates, how much will she receive for each?

- 5 What is the total cost of the petrol shown on the diagram at right?

- 6 Block chocolate is sold in the following sizes.

A 100 g for \$3.20

B 150 g for \$3.95

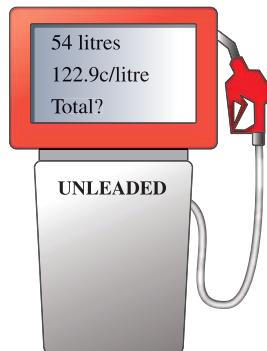
C 250 g for \$6.90

D 300 g for \$8.60

Which is the best buy?

- 7 The following table shows the number of items sold in one year by the HiProduct Company in each of the locations.

In which location was the highest number of items sold per salesperson?



Location	Number of salespeople	Number of items sold
North	52	49 192
Northwest	36	61 164
City	34	23 154
East	20	50 518
South	23	48 048

- 8 A customer has to wait 8 days for a leaking pipe to be repaired. In 1 hour the pipe loses 4 litres of water. The amount of water that will be lost while the customer is waiting is approximately:

A 760 L

B 770 L

C 780 L

D 790 L

- 9 Each number in a sequence is multiplied by 7, and then 1 is added. The first three numbers in the new pattern are 15, 43, 71. What is the fifth number in the original sequence?

A 99

B 14

C 127

D 18

- 10** Molecules generally are approximately one billionth of a metre in size. Which one of the following represents that measurement?

A 1 000 000 000      B 100 000 000  
C 0.0 000 000 001      D 0.000 000 001

- 11** The area of a square is  $9 \text{ cm}^2$ . If the length of each side is doubled, what is the area now?

A  $18 \text{ cm}^2$       B  $36 \text{ cm}^2$       C  $81 \text{ cm}^2$       D  $324 \text{ cm}^2$

- 12** If each small cake needs 10 g of chocolate and 50 g of rice crispies, how many grams of rice crispies are needed for 50 g of chocolate?

- 13** A ship's container is loaded with nine boxes each weighing  $1\frac{1}{4}$  tonnes. If the container itself weighs 850 kg, what is the total weight of the loaded container?

A 2100 kg      B 10 075 kg  
C 11 250 kg      D 12 100 kg



- 14** Jodie earns \$15.00 an hour for 40 hours and \$17.50 per hour for any hours of overtime. During one week she earned \$810. How many hours did she work?

- 15** A survey of 300 students in Year 7 looked at the most popular reality TV shows. How many watched *Biggest loser*?

TV show	Number of people
<i>Australian idol</i>	177
<i>Biggest loser</i>	
<i>So you think you can dance</i>	20%
<i>Who do you think you are?</i>	63

A 40      B 20      C 0      D 54

- 16** Blueberries cost \$14 per kg. You paid \$1.75 for a punnet. How many grams of blueberries did you purchase?

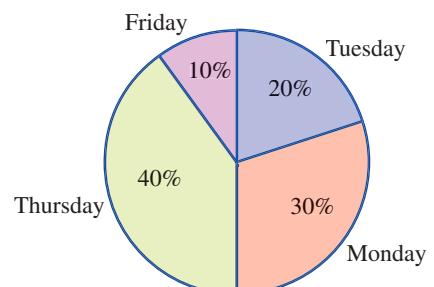
A 125 g      B 0.75 g      C 200 g      D 175 g

- 17** There are 200 Year 7 students. The diagram at right shows the percentage of students who were late on four days of the week.  
Which day had the most latecomers? How many students were late on that day?

- 18** Hayden purchased some content for his MP3 player using the internet. He bought 6 songs at \$1.70 each and 4 music videos at \$3.20 each. If he used his \$20 internet gift card, how much extra did he need to pay?

- 19** The hospital has just installed a new scanner to help in the diagnosis of cancer. The 'state of the art' machine detects a tumour as small as  $0.5 \text{ cm}^3$ . Cancers are made of individual cells, each of which is  $0.000\ 0001 \text{ cm}^3$ . How many cells are needed in a tumour before it is detected by the new scanner?

A  $5 \times 0.000\ 0001$       B  $0.5 \times 100\ 000$   
C  $5 \times 100\ 000$       D  $50 \times 100\ 000$



- 20** A cancer doubles in size every five days. Beginning from one cell, after 50 days how many cells will the cancer contain?

A 10

B 100

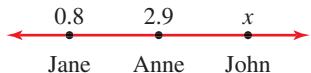
C 512

D 1024

- 21** A room measures 7 m by 3 m. If 4 pages of *The Age* newspaper measure one square metre, how many pages are needed to cover the floor area with no overlap?

- 22** What fraction is halfway between one-quarter and one-half?

- 23** Anne is standing halfway between her two friends on a number line at 2.9 m. Jane is at a position 0.8 on the line and John is at  $x$ . What is John's position?



- 24** A square has an area of 36 square cm. If the sides are enlarged by a factor of 5, what is the area of the new square?

A 188 cm<sup>2</sup>

B 150 cm<sup>2</sup>

C 900 cm<sup>2</sup>

D 875 cm<sup>2</sup>

- 25** Ten teenagers were weighed in kilograms as follows:

47, 53, 51, 62, 58, 53, 48, 52, 67, 54.

What is the mean of these measurements?



- 26** A cube is made up of 125 small cubes. How many cubes will make up one face?

A 5

B 10

C 15

D 25

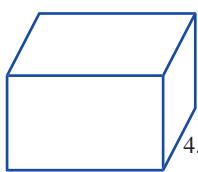
- 27** How much wire is needed to create this cuboid?

A 17.1 cm

B 34.2 cm

C 68.4 cm

D 94.64 cm



- 28** Think of two 2-digit numbers, both less than seventy five.

Both numbers have factors of 6, 3 and 5. One number is half the size of the other number. What are the two numbers?

- 29** Which is the odd one out in the following?

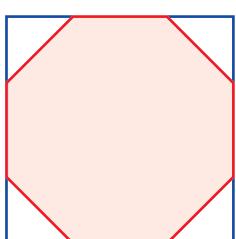
A 0.8

B  $\frac{4}{5}$

C 80%

D  $\frac{16}{80}$

- 30** The regular octagon shown has an area of 125 cm<sup>2</sup>. The square surrounding the octagon has a perimeter of 60 cm. What is the area of one triangle if all the triangles are the same size?



# 1D Set 1D

## Non-calculator

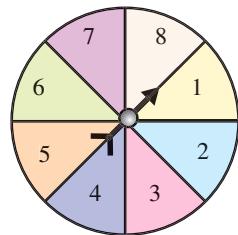
- 1 You spin the following spinner once. What is the probability of selecting a prime number?

A  $\frac{1}{8}$

C  $\frac{4}{8}$

B  $\frac{5}{8}$

D  $\frac{6}{8}$



- 2 In the graph below, when was the biggest change in temperature?

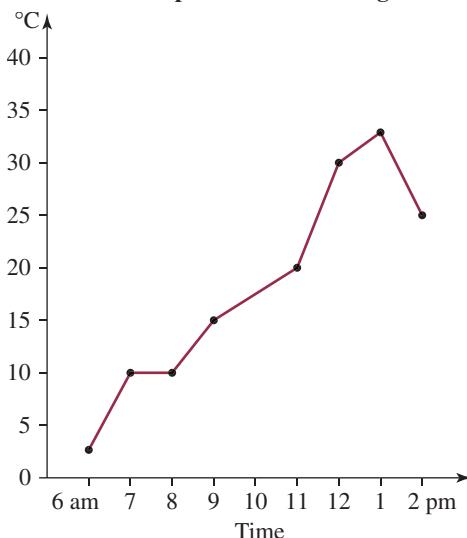
A 6 am–7 am

B 1 pm–2 pm

C 11 am–12 pm

D 10 am–11 am

Temperatures in Geelong



- 3 Jason packed  $5\frac{1}{4}$  shelves in the supermarket and Charlie packed  $8\frac{1}{2}$  shelves. How many more shelves did Charlie pack?

A  $3\frac{1}{2}$

B  $2\frac{1}{4}$

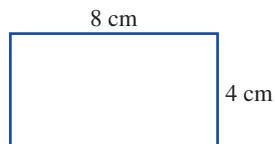
C  $2\frac{1}{2}$

D  $3\frac{1}{4}$

- 4 A class contains 12 boys and 18 girls. Five have red hair, 7 have brown hair and the rest have blond hair. What is the probability that a student chosen has blond hair?

- 5 A cuboid is made from 1-cm cubes, stacked in layers on the rectangular base shown.

If there are 5 layers, how many cubes will be used?



- 6 The diagram shows a cubic object whose side lengths are 20 cm. A smaller cube with a side length of 4 cm has been cut out of the larger cube.

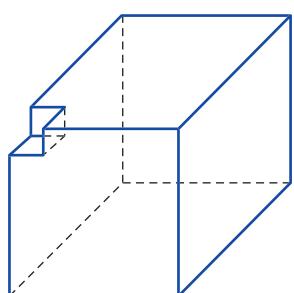
What is the volume of the object that is left?

A  $16 \text{ cm}^3$

B  $56 \text{ cm}^3$

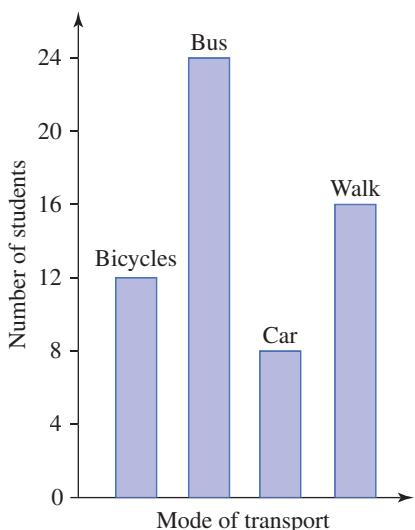
C  $736 \text{ cm}^3$

D  $7936 \text{ cm}^3$



- 7 The graph at right shows how 60 students travel to school.

How many students travel on wheels?



- 8 A rectangle is twice as long as it is wide. The perimeter of the rectangle is 60 cm. What is the area of the rectangle?

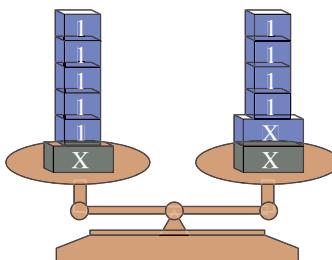
A 100 cm<sup>2</sup>      B 200 cm<sup>2</sup>      C 400 cm<sup>2</sup>      D 60 cm<sup>2</sup>

- 9 Suem bought a packet of chocolates and shared it with six friends.

They each received 4 chocolates and three were left over. If you share the same number of chocolates equally with five friends, how many chocolates will be left over?

A 2      B 1      C 4      D 0

- 10 What is the equation represented by the following balance?



- 11 The Temple of Kukulcan (El Castillo) in Mexico sits on a pyramid-shaped platform. The height of the temple is 6 m and the height to the top of the platform on which it sits is 24 m.

What is the ratio of the height of the platform to the height of the temple?

A 1 : 6      B 6 : 1  
C 6 : 24      D 4 : 1

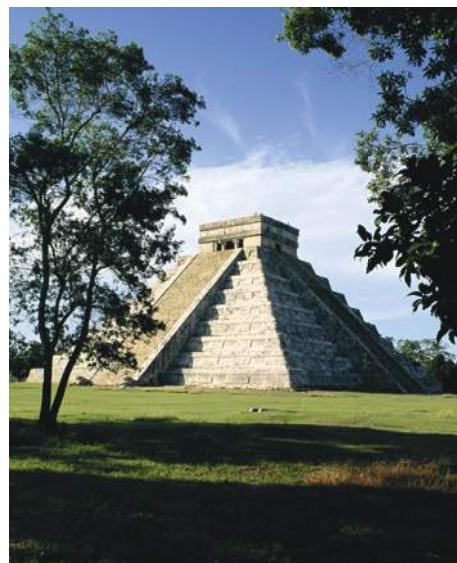
- 12 Which two numbers will have a difference of 2.5?

-3, -2.5, -2, -1.5, -1, -0.5

- 13 Julia walked to her friend's house and arrived at 4.10 pm. It took one hour and thirty-five minutes. What time did she begin her walk?

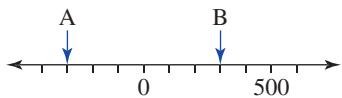
- 14 What is the value of  $3 + 6 \div 3 \times 2$ ?

A 6      B 4  
C 7      D 1.5



- 15 What is the range between A and B?

A 600      B 700  
C 800      D 900



- 16 To make a batch of chocolate mousse, you need 20 g of chocolate. How many batches can you make from 280 g of chocolate?

- 17 On a map, a distance of 30 km is represented by 24 cm. What actual distance would be represented by 16 cm on a map?

- 18 Which of the following is equivalent to  $\frac{3}{4}$ ?

A  $\frac{21}{28}$       B  $\frac{8}{40}$       C  $\frac{33}{45}$       D  $\frac{13}{14}$

- 19 A plane leaves Brisbane at 0935 and arrives in Perth at 1342 (Brisbane time). How long did the trip take?

A 3 hours 42 minutes      B 3 hours 72 minutes  
C 4 hours 28 minutes      D 4 hours 7 minutes

- 20 A kindergarten has 120 students. On one day, 42 students were absent. The fraction of students who were present is closest to:

A one-third      B one-quarter      C two-thirds      D three-quarters

- 21 In this recipe, how many more teaspoons of sugar than salt are needed?

**Marinade for chicken wings**

1 cup soy sauce  
 $\frac{1}{2}$  cup maple syrup  
5 cloves of garlic  
 $1\frac{1}{4}$  teaspoons of salt  
 $1\frac{2}{3}$  teaspoon brown sugar  
2 tablespoons of vinegar

- 22 The attendance at a Geelong Cats football match was given as 38 479. Rounded off to the nearest thousand, this number is:

A 30 000      B 38 500      C 38 000      D 40 000

- 23 The following temperatures were measured in cities worldwide.

What was the largest difference in temperature between any two cities?

Cities worldwide	Temp
Athens	37 °C
Cairo	17 °C
Hobart	8 °C
Stockholm	5 °C
Anchorage	-9 °C
New York	7 °C
Helsinki	-5 °C
Paris	18 °C
Rome	35 °C
Perth	23 °C

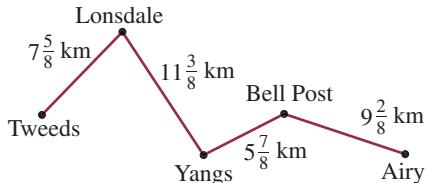
- 24** The temperature at midnight was  $-3^{\circ}\text{C}$ . By 8 am it had risen by  $6^{\circ}\text{C}$ . What was the temperature at 8 am?

A  $6^{\circ}\text{C}$       B  $9^{\circ}\text{C}$       C  $3^{\circ}\text{C}$       D  $8^{\circ}\text{C}$

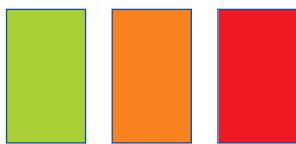
- 25** At 9 pm, the temperature was  $2^{\circ}\text{C}$ . By midnight, the temperature had dropped to  $-5^{\circ}\text{C}$ . By how many degrees had the temperature fallen?

A  $5^{\circ}\text{C}$       B  $-5^{\circ}\text{C}$       C  $3^{\circ}\text{C}$       D  $7^{\circ}\text{C}$

- 26** From Yangs, how much further is it to Lonsdale than to Bell Post? (Give your answer in simplest form.)



- 27** A coloured card is chosen and a coin is tossed. The results recorded. How many different combinations are possible?



A 3

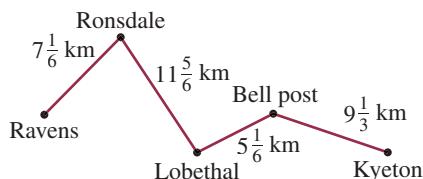
B 9



C 14

D 21

- 28** How far is it from Lobethal to Kyeton?



- 29** Johan Vaaler was the first person to patent the paperclip (1899). If this paperclip is made from 12.5 cm of wire, how much wire do you need to fill a box containing 200 paper clips?

A 250 cm

B 250 m

C 25 m

D 2.5 m



- 30** Which one of the following is another way of writing  $4 \times 4 \times 4$ ?

A  $4^3$

B  $4 + 4 + 4$

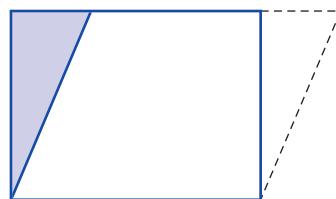
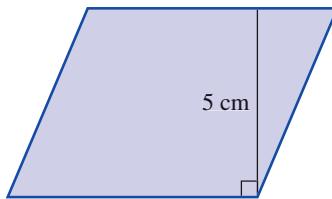
C  $3 \times 4$

D  $3^4$

# 1E Set 1E

## Calculator allowed

- 1 In May 2008 the number of students at a school was 1200. Given that the number of pupils in May 2009 was 1320, find the percentage increase in the number of students.
- 2 A carpenter is looking for the smallest nail diameter in order to construct a cabinet. Choose the smallest diameter from the following sizes:
  - A 0.22 mm
  - B 0.109 mm
  - C 0.072 mm
  - D 0.284 mm
- 3 Jamie earns \$26 from mowing lawns and \$27 from delivering pamphlets each week. Jamie also receives \$20 per week in pocket money from his parents. Each week he keeps \$27 in spending money and banks the rest.  
Calculate the amount that Jamie banks each week.
- 4 Twenty students were surveyed on the type of music they liked. The results were:
  - $\frac{1}{5}$  liked pop
  - $\frac{1}{10}$  liked rock
  - $\frac{1}{4}$  liked hip hop
  - $\frac{1}{20}$  liked classical
  - the rest of the students liked more than one type.  
How many students liked more than one type of music?
- 5 A rectangular luggage label has an area of  $96 \text{ cm}^2$  and a width of 8 cm. What is the perimeter of the label?  
**A** 40 cm      **B** 20 cm      **C** 12 cm      **D** 32 cm
- 6 The area of this parallelogram is  $40 \text{ cm}^2$ . What will be the perimeter of the rectangle created by moving the right-angled triangle as shown?



- 7 A computer costs \$1200. It is reduced by 5% in a sale. What is the sale price of the computer?  
**A** \$1000      **B** \$1195      **C** \$1140      **D** \$60
- 8 Gordon is given a new PIN for his debit card. He calculates that if he divides his PIN by his age, which is 14, and then adds his house number, which is 105, the answer will be 225. What is the PIN?



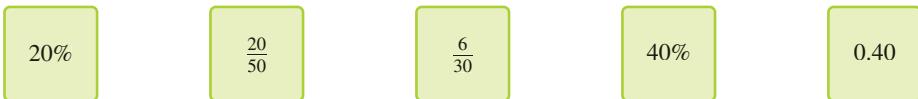
- 9 A skip that holds 3.6 cubic metres has been delivered. You can use a wheelie bin with a capacity of 240 L to transport your rubbish to the skip. How many trips will you make to fill the skip? ( $1 \text{ m}^3 = 1000 \text{ litres}$ .)

A 15      B 14      C 150      D 140

- 10 I buy 12 CDs at a cost of \$65.00. If I return 25% of them to the shop, how much money will I get back?

A \$3.00      B \$59.58      C \$16.25      D \$21.67

- 11 How many of these cards show values the same as  $\frac{2}{5}$ ?



A 4      B 3      C 2      D 1

- 12 When  $2^8$  is divided by  $2^4$ , the answer is:

A  $2^2$       B  $2^4$       C  $2^{12}$       D  $4^4$

- 13 Which of the following has the largest value?

A  $3^2 \times 3^2$       B  $3^3 \times 3^2$       C  $\frac{3^8}{3^2}$       D  $\frac{3^2}{3^8}$

- 14 The table below shows the time that Angela spent running this week.

Day	Time
Monday	35 minutes
Tuesday	25 minutes
Wednesday	30 minutes
Thursday	42 minutes
Friday	50 minutes
Saturday	28 minutes

What was the average time spent running?

A 210 minutes      B 50 minutes      C 35 minutes      D 30 minutes

- 15 Justin buys some ‘Footy’ cards for \$8.95 and soft drink for \$3.80. He pays with a \$20 note. How much change does he receive?

A \$6.75      B \$7.80      C \$7.25      D \$7.76

- 16 The students of Year 7 did a survey on how they travel to school, and recorded their data in a pictograph.

How many students are in Year 7?

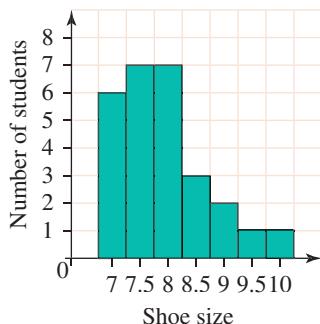
= 6 students      = 3 students

Walk	Bus	Ride	Train	Car
------	-----	------	-------	-----

A 150      B 147      C 141      D 132

- 17** The bar graph represents the shoe sizes of 28 Year 7 students in a mathematics class. What percentage of students has a shoe size greater than 7.5?

A 50%  
B 30%  
C 60%  
D 30%



- 18** Ann, Meg and Sally shared a pizza. Ann ate  $\frac{1}{4}$  and Meg ate  $\frac{1}{6}$  of the pizza. What was left for Sally?

A  $\frac{1}{5}$       B  $\frac{4}{5}$       C  $\frac{5}{12}$       D  $\frac{7}{12}$

- 19** You buy 3.5 kg of chicken at \$15.30 per kg and 2.5 kg of potatoes at 90 cents per kg. You have only \$40. You try to work out by how much you are short.

Which number sentence best describes your problem?

A  $(3.5 \times 15.30 + 2.5 \times 90) - 40$   
B  $40 - 3.5 \times 15.30 + 2.5 \times 0.90$   
C  $3.5 + 2.5 \times 15.30 \times 0.90 - 40$   
D  $40 - (3.5 \times 15.30 + 2.5 \times 0.90)$

- 20** The planet Mercury has a diameter that is 38% less than the planet Earth's diameter, which is 12 756 km. What is the diameter of Mercury? (Round to the nearest kilometer.)

- 21** The table shows the prices for admission to the museum.

Admission	
Adults	\$19.80
Children	\$12.25
Family (2 adults and 2 children)	\$49.75

How much do you save by buying a family ticket instead of buying 2 adults and 2 children?

A \$14.35      B \$17.75      C \$10.75      D \$7.55

- 22** The maximum temperatures in Perth and at Mt Buller were recorded for one week.

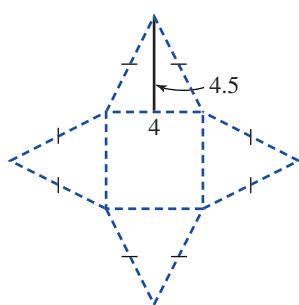
	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Perth	16	12	14	10	8	9	11
Mt Buller	-3	-1	-6	-2	-3	-1	-4

On which day is the temperature difference the greatest?

A Monday      B Wednesday      C Friday      D Sunday

- 23** What is the area of this star-shaped diagram?

A 56 unit<sup>2</sup>  
B 52 unit<sup>2</sup>  
C 61 unit<sup>2</sup>  
D 96 unit<sup>2</sup>



- 24** The following table shows the ages of people on a bus.

	0–19	20–29	30–39	40–49	50–59	60–69	70–79	>79
Male	3	1	5	2	0	4	1	0
Female	2	2	7	1	3	4	0	1

Which is the closest percentage of people on the bus who were less than 30 years old?

- A 20%      B 22%      C 24%      D 26%

- 25** Fill in the missing terms in the following number pattern.

45, \_\_\_\_\_, 37, 33, \_\_\_\_\_, \_\_\_\_\_, ...

- 26** I have a dripping tap that increases my water consumption by 3%. I used 64 kilolitres over a 3-month period and my tap was constantly dripping over that period of time. How many litres could I have saved by fixing my tap?

- 27** The average bath uses two-and-a-half times as much water as a three-minute shower. If the average shower uses 15 litres of water per minute, how many more litres are used by having a bath?

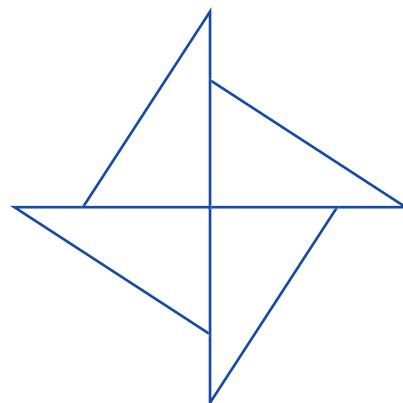
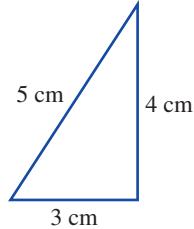
- A 80 L      B 67.5 L  
C 77 L      D 60.5 L

- 28** At 10 drips per minute, a leaking tap wastes 3000 litres of water in a year. At this rate, how many millilitres of water are wasted per drip? (Round to 2 decimal places.)

- A 1.92 mL      B 13 mL  
C 0.57 mL      D 14 mL

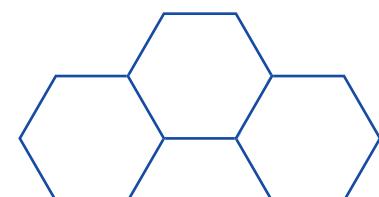
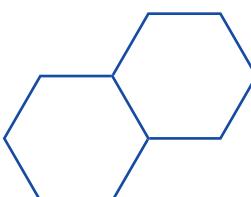


- 29** A star is made by combining four copies of the single triangle shown. What is the star's perimeter?



- A 32 cm      B 26 cm      C 24 cm      D 20 cm

- 30** You have a number of regular hexagon tiles with sides of 6 cm. If 6 tiles are joined in the pattern presented, what will be the perimeter?

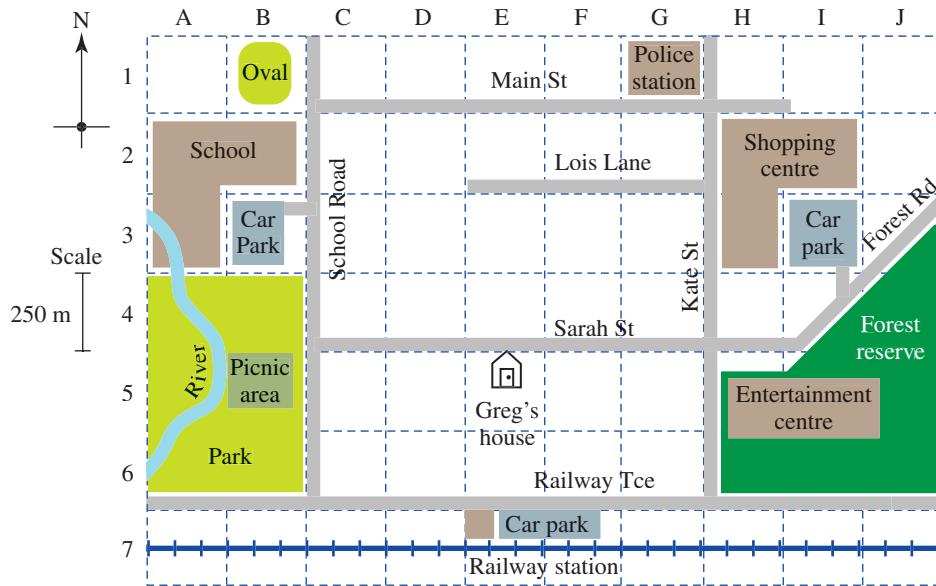


- A 172 cm      B 156 cm      C 164 cm      D 160 cm

# 1F Set 1F

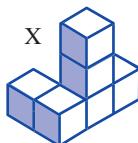
## Non-calculator

Use the following map to answer Questions 1, 2 and 3.



- 1 What is the direct (straight line) distance from the front of Greg's house to the car park entrance near the school?  
**A** 1000 m      **B** 750 m      **C** 625 m      **D** 500 m
- 2 I meet a friend at E6. We walk west 0.75 km, then 1.25 km north and finally 1 km east. Where are we on the map?  
**A** D2      **B** E2      **C** F2      **D** F1
- 3 Which of the following directions will I need to follow to get to the railway station from the house?  
**A** Turn left and walk 625 m, turn left and walk 500 m, turn left and walk 250 m.  
**B** Turn right and walk 625 m, turn left and walk 500 m, turn left and walk 500 m.  
**C** Turn left and walk 625 m, turn left and walk 500 m, turn left and walk 500 m.  
**D** Turn left and walk 625 m, turn right and walk 500 m, turn left and walk 500 m.

- 4 How many of the shapes below are the same as X?



**A** 0



**B** 1



**C** 2



**D** 3



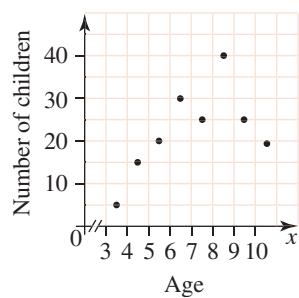
- 5 From the graph, how many of the children are less than 7 years of age?

**A** 30

**B** 25

**C** 70

**D** 27



- 6 Students were asked to state how many SMSs they sent in one day from the tally chart, what are the values of  $x$ ,  $y$  and  $z$ ?

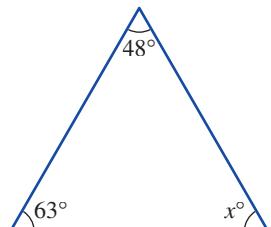
Number of SMS	Tally	Frequency
0		3
1		$x$
2		$y$
3		$z$
4		4
5		6

- A 7, 9 and 16      B 6, 8 and 15      C 8, 10 and 17      D 7, 18 and 48
- 7 What is the value in degrees of the angle  $x$  in the triangle shown at left?

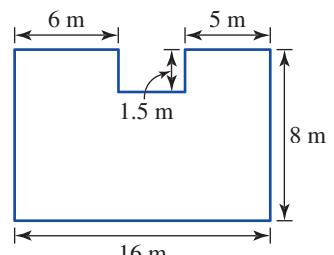
- 8 Jacinta is painting her bedroom. The area of the walls is  $59 \text{ m}^2$ , and each can of paint will cover an area of about  $11 \text{ m}^2$ . Estimate how many cans of paint Jacinta will need to buy.

- A 4 cans      B 1 can  
C 5 cans      D 6 cans

- 9 The diagram shows the plan view of a pool. What is its perimeter?  
A 36.5 m      B 51 m  
C 46 m      D 62 m



- 10 What is the reading on this scale?  
A 1.50      B 1.55  
C 1.75      D 1.8

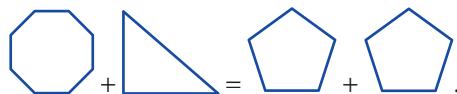
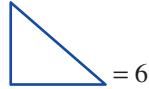


- 11 Which of the following is most likely to be the diameter of a dinner plate?  
A 12 cm      B 25 cm  
C 50 cm      D 75 cm

- 12 You arrive at a backpackers on 29 March and leave on 3 April. If you pay \$45 per night, what is your total bill?  
A \$225      B \$270  
C \$315      D \$45



**13** Let



What number is represented by

**A** 6

**B** 7

**C** 8

**D** 9

**14** In a classroom,  $\frac{6}{10}$  of the students are female. What percentage are boys?

**A** 60%

**B** 50

**C** 35%

**D** 40%

**15** A map has a scale of 1 : 35 000. How much does 1 cm on the map represent in real life?

**A** 350 m

**B** 3 m

**C** 3500 m

**D** 3500 m

**16** You divided 60 sweets between your two brothers in the ratio of their ages. If Nyg is 7 and Tyng is 5, how many does Tyng receive?

**A** 25

**B** 35

**C** 5

**D** 12

**17** One angle of an isosceles triangle measures  $40^\circ$ . If the other two angles are the equal angles, find the measure of each of those angles.

**A**  $50^\circ$

**B**  $140^\circ$

**C**  $70^\circ$

**D**  $40^\circ$

**18** If X represents the number of stamps that Alice had, and Alice then gave 15 stamps to her friend, which expression shows how many stamps Alice has now?

**A**  $15 + X$

**B**  $15 - X$

**C**  $X - 15$

**D**  $X \div 15$



**19** What are the whole numbers that make  $9 - \underline{\quad} > 4$  true?

**A** 0, 1, 2, 3, 4, 5

**B** 0, 1, 2, 3, 4

**C** 0, 1, 2

**D** 5

**20** A calculator display shows 4.33333. Which problem could have given that answer on the calculator?

**A**  $4 + .3$

**B**  $4 \times .3$

**C**  $33\,333 + .4$

**D**  $\frac{13}{3}$

**21** You were given 4 cards each with a fraction on it. Can you order them from the smallest to the largest?



**A**  $\frac{1}{12}, \frac{1}{3}, \frac{3}{6}, \frac{3}{4}$

**B**  $\frac{1}{3}, \frac{3}{4}, \frac{3}{6}, \frac{1}{12}$

**C**  $\frac{1}{12}, \frac{1}{3}, \frac{3}{4}, \frac{3}{6}$

**D**  $\frac{1}{3}, \frac{1}{12}, \frac{3}{6}, \frac{3}{4}$

- 22** 120 people attended the Year 7 production. Tickets cost \$16.70. Half of the people bought programs at \$4.00.  
How much money was collected?



- 23** The table below shows the number of students in a Year 7 class that own mobile phones.

	<b>Own mobile</b>	<b>Do not own mobile</b>
Boys	8	10
Girls	10	4

What is the probability of choosing a girl who owns a mobile phone?

- A  $\frac{7}{16}$       B  $\frac{5}{7}$       C  $\frac{9}{16}$       D  $\frac{5}{9}$

- 24** Which one of the following does not represent  $12 \times 6$ ?

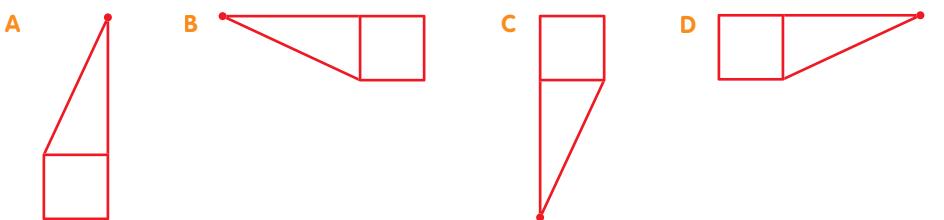
- A  $10 \times 6 + 2 \times 6$   
B  $7 \times 6 + 5 \times 6$   
C  $12 \times 4 + 12 \times 2$   
D  $12 \times 3 + 12 \times 2$

- 25** \$5 is taken off the price of all items for one day only.

How much would you pay for items worth \$32, \$48.85, \$57.95, \$114, \$151.75, \$201.65?

- A \$601.20      B \$575.20      C \$600.35      D \$576.20

- 26** When you rotate the following diagram anticlockwise  $90^\circ$  about the point, which of the following will be the result?



- 27** Which of these has the least value?

- A 20%      B  $\frac{3}{5}$       C  $\frac{1}{10}$       D 0.3

- 28** Hector is forty-nine years old. He was born on 29 February. How many times has he had a birthday?



- 29** A rope measures  $3\frac{2}{3}$  m. Which of the following has the same measurement?

A  $\frac{7}{3}$  m

B  $\frac{8}{3}$  m

C  $\frac{10}{3}$  m

D  $\frac{11}{3}$  m

- 30** Let

$$\text{a } \triangle + \triangle + \textcirclearrowleft = 600$$

$$\text{b } \triangle + \textcirclearrowleft = 350.$$

What do these shapes total when they are combined?

$$\triangle + \triangle + \triangle + \textcirclearrowleft = ?$$

A 750

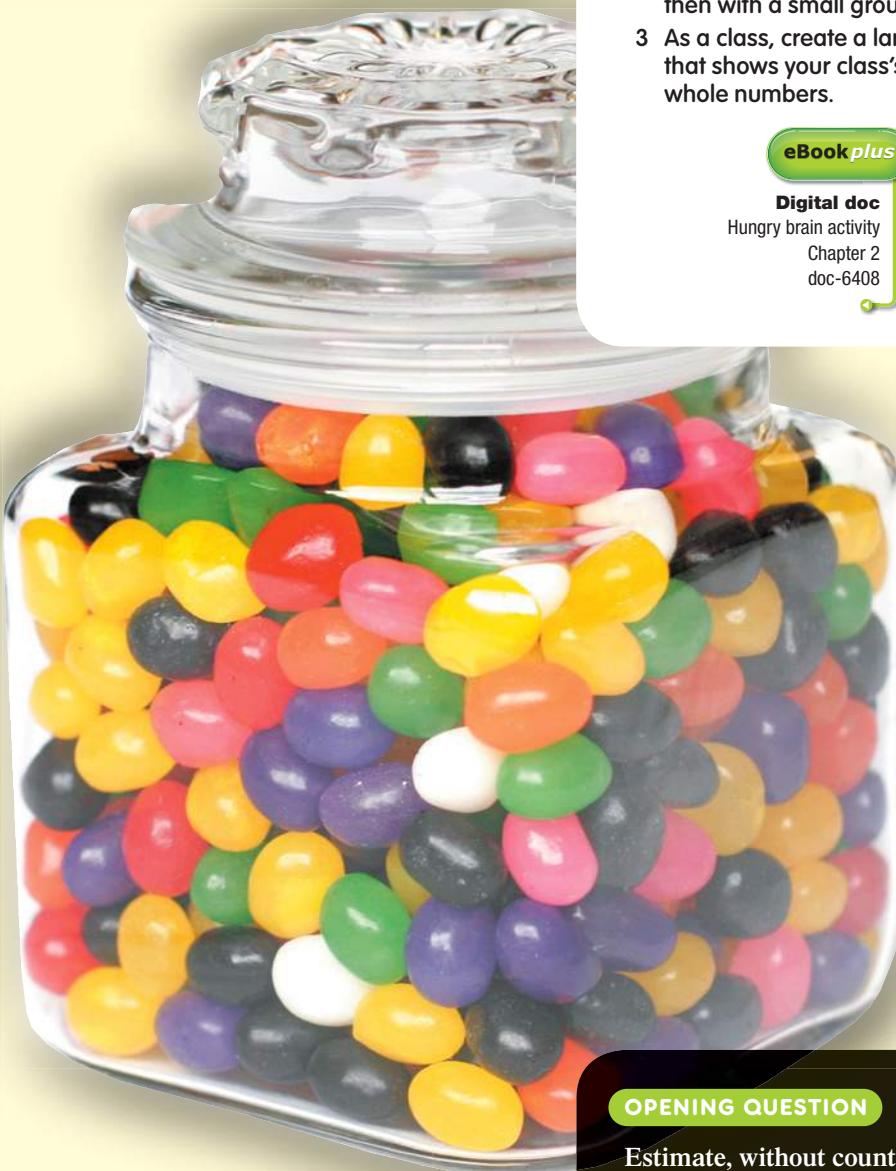
B 850

C 950

D 1050

## 2

# Positive integers



- 2A** Place value
- 2B** Adding and subtracting positive integers
- 2C** Multiplying and dividing positive integers
- 2D** Long division
- 2E** Order of operations
- 2F** Factors and multiples
- 2G** Lowest common multiple and highest common factor
- 2H** Estimation

## WHAT DO YOU KNOW?

- 1 List what you know about whole numbers. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of whole numbers.

eBook plus

Digital doc

Hungry brain activity

Chapter 2

doc-6408

## OPENING QUESTION

Estimate, without counting, the number of lollies in the jar.

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can get extra help by completing the matching SkillsHEET located on your eBookPLUS.

- Place value**
- Digital doc**  
SkillsHEET 2.1  
doc-6409
- 1 What is the place value of the digit shown in red?  
**a** 362      **b** 59 472  
**c** 7308      **d** 238 946
- Ascending and descending order**
- Digital doc**  
SkillsHEET 2.2  
doc-6410
- 2 **a** Place the following numbers in ascending order.  
 919, 99, 991, 199, 19, 91  
**b** Place the following numbers in descending order.  
 12, 102, 21, 120, 201, 112, 121
- Adding and subtracting whole numbers less than 20**
- Digital doc**  
SkillsHEET 2.3  
doc-6411
- 3 Write the answer to each of the following.  
**a**  $7 + 8$       **b**  $12 + 15$   
**c**  $9 + 17$   
**d**  $9 - 5$       **e**  $18 - 11$       **f**  $12 - 3$
- Times tables**
- Digital doc**  
SkillsHEET 2.4  
doc-6412
- 4 Find the answer to each of these as quickly as possible.  
**a**  $4 \times 8$       **b**  $6 \times 5$       **c**  $9 \times 7$   
**d**  $12 \times 3$       **e**  $8 \times 2$       **f**  $11 \times 6$
- Multiplying whole numbers**
- Digital doc**  
SkillsHEET 2.5  
doc-6413
- 5 Work out the answer to each of the following.  
**a**  $45 \times 7$       **b**  $23 \times 14$       **c**  $157 \times 36$
- Dividing whole numbers**
- Digital doc**  
SkillsHEET 2.6  
doc-6414
- 6 Work out the answer to each of the following.  
**a**  $56 \div 4$       **b**  $\overline{)6\overline{)979}}$       **c**  $\frac{651}{7}$
- Order of operations**
- Digital doc**  
SkillsHEET 2.7  
doc-6415
- 7 Find the value of each of the following.  
**a**  $3 + 2 \times 8$       **b**  $6 \times 5 - 4$       **c**  $8 \div 2 + 3 \times 6$
- Multiples**
- Digital doc**  
SkillsHEET 2.8  
doc-6416
- 8 **a** Write the first five numbers for a sequence where you are counting by fours.  
**b** Write the first five numbers for a sequence where you are counting by sixes.
- Factor pairs**
- Digital doc**  
SkillsHEET 2.9  
doc-6417
- 9 Find the missing factor in each of the following factor pairs of 16:  
**a** \_\_\_\_ and 16      **b** 2 and \_\_\_\_      **c** 4 and \_\_\_\_
- Rounding to the first (leading) digit**
- Digital doc**  
SkillsHEET 2.10  
doc-6418
- 10 Round each of the following numbers to the first digit.  
**a** 463 (*Hint:* Is 463 closer to 400 or 500?)  
**b** 2401  
**c** 68

## 2A Place value

- Our number system is based on the number 10, and is known as the Hindu–Arabic system.
- The symbols used today, called **digits**, are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- The advantage of the Hindu–Arabic system over other number systems is that **place value**, or the position of the digit, is important. Each symbol represents a different value depending on where it is placed in the number. This makes the four number operations — adding, subtracting, multiplying and dividing — much easier than they are in other systems.
- This number system also has a symbol for zero, which many number systems do not have. This symbol (0) is very important for establishing the place value in many numbers.
- The place value of each column is shown below. Working from the right, each column has a place value 10 times as great as the one before it.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
1 000 000	100 000	10 000	1000	100	10	1

### WORKED EXAMPLE 1

Write the following numbers in expanded notation.

a 59 176

b 108 009

#### THINK

- a 1 Read the number to yourself, stating the place values.  
2 Write the number as the sum of each place value.
- b 1 Read the number to yourself, stating the place values.  
2 Write the number as the sum of each place value.

#### WRITE

a  $59\,176 = 50\,000 + 9000 + 100 + 70 + 6$

b  $108\,009 = 100\,000 + 8000 + 9$

- Numbers are ordered according to their place values. For whole numbers, the number with the most digits is the greatest in value because it will have the highest place value.
- If two numbers have the same number of digits, then the digits with the highest place value are compared. If they are equal, the next higher place values are compared, and so on.

### WORKED EXAMPLE 2

Write the following numbers in descending order.

858    58    85    8588    5888    855

#### THINK

- 1 Look for the numbers with the most digits.
- 2 There are two numbers with 4 digits. The number with the higher digit in the thousands column is larger; the other is placed second.
- 3 Compare the two numbers with 3 digits. Both have the same hundreds and tens values so compare the units values.
- 4 Compare the two 2-digit numbers.
- 5 Write the answer.

#### WRITE

8588, 5888, 858, 855, 85, 58

**REMEMBER**

1. Numbers are organised by place value.
2. The first place value is the units. Each place value to the left of this column has a place value 10 times as great as the value in the previous column.
3. Numbers can be placed in order by comparing their place values. For whole numbers, the more digits, the greater the number. If two numbers have the same number of digits, they are ordered by comparing the digits with the highest place value.

**EXERCISE****2A****Place value****INDIVIDUAL PATHWAYS****eBookplus****Activity 2-A-1**Whole numbers  
doc-0275**Activity 2-A-2**More whole numbers  
doc-0276**Activity 2-A-3**Advanced whole  
numbers  
doc-0277**FLUENCY**

- 1 WEI** Write the following numbers in expanded notation.

<b>a</b> 925	<b>b</b> 1062	<b>c</b> 28 469	<b>d</b> 43
<b>e</b> 502 039	<b>f</b> 800 002	<b>g</b> 1 080 100	<b>h</b> 22 222

- 2** Write the following numbers in words.

<b>a</b> 765	<b>b</b> 9105	<b>c</b> 90 450	<b>d</b> 100 236
--------------	---------------	-----------------	------------------

- 3** Write the numeral for each of the following.

<b>a</b> Four hundred and ninety-five
<b>b</b> Two thousand, six hundred and seventy
<b>c</b> Twenty-four thousand
<b>d</b> One hundred and nine thousand, six hundred and five

- 4** Give the value of the digits (in brackets) in the distances to the following destinations in these road signs. Use words to state these values.

**a**

- i** Broken Hill (2)
- ii** Yancannia (4)
- iii** White Cliffs (8)

**c**

- i** Katherine (0)
- ii** Alice Springs (2)

**b**

- i** Geelong (2)
- ii** Ballarat (1)
- iii** Bendigo (3)

**d**

- i** Balladonia (8)
- ii** Eucla (7)
- iii** Adelaide (1)

## UNDERSTANDING

- 5 a MC** Which of the following numbers is the largest?  
**A** 4884      **B** 4488      **C** 4848      **D** 4844      **E** 4888
- b** Justify your answer to part **a** using mathematical reasoning.
- 6 MC** Which of the following numbers is the smallest?  
**A** 4884      **B** 4488      **C** 4848      **D** 4844      **E** 4888
- 7 WE2** In each of the following, write the numbers in *descending* order.  
**a** 8569, 742, 48 987, 28, 647      **b** 47 890, 58 625, 72 167, 12 947, 32 320  
**c** 6477, 7647, 7476, 4776, 6747      **d** 8088, 8800, 8080, 8808, 8008, 8880
- 8** In each of the following, write the numbers in *ascending* order.  
**a** 58, 9, 743, 68 247, 1 258 647      **b** 78 645, 58 610, 60 000, 34 108, 84 364  
**c** 9201, 2910, 1902, 9021, 2019, 1290      **d** 211, 221, 212, 1112, 222, 111
- 9** Did you know that we can use the abbreviation K to represent 1000? For example, \$50 000 can be written as \$50K.  
**a** What amounts do each of the following represent?  
**i** \$6K      **ii** \$340K      **iii** \$58K  
**b** Write the following using K as an abbreviation.  
**i** \$430 000      **ii** \$7000      **iii** \$800 000  
**c** Find a job or real estate advertisement that uses this notation.

## REFLECTION

What strategies will you use to place numbers in order?

## 2B Adding and subtracting positive integers

- **Integers** are positive and negative whole numbers, including zero. Integers are denoted by the letter Z.

### Addition of positive integers

- To add larger numbers, write them in columns according to place value and then add them.

## WORKED EXAMPLE 3

Arrange these numbers in columns, then add them.

$$1462 + 78 + 316$$

## THINK

- Set out the sum in columns.
- Add the digits in the units column in your head ( $2 + 8 + 6 = 16$ ). Write the 6 in the units column of your answer and carry the 1 to the tens column, as shown in red.
- Now add the digits in the tens column ( $1 + 6 + 7 + 1 = 15$ ). Write the 5 in the tens column of your answer and carry the 1 to the hundreds column, as shown in orange.
- Add the digits in the hundreds column ( $1 + 4 + 3 = 8$ ). Write 8 in the hundreds column of your answer, as shown in green. There is nothing to carry.
- There is only a 1 in the thousands column. Write 1 in the thousands column of your answer.

## WRITE

$$\begin{array}{r}
 1462 \\
 78 \\
 +316 \\
 \hline
 1856
 \end{array}$$

**WORKED EXAMPLE 4**

Mentally perform the addition  $27 + 19 + 141 + 73$  by finding suitable pairs of numbers.

**THINK**

- 1 Write the question.
- 2 Look for pairs of numbers that can be added to make a multiple of 10. Reorder the sum, pairing these numbers.
- 3 Add the number pairs.
- 4 Complete the addition.

**WRITE**

$$\begin{aligned} 27 + 19 + 141 + 73 \\ = (27 + 73) + (141 + 19) \\ = 100 + 160 \\ = 260 \end{aligned}$$

**Commutative and associative properties for addition**

- The **Commutative law** for addition means that you can add in any order. For example,  $3 + 4$  gives the same result as  $4 + 3$ .
- The **Associative law** for addition, simply stated, is that the order in which additions are calculated is not important and does not change the result. For example,  $2 + 3 + 7$  could be calculated as  $5 + 7 = 12$  or  $2 + 10 = 12$ . This is a very useful property when performing mental calculations.

**Subtraction of positive integers**

- Subtraction is the inverse of addition. To evaluate  $800 - 360$  you could ask the question: ‘What do I need to add to 360 to give 800?’ The answer is 440.
- To subtract numbers in a more formal manner, the digits are lined up vertically according to place value, as we saw for addition.
- The most commonly used method of subtraction is called the **decomposition method**. This is because the larger number is decomposed, or taken apart. The 10 which is added to the top number is taken from the previous column of the same number. So  $32 - 14$  is written as

$$\begin{array}{r} 32 \\ - 14 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 30 + 2 \\ - (10 + 4) \\ \hline \end{array}$$

and becomes

$$\begin{array}{r} 2\cancel{3}^2 12 \\ - 1\ 4 \\ \hline \end{array} \quad \begin{array}{r} 20 + 12 \\ - (10 + 4) \\ \hline \end{array}$$

Now 4 can be taken from 12  
and 10 from 20 to give 18.

$$\begin{array}{r} \cancel{2}^2 12 \\ - 1\ 4 \\ \hline 18 \end{array}$$

- Subtraction is neither *commutative* nor *associative* since the order in which the subtraction is performed is very important. For example,  $6 - 3$  does not give the same result as  $3 - 6$ .

**WORKED EXAMPLE 5**

Evaluate: a  $6892 - 467$

b  $3000 - 467$ .

**THINK**

- a 1 Since 7 cannot be subtracted from 2, take one ten from the tens column of the larger number and add it to the units column of the same number. So the 2 becomes 12, and the 9 tens become 8 tens.

**WRITE**

$$\begin{array}{r} 6\ 8\cancel{9}^8\ 12 \\ - 4\ 6\ 7 \\ \hline 6\ 4\ 2\ 5 \end{array}$$

- 2** Subtract the 7 units from the 12 units ( $12 - 7 = 5$ ).
  - 3** Subtract 6 tens from the 8 remaining tens ( $8 - 6 = 2$ ).
  - 4** Subtract 4 hundreds from the 8 hundreds ( $8 - 4 = 4$ ).
  - 5** Subtract 0 thousands from the 6 thousands ( $6 - 0 = 6$ ).
- b**
- 1** Since 7 cannot be taken from 0, 0 needs to become 10.
  - 2** We cannot take 10 from the tens column, as it is also 0. The first column that we can take anything from is the thousands, so 3000 is decomposed to 2 thousands, 9 hundreds, 9 tens and 10 units.
  - 3** Subtract the units ( $10 - 7 = 3$ ).
  - 4** Subtract the tens ( $9 - 6 = 3$ ).
  - 5** Subtract the hundreds ( $9 - 4 = 5$ ).
  - 6** Subtract the thousands ( $2 - 0 = 2$ ).

$$\begin{array}{r} \textcolor{blue}{2} \cancel{3} \cancel{0} \cancel{0} \cancel{1} \cancel{0} \\ - \quad \textcolor{blue}{4} \textcolor{red}{6} \textcolor{blue}{7} \\ \hline \textcolor{blue}{2} \textcolor{red}{5} \textcolor{blue}{3} \textcolor{red}{3} \end{array}$$

**REMEMBER**

1. Addition and subtraction are the inverses of each other.
2. When we are adding and subtracting, it is important to line up the numbers vertically so that the digits of the same place value are in the same column.
3. When we are adding numbers, the order in which they are added is not important. To simplify an addition, we can find suitable pairs of numbers that will add to a multiple of 10, 100 and so on.
4. Large numbers are subtracted by decomposing the larger number into parts.

**EXERCISE****2B****Adding and subtracting positive integers****INDIVIDUAL PATHWAYS****eBookplus****Activity 2-B-1**Subtracting positive integers  
doc-0278**Activity 2-B-2**Adding and subtracting positive integers  
doc-0279**Activity 2-B-3**Advanced addition and subtraction of positive integers  
doc-0280**FLUENCY**

- 1** Answer these questions, doing the working in your head.

<b>a</b> $7 + 8 =$	<b>b</b> $18 + 6 =$	<b>c</b> $20 + 17 =$
<b>d</b> $80 + 41 =$	<b>e</b> $195 + 15 =$	<b>f</b> $227 + 13 =$
<b>g</b> $420 + 52 =$	<b>h</b> $1000 + 730 =$	<b>i</b> $7300 + 158 =$
<b>j</b> $17\ 000 + 1220 =$	<b>k</b> $125\ 000 + 50\ 000 =$	<b>l</b> $2 + 8 + 1 + 9 =$
<b>m</b> $6 + 8 + 9 + 3 + 2 + 4 + 1 + 7 =$ <b>n</b> $12 + 5 + 3 + 7 + 15 + 8 =$		

- 2** Add these numbers, setting them out in columns as shown. Check your answers using a calculator.

<b>a</b> $\begin{array}{r} 34 \\ + 65 \\ \hline \end{array}$	<b>b</b> $\begin{array}{r} 482 \\ + 517 \\ \hline \end{array}$	<b>c</b> $\begin{array}{r} 1418 \\ + 2765 \\ \hline \end{array}$
<b>d</b> $\begin{array}{r} 68\ 069 \\ 317 \\ 8 \\ + 4254 \\ \hline \end{array}$	<b>e</b> $\begin{array}{r} 696 \\ 3\ 421\ 811 \\ + 63\ 044 \\ \hline \end{array}$	<b>f</b> $\begin{array}{r} 399 \\ 1489 \\ 2798 \\ + 8943 \\ \hline \end{array}$

- 3 WE3** Arrange these numbers in columns, then add them.

- |   |                        |   |                                 |
|---|------------------------|---|---------------------------------|
| a | 137 + 841              | b | 723 + 432                       |
| c | 149 + 562 + 55         | d | 47 + 198 + 12                   |
| e | 376 + 948 + 11         | f | 8312 + 742 + 2693               |
| g | 8 + 12 972 + 59 + 1423 | h | 465 + 287 390 + 45 012 + 72 + 2 |
| i | 1 700 245 + 378 + 930  | j | 978 036 + 67 825 + 7272 + 811   |

Check your answers with a calculator.

- 4 WE4** Mentally perform each of the following additions by pairing numbers together.

- |   |                              |   |                                     |
|---|------------------------------|---|-------------------------------------|
| a | 56 + 87 + 24 + 13            | b | 69 + 45 + 55 + 31                   |
| c | 74 + 189 + 6 + 11            | d | 254 + 187 + 6 + 13                  |
| e | 98 + 247 + 305 + 3 + 95 + 42 | f | 180 + 364 + 59 + 141 + 47 + 20 + 16 |

- 5** Answer these questions without using a calculator.

- |   |                       |   |                |   |              |
|---|-----------------------|---|----------------|---|--------------|
| a | 11 – 5                | b | 20 – 12        | c | 53 – 30      |
| d | 100 – 95              | e | 87 – 27        | f | 150 – 25     |
| g | 820 – 6               | h | 1100 – 200     | i | 1700 – 1000  |
| j | 22 000 – 11 500       | k | 100 – 20 – 10  | l | 75 – 25 – 15 |
| m | 1000 – 50 – 300 – 150 | n | 80 – 8 – 4 – 5 | o | 24 – 3 – 16  |
| p | 54 – 28               | q | 78 – 39        |   |              |

- 6** Answer these questions which involve adding and subtracting whole numbers.

- |   |                     |   |                              |   |                   |
|---|---------------------|---|------------------------------|---|-------------------|
| a | 10 + 8 – 5 + 2 – 11 | b | 40 + 15 – 35                 | c | 16 – 13 + 23      |
| d | 120 – 40 – 25       | e | 53 – 23 + 10                 | f | 15 + 45 + 25 – 85 |
| g | 100 – 70 + 43       | h | 1000 – 400 + 250 + 150 + 150 |   |                   |

- 7 WE5** Evaluate:

- |   |           |   |                  |   |                 |
|---|-----------|---|------------------|---|-----------------|
| a | 98 – 54   | b | 167 – 132        | c | 47 836 – 12 713 |
| d | 149 – 63  | e | 642 803 – 58 204 | f | 3642 – 1811     |
| g | 664 – 397 | h | 12 900 – 8487    | i | 69 000 – 3561   |

Check your answers using a calculator.

- 8** Evaluate:

- |   |                        |
|---|------------------------|
| a | 406 564 – 365 892      |
| b | 2683 – 49              |
| c | 70 400 – 1003          |
| d | 64 973 – 8797          |
| e | 27 321 – 25 768        |
| f | 518 362 – 836          |
| g | 812 741 – 462 923      |
| h | 23 718 482 – 4 629 738 |

Check your answers using a calculator.

### eBookplus

#### Digital docs

Spreadsheet  
Adding numbers

doc-6419

Spreadsheet  
Subtracting numbers

doc-2086



### UNDERSTANDING

- 9** Julia was performing in a ballet and needed to buy a tutu, pointe shoes and white tights. How much did she spend in total on her costume?
- 10** The Sydney telephone directory has 1544 pages in the A–K book and 1488 pages in the L–Z book. How many pages does it have in total?

- 11** Nathan has taken his sister and her friends to lunch for her birthday. What will be the total cost of the items shown?

- 12** Hussein's family drove from Melbourne to Perth. In the first 2 days they drove from Melbourne to Adelaide, a distance of 738 kilometres. After a couple of days' sightseeing in Adelaide, Hussein's family took a day to drive 321 kilometres to Port Augusta, and another to drive the 468 kilometres to Ceduna.

They drove 476 kilometres to Norseman the following day, then took 3 more days to travel the remaining 1489 kilometres to Perth.

- a** How many days did it take for Hussein's family to drive from Melbourne to Perth?
- b** How far is Norseman from Melbourne?
- c** How many kilometres is Perth from Melbourne?
- d** How far is Adelaide from Perth?

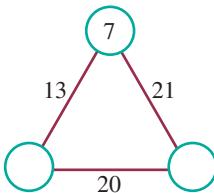
- 13** Of all the world's rivers, the Amazon in South America and the Nile in Africa are the two longest. The Amazon is 6437 kilometres in length and the Nile is 233 kilometres longer than the Amazon. How long is the Nile River?

- 14** An **arithmagon** is a triangular figure in which the two numbers at the end of each line add to the number along the line.

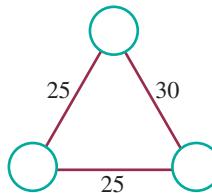
An example is shown at right.

Work in pairs to try and solve each of these arithmagons.

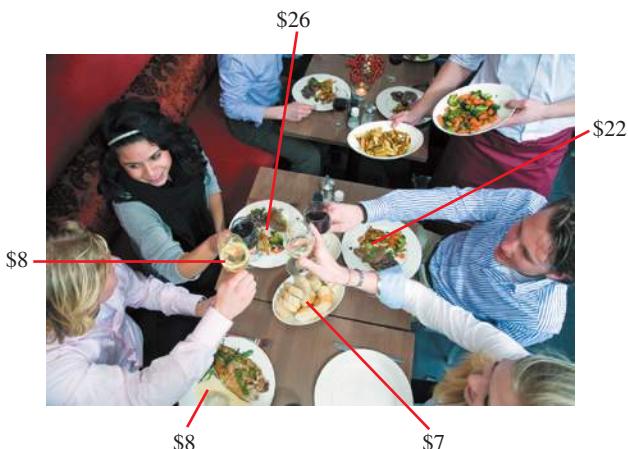
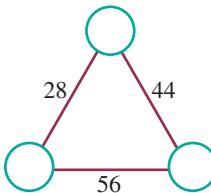
**a**



**b**



**c**



- 15** Hayden received a box of 36 chocolates. He ate 3 on Monday, 11 on Tuesday and gave 7 away on Wednesday. How many did he have left?

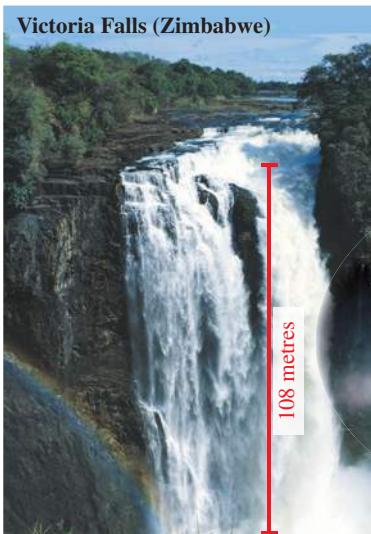
- 16** A crowd of 87 043 attended the 2009 AFL match between Carlton and Richmond. If 45 492 people supported Carlton and the rest supported Richmond, how many supporters did Richmond have?

- 17** A school bus left Laurel High School with 31 students aboard. Thirteen of these passengers alighted at Hardy Railway Station. The bus collected 24 more students at Hardy High School and a further 11 students disembarked at Laurel swimming pool. How many students were still on the bus?

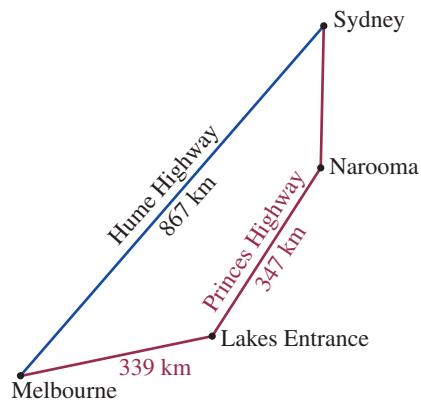
- 18** The most commonly spoken language in the world is Mandarin, spoken by approximately 575 000 000 people (in north and east central China). Approximately 360 000 000 people speak English and 140 000 000 Spanish.

- a** How many more people speak Mandarin than English?
- b** How many more people speak English than Spanish?

- 19** The photographs show three of the highest waterfalls in the world. How much higher are the:
- Victoria Falls than the Iguazu Falls?
  - Iguazu Falls than the Niagara Falls?
  - Victoria Falls than the Niagara Falls?
  - Explain how you obtained your answers.



- 20** Lucy and Ty were driving from Melbourne to Sydney for a holiday. The distance via the Hume Highway is 867 kilometres, but they chose the more scenic Princes Highway even though the distance is 1039 kilometres. They drove to Lakes Entrance the first day (339 kilometres), a further 347 kilometres to Narooma on the second day and arrived in Sydney on the third day.
- How much further is Melbourne to Sydney via the Princes Highway than via the Hume Highway?
  - How far did Lucy and Ty travel on the third day?



### REASONING

- 21** Can you fill in the blanks? The \* can represent any digit.

a 
$$\begin{array}{r} 6*8 *2* \\ - 488\ 417 \\ \hline *49\ 9*4 \end{array}$$

b 
$$\begin{array}{r} 3*9* \\ - *6*5 \\ \hline 1*07 \end{array}$$

- 22** Without using a calculator, and in less than 10 seconds, find the answer to  $6\ 849\ 317 - 999\ 999$ .

- 23** A beetle has fallen into a hole that is 15 metres deep. It is able to climb a distance of 3 metres during the day but at night the beetle is tired and must rest. However, during the night it slides back 1 metre. How many days will it take the beetle to reach the top of the hole to freedom?

- 24** Each letter below stands for a digit from 1 to 9. Find the value of each letter so that the addition statement is true.

BIG + JOKE = HAHA

### eBookplus

Digital doc  
Worksheet 2.1  
doc-0273

### REFLECTION

In what situations do you use addition and subtraction of positive integers in everyday life?

## 2C Multiplying and dividing positive integers

### Multiplying positive integers

- Short multiplication can be used when multiplying a large number by a single digit number.

#### WORKED EXAMPLE 6

Calculate  $1456 \times 5$ .

##### THINK

- 1 Multiply the units ( $5 \times 6 = 30$ ). Write the 0 and carry the 3 to the tens column.
- 2 Multiply the tens digit of the question by 5 and add the carried number ( $5 \times 5 + 3 = 28$ ). Write the 8 in the tens column and carry the 2 to the hundreds column.
- 3 Multiply the hundreds digit by 5 and add the carried number ( $5 \times 4 + 2 = 22$ ). Write the last 2 in the hundreds column and carry the other 2 to the thousands column.
- 4 Multiply the thousands digit by 5 and add the carried number ( $5 \times 1 + 2 = 7$ ). Write 7 in the thousands column of the answer.

##### WRITE

$$\begin{array}{r}
 2124^35\ 6 \\
 \times \quad \quad \quad 5 \\
 \hline
 7280
 \end{array}$$

- The **Distributive law** only applies to multiplication. It is defined as  $a(b + c) = ab + ac$ . Put simply, it helps with mental calculation because it means that  $8 \times 13$  can be thought of as

$$\begin{aligned}
 8 \times 13 &= 8 \times (3 + 10) \\
 &= 8 \times 3 + 8 \times 10 \\
 &= 24 + 80 \\
 &= 104
 \end{aligned}$$

### Mental strategies for multiplication

- In many cases it is not practical to use pen and paper or even a calculator to perform multiplication.
- Some mental strategies can make multiplication easier.
- Multiplication is **commutative** because the order in which numbers are multiplied is not important. For example,  $3 \times 8 \times 6 = 6 \times 3 \times 8$ .
- Multiplication is **associative** because larger numbers can be paired up for multiplication.

$$\begin{aligned}
 \text{For example, } 2 \times 17 \times 5 &= 17 \times (2 \times 5) \\
 &= 17 \times 10 \\
 &= 170
 \end{aligned}$$

By finding the multiplication pair that made 10, this question has been made simpler.

#### WORKED EXAMPLE 7

Use mental strategies to calculate  $4 \times 23 \times 25$ .

##### THINK

- 1 Write the question.
- 2 Rearrange it looking for a number pair that makes a simpler multiplication.
- 3 Mentally calculate  $4 \times 25$ .
- 4 Mentally calculate the final answer.

##### WRITE

$$\begin{aligned}
 4 \times 23 \times 25 &= 23 \times (4 \times 25) \\
 &= 23 \times 100 \\
 &= 2300
 \end{aligned}$$

**WORKED EXAMPLE 8**

Use a mental strategy to calculate  $34 \times 200$ .

**THINK**

- 1 Write the question.
- 2 Break 200 into  $2 \times 100$ .
- 3 Calculate  $34 \times 2$ .
- 4 Calculate  $68 \times 100$ .

**WRITE**

$$\begin{aligned}34 \times 200 \\= 34 \times 2 \times 100 \\= 68 \times 100 \\= 6800\end{aligned}$$

- If both numbers are **multiples** of 10, 100 and so on, ignore the zeros, multiply the remaining numbers then add the total number of zeros to the answer. For example,  $900 \times 6000 = 5400000$ .
- Consider the multiplication  $9 \times 58$ . This multiplication can be regarded as:  $10 \times 58 - 1 \times 58$ . Using this, the answer can be mentally calculated by multiplying 58 by 10 then subtracting 58 from the answer.

**WORKED EXAMPLE 9**

Use mental strategies to calculate  $77 \times 9$ .

**THINK**

- 1 Write the question.
- 2 Use the strategy of ‘multiply by 10’.
- 3 Calculate  $77 \times 10$  and subtract 77.

**WRITE**

$$\begin{aligned}77 \times 9 \\= 77 \times 10 - 77 \\= 770 - 77 \\= 693\end{aligned}$$

**eBookplus****eLesson**  
Identifying patterns  
eles-0745**Dividing positive integers**

- Division is the inverse of multiplication. To evaluate  $56 \div 8$ , you could ask the question: ‘By which number do I multiply 8 to get 56?’ The answer is 7.
- Short division can be used when dividing by numbers up to 12.

**WORKED EXAMPLE 10**

Calculate  $89656 \div 8$ .

**THINK**

- 1 Divide 8 into the first digit and carry the remainder to the next digit; 8 goes into 8 once. Write 1 above the 8 as shown. There is no remainder.
- 2 Divide 8 into the second digit and carry the remainder to the next digit; 8 goes into 9 once with 1 left over. Write 1 above the 9 and carry 1 to the hundreds column.
- 3 Divide 8 into the third digit and carry the remainder to the next digit; 8 goes into 16 twice with no remainder. Write 2 above the 6 as shown.
- 4 Divide 8 into the fourth digit and carry the remainder to the next digit; 8 doesn’t go into 5. Write the 0 above the 5. Carry 5 to the next digit.
- 5 Divide 8 into 56; 8 goes into 56 seven times.
- 6 Write the answer.

**WRITE**

$$\begin{array}{r}11207 \\8\overline{)89656}\end{array}$$

$$89656 \div 8 = 11207$$

## Dividing numbers that are multiples of 10

- When we are dividing numbers that are multiples of 10, an equivalent number of zeroes can be cancelled from both the **dividend** and the **divisor**.
- The dividend is the quantity to be divided.
- The divisor is the number by which the dividend is to be divided.

### WORKED EXAMPLE 11

Calculate  $48\ 000 \div 600$ .

#### THINK

- Write the question.
- Write the question as a fraction.
- Cancel as many zeros as possible, crossing off the same number in both numerator and denominator.
- Perform the division.
- Write your answer.

#### WRITE

$$48\ 000 \div 600$$

$$\begin{aligned} &= \frac{48\ 000}{600} \\ &= \frac{480}{6} \\ &= \frac{080}{6} \end{aligned}$$

$$48\ 000 \div 600 = 80$$

#### REMEMBER

- The basis for all multiplication work is to know your multiplication tables up to 12.
- You should know how to perform both short and long multiplication.
- Some multiplications can be done mentally using various strategies.
  - Look for a multiplication pair that will make a multiple of 10, 100, and so on.
  - To multiply numbers that are multiples of 10, ignore the zeros, perform the multiplications, and then add the total number of zeros to your answer.
  - To multiply by a number such as 9, multiply by 10 then subtract the number.
- Division is the inverse of multiplication.
- Use short division when dividing by numbers up to 12 (or higher, if you know the tables for it; for example 13, 15, 20).
- When dividing numbers that are multiples of 10, write the question as a fraction, cancel as many zeroes as possible and then divide.
- Addition and multiplication are commutative and associative.
- Multiplication is commutative, associative and distributive.
- Subtraction and division are not commutative, associative or distributive.

#### EXERCISE

### 2C

## Multiplying and dividing positive integers

#### INDIVIDUAL PATHWAYS

eBook plus

#### Activity 2-C-1

Division bingo  
doc-0281

#### Activity 2-C-2

More division bingo  
doc-0282

#### FLUENCY

- Write the answer to each of the following without using a calculator.
 

<b>a</b>	$4 \times 3$	<b>b</b>	$9 \times 5$	<b>c</b>	$2 \times 11$
<b>d</b>	$8 \times 7$	<b>e</b>	$12 \times 8$	<b>f</b>	$10 \times 11$
<b>g</b>	$6 \times 9$	<b>h</b>	$12 \times 11$	<b>i</b>	$9 \times 8$
- Multiply the following without using a calculator.
 

<b>a</b>	$13 \times 2$	<b>b</b>	$15 \times 3$	<b>c</b>	$25 \times 2$
<b>d</b>	$3 \times 13$	<b>e</b>	$25 \times 4$	<b>f</b>	$45 \times 2$
<b>g</b>	$16 \times 2$	<b>h</b>	$35 \times 2$	<b>i</b>	$14 \times 3$

INDIVIDUAL  
PATHWAYS

eBookplus

**Activity 2-C-3**  
 Advanced division  
 bingo  
 doc-0283

j  $21 \times 3$   
 m  $3 \times 4 \times 6$   
 p  $5 \times 6 \times 3$

k  $54 \times 2$   
 n  $2 \times 5 \times 9$   
 q  $5 \times 4 \times 5$

l  $25 \times 3$   
 o  $3 \times 3 \times 3$   
 r  $8 \times 5 \times 2$

- 3 **WE6** Calculate these using short multiplication.

a  $16 \times 8$   
 d  $857 \times 3$   
 g  $7888 \times 8$   
 j  $10\ 597 \times 6$

b  $29 \times 4$   
 e  $4920 \times 5$   
 h  $472 \times 4$   
 k  $34\ 005 \times 11$

c  $137 \times 9$   
 f  $15\ 984 \times 7$   
 i  $2015 \times 8$   
 l  $41\ 060 \times 12$

Check your answers using a calculator.

- 4 Calculate these using long multiplication.

a  $52 \times 44$   
 d  $16 \times 57$   
 g  $407 \times 53$   
 j  $19 \times 256\ 340$

b  $97 \times 31$   
 e  $173 \times 41$   
 h  $47 \times 2074$   
 k  $57\ 835 \times 476$

c  $59 \times 28$   
 f  $850 \times 76$   
 i  $80\ 055 \times 27$   
 l  $8027 \times 215$

Check your answers using a calculator.

- 5 **WE7** Use mental strategies to calculate each of the following.

a  $2 \times 8 \times 5$   
 d  $50 \times 45 \times 2$

b  $2 \times 4 \times 5 \times 6$   
 e  $2 \times 9 \times 50$

c  $4 \times 19 \times 25$   
 f  $4 \times 67 \times 250$

- 6 **WE8** Use mental strategies to calculate each of the following.

a  $45 \times 20$   
 d  $84 \times 200$

b  $61 \times 30$   
 e  $500 \times 19$

c  $62 \times 50$   
 f  $86 \times 2000$

- 7 Find each of the following.

a  $200 \times 40$   
 d  $90 \times 80$   
 g  $900\ 000 \times 7000$   
 j  $9000 \times 6000$

b  $30 \times 700$   
 e  $120 \times 400$   
 h  $120\ 000 \times 1200$   
 k  $4000 \times 110$

c  $600 \times 800$   
 f  $1100 \times 5000$   
 i  $800 \times 7000$   
 l  $12\ 000 \times 1100$

- 8 **WE9** Use mental strategies to calculate each of the following.

a  $34 \times 9$   
 d  $75 \times 99$

b  $83 \times 9$   
 e  $24 \times 19$

c  $628 \times 9$   
 f  $26 \times 8$

- 9 a Calculate  $56 \times 100$ .

- b Calculate  $56 \times 10$ .

- c Use your answers to parts a and b to calculate the answer to  $56 \times 90$ .

- 10 Use the method demonstrated in question 9 to calculate each of the following.

a  $48 \times 90$   
 d  $32 \times 900$

b  $74 \times 90$   
 e  $45 \times 80$

c  $125 \times 90$   
 f  $72 \times 800$

- 11 a Calculate  $25 \times 6$ .

- b Multiply your answer to part a by 2.

- c Now calculate  $25 \times 12$ .

- d Use the answers to parts a, b and c to describe a method for mentally multiplying by 12.

- 12 Use the method that you discovered in question 11 to mentally calculate the value of each of the following.

a  $15 \times 12$   
 d  $105 \times 12$   
 g  $11 \times 18$

b  $70 \times 12$   
 e  $25 \times 14$   
 h  $34 \times 20$

c  $18 \times 12$   
 f  $40 \times 16$

- 13 a What is the value of  $9 \times 10$ ?

- b What is the value of  $9 \times 3$ ?

- c Calculate the value of  $9 \times 13$ .

- d Use the answers to parts a, b and c above to describe a method for mentally multiplying by 13.

**eBook plus****Digital docs**

Spreadsheet

**Multiplying numbers**

doc-2087

Spreadsheet

**Tangle tables**

doc-2088

- 14** Use the method that you discovered in question **13** to calculate the value of each of the following.

**a**  $25 \times 13$       **b**  $30 \times 13$   
**c**  $24 \times 13$       **d**  $102 \times 13$

- 15** Evaluate these divisions without using a calculator. There should be no remainder.

<b>a</b> $24 \div 6$	<b>b</b> $24 \div 8$	<b>c</b> $36 \div 9$
<b>d</b> $72 \div 8$	<b>e</b> $49 \div 7$	<b>f</b> $96 \div 12$
<b>g</b> $108 \div 9$	<b>h</b> $56 \div 7$	<b>i</b> $16 \div 4$
<b>j</b> $28 \div 7$	<b>k</b> $40 \div 2$	<b>l</b> $26 \div 2$
<b>m</b> $45 \div 15$	<b>n</b> $32 \div 16$	<b>o</b> $27 \div 3 \div 3$
<b>p</b> $96 \div 8 \div 6$	<b>q</b> $48 \div 12 \div 2$	<b>r</b> $72 \div 2 \div 9$
<b>s</b> $56 \div 7 \div 4$	<b>t</b> $100 \div 2 \div 10$	<b>u</b> $90 \div 3 \div 2$

Check your answers using multiplication.

- 16** Perform these calculations which involve a combination of multiplication and division. Always work from left to right.

<b>a</b> $4 \times 5 \div 2$	<b>b</b> $9 \times 8 \div 12$	<b>c</b> $80 \div 10 \times 7$
<b>d</b> $45 \div 9 \times 7$	<b>e</b> $144 \div 12 \times 7$	<b>f</b> $120 \div 10 \times 5$
<b>g</b> $4 \times 9 \div 12$	<b>h</b> $121 \div 11 \times 4$	<b>i</b> $81 \div 9 \times 6$

- 17** **WE10** Calculate each of the following using short division.

<b>a</b> $3\overline{)1455}$	<b>b</b> $4\overline{)27\ 768}$	<b>c</b> $7\overline{)43\ 456}$
<b>d</b> $9\overline{)515\ 871}$	<b>e</b> $11\overline{)30\ 371}$	<b>f</b> $8\overline{)640\ 360}$
<b>g</b> $3\overline{)255\ 194}$	<b>h</b> $6\overline{)516\ 285}$	<b>i</b> $12\overline{)103\ 717}$
<b>j</b> $7\overline{)6\ 328\ 530}$	<b>k</b> $5\overline{)465\ 777}$	<b>l</b> $8\overline{)480\ 594}$

Check your answers using a calculator.

- 18** **WE11** Divide these numbers, which are multiples of ten.

<b>a</b> $4200 \div 6$	<b>b</b> $700 \div 70$
<b>c</b> $210 \div 30$	<b>d</b> $720\ 000 \div 800$
<b>e</b> $8100 \div 900$	<b>f</b> $4\ 000\ 000 \div 8000$
<b>g</b> $600\ 000 \div 120$	<b>h</b> $560 \div 80$
<b>i</b> $880\ 000 \div 1100$	

**UNDERSTANDING**

- 19** John wants to make a telephone call to his friend Rachel who lives in San Francisco. The call will cost him \$3 per minute. If John speaks to Rachel for 24 minutes:

- a** what will the call cost?  
**b** what would John pay if he made this call every month for 2 years?

- 20** Chris is buying some generators. The generators cost \$12 000 each and she needs 11 of them. How much will they cost her?

- 21** Julie was saving money to buy a digital camera.

She was able to save \$75 each month.

- a** How much did she save after 9 months?  
**b** How much did she save over 16 months?  
**c** If Julie continued to save at the same rate, how much will she save over a period of 3 years?

- 22** A car can travel 14 kilometres using 1 litre of fuel. How far could it travel with 35 litres?

**eBook plus****Digital doc**

Spreadsheet

**Dividing numbers**

doc-2089



- 23** As Todd was soaking in the bath, he was contemplating how much water was in the bath. If Todd used 85 litres of water each time he bathed and had a bath every week:
- how much bath water would Todd use in 1 year?
  - how much would he use over a period of 5 years?
- 24** In 1995, a team of British soldiers at Hameln, Germany, constructed the fastest bridge ever built. The bridge spanned an 8-metre gap and it took the soldiers 8 minutes and 44 seconds to build it. How many seconds did it take them to build it?
- 25** You are helping your Dad build a fence around your new swimming pool. He estimates that each metre of fence will take 2 hours and cost \$65 to build.
- How long will it take you and your Dad to build a 17-metre fence?
  - How much will it cost to build a 17-metre fence?
  - How much would it cost for a 29-metre fence?
- 26** Narissa does a paper round each morning before school. She travels 2 kilometres each morning on her bicycle, delivers 80 papers and is paid \$35. She does her round each weekday.
- How far does she travel in 1 week?
  - How much does she get paid in 1 week?
  - How far does she travel in 12 weeks?
  - How much would she be paid over 52 weeks?
  - How many papers would she deliver in 1 week?
  - How many papers would she deliver in 52 weeks?
- 27** **a** What is the value of  $580 \div 10$ ? **b** Halve your answer to part **a**.  
**c** Now find the value of  $580 \div 20$ . **d** Use the answers to parts **a**, **b** and **c** to describe a method for mentally dividing by 20.
- 28** Use the method that you discovered in question 7 to evaluate:  
**a**  $4280 \div 20$       **b**  $1840 \div 20$       **c**  $790 \div 20$       **d**  $960 \div 30$
- 29** Spiro travels 140 kilometres per week travelling to and from work. If Spiro works 5 days per week:
- how far does he travel each day?
  - what distance is his work from home?
- 30** Kelly works part time at the local pet shop. Last year she earned \$2496.
- How much did she earn each month?
  - How much did she earn each week?
- 31** At the milk processing plant, the engineer asked Farid how many cows he had to milk each day. Farid said he milked 192 cows because he obtained 1674 litres of milk each day and each cow produced 9 litres. Does Farid really milk 192 cows each day? If not, calculate how many cows he does milk.
- 32** When Juan caters for a celebration such as a wedding he fills out a form for the client to confirm the arrangements. Juan has been called to answer the telephone so it has been left to you to fill in the missing details. Copy and complete this planning form.

Celebration type	Wedding
Number of guests	152
Number of people per table	8
Number of tables required	
Number of courses for each guest	4
Total number of courses to be served	
Number of courses each waiter can serve	80
Number of waiters required	
Charge per guest	\$55
Total charge for catering	



- 33** Janet is a land developer and has bought 10 450 square metres of land. She intends to subdivide the land into 11 separate blocks.
- How many square metres will each block be?
  - If she sells each block for \$72 250, how much will she receive for the subdivided land?
- 34** Shea has booked a beach house for a week over the summer period for a group of 12 friends. The house costs \$1344 for the week. If all 12 people stayed for 7 nights, how much will the house cost each person per night?

**REASONING**

- 35** In AFL football, a goal scores 6 points and a behind scores 1 point. Find a score which is the same as the product of the number of goals and the number of behinds. For example, 2 goals 12 behinds =  $2 \times 6 + 12 = 24$  points. Also  $2 \times 12 = 24$ . Find two other similar results.

- 36** a Consider numbers with 2 identical digits multiplied by 99. Work out each of the following.

$$11 \times 99 =$$

$$22 \times 99 =$$

$$33 \times 99 =$$

Can you see a pattern? Without using long multiplication or a calculator, write down the answers to  $44 \times 99$ ,  $55 \times 99$ ,  $66 \times 99$ ,  $77 \times 99$ ,  $88 \times 99$  and  $99 \times 99$ .

- b Try it again but this time multiply numbers with 3 identical digits by 99. Use only long multiplication or a calculator with the first 3 calculations. Look for a pattern and then write down the answers to the remaining multiplications.
- c What about numbers with 4 or 5 identical digits which are multiplied by 99? Try these as well.

**REFLECTION**

How are multiplication and division related?

## 2D Long division

- Long division uses the same process as short division, but all the calculations are recorded.
- As a general rule, when dividing by numbers greater than 12, use long division.

**WORKED EXAMPLE 12**

Calculate  $2685 \div 15$ .

**THINK**

- $15 > 12$ , so long division is needed.
- As  $15 > 2$ , divide 15 into the first 2 digits, as shown in red.
- To find the remainder, multiply 1 by 15, write the result under the digits 26 and subtract.

**WRITE**

$$\begin{array}{r} 1 \\ 15 ) 2685 \end{array}$$

$$\begin{array}{r} 1 \\ 15 ) 2685 \\ - 15 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 1 \\ 15 ) 2685 \\ - 15 \\ \hline 11 \end{array}$$





- 4 Bring the remaining digits down next to the 11.

$$\begin{array}{r} 1 \\ 15 \overline{) 2685} \\ -15 \downarrow \downarrow \\ \hline 1185 \end{array}$$

- 5 As  $15 > 11$ , divide 15 into the first 3 digits.

$$\begin{array}{r} 17 \\ 15 \overline{) 2685} \\ -15 \\ \hline 1185 \end{array}$$

- 6 Multiply 7 by 15 and write the result under the digits 118. Subtract the numbers and bring the 5 down next to the 3.

$$\begin{array}{r} 17 \\ 15 \overline{) 2685} \\ -15 \\ \hline 1185 \\ -105 \downarrow \\ \hline 135 \end{array}$$

- 7 Divide 15 into 135 and repeat.

$$\begin{array}{r} 179 \\ 15 \overline{) 2685} \\ -15 \\ \hline 1185 \\ -105 \\ \hline 135 \\ -135 \\ \hline 0 \end{array}$$

- 8 Write the answer.

$$22685 \div 15 = 179$$

### WORKED EXAMPLE 13

Calculate  $2297 \div 17$ .

#### THINK

- 1  $17 > 12$ , so long division is needed.

#### WRITE

$$17 \overline{) 2297}$$

- 2 Divide 17 into the first 2 digits. Multiply this result by 17 and subtract.

$$\begin{array}{r} 1 \\ 17 \overline{) 2297} \\ -17 \\ \hline 5 \end{array}$$

- 3 Bring the remaining digits down next to the 5.

$$\begin{array}{r} 1 \\ 17 \overline{) 2297} \\ -17 \downarrow \\ \hline 597 \end{array}$$

- 4 Divide 17 into the first 2 digits. Multiply this result by 17 and subtract.

$$\begin{array}{r} 13 \\ 17 \overline{) 2297} \\ -17 \\ \hline 59 \\ -51 \\ \hline 8 \end{array}$$

- 5 Bring the remaining digit 7 down next to the 8.

$$\begin{array}{r} 13 \\ 17 \overline{) 2297} \\ -17 \\ \hline 59 \\ -51 \\ \hline 87 \end{array}$$

- 6 Divide 17 into 87 and repeat.

$$\begin{array}{r} 135 \\ 17 \overline{) 2297} \\ -17 \\ \hline 59 \\ -51 \\ \hline 87 \\ -85 \\ \hline 2 \end{array}$$

- 7 Write the answer.

$$2297 \div 17 = 135 \text{ remainder } 2 \text{ or}$$

$$2297 \div 17 = 135\frac{2}{17}$$

#### REMEMBER

1. Long division uses the same processes as short division, but all the steps are recorded.
2. Use long division when the divisor is greater than 12.

## EXERCISE

## 2D Long division

## INDIVIDUAL PATHWAYS

**eBookplus****Activity 2-D-1**

Long division

doc-1096

**Activity 2-D-2**

More long division

doc-1097

**Activity 2-D-3**

Advanced long

division

doc-1098

## FLUENCY

- 1 WE12** Using long division, evaluate the following.

a  $195 \div 13$

b  $308 \div 22$

c  $544 \div 17$

d  $589 \div 31$

e  $9683 \div 23$

f  $8554 \div 13$

g  $5525 \div 17$

h  $9050 \div 25$

i  $1302 \div 21$

j  $11\ 004 \div 2$

- 2 WE13** Using long division, evaluate the following.

a  $847 \div 13$

b  $951 \div 15$

c  $1210 \div 17$

d  $1600 \div 19$

e  $5050 \div 41$

f  $8289 \div 33$

g  $6831 \div 21$

h  $7721 \div 31$

i  $14\ 997 \div 23$

j  $20\ 011 \div 27$

## UNDERSTANDING

- 3** Complete the following long division problems.

a  $\begin{array}{r} 2\Box 6 \\ 19)448\Box \end{array}$

b  $\begin{array}{r} 48\Box \\ 17)82\Box 9 \end{array}$

c  $\begin{array}{r} 2\Box 3 \text{ r } 17 \\ 21)512\Box \end{array}$

**eBookplus**

**Weblink**  
Long division  
game

## REFLECTION

Why would you use long division instead of short division?

## 2E Order of operations

- In mathematics, conventions are followed so that we all have a common understanding of mathematical operations.
- Consider the question  $6 + 6 \div 3$ . If you perform the addition first the answer is 4. If you perform the division first the answer is 8. The correct answer is 8.
- There is a set order in which mathematicians calculate problems. The order is:
  1. Brackets
  2. Indices or square roots
  3. Division and Multiplication (from left to right)
  4. Addition and Subtraction (from left to right).
- The acronym **BIDMAS** can be used to remember the correct order of operations.

## WORKED EXAMPLE 14

Calculate  $6 + 12 \div 4$ .

## THINK

- 1 Write the question.
- 2 Perform the division before the addition.
- 3 Calculate the answer.

## WRITE

$$\begin{aligned} 6 + 12 \div 4 \\ = 6 + 3 \\ = 9 \end{aligned}$$

**WORKED EXAMPLE 15****Calculate:**

**a**  $12 \div 2 + 4 \times (4 + 6)$

**b**  $80 \div \{(11 - 2) \times 2\} + 2\}$

**THINK**

- a**
- 1 Write the question.
  - 2 Remove the brackets by working out the addition inside.
  - 3 Perform the division and multiplication next, working from left to right.
  - 4 Complete the addition last and calculate the answer.
- b**
- 1 Write the question.
  - 2 Remove the innermost brackets by working out the difference of 11 and 2.
  - 3 Remove the next pair of brackets by working out the multiplication inside them.
  - 4 Remove the final pair of brackets by working out the addition inside them.
  - 5 Perform the division last and calculate the answer.

**WRITE**

**a**  $12 \div 2 + 4 \times (4 + 6)$

$= 12 \div 2 + 4 \times 10$

$= 6 + 40$

$= 46$

**b**  $80 \div \{(11 - 2) \times 2\} + 2\}$

$= 80 \div \{[9 \times 2] + 2\}$

$= 80 \div \{18 + 2\}$

$= 80 \div 20$

$= 4$

**WORKED EXAMPLE 16****Insert one set of brackets in the appropriate place to make the following statement true.**

$3 \times 10 - 8 \div 2 + 4 = 7$

**THINK**

- 1 Write the left-hand side of the equation.
- 2 Place one set of brackets around the first two values.
- 3 Perform the multiplication inside the bracket.
- 4 Perform the division.
- 5 Perform the subtraction and addition working from left to right.  
*Note:* Since this is not the answer, the above process must be repeated.
- 6 Place one set of brackets around the second and third values.
- 7 Perform the subtraction inside the bracket.
- 8 Perform the multiplication and division working from left to right.
- 9 Perform the addition last and calculate the answer. This is correct, so the brackets are in the correct place.

**WRITE**

$3 \times 10 - 8 \div 2 + 4$

$= (3 \times 10) - 8 \div 2 + 4$

$= 30 - 8 \div 2 + 4$

$= 30 - 4 + 4$

$= 26 + 4$

 $= 30$  Since this is not equal to 7 we must place the brackets in a different position.

$3 \times 10 - 8 \div 2 + 4$

$= 3 \times (10 - 8) \div 2 + 4$

$= 3 \times 2 \div 2 + 4$

$= 6 \div 2 + 4$

$= 3 + 4$

$= 7$

**REMEMBER**

1. The operations inside brackets are always calculated first.
2. If there is more than one set of brackets, calculate the operations inside the innermost brackets first.
3. Multiplication and division operations are calculated in the order that they appear.
4. Addition and subtraction operations are calculated in the order that they appear.

**EXERCISE****2E****Order of operations****INDIVIDUAL PATHWAYS****eBookplus****Activity 2-E-1**Order of operations  
doc-0284**Activity 2-E-2**More order of  
operations  
doc-0285**Activity 2-E-3**Advanced order of  
operations  
doc-0286**FLUENCY**

- 1 WE14, 15** Calculate each of these, following the order of operations rules.

- |   |  |  |
|---|--|--|
| <b>a</b> $3 + 4 \div 2$                       | <b>b</b> $8 + 1 \times 1$                      | <b>c</b> $24 \div (12 - 4)$                  |
| <b>d</b> $15 \times (17 - 15)$                | <b>e</b> $11 + 6 \times 8$                     | <b>f</b> $30 - 45 \div 9$                    |
| <b>g</b> $56 \div (7 + 1)$                    | <b>h</b> $12 \times (20 - 12)$                 | <b>i</b> $3 \times 4 + 23 - 10 - 5 \times 2$ |
| <b>j</b> $42 \div 7 \times 8 - 8 \times 3$    | <b>k</b> $10 + 40 \div 5 + 14$                 | <b>l</b> $81 \div 9 + 108 \div 12$           |
| <b>m</b> $16 + 12 \div 2 \times 10$           | <b>n</b> $(18 - 15) \div 3 \times 27$          | <b>o</b> $4 + (6 + 3 \times 9) - 11$         |
| <b>p</b> $52 \div 13 + 75 \div 25$            | <b>q</b> $(12 - 3) \times 8 \div 6$            | <b>r</b> $88 \div (24 - 13) \times 12$       |
| <b>s</b> $(4 + 5) \times (20 - 14) \div 2$    | <b>t</b> $(7 + 5) - (10 + 2)$                  |  |
| <b>u</b> $\{(16 + 4) \div 4\} - 2\} \times 6$ | <b>v</b> $60 \div \{[(12 - 3) \times 2] + 2\}$ |  |

**UNDERSTANDING**

- 2 WE16** Insert one set of brackets in the appropriate place to make these statements true.

- |   |   |
|---|---|
| <b>a</b> $12 - 8 \div 4 = 1$                    | <b>b</b> $4 + 8 \times 5 - 4 \times 5 = 40$       |
| <b>c</b> $3 + 4 \times 9 - 3 = 27$              | <b>d</b> $3 \times 10 - 2 \div 4 + 4 = 10$        |
| <b>e</b> $12 \times 4 + 2 - 12 = 60$            | <b>f</b> $17 - 8 \times 2 + 6 \times 11 - 5 = 37$ |
| <b>g</b> $10 \div 5 + 5 \times 9 \times 9 = 81$ | <b>h</b> $18 - 3 \times 3 \div 5 = 9$             |

- 3**  $20 - 6 \times 3 + 28 \div 7$  is equal to:

- |             |             |            |            |            |
|-------------|-------------|------------|------------|------------|
| <b>A</b> 46 | <b>B</b> 10 | <b>C</b> 6 | <b>D</b> 4 | <b>E</b> 2 |
|-------------|-------------|------------|------------|------------|

- 4** The two signs marked with \* in the equation  $7 * 2 * 4 - 3 = 12$  are:

- |               |               |               |               |               |
|---------------|---------------|---------------|---------------|---------------|
| <b>A</b> -, + | <b>B</b> +, × | <b>C</b> =, ÷ | <b>D</b> ÷, × | <b>E</b> ×, ÷ |
|---------------|---------------|---------------|---------------|---------------|

- 5** Insert brackets if necessary to make each statement true.

- |   |   |
|---|---|
| <b>a</b> $6 + 2 \times 4 - 3 \times 2 = 10$ | <b>b</b> $6 + 2 \times 4 - 3 \times 2 = 26$ |
| <b>c</b> $6 + 2 \times 4 - 3 \times 2 = 16$ | <b>d</b> $6 + 2 \times 4 - 3 \times 2 = 8$  |

**REASONING**

- 6** James and Liz discovered that they had different answers to the same question, which was to calculate  $6 + 6 \div 3$ . James thought the answer was 8. Liz thought the answer was 4. Who was correct, James or Liz?
- 7** What is the smallest number of pebbles greater than 10 for which grouping them in heaps of 7 leaves 1 extra and grouping them in heaps of 5 leaves 3 extra?



- 8** Choose a digit from 2 to 9. Write it six times. For example, if 4 is chosen the number is 444 444. Divide the six-digit number by 33. Next divide the result by 37 and finally divide this last result by 91. What is the final result?

Try this again with another six-digit number formed as before. (Divide by 33, then 37, then 91.) What is your final result in this case? Try to explain how this works.

## 2F Factors and multiples

### Multiples

- A **multiple** of a number is the answer obtained when that number is multiplied by another whole number. For example, all numbers in the 5 times table are multiples of 5: so 5, 10, 15, 20, 25, ... are all multiples of 5.

#### WORKED EXAMPLE 17

List the first five multiples of 7.

##### THINK

- First multiple is the number  $\times$  1; that is,  $7 \times 1$ .
- Second multiple is the number  $\times$  2; that is,  $7 \times 2$ .
- Third multiple is the number  $\times$  3; that is  $7 \times 3$ .
- Fourth multiple is the number  $\times$  4; that is,  $7 \times 4$ .
- Fifth multiple is the number  $\times$  5; that is,  $7 \times 5$ .
- Answer the question.

##### WRITE

The first five multiples of 7 are 7, 14, 21, 28, 35.

#### WORKED EXAMPLE 18

Write the numbers in the list that are multiples of 8.

18, 8, 80, 100, 24, 60, 9, 40

##### THINK

- The biggest number in the list is 100. List multiples of 8 using the 8 times table just past 100; that is,  $8 \times 1 = 8$ ,  $8 \times 2 = 16$ ,  $8 \times 3 = 24$ ,  $8 \times 4 = 32$ ,  $8 \times 5 = 40$ , and so on.
- Write any multiples that appear in the list.

##### WRITE

8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104

Numbers in the list that are multiples of 8 are 8, 24, 40, 80.

### Factors

- A **factor** is a whole number that divides exactly into another whole number, with no remainder. For example, the number 4 is a factor of 8 because 4 divides into 8 twice with no remainder, or  $8 \div 4 = 2$ .
- If one number is divisible by another number, the second number divides exactly into the first number.
- Factors of a number can be written as factor pairs. These are pairs of numbers that multiply to equal a certain number.

**WORKED EXAMPLE 19****Find all the factors of 14.****THINK**

- 1 1 is a factor of every number and the number itself is a factor; that is,  $1 \times 14 = 14$ .
- 2 14 is an even number so 14 is divisible by 2; therefore 2 is a factor. Divide the number by 2 to find the other factor ( $14 \div 2 = 7$ ).
- 3 Write a sentence placing the factors in order from smallest to largest.

**WRITE**

1, 14

2, 7

The factors of 14 are 1, 2, 7 and 14.

**WORKED EXAMPLE 20****List the factor pairs of 30.****THINK**

- 1 1 and the number itself are factors; that is,  $1 \times 30 = 30$ .
- 2 30 is an even number so 2 and 15 are factors; that is,  $2 \times 15 = 30$ .
- 3 Divide the next smallest number into 30. Therefore, 3 and 10 are factors; that is,  $3 \times 10 = 30$ .
- 4 30 ends in 0 so 5 divides evenly into 30, that is,  $5 \times 6 = 30$ .
- 5 List the factor pairs.

**WRITE**

1, 30

2, 15

3, 10

5, 6

The factor pairs of 30 are 1, 30; 2, 15; 3, 10 and 5, 6.

**REMEMBER**

- 1 A multiple of a number is the answer obtained when that number is multiplied by another whole number. For example, all numbers in the 6 times table are multiples of 6; that is: 6, 12, 18, 24, 30, ...
- 2 A factor is a whole number that divides exactly into another whole number with no remainder.
- 3 The factors of a number can be found using factor pairs. One of the pairs may be a single number that multiplies by itself. The number 5 is a factor of 25 because  $5 \times 5 = 25$ . The number and itself are always factors of a number.

**EXERCISE****2F Factors and multiples****INDIVIDUAL PATHWAYS****eBook plus****Activity 2-F-1**Factors and multiples  
doc-0287**FLUENCY**

- 1 WE17** List the first five multiples of the following numbers.

**a** 3**b** 6**c** 100**d** 11**e** 15**f** 4**g** 21**h** 25**i** 14**j** 12**k** 50**l** 30**m** 33**n** 120**o** 45**p** 72

## INDIVIDUAL PATHWAYS

## eBookplus

**Activity 2-F-2**

More factors and multiples  
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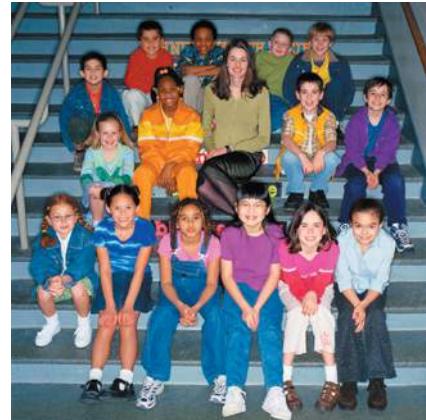
**Activity 2-F-3**

Advanced factors and multiples  
doc-0289

- 2 WE18** Write the numbers in the following list that are multiples of 10.  
10, 15, 20, 100, 38, 62, 70
- 3** Write the numbers in the following list that are multiples of 7.  
17, 21, 7, 70, 47, 27, 35
- 4** Write the numbers in the following list that are multiples of 16.  
16, 8, 24, 64, 160, 42, 4, 32, 1, 2, 80
- 5** Write the numbers in the following list that are multiples of 35.  
7, 70, 95, 35, 140, 5, 165, 105, 700
- 6** The numbers 16, 40 and 64 are all multiples of 8. Find three more multiples of 8 that are less than 100.
- 7** List the multiples of 9 that are less than 100.
- 8** List the multiples of 6 between 100 and 160.
- 9 MC**
- a** The first three multiples of 9 are:  
**A** 1, 3, 9      **B** 3, 6, 9      **C** 9, 18, 27      **D** 9, 18, 81      **E** 18, 27, 36
  - b** The first three multiples of 15 are:  
**A** 15, 30, 45      **B** 30, 45, 60      **C** 1, 15, 30      **D** 45      **E** 3, 5, 15
  - c** Justify your answer to part **b** using mathematical reasoning.
- 10 WE19** Find all the factors of each of the following numbers.
- |             |              |              |              |
|-------------|--------------|--------------|--------------|
| <b>a</b> 12 | <b>b</b> 8   | <b>c</b> 40  | <b>d</b> 35  |
| <b>e</b> 28 | <b>f</b> 60  | <b>g</b> 100 | <b>h</b> 72  |
| <b>i</b> 39 | <b>j</b> 85  | <b>k</b> 76  | <b>l</b> 69  |
| <b>m</b> 99 | <b>n</b> 250 | <b>o</b> 51  | <b>p</b> 105 |
- 11 WE20** List the factor pairs of:  
**a** 20      **b** 18      **c** 36      **d** 132
- 12** If 3 is a factor of 12, state the smallest number greater than 12 which has 3 as one of its factors.
- 13 MC**
- a** A factor pair of 24 is:  
**A** 2, 4      **B** 4, 6      **C** 6, 2      **D** 2, 8      **E** 3, 9
  - b** A factor pair of 42 is:  
**A** 6, 7      **B** 20, 2      **C** 21, 1      **D** 16, 2      **E** 6, 8
- 14** Which of the numbers 3, 4, 5 and 11 are factors of 2004?

## UNDERSTANDING

- 15** Alex and Nadia were having races running down a flight of stairs. Nadia took the stairs two at a time while Alex took the stairs three at a time. In each case, they reached the bottom with no steps left over.
- a** How many steps are there in the flight of stairs? List three possible answers.
  - b** What is the smallest number of steps there could be?
  - c** If Alex can also take the stairs five at a time with no steps left over, what is the smallest number of steps in the flight of stairs?
- 16** Connie Pythagoras is trying to organise her Year 4 class into rows for their class photograph. If Ms Pythagoras wishes to organise the 20 students into rows containing equal numbers of students, what possible arrangements can she have?  
*Note:* Ms Pythagoras will not be in the photograph.



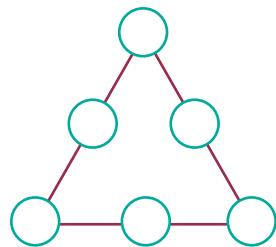
- 17** Place the first 6 multiples of 3 into the triangle at right, so the numbers in each side of the triangle add up to 27. Use each number once only.

- 18 a MC** Tilly Tyler has 24 green bathroom tiles left over. If she wants to use them all on the wall behind the kitchen sink (without breaking any) which of the following arrangements would be suitable?

- I 4 rows of 8 tiles      II 2 rows of 12 tiles  
 III 4 rows of 6 tiles      IV 6 rows of 5 tiles  
 V 3 rows of 8 tiles

- A I and II      B I, II and III  
 C II, IV and V      D II, III and V  
 E V

**b** Justify your answer to part **a** using mathematical reasoning.



### REASONING

- 19** I am a 2-digit number that can be divided by 3 with no remainder. The sum of my digits is a multiple of 4 and 6. My first digit is double my second digit. What number am I?
- 20** Find a 2-digit number such that if you subtract 3 from it, the result is a multiple of 3; if you subtract 4 from it, the result is a multiple of 4 and if you subtract 5 from it, the result is a multiple of 5.
- 21** In a class election with 3 candidates, the winner beat the other 2 candidates by 3 and 6 votes respectively. If 27 votes were cast, how many votes did the winner receive?
- 22** What number am I? I am a multiple of 5 with factors of 6, 4 and 3. The sum of my digits is 6.
- 23** My age is a multiple of 3 and a factor of 60. The sum of my digits is 3. How old am I? (There are two possible answers.)
- 24** A room measures 550 centimetres by 325 centimetres. What would be the side length of the largest square tile that can be used to tile the floor without any cutting?
- a** Try this question now.  
*(Hint: Find the factors of 550 and 325.)*
- b** How many tiles would fit on the floor along the wall 550 centimetres long?
- c** How many tiles would fit on the floor along the wall 325 centimetres long?
- d** How many floor tiles would be needed for this room?



### REFLECTION

Does a number have more factors than multiples? Explain.

## 2G Lowest common multiple and highest common factor

- The **lowest common multiple (LCM)** of two or more numbers is the smallest number that has those numbers as factors. For example, 20 is a common multiple of 5 and 10, but 10 is the lowest common multiple of 5 and 10.

- A **common factor** is a number that is a factor of two or more given numbers. For example, for the numbers 6 and 10, the number 2 is a common factor because 2 is a factor of 6, and 2 is a factor of 10.
- The **highest common factor (HCF)** is the largest of the common factors.

**WORKED EXAMPLE 21**

- Find the common factors of 8 and 24 by:
  - listing the factors of 8
  - listing the factors of 24
  - listing the factors common to both 8 and 24.
- State the highest common factor of 8 and 24.

**THINK**

- Find the pairs of factors of 8.
  - Write them in order.
- Find the pairs of factors of 24.
  - Write them in order.
- Write the common factors.

- b Find the highest common factor.

**WRITE**

- $1, 8 \quad 2, 4$   
Factors of 8 are 1, 2, 4, 8.
  - $1, 24 \quad 2, 12 \quad 3, 8 \quad 4, 6$   
Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
  - Common factors are 1, 2, 4, 8.
- HCF is 8.

Factors can make it easier to multiply numbers mentally. The following examples show the thought processes required.

**WORKED EXAMPLE 22**

Find  $12 \times 15$  using factors.

**THINK**

- Write the question.
- Write one of the numbers as a factor pair.
- Multiply the first two numbers.
- Find the answer.

**WRITE**

$$\begin{aligned} 12 \times 15 \\ = 12 \times 5 \times 3 \\ = 60 \times 3 \\ = 180 \end{aligned}$$

**WORKED EXAMPLE 23**

Use factors to evaluate  $32 \times 25$ .

**THINK**

- Write the question.
- Rewrite the question so that one pair of factors is easy to multiply. The number 4 is a factor of 32 and  $4 \times 25 = 100$ .
- Find the answer.

**WRITE**

$$\begin{aligned} 32 \times 25 \\ = 8 \times 4 \times 25 \\ \\ = 8 \times 100 \\ = 800 \end{aligned}$$

**REMEMBER**

1. If two or more numbers have the same multiple, it is called a common multiple.
2. The smallest number that 2 or more numbers can divide into is called the lowest common multiple of those numbers.
3. If two or more numbers have the same factor, it is called a common factor.
4. The highest common factor or HCF of 2 or more numbers is the largest factor that divides into all of the given numbers, with no remainder.

**EXERCISE****2G****Lowest common multiple and highest common factor****INDIVIDUAL PATHWAYS****eBook plus****Activity 2-G-1**

Lowest common multiples and highest common factors  
doc-0290

**Activity 2-G-2**

More lowest common multiples and highest common factors  
doc-0291

**Activity 2-G-3**

Advanced lowest common multiples and highest common factors  
doc-0292

**FLUENCY**

- 1 a List the first ten multiples of 4.  
b List the first ten multiples of 6.  
c In your lists, circle the multiples that 4 and 6 have in common (that is, circle the numbers that appear in both lists).  
d What is the *lowest multiple* that 4 and 6 have in *common*? This is the Lowest Common Multiple of 4 and 6, known as the LCM.
- 2 a List the first six multiples of 3.  
b List the first six multiples of 9.  
c Circle the multiples that 3 and 9 have in common.  
d What is the lowest common multiple of 3 and 9?
- 3 Find the LCM of each of the following pairs of numbers.  

<b>a</b> 3 and 6	<b>b</b> 6 and 15	<b>c</b> 7 and 10
<b>d</b> 12 and 16	<b>e</b> 4 and 15	
- 4 Answer true (T) or false (F) to each of the following statements.  

<b>a</b> 20 is a multiple of 10 and 2 only.	<b>b</b> 15 and 36 are both multiples of 3.	<b>c</b> 60 is a multiple of 2, 3, 6, 10 and 12.
<b>d</b> 100 is a multiple of 2, 4, 5, 10, 12 and 25.		
- 5 a **WE21** List the factors of 21.  
b List the factors of 56.  
c Find the highest common factor of 21 and 56.
- 6 a List the factors of 27.  
b List the factors of 15.  
c Find the highest common factor of 27 and 15.
- 7 a List the factors of 7.  
b List the factors of 28.  
c Find the highest common factor of 7 and 28.
- 8 a List the factors of 48.  
b List the factors of 30.  
c Find the HCF of 48 and 30.
- 9 Find the highest common factor of 9 and 36.
- 10 Find the highest common factor of 42 and 77.
- 11 Find the highest common factor of 36 and 64.
- 12 Find the highest common factor of 45, 72 and 108.

- 13 WE22** Find the following using factors.

a  $12 \times 25$   
d  $11 \times 16$   
g  $20 \times 15$

b  $12 \times 35$   
e  $11 \times 14$   
h  $20 \times 18$

c  $12 \times 55$   
f  $11 \times 15$   
i  $30 \times 21$

- 14 WE23** Use factors to evaluate the following.

a  $36 \times 25$   
d  $72 \times 25$   
g  $56 \times 50$

b  $44 \times 25$   
e  $124 \times 25$   
h  $48 \times 125$

c  $24 \times 25$   
f  $132 \times 25$   
i  $52 \times 250$

### UNDERSTANDING

- 15** Kate goes to the gym every second evening, while Ian goes every third evening.  
 a How many days will it be before both attend the gym again on the same evening?  
 b Explain how this answer relates to the multiples of 2 and 3.

- 16** Vinod and Elena are riding around a mountain bike trail. Each person completes one lap in the time shown on the stopwatches.  
 a If they both begin cycling from the starting point at the same time, how long will it be before they pass this starting point again at exactly the same time?  
 b Relate your answer to the multiples of 5 and 7.



- 17** A warehouse owner employs Bob and Charlotte as security guards. Each security guard checks the building at midnight. Bob then checks the building every 4 hours, and Charlotte every 6 hours.

- a How long will it be until both Bob and Charlotte are next at the warehouse at the same time?  
 b Relate your answer to the multiples of 4 and 6.



- 18** Two smugglers, Bill Bogus and Sally Seadog have set up signal lights that flash continuously across the ocean. Bill's light flashes every 5 seconds and Sally's light flashes every 4 seconds. If they both start together, how long will it take for both lights to flash again at the same time?

- 19** Twenty students in Year 7 were each given a different number from 1 to 20 and then asked to sit in numerical order in a circle. Three older girls — Milly, Molly and Mandy — came to distribute jelly beans to the class. Milly gave red jelly beans to every second student, Molly gave green jelly beans to every third student and Mandy gave yellow jelly beans to every fourth student.

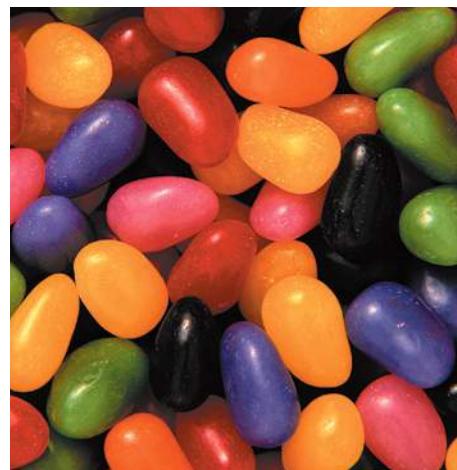
- a** Which student had jelly beans of all 3 colours?
- b** How many students received exactly 2 jelly beans?
- c** How many students did not receive any jelly beans?

- 20** Wah needs to cut tubing into the largest pieces of equal length that he can, without having any offcuts left over. He has three sections of tubing; one 6 metres long, another 9 metres long and the third 15 metres long.

- a** How long should each piece of tubing be?
- b** How many pieces of tubing will Wah end up with?

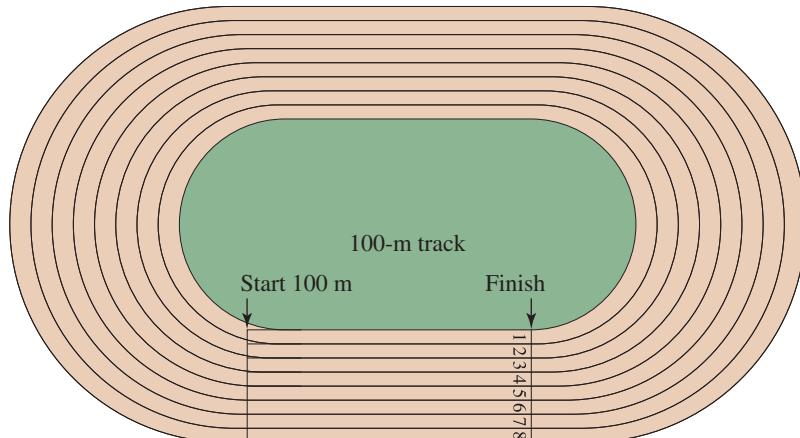
- 21** Mario, Luigi, Zoe and Daniella are playing on a playstation. Mario takes 2 minutes to play one level of the game, Luigi takes 3 minutes, Zoe takes 4 minutes and Daniella takes 5 minutes. They have 12 minutes to play.

- a** If they play continuously, which of the players would be in the middle of a game as time ran out?
- b** After how many minutes did this player begin the last game?



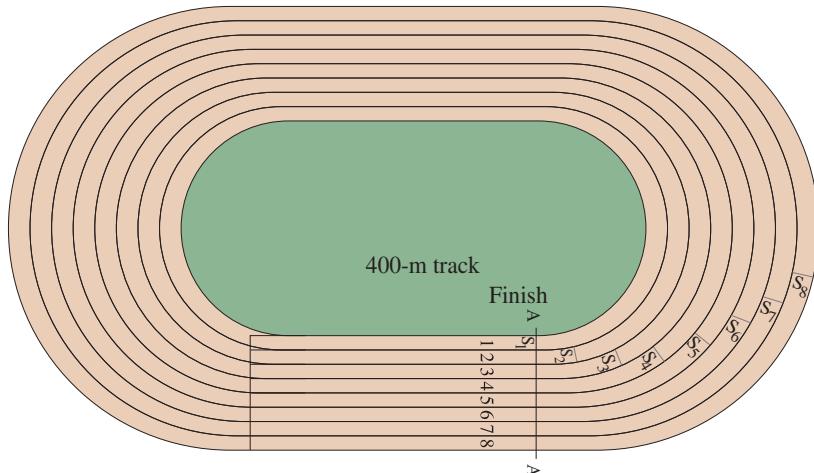
### REASONING

- 22** The runners in a 100-m race are lined up next to each other while those in a 400-m race are staggered around the track. Look at the diagram of a standard 400-m athletic track and take note of both the starting position and finish line for a 100-m race.



- What two shapes is the track made up of?
- Explain why the starting line for a 100-m race has been positioned in the extended section.
- Explain why the finish line for a 100-m race has been positioned where it is.

Track events such as the 200-m, 400-m and 800-m races have staggered starting positions. The diagram illustrates the position of the starting blocks  $S_1$ – $S_8$  for each lane in the 400-m race.



- Why is there a need to stagger the starting blocks in the 200-m, 400-m and 800-m track events?
- A runner completing one lap of the inside lane (lane 1) runs a distance of 400 m, while a runner completing one lap in the next lane (lane 2) runs a distance of 408 m. If this pattern continues, determine how far runners in lanes 3–8 run. Enter these results in the appropriate column in the table.

Lane number	Distance travelled (m)	Difference
1	400 m	
2	408 m	
3		
4		
5		
6		
7		
8	456 m	

- Calculate the difference between the distances travelled by the runners in each of the lanes compared to the distance travelled by the runner in lane 1. Enter these results in the appropriate column in the table above.
- What do you notice about the values obtained in question 6?

#### REFLECTION

What strategies will you use to help you remember the difference between highest common factors and lowest common multiples?

## 2H Estimation

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elesson  
Estimating and rounding  
eles-0822

- An estimate is not the same as a guess, because it is based on information.
- Estimation is useful when we are working with calculators. By mentally estimating an approximate answer, we are more likely to notice a data entry error.
- To estimate the answer to a mathematical problem, round the numbers to the first digit and find an approximate answer.
- Estimations can be made when multiplying, dividing, adding or subtracting. They can also be used when there is more than one operation in the same question.

### Rounding

- If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same.
- If the second digit is 5, 6, 7, 8 or 9, the first digit is rounded up.
- If rounding to the first digit:  
6512 would be rounded to 7000 as it is closer to 7000  
6397 would be rounded to 6000 as it is closer to 6000  
6500 would be rounded to 7000. It is exactly halfway between 6000 and 7000. So to avoid confusion, if it is halfway the number is rounded up.

#### WORKED EXAMPLE 24

Estimate  $48\ 921 \times 823$  by rounding to the first digit.

##### THINK

- Write the question.
- Round each part of the question to the first digit.
- Multiply.

##### WRITE

$$\begin{aligned}48\ 921 \times 823 \\= 50\ 000 \times 800 \\= 40\ 000\ 000\end{aligned}$$

#### REMEMBER

- An estimate is not the same as a guess because it is based on information.
- Rounding
  - If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same.
  - If the second digit is 5, 6, 7, 8 or 9, the first digit is rounded up.

#### EXERCISE

## 2H Estimation

#### FLUENCY

INDIVIDUAL PATHWAYS

eBookplus

**Activity 2-H-1**

Estimation  
doc-0293

**Activity 2-H-2**

More estimation  
doc-0294

- 1 **WE24** Estimate  $67\ 451 \times 432$  by rounding to the first digit.

- 2 Copy and complete the following table by rounding the numbers to the first digit. The first row has been completed as an example.
- In the column headed ‘Estimate’, round each number to the leading digit.
  - In the column headed ‘Estimated Answer’, calculate the answer.
  - In the column headed ‘Prediction’, guess whether the actual answer will be higher or lower than your estimate.
  - Use a calculator to work out the actual answer and record it in the final column titled ‘Calculation’ to determine whether it is higher or lower than your estimate.

INDIVIDUAL  
PATHWAYS**eBookplus**
**Activity 2-H-3**  
 Advanced estimation  
 doc-0295

		Estimate	Estimated answer	Prediction	Calculation*
Example	$4129 \div 246$	$4000 \div 200$	20	Lower	16.784 553 so lower
a	$487 + 962$				
b	$33\ 041 + 82\ 629$				
c	$184\ 029 + 723\ 419$				
d	$1127 + 6302$				
e	$29 + 83$				
f	$55\ 954 + 48\ 312$				
g	$93\ 261 - 37\ 381$				
h	$321 - 194$				
i	$468\ 011 - 171\ 962$				
j	$942\ 637 - 389\ 517$				
k	$64\ 064 - 19\ 382$				
l	$89\ 830 - 38\ 942$				
m	$36 \times 198$				
n	$8631 \times 9$				
o	$87 \times 432$				
p	$623 \times 12\ 671$				
q	$29\ 486 \times 39$				
r	$222 \times 60$				
s	$31\ 690 \div 963$				
t	$63\ 003 \div 2590$				
u	$867\ 910 \div 3300$				
v	$8426 \div 3671$				
w	$69\ 241 \div 1297$				
x	$37\ 009 \div 180$				

\* Is the actual answer higher or lower than the estimate?

**3 MC**

- a The best estimate of  $4372 + 2587$  is:
- A 1000      B 5527      C 6000      D 7000      E 7459
- b The best estimate of  $672 \times 54$  is:
- A 30 000      B 35 000      C 36 000      D 40 000      E 42 000
- c The best estimate of  $67\ 843 \div 365$  is:
- A 150      B 175      C 200      D 230      E 250

- 4** Estimate the answers to each of these.

- |                                |                              |
|--------------------------------|------------------------------|
| <b>a</b> $5961 + 1768$         | <b>b</b> $432 - 192$         |
| <b>c</b> $48\ 022 \div 538$    | <b>d</b> $9701 \times 37$    |
| <b>e</b> $98\ 631 + 608\ 897$  | <b>f</b> $6501 + 3790$       |
| <b>g</b> $11\ 890 - 3642$      | <b>h</b> $83\ 481 \div 1751$ |
| <b>i</b> $112\ 000 \times 83$  | <b>j</b> $66\ 501 \div 738$  |
| <b>k</b> $392 \times 113\ 486$ | <b>l</b> $12\ 476 \div 24$   |

- 5** **(MC)** Su-Lin was using her calculator to answer some mathematical questions, but found she obtained a different answer each time she performed the same calculation. Using your estimation skills, predict which of Su-Lin's answers is most likely to be correct.

- |                             |                  |                  |                 |                     |                  |
|-----------------------------|------------------|------------------|-----------------|---------------------|------------------|
| <b>a</b> $217 \times 489$   | <b>A</b> 706     | <b>B</b> 106 113 | <b>C</b> 13 203 | <b>D</b> 19 313     | <b>E</b> 105 203 |
| <b>b</b> $89\ 344 \div 256$ | <b>A</b> 39      | <b>B</b> 1595    | <b>C</b> 89 088 | <b>D</b> 349        | <b>E</b> 485     |
| <b>c</b> $78 \times 6703$   | <b>A</b> 522 834 | <b>B</b> 52 260  | <b>C</b> 6781   | <b>D</b> 56 732 501 | <b>E</b> 51 624  |
| <b>d</b> $53\ 669 \div 451$ | <b>A</b> 10      | <b>B</b> 1076    | <b>C</b> 53 218 | <b>D</b> 119        | <b>E</b> 183     |

- 6** Julian is selling tickets for his school's theatre production of South Pacific. So far he has sold 439 tickets for Thursday night's performance, 529 for Friday's and 587 for Saturday's. The costs of the tickets are \$9.80 for adults and \$4.90 for students.

- a** Round the figures to the first digit to estimate the number of tickets Julian has sold so far.
- b** If approximately half the tickets sold were adult tickets and the other half were student tickets, estimate how much money has been received so far by rounding the cost of the tickets to the first digit.

- 7** During the show's intermission, Jia is planning to run a stall selling hamburgers to raise money for the school. She has priced the items she needs and made a list in order to estimate her expenses.

- a** By rounding the item price to the first digit, use the table below to estimate how much each item will cost Jia for the quantity she requires.
- b** Estimate what Jia's total shopping bill will be.
- c** If Jia sells 300 hamburgers over the 3 nights for \$2 each, how much money will she receive for the hamburgers?
- d** Approximately how much money will Jia raise through selling hamburgers over the 3 nights?



Item	Item price	Quantity required	Estimated cost
Bread rolls	\$2.90/dozen	25 packets of 12	
Hamburgers	\$2.40/dozen	25 packets of 12	
Tomato sauce	\$1.80/litre	2 litres	
Margarine	\$2.20/tub	2 tubs	
Onions	\$1.85/kilogram	2 kilograms	
Tomatoes	\$3.50/kilogram	2 kilograms	
Lettuce	\$1.10 each	5 lettuces	

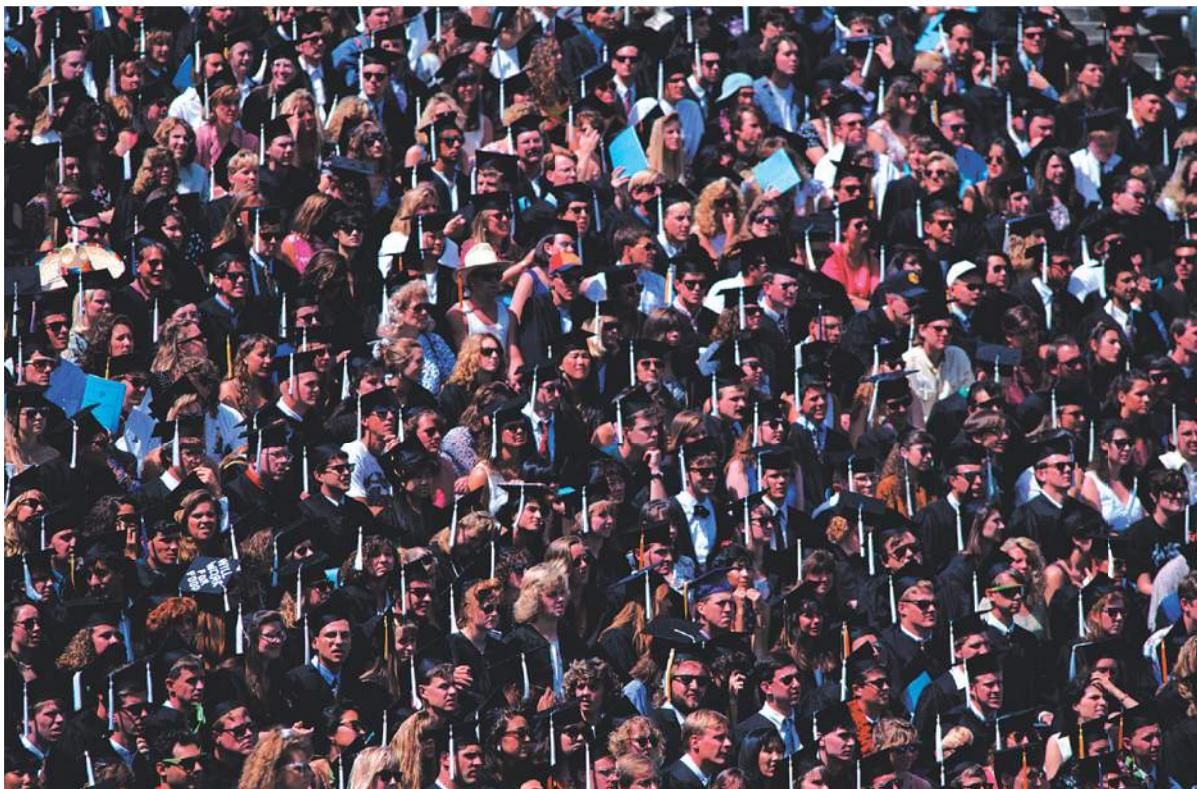
**eBookplus****Digital doc**  
Worksheet 2.2  
doc-0274**Weblink****Rounding quiz**

**REASONING**

- 8 Estimate the number of graduating students shown in the photograph below. (Do not count all of the students in the photograph.) If the hall holds 12 times this number, estimate the total capacity of the hall. Show all working and write a sentence explaining how you solved this problem.

**REFLECTION**

When is it important that you do not estimate calculations by rounding numbers?



# Summary

## Place value

- Numbers are organised by place value.
- The first place value is the units. Each place value to the left of this column has a place value 10 times as great as the value in the previous column.
- Numbers can be placed in order by comparing their place values.

## Adding and subtracting positive integers

- Addition and subtraction are the inverse of each other.
- When we are adding and subtracting, it is important to line up the numbers vertically.
- When we are adding numbers, the order in which they are added is not important.
- Large numbers are subtracted by decomposing the larger number into parts.

## Multiplying and dividing positive integers

- The basis for all multiplication work is to know your multiplication tables up to 12.
- Some multiplications can be done mentally using various strategies.
- Division is the inverse of multiplication.
- Use short division when dividing by numbers up to 12.
- When dividing numbers that are multiples of 10, write the question as a fraction, cancel as many zeroes as possible and then divide.
- Addition and multiplication are commutative and associative.
- Multiplication is commutative, associative and distributive.
- Subtraction and division are not commutative, associative or distributive.

## Long division

- Long division uses the same processes as short division, but the steps are written down.
- Use long division when the divisor is greater than 12.

## Order of operations

- The operations inside brackets are always calculated first.
- If there is more than one set of brackets, calculate operations inside the innermost brackets first.
- Multiplication and division operations are calculated in the order that they appear.
- Addition and subtraction operations are calculated in the order that they appear.

## Factors and multipliers

- A multiple of a number is the answer obtained when that number is multiplied by another whole number.
- A factor is a whole number that divides exactly into another whole number with no remainder.
- The factors of a number can be found using factor pairs.

## Lowest common multiple and highest common factor

- If two or more numbers have the same multiple, it is called a common multiple.
- The smallest number that 2 or more numbers can divide into is called the lowest common multiple of those numbers.
- If two or more numbers have the same factor, it is called a common factor.
- The highest common factor or HCF of 2 or more numbers is the largest factor that divides into all of the given numbers, with no remainder.

**Estimation**

- When rounding, if the second digit is 0, 1, 2, 3 or 4, the first digit stays the same. If the second digit is 5, 6, 7, 8 or 9, the first digit is rounded up.

**MAPPING YOUR UNDERSTANDING**

Using terms from the summary, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1** State the place value of the digit shown in red in each of the following.
  - a** 74 03<sup>7</sup>
  - b** 541 910
  - c** 1 904 000
  - d** 290
- 2** Write each of the following numbers using expanded notation.
  - a** 392
  - b** 4109
  - c** 42 001
  - d** 120 000
- 3** List the numbers 394, 349, 943, 934, 3994, 3499 in ascending order.
- 4** List the numbers 1011, 101, 110, 1100, 1101 in descending order.
- 5** Add these numbers.
  - a** 43 + 84
  - b** 139 + 3048
  - c** 3488 + 91 + 4062
  - d** 3 486 208 + 38 645 + 692 803
- 6** Calculate each of the following.
  - a** 20 – 12 + 8 – 14
  - b** 35 + 15 + 5 – 20
  - c** 300 – 170 + 20
  - d** 18 + 10 – 3 – 11
- 7** Complete these subtractions.
  - a** 688 – 273
  - b** 400 – 183
  - c** 68 348 – 8026
  - d** 46 234 – 8476
  - e** 286 005 – 193 048
  - f** 1 370 000 – 1 274 455
- 8** Use mental strategies to multiply each of the following.
  - a**  $2 \times 15 \times 5$
  - b**  $4 \times 84 \times 25$
  - c**  $62 \times 20$
  - d**  $56 \times 300$
  - e**  $67 \times 9$
  - f**  $31 \times 19$
- 9** Calculate each of these using short division.
  - a**  $4172 \div 7$
  - b**  $101\ 040 \div 12$
  - c**  $15\ 063 \div 3$

- 10** Calculate each of these.

- a**  $6 \times 4 \div 3$
- b**  $4 \times 9 \div 12$
- c**  $49 \div 7 \times 12$
- d**  $81 \div 9 \times 5$
- e**  $6 \times 3 \div 9 \div 2$
- f**  $12 \div 2 \times 11 \div 3$

- 11** Divide these multiples of 10.

- a**  $84\ 000 \div 120$
- b**  $4900 \div 700$
- c**  $12\ 300 \div 30$

- 12** Evaluate the following using long division:

- a**  $4706 \div 13$
- b**  $6867 \div 21$
- c**  $10\ 744 \div 17$
- d**  $3762 \div 19$

- 13** Evaluate the following using long division:

- a**  $5702 \div 17$
- b**  $6932 \div 23$
- c**  $11\ 852 \div 13$
- d**  $13\ 187 \div 29$

- 14** Write the rules for the order of operations.

- 15** Follow the rules for the order of operations to calculate each of the following.

- a**  $35 \div (12 - 5)$
- b**  $11 \times 3 + 5$
- c**  $8 \times 3 \div 4$
- d**  $5 \times 12 - 11 \times 5$
- e**  $(6 + 4) \times 7$
- f**  $6 + 4 \times 7$
- g**  $3 \times (4 + 5) \times 2$
- h**  $5 + [21 - (5 \times 3)] \times 4$

- 16** By rounding each number to its first digit, estimate the answer to each of the calculations.

- a**  $6802 + 7486$
- b**  $8914 - 3571$
- c**  $5304 \div 143$
- d**  $5706 \times 68$
- e**  $49\ 581 + 73\ 258$
- f**  $17\ 564 - 10\ 689$
- g**  $9480 \div 2559$
- h**  $289 \times 671$

- 17** List the first 5 multiples of each number.

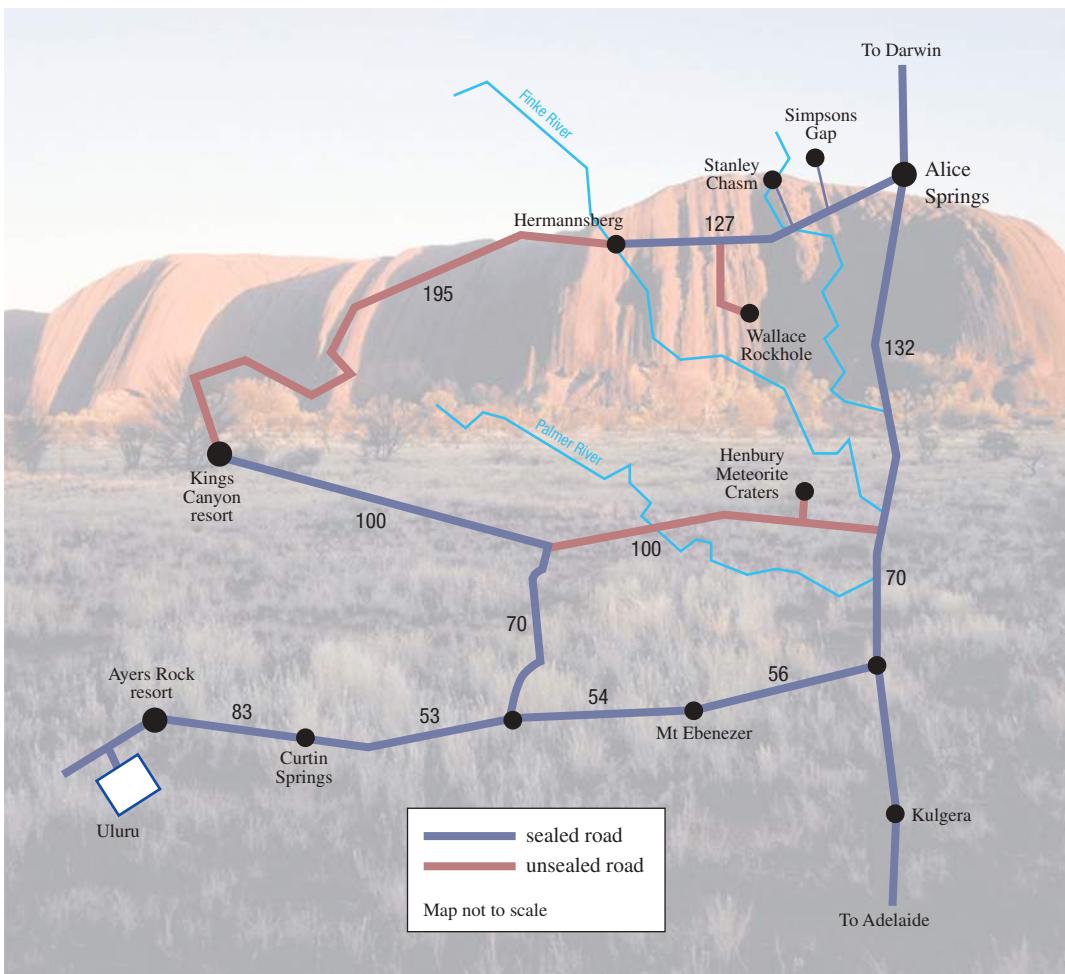
- a** 11
- b** 100
- c** 5
- d** 20
- e** 13
- f** 35

- 18** Find the lowest common multiple (LCM) of the following pairs of numbers.
- 3 and 12
  - 6 and 15
  - 4 and 7
  - 5 and 8
- 19** Find all the factors of each of the following numbers.
- 16
  - 27
  - 50
  - 42
  - 36
  - 72

- 20** List the factor pairs of the following numbers.
- 24
  - 40
  - 48
  - 21
  - 99
  - 100

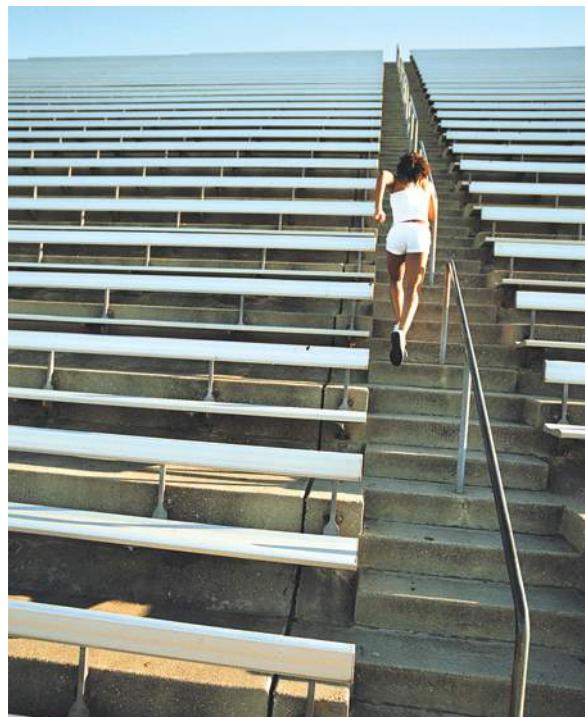
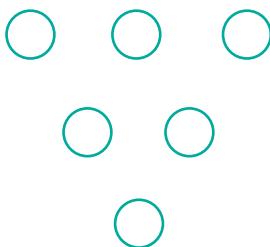
### PROBLEM SOLVING

- 1** Uluru is a sacred Aboriginal site. The map below shows some roads between Uluru and Alice Springs. The distances (in kilometres) along particular sections of road are indicated.



- How far is Kings Canyon resort from Ayers Rock resort near Uluru?
- What is the shortest distance by road if you are travelling from Kings Canyon resort to Alice Springs?
- If you are in a hire car, you must travel only on sealed roads. Calculate the distance you need to travel if driving from Kings Canyon resort to Alice Springs.

- 2** In summer, an ice-cream factory operates 16 hours a day and makes 28 ice-creams each hour.
- How many ice-creams are produced each day?
  - If the factory operates 7 days a week, how many ice-creams are produced in one week?
  - If there are 32 staff who run the machines over a week, how many ice-creams would each person produce?
- 3** Hung and Frank are cyclists who are riding around a track. They ride past the finish line together, and from then Hung takes 25 seconds to complete a lap and Frank takes 40 seconds.
- How long will it be until they next pass the finish line together?
  - How many laps will each have ridden when this occurs?
- 4** When you add two even positive integers, the answer is even; the sum of an even and an odd positive integer is an odd integer; two odd positive integers add to give an even integer. What happens if you perform multiplication on these types of integers? Give examples to support your statements. Write a general statement to summarise what would result when you multiplied more than two positive integers.
- 5** Joe, Claire and Daniela were having races up and down a flight of stairs. Joe took the stairs three at a time, Daniela took the stairs two at a time while Claire took the stairs four at a time. In each case, they reached the bottom with no steps left over.
- How many steps are there in the flight of stairs?  
List three possible answers.
  - What is the smallest number of steps there could be?
  - If Joe can also take the stairs five at a time with no steps left over, what is the smallest number of steps in the flight of stairs?
- 6** Place the numbers 1 to 6 in these circles so that the number in every circle below two higher circles shows the difference between the two numbers in the higher circles



- 7** A perfect number is one whose factors (all except the number itself) add to the number. For example, 6 is a perfect number because  $1 + 2 + 3$  (the sum of its factors, excluding 6) is equal to 6. Show why 496 is a perfect number.

- 8 Complete this subtraction by filling in the missing numbers in the boxes.

$$\begin{array}{r} 8 & 2 & \boxed{\phantom{0}} & 0 \\ - 3 & \boxed{\phantom{0}} & 8 & \boxed{\phantom{0}} \\ \hline \boxed{\phantom{0}} & 5 & 5 & 9 \end{array}$$

- 9 At a confectionary factory, a machine dispenses 760 lollies into 8 jars. Assuming that an equal number of lollies is dispensed into each jar, find the number in each jar.
- 10 Arrange the digits 0, 2, 4, 5, and 8 to form the smallest possible 5-digit number. You must use each digit once and only once.
- 11 In a race, one dirt bike rider completes each lap in 40 seconds while another completes it in 60 seconds. How long after the start of the race will the two bikes pass the starting point together?



- 12 There are sets of numbers known as *Hailstone numbers*. Here's an example. Pick any integer and enter it into your calculator. If it is an even number, divide by 2. If it is an odd number, multiply by 3, then add 1. Repeat the process with the new number over and over. Explain what happens in the long run, and why these numbers would be called *Hailstone numbers*.
- 13 Julie sells 8 bottles of soft drink for \$3 each and 12 bottles of water for \$2 each.
- Write a calculation that will find the total value of Julie's sales.
  - Find the total value of Julie's sales.
- 14 At a football match, Richard estimates the crowd to be 20 000 people. If Richard's estimate is correct to the nearest 1000 people, what is the greatest number of people that could possibly be at the football match.

**eBook plus**
**Interactivities**

Test yourself  
 Chapter 2  
 int-1814  
 Word search  
 Chapter 2  
 int-2762  
 Crossword  
 Chapter 2  
 int-2763

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 2 (doc-6408) (*page 25*)

**Are you ready?****Digital docs (page 26)**

- SkillsHEET 2.1 (doc-6409) Place value
- SkillsHEET 2.2 (doc-6410) Ascending and descending order
- SkillsHEET 2.3 (doc-6411) Adding and subtracting whole numbers less than 20
- SkillsHEET 2.4 (doc-6412) Times tables
- SkillsHEET 2.5 (doc-6413) Multiplying whole numbers
- SkillsHEET 2.6 (doc-6414) Dividing whole numbers
- SkillsHEET 2.7 (doc-6415) Order of operations
- SkillsHEET 2.8 (doc-6416) Multiples
- SkillsHEET 2.9 (doc-6417) Factor pairs
- SkillsHEET 2.10 (doc-6418) Rounding to the first (leading) digit

**2A Place value****Digital docs (page 28)**

- Activity 2-A-1 (doc-0275) Whole numbers
- Activity 2-A-2 (doc-0276) More whole numbers
- Activity 2-A-3 (doc-0277) Advanced whole numbers

**2B Adding and subtracting positive integers****Digital docs**

- Activity 2-B-1 (doc-0278) Subtracting positive integers (*page 31*)
- Activity 2-B-2 (doc-0279) Adding and subtracting positive integers (*page 31*)
- Activity 2-B-3 (doc-0280) Advanced addition and subtraction of positive integers (*page 31*)
- Spreadsheet Adding numbers (doc-6419) (*page 32*)
- Spreadsheet Subtracting numbers (doc-2086) (*page 32*)
- Worksheet 2.1 (doc-0273) (*page 34*)

**2C Multiplying and dividing positive integers****eLesson**

- Identifying patterns (eles-0745) (*page 36*)

**Digital docs**

- Activity 2-C-1 (doc-0281) Division bingo (*page 37*)
- Activity 2-C-2 (doc-0282) More division bingo (*page 37*)
- Activity 2-C-3 (doc-0283) Advanced division bingo (*page 38*)
- Spreadsheet Multiplying numbers (doc-2087) (*page 39*)
- Spreadsheet Tangle tables (doc-2088) (*page 39*)
- Spreadsheet Dividing numbers (doc-2089) (*page 39*)

**Weblink**

- Fruit shoot game (*page 41*)

**2D Long division****Digital docs (page 44)**

- Activity 2-D-1 (doc-1096) Long division
- Activity 2-D-2 (doc-1097) More long division
- Activity 2-D-3 (doc-1098) Advanced long division

**Weblink**

- Long division game (*page 44*)

**2E Order of operations****Digital docs (page 46)**

- Activity 2-E-1 (doc-0284) Order of operations
- Activity 2-E-2 (doc-0285) More order of operations
- Activity 2-E-3 (doc-0286) Advanced order of operations
- Spreadsheet Four operations (doc-2090)

**2F Factors and multiples****Digital docs**

- Activity 2-F-1 (doc-0287) Factors and multiples (*page 48*)
- Activity 2-F-2 (doc-0288) More factors and multiples (*page 49*)
- Activity 2-F-3 (doc-0289) Advanced factors and multiples (*page 49*)

**2G Lowest common multiple and highest common factor****Digital docs (page 52)**

- Activity 2-G-1 (doc-0290) Lowest common multiples and highest common factors
- Activity 2-G-2 (doc-0291) More lowest common multiples and highest common factors
- Activity 2-G-3 (doc-0292) Advanced lowest common multiples and highest common factors

**2H Estimation****eLesson**

- Estimating and rounding (eles-0822) (*page 56*)

**Digital docs**

- Activity 2-H-1 (doc-0293) Estimation (*page 56*)
- Activity 2-H-2 (doc-0294) More estimation (*page 56*)
- Activity 2-H-3 (doc-0295) Advanced estimation (*page 57*)
- Worksheet 2.2 (doc-0274) (*page 58*)

**Weblink**

- Rounding quiz (*page 58*)

**Chapter review****interactivities (page 65)**

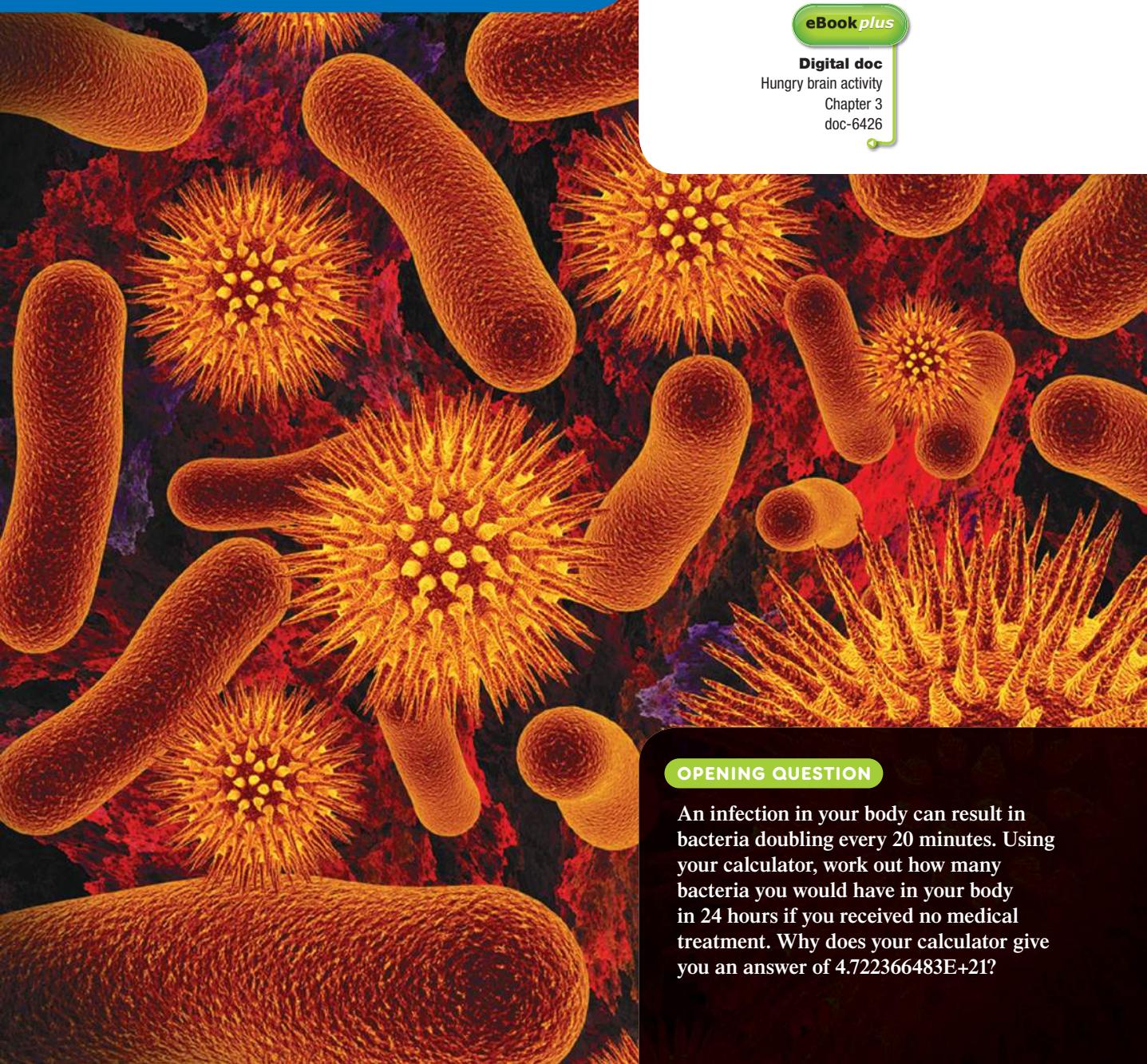
- Test yourself Chapter 2 (int-1814) Take the end-of-chapter test to test your progress.
- Word search Chapter 2 (int-2762)
- Crossword Chapter 2 (int-2763)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

## 3

# Indices and primes



- 3A** Index notation
- 3B** Prime numbers and composite numbers
- 3C** Squares and square roots
- 3D** Cubes and cube roots

## WHAT DO YOU KNOW?

- 1 List what you know about indices and prime numbers. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of indices and prime numbers.

eBook plus

Digital doc

Hungry brain activity

Chapter 3

doc-6426

## OPENING QUESTION

An infection in your body can result in bacteria doubling every 20 minutes. Using your calculator, work out how many bacteria you would have in your body in 24 hours if you received no medical treatment. Why does your calculator give you an answer of  $4.722366483\text{E}+21$ ?

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can obtain extra help by completing the matching SkillsHEET located on your eBookPLUS.



## Finding the square of a number

- 1 Find the value of each of the following.

- a  $4 \times 4$
- b  $12 \times 12$
- c  $7 \times 7$

**Digital doc**  
SkillsHEET 3.1  
doc-6427



## Finding the square root of a number

- 2 Find the value of the missing number represented by the symbol  $\Delta$  in each of the following.

- a If  $6 \times \Delta = 36$  then  $\Delta = ?$
- b If  $\Delta \times 9 = 81$  then  $\Delta = ?$
- c If  $\Delta \times \Delta = 25$  then  $\Delta = ?$
- d If  $\Delta \times \Delta = 64$  then  $\Delta = ?$

**Digital doc**  
SkillsHEET 3.2  
doc-6428



## Finding the cube of a number

- 3 Find the value of each of the following.

- a  $5 \times 5 \times 5$
- b  $11 \times 11 \times 11$
- c  $3 \times 3 \times 3$

**Digital doc**  
SkillsHEET 3.3  
doc-6429



## Finding the cube root of a number

- 4 Find the value of the missing number represented by the symbol  $\Delta$  in each of the following.

- a If  $6 \times \Delta \times \Delta = 216$  then  $\Delta = ?$
- b If  $\Delta \times 9 \times 9 = 729$  then  $\Delta = ?$
- c If  $\Delta \times \Delta \times \Delta = 8$  then  $\Delta = ?$
- d If  $\Delta \times \Delta \times \Delta = 64$  then  $\Delta = ?$

**Digital doc**  
SkillsHEET 3.4  
doc-6430



## Order of operations with squares and square roots

- 5 Evaluate each of the following.

- a  $10 \times 10 + 7$
- b  $9 \times 9 + 2 \times 2$
- c  $35 - 3 \times 3$

**Digital doc**  
SkillsHEET 3.5  
doc-6431



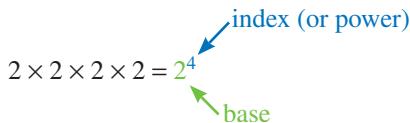
## Consecutive numbers

- 6 a Write three consecutive numbers, beginning with 5.  
 b Write four consecutive even numbers, beginning with 14.  
 c Write three consecutive odd numbers, beginning with 37.

**Digital doc**  
SkillsHEET 3.6  
doc-6432

## 3A Index notation

- An index is a shorthand way of writing a **repeated multiplication**.
- The **base** is the number that is being multiplied and the **index (or power)** is the number of times it is multiplied.



### WORKED EXAMPLE 1

**Write the following using index notation.**

a  $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

b  $3 \times 3 \times 3 \times 3 \times 7 \times 7$

#### THINK

- a 1 Write the multiplication.  
2 Write the number being multiplied as the base and the number of times it is written as the index.
- b 1 Write the multiplication.  
2 Write the number being multiplied as the base and the number of times it is written as the index.

#### WRITE

$$\begin{aligned} \text{a } 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ = 5^7 \end{aligned}$$

$$\begin{aligned} \text{b } 3 \times 3 \times 3 \times 3 \times 7 \times 7 \\ = 3^4 \times 7^2 \end{aligned}$$

- By using place value, you can write numbers in expanded form with index notation.  
 $2700 = 2000 + 700 = 2 \times 10 \times 10 \times 10 + 7 \times 10 \times 10 = 2 \times 10^3 + 7 \times 10^2$

### WORKED EXAMPLE 2

**Write the following numbers in expanded form using index notation.**

a 59 176

b 108 009

#### THINK

- a 1 Read the number to yourself, stating the place values.  
2 Write the number as the sum of each place value.  
3 Write each place value in multiples of 10.  
4 Write each place value in index notation.
- b 1 Read the number to yourself stating the place values.  
2 Write the number as the sum of each place value.  
3 Write each place value in multiples of 10.  
4 Write each place value in index notation.

#### WRITE

$$\begin{aligned} \text{a } & 59\,176 = 50\,000 + 9\,000 + 100 + 70 + 6 \\ & 59\,176 = 5 \times 10 \times 10 \times 10 \times 10 + 9 \times 10 \\ & \quad \times 10 \times 10 + 1 \times 10 \times 10 + 7 \\ & \quad \times 10 + 6 \\ & 59\,176 = 5 \times 10^4 + 9 \times 10^3 + 1 \times 10^2 + 7 \\ & \quad \times 10^1 + 6 \end{aligned}$$

$$\begin{aligned} \text{b } & 108\,009 = 100\,000 + 8\,000 + 9 \\ & 108\,009 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \\ & \quad + 8 \times 10 \times 10 \times 10 + 9 \\ & 108\,009 = 1 \times 10^5 + 8 \times 10^3 + 9 \end{aligned}$$

**WORKED EXAMPLE 3**

Simplify each of the following by first writing as a repeated multiplication and then using index notation.

a  $3^4 \times 3^6$

b  $(4^3)^3$

c  $(3 \times 5)^2$

**THINK**

- a 1 Write the question.  
 2 Write the question using repeated multiplication (that is, in expanded form).  
 3 Write the repeated multiplication using index notation.  
*Note:*  $3^4 \times 3^6 = 3^{10}$ .
- b 1 Write the question.  
 2 Write the question using repeated multiplication (that is, in expanded form).  
 3 Write the repeated multiplication using index notation.  
*Note:*  $(4^3)^3 = 4^9$ .
- c 1 Write the question.  
 2 Write the question using repeated multiplication (that is, in expanded form).  
 3 Write the repeated multiplication using index notation.  
*Note:*  $(3 \times 5)^2 = 3^2 \times 5^2$ .

**WRITE**

$$\begin{aligned} \text{a } 3^4 \times 3^6 &= (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3) \\ &= 3 \times 3 \\ &= 3^{10} \\ \\ \text{b } (4^3)^3 &= 4^3 \times 4^3 \times 4^3 \\ &= (4 \times 4 \times 4) \times (4 \times 4 \times 4) \times (4 \times 4 \times 4) \\ &= 4 \times 4 \\ &= 4^9 \\ \\ \text{c } (3 \times 5)^2 &= (3 \times 5) \times (3 \times 5) \\ &= 3 \times 5 \times 3 \times 5 \\ &= 3 \times 3 \times 5 \times 5 \\ &= 3^2 \times 5^2 \end{aligned}$$

**REMEMBER**

- Index notation is a shorthand way of writing a repeated multiplication.
- The number being multiplied is the base of the expression; the number of times it has been written is the index (or power).

**EXERCISE****3A Index notation****INDIVIDUAL PATHWAYS****eBook plus****Activity 3-A-1**  
Famous astronomer  
doc-1675**Activity 3-A-2**  
Famous mathematician  
doc-1676**Activity 3-A-3**  
Famous physicist  
doc-1677**FLUENCY**

- 1 **WE1a** Write  $7 \times 7 \times 7 \times 7$  using index notation.
- 2 Write each of the following using index notation.
- $2 \times 2 \times 2 \times 2$
  - $8 \times 8 \times 8 \times 8 \times 8 \times 8$
  - $10 \times 10 \times 10 \times 10 \times 10$
  - $3 \times 3 \times 3$
  - $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
  - $13 \times 13 \times 13$
  - $12 \times 12 \times 12 \times 12 \times 12 \times 12$
  - $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
- 3 **WE1b** Write  $4 \times 4 \times 4 \times 4 \times 4 \times 6 \times 6 \times 6$  using index notation.

4 Write the following using index notation.

- |   |  |
|---|--|
| <b>a</b> $2 \times 2 \times 3$                              | <b>b</b> $3 \times 3 \times 3 \times 3 \times 2 \times 2$          |
| <b>c</b> $5 \times 5 \times 2 \times 2 \times 2 \times 2$   | <b>d</b> $7 \times 2 \times 2 \times 2$                            |
| <b>e</b> $5 \times 11 \times 11 \times 3 \times 3 \times 3$ | <b>f</b> $13 \times 5 \times 5 \times 5 \times 7 \times 7$         |
| <b>g</b> $2 \times 2 \times 2 \times 3 \times 3 \times 5$   | <b>h</b> $3 \times 3 \times 2 \times 2 \times 5 \times 5 \times 5$ |

5 Write  $6^5$  using a repeated multiplication (that is, in expanded form).

6 Write each of the following using repeated multiplication (that is, in expanded form).

- |                 |                |                |
|-----------------|----------------|----------------|
| <b>a</b> $11^3$ | <b>b</b> $4^9$ | <b>c</b> $5^6$ |
|-----------------|----------------|----------------|

7 **WE2** Write the following numbers in expanded form using index notation.

- |                 |                 |                  |
|-----------------|-----------------|------------------|
| <b>a</b> 300    | <b>b</b> 4500   | <b>c</b> 6785    |
| <b>d</b> 10 000 | <b>e</b> 31 702 | <b>f</b> 200 301 |

8 **WE3** Simplify each of the following by first writing as a repeated multiplication and then using index notation.

- |                            |                           |                           |
|----------------------------|---------------------------|---------------------------|
| <b>a</b> $5^8 \times 5^3$  | <b>b</b> $7^2 \times 7^6$ | <b>c</b> $4^8 \times 4^5$ |
| <b>d</b> $(2^7)^3$         | <b>e</b> $(11^3)^8$       | <b>f</b> $(9^5)^3$        |
| <b>g</b> $(3 \times 13)^6$ | <b>h</b> $(2 \times 5)^4$ | <b>i</b> $(5 \times 7)^5$ |

### UNDERSTANDING

9 **a MC** Which of the following expressions has the greatest value?

- |                |                |                |                |                |
|----------------|----------------|----------------|----------------|----------------|
| <b>A</b> $2^8$ | <b>B</b> $8^2$ | <b>C</b> $3^4$ | <b>D</b> $4^3$ | <b>E</b> $9^2$ |
|----------------|----------------|----------------|----------------|----------------|

**b** Justify your answer using mathematical reasoning.

10 **MC** The value of  $4^4$  is:

- |            |             |             |              |              |
|------------|-------------|-------------|--------------|--------------|
| <b>A</b> 8 | <b>B</b> 16 | <b>C</b> 64 | <b>D</b> 256 | <b>E</b> 484 |
|------------|-------------|-------------|--------------|--------------|

11 Find the value of  $2^3 \times 3^2$ .

12 Evaluate each of the following.

- |                           |                                 |                               |
|---------------------------|---------------------------------|-------------------------------|
| <b>a</b> $3^4 \times 4^3$ | <b>b</b> $3^5 + 9^3$            | <b>c</b> $8^3 \div 2^5$       |
| <b>d</b> $6^4 - 9^3$      | <b>e</b> $5^3 + 2^5 \times 9^2$ | <b>f</b> $2^7 - 4^5 \div 2^6$ |

### REASONING

13 We know that  $12^2 = 144$  and  $21^2 = 441$ . It is also true that  $13^2 = 169$  and  $31^2 = 961$ . If  $14^2 = 196$  will  $41^2 = 691$ ? Try to justify your answer without calculating  $41^2$ .

14 There is a shortcut that allows you to square any number that ends in 5. Look at the following examples and see if you can identify the rule. Use your rule to evaluate  $75^2$  and  $95^2$ .

$$15^2 = 225, 25^2 = 625, 35^2 = 1225, 45^2 = 2025$$

### REFLECTION

Why do we use index notation?

## 3B Prime numbers and composite numbers

### Prime numbers

- A **prime number** is a counting number that has exactly 2 factors, itself and 1. (Counting numbers are 1, 2, 3, 4, ...)
- The number 3 is a prime number. Its only factors are 3 and 1.
- The number 2 is the only even prime number. Its only factors are 2 and 1.
- A **composite number** is one which has more than two factors.
- The number 4 is composite. Its factors are 1, 2 and 4.
- The number 1 is a special number. It is neither a prime number nor a composite number because it has only one factor, 1.

**WORKED EXAMPLE 4**

List the prime numbers between 50 and 70.

**THINK**

- 1 The only even prime number is 2. The prime numbers between 50 and 70 will be odd. Numbers ending in 5 are divisible by 5 so 55 and 65 are not primes.
- 2 Check the remaining odd numbers between 20 and 40: **51 = 3 × 17**  
**53 is prime**  
**57 = 3 × 19**  
**61 is prime**  
**63 = 7 × 9**  
**67 is prime**  
**69 = 3 × 23**

**WRITE**

The prime numbers are 53, 57, 63, 69.

**WORKED EXAMPLE 5**

State whether the following numbers are prime or composite.

- a** 45                    **b** 37                    **c** 86

**THINK**

- a** Factors of 45 are 1, 3, 5, 9, 15 and 45.  
**b** The only factors of 37 are 1 and 37.  
**c** All even numbers except 2 are composite.

**WRITE**

- a** 45 is composite.  
**b** 37 is prime.  
**c** 86 is composite.

**Composite numbers and factor trees**

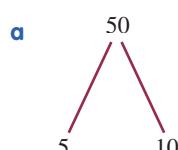
- Every composite number can be written as the product of powers of prime numbers; for example,  $12 = 2 \times 2 \times 3 = 2^2 \times 3$ .
- A **factor tree** shows the **prime factors** of a composite number.
- Each branch shows a factor of all the numbers above it.
- The last numbers in a factor tree are all prime numbers.

**WORKED EXAMPLE 6**

- a** Find the prime factors of 50 by drawing a factor tree.  
**b** Write 50 as a product of its prime factors.

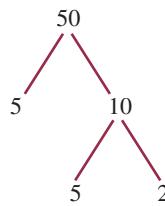
**THINK**

- a** 1 Find a factor pair of the given number and begin the factor tree ( $50 = 5 \times 10$ ).

**WRITE**

- 2** If a branch is prime, no other factors can be found (5 is prime). If a branch is composite, find factors of that number; 10 is composite so  $10 = 5 \times 2$ .

- 3** Continue until all branches end in a prime number, then stop.
- 4** Write the prime factors.
- b** Write 50 as a product of prime factors found in part **a**.



The prime factors of 50 are 2 and 5.

**b**  $50 = 5 \times 5 \times 2$   
 $= 5^2 \times 2$

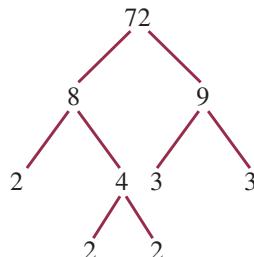
### WORKED EXAMPLE 7

Write 72 as a product of its prime factors in index form.

#### THINK

- 1** Draw a factor tree. When all factors are prime numbers you have found the prime factors.

#### WRITE



- 2** Write 72 as a product of its prime factors in index form.

$72 = 2^3 \times 3^2$

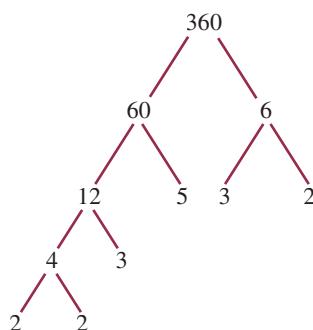
### WORKED EXAMPLE 8

Write 360 as a product of prime factors using index notation.

#### THINK

- 1** Find a factor pair and begin the factor tree. If the number on the branch is a prime number, stop. If not, continue until a prime number is reached.

#### WRITE



- 2** Write the number as a product of prime factors.
- 3** Write your answer using index notation.

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$   
 $360 = 2^3 \times 3^2 \times 5$

## Lowest common multiple and highest common factors using prime factorisation

- It can be difficult to find the lowest common multiple (LCM) or the highest common factor (HCF) of large numbers by inspection alone.
- By expressing large numbers as products of their prime factors, it can be easier to determine their LCM or HCF.
- The HCF of two numbers expressed as the products of their prime factors is the product of the prime factors common to both numbers.
- The LCM of two numbers expressed as the products of their prime factors is the product of the prime factors that are factors of either number.

### WORKED EXAMPLE 9

Determine the highest common factor and lowest common multiple of 270 and 900.

#### THINK

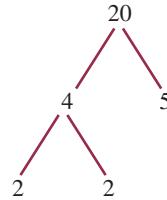
- Write 270 and 900 as products of their prime factors.
- Circle the prime factors common to both numbers.
- For the HCF, multiply the prime factors common to both numbers.
- For the LCM, multiply the prime factors that are factors of either number.

#### WRITE

$$\begin{aligned} 270 &= 2 \times 3 \times 3 \times 3 \times 5 \\ 900 &= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \\ 270 &= (2) \times (3) \times (3) \times 3 \times 5 \\ 900 &= 2 \times (2) \times (3) \times (3) \times 5 \times 5 \\ 2 \times 3 \times 3 \times 5 &= 90 \\ \text{The HCF of 270 and 900 is 90.} \\ 2 \times 3 \times 3 \times 5 \times 2 \times 3 \times 5 &= 2700 \\ \text{The LCM of 270 and 900 is 2700.} \end{aligned}$$

#### REMEMBER

- A *prime number* is a whole number which has exactly two factors, itself and 1.
- A *composite number* is one which has more than two factors.
- The number 1 is neither a prime number nor a composite number.
- A *factor tree* shows the *prime factors* of a composite number.
- The last numbers in the factor tree are all prime numbers, therefore they are prime factors of the original number.
- Every composite number can be written as a product of prime factors; for example,  $20 = 2 \times 2 \times 5$ .
- The product of prime factors can be written in shorter form using index notation.
- Lowest common multiples and highest common factors can be determined by expressing numbers as the products of their prime factors.



### EXERCISE

## 3B Prime numbers and composite numbers

#### FLUENCY

- WE4** Find four prime numbers that are between 20 and 40.
  - WE5** State whether each of the following numbers is prime or composite.
- |             |             |              |              |
|-------------|-------------|--------------|--------------|
| <b>a</b> 9  | <b>b</b> 13 | <b>c</b> 27  | <b>d</b> 55  |
| <b>e</b> 41 | <b>f</b> 64 | <b>g</b> 49  | <b>h</b> 93  |
| <b>i</b> 51 | <b>j</b> 79 | <b>k</b> 243 | <b>l</b> 101 |

INDIVIDUAL  
PATHWAYS

## eBookplus

**Activity 3-B-1**Prime numbers and composite numbers  
doc-1678**Activity 3-B-2**More prime numbers and composite numbers  
doc-1679**Activity 3-B-3**Advanced prime numbers and composite numbers  
doc-1680

- 3 WE6** i Find the prime factors of each of the following numbers by drawing a factor tree.  
ii Write each one as a product of its prime factors.
- |       |      |       |
|-------|------|-------|
| a 15  | b 30 | c 24  |
| d 100 | e 49 | f 72  |
| g 18  | h 56 | i 45  |
| j 84  | k 98 | l 112 |
- 4 WE7** i Find the prime factors of the following numbers by drawing a factor tree.  
ii Express the number as a product of its prime factors.
- |        |       |       |
|--------|-------|-------|
| a 40   | b 35  | c 32  |
| d 121  | e 110 | f 150 |
| g 3000 | h 64  | i 96  |
| j 196  | k 90  | l 75  |
- 5** Find the prime factors of each of the following numbers.
- |       |       |      |
|-------|-------|------|
| a 48  | b 200 | c 42 |
| d 81  | e 18  | f 39 |
| g 27  | h 300 | i 60 |
| j 120 | k 50  | l 80 |
- 6 WE8** Write the following as a product of prime factors using index notation.
- |       |       |      |
|-------|-------|------|
| a 60  | b 50  | c 75 |
| d 220 | e 192 | f 72 |
| g 124 | h 200 |      |

## UNDERSTANDING

- 7 WE9** By expressing the following pairs of numbers as products of their prime factors, determine their lowest common multiple and their highest common factor.
- |               |             |               |
|---------------|-------------|---------------|
| a 36 and 84   | b 48 and 60 | c 120 and 400 |
| d 220 and 800 |             |               |
- 8** Can you find four prime numbers that are even? Explain.
- 9** Answer true (T) or false (F) for each of the following.
- |   |  |
|---|--|
| a All odd numbers are prime numbers.              | b No even numbers are prime numbers.           |
| c 1, 2, 3 and 5 are the first four prime numbers. | d A prime number has two factors only.         |
| e 2 is the only even prime number                 | f The sum of two prime numbers is always even. |
| g The product of two prime numbers is always odd. | h There are no consecutive prime numbers.      |
- 10 MC** a The number of primes less than 10 is:
- |     |     |     |     |     |
|-----|-----|-----|-----|-----|
| A 4 | B 3 | C 5 | D 2 | E 1 |
|-----|-----|-----|-----|-----|
- b The first three prime numbers are:
- |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|
| A 1, 3, 5 | B 2, 3, 4 | C 2, 3, 5 | D 3, 5, 7 | E 2, 5, 7 |
|-----------|-----------|-----------|-----------|-----------|
- c The number 15 can be written as the sum of two prime numbers. These are:
- |          |          |          |
|----------|----------|----------|
| A 3 + 12 | B 1 + 14 | C 13 + 2 |
| D 7 + 8  | E 9 + 6  |          |
- d Factors of 12 that are prime numbers are:
- |               |                     |        |
|---------------|---------------------|--------|
| A 1, 2, 3, 4  | B 2, 3, 6           | C 2, 3 |
| D 2, 4, 6, 12 | E 1, 2, 3, 4, 6, 12 |        |
- 11** Twin primes are pairs of primes that are separated from each other by one even number. For example, 3 and 5 are twin primes. Find two more pairs of twin primes.

- 12** **a** Which of the numbers 2, 3, 4, 5, 6 and 7 cannot be the difference between two consecutive prime numbers? Explain.
- b** For each of the numbers that can be a difference between two consecutive primes, give an example of a pair of primes less than 100 with such a difference.
- 13** The following numbers are not primes. Each of them is the product of two primes. Find the two primes in each case.
- a** 365
- b** 187
- 14** An easy way to find prime numbers is to use the ‘Sieve of Eratosthenes’. Eratosthenes discovered a simple method of sifting out all of the composite numbers so that only prime numbers are left. Alternatively, you can use the Excel file on your eBookPLUS.

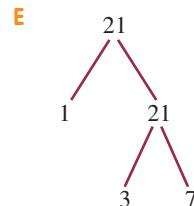
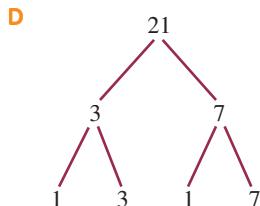
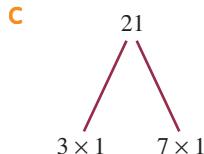
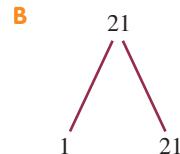
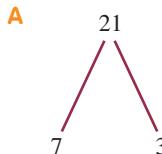
You can follow the steps below to find all prime numbers between 1 and 100.

- a** Copy the numbers from 1 to 100 in a grid as shown below. Use 1-centimetre square grid paper.

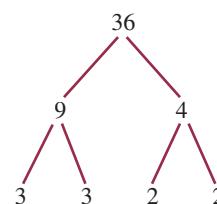
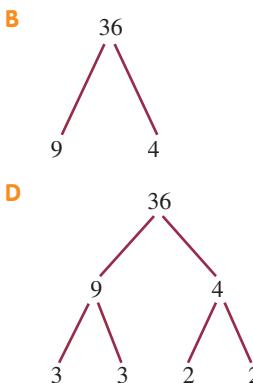
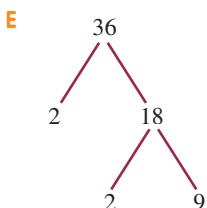
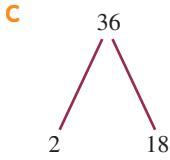
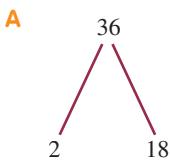
X	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- b** Cross out 1 as shown. It is not a prime number.
- c** Circle the first prime number, 2. Then cross out all of the multiples of 2.
- d** Circle the next prime number, 3. Now cross out all of the multiples of 3 that have not already been crossed out.
- e** The number 4 is already crossed out. Circle the next prime number, 5. Cross out all of the multiples of 5 that are not already crossed out.
- f** The next number that is not crossed out is 7. Circle 7 and cross out all of the multiples of 7 that are not already crossed out.
- g** Do you need to check the multiples of any primes greater than 7? Why or why not?

- 15** **MC** **a** A factor tree for 21 is:



- b A factor tree for 36 is:



- c The prime factors of 16 are:

A 1, 2  
D 1, 2, 4, 8, 16

B 1, 2, 4  
E 2, 4, 8

C 2

- d The prime factors of 28 are:

A 1, 28  
D 1, 2, 7

B 2, 7  
E 2, 7, 14

C 1, 2, 14

### REASONING

#### eBookplus

**Digital docs**  
WorkSHEET 3.1  
doc-1687

16 What is the largest three-digit prime number in which each digit is a prime number?

17 Find a prime number greater than 10 where the sum of the digits equals 11.

18 My age is a prime number. I am older than 50. The sum of the digits in my age is also a prime number. If you add a multiple of 13 to my age the result is 100. How old am I?

#### REFLECTION

What strategies will you use to help recall the difference between prime and composite numbers?

Prime number sequence: 1–1000

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997

#### eBookplus

**Weblink**  
Prime number game

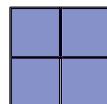
# 3C Squares and square roots

## Square numbers

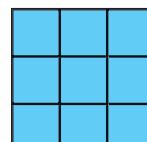
- Square numbers are numbers that can be arranged in a square, as shown below.



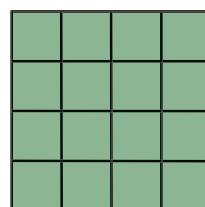
$$1^2 = 1$$



$$2^2 = 4$$



$$3^2 = 9$$



$$4^2 = 16$$

- The first square number, 1, equals  $1 \times 1$ .
- The second square number, 4, equals  $2 \times 2$ .
- The third square number, 9, equals  $3 \times 3$ .
- The fourth square number, 16, equals  $4 \times 4$ .
- If this pattern is continued, any square number can be found by multiplying the position of the square number by itself. This is known as *squaring a number* and is written by an index (or power) of 2. For example,  $3 \times 3$  can be written as  $3^2$ .

### WORKED EXAMPLE 10

Find the square numbers between 90 and 150.

#### THINK

- 1 Use your knowledge of tables to find the first square number after 90.
- 2 Find the square numbers which come after that one but before 150.
- 3 Write the answer in a sentence.

#### WRITE

$$10^2 = 10 \times 10 = 100$$

$$11^2 = 11 \times 11 = 121$$

$$12^2 = 12 \times 12 = 144$$

$$13^2 = 13 \times 13 = 169 \quad (\text{too big})$$

The square numbers between 90 and 150 are 100, 121 and 144.

**WORKED EXAMPLE 11**

Write the two whole numbers between which  $5.7^2$  will lie.

**THINK**

- 1 Write the whole numbers either side of 5.7.
- 2 Consider the square of each whole number.
- 3 Simplify  $5^2$  and  $6^2$  then write the answer in a sentence.
- 4 Verify your answer with a calculator.

**WRITE**

$5.7$  is between 5 and 6.  
 $5.7^2$  is between  $5^2$  and  $6^2$ .  
So  $5.7^2$  is between 25 and 36.

**Square roots**

- Evaluating the **square root** of a number is the opposite of squaring a number.
- By writing  $4^2 = 16$ , we mean that 4 multiplied by itself is equal to 16. The opposite of this is to say that the square root of 16 is 4, or  $\sqrt{16} = 4$ .
- Using the illustrations of the squares above, a square of area 16 square units must have a side length of 4 units.
- To find the square roots of larger numbers, it helps to break the number up as a product of two smaller square roots with which we are more familiar. For example,

$$\begin{aligned}\sqrt{900} &= \sqrt{9} \times \sqrt{100} \\ &= 3 \times 10 \\ &= 30\end{aligned}$$

**WORKED EXAMPLE 12**

Find: a  $\sqrt{49}$  b  $\sqrt{3600}$ .

**THINK**

- a Find a number which when multiplied by itself gives 49 ( $7 \times 7 = 49$ ).
- b 1 Write 3600 as the product of two smaller numbers for which we can find the square root.  
2 Take the square root of each of these numbers.  
3 Find the product.

**WRITE**

a  $\sqrt{49} = 7$   
b  $\sqrt{3600} = \sqrt{36} \times \sqrt{100}$   
 $= 6 \times 10$   
 $= 60$

**WORKED EXAMPLE 13**

Between which two numbers will  $\sqrt{74}$  lie?

**THINK**

- 1 Write the square numbers either side of 74.
- 2 Consider the square root of each number.
- 3 Simplify  $\sqrt{64}$  and  $\sqrt{81}$ .
- 4 Verify your answer with a calculator.

**WRITE**

$74$  is between 64 and 81.  
 $\sqrt{74}$  is between  $\sqrt{64}$  and  $\sqrt{81}$ .  
So  $\sqrt{74}$  is between 8 and 9.

## REMEMBER

- Square numbers are written using an index (or power) of 2 and are found by multiplying the number by itself.
- The opposite of squaring a number is finding the square root of a number.
- To find the square root of a number we can use our knowledge of square numbers or use a calculator.

## EXERCISE

## 3C

## Squares and square roots

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 3-C-1

Squares and square roots  
doc-1681

## Activity 3-C-2

More squares and square roots  
doc-1682

## Activity 3-C-3

Advanced squares and square roots  
doc-1683

## FLUENCY

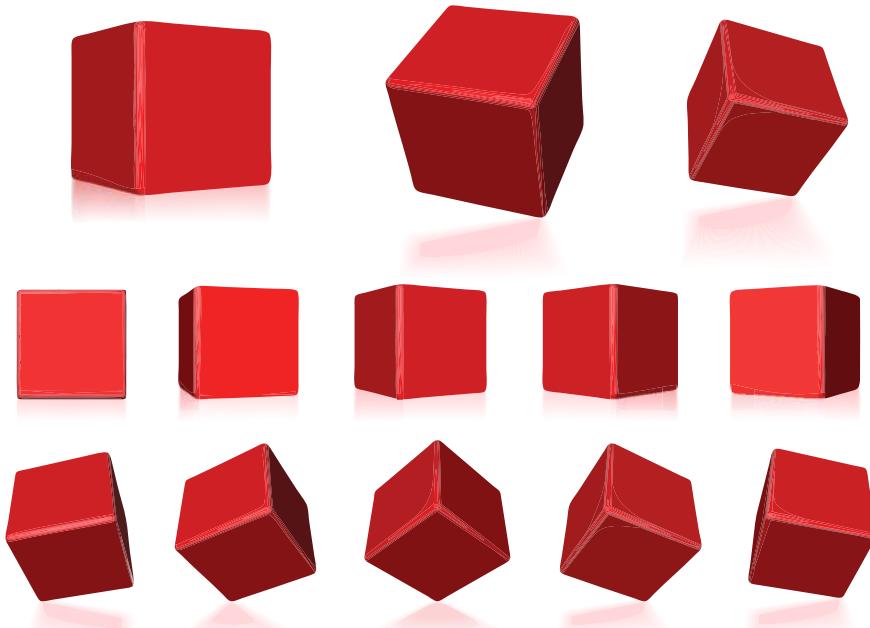
- 1 Evaluate and verify your answers with a calculator.
- a  $8^2$       b  $11^2$       c  $15^2$       d  $25^2$
- 2 **WE10** a Find the square numbers between 50 and 100.  
b Find the square numbers between 160 and 200.
- 3 **WE11** Write two whole square numbers between which each of the following will lie.
- a  $6.4^2$       b  $7.8^2$       c  $9.2^2$       d  $12.5^2$
- 4 **WE12a** Find:
- a  $\sqrt{25}$       b  $\sqrt{81}$       c  $\sqrt{144}$       d  $\sqrt{400}$
- 5 **WE12b** Find and verify your answers with a calculator.
- a  $\sqrt{4900}$       b  $\sqrt{14\,400}$       c  $\sqrt{360\,000}$       d  $\sqrt{160\,000}$
- 6 **WE13** Between which two numbers will  $\sqrt{60}$  lie?
- 7 Write the two whole numbers between which each of the following will lie.
- a  $\sqrt{14}$       b  $\sqrt{90}$       c  $\sqrt{200}$       d  $\sqrt{2}$

## UNDERSTANDING

- 8 a Find the even square numbers between 10 and 70.  
b Find the odd square numbers between 50 and 120.
- 9 **MC** For which of the following square roots can we calculate an exact answer?
- A  $\sqrt{10}$       B  $\sqrt{25}$       C  $\sqrt{50}$       D  $\sqrt{75}$       E  $\sqrt{82}$
- 10 **MC** For which of the following square roots can we NOT calculate the exact value?
- A  $\sqrt{160}$       B  $\sqrt{400}$       C  $\sqrt{900}$       D  $\sqrt{2500}$       E  $\sqrt{3600}$
- 11 Evaluate the following. Verify your answers with a calculator.
- a  $2^2 + \sqrt{25}$   
 b  $9^2 - \sqrt{36}$   
 c  $5^2 \times 2^2 \times \sqrt{49}$   
 d  $3^2 + 2^2 \times \sqrt{16}$   
 e  $3^2 - 2^2 \div \sqrt{4} + \sqrt{49}$   
 f  $\sqrt{9} \times 4^2 - \sqrt{144} \div 2$

## REASONING

- 12 Explain why we cannot find the exact value of  $\sqrt{10}$ .
- 13 Megan has 3 game scores that happen to be square numbers. The first 2 scores have the same three digits. The total of the 3 scores is 590. What are the 3 scores?
- 14 The difference of the squares of two consecutive odd numbers is 32. What are the two odd numbers?



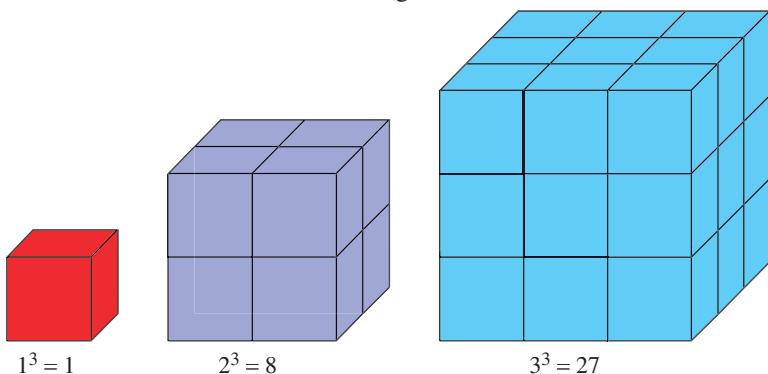
## REFLECTION

Because square numbers form a shape (or figure), they are called **figurate numbers**. Can you think of any other figurate numbers?

## 3D Cubes and cube roots

### Cubic numbers

- **Cubic numbers** are numbers that can be arranged in a cube, as shown below.



- The first cubic number, 1, equals  $1 \times 1 \times 1$ .
- The second cubic number, 8, equals  $2 \times 2 \times 2$ .
- The third square number, 27, equals  $3 \times 3 \times 3$ .
- If this pattern is continued, any cubic number can be found by writing the position of the cubic number 3 times and multiplying. This is known as *cubing a number* and is written by an index (or power) of 3. For example,  $4^3 = 4 \times 4 \times 4 = 64$ .

**WORKED EXAMPLE 14****Find the value of  $5^3$ .****THINK**

- Write  $5^3$  as the product of three lots of 5.
- Evaluate.

**WRITE**

$$\begin{aligned} 5^3 &= 5 \times 5 \times 5 \\ &= 125 \end{aligned}$$

**Cube roots**

- The opposite of cubing a number is finding the **cube root** of a number.
- The cube root is found by looking for a number that can be written three times and multiplied to produce the given number.
- The cube root symbol is similar to the square root symbol but with a small 3 written in front, as shown:  $\sqrt[3]{\phantom{x}}$ .
- From the worked example, we can see that  $\sqrt[3]{125} = 5$ .

**WORKED EXAMPLE 15****Find  $\sqrt[3]{27}$ .****THINK**

Look for a number that when written three times and multiplied gives 27.

**WRITE**

$$\begin{aligned} 27 &= 3 \times 3 \times 3 \\ \sqrt[3]{27} &= 3 \end{aligned}$$

**REMEMBER**

- When a number is written three times and multiplied, this is called *cubing a number* and is written using an index (or power) of 3.
- The opposite of cubing a number is finding the cube root of a number. This is calculated using our knowledge of cubic numbers or using a calculator.

**EXERCISE****3D Cubes and cube roots****FLUENCY**

- WE14** Find the value of  $4^3$ .
- Find the value of:
 

<b>a</b> $2^3$	<b>b</b> $3^3$	<b>c</b> $6^3$	<b>d</b> $10^3$
----------------	----------------	----------------	-----------------
- Write the first 10 cubic numbers.
- WE15** Find  $\sqrt[3]{8}$ .

**INDIVIDUAL PATHWAYS****eBook plus****Activity 3-D-1**Cubes and cube roots  
doc-1684**Activity 3-D-2**More cubes and cube roots  
doc-1685**Activity 3-D-3**Advanced cubes and cube roots  
doc-1686**eBook plus****Digital doc**WorkSHEET 3.2  
doc-1688**UNDERSTANDING**

- 5** Find each of the following. Verify your answers with a calculator.

a  $\sqrt[3]{64}$

b  $\sqrt[3]{216}$

c  $\sqrt[3]{343}$

d  $\sqrt[3]{729}$

(Hint: Use your answer to question 3.)

**REASONING**

- 6** The first 5 square numbers are 1, 4, 9, 16, 25. If we find the difference between these numbers, we get  $4 - 1 = 3$ ,  $9 - 4 = 5$ ,  $16 - 9 = 7$  and  $25 - 16 = 9$ . These numbers all differ by 2. Representing this in a table, we get:

Square numbers	1	4	9	16	25
First difference		3	5	7	9
Second difference			2	2	2

Repeat this process for the first 6 cubic numbers. How many times did you need to find the difference until they were equal?

If you look at  $1^4, 2^4, 3^4, 4^4, \dots$ , how many differences would you need to find until it they were equal?

**REFLECTION**

What would be the first 4 numbers that could be arranged as a triangle-based pyramid (all triangles equilateral)?

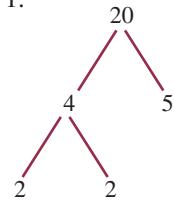
# Summary

## Index notation

- Index notation is a shorthand way of writing a repeated multiplication.
- The number being multiplied is the base of the expression; the number of times it has been written is the index (or power).

## Prime numbers and composite numbers

- A *prime number* is a whole number which has exactly two factors, itself and 1.
- A *composite number* is one which has more than two factors.
- The number 1 is neither a prime number nor a composite number.
- A *factor tree* shows the *prime factors* of a composite number.
- The last numbers in the factor tree are all prime numbers, therefore they are prime factors of the original number.
- Every composite number can be written as a product of prime factors; for example,  $20 = 2 \times 2 \times 5$ .
- The product of prime factors can be written in shorter form using index notation.
- Lowest common multiples and highest common factors can be determined by expressing numbers as the products of their prime factors.



## Squares and square roots

- Square numbers are written using an index (or power) of 2 and are found by multiplying the number by itself.
- The opposite of squaring a number is finding the square root of a number.
- To find the square root of a number we can use our knowledge of square numbers or use a calculator.

## Cubes and cube roots

- When a number is written three times and multiplied, this is called *cubing a number* and is written using an index (or power) of 3.
- The opposite of cubing a number is finding the cube root of a number. This is calculated using our knowledge of cubic numbers or using a calculator.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1** Evaluate:
 

<b>a</b> $6^2$	<b>b</b> $14^2$
<b>c</b> $19^2$	<b>d</b> $80^2$
- 2** Write the two whole numbers between which each of the following will lie.
 

<b>a</b> $3.8^2$	<b>b</b> $5.1^2$
<b>c</b> $10.6^2$	<b>d</b> $15.2^2$
- 3** Find:
 

<b>a</b> $\sqrt{49}$	<b>b</b> $\sqrt{256}$
<b>c</b> $\sqrt{900}$	<b>d</b> $\sqrt{1369}$
- 4** Evaluate:
 

<b>a</b> $4^3$	<b>b</b> $7^3$
<b>c</b> $15^3$	<b>d</b> $30^3$
- 5** Find each of the following.
 

<b>a</b> $\sqrt[3]{27}$	<b>b</b> $\sqrt[3]{125}$
<b>c</b> $\sqrt[3]{1000}$	<b>d</b> $\sqrt[3]{8000}$
- 6** Write the following using index notation.
 

<b>a</b> $2 \times 3 \times 3 \times 3$
<b>b</b> $5 \times 5 \times 6 \times 6 \times 6 \times 6$
<b>c</b> $2 \times 5 \times 5 \times 5 \times 9 \times 9 \times 9$
- 7** Use your calculator to evaluate:
 

<b>a</b> $3^5$	<b>b</b> $7^3$
<b>c</b> $8^4$	<b>d</b> $11^5$
- 8** List all of the prime numbers less than 30.
- 9** How many single digit prime numbers are there?
- 10** Find the prime number which comes next after 50.
- 11** Find the prime factors of:
 

<b>a</b> 99	<b>b</b> 63
<b>c</b> 125	<b>d</b> 124
- 12** Express 280 as a product of prime factors.
- 13** Express 144 as a product of prime factors in index form.
- 14** Write the following numbers in expanded form using index notation.
 

<b>a</b> 1344
<b>b</b> 30 601
- 15** Determine the highest common factor and lowest common multiple of the following pairs of numbers by first expressing each number as a product of its prime numbers.
 

<b>a</b> 120 and 384
<b>b</b> 72 and 224

## PROBLEM SOLVING

- 1** I am a three-digit number. I am divisible by 6. My middle digit is a prime number. The sum of my digits is 9. I am between 400 and 500. My digits are in descending order. What number am I?
- 2** Explain why we cannot find the exact value of  $\sqrt{10}$ .
- 3** Write down the following numbers in the order of size, starting with the smallest.
 

$5, \sqrt{64}, 2, \sqrt[3]{27}$
- 4** Find the smallest possible number which, when multiplied by 120, will give a result that is a perfect square.  
Now find the smallest possible number that will give a result that is a perfect cube.
- 5** List the prime numbers up to 100. Some prime numbers are separated by a difference of 2 or 6. (For example,  $5 - 3 = 2$  and  $19 - 17 = 2$ .)  
  - Find the prime number pairs up to 100 that have a difference of 2.
  - Find the prime number pairs up to 100 that have a difference of 6.
  - Why are there no prime number pairs with a difference of 7?
- 6** What is the smallest natural number that is both a perfect square and a perfect cube? What is the next smallest number?
- 7** The last digit of  $4^2$  is 6. What is the last digit of  $4^{100}$ ?
- 8** Complete the following sequence 2, 3, 5, 7, 11, 13, \_\_\_, \_\_\_, \_\_\_
- 9** Write each of the numbers 250, 375 and 625 as a product of prime factors in index form.  
Find the highest common *prime* factor.  
Find the HCF.
- 10** A motor boat requires an engine service every 5000 nautical miles, refueling every 300 nautical miles and an oil change after 2250 nautical miles.  
How many nautical miles will the motor boat have travelled before all three services are required simultaneously?



- 11** By folding a sheet of paper in half, you divide it into 2 regions.
- If you fold it in half again, how many regions are there?
  - Set up a table similar to the one below and fill it in. How many times can you fold the page in half?
  - How does the number of regions increase with each new fold?
  - Write a formula for the number of regions ( $R$ ) for  $n$  folds.

<b><i>n</i> (number of folds)</b>	<b><i>R</i> (number of regions made)</b>
0	1
1	2
2	4
3	
4	
5	
6	

- 12** Look at this table. As you read down each column, you'll notice that the equations follow a pattern. Continue the patterns to complete the table.

$2^5 = 32$	$3^5 = 243$
$2^4 = 16$	$3^4 = 81$
$2^3 = 8$	$3^3 = 27$
$2^2 =$	$3^2 =$
$2^1 =$	$3^1 =$
$2^0 =$	$3^0 =$
$2^{-1} =$	$3^{-1} =$

What do you suppose  $5^0$  equals?  
What does  $2^{-2}$  equal?

eBook plus

#### Interactivities

- Test yourself  
Chapter 3  
int-2737
- Word search  
Chapter 3  
int-2587
- Crossword  
Chapter 3  
int-2588

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 3 (doc-6426) (*page 67*)

**Are you ready?****Digital docs** (*page 68*)

- SkillSHEET 3.1 (doc-6427) Finding the square of a number
- SkillSHEET 3.2 (doc-6428) Finding the square root of a number
- SkillSHEET 3.3 (doc-6429) Finding the cube of a number
- SkillSHEET 3.4 (doc-6430) Finding the cube root of a number
- SkillSHEET 3.5 (doc-6431) Order of operations with squares and square roots
- SkillSHEET 3.6 (doc-6432) Consecutive numbers

**3A Index notation****Digital docs** (*page 70*)

- Activity 3-A-1 (doc-1675) Famous astronomer
- Activity 3-A-2 (doc-1676) Famous mathematician
- Activity 3-A-3 (doc-1677) Famous physicist

**3B Prime numbers and composite numbers****Digital docs**

- Activity 3-B-1 (doc-1678) Prime numbers and composite numbers (*page 75*)
- Activity 3-B-2 (doc-1679) More prime numbers and composite numbers (*page 75*)
- Activity 3-B-3 (doc-1680) Advanced prime numbers and composite numbers (*page 75*)

- Spreadsheet Sieve of Eratosthenes (doc-1689) (*page 76*)

- WorkSHEET 3.1 (doc-1687) (*page 77*)

**Weblink**

- Prime number game (*page 77*)

**3C Squares and square roots****Digital docs** (*page 80*)

- Activity 3-C-1 (doc-1681) Squares and square roots
- Activity 3-C-2 (doc-1682) More squares and square roots
- Activity 3-C-3 (doc-1683) Advanced squares and square roots

**3D Cubes and cube roots****Digital docs** (*page 83*)

- Activity 3-D-1 (doc-1684) Cubes and cube roots
- Activity 3-D-2 (doc-1685) More cubes and cube roots
- Activity 3-D-3 (doc-1686) Advanced cubes and cube roots
- WorkSHEET 3.2 (doc-1688) (*page 83*)

**Chapter review****Interactivities** (*page 86*)

- Test yourself Chapter 3 (int-2737) Take the end-of-chapter test to test your progress.
- Word search Chapter 3 (int-2587)
- Crossword Chapter 3 (int-2588)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# ICT ACTIVITY

## projects plus

### We are not alone!

SEARCHLIGHT ID: PRO-0096



#### Scenario

'ALERT! ALERT! Those alien Earthlings have finally discovered our planet! We must send them a message of peace immediately! WHAT? They do not use our language? Find one that they will understand. Look at their number systems, surely they must have one!'

The year is 2123AD. Earthlings have colonised the Moon and Mars. Earth scientists have now discovered that we are not alone in the Universe. Not much is known about the inhabitants of the planet, but new

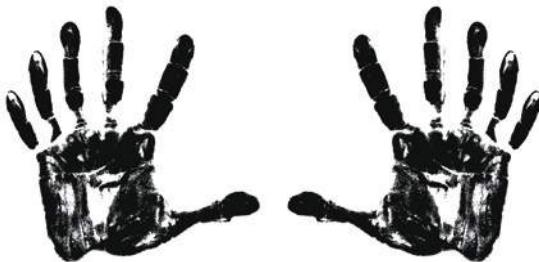
images from the planet indicate that the planet's inhabitants have two hands like Earthlings. At this stage the images are not clear enough to show how many fingers they have. Scientists are assuming that they may have developed a number system based on the number of fingers on each hand, just like the Earthlings did centuries ago.



Your research centre has the best mathematical team on Earth, so it will be up to you to communicate with these inhabitants and the best way is by numbers. Until more detailed images arrive you must research and prepare for whatever number system may be required.

## Task

You will need to research the development of number systems as part of your background knowledge leading to an understanding of these number systems. From your research, you will create an e-poster that will inform your partner of your findings. You will then create a worksheet to ensure your partner's understanding of the number systems you have researched.



### SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft Word
- Microsoft PowerPoint
- Microsoft Publisher
- Microsoft FrontPage
- Crossword maker

## Process

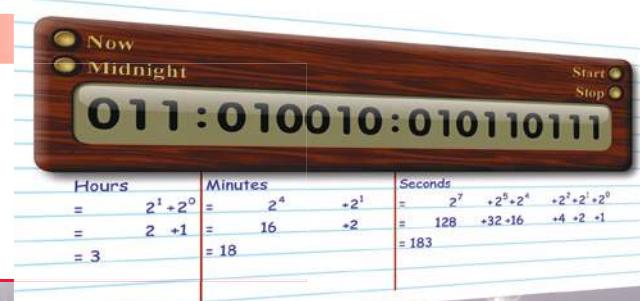
- Open the ProjectsPLUS application for this chapter in your eBookPLUS. Watch the introductory video lesson, click the 'Start Project' button and then set up your project team. You will invite another member of your class to form a partnership. Save your settings and the project will be launched.
- Team member 1 will research the ancient number systems of the Mayans, Romans and Egyptians and report back to your partner.
- Team member 2 will research the modern era number systems of the Chinese, Hindu/Arabic and Binary and report back to your partner.
- Visit your Media centre and view the selection of images of other number systems.
- Navigate to your Research Forum.
- Follow the Weblinks given to find out how various cultures throughout history developed and used their number systems.
- Navigate to the Worksheets, and complete a Worksheet for each of the different number systems. Submit these completed Worksheets to your teacher for assessment.
- Create an e-poster for each number system that helps explain that number system.
- Create a quiz of 20 questions and solutions to test your class's understanding of these number systems.
- Complete the quiz provided by your partner.
- Write a report that includes your quiz, the results of your quiz and a critical evaluation of the strengths and weaknesses of your quiz. Submit this to your teacher for assessment.
- Remember, Earth needs to be safe — and to do that, communication is vital!

## Interactivity

### THE BINARY CLOCK

SEARCHLIGHT ID: int-2355

There have been many number systems developed throughout history and within different cultures. Use the interactivity *The binary clock* to investigate the binary number system.





## 4

# Rational numbers



- 4A** What are rational numbers?
- 4B** Simplifying rational numbers
- 4C** Mixed numbers and improper fractions
- 4D** Adding and subtracting rational numbers
- 4E** Multiplying rational numbers
- 4F** Dividing rational numbers
- 4G** Fractions as ratios
- 4H** Working with mixed numbers

## WHAT DO YOU KNOW?

- 1 List what you know about fractions.  
Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of fractions.

eBook plus

Digital doc

Hungry brain activity  
Chapter 4  
doc-6436

## OPENING QUESTION

If 8 people wanted to share the block of chocolate fairly, how many pieces would they each receive?

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can get extra help by completing the matching SkillsHEET located on your eBookPLUS.



## Types of fractions

**Digital doc**

SkillsHEET 4.1

doc-6437

- 1 State whether each of the following is a proper fraction, an improper fraction or a mixed number.

a  $\frac{5}{6}$

b  $\frac{9}{4}$

c  $3\frac{1}{2}$

d  $\frac{3}{2}$

e  $1\frac{3}{4}$

f  $\frac{11}{20}$



## Equivalent fractions I

**Digital doc**

SkillsHEET 4.2

doc-6438

- 2 a Form an equivalent fraction to  $\frac{2}{3}$  by multiplying the numerator and denominator by 5.  
b Form an equivalent fraction to  $\frac{24}{28}$  by dividing the numerator and denominator by 4.



## Equivalent fractions II

**Digital doc**

SkillsHEET 4.3

doc-6439

- 3 Copy and complete the following equivalent fractions by filling in the gaps.

$$\frac{1}{3} = \frac{2}{\underline{\hspace{1cm}}} = \frac{\underline{\hspace{1cm}}}{9} = \frac{12}{\underline{\hspace{1cm}}}$$



## Lowest common multiple

**Digital doc**

SkillsHEET 4.4

doc-6440

- 4 a List the first eight multiples of 3.  
b List the first eight multiples of 4.  
c Which multiples are common (the same) in both lists?  
d What is the lowest common multiple of 3 and 4?



## Ordering fractions

**Digital doc**

SkillsHEET 4.5

doc-6441

- 5 Write these fractions in ascending order (from smallest to largest).

a  $1, \frac{4}{5}, \frac{1}{5}, \frac{3}{5}, \frac{7}{5}$

b  $\frac{11}{12}, 0, \frac{7}{12}, \frac{13}{12}, \frac{5}{12}$



## Highest common factor

**Digital doc**

SkillsHEET 4.6

doc-6442

- 6 a List the factors of 18.  
b List the factors of 12.  
c Which factors are common (the same) in both lists?  
d What is the highest common factor of 18 and 12?



## Converting an improper fraction to a mixed number

**Digital doc**

SkillsHEET 4.7

doc-6443

- 7 Use a diagram to express  $\frac{5}{2}$  as a mixed number.



## Converting a mixed number to an improper fraction

**Digital doc**

SkillsHEET 4.8

doc-6444

- 8 Use a diagram to express  $1\frac{1}{3}$  as an improper fraction.



## Adding and subtracting fractions with the same denominator

**Digital doc**

SkillsHEET 4.9

doc-6445

- 9 Find:

a  $\frac{5}{6} + \frac{3}{6}$

b  $\frac{7}{12} - \frac{5}{12}$

c  $1 - \frac{1}{3}$

d  $4 - \frac{1}{4}$

## 4A What are rational numbers?

### eBook plus

eLesson  
Types of  
fractions  
eles-0002

### Understanding fractions

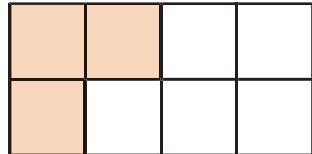
- **Rational numbers** are numbers that can be written as fractions.
  - **Fractions** are used to describe parts of a whole.
  - The **numerator**, or top number of the fraction, shows how many parts are required. The numerator must be an integer.
  - The **denominator**, or bottom number, shows the number of parts into which the whole can be divided. The denominator must be an integer, but cannot be zero.
  - The horizontal bar separating the numerator from the denominator is called the **vinculum**.
- 
- A **proper fraction** has a numerator that is smaller than the denominator; for example  $\frac{2}{3}$ .
  - An **improper fraction** has a numerator that is larger than the denominator; for example  $\frac{5}{4}$ .
  - A **mixed number** is made up of a number and a fraction; for example  $1\frac{3}{5}$ .

### Expressing one quantity as a fraction of another

- To express one quantity as a fraction of another, write the first quantity as a fraction of the second quantity.

#### WORKED EXAMPLE 1

- What fraction of the rectangle shown has been shaded?
- Express the number of unshaded squares as a fraction of the number of squares in total.



#### THINK

- Count how many equal parts the rectangle has been divided into.
  - State how many parts are shaded.
  - Write the answer in a sentence.
- Count the number of unshaded squares.
  - State the number of squares in total.
  - Write the first quantity as a fraction of the second quantity and write the answer in words.

#### WRITE

- Total number of parts = 8  
3 parts are shaded.  
 $\frac{3}{8}$  of the rectangle has been shaded.
- There are 5 unshaded squares.  
There are 8 squares in total.  
The number of unshaded squares as a fraction of the total number of squares in the rectangle is  $\frac{5}{8}$ .

### Equivalent fractions

- Equivalent fractions are equal fractions. For example,  $\frac{1}{2}, \frac{5}{10}, \frac{10}{20}, \frac{8}{16}$ . All of these fractions can be simplified to  $\frac{1}{2}$ .

**WORKED EXAMPLE 2**

For the equivalent fractions  $\frac{4}{5} = \frac{8}{10}$  find the number which has been used to multiply both numerator and denominator.

**THINK**

- 1 What number is 4 multiplied by to equal 8? ( $4 \times \underline{\quad} = 8$ )  
What number is 5 multiplied by to equal 10?  
( $5 \times \underline{\quad} = 10$ )
- 2 Write the answer.

**WRITE**

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

The number used is 2.

**WORKED EXAMPLE 3**

From the list, find those fractions which are equivalent to  $\frac{1}{2}$ :  $\frac{3}{9}$ ,  $\frac{3}{6}$ ,  $\frac{9}{18}$ ,  $\frac{10}{15}$ ,  $\frac{7}{14}$ ,  $\frac{17}{34}$ .

**THINK**

- 1 Multiply the numerator and denominator of  $\frac{1}{2}$  by the numerator of the first fraction (3) to check whether the new fraction is in the list.
- 2 Multiply the numerator and denominator of  $\frac{1}{2}$  by the next different numerator (9) and check whether the new fraction is in the list.
- 3 Continue until all fractions have been considered.
- 4 Write the equivalent fractions.

**WRITE**

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1 \times 9}{2 \times 9} = \frac{9}{18}$$

$$\frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

$$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

$$\frac{1 \times 17}{2 \times 17} = \frac{17}{34}$$

From the given list the equivalent fractions of  $\frac{1}{2}$  are  $\frac{3}{6}$ ,  $\frac{9}{18}$ ,  $\frac{7}{14}$ ,  $\frac{17}{34}$ .

**WORKED EXAMPLE 4**

Write the sequence of the first three equivalent fractions for  $\frac{2}{3}$ .

**THINK**

- 1 Find the first three equivalent fractions in the sequence by multiplying both the numerator and denominator by 2, 3 and 4.
- 2 Write the sequence of equivalent fractions.

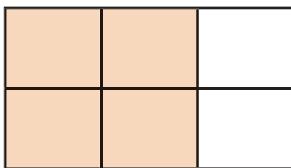
**WRITE**

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{2 \times 3}{3 \times 3} = \frac{2 \times 4}{3 \times 4}$$

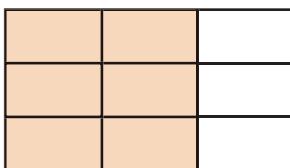
$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

- There are other fractions that are equivalent to  $\frac{2}{3}$ . These can be found by multiplying the numerator and denominator by other numbers.

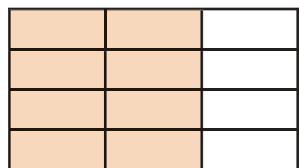
- Equivalent fractions can also be shown using diagrams. Equivalent fractions for  $\frac{2}{3}$  are shown below.



$$\frac{2}{3} = \frac{4}{6}$$



$$\frac{2}{3} = \frac{6}{9}$$

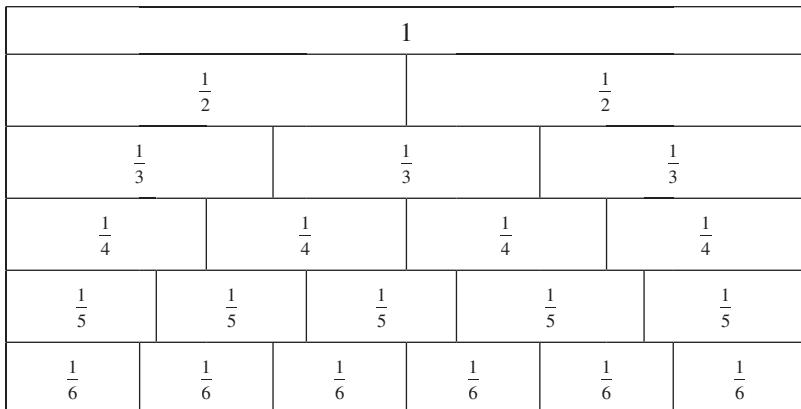


$$\frac{2}{3} = \frac{8}{12}$$

- Note that the same amount of the rectangle has been shaded in each case.

## Comparing fractions

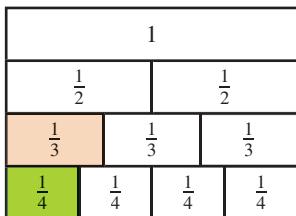
- If we cut a block into 4 pieces, then 1 piece is less than 2 pieces. This means that  $\frac{1}{4}$  of the block is smaller than  $\frac{2}{4}$ .
- Fraction walls, or number lines, are useful tools when comparing fractions. An example of a fraction wall is shown below.



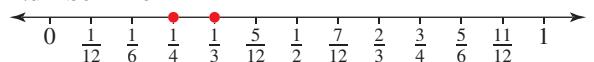
- With a diagram, such as a fraction wall or a number line, we can also see that  $\frac{1}{4}$  is less than  $\frac{1}{3}$ . That is, if we divided a block into 4 equal parts, each part would be smaller than if we divided it into 3 equal parts.

**eBook plus**  
Interactivity  
Fraction walls  
int-0002

Fraction wall



Number line



- If the numerators are the same, the smaller fraction is the one with the larger denominator. For example,  $\frac{1}{7}$  is less than  $\frac{1}{6}$  and  $\frac{3}{10}$  is less than  $\frac{3}{7}$ .
- Fractions that do not have the same denominator or numerator can be compared by using equivalent fractions.
- The fractions need to be rewritten so that they have the same denominator.
- Look at the multiples of both denominators and choose the lowest multiple that is common to both.

## WORKED EXAMPLE 5

- a Which is the bigger fraction,  $\frac{2}{3}$  or  $\frac{3}{5}$ ?  
 b Justify your answer using a fraction wall and a number line.

## THINK

- a 1 Find the lowest common multiple of the denominators. First list the multiples of 3 and 5. Identify the lowest number that is common to both lists.
- 2 Write each fraction as an equivalent fraction using the lowest common multiple (15) as the denominator.
- 3 Decide which is bigger by comparing the numerators of the equivalent fractions ( $\frac{10}{15}$  and  $\frac{9}{15}$ ).
- 4 Answer the question.

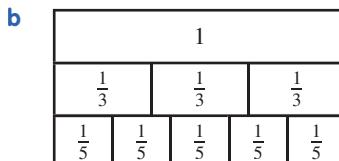
- b 1 Create a fraction wall showing thirds and fifths.

## WRITE

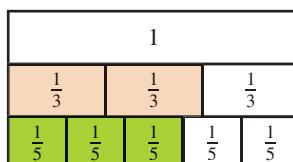
- a Multiples of 3 are 3, 6, 9, 12, 15, 18, ...  
 Multiples of 5 are 5, 10, 15, 20, ...  
 The lowest common multiple is 15.

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15} \text{ and } \frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

$\frac{2}{3}$  is bigger than  $\frac{3}{5}$ .



- 2 Shade  $\frac{2}{3}$  and  $\frac{3}{5}$ .

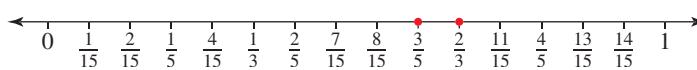
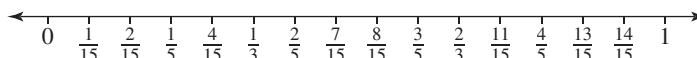


- 3 Compare the lengths of the shaded areas to compare the fractions. Answer the question in words.

- 4 Draw a number line showing 0 to 1 in intervals of  $\frac{1}{15}$  (found by using the lowest common multiple of the denominators).

- 5 Mark  $\frac{2}{3}$  and  $\frac{3}{5}$ .

- 6 Answer the question in words.



$\frac{2}{3}$  is bigger than  $\frac{3}{5}$ .

- The answer to Worked example 5 can be written as  $\frac{2}{3} > \frac{3}{5}$ .
- The symbol ‘ $>$ ’ means *is bigger than* or *is greater than*. The symbol ‘ $<$ ’ means *is smaller than* or *is less than*.
- When we write  $\frac{1}{2} < \frac{3}{4}$ , it means that  $\frac{1}{2}$  is less than  $\frac{3}{4}$ .

**WORKED EXAMPLE 6**

Insert the appropriate symbol,  $<$  or  $>$ , between each pair of fractions to make a true statement.

a  $\frac{6}{7}$      $\frac{7}{8}$

b  $\frac{7}{12}$      $\frac{5}{9}$

**THINK**

- a 1 Find the lowest common multiple of the denominators.

- 2 Write each fraction as an equivalent fraction using the lowest common multiple (56) as the denominator.

- 3 Decide which is bigger by comparing the numerators of the equivalent fractions.

- 4 Answer the question.

- b 1 Find the lowest common multiple of the denominators.

- 2 Write each fraction as an equivalent fraction using the lowest common multiple (36) as the denominator.

- 3 Decide which is bigger by comparing the numerators of the equivalent fractions.

- 4 Answer the question.

**WRITE**

- a Multiples of 7 are:

7, 14, 21, 28, 35, 42, 49, 56, 63, ...

- Multiples of 8 are:

8, 16, 24, 32, 40, 48, 56, 64, ...

The lowest common multiple is 56.

$$\frac{6}{7} = \frac{6 \times 8}{7 \times 8} = \frac{48}{56} \text{ and } \frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56}$$

$\frac{6}{7}$  is less than  $\frac{7}{8}$ .

$$\frac{6}{7} < \frac{7}{8}$$

- b Multiples of 12 are: 12, 24, 36, 48, ...

Multiples of 9 are: 9, 18, 27, 36, 45, ...

The lowest common multiple is 36.

$$\frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36} \text{ and } \frac{5}{9} = \frac{5 \times 4}{9 \times 4} = \frac{20}{36}$$

$\frac{7}{12}$  is greater than  $\frac{5}{9}$ .

$$\frac{7}{12} > \frac{5}{9}$$

**REMEMBER**

- Equivalent fractions* can be found by multiplying or dividing both the numerator and the denominator by the same non-zero number.
- The *numerator* is the top part of the fraction.
- The *denominator* is the bottom part of the fraction.
- Different fractions can be compared or ordered by writing each one with the same denominator. This is found using the lowest common multiple (LCM) of the denominators.
- The symbol ‘ $>$ ’ means *is greater than*, the symbol ‘ $<$ ’ means *is less than*.

## EXERCISE

## 4A What are rational numbers?

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 4-A-1

Fractions  
doc-1700

## Activity 4-A-2

More fractions  
doc-1701

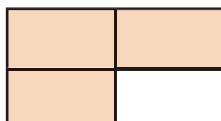
## Activity 4-A-3

Advanced fractions  
doc-1702

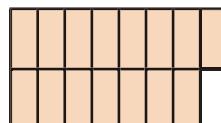
## FLUENCY

- 1 WE1a What fraction of each of the following rectangles has been shaded?

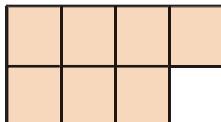
a



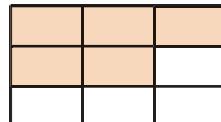
b



c



d



- 2 What fraction of each of these flags is coloured red?

a



b

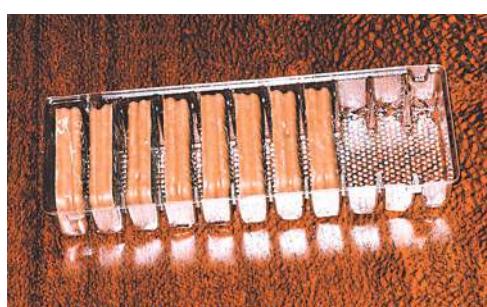


- c Do you recognise these flags? Can you name the country to which each flag belongs?

- 3 WE1b a What fraction of the total number of pieces of fruit is made up of:

i bananas?                    ii oranges?

- b What fraction of chocolate biscuits in this packet has been eaten?



- 4 WE2 For the equivalent fractions below find the number which has been used to multiply both numerator and denominator.

a  $\frac{3}{8} = \frac{9}{24}$

b  $\frac{1}{4} = \frac{3}{12}$

c  $\frac{3}{5} = \frac{12}{20}$

d  $\frac{2}{3} = \frac{10}{15}$

e  $\frac{5}{6} = \frac{30}{36}$

f  $\frac{9}{10} = \frac{81}{90}$

g  $\frac{7}{8} = \frac{77}{88}$

h  $\frac{7}{8} = \frac{84}{96}$

i  $\frac{2}{5} = \frac{14}{35}$

- 5 WE3** From the list, find those fractions which are equivalent to the fractions marked in red.

a  $\frac{2}{3}, \frac{7}{9}, \frac{20}{30}, \frac{5}{8}, \frac{12}{16}, \frac{14}{21}, \frac{40}{60}$

c  $\frac{7}{8}, \frac{17}{18}, \frac{40}{45}, \frac{56}{64}, \frac{14}{18}, \frac{18}{19}, \frac{21}{24}$

b  $\frac{4}{5}, \frac{12}{15}, \frac{15}{20}, \frac{36}{45}, \frac{16}{20}, \frac{28}{35}, \frac{80}{100}$

d  $\frac{7}{10}, \frac{18}{25}, \frac{35}{50}, \frac{14}{21}, \frac{21}{30}, \frac{14}{20}, \frac{140}{200}$

- 6** Fill in the gaps.

a  $\frac{3}{5} = \frac{9}{\underline{\hspace{1cm}}} = \frac{\underline{\hspace{1cm}}}{20} = \frac{35}{\underline{\hspace{1cm}}}$

c  $\frac{1}{4} = \frac{6}{\underline{\hspace{1cm}}} = \frac{6}{28} = \frac{3}{\underline{\hspace{1cm}}}$

b  $\frac{3}{4} = \frac{\underline{\hspace{1cm}}}{40} = \frac{18}{36} = \frac{18}{\underline{\hspace{1cm}}}$

d  $\frac{5}{6} = \frac{15}{\underline{\hspace{1cm}}} = \frac{40}{42} = \frac{40}{\underline{\hspace{1cm}}}$

### UNDERSTANDING

- 7 WE4** Write a sequence of the first three equivalent fractions for each of the following.

a  $\frac{5}{6}$

b  $\frac{5}{8}$

c  $\frac{3}{10}$

d  $\frac{2}{3}$

e  $\frac{1}{8}$

f  $\frac{7}{8}$

- 8 WE5** Which is the bigger fraction? Justify your answer using a fraction wall or number line.

a  $\frac{2}{5}$  or  $\frac{3}{5}$

b  $\frac{5}{8}$  or  $\frac{7}{8}$

c  $\frac{1}{5}$  or  $\frac{1}{6}$

d  $\frac{1}{8}$  or  $\frac{1}{10}$

e  $\frac{1}{2}$  or  $\frac{3}{10}$

f  $\frac{3}{4}$  or  $\frac{3}{5}$

g  $\frac{2}{5}$  or  $\frac{5}{8}$

h  $\frac{3}{5}$  or  $\frac{5}{8}$

i  $\frac{4}{5}$  or  $\frac{5}{8}$

- 9 WE6** Insert the appropriate symbol,  $<$  or  $>$ , between each pair of fractions to make a true statement.

a  $\frac{5}{8} \quad \frac{3}{8}$

b  $\frac{5}{6} \quad \frac{7}{6}$

c  $\frac{3}{5} \quad \frac{1}{4}$

d  $\frac{3}{4} \quad \frac{3}{5}$

e  $\frac{1}{5} \quad \frac{1}{4}$

f  $\frac{3}{10} \quad \frac{2}{5}$

- 10 MC** a Which fraction is smaller than  $\frac{5}{8}$ ?

A  $\frac{7}{8}$

B  $\frac{17}{24}$

C  $\frac{11}{12}$

D  $\frac{3}{5}$

E  $\frac{13}{16}$

- b Which fraction is equivalent to  $\frac{3}{4}$ ?

A  $\frac{3}{8}$

B  $\frac{6}{8}$

C  $\frac{9}{16}$

D  $\frac{12}{24}$

E  $\frac{15}{24}$

- 11** Write the following fractions with the same denominator and then write them in ascending order. (Ascending means from smallest to largest.)

a  $\frac{3}{10}, \frac{1}{2}, \frac{1}{5}$

b  $\frac{3}{8}, \frac{1}{2}, \frac{1}{3}$

c  $\frac{2}{3}, \frac{4}{5}, \frac{6}{15}$

d  $\frac{3}{4}, \frac{2}{3}, \frac{7}{15}$

- 12** Four hungry teenage girls ordered pizzas, but none

could eat the whole pizza. Sally Seafood ate  $\frac{6}{8}$  of her

seafood pizza. Hanna Hawaiian ate  $\frac{1}{2}$  of her

Hawaiian pizza. Sonya Special ate  $\frac{3}{4}$  of her

special pizza. Vanessa Vegetarian ate  $\frac{5}{6}$  of her

vegetarian pizza.

- a Draw the four pizzas and shade the amount that each girl ate.

- b Which girl ate the most pizza?

- c Who ate more pizza, Sally Seafood or Sonya Special?



### REFLECTION

Why is it important to express one quantity as a fraction of another?

## 4B Simplifying rational numbers

- Generally, fractions are written in their simplest form. In other words, we reduce the fraction to its lowest equivalent form.
- The best number to divide by is the highest common factor of the numerator and the denominator.
- The highest common factor (HCF) of two numbers is the largest factor of both numbers.
- Consider the numbers 8 and 12:
  - Factors of 8: 1, 2, 4, 8.
  - Factors of 12: 1, 2, 3, 4, 6, 12.
  - The common factors are: 1, 2, 4.
  - The highest common factor is 4.
- Dividing the numerator and denominator by the same number is called *cancelling* the common factor.

### WORKED EXAMPLE 7

Write  $\frac{3}{6}$  in simplest form.

#### THINK

- 1 Write the fraction and find the highest common factor (HCF) or the largest number that is a factor of both the numerator and the denominator (3).
- 2 Cancel by dividing the numerator and denominator by this factor ( $3 \div 3 = 1$ ,  $6 \div 3 = 2$ ).
- 3 Write the answer in simplest form.

#### WRITE

$$\begin{aligned} & \frac{3}{6} \\ &= \frac{\cancel{3}^1}{\cancel{6}^2} \\ &= \frac{1}{2} \end{aligned}$$

- If the largest factor is not used, the process can be repeated to find the simplest fraction.
- To simplify a mixed number, leave the whole number and simplify the fraction.

### WORKED EXAMPLE 8

Simplify  $7\frac{40}{64}$ .

#### THINK

- 1 Write the mixed number and think of the HCF or largest number that divides into both numerator and denominator; that is, the number 8.
- 2 Divide both numerator and denominator by that factor ( $40 \div 8 = 5$ ,  $64 \div 8 = 8$ ).
- 3 Write your answer as a mixed number in simplest form.

#### WRITE

$$\begin{aligned} & 7\frac{40}{64} \\ &= 7\frac{\cancel{40}^5}{\cancel{64}^8} \\ &= 7\frac{5}{8} \end{aligned}$$

#### REMEMBER

1. To simplify a fraction, divide both the numerator and denominator by the highest common factor or HCF.
2. To simplify a mixed number, leave the whole number and simplify the fraction.

## EXERCISE

## 4B

## Simplifying rational numbers

## INDIVIDUAL PATHWAYS

## eBook plus

**Activity 4-B-1**  
Equivalent fractions  
dominoes  
doc-1703

**Activity 4-B-2**  
More equivalent  
fractions dominoes  
doc-1704

**Activity 4-B-3**  
Tricky equivalent  
fractions dominoes  
doc-1705

## eBook plus

**Interactivity**  
Equivalent  
fractions  
int-0001

## eBook plus

**Digital doc**  
Spreadsheet  
Simplifying a  
fraction  
doc-1729

## FLUENCY

- 1 **WE7** Write the following fractions in simplest form.

a  $\frac{5}{10}$

b  $\frac{8}{12}$

c  $\frac{21}{24}$

d  $\frac{48}{60}$

e  $\frac{28}{35}$

f  $\frac{18}{24}$

g  $\frac{81}{90}$

h  $\frac{49}{56}$

i  $\frac{100}{120}$

j  $\frac{48}{50}$

k  $\frac{63}{72}$

l  $\frac{49}{70}$

m  $\frac{33}{36}$

n  $\frac{22}{50}$

o  $\frac{21}{56}$

- 2 **WE8** Simplify.

a  $4\frac{21}{35}$

b  $7\frac{45}{54}$

c  $10\frac{10}{20}$

d  $5\frac{7}{42}$

e  $3\frac{56}{64}$

f  $1\frac{44}{48}$

g  $6\frac{21}{28}$

h  $5\frac{16}{48}$

i  $3\frac{11}{55}$

j  $2\frac{16}{64}$

- 3 **MC** The fraction  $\frac{26}{28}$  is the same as:

A  $\frac{2}{3}$

B  $\frac{2}{6}$

C  $\frac{3}{8}$

D  $\frac{9}{12}$

E  $\frac{13}{14}$

## UNDERSTANDING

- 4 Kylie's netball team scored 28 goals. Kylie scored 21 of her team's goals. What fraction of the team's goals did Kylie score? Simplify the answer.

- 5 David's basketball team scored 36 goals. David scored 20 of the team's goals.

a What fraction of the team's goals did David score? Simplify the answer.

b What fraction of the team's goals did the rest of the team score? Simplify the answer.

- 6 Year 7 students at Springfield High School ran a car wash to raise money for the local hospital. They raised a total of \$1000 and drew up a table showing how much money each class raised.

7A \$200

7B \$150

7C \$320

7D \$80

7E \$250

Express as a simple fraction how much of the total each class raised.

- 7 Below are the results of a book-reading competition.

Mark 5 books

David 10 books

Samantha 8 books

Jules 6 books

Ahmed 4 books

Darren 7 books

a How many books were read in total?

b For each contestant, record the number of books read as a fraction of the total number of books read. Where possible, reduce the fractions to their simplest forms.

- 8 a Can you arrange the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to form a fraction which is equivalent to  $\frac{1}{2}$ ? For example, one way is  $\frac{7329}{14658}$ . Can you find two others?

*Hint:* Try starting with a numerator beginning with 72 for one of them and 67 for the other.

b Can you find a way to arrange the nine digits so that they form a fraction equivalent to  $\frac{1}{3}$ ?

- 9 The Geelong train to Melbourne runs on time 7 out of every 9 runs. The Melbourne train to Geelong runs on time 11 out of 15 runs. Explain which train is more reliable and by how much.



## REFLECTION

What strategies will you use to write a fraction in simplest form?

## 4C Mixed numbers and improper fractions

- An improper fraction has a numerator larger than the denominator; for example  $\frac{3}{2}$ .
- A mixed number is made up of a whole number and a fraction; for example  $1\frac{3}{4}, 2\frac{1}{2}$ .

### Change improper fractions to mixed numbers

- An improper fraction can be changed to a mixed number by dividing the denominator into the numerator and writing the remainder over the same denominator.

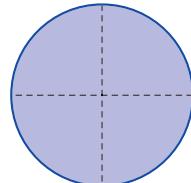
#### WORKED EXAMPLE 9

Draw a diagram to show  $\frac{5}{4}$  as parts of a circle, then write the improper fraction as a mixed number.

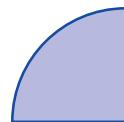
##### THINK

- Draw a whole circle and divide it into the number of parts shown by the denominator.
- Find the number of parts left over and draw them.
- Write the improper fraction as a mixed number.

##### WRITE



This is 4 quarters or  $\frac{4}{4}$ .



Extra  $\frac{1}{4}$  needed.

$$\frac{5}{4} = 1\frac{1}{4}$$

- If the improper fraction had a larger numerator, it might be necessary to draw more circles and the whole number part would then be greater than 1.

#### WORKED EXAMPLE 10

Express  $\frac{11}{5}$  as a mixed number.

##### THINK

- Write the improper fraction.
- Determine how many times the denominator can be divided into the numerator and what the remainder is. The whole number is part of the answer and the remainder becomes the numerator of the fractional part with the same denominator as the original improper fraction.
- Write the answer.

##### WRITE

$$\begin{aligned}\frac{11}{5} \\ = 11 \div 5 \\ = 2 \text{ remainder } 1\end{aligned}$$

$$= 2\frac{1}{5}$$

### Changing mixed numbers to improper numbers

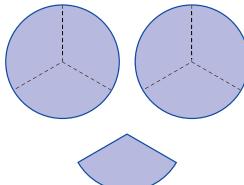
- A mixed number can be changed to an improper fraction by first multiplying the whole number with the denominator then adding the numerator. The denominator stays the same.

**WORKED EXAMPLE 11**

Draw a diagram to show  $2\frac{1}{3}$  as pieces of circle and then write  $2\frac{1}{3}$  as an improper fraction.

**THINK**

- 1 Draw two whole circles and  $\frac{1}{3}$  of a circle.
- 2 Divide the whole circles into thirds.
- 3 Count the number of thirds and write the mixed number as an improper fraction.

**WRITE**

$$2\frac{1}{3} = \frac{7}{3}$$

**WORKED EXAMPLE 12**

Express  $2\frac{3}{4}$  as an improper fraction.

**THINK**

- 1 Write the mixed number.
- 2 Multiply the whole number with the denominator then add the numerator. The result becomes the numerator and the denominator stays the same.
- 3 Evaluate the top line of the fraction.
- 4 Write the answer.

**WRITE**

$$\begin{aligned} 2\frac{3}{4} &= \frac{2 \cdot 4 + 3}{4} \\ &= \frac{8 + 3}{4} \\ &= \frac{11}{4} \end{aligned}$$

**REMEMBER**

1. An *improper fraction* has a numerator that is larger than the denominator.
2. A *mixed number* is made up of a whole number and a fraction.
3. An improper fraction can be changed to a mixed number by dividing the denominator into the numerator and writing the remainder over the same denominator.
4. A mixed number can be changed to an improper fraction by first multiplying the whole number with the denominator then adding the numerator. The denominator stays the same.

**EXERCISE****4C Mixed numbers and improper fractions****FLUENCY**

- 1 WE9** Draw a diagram to show the following improper fractions as pieces of a circle, then write each improper fraction as a mixed number.

a  $\frac{5}{2}$

b  $\frac{4}{3}$

c  $\frac{11}{6}$

d  $\frac{13}{8}$

e  $\frac{12}{4}$

f  $\frac{7}{6}$

**INDIVIDUAL PATHWAYS****eBookplus****Activity 4-C-1**Mixed numbers and  
improper fractions

doc-1706

**Activity 4-C-2**More mixed numbers  
and improper  
fractions

doc-1707

**Activity 4-C-3**Tricky mixed  
numbers and  
improper fractions

doc-1708

- 2 a** Sarah ate  $\frac{3}{2}$  or 3 halves of an apple. Draw the amount of apple that Sarah ate and express  $\frac{3}{2}$  as a mixed number.

- b** Dean ate  $\frac{9}{4}$  or 9 quarters of pizza. Draw the amount of pizza that Dean ate and express  $\frac{9}{4}$  as a mixed number.

- 3 WE10** Express these improper fractions as mixed numbers.

**a**  $\frac{7}{5}$

**b**  $\frac{11}{4}$

**c**  $\frac{21}{2}$

**d**  $\frac{39}{10}$

**e**  $\frac{51}{12}$

**f**  $\frac{92}{11}$

**g**  $\frac{29}{13}$

**h**  $\frac{23}{8}$

**i**  $\frac{100}{3}$

**j**  $\frac{25}{2}$

**k**  $\frac{20}{3}$

**l**  $\frac{10}{3}$

- 4** Change these improper fractions to mixed numbers.

**a**  $\frac{8}{5}$

**b**  $\frac{27}{8}$

**c**  $\frac{11}{2}$

**d**  $\frac{97}{9}$

**e**  $\frac{58}{7}$

**f**  $\frac{117}{10}$

**g**  $\frac{67}{8}$

**h**  $\frac{74}{9}$

**i**  $\frac{31}{12}$

**j**  $\frac{89}{6}$

- 5 MC a** Which of the following is the same as  $\frac{61}{8}$ ?

**A**  $5\frac{5}{8}$

**B**  $6\frac{5}{8}$

**C**  $7\frac{5}{8}$

**D**  $8\frac{5}{8}$

**E**  $9\frac{5}{8}$

- b** Which of the following is the same as  $\frac{73}{10}$ ?

**A**  $3\frac{7}{10}$

**B**  $7\frac{3}{10}$

**C**  $7\frac{7}{10}$

**D**  $3\frac{3}{10}$

**E**  $7\frac{4}{10}$

- c** Which of the following is the same as  $\frac{48}{5}$ ?

**A**  $8\frac{4}{5}$

**B**  $5\frac{5}{9}$

**C**  $9\frac{5}{9}$

**D**  $9\frac{3}{5}$

**E**  $9\frac{4}{9}$

- 6 WE11** Draw a diagram to show the following mixed numbers as pieces of a circle and then write each one as an improper fraction.

**a**  $1\frac{1}{8}$

**b**  $3\frac{3}{4}$

**c**  $2\frac{1}{6}$

**d**  $1\frac{1}{6}$

**e**  $5\frac{1}{4}$

**f**  $1\frac{3}{5}$

- 7 WE12** Express the following mixed numbers as improper fractions.

**a**  $1\frac{1}{7}$

**b**  $1\frac{4}{5}$

**c**  $2\frac{2}{3}$

**d**  $3\frac{1}{3}$

**e**  $6\frac{1}{4}$

**f**  $4\frac{1}{9}$

**g**  $11\frac{1}{2}$

**h**  $3\frac{7}{8}$

**i**  $7\frac{4}{5}$

**j**  $9\frac{2}{5}$

- 8** Change these mixed numbers to improper fractions.

**a**  $3\frac{1}{2}$

**b**  $4\frac{1}{5}$

**c**  $5\frac{4}{5}$

**d**  $6\frac{5}{7}$

**e**  $2\frac{9}{10}$

**f**  $3\frac{7}{12}$

**g**  $5\frac{3}{5}$

**h**  $9\frac{4}{7}$

**i**  $1\frac{10}{11}$

**j**  $8\frac{5}{6}$

- 9 a MC** Which of the following is the same as  $10\frac{2}{7}$ ?

**A**  $\frac{27}{10}$

**B**  $\frac{72}{10}$

**C**  $\frac{10}{27}$

**D**  $\frac{72}{7}$

**E**  $\frac{72}{2}$

- b** Which of the following is the same as  $8\frac{5}{6}$ ?

**A**  $\frac{85}{6}$

**B**  $\frac{58}{6}$

**C**  $\frac{56}{8}$

**D**  $\frac{53}{6}$

**E**  $\frac{68}{5}$

- c** Which of the following is the same as  $7\frac{9}{11}$ ?

**A**  $\frac{86}{11}$

**B**  $\frac{99}{7}$

**C**  $\frac{79}{11}$

**D**  $\frac{97}{11}$

**E**  $\frac{77}{9}$

**eBookplus****Digital doc**

Spreadsheet

Converting

improper

fractions

to mixed

numbers

doc-1725

## UNDERSTANDING

- 10** Kim and Carly arrived home from school quite hungry and cut up some fruit. Kim ate 9 quarters of an apple and Carly ate 11 quarters. How many apples did they each eat?
- 11** Daniel was selling pieces of quiche at the school fete. Each quiche was divided into 8 pieces. If Daniel sold 51 pieces, how many quiches did Daniel sell?
- 12** It was Cecilia's responsibility to supply oranges for her soccer team at half-time. If 19 players ate  $\frac{1}{4}$  orange each, how many oranges were eaten?
- 13** Insert the appropriate < or > sign between each pair of fractions to make a true statement.

**a**  $\frac{8}{5} \quad 1\frac{2}{5}$

**b**  $\frac{19}{2} \quad 8\frac{1}{2}$

**c**  $\frac{33}{32} \quad 1\frac{3}{32}$

**d**  $\frac{15}{2} \quad 5\frac{1}{2}$

**e**  $3\frac{3}{4} \quad \frac{19}{4}$

**f**  $7\frac{1}{7} \quad \frac{65}{7}$

**g**  $2\frac{1}{3} \quad \frac{8}{3}$

**h**  $1\frac{16}{17} \quad \frac{35}{17}$

**i**  $4\frac{3}{8} \quad \frac{36}{8}$



- 14** **a** Amy made apple pies for her Grandma's 80th birthday party. She divided each pie into 6 pieces, and after the party she noted that  $4\frac{1}{6}$  pies had been eaten. How many pieces had been eaten?
- b** Amy's cousin Ria provided cordial for the same party, and calculated that she could make 20 drinks from each bottle. At the end of the party  $3\frac{17}{20}$  bottles had been used. How many drinks had been consumed?
- 15** A catering company provided 12 quiches which were each cut into 8 pieces for a luncheon. At the end of the function there were 12 pieces of quiche left over. How many whole quiches were eaten?

## REFLECTION

What strategies will you use to write an improper fraction as a mixed number?

## 4D Adding and subtracting rational numbers

eBook plus

**Interactivity**  
Adding and  
subtracting  
fractions  
int-2357

### Adding and subtracting proper fractions

- Fractions can be added and subtracted if they have the same denominator.
- Remember to simplify your answer if possible.

## WORKED EXAMPLE 13

Find  $\frac{5}{9} + \frac{1}{9}$ .

## THINK

- Write the question.
- Add the numerators since the denominators are the same.
- Simplify the answer if possible by cancelling the highest common factor ( $6 \div 3 = 2$ ,  $9 \div 3 = 3$ ).

## WRITE

$$\begin{aligned}\frac{5}{9} + \frac{1}{9} \\ = \frac{6}{9} \\ = \frac{6^2}{9^3} \\ = \frac{2}{3}\end{aligned}$$

- If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.

**WORKED EXAMPLE 14**

Find  $\frac{5}{6} - \frac{1}{12}$ , expressing the answer in simplest form.

**THINK**

- Find multiples of each denominator.
- Find the lowest common multiple (LCM) of 6 and 12 (12).
- Rewrite the fractions if necessary so that they both have the LCM as the new denominator.
- Subtract the numerators since the denominators are the same.
- Simplify your answer if possible by cancelling the highest common factor ( $9 \div 3 = 3$ ,  $12 \div 3 = 4$ ).

**WRITE**

Multiples of 6 are 6, 12...  
Multiples of 12 are 12...

LCM is 12.

$$\begin{aligned}\frac{5}{6} - \frac{1}{12} &= \frac{5 \times 2}{6 \times 2} - \frac{1}{12} \\ &= \frac{10}{12} - \frac{1}{12} \\ &= \frac{9}{12} \\ &= \frac{9^{\cancel{3}}}{12^{\cancel{4}}} \\ &= \frac{3}{4}\end{aligned}$$

- With practice, many of these steps can be done in your head and the solution written as follows.

$$\begin{aligned}\frac{5}{6} - \frac{1}{12} &= \frac{10}{12} - \frac{1}{12} \\ &= \frac{9}{12} \\ &= \frac{3}{4}\end{aligned}$$

**REMEMBER**

- Fractions can be added and subtracted if they have the same denominator.
- If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.

**EXERCISE****4D****Adding and subtracting rational numbers****FLUENCY**

- 1 WE13** Find:

a  $\frac{1}{5} + \frac{3}{5}$

b  $\frac{9}{11} - \frac{3}{11}$

c  $\frac{3}{6} + \frac{2}{6}$

d  $\frac{20}{50} + \frac{11}{50}$

e  $\frac{15}{25} - \frac{6}{25}$

f  $\frac{12}{12} - \frac{1}{12}$

g  $\frac{5}{8} - \frac{1}{8} + \frac{3}{8}$

h  $\frac{4}{7} + \frac{5}{7} - \frac{3}{7}$

i  $\frac{33}{100} + \frac{7}{100} - \frac{11}{100}$

**INDIVIDUAL PATHWAYS**

eBook plus

**Activity 4-D-1**

How much honey?  
doc-1709

INDIVIDUAL  
PATHWAYS

## eBookplus

## Activity 4-D-2

How fast is it?  
doc-1710

## Activity 4-D-3

How much milk?  
doc-1711

- 2 Find the following, expressing the answers in simplest form.

a  $\frac{5}{12} + \frac{1}{12}$

b  $\frac{7}{16} - \frac{3}{16}$

c  $\frac{9}{15} - \frac{4}{15}$

d  $\frac{37}{100} + \frac{13}{100}$

e  $\frac{9}{4} - \frac{7}{4}$

f  $\frac{4}{7} + \frac{5}{7} - \frac{2}{7}$

g  $\frac{13}{16} + \frac{9}{16} - \frac{10}{16}$

h  $\frac{11}{28} + \frac{10}{28}$

i  $\frac{21}{81} + \frac{21}{81} + \frac{30}{81}$

- 3 WE14 Find the following, expressing the answers in simplest form.

a  $\frac{1}{2} + \frac{1}{4}$

b  $\frac{1}{2} - \frac{1}{4}$

c  $\frac{1}{3} + \frac{2}{21}$

d  $\frac{1}{8} + \frac{20}{32}$

e  $\frac{9}{18} + \frac{1}{6}$

f  $\frac{17}{40} + \frac{1}{2}$

g  $\frac{7}{9} - \frac{8}{27}$

h  $\frac{31}{35} - \frac{3}{7}$

i  $\frac{8}{10} - \frac{28}{70}$

- 4 Find the lowest common multiple of each of the following pairs of numbers.

a 2 and 3

b 3 and 5

c 5 and 10

d 4 and 6

e 5 and 6

f 9 and 6

g 4 and 8

h 6 and 8

i 7 and 5

- 5 Using the lowest common multiples that were found in question 4 as denominators, add or subtract these fractions.

a  $\frac{1}{2} - \frac{1}{3}$

b  $\frac{2}{3} - \frac{2}{5}$

c  $\frac{3}{5} + \frac{3}{10}$

d  $\frac{3}{4} - \frac{1}{6}$

e  $\frac{4}{5} - \frac{1}{6}$

f  $\frac{1}{9} + \frac{5}{6}$

g  $\frac{1}{4} + \frac{3}{8}$

h  $\frac{5}{6} - \frac{3}{8}$

i  $\frac{5}{7} - \frac{2}{5}$

- 6 Answer the following questions by first finding the lowest common denominator. Simplify your answer if necessary.

a  $\frac{2}{3} + \frac{1}{4}$

b  $\frac{5}{7} - \frac{3}{8}$

c  $\frac{4}{5} - \frac{3}{4}$

d  $\frac{7}{10} + \frac{2}{15}$

e  $\frac{5}{8} - \frac{5}{12}$

f  $\frac{7}{9} - \frac{1}{2}$

g  $\frac{5}{12} + \frac{2}{9}$

h  $\frac{7}{11} + \frac{1}{4}$

i  $\frac{4}{5} - \frac{2}{11}$

j  $\frac{5}{6} + \frac{7}{10}$

k  $\frac{3}{10} - \frac{1}{20}$

l  $\frac{7}{8} - \frac{3}{5}$

m  $\frac{2}{3} + \frac{4}{5}$

n  $\frac{5}{7} + \frac{3}{4}$

o  $\frac{5}{9} - \frac{2}{7}$

p  $\frac{2}{3} - \frac{1}{13}$

q  $\frac{11}{12} - \frac{7}{10}$

r  $\frac{4}{7} + \frac{5}{6}$

s  $\frac{4}{5} - \frac{2}{3}$

t  $\frac{6}{11} - \frac{1}{3}$

- 7 MC a The lowest common denominator of  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{6}$  is:

A 16      B 24      C 18      D 12      E 72

- b The lowest common denominator of  $\frac{1}{2}$ ,  $\frac{1}{4}$  and  $\frac{1}{5}$  is:

A 8      B 10      C 20      D 30      E 40

## UNDERSTANDING

- 8 Answer the following:

a  $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

b  $\frac{3}{10} + \frac{4}{15} - \frac{1}{6}$

c  $\frac{7}{8} - \frac{3}{4} + \frac{4}{6}$

d  $\frac{11}{12} - \frac{8}{15} - \frac{3}{20}$

e  $\frac{1}{16} + \frac{1}{8} + \frac{1}{4}$

f  $\frac{5}{6} - \frac{7}{18} - \frac{1}{9}$

g  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

h  $\frac{3}{4} + \frac{1}{3} + \frac{5}{6}$

- 9 Angela was a little hungry the day before her birthday, and spied the cake to be eaten the following day. She decided that if she ate only  $\frac{1}{10}$  of it, her Mum might not notice. However, she was still hungry and took another  $\frac{1}{6}$ .

a How much of the cake did Angela eat before her birthday?

b How much of the cake was left for her birthday?

## eBookplus

## Digital docs

Spreadsheet

Adding and

subtracting

fractions

doc-1726

WorkSHEET 4.1

doc-1721

- 10** Bread needed to be cooked at  $200^{\circ}\text{C}$  for  $\frac{3}{4}$  hour then a further  $\frac{1}{2}$  hour at  $150^{\circ}\text{C}$ . What is the total cooking time in hours?
- 11** Robert, Jason and Luke play football in the same team. Last Saturday, Robert kicked  $\frac{1}{4}$  of the team's goals, Jason kicked  $\frac{1}{3}$  and Luke  $\frac{5}{12}$ . What fraction of the team's goals were kicked by:
- Jason and Luke?
  - Robert and Jason?
  - the three boys together?
- 12** Jacinta and her friend decided to climb the tree in her Nan's front yard. It took her 10 minutes to climb halfway up, 10 more minutes to climb another sixth of the way and yet another 10 minutes to reach a further tenth of the way. Here she sat admiring the view for 5 minutes before beginning her descent.
- What fraction of the height of the tree had Jacinta climbed after 20 minutes?
  - What fraction of the tree did Jacinta climb in total?


**REFLECTION**

What difficulties might you experience when adding and subtracting fractions with different denominators? How could you overcome them?

**4E**

## Multiplying rational numbers

**eBookplus**

**Interactivity**  
Repeated addition  
int-2738

- It has been seen in an earlier chapter that multiplication by a whole number can be thought of as repeated addition.  
For example,  $5 \times 6 = 5 + 5 + 5 + 5 + 5 + 5 = 30$
- The same principle applies to multiplying a whole number by a fraction.  
For example,  $\frac{1}{5} \times 6 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 1\frac{1}{5}$
- To multiply two fractions we need to use an alternative approach.
- To multiply fractions, multiply the numerators and then multiply the denominators.
- It is important to note that the denominators do not need to be the same.

**WORKED EXAMPLE 15**

Find  $\frac{1}{5} \times \frac{3}{4}$ .

**THINK**

- Write the question.
- Write as one fraction.
- Multiply the numerators, then the denominators and simplify your answer if appropriate.

**WRITE**

$$\begin{aligned} \frac{1}{5} \times \frac{3}{4} \\ = \frac{1 \times 3}{5 \times 4} \\ = \frac{3}{20} \end{aligned}$$

- Before being multiplied, fractions can be simplified by dividing both the numerator and denominator with their highest common factor.

- This is easier than working with large numbers.
- Remember that dividing both numerator and denominator by the same number is often called *cancelling*.

**WORKED EXAMPLE 16**Find  $\frac{2}{3} \times \frac{9}{10}$ .**THINK**

- 1 Write the question.
- 2 Write the two fractions as one keeping the multiplication signs.
- 3 Divide the terms in the numerator and denominator by their highest common factor.  
*Note:* This may be done vertically or diagonally.
- 4 Multiply the numerators, then the denominators and simplify your answer if appropriate.

**WRITE**

$$\begin{aligned}\frac{2}{3} \times \frac{9}{10} \\ = \frac{1\cancel{2} \times 9^3}{\cancel{3} \times 10^5} \\ = \frac{3}{5}\end{aligned}$$

**WORKED EXAMPLE 17**Find  $\frac{1}{3} \times 12$  by using the method of repeated addition.**THINK**

- 1 Write the question.
- 2 Write the expression as a repeated addition, that is, sum 12 thirds.
- 3 Add the numerators.
- 4 Simplify your answer.

**WRITE**

$$\begin{aligned}\frac{1}{3} \times 12 \\ = \frac{1}{3} + \frac{1}{3} \\ = \frac{12}{3} \\ = 4\end{aligned}$$

**Using the word of**

- Sometimes the word *of* can mean the same as *multiply*.
- It might take a third of an hour (60 minutes) to cook a pizza. A third of an hour, or  $\frac{1}{3}$  of 60 minutes, is the same as  $\frac{1}{3} \times \frac{60}{1} = 20$  minutes. It takes 20 minutes to cook the pizza.
- If a customer wants to buy  $\frac{1}{4}$  of the remaining 8 pizzas, it is the same as  $\frac{1}{4} \times \frac{8}{1} = 2$  pizzas. The customer buys 2 pizzas.

**WORKED EXAMPLE 18**Find  $\frac{2}{3}$  of  $\frac{1}{4}$ .**THINK**

- 1 Write the question.
- 2 Change ‘of’ to ‘×’ and cancel if appropriate.
- 3 Perform the multiplication.

**WRITE**

$$\begin{aligned}\frac{2}{3} \text{ of } \frac{1}{4} \\ = \frac{\cancel{2}^1}{\cancel{3}^1} \times \frac{1}{4^2} \\ = \frac{1}{6}\end{aligned}$$

## WORKED EXAMPLE 19

Mum put a 2-litre carton of flavoured milk in the refrigerator. When the children came home from school, Joanna drank one quarter of the milk, Kieren drank one half of it and Daisy drank one sixth of it. What fraction of the milk, in litres, did each person drink?

## THINK

- 1 Write the fraction of the milk that Joanna drank and simplify the answer.
- 2 Perform the multiplication.
- 3 Write the fraction of the milk that Kieren drank and simplify the answer.
- 4 Perform the multiplication.
- 5 Write the fraction of the milk that Daisy drank and simplify the answer.
- 6 Perform the multiplication.
- 7 Write a sentence giving the answer in litres.

## WRITE

$$\text{Joanna: } \frac{1}{4} \text{ of } 2 = \frac{1}{4} \times 2$$

$$= \frac{1}{2} \cancel{\frac{1}{4}} \times \frac{2^1}{1}$$

$$= \frac{1}{2}$$

$$\text{Kieren: } \frac{1}{2} \text{ of } 2 = \frac{1}{2} \times 2$$

$$= \frac{1}{1} \cancel{\frac{1}{2}} \times \frac{2^1}{1}$$

$$= 1$$

$$\text{Daisy: } \frac{1}{6} \text{ of } 2 = \frac{1}{6} \times 2$$

$$= \frac{1}{3} \cancel{\frac{1}{6}} \times \frac{2^1}{1}$$

$$= \frac{1}{3}$$

Joanna drank a half a litre, Kieren drank 1 litre and Daisy drank a third of a litre of milk.

## REMEMBER

1. To multiply fractions, multiply the numerators and then multiply the denominators.
2. When multiplying fractions the denominators do not need to be the same.
3. Fractions can be simplified (or cancelled) before being multiplied.
4. A whole number can be written as a fraction with a denominator of 1.
5. In a problem involving fractions, ‘of’ can be replaced with ‘multiply’, then the fractions can be multiplied in the usual way.

## EXERCISE

## 4E Multiplying rational numbers

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 4-E-1

Fraction multiplication  
doc-1712

## Activity 4-E-2

More fraction multiplication  
doc-1713

## FLUENCY

- 1 WE15 Find:

a  $\frac{1}{2} \times \frac{1}{4}$

b  $\frac{1}{3} \times \frac{2}{3}$

c  $\frac{2}{5} \times \frac{3}{5}$

d  $\frac{5}{9} \times \frac{4}{3}$

e  $\frac{11}{13} \times \frac{1}{2}$

f  $\frac{6}{7} \times \frac{9}{11}$

g  $\frac{11}{12} \times \frac{11}{12}$

h  $\frac{8}{9} \times \frac{5}{9}$

i  $\frac{5}{8} \times \frac{1}{3}$

j  $\frac{2}{3} \times \frac{2}{3}$

k  $\frac{5}{6} \times \frac{11}{12}$

l  $\frac{5}{11} \times \frac{5}{12}$

m  $\frac{7}{8} \times \frac{7}{8}$

n  $\frac{2}{3} \times \frac{4}{13}$

o  $\frac{3}{5} \times \frac{3}{10}$

p  $\frac{4}{7} \times \frac{2}{3}$

**INDIVIDUAL PATHWAYS****eBookplus**

**Activity 4-E-3**  
Advanced fraction multiplication  
doc-1714

**eBookplus**

**Digital doc**  
Spreadsheet  
Multiplying fractions  
doc-1727

**2 WE16** Find:

a  $\frac{4}{8} \times \frac{3}{9}$

e  $\frac{10}{11} \times \frac{22}{25}$

i  $\frac{15}{27} \times \frac{36}{45}$

m  $\frac{42}{54} \times \frac{12}{49}$

b  $\frac{2}{5} \times \frac{1}{2}$

f  $\frac{24}{27} \times \frac{9}{8}$

j  $\frac{8}{49} \times \frac{14}{16}$

n  $\frac{5}{25} \times \frac{12}{48}$

c  $\frac{3}{7} \times \frac{5}{6}$

g  $\frac{18}{32} \times \frac{64}{72}$

k  $\frac{4}{9} \times \frac{6}{8}$

o  $\frac{18}{24} \times \frac{12}{36}$

d  $\frac{3}{8} \times \frac{4}{15}$

h  $\frac{48}{56} \times \frac{24}{60}$

l  $\frac{6}{12} \times \frac{8}{10}$

**3 WE17** Find:

a  $\frac{2}{3} \times 3$

b  $\frac{5}{14} \times 7$

c  $\frac{15}{22} \times 11$

d  $\frac{5}{16} \times 4$

**4 WE18** Find:

a  $\frac{1}{2}$  of  $\frac{1}{4}$

b  $\frac{3}{4}$  of  $\frac{2}{3}$

c  $\frac{5}{6}$  of  $\frac{5}{6}$

d  $\frac{9}{10}$  of  $\frac{5}{7}$

e  $\frac{8}{9}$  of  $\frac{1}{4}$

f  $\frac{5}{7}$  of  $\frac{7}{25}$

g  $\frac{4}{5}$  of 20

h  $\frac{7}{8}$  of 32

i  $\frac{9}{10}$  of 50

j  $\frac{2}{5}$  of 45

k  $\frac{3}{5}$  of 25

l  $\frac{8}{9}$  of 81

m  $\frac{3}{4}$  of 6

n  $\frac{9}{10}$  of 15

o  $\frac{1}{12}$  of 10

p  $\frac{3}{7}$  of 12

**5 WE19** Jamie's mum made 6 litres of cordial for his birthday party. Emily drank  $\frac{1}{3}$  of the cordial, Tracy drank  $\frac{1}{12}$  and Jonathan drank  $\frac{1}{18}$ . How much cordial, in litres, did each person drink?

- 6** Zoe and Sarah play basketball with the Sharp Shooters. The games are played for 40 minutes. Zoe played  $\frac{4}{5}$  of last week's game and Sarah played  $\frac{7}{8}$ . How many minutes of the game did:
- Zoe play?
  - Sarah play?

- 7** Mark's uncle gave him a family-size block of chocolate. He divided it into thirds to share with his two sisters. His friends Tom and Nick then arrived to visit, and Mark shared his portion of the chocolate with them, so that they each had the same amounts.

- Draw a block of chocolate with  $9 \times 6$  pieces, and shade Mark's third of the block. Then draw lines to represent Tom's share.
- What fraction of the block of chocolate did Nick receive?



- 8** A car's petrol tank holds 48 litres of fuel.
- If the tank was full at the start of a trip, what fraction of the tank has been used?
  - How many litres of petrol have been used?
- 9** a Draw a quarter of a pie.  
b Shade half of this piece of pie.  
c What fraction of the whole pie is shaded?  
d Complete the mathematical sentence  $\frac{1}{4} \times \frac{1}{2} = \underline{\hspace{2cm}}$  to show how the answer was found.
- 10** a Find how many half pizzas in 1 pizza.  
b Find how many half pizzas in 2 whole pizzas.  
c Find how many half pizzas in 5 whole pizzas.

**REASONING**

- 11** Gustave's monthly take-home pay is \$2400. From this he spends a quarter on his home loan payments, one half on food and drink and one sixth on clothing. One half of the remainder goes into his savings account. How much money does Gustave put into his savings account each month?

- 12** To make a loaf of fruit bread, the fruit needs to be added after the second rise stage and the top of the bread needs to be glazed two-thirds of the way through the bake stage. The recipe shows you how long each stage of the total cycle takes.



Stage	Length of time
First knead	5 minutes
Second knead	20 minutes
First rise then punch down	40 minutes
Second rise then punch down	25 minutes
Third rise	50 minutes
Bake	40 minutes

- a After what length of time should the fruit be added?
- b Express this as a fraction of the total time in the bread maker.
- c After what length of time should the top of the bread be glazed?

#### REFLECTION

Is repeated addition an efficient method of multiplying fractions?  
Explain your reasoning.

## 4F Dividing rational numbers

### eBook plus

eLesson  
Dividing  
fractions  
eles-0003

- The **reciprocal** of a fraction is found by tipping the fraction upside down. So  $\frac{3}{5}$  is the reciprocal of  $\frac{5}{3}$ .
- A whole number can be written as a fraction by putting it over 1. This means that  $\frac{1}{4}$  is the reciprocal of 4 or  $\frac{4}{1}$ .

### WORKED EXAMPLE 20

Find the reciprocal of  $\frac{2}{3}$ .

#### THINK

Turn the fraction upside down and write the answer in a sentence.

#### WRITE

The reciprocal of  $\frac{2}{3}$  is  $\frac{3}{2}$ .

- To find the reciprocal of a mixed number, express it as an improper fraction first.

## WORKED EXAMPLE 21

**Find the reciprocal of  $1\frac{2}{3}$ .**

**THINK**

- 1 Write  $1\frac{2}{3}$  as an improper fraction.
- 2 Turn it upside down and write the answer in a sentence.

**WRITE**

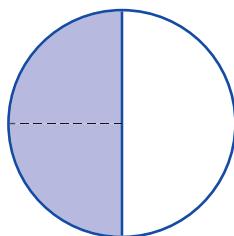
$$1\frac{2}{3} = \frac{5}{3}$$

The reciprocal of  $1\frac{2}{3}$  is  $\frac{3}{5}$ .

**eBook plus**

**Interactivity**  
Dividing fractions  
int-2358

- To find out how many quarters there are in one half, we could draw a diagram and find that there are 2 quarters.
- This means that  $\frac{1}{2} \div \frac{1}{4} = 2$ .
- The answer could also have been found by changing the division sign to multiplication and tipping the second fraction upside down.
- Division by a number is the same as multiplication by its reciprocal.



## WORKED EXAMPLE 22

**Find  $\frac{2}{3} \div \frac{4}{9}$ .**

**THINK**

- 1 Write the question.
- 2 Change the division sign to a multiplication sign and tip the second fraction.
- 3 Cancel if possible.
- 4 Perform the multiplication.
- 5 Simplify your answer.

**WRITE**

$$\begin{aligned} \frac{2}{3} \div \frac{4}{9} &= \frac{2}{3} \times \frac{9}{4} \\ &= \frac{2^1}{3^1} \times \frac{9^3}{4^4} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$

**REMEMBER**

1. The *reciprocal* of a fraction is found by tipping the fraction upside down.
2. The reciprocal of a mixed number is found by expressing it as an improper fraction, then tipping it upside down.
3. A whole number can be written as a fraction by putting it over 1.
4. Division by a number is the same as multiplication by its reciprocal.
5. To divide fractions:
  - (a) change the division sign to a multiplication sign
  - (b) tip the second fraction (that is, take the reciprocal of the fraction that follows the division sign)
  - (c) perform the multiplication.

## EXERCISE

## 4F Dividing rational numbers

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 4-F-1

Just flip it  
doc-1715

## Activity 4-F-2

More just flip it  
doc-1716

## Activity 4-F-3

Advanced just flip it  
doc-1717

## FLUENCY

- 1 WE20** Find the reciprocals of each of the following.

**a**  $\frac{3}{4}$

**b**  $\frac{2}{7}$

**c**  $\frac{5}{3}$

**d**  $\frac{5}{12}$

**e**  $\frac{6}{5}$

**f**  $\frac{9}{2}$

**g**  $\frac{5}{3}$

**h**  $\frac{10}{3}$

**i**  $\frac{1}{5}$

**j**  $\frac{1}{12}$

**k**  $\frac{3}{15}$

**l**  $\frac{2}{22}$

**m** 5

**n** 20

**o** 13

**p** 1

- 2 WE21** Find the reciprocals of these mixed numbers.

**a**  $1\frac{2}{3}$

**b**  $4\frac{3}{7}$

**c**  $7\frac{1}{2}$

**d**  $4\frac{2}{5}$

**e**  $9\frac{2}{7}$

**f**  $3\frac{6}{7}$

**g**  $3\frac{3}{8}$

**h**  $5\frac{1}{3}$

**i**  $10\frac{2}{9}$

**j**  $6\frac{9}{10}$

- 3** Multiply each of these numbers by its reciprocal.

**a**  $\frac{5}{7}$

**b**  $\frac{2}{9}$

**c**  $\frac{3}{5}$

**d**  $3\frac{2}{7}$

**e**  $2\frac{1}{8}$

**f**  $3\frac{3}{4}$

- 4 WE22** Find:

**a**  $\frac{1}{4} \div \frac{2}{3}$

**b**  $\frac{8}{9} \div \frac{7}{6}$

**c**  $\frac{9}{10} \div \frac{1}{2}$

**d**  $\frac{1}{12} \div \frac{2}{3}$

**e**  $\frac{4}{11} \div \frac{5}{6}$

**f**  $\frac{7}{12} \div \frac{9}{11}$

**g**  $\frac{10}{3} \div \frac{15}{21}$

**h**  $\frac{2}{7} \div \frac{20}{21}$

**i**  $\frac{8}{15} \div \frac{16}{15}$

**j**  $\frac{4}{9} \div \frac{4}{9}$

**k**  $\frac{4}{7} \div 10$

**l**  $\frac{5}{8} \div 20$

**m**  $\frac{4}{3} \div 12$

**n**  $\frac{15}{11} \div 5$

**o**  $\frac{7}{9} \div 30$

## eBook plus

## UNDERSTANDING

Digital doc  
Spreadsheet  
Dividing fractions  
doc-1728

- 5** What is the value of this expression?

$$\frac{1}{2 + \frac{1}{2\frac{1}{2}}}$$

## REASONING

- 6** Millicent is doing some baking. Her recipe calls for  $\frac{1}{4}$  cup of sugar. If she has a 2-kg bag of sugar, and if 1 cup of sugar weighs 230 g, how many batches of the recipe can she make?
- 7** Cakes for all Occasions makes birthday cakes cut to serve 24 people. Arabella takes one to school to celebrate her birthday, but there are 30 people in her class. What fraction of the original slices will each student receive?

## REFLECTION

What strategies will you use to divide fractions?

## 4G Fractions as ratios

### Introduction to ratios

- Ratios are used in many aspects of everyday life. They are used to compare *quantities of the same kind*.

Where might you hear the following comparisons? What are the quantities being compared?



- Michael Schumacher's F1 car is twice as fast as Mick's delivery van.
  - You will need 2 buckets of water for every  $\frac{1}{2}$  bucket of sand.
  - The fertiliser contains 3 parts of phosphorus to 2 parts of potassium.
  - The Tigers team finished the season with a win : loss ratio of 5 to 2.
  - Mix 1 teaspoon of salt with 4 teaspoons of flour.
- In the last example we are considering the ratio of salt (1 teaspoon) to flour (4 teaspoons). We write  $1 : 4$  and say, ‘one is to four’; we are actually mixing a total of 5 teaspoons. Ratios can also be written in fraction form:

$$1 : 4 \Leftrightarrow \frac{1}{4}$$

- Note:** Since the ratios compare quantities of the same kind, they do not have a name or unit of measurement. That is, we write the ratio of salt to flour as  $1 : 4$ , not 1 teaspoon : 4 teaspoons. The order of the numbers in a ratio is important. In the example of the ratio of salt and flour,  $1 : 4$  means 1 unit (for example a teaspoon) of salt to 4 units of flour. The amount of flour is 4 times as large as the amount of salt. On the contrary, the ratio  $4 : 1$  means 4 units of salt to 1 unit of flour, which means the amount of salt is 4 times as large as the amount of flour.

**WORKED EXAMPLE 23**

Look at the completed game of ‘noughts and crosses’ at right and write down the ratios of:

- crosses to noughts
- noughts to unmarked spaces.

X	O	
X	X	X
O		O

**THINK**

- Count the number of crosses and the number of noughts. Write the 2 numbers as a ratio (the number of crosses must be written first).
- Count the number of noughts and the number of unmarked spaces. Write the 2 numbers as a ratio, putting them in the order required (the number of noughts must be written first).

**WRITE**

a  $4 : 3$

b  $3 : 2$

- Before ratios are written, the numbers must be expressed in *the same units of measurement*. Once the units are the same, they can be omitted. When choosing which of the quantities to convert, keep in mind that ratios *contain only whole numbers*.

**WORKED EXAMPLE 24**

Rewrite the following statement as a ratio: 7 mm to 1 cm.

**THINK**

- Express both quantities in the same units. To obtain whole numbers, convert 1 cm to mm (rather than 7 mm to cm).
- Omit the units and write the 2 numbers as a ratio.

**WRITE**

7 mm to 1 cm  
7 mm to 10 mm

$7 : 10$

**Simplifying ratios**

- When the numbers in a ratio are multiplied, or divided by the same number to obtain another ratio, these two ratios are said to be *equivalent*. (This is similar to the process of obtaining equivalent fractions.)  
For instance, the ratios  $2 : 3$  and  $4 : 6$  are equivalent, as the second ratio can be obtained by multiplying both numbers of the first ratio by 2.
- Ratios  $10 : 5$  and  $2 : 1$  are also equivalent, as the second ratio is obtained by dividing both numbers of the first ratio by 5.
- Like fractions, ratios are usually written in simplest form; that is, reduced to lowest terms. This is achieved by dividing each number in the ratio by the highest common factor (HCF).

**WORKED EXAMPLE 25**

Express the ratio  $16 : 24$  in simplest form.

**THINK**

- Copy the ratio in your workbook. What is the largest number, by which both 16 and 24 can be divided (that is, the highest common factor of 16 and 24)? It is 8.
- Divide both 16 and 24 by 8 to obtain an equivalent ratio in simplest form.

**WRITE**

$16 : 24$

$2 : 3$

**WORKED EXAMPLE 26**

Write the ratio of 1.5 m to 45 cm in simplest form.

**THINK**

- 1 Write down the question.
- 2 Express both quantities in the same units by changing 1.5 m into cm.  
(1 m = 100 cm)
- 3 Omit the units and write the 2 numbers as a ratio.
- 4 Simplify the ratio by dividing both 150 and 45 by 15 — the HCF.

**WRITE**

- 1.5 m to 45 cm  
150 cm to 45 cm  
150 : 45  
10 : 3

**WORKED EXAMPLE 27**

Simplify the following ratios.

a  $\frac{2}{5} : \frac{7}{10}$       b  $\frac{5}{6} : \frac{5}{8}$

**THINK**

- a 1 Write the fractions in ratio form.
- 2 Write equivalent fractions using the lowest common denominator: in this case 10.
- 3 Multiply both fractions by 10.
- 4 Check if the remaining whole numbers that form the ratio can be simplified. In this case they cannot.
- b 1 Write the fractions in ratio form.
- 2 Write equivalent fractions using the lowest common denominator. In this case 24.
- 3 Multiply both fractions by 24.
- 4 Check if the remaining whole numbers that form the ratio can be simplified. In this case divide each by the HCF of 5.

**WRITE**

a  $\frac{2}{5} : \frac{7}{10}$   
 $= \frac{4}{10} : \frac{7}{10}$   
 $= 4 : 7$

b  $\frac{5}{6} : \frac{5}{8}$   
 $= \frac{20}{24} : \frac{15}{24}$   
 $= 20 : 15$   
 $= 4 : 3$

## Dividing in a given ratio

■ Consider the following situation:

Isabel and Rachel decided to buy a \$10 lottery ticket. Isabel had only \$3 so Rachel put in the other \$7. The ticket won the first prize of \$500 000. How are the girls going to share the prize? Is it fair that they would get equal shares?

In all fairness, it should be shared in the ratio 3 : 7.

**WORKED EXAMPLE 28**

Share the amount \$500 000 in the ratio 3 : 7.

**THINK**

- 1 Calculate the total number of parts in the ratio.
- 2 The first share represents 3 parts out of a total of 10, so find  $\frac{3}{10}$  of the total amount.
- 3 The second share represents 7 parts out of a total of 10, so find  $\frac{7}{10}$  of the total amount.

**WRITE**

$$\begin{aligned}\text{Total number of parts} &= 3 + 7 \\ &= 10\end{aligned}$$

$$\begin{aligned}\text{First share} &= \frac{3}{10} \times \$500\,000 \\ &= \$150\,000\end{aligned}$$

$$\begin{aligned}\text{Second share} &= \frac{7}{10} \times \$500\,000 \\ &= \$350\,000\end{aligned}$$

- We can mentally check our answer by adding the two shares together. The total should be the same as the amount that was originally divided.

**WORKED EXAMPLE 29**

Concrete mixture for a footpath was made up of 1 part of cement, 2 parts of sand and 4 parts of blue metal. How much sand was used to make  $4.2 \text{ m}^2$  of concrete?

**THINK**

- 1 Find the total number of parts.
- 2 There are 2 parts of sand to be used in the mixture, so find  $\frac{2}{7}$  of the total amount of concrete made.

**WRITE**

$$\begin{aligned}\text{Total number of parts} &= 1 + 2 + 4 \\ &= 7\end{aligned}$$

$$\begin{aligned}\text{Amount of sand} &= \frac{2}{7} \times 4.2 \text{ m}^2 \\ &= 1.2 \text{ m}^2\end{aligned}$$

**REMEMBER**

1. Ratios compare quantities of the same kind.
2. The ratios themselves do not have a name or unit of measurement.
3. The order of the numbers in a ratio is important.
4. Before ratios are written, the numbers must be expressed in the *same units of measurement*.
5. Ratios contain only whole numbers.
6. If each number in a ratio is multiplied, or divided by the same number, the equivalent (or equal) ratio is formed.
7. It is customary to write ratios in the simplest form. This is achieved by dividing each number in the ratio by the highest common factor (HCF).
8. To form a ratio, using fractions, convert the fractions so that they have a common denominator and then write the ratio of the numerators.
9. To share a certain amount in a given ratio, find the total number of shares (parts) first. The size of each share is given by the fraction this share represents out of the total number of shares.

## EXERCISE

## 4G

## Fractions as ratios

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 4-G-1

Ratios

doc-6433

## Activity 4-G-2

More ratios

doc-6434

## Activity 4-G-3

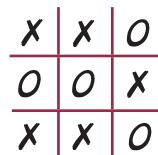
Advanced ratios

doc-6435

## FLUENCY

- 1 **WE23** Look at the completed game of ‘noughts and crosses’ and write down the ratios of:

- a noughts to crosses
- b crosses to noughts
- c crosses to total number of spaces
- d total number of spaces to noughts
- e noughts in the top row to crosses in the bottom row.



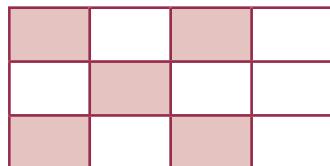
- 2 Look at the coloured circles on the right then write the following ratios.

- |                          |                          |
|--------------------------|--------------------------|
| a Black : red            | b Red : black            |
| c Aqua : black           | d Black : aqua           |
| e Aqua : red             | f Black : (red and aqua) |
| g Aqua : (black and red) | h Black : total circles  |
| i Aqua : total circles   | j Red : total circles    |



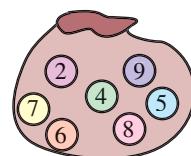
- 3 For the diagram shown, write down the following ratios.

- a Shaded parts : unshaded parts
- b Unshaded parts : shaded parts
- c Shaded parts : total parts



- 4 In the bag of numbers shown to the right, write down the ratios of:

- a even numbers to odd numbers
- b prime numbers to composite numbers
- c numbers greater than 3 to numbers less than 3
- d multiples of 2 to multiples of 5
- e numbers divisible by 3 to numbers not divisible by 3.



- 5 **WE24** Rewrite each of the following statements as a ratio.

- |                              |                             |
|------------------------------|-----------------------------|
| a 3 mm to 5 mm               | b 6 s to 19 s               |
| c \$4 to \$11                | d 7 teams to 9 teams        |
| e 1 goal to 5 goals          | f 9 boys to 4 boys          |
| g 3 weeks to 1 month         | h 3 mm to 1 cm              |
| i 17 seconds to 1 minute     | j 53 cents to \$1           |
| k 11 cm to 1 m               | l 1 g to 1 kg               |
| m 1 L to 2 kL                | n 7 hours to 1 day          |
| o 5 months to 1 year         | p 1 km to 27 m              |
| q 7 apples to 1 dozen apples | r 13 pears to 2 dozen pears |
| s 3 females to 5 males       | t 1 teacher to 22 students  |

- 6 **MC** In each of the following situations, choose the best ratio approximation.

- a Of the 90 000 people who attended the test match, 23 112 were females.  
The number of males to females is approximately:

- A 1 : 4      B 4 : 1      C 3 : 1      D 1 : 3

- b A Concorde jet travels at 1100 km/h while a Cessna travels at 210 km/h. Compare their speeds.

- A 2 : 1      B 3 : 1      C 4 : 1      D 5 : 1

- c A house and land package is sold for \$220 000. If the land was valued at \$90 000, compare the land and house values.

- A 1 : 2      B 1 : 3      C 2 : 5      D 2 : 3

- d In a kilogram of fertiliser, there are 550 g of phosphorus. Compare the amount of phosphorus to other components of the fertiliser.

- A 1 : 2      B 4 : 3      C 3 : 1      D 5 : 1

- e Sasha saves \$120 out of his take-home pay of \$700 each fortnight. Compare his savings with his expenses.

A 7 : 12

B 1 : 5

C 1 : 3

D 1 : 7

- 7 **WE25** Express each ratio in simplest form.

a 2 : 4

b 3 : 9

c 5 : 10

d 6 : 18

e 12 : 16

f 15 : 18

g 24 : 16

h 21 : 14

i 25 : 15

j 13 : 26

k 15 : 35

l 27 : 36

m 36 : 45

n 42 : 28

o 45 : 54

p 50 : 15

q 56 : 64

r 75 : 100

s 84 : 144

t 88 : 132

- 8 Complete the patterns of equivalent ratios.

a 1 : 3

b 2 : 1

c 2 : 3

d 64 : 32

e 48 : 64

2 : 6

4 : 2

4 : 6

\_\_ : 16

24 : \_\_

\_\_ : 9

\_\_ : 4

6 : \_\_

\_\_ : 8

12 : \_\_

\_\_ : 12

\_\_ : 8

\_\_ : 12

8 : \_\_

\_\_ : 8

5 : \_\_

20 : \_\_

\_\_ : 24

\_\_ : 1

\_\_ : \_\_

- 9 **WE26** Write the following ratios in simplest form.

a 8 cm to 12 cm

b \$6 to \$18

c 50 s to 30 s

d 80 cm to 2 m

e 75 cents to \$3

f 2 h to 45 min

g 300 mL to 4 L

h 500 g to 2.5 kg

i 45 mm to 2 cm

j \$4 to \$6.50

k 2500 L to 2 kL

l 2500 m to 2 km

m 30 cents to \$1.50

n 2 h 45 min to 30 min

o 200 m to 0.5 km

p 0.8 km to 450 m

q  $1\frac{1}{2}$  min to 300 s

r 1.8 cm to 12 mm

s 3500 mg to 1.5 g

t \$1.75 to \$10.50

- 10 Compare the following, using a mathematical ratio (in simplest form):

a The Hawks won 8 games while the Lions won 10 games.

b This jar of coffee costs \$4 but that one costs \$6.

c While Joanne made 12 hits, Holly made 8 hits.

d In the first innings, Ian scored 48 runs and Adam scored 12 runs.

e During the race, Rebecca's average speed was 200 km/h while Donna's average speed was 150 km/h.

f In the basketball match, the Tigers beat the Magic by 105 points to 84 points.

g The capacity of the plastic bottle is 250 mL and the capacity of the glass container is 2 L.

h Joseph ran the 600 m in 2 minutes but Maya ran the same distance in 36 seconds.

i In the movie audience, there were 280 children and 35 adults.

j On a page in the novel 'Moby Dick' there are 360 words. Of these, 80 begin with a vowel.

- 11 **WE27** Write the following ratios in simplest form.

a  $\frac{1}{3}$  to  $\frac{2}{3}$ b  $\frac{5}{7}$  to  $\frac{6}{7}$ c  $\frac{1}{4} : \frac{1}{2}$ d  $\frac{2}{5} : \frac{5}{6}$ e  $\frac{4}{5}$  to  $\frac{9}{20}$ f  $\frac{2}{3} : \frac{3}{5}$ g  $\frac{3}{10} : 1$ h  $1\frac{2}{3}$  to  $\frac{1}{3}$ i  $1\frac{1}{4} : 1\frac{1}{2}$ j  $3\frac{1}{3}$  to  $2\frac{1}{2}$ k  $1 : 1\frac{3}{5}$ l  $3\frac{1}{4}$  to  $2\frac{4}{5}$ 

- 12 **WE28** Share the amount of \$1000 in the following ratios.

a 2 : 3

b 3 : 1

c 1 : 4

d 1 : 1

e 3 : 5

f 5 : 3

g 3 : 7

h 9 : 1

i 7 : 13

j 9 : 11

- 13 Rosa and Mila bought a lottery ticket costing \$10. How should they share the first prize of \$50 000 if their respective contributions were:

a \$2 and \$8?

b \$3 and \$7?

c \$4 and \$6?

d \$5 and \$5?

e \$2.50 and \$7.50?

## UNDERSTANDING

- 14 WE29** Concrete mixture is made up of 1 part cement, 2 parts sand and 4 parts blue metal.
- How much sand is needed for  $7 \text{ m}^3$  of concrete?
  - How much cement is needed for  $3.5 \text{ m}^3$  of concrete?
  - How much blue metal is required for  $2.8 \text{ m}^3$  of concrete?
  - How much sand is used for  $5.6 \text{ m}^3$  of concrete?
  - How much cement is needed to make  $8.4 \text{ m}^3$  of concrete?
- 15** Three friends buy a Lotto ticket costing \$20. How should they share the first prize of \$600 000, if they each contribute:
- \$3, \$7 and \$10?
  - \$6, \$6 and \$8?
  - \$1, \$8 and \$11?
  - \$5, \$6 and \$9?
  - \$5, \$7.50 and \$7.50?
- 16** Three angles of a triangle are in the ratio  $1 : 2 : 3$ . What is the magnitude of each angle?
- 17** In a school, the ratio of girls in Years 8, 9 and 10 is  $6 : 7 : 11$ . If there are 360 girls in the school:
- How many Year 8 girls are there?
  - How many more Year 10 girls are there than Year 8 girls?
- 18** In a moneybox, there are 5 cent, 10 cent and 20 cent coins in the ratio  $8 : 5 : 2$ . If there are 225 coins altogether:
- how many 5 cent coins are there?
  - how many more 10 cent coins than 20 cent coins are there?
  - what is the total value of the 5 cent coins?
  - what is the total value of the coins in the moneybox?



## REASONING

- 19** In a family, 3 children receive their allowances in the ratio of their ages, which are 15 years, 12 years and 9 years. If the total of the allowances is \$30, how much does each child receive? Explain your answer.
- 20** The angles of a quadrilateral are in the ratio  $2 : 3 : 4 : 6$ . What is the difference in magnitude between the smallest and largest angles? Provide reasons for your answer.

## REFLECTION

Why is it important to be able to write ratios as fractions?

## 4H Working with mixed numbers

### Adding and subtracting mixed numbers

- There are a number of ways of adding and subtracting mixed numbers.
- All mixed numbers are added or subtracted by first changing them to improper fractions.
- Use equivalent fractions to write the fractions with the same denominator.
- Remember to simplify the answer if possible.

#### WORKED EXAMPLE 30

Find  $2\frac{3}{4} + 1\frac{1}{2}$ .

## THINK

- 1 Write the question.
- 2 Change each mixed number into an improper fraction.

## WRITE

$$\begin{aligned} 2\frac{3}{4} + 1\frac{1}{2} \\ = \frac{11}{4} + \frac{3}{2} \\ = \frac{11}{4} + \frac{3 \times 2}{2 \times 2} \end{aligned}$$



- THINK**
- 3 Write both fractions with the same denominator using equivalent fractions.
  - 4 Add the fractions.
  - 5 Write the answer as a mixed number if appropriate.
- WRITE**
- $$\begin{aligned}
 &= \frac{11}{4} + \frac{6}{4} \\
 &= \frac{17}{4} \\
 &= 4\frac{1}{4}
 \end{aligned}$$

**WORKED EXAMPLE 31**

Find  $3\frac{1}{5} - 1\frac{3}{4}$ .

**THINK**

- 1 Write the question.
- 2 Change each mixed number into an improper fraction.
- 3 Write both fractions with the same denominator using equivalent fractions.
- 4 Subtract the second fraction from the first.
- 5 Write the answer as a mixed number if appropriate.

**WRITE**

$$\begin{aligned}
 &3\frac{1}{5} - 1\frac{3}{4} \\
 &= \frac{16}{5} - \frac{7}{4} \\
 &= \frac{16 \times 4}{5 \times 4} - \frac{7 \times 5}{4 \times 5} \\
 &= \frac{64}{20} - \frac{35}{20} \\
 &= \frac{39}{20} \\
 &= 1\frac{19}{20}
 \end{aligned}$$

- Estimation can be used to find approximate answers to addition and subtraction of mixed numbers.
- This allows you to check easily whether your answer to a calculation is reasonable.

**WORKED EXAMPLE 32**

Find  $3\frac{1}{3} + 2\frac{1}{2}$  and check the answer by estimation.

**THINK**

- 1 Write the question.
- 2 Change each mixed number into an improper fraction.
- 3 Write both fractions with the same denominator using equivalent fractions.
- 4 Add the fractions.
- 5 Write the answer as a mixed number if appropriate.

**WRITE**

$$\begin{aligned}
 &3\frac{1}{3} + 2\frac{1}{2} \\
 &= \frac{10}{3} + \frac{5}{2} \\
 &= \frac{10}{3} \quad \frac{2}{2} + \frac{5}{2} \quad \frac{3}{3} \\
 &= \frac{20}{6} + \frac{15}{6} \\
 &= \frac{35}{6} \\
 &= 5\frac{5}{6}
 \end{aligned}$$

- 6 Check the answer by adding the whole numbers to find an approximation.

$$3\frac{1}{3} + 2\frac{1}{2} > 3 + 2$$

$$3\frac{1}{3} + 2\frac{1}{2} > 5$$

The approximation shows that the answer we obtain should be greater than 5.

## Multiplying mixed numbers

- Mixed numbers must be changed into improper fractions before multiplying.
- Once the mixed numbers are written as improper fractions, multiply as normal.
- Remember to simplify your answer if possible.

### WORKED EXAMPLE 33

Find  $1\frac{1}{2} \times 3\frac{3}{5}$ .

#### THINK

- 1 Write the question.
- 2 Change mixed numbers to improper fractions and cancel if possible.
- 3 Multiply the numerators and then the denominators.
- 4 Simplify your answer.

#### WRITE

$$\begin{aligned} 1\frac{1}{2} \times 3\frac{3}{5} \\ = \frac{3}{2} \times \frac{18}{5} \\ = \frac{27}{5} \\ = 5\frac{2}{5} \end{aligned}$$

## Dividing mixed numbers

- To divide mixed numbers, first change them to improper fractions.
- Once the fractions are written as improper fractions, divide as normal.

### WORKED EXAMPLE 34

Find  $1\frac{1}{2} \div 3\frac{2}{5}$ .

#### THINK

- 1 Write the question.
- 2 Change the mixed numbers to improper fractions.
- 3 Change the  $\div$  to  $\times$  and tip the second fraction.
- 4 Multiply the numerators, then the denominators and simplify your answer if necessary.

#### WRITE

$$\begin{aligned} 1\frac{1}{2} \div 3\frac{2}{5} \\ = \frac{3}{2} \div \frac{17}{5} \\ = \frac{3}{2} \times \frac{5}{17} \\ = \frac{15}{34} \end{aligned}$$

**REMEMBER**

1. Fractions can be added and subtracted if they have the same denominator.
2. If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.
3. Mixed numbers can be added or subtracted by first changing them to improper fractions.
4. To multiply or divide mixed numbers, write them as improper fractions first.
5. Improper fractions can be multiplied and divided as normal.

**EXERCISE****4H****Working with mixed numbers****INDIVIDUAL PATHWAYS****eBook plus****Activity 4-H-1**Working with mixed numbers  
doc-6518**Activity 4-H-2**More working with mixed numbers  
doc-6519**Activity 4-H-3**Advanced working with mixed numbers  
doc-6520**FLUENCY**

- 1 WE30** Find:

a  $1\frac{1}{5} + 3\frac{1}{2}$

b  $3\frac{2}{3} - 2\frac{1}{2}$

c  $5\frac{2}{3} - 2\frac{1}{4}$

d  $9\frac{3}{8} + 4$

e  $2\frac{3}{5} - 1\frac{1}{2}$

f  $3\frac{1}{2} + 2\frac{3}{4}$

- 2 WE31** Find:

a  $7 - 5\frac{2}{3}$

b  $2\frac{1}{3} - \frac{5}{6}$

c  $3\frac{1}{2} - 1\frac{7}{8}$

d  $4\frac{3}{5} - 2\frac{9}{10}$

e  $5\frac{1}{4} - 1\frac{7}{12}$

f  $5\frac{3}{4} - 3\frac{5}{6}$

- 3** Find:

a  $\frac{1}{5} + 2\frac{2}{7}$

b  $\frac{5}{8} + 3\frac{3}{4} - \frac{1}{2}$

c  $2\frac{2}{9} + 1\frac{1}{3} + \frac{1}{6}$

d  $4\frac{2}{3} + 1\frac{3}{8} - 3$

- 4 WE32** Find the following and check the answers by estimation.

a  $5\frac{1}{3} + 2\frac{1}{4}$

b  $1\frac{1}{6} + 2\frac{1}{2}$

c  $3\frac{1}{3} + 1\frac{1}{6}$

d  $6\frac{1}{2} - 2\frac{1}{4}$

e  $4\frac{3}{5} - 1\frac{3}{10}$

f  $7\frac{5}{6} - 3\frac{1}{3}$

- 5 WE33** Find:

a  $1\frac{1}{4} \times \frac{2}{3}$

b  $\frac{3}{4} \times 1\frac{5}{8}$

c  $1\frac{1}{2} \times 1\frac{1}{2}$

d  $2\frac{5}{8} \times 1\frac{3}{4}$

e  $2\frac{1}{5} \times 3\frac{1}{2}$

f  $10\frac{9}{10} \times \frac{2}{3}$

g  $2\frac{1}{4} \times 2\frac{1}{2}$

h  $\frac{4}{5} \times 1\frac{6}{7}$

i  $\frac{9}{10} \times 3\frac{2}{3}$

j  $4\frac{1}{3} \times \frac{3}{5}$

k  $1\frac{1}{2} \times 12$

l  $2\frac{3}{4} \times 6$

m  $5 \times 3\frac{1}{2}$

n  $8 \times 3\frac{1}{4}$

o  $4\frac{3}{5} \times 2\frac{1}{10}$

p  $3\frac{5}{6} \times 9\frac{5}{7}$

q  $5\frac{1}{5} \times 6\frac{2}{3}$

r  $3\frac{3}{4} \times 2\frac{2}{3}$

s  $\frac{7}{8} \times 3\frac{1}{3}$

t  $6\frac{2}{5} \times 4\frac{1}{7}$

- 6 WE34** Find:

a  $1\frac{1}{4} \div \frac{3}{4}$

b  $9 \div 4\frac{1}{2}$

c  $\frac{9}{4} \div 3\frac{1}{2}$

**d**  $\frac{10}{3} \div 3\frac{1}{3}$

**e**  $5 \div 2\frac{3}{4}$

**f**  $7\frac{4}{9} \div \frac{1}{3}$

**g**  $\frac{1}{8} \div 2\frac{1}{2}$

**h**  $1\frac{1}{5} \div 2\frac{1}{3}$

**i**  $5\frac{6}{7} \div 2\frac{3}{4}$

**j**  $1\frac{1}{2} \div \frac{1}{2}$

**k**  $10\frac{9}{10} \div 4$

**l**  $4\frac{1}{2} \div 3\frac{6}{7}$

**m**  $3\frac{2}{7} \div 2\frac{1}{7}$

**n**  $4\frac{2}{3} \div \frac{7}{6}$

**o**  $3\frac{5}{8} \div 1\frac{3}{4}$

### UNDERSTANDING

- 7** One-third of a litre of cordial is mixed with  $1\frac{1}{2}$  litres of water. How many litres of drink have been made?
- 8** The lengths of bolts and nails are often measured in inches. For the bolt shown, find the length of the threaded section if the total length of the bolt is  $3\frac{1}{4}$  inches.
- 
- 9** Julia is planning a holiday to the US. She has 5 months and has worked out the following itinerary. She will be in California for  $1\frac{1}{2}$  months, in Colorado for  $1\frac{2}{3}$  months and in Florida for  $\frac{3}{4}$  of a month. The other state she will be visiting is New York. How much time will she spend in New York?
- 10** James is building a house. Before the frame can be assembled the footings need to be dug. The digging will take  $3\frac{1}{2}$  days. The concrete slab that is then needed will take  $2\frac{3}{4}$  days to pour and set. How long will it be before James can start assembling the frame?
- 11** Tim was looking for something to do one Saturday afternoon and his dad suggested he cook something for afternoon tea.

Tim found the recipe for peanut butter muffins.

#### Ingredients:

$\frac{1}{4}$  cup sugar,  $\frac{1}{4}$  cup margarine,  
 $\frac{1}{2}$  cup peanut butter, 2 eggs,  
 $1\frac{1}{2}$  cups milk,  $2\frac{1}{2}$  cups self-raising flour,  
 $\frac{1}{4}$  teaspoon baking soda



#### Method:

Blend the sugar, margarine and peanut butter. Beat in the eggs and milk. Add the self-raising flour and baking soda. Place the mixture in greased muffin pans and sprinkle with cinnamon sugar. Bake for 15–20 minutes in a 200 °C oven.

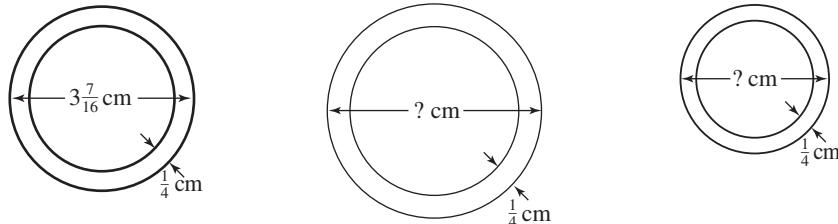
This recipe makes 10 muffins, but Tim wants enough mixture to make 15. He needs to multiply the quantities by  $1\frac{1}{2}$ . Write the quantities of each ingredient Tim needs to make 15 muffins.

- 12** Felicity is dividing  $1\frac{1}{2}$  kilograms of mince steak into 8 hamburger portions. How many kilograms of mince will be in each hamburger portion?
- 13** Ned spends his school holidays helping his father with the shearing.
- a** It takes Ned  $\frac{1}{4}$  hour to shear one sheep. How many sheep will Ned shear in  $5\frac{1}{2}$  hours?
  - b** Ned's father, Wesley, shears a sheep every  $\frac{1}{12}$  of an hour. How many sheep would Wesley shear if he worked continuously for 8 hours?
- 14** Andrew bought  $7\frac{3}{4}$  kg of prawns for a party and  $\frac{3}{5}$  of them were eaten. How many kilograms of prawns were left over?
- 15** Teagan buys  $8\frac{3}{4}$  kg of sugar. She wants to store it in containers that can hold  $1\frac{1}{4}$  kg. How many containers will she need?

- 16** Len works as a train driver and is paid \$21 per hour. If Len works on a public holiday, he is paid double time and a half, which means he earns  $2\frac{1}{2}$  times his normal hourly rate. Calculate what Len earns per hour worked on a public holiday.

**REASONING**

- 17** To make the casing for a simple collapsible telescope you need two cylinders, one of which fits inside the other. One cylinder has measurements as shown below left.



- a** What is the width of a cylinder (see middle diagram above) which would slide over the one shown above left?
- b** What is the width of a cylinder (see diagram above right) which would slide into the one shown above left?
- 18** Michelle can run one lap of a cross-country course in  $\frac{1}{5}$  hour.
- a** How many laps could she complete in  $1\frac{1}{3}$  hours? (Assume she can keep up the same pace.)
- b** How many minutes does it take Michelle to run one lap?
- 19** Five-eighths of the class were girls and  $\frac{2}{3}$  of the girls were brunette. What fraction of the class are brunette girls? If there were 24 people in the class, how many are brunette girls?

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**Digital doc**  
WorkSHEET 4.2  
doc-1722

**REFLECTION**

How could you convince someone that  $\frac{1}{2}$  of  $3\frac{1}{3}$  is  $1\frac{2}{3}$ ?

# Summary

## What are rational numbers?

- *Equivalent fractions* can be found by multiplying or dividing both the numerator and the denominator by the same non-zero number.
- The *numerator* is the top part of the fraction.
- The *denominator* is the bottom part of the fraction.
- Different fractions can be compared or ordered by writing each one with the same denominator. This is found using the lowest common multiple (LCM) of the denominators.
- The symbol ‘>’ means *is greater than*, the symbol ‘<’ means *is less than*.

## Simplifying rational numbers

- To simplify a fraction, divide both the numerator and denominator by the highest common factor or HCF.
- To simplify a mixed number, leave the whole number and simplify the fraction.

## Mixed numbers and improper fractions

- An *improper fraction* has a numerator that is larger than the denominator.
- A *mixed number* is made up of a whole number and a fraction.
- An improper fraction can be changed to a mixed number by dividing the denominator into the numerator and writing the remainder over the same denominator.
- A mixed number can be changed to an improper fraction by first multiplying the whole number with the denominator then adding the numerator. The denominator stays the same.

## Adding and subtracting rational numbers

- Fractions can be added and subtracted if they have the same denominator.
- If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.

## Multiplying rational numbers

- To multiply fractions, multiply the numerators and then multiply the denominators.
- When multiplying fractions the denominators do not need to be the same.
- Fractions can be simplified (or cancelled) before being multiplied.
- A whole number can be written as a fraction with a denominator of 1.
- In a problem involving fractions, ‘of’ can be replaced with ‘multiply’, then the fractions can be multiplied in the usual way.

## Dividing rational numbers

- The *reciprocal* of a fraction is found by tipping the fraction upside down.
- The reciprocal of a mixed number is found by expressing it as an improper fraction, then tipping it upside down.
- A whole number can be written as a fraction by putting it over 1.
- Division by a number is the same as multiplication by its reciprocal.
- To divide fractions:
  - (a) change the division sign to a multiplication sign
  - (b) tip the second fraction (that is, take the reciprocal of the fraction that follows the division sign)
  - (c) perform the multiplication.

**Fractions as ratios**

- Ratios compare quantities of the same kind.
- The ratios themselves do not have a name or unit of measurement.
- The order of the numbers in a ratio is important.
- Before ratios are written, the numbers must be expressed in the *same units of measurement*.
- Ratios contain only whole numbers.
- If each number in a ratio is multiplied, or divided by the same number, the equivalent (or equal) ratio is formed.
- It is customary to write ratios in the simplest form. This is achieved by dividing each number in the ratio by the highest common factor (HCF).
- To form a ratio, using fractions, convert the fractions so that they have a common denominator and then write the ratio of the numerators.
- To share a certain amount in a given ratio, find the total number of shares (parts) first. The size of each share is given by the fraction this share represents out of the total number of shares.

**Working with mixed numbers**

- Fractions can be added and subtracted if they have the same denominator.
- If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.
- Mixed numbers can be added or subtracted by first changing them to improper fractions.
- To multiply or divide mixed numbers, write them as improper fractions first.
- Improper fractions can be multiplied and divided as normal.

**MAPPING YOUR UNDERSTANDING**

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 What fraction of the eggs in this carton are brown?



- 2 Write the sequence of the first three equivalent fractions for the following.

a  $\frac{3}{8}$

c  $\frac{5}{11}$

b  $\frac{4}{9}$

d  $\frac{6}{7}$

- 3 Fill the gaps to make equivalent fractions.

a  $\frac{1}{2} = \frac{3}{\square} = \frac{3}{4}$

c  $\frac{3}{4} = \frac{9}{\square} = \frac{27}{28}$

e  $\frac{1}{4} = \frac{5}{\square} = \frac{5}{100}$

b  $\frac{1}{3} = \frac{3}{\square} = \frac{15}{\square}$

d  $\frac{2}{5} = \frac{\square}{10} = \frac{16}{\square}$

f  $\frac{3}{10} = \frac{3}{\square} = \frac{300}{100}$

- 4 Ricky, Leo and Gabriel were the highest goal scorers for their local football team throughout the season. Ricky scored  $\frac{2}{7}$  of the team's goals, Leo scored  $\frac{1}{3}$  of the goals and Gabriel scored  $\frac{1}{4}$  of the goals.

a Who scored most of the goals?

b Who scored more goals; Ricky or Gabriel?

- 5 Simplify these fractions.

a  $\frac{42}{49}$

c  $\frac{18}{20}$

e  $\frac{36}{72}$

g  $\frac{75}{125}$

b  $\frac{81}{108}$

d  $\frac{21}{28}$

f  $\frac{50}{150}$

h  $\frac{2}{80}$

- 6 Simplify the following mixed numbers.

a  $6\frac{8}{10}$

c  $4\frac{15}{20}$

b  $3\frac{6}{9}$

d  $1\frac{20}{100}$

- 7 What fraction of the months of the year begin with the letter J? Express your answer in simplest form.

- 8 Year 7 students at Camberwell Secondary College raised a total of \$1600 for the Royal Childrens Hospital Good Friday appeal. The table above right shows how much money each class raised.

Class	7.1	7.2	7.3	7.4	7.5	7.6
Amount raised (\$)	\$200	\$400	\$250	\$250	\$280	\$220

Express as a simple fraction how much of the total each class raised.

- 9 Convert these improper fractions to mixed numbers.

a  $\frac{16}{3}$

c  $\frac{12}{7}$

e  $\frac{5}{4}$

g  $\frac{55}{9}$

b  $\frac{21}{5}$

d  $\frac{11}{2}$

f  $\frac{80}{7}$

h  $\frac{72}{10}$

- 10 Express these mixed numbers as improper fractions.

a  $2\frac{3}{4}$

c  $3\frac{5}{7}$

e  $3\frac{2}{3}$

g  $3\frac{11}{12}$

b  $9\frac{7}{8}$

d  $5\frac{5}{6}$

f  $9\frac{9}{10}$

h  $8\frac{1}{2}$

- 11 Marcella was in charge of baking and selling lemon slices at the school fete. Each tray she baked contained 15 lemon slices. If Marcella sold 68 lemon slices, how many empty trays were there?

- 12 Find:

a  $\frac{5}{6} + \frac{1}{8}$

c  $\frac{12}{5} - \frac{4}{3}$

b  $\frac{7}{9} - \frac{2}{5}$

- 13 Find:

a  $\frac{4}{7} \times \frac{5}{9}$

c  $\frac{25}{36} \times \frac{18}{20}$

e  $\frac{4}{5}$  of 55

g  $\frac{7}{9}$  of  $\frac{18}{84}$

b  $\frac{2}{5} \times \frac{3}{5}$

d  $\frac{56}{81} \times \frac{18}{64}$

f  $\frac{2}{11}$  of 121

h  $\frac{1}{12}$  of 200

- 14 Write the reciprocals of:

a  $\frac{5}{2}$

c  $\frac{1}{9}$

b  $\frac{2}{7}$

d 12

**e** 124**f**  $\frac{1}{4}$ **g**  $5\frac{3}{4}$ **h**  $8\frac{1}{2}$ **i**  $7\frac{2}{5}$ **j**  $2\frac{4}{9}$ **k**  $3\frac{5}{8}$ **l**  $4\frac{1}{5}$ **15** Find:

**a**  $\frac{4}{5} \div \frac{8}{15}$

**b**  $\frac{9}{10} \div \frac{27}{40}$

**c**  $\frac{7}{9} \div 4$

**d**  $\frac{3}{7} \div 10$

**16** Calculate:

**a**  $1\frac{7}{8} + 2\frac{5}{6}$

**b**  $10\frac{3}{7} - 8\frac{1}{2}$

**c**  $6\frac{1}{3} - 2\frac{5}{9}$

**d**  $5\frac{1}{2} - 2\frac{5}{6}$

**e**  $3\frac{5}{8} + 2\frac{7}{10}$

**f**  $8\frac{2}{9} - 4\frac{5}{8}$

**g**  $12\frac{4}{7} - 10\frac{9}{10}$

**h**  $12\frac{6}{11} - 11\frac{7}{9}$

**17** Calculate the following and verify each answer using a calculator.

**a**  $1\frac{1}{5} + 3\frac{1}{3} - \frac{13}{15}$

**b**  $4\frac{7}{9} - 1\frac{5}{6} + 3\frac{1}{3}$

**18** Calculate:

**a**  $4\frac{2}{5} \times 3\frac{2}{11}$

**b**  $10\frac{2}{7} \times 6\frac{2}{9}$

**19** Calculate:

**a**  $\frac{4}{9} \times 2\frac{1}{4} \times \frac{3}{5}$

**b**  $3\frac{5}{12} \times 1\frac{3}{5} \times 2\frac{1}{2}$

**c**  $\frac{6}{17} \times 1\frac{1}{4} \times 3\frac{2}{5}$

**20** Calculate:

**a**  $1\frac{1}{2} \div 4\frac{4}{5}$

**b**  $6\frac{2}{3} \div 4\frac{1}{6}$

**c**  $4\frac{1}{5} \div 2\frac{7}{8}$

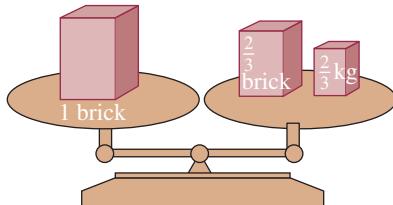
**d**  $10\frac{1}{3} \div 12\frac{5}{7}$

**e**  $6\frac{2}{7} \div 4\frac{1}{5}$

**21** Divide \$25 in the ratio 2 : 3**22** Three people share a lotto prize of \$6600 in the ratio 4 : 5 : 6. What is the difference between the smallest and largest shares?**PROBLEM SOLVING**

- 1** Anthony's monthly take-home pay is \$2800. From this he spends a quarter on his home loan payments, one-half on food and drink and one-seventh on clothing. One-half of the remainder goes into his savings account. How much money does Anthony put into his savings account each month?

- 2** On a balance scale, a brick exactly balances  $\frac{2}{3}$  of a brick and a  $\frac{2}{3}$ -kg weight. How heavy is the brick?



- 3** The temperature rose from  $21\frac{1}{2}^{\circ}\text{C}$  to  $21\frac{3}{4}^{\circ}\text{C}$ . What was the temperature rise?

- 4** You need a piece of trim  $7\frac{5}{8}$  m long to finish an upholstery job. You have a piece that is  $\frac{31}{4}$  m left over from another job. Will the trim be enough to complete the job?

- 5** Complete the following:

**a**  $\frac{5}{12} = \frac{1}{4} + ?$

**b**  $\frac{5}{12} = \frac{2}{3} - ?$

**c**  $\frac{5}{12} = \frac{3}{4} \times ?$

**d**  $\frac{5}{12} = \frac{4}{5} \div ?$

- 6** James, Jessica, Will, Alice and Alex held a lunchtime sausage sizzle and made a profit of \$320. Since some students had worked harder than others, it was agreed the money should be split as follows: Alice gets one quarter and Will gets one fifth. James gets the average of Alice's and Will's shares. If Jessica and Alex split the remainder, determine the amount they each receive.

- 7** Explain how you can determine a fraction whose numerator and denominator have a product of 252, and which is equivalent to  $\frac{4}{7}$ .

- 8** Tomas buys a 24-piece pizza and eats 8 pieces. Drew buys a 16-piece pizza of the same size, and eats 5 pieces. Who ate the most pizza? Explain your reasoning.

- 9** An Egyptian fraction is one that can be written as the sum of a set of different unit fractions; for example,  $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ .

(Note that the numerator of each fraction is 1.)

Write  $\frac{4}{5}$  as the sum of a set of different unit fractions.

- 10** An experienced cabinet-maker can make a large dining table in 3 hours. A novice can do the job in 8 hours. How long will it take them to do the job working together?

- 11** The approximate lifespan of a dollar note is  $1\frac{1}{2}$  years. A coin lasts about  $16\frac{2}{3}$  times as long. How long is a coin in circulation?

- 12** A skydiver opens his parachute at 846 m above ground level. He has already fallen five-sevenths of the distance to the ground. How high was the plane from which he jumped?

- 13** Mr Thompson earned \$2400 a month. Recently, he had a pay increase. He now earns  $1\frac{1}{4}$  times his old salary. What is his new salary?

Mr Thompson's colleague, Mr Goody, earned \$3000 per month and had a pay cut. He now gets  $\frac{1}{6}$  less of his old salary. What will be his new salary?

- 14** Li was playing his vinyl record collection. It took  $3\frac{1}{3}$  minutes to play a record at 45 revolutions per minute (rpm). Li was unsure of which speed to use, and he mistakenly played the record at  $33\frac{1}{3}$  rpm. How long did it take to play the record at this incorrect speed?

- 15** A shopping centre has a car park that has 250 spaces. Of these,  $\frac{3}{10}$  are suitable for small cars only. The remainder of spaces are for standard cars. Last Friday small cars took up  $\frac{14}{15}$  of their allocated spots but overall the car park was  $\frac{7}{10}$  full. Determine the fraction of standard car spaces that were filled.

- 16** Byron needs  $\frac{2}{5}$  kg of catalyst mixed with  $4\frac{1}{2}$  kg of resin to create one surfboard.

a If he has  $2\frac{1}{4}$  kg of catalyst, and  $20\frac{1}{4}$  kg of resin, how many whole surfboards can Byron build?

b Calculate how much more catalyst and resin Byron needs to build one more surfboard. Explain your reasoning.

### eBookplus

#### Interactivities

Test yourself

Chapter 4

int-1816

Word search

Chapter 4

int-2589

Crossword

Chapter 4

int-2590

**Chapter Opener****Digital docs** (page 91)

- Hungry brain activity Chapter 4 (doc-6436)

**Are you ready?****Digital docs** (page 92)

- SkillsHEET 4.1 (doc-6437) Types of fractions
- SkillsHEET 4.2 (doc-6438) Equivalent fractions I
- SkillsHEET 4.3 (doc-6439) Equivalent fractions II
- SkillsHEET 4.4 (doc-6440) Lowest common multiple
- SkillsHEET 4.5 (doc-6441) Ordering fractions
- SkillsHEET 4.6 (doc-6442) Highest common factor
- SkillsHEET 4.7 (doc-6443) Converting an improper fraction to a mixed number
- SkillsHEET 4.8 (doc-6444) Converting a mixed number to an improper fraction
- SkillsHEET 4.9 (doc-6445) Adding and subtracting fractions with the same denominator

**4A What are rational numbers?****Interactivity**

- Fraction walls (int-0002) (page 95)

**eLesson**

- Types of fractions (eles-0002) (page 93)

**Digital docs** (page 98)

- Activity 4-A-1 (doc-1700) Fractions
- Activity 4-A-2 (doc-1701) More fractions
- Activity 4-A-3 (doc-1702) Advanced fractions

**4B Simplifying rational numbers****Digital docs** (page 101)

- Activity 4-B-1 (doc-1703) Equivalent fractions dominoes
- Activity 4-B-2 (doc-1704) More equivalent fractions dominoes
- Activity 4-B-3 (doc-1705) Advanced equivalent fractions dominoes
- Spreadsheet Simplifying a fraction (doc-1729)

**Interactivity**

- Equivalent fractions (int-0001) (page 101)

**4C Mixed numbers and improper fractions****Digital docs** (page 104)

- Activity 4-C-1 (doc-1706) Mixed numbers and improper fractions
- Activity 4-C-2 (doc-1707) More mixed numbers and improper fractions
- Activity 4-C-3 (doc-1708) Tricky mixed numbers and improper fractions
- Spreadsheet Converting improper fractions to mixed numbers (doc-1725)

**4D Adding and subtracting rational numbers****Digital docs**

- Activity 4-D-1 (doc-1709) How much honey? (page 106)
- Activity 4-D-2 (doc-1710) How fast it it? (page 107)

- Activity 4-D-3 (doc-1711) How much milk? (page 107)
- Spreadsheet Adding and subtracting fractions (doc-1726) (page 107)
- WorkSHEET 4.1 (doc-1721) (page 107)

**Interactivity**

- Adding and subtracting fractions (int-2357) (page 105)

**4E Multiplying rational numbers****Digital docs**

- Activity 4-E-1 (doc-1712) Fraction multiplication (page 110)
- Activity 4-E-2 (doc-1713) More fraction multiplication (page 110)
- Activity 4-E-3 (doc-1714) Advanced fraction multiplication (page 111)
- Spreadsheet Multiplying fractions (doc-1727) (page 111)

**Interactivity**

- Repeated addition (int-2738) (page 108)

**4F Dividing rational numbers****eLesson**

- Dividing fractions (eles-0003) (page 112)

**Digital docs** (page 114)

- Activity 4-F-1 (doc-1715) Just flip it
- Activity 4-F-2 (doc-1716) More just flip it
- Activity 4-F-3 (doc-1717) Advanced just flip it
- Spreadsheet Dividing fractions (doc-1728)

**Interactivity**

- Dividing fractions (int-2358) (page 113)

**4G Fractions as ratios****Digital docs** (page 119)

- Activity 4-G-1 (doc-6433) Ratios
- Activity 4-G-2 (doc-6434) More ratios
- Activity 4-G-3 (doc-6435) Advanced ratios

**4H Working with mixed numbers****Digital docs**

- Activity 4-H-1 (doc-6518) Working with mixed numbers (page 124)
- Activity 4-H-2 (doc-6519) More working with mixed numbers (page 124)
- Activity 4-H-3 (doc-6520) Advanced working with mixed numbers (page 124)
- WorkSHEET 4.2 (doc-1722) (page 126)

**Chapter review****Interactivities** (page 131)

- Test yourself Chapter 4: (int-1816) Take the end-of-chapter test to test your progress.
- Word search Chapter 4 (int-2589)
- Crossword Chapter 4 (int-2590)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

## 5

# Geometry



- 5A Measuring angles
- 5B Constructing angles with a protractor
- 5C Types of angles and naming angles
- 5D Triangles
- 5E Quadrilaterals and their properties
- 5F Parallel and perpendicular lines

**WHAT DO YOU KNOW?**

- 1 List what you know about angles, triangles and quadrilaterals. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of angles, triangles and quadrilaterals.

**eBook plus****Digital doc**

Hungry brain activity

Chapter 5

doc-6446

**OPENING QUESTION**

What types of angles, triangles and quadrilaterals can be found in this framework?

# Are you ready?

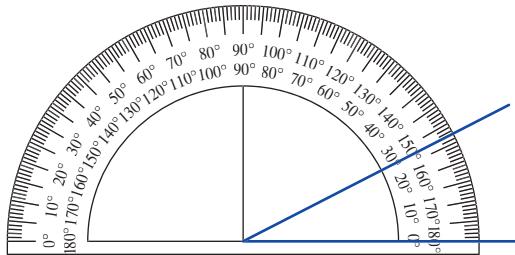
Try the questions below. If you have difficulty with any of them, you can get extra help by completing the matching SkillsHEET located on your eBookPLUS.



**Digital docs**  
SkillsHEET 5.1  
doc-6447

## Reading the scale on a protractor

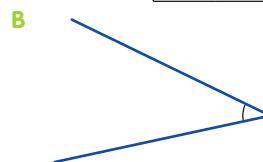
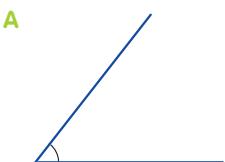
- 1 What is the size of the angle shown in the diagram at right?



**Digital docs**  
SkillsHEET 5.2  
doc-6448

## Comparing the sizes of angles

- 2 Consider the three angles shown.



- a Which diagram shows the largest angle?      b Which diagram shows the smallest angle?

## Estimating the size of an angle

- 3 Choose one of the following alternatives to complete the sentences below about the angles in question 2.

- i 10° and 20°      ii 30° and 40°      iii 45° and 55°  
iv 70° and 80°      v 90° and 100°      vi 115° and 125°

- a The size of the angle in diagram A can be estimated to be between \_\_\_\_\_.  
b The size of the angle in diagram B can be estimated to be between \_\_\_\_\_.  
c The size of the angle in diagram C can be estimated to be between \_\_\_\_\_.



**Digital docs**  
SkillsHEET 5.3  
doc-6449

## Measuring angles with a protractor

- 4 Measure the size of each angle in question 2 with a protractor.



**Digital docs**  
SkillsHEET 5.4  
doc-6450

## Constructing angles with a protractor

- 5 Construct an angle of 50° using a protractor.



**Digital docs**  
SkillsHEET 5.5  
doc-6451

## Classifying angles

- 6 Angles can be classified according to their size. Match the name of the angle to the given sizes.

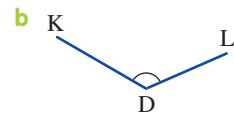
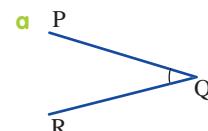
- |                        |          |                         |              |
|------------------------|----------|-------------------------|--------------|
| a between 0° and 90°   | A reflex | d 180°                  | D revolution |
| b 90°                  | B acute  | e between 180° and 360° | E straight   |
| c between 90° and 180° | C right  | f 360°                  | F obtuse     |



**Digital docs**  
SkillsHEET 5.6  
doc-6452

## Naming angles

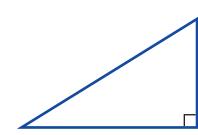
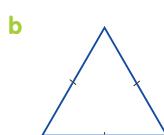
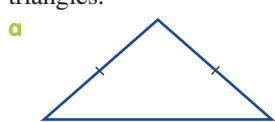
- 7 Use three letters to name each of the following angles.



**Digital docs**  
SkillsHEET 5.7  
doc-6453

## Classifying triangles according to the lengths of their sides

- 8 A triangle can be classified as equilateral, isosceles or scalene, classify each of the following triangles.



**Digital docs**  
SkillsHEET 5.8  
doc-6454

## Classifying triangles according to the size of their angles

- 9 A triangle can also be classified according to the size of its angles. Match one of the following descriptions to each of the triangles in question 8.

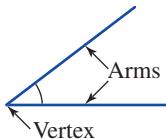
- i acute-angled

- ii right-angled

- iii obtuse-angled

## 5A Measuring angles

- An **angle** is made when two lines meet at a point.
- The point where the two lines meet is called a **vertex** and the lines are called **arms** of the angle.

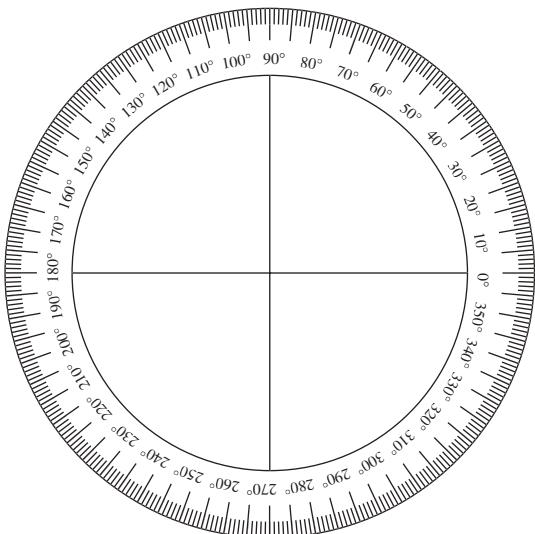


- Angles are measured in degrees.
- One full revolution is equal to 360 degrees.
- The symbol for degrees is  $^\circ$ .
- The device for measuring angles is called a **protractor**. There are two types of protractors: circular and semicircular.



### The circular protractor

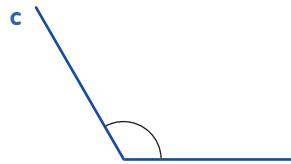
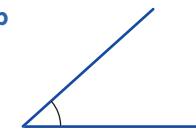
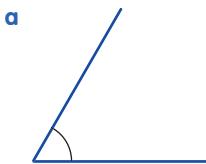
- Circular protractors are marked from  $0^\circ$  to  $360^\circ$ .



- To measure angles using a circular protractor, follow these steps.
- Step 1:** Place the centre of the protractor on the vertex of the angle.  
**Step 2:** Match the  $0^\circ$  horizontal centre line with one of the arms of the angle.  
**Step 3:** Read the size of the angle indicated by the other arm.

## WORKED EXAMPLE 1

First estimate and then measure the size of each of the following angles using a circular protractor. Comment on the accuracy of the estimate.



## THINK

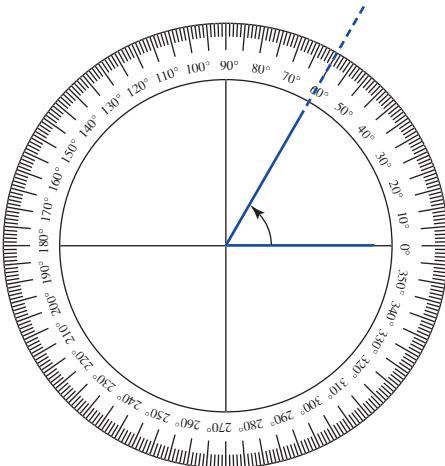
- a** 1 Estimate the size of the angle. Write the estimate.  
*Note:* The angle is between  $0^\circ$  and  $90^\circ$  but closer to  $90^\circ$ .
- 2 Place the centre of the protractor on the vertex of the angle.
  - 3 Match the horizontal line that passes through the centre of the protractor and points to  $0^\circ$  with one of the arms of the angle.
  - 4 Read the size of the angle indicated by the other arm. (You may need to extend the line so that you can read it on the protractor.)

- 5 Comment on the accuracy of the estimate.

- b** 1 Estimate the size of the angle. Write the estimate.  
*Note:* The angle appears to be approximately halfway between  $0^\circ$  and  $90^\circ$ .
- 2 Repeat steps 2–5 from part **a**.

## WRITE/MEASURE

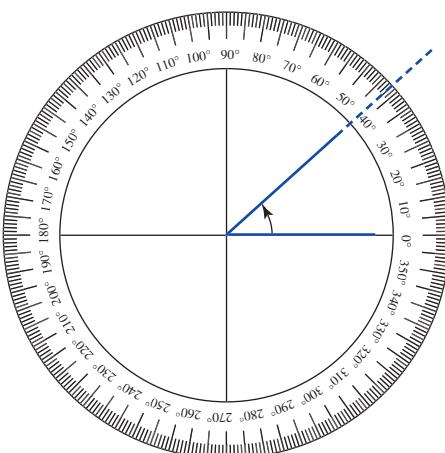
- a** The estimate is approximately  $60^\circ$ .



The size of this angle is  $60^\circ$ .

The estimate was an accurate one.

- b** The estimate is approximately  $45^\circ$ .



The size of this angle is  $42^\circ$ .

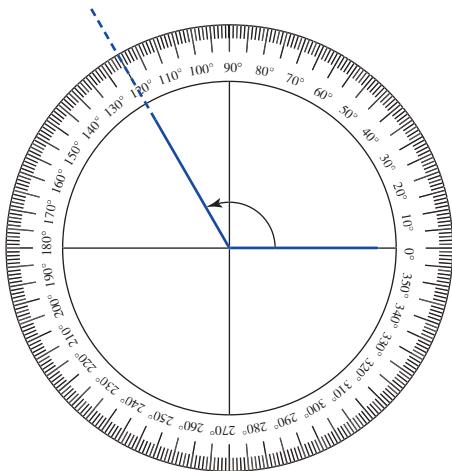
The estimate was a reasonable one.

- c 1 Estimate the size of the angle. Write the estimate.

Note: The angle is between  $90^\circ$  and  $180^\circ$  but closer to  $90^\circ$ .

- 2 Repeat steps 2–5 from part a.

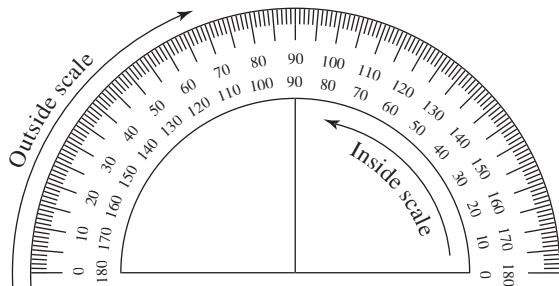
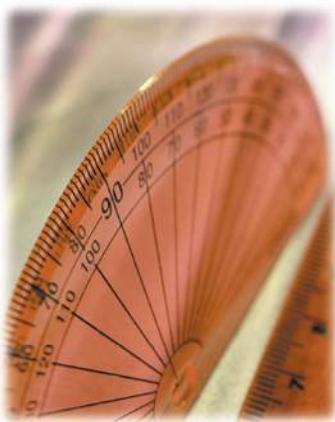
c The estimate is approximately  $115^\circ$ .



The size of this angle is  $120^\circ$ .  
The estimate was a reasonable one.

## The semicircular protractor

- Semicircular protractors have two scales: inner and outer.
- Each scale is marked from  $0^\circ$  to  $180^\circ$ .
- Either of the two scales can be used for measuring angles.



- To measure angles that are less than  $180^\circ$ , follow these steps.

**Step 1:** Line up the centre of the protractor's base line with the vertex (that is, where the vertical  $90^\circ$  line intersects with horizontal  $180^\circ$  base line).

**Step 2:** Match the base of the protractor with one of the arms of the angle.

**Step 3:** Use the scale on the base starting at 0 to read the number of degrees spanned by the angle.

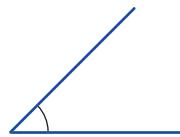
- To measure angles which are greater than  $180^\circ$ , follow these steps.

**Step 1:** Measure the angle enclosed by the straight lines (that is, the one that is smaller than  $180^\circ$ ).

**Step 2:** Subtract the number obtained in step (1) from  $360^\circ$ .

**WORKED EXAMPLE 2**

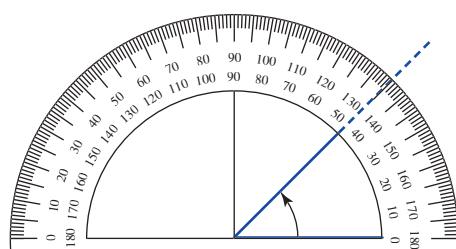
First estimate and then measure the size of the angle shown at right using a semicircular protractor. Comment on the accuracy of the estimate.

**THINK**

- 1 Estimate the size of the angle. Write the estimate.  
*Note:* The angle appears to be approximately halfway between  $0^\circ$  and  $90^\circ$ .
- 2 Line up the centre of the protractor's baseline with the vertex (that is, where the vertical,  $90^\circ$  line, intersects with the horizontal  $180^\circ$  baseline).
- 3 Match the base of the protractor with one of the arms of the angle.  
(You may need to extend the line so that you can read it on the protractor.)
- 4 Use the scale on the base starting at  $0^\circ$  (in this case an inner scale) to read the number of degrees spanned by the angle.
- 5 Comment on the accuracy of the estimate.

**WRITE/MEASURE**

The estimate is approximately  $45^\circ$ .

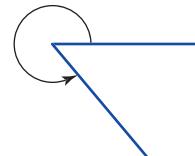


The size of this angle is  $45^\circ$ .

The estimate was an accurate one.

**WORKED EXAMPLE 3**

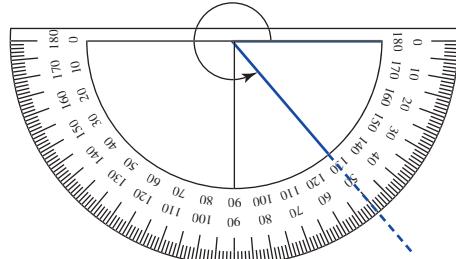
First estimate and then measure the size of the angle shown at right using a semicircular protractor. Comment on the accuracy of the estimate.

**THINK**

- 1 Estimate the size of the angle. Write the estimate.  
*Note:* The angle appears to be approximately halfway between  $270^\circ$  and  $360^\circ$ .
- 2 Measure the angle enclosed by the straight lines.
- 3 Subtract  $50^\circ$  from  $360^\circ$  to find the required angle.
- 4 Comment on the accuracy of the estimate.

**WRITE/MEASURE**

The estimate is approximately  $315^\circ$ .



The smaller angle is  $50^\circ$ .

Required angle:  $360^\circ - 50^\circ = 310^\circ$

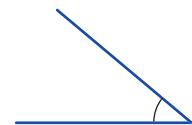
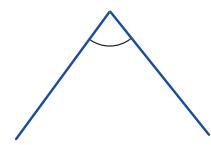
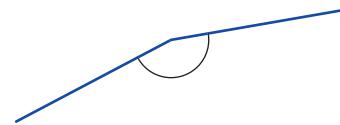
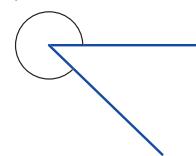
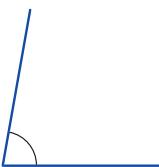
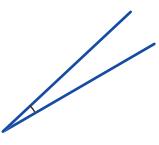
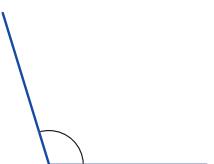
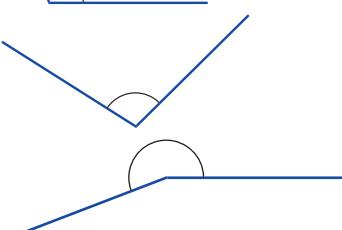
The estimate was a reasonable one.

**REMEMBER**

- When two lines meet at a point, an *angle* is formed.
- The point where the lines meet is called a *vertex* and the lines are called the *arms* of the angle.
- All angles are measured in degrees using a protractor.
- Circular protractors are marked from  $0^\circ$  to  $360^\circ$ . If the centre of the protractor is placed on the vertex and one of the arms points at  $0^\circ$ , the other arm will point to the angle being measured.
- Semicircular protractors have two scales (inner and outer) which are marked from  $0^\circ$  to  $180^\circ$ . Either of these two scales can be used for measuring angles. Angles that are less than  $180^\circ$  can be measured in the same way as with circular protractors. To measure an angle that is larger than  $180^\circ$ , the angle enclosed by the arms (that is, the one that is less than  $180^\circ$ ) is measured first and is then subtracted from  $360^\circ$ .
- It is a good idea to estimate the size of an angle before measuring it.

**EXERCISE****5A****Measuring angles****INDIVIDUAL PATHWAYS****eBook plus****Activity 5-A-1**Measuring angles  
doc-1737**Activity 5-A-2**More measuring  
angles  
doc-1738**Activity 5-A-3**Advanced  
measuring angles  
doc-1739**FLUENCY**

- 1 WE1, 2, 3** First estimate and then measure the size of each of the following angles using a protractor. Comment on the accuracy of the estimate.

**a****c****e****g****i****k****b****f****h****j****l****eBook plus****Interactivity**  
Digital  
protractor  
int-2348

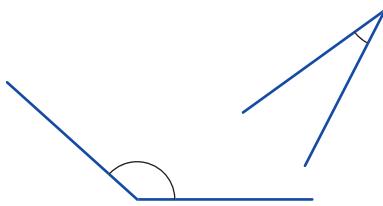
2 Comment on the accuracy of your measurement compared with the answer in the text.

3 MC a The size of the angle shown is closest to:

- A  $20^\circ$       B  $30^\circ$       C  $40^\circ$   
 D  $150^\circ$       E  $140^\circ$

b The size of the angle shown is closest to:

- A  $42^\circ$       B  $138^\circ$       C  $142^\circ$   
 D  $145^\circ$       E  $38^\circ$



### UNDERSTANDING

4 In each of the photographs, measure the indicated angle.

a



b



c



d



e



f



5 The recommended slope for wheelchair access is  $4^\circ$  from the horizontal. An angle greater than this makes it more difficult for the person in the wheelchair to travel up the ramp.



- a Would you rate the slope of the bottom section of the ramp as ‘difficult’, ‘as recommended’ or ‘easy’? Why?  
 b How would you rate the second part of the ramp?

**REASONING**

- 6 Study the cartoon of the golfer to help you answer the questions below.

1



2



3



4



5



6



7



8



- a Copy the following table.

Diagram	Estimate of angle	Measured angle	Difference
1			
2			
3			
4			

- b Without using a protractor, estimate the size of each angle in the first four diagrams of the golf sequence above and complete the second column of the table.  
 c Explain how you obtained an estimate. What steps did you follow?  
 d Complete the third column by measuring each angle with a protractor.  
 e Calculate the difference between your estimate and the actual value of each angle.  
 f Describe how you could improve your skills in estimating the size of an angle.  
 g Try these improvements for estimating the size of the angles in the remaining four diagrams. Copy and complete the following table.

Diagram	Estimate of angle	Measured angle	Difference
5			
6			
7			
8			

- h Has the difference between the estimated and actual value for each angle become smaller? Have your estimating skills improved?

**REFLECTION**

Where in everyday life may I need to use my skills of measuring angles?

## 5B Constructing angles with a protractor

- Protractors can be used not only for measuring, but also for constructing angles.
- To construct an angle using a circular protractor, follow the steps below.
  - Step 1:** Draw a straight line.
  - Step 2:** Put a dot at one end of the line. (The line now represents one of the arms and the dot represents the vertex of the angle.)
  - Step 3:** Place the protractor so that its centre is on the vertex and  $0^\circ$  is on the baseline.
  - Step 4:** Find the required angle on the scale and mark a small dot at the edge of the protractor.
  - Step 5:** Join the small dot with the vertex to form the second arm of the angle.
  - Step 6:** Label the angle.
- If the angle to be constructed is between  $0^\circ$  and  $180^\circ$ , a semicircular protractor can be used in exactly the same way as the circular one.

### WORKED EXAMPLE 4

Construct each of the following angles.

a  $50^\circ$

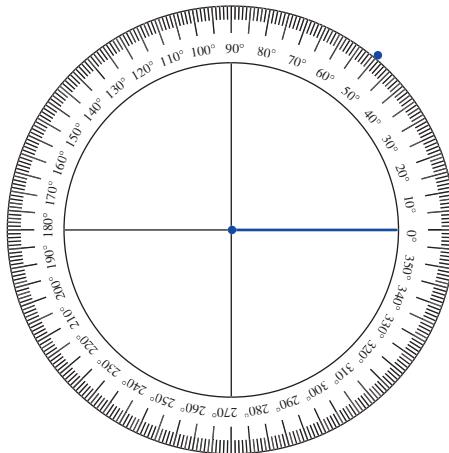
b  $152.5^\circ$

#### THINK

- a 1 Draw a baseline. Put a dot at one end. This is to be the vertex of the angle.
- 2 Place the protractor so that its centre is on the vertex and  $0^\circ$  is on the baseline.
- 3 Find  $50^\circ$  on the scale and mark a small dot at the edge of the protractor.

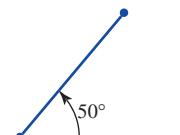
#### WRITE/DRAW

a 



- 4 Join the small dot and the vertex with a straight line.

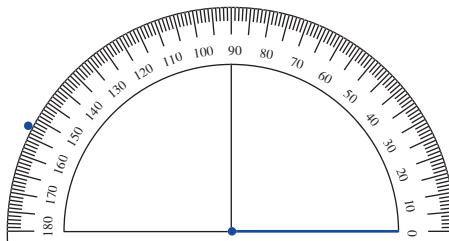
- 5 Label the angle.



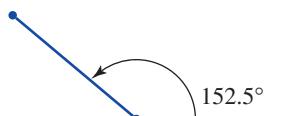
- b 1 Repeat steps 1 and 2 from part a.

- 2 Find  $152.5^\circ$  on the scale and mark a small dot at the edge of the protractor. Remember to start at  $0^\circ$  and, using the inside scale, move in an anticlockwise direction.

b 



- 3 Join the vertex and the dot where you have marked the  $152.5^\circ$  with a straight line.
- 4 Label the angle.



■ To construct an angle greater than  $180^\circ$  using a semicircular protractor, follow the steps below.

**Step 1:** Subtract the given angle from  $360^\circ$ .

**Step 2:** Construct the resulting angle (which will be less than  $180^\circ$ ).

**Step 3:** Label the required angle (it is ‘outside’ the one that has been constructed).

### WORKED EXAMPLE 5

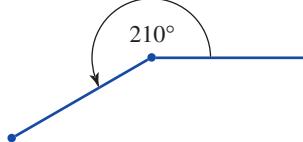
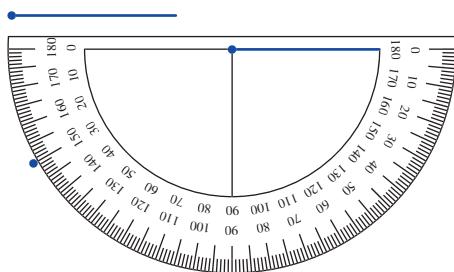
**Construct an angle of  $210^\circ$  using a semicircular protractor.**

#### THINK

- 1 Subtract  $210^\circ$  from  $360^\circ$ .
- 2 Draw a baseline. Put a dot at one end. This will become the vertex of the angle.
- 3 Place the protractor upside down on the baseline so that its centre is on the vertex.
- 4 Start at  $0^\circ$  and, using the outside scale, move in a clockwise direction; mark the position of  $150^\circ$  with a small dot.
- 5 Remove the protractor and join the vertex to the small dot representing  $150^\circ$ .
- 6 Mark the ‘outside’ angle as  $210^\circ$ .

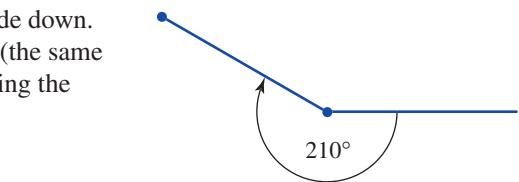
#### WRITE/DRAW

$$360^\circ - 210^\circ = 150^\circ$$



■ It is not necessary to place the protractor upside down.

The diagram at right shows the angle of  $210^\circ$  (the same as in Worked example 5), constructed by placing the protractor in the upright position.



#### REMEMBER

1. To construct an angle, follow the steps given below.
  - (a) Rule a straight line.
  - (b) Put a dot at one end of the line. (The line represents one of the arms and the dot represents the vertex of the angle.)
  - (c) Place the protractor so that its centre is on the vertex and  $0^\circ$  is on the baseline.
  - (d) Find the required angle on the scale and mark a small dot at the edge of the protractor.
  - (e) Join the small dot with the vertex to form the second arm of the angle.
  - (f) Label the angle.

2. To construct an angle that is larger than  $180^\circ$ , using a semicircular protractor:
- subtract the given angle from  $360^\circ$ .
  - construct the resulting angle (it will be less than  $180^\circ$ ).
  - label the required angle: it is ‘outside’ the one that has been constructed.

**EXERCISE****5B****Constructing angles with a protractor****INDIVIDUAL PATHWAYS****eBookplus****Activity 5-B-1**Constructing angles  
doc-1740**Activity 5-B-2**More constructing  
angles  
doc-1741**Activity 5-B-3**Advanced  
constructing angles  
doc-1742**eBookplus****Interactivity**  
**Bisectors**  
int-2347**FLUENCY**

- 1 **WE4a** Construct each of the following angles.

<b>a</b> $15^\circ$	<b>b</b> $9^\circ$	<b>c</b> $53^\circ$	<b>d</b> $75.5^\circ$
<b>e</b> $45^\circ$	<b>f</b> $40.8^\circ$	<b>g</b> $88^\circ$	<b>h</b> $76^\circ$

- 2 **WE4b** Construct each of the following angles.

<b>a</b> $96^\circ$	<b>b</b> $92^\circ$	<b>c</b> $165.2^\circ$	<b>d</b> $143^\circ$
<b>e</b> $140^\circ$	<b>f</b> $156.4^\circ$	<b>g</b> $127^\circ$	<b>h</b> $149^\circ$

- 3 **WES** Construct each of the following angles using a semicircular protractor.

<b>a</b> $185^\circ$	<b>b</b> $210.5^\circ$	<b>c</b> $235^\circ$	<b>d</b> $260^\circ$
<b>e</b> $243^\circ$	<b>f</b> $192^\circ$	<b>g</b> $249.8^\circ$	<b>h</b> $214^\circ$

- 4 **MC** To construct an angle of  $212^\circ$  using a semicircular protractor, we need to construct an angle less than  $180^\circ$  first and then label the ‘outside’ angle. The size of the first angle to be constructed is:

**A**  $212^\circ$       **B**  $58^\circ$       **C**  $148^\circ$       **D**  $112^\circ$       **E** None of these

- 5 Construct each of the following angles.

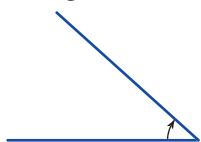
<b>a</b> $295^\circ$	<b>b</b> $269^\circ$
<b>c</b> $307^\circ$	<b>d</b> $349^\circ$
<b>e</b> $328^\circ$	<b>f</b> $300^\circ$
<b>g</b> $345^\circ$	<b>h</b> $358^\circ$

**REFLECTION**

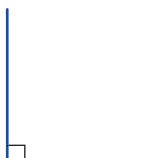
In what situations would a semicircular protractor be easier to use than a circular one?

**5C****Types of angles and naming angles****Types of angles**

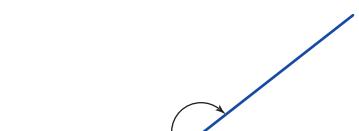
- Angles can be classified according to their size.



An **acute angle** is greater than  $0^\circ$ , but less than  $90^\circ$ .



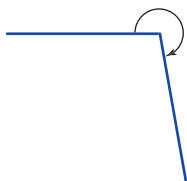
A **right angle** is an angle that equals exactly  $90^\circ$ .



An **obtuse angle** is greater than  $90^\circ$  but less than  $180^\circ$ .



A **straight angle** equals exactly  $180^\circ$ .



A **reflex angle** is greater than  $180^\circ$  but less than  $360^\circ$ .



A **revolution** or a **perigon** is an angle of  $360^\circ$  (a full circle).

- Angles that sum to  $90^\circ$  are called **complementary angles**.  
■ Angles that sum to  $180^\circ$  are called **supplementary angles**.

**WORKED EXAMPLE 6**

Classify each of the following angles according to their size.

- a  $115^\circ$       b  $27^\circ$       c  $300^\circ$

**THINK**

- a The given angle is larger than  $90^\circ$ , but smaller than  $180^\circ$ , so classify it accordingly.
- b The given angle is between  $0^\circ$  and  $90^\circ$ , so classify it accordingly.
- c The given angle is larger than  $180^\circ$ , but less than  $360^\circ$ , so classify it accordingly.

**WRITE**

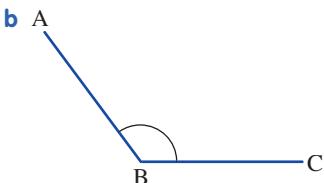
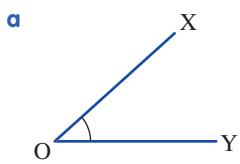
- a  $115^\circ$  is an obtuse angle.
- b  $27^\circ$  is an acute angle.
- c  $300^\circ$  is a reflex angle.

**Naming angles**

- Angles can be named using the capital letters of the English alphabet.
- A common way to name an angle is to use three letters: two letters to represent the arms of the angle, and a third letter to represent its vertex. The letter representing the vertex is always placed in the middle (between the two letters representing the arms).
- Instead of writing the word ‘angle’, we use the symbol  $\angle$ .

**WORKED EXAMPLE 7**

Name each of the following angles.

**THINK**

- a Name the angle by starting either from X, or from Y, and placing O (the letter for the vertex) in the middle. Remember to place the angle symbol ( $\angle$ ) before the letters.
- b Name the angle by starting either from A, or from C, and placing B (the letter representing the vertex) in the middle. Remember to place the angle symbol ( $\angle$ ) before the letters.

**WRITE**

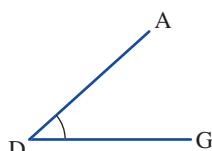
- a  $\angle X O Y$  or  $\angle Y O X$
- b  $\angle A B C$  or  $\angle C B A$

**WORKED EXAMPLE 8**

Draw the acute angle  $\angle ADG$ .

**THINK**

- 1 Construct any acute angle (unless specified otherwise).
- 2 Letter D is in the middle, so place it at the vertex.
- 3 Place letter A at one arm of the angle and letter G at the other. (Note that it does not matter which arm is represented by AD and GD.)

**DRAW**

## REMEMBER

1. Angles can be classified according to their size.

Angle size	Diagram	Name of angle
Between $0^\circ$ and $90^\circ$		Acute
$90^\circ$		Right
Between $90^\circ$ and $180^\circ$		Obtuse

Angle size	Diagram	Name of angle
$180^\circ$		Straight
Between $180^\circ$ and $360^\circ$		Reflex
$360^\circ$		Revolution

2. An angle can be named using three capital letters from the English alphabet: two for the arms and one for the vertex of the angle.  
 3. The letter representing the vertex of the angle is always placed in the middle. Use the symbol  $\angle$  instead of the word ‘angle’ before the three letters.

## EXERCISE

## 5C

## Types of angles and naming angles

## INDIVIDUAL PATHWAYS

## eBookplus

**Activity 5-C-1**  
Types of angles and naming angles  
doc-1743

**Activity 5-C-2**  
More types of angles and naming angles  
doc-1744

**Activity 5-C-3**  
Advanced types of angles and naming angles  
doc-1745

## FLUENCY

- 1 **WE6** Classify each of the following angles according to its size.

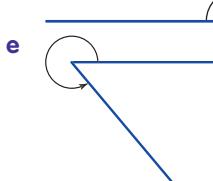
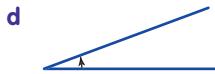
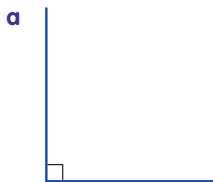
- |               |                 |                         |               |
|---------------|-----------------|-------------------------|---------------|
| a $12^\circ$  | b $215.3^\circ$ | c $98^\circ$            | d $156^\circ$ |
| e $180^\circ$ | f $62^\circ$    | g $355.2^\circ$         | h $90^\circ$  |
| i $4.8^\circ$ | j $360^\circ$   | k $100^\circ$           | l $45^\circ$  |
| m $82^\circ$  | n $270^\circ$   | o $36\frac{1}{2}^\circ$ | p $195^\circ$ |

- 2 The following list gives values of particular angles.

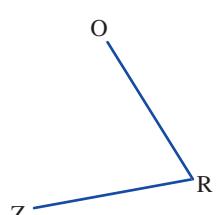
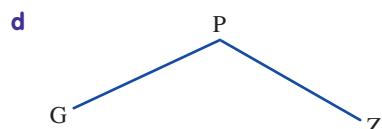
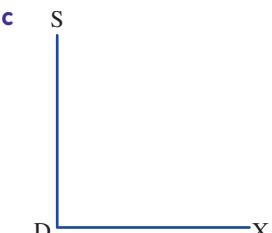
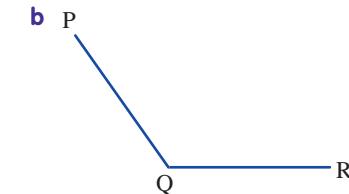
$3^\circ, 45^\circ, 65^\circ, 123^\circ, 69^\circ, 234^\circ, 90^\circ, 360^\circ, 300^\circ, 270^\circ, 165^\circ, 210^\circ, 180^\circ$

- a How many of these are acute angles? List them.
- b How many of these are obtuse angles? List them.
- c Is there a right angle in the list?
- d Is there a straight angle?
- e How many reflex angles are there? List them.
- f Is there a full revolution? What is its value?
- g What is the complement of  $65^\circ$ ?
- h What is the supplement of  $123^\circ$ ?

- 3 Consider the diagrams below and write down the type of angle shown in each case.



**4** **WE7** Name each of the following angles.



**5** **WE8** Draw each of the following angles, to the type as specified in brackets.

a  $\angle CDE$  (acute)

b  $\angle TRE$  (obtuse)

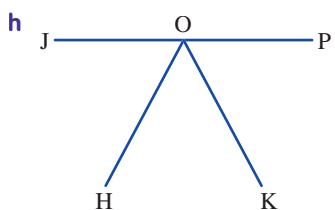
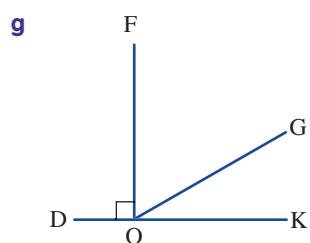
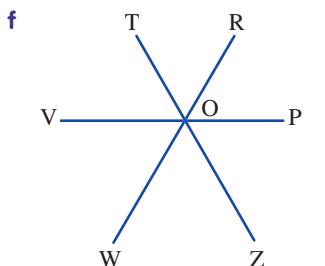
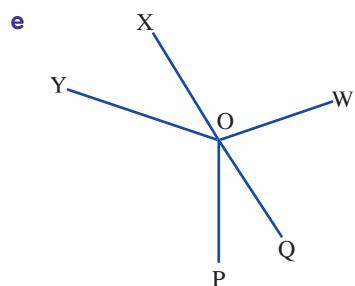
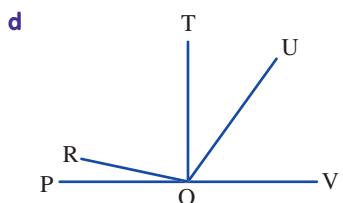
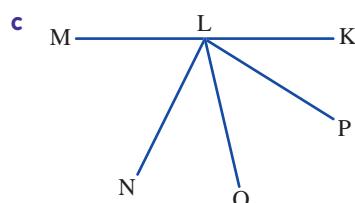
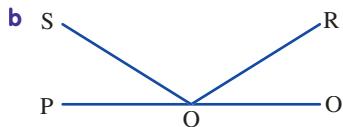
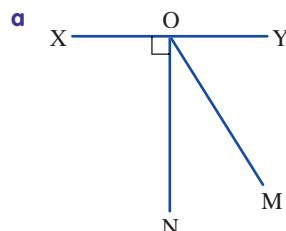
c  $\angle QAS$  (straight)

d  $\angle FGH$  (reflex)

e  $\angle KJF$  (right)

f  $\angle NBV$  (acute)

**6** Give the letter name of each acute and obtuse angle in the following diagrams.



## UNDERSTANDING

- 7 State the type of angle shown in each photograph.

a



b



c



d



- 8 State the type of angle that the minute hand of a clock sweeps while moving for the period of:

a 10 minutes  
c 20 minutes  
e 45 minutes

b 15 minutes  
d 30 minutes  
f 1 hour.

- 9 At various times of the day the hands of a clock form two angles: the ‘inside’ angle (the smaller one) and the ‘outside’ angle (the larger one).

- a State the type of the inside angle, formed by the hands of the clock at:

- |                |                     |
|----------------|---------------------|
| i 1 o'clock    | ii 20 minutes to 12 |
| iii 6 o'clock  | iv 9 o'clock        |
| v quarter to 4 | vi half past 10.    |

- b Write two different times when the inside angle of the clock is:

- i acute      ii obtuse  
iii right.

- 10 Angles are very important in engineering and architecture. Three angles have been marked in the photograph shown.

- a Classify each angle.  
b Use a protractor to measure each angle.  
c With a partner, locate as many other angles as you can that are the same size as the marked angles 1, 2 and 3.



- 11 In each photograph below, acute, right-angled and obtuse angles have been used.

- Name an acute angle.
- Name two right angles.
- Name an obtuse angle.
- Can you name a reflex angle?

a



b



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Digital doc  
WorkSHEET 5.1  
doc-1755

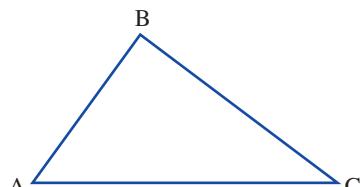
**REFLECTION**

Why is it important to always put the letter representing the vertex in the middle of the angle's name?

## 5D Triangles

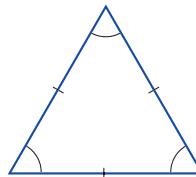
### Types of triangles

- Every triangle has three angles and three sides.
- Capital letters of the English alphabet are used at each vertex (in any order) to name triangles.
- The symbol  $\Delta$  is used in place of the word ‘triangle’. This triangle can be named  $\Delta ABC$ . (It can also be named  $\DeltaBCA$ ,  $\Delta CAB$ ,  $\Delta ACB$ ,  $\Delta BAC$  or  $\Delta CBA$ .)
- Identical marks on the sides of the triangle are used to indicate that the sides have the same length. Likewise, identical marks on the angles indicate that the angles are equal in size.

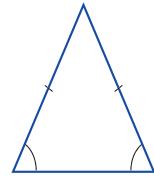


## Classifying triangles according to the length of their sides

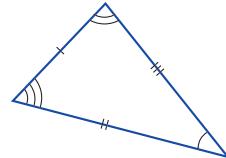
- According to their side lengths, triangles can be classified as either **equilateral**, **isosceles**, or **scalene**.
- An *equilateral triangle* has all sides equal in length. (All angles of an equilateral triangle are also equal in size.)



- An *isosceles triangle* has two sides of equal length. (The angles adjacent to the unequal third side are also equal in size.)

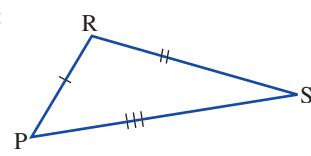
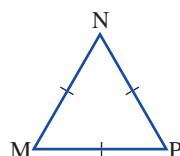
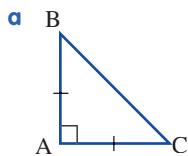


- A *scalene triangle* has no equal sides. (All angles in a scalene triangle are different in size.)



### WORKED EXAMPLE 9

Classify each of these triangles according to the lengths of its sides.



#### THINK

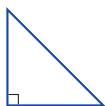
- Sides AB and AC have identical markings on them, which indicate that they are of equal length. So  $\triangle ABC$  has 2 equal sides. Classify it accordingly.
- The 3 sides of  $\triangle MNP$  have identical markings on them, which means that all 3 sides are equal in length. Classify this triangle.
- All 3 sides of  $\triangle PRS$  are marked differently. Therefore, no sides in this triangle are equal in length. Use this information to classify the triangle.

#### WRITE

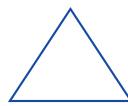
- $\triangle ABC$  is an isosceles triangle.
- $\triangle MNP$  is an equilateral triangle.
- $\triangle PRS$  is a scalene triangle.

## Classifying triangles according to the size of their angles

- According to the size of their angles, triangles can be classified as being *right-angled*, *acute-angled*, or *obtuse-angled*.



A **right-angled triangle** has one of its angles equal to  $90^\circ$  (one of its angles is a right angle). A small square in the corner marks the right angle.



An **acute-angled triangle** has all angles smaller than  $90^\circ$  (all three angles are acute).



An **obtuse-angled triangle** has one angle greater than  $90^\circ$  (one angle is obtuse).

**WORKED EXAMPLE 10**

Classify each of the triangles in Worked example 9 according to the size of its angles.

**THINK**

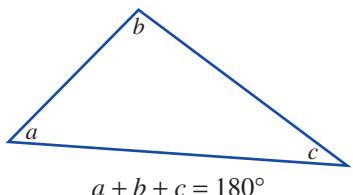
- In  $\triangle ABC$ ,  $\angle CAB$  is marked as the right angle, so classify it accordingly.
- In  $\triangle MNP$  all angles are less than  $90^\circ$ , so classify this triangle.
- In  $\triangle PRS$ ,  $\angle PRS$  is greater than  $90^\circ$ ; that is, it is obtuse. Use this information to classify the triangle.

**WRITE**

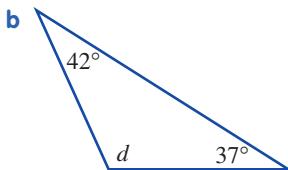
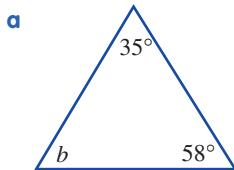
- $\triangle ABC$  is a right-angled triangle.
- $\triangle MNP$  is an acute-angled triangle.
- $\triangle PRS$  is an obtuse-angled triangle.

**Angles in a triangle**

- The sum of the three angles in any triangle is always  $180^\circ$ .

**WORKED EXAMPLE 11**

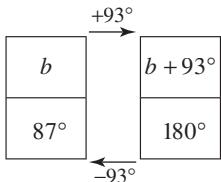
Find the value of the pronumeral in each of the following triangles.

**THINK**

- 1 The sum of the three angles ( $b$ ,  $35^\circ$  and  $58^\circ$ ) must be  $180^\circ$ . Write this as an equation.
- 2 Simplify by adding  $35^\circ$  and  $58^\circ$  together.
- 3 Use inspection or backtracking to solve for  $b$ .

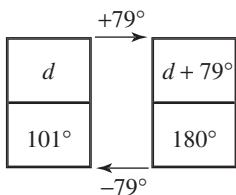
**WRITE**

$$\begin{aligned} \text{a } b + 35^\circ + 58^\circ &= 180^\circ \\ b + 93^\circ &= 180^\circ \\ b &= 180^\circ - 93^\circ \\ b &= 87^\circ \end{aligned}$$



- 1 The sum of the three angles ( $d$ ,  $37^\circ$  and  $42^\circ$ ) must be  $180^\circ$ . Write this as an equation.
- 2 Simplify by adding  $37^\circ$  and  $42^\circ$  together.
- 3 Use inspection or backtracking to solve for  $d$ .

$$\begin{aligned} \text{b } d + 37^\circ + 42^\circ &= 180^\circ \\ d + 79^\circ &= 180^\circ \\ d &= 180^\circ - 79^\circ \\ d &= 101^\circ \end{aligned}$$

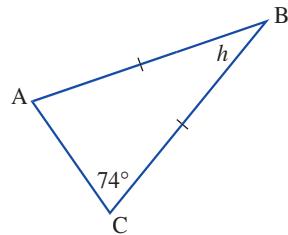


## WORKED EXAMPLE 12

Find the value of the pronumeral in the following triangle.

## THINK

- 1 The markings on the diagram indicate that  $\triangle ABC$  is isosceles with  $AB = BC$ . Therefore, the angles at the base are equal in size; that is,  $\angle BCA = \angle BAC = 74^\circ$ .
- 2 All 3 angles in a triangle must add up to  $180^\circ$ .
- 3 Simplify.
- 4 Solve for  $h$ , that is, subtract  $148^\circ$  from  $180^\circ$ .



## WRITE

$$\angle BCA = \angle BAC = 74^\circ$$

$$\angle ABC + \angle BAC + \angle BCA = 180^\circ$$

$$h + 74^\circ + 74^\circ = 180^\circ$$

$$h + 148^\circ = 180^\circ$$

$$h = 180^\circ - 148^\circ$$

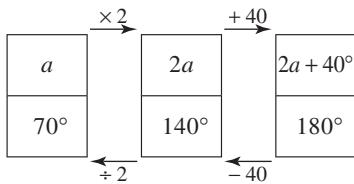
$$h = 32^\circ$$

## WORKED EXAMPLE 13

Find the value of the pronumeral in the following triangle.

## THINK

- 1 From the diagram we can see that  $\triangle MNP$  is isosceles with  $MN = NP$ . Hence,  $\angle NPM = \angle NMP = a$ .
- 2 Form an equation by putting the sum of the angles on one side and  $180^\circ$  on the other side of the equals sign.
- 3 Simplify by collecting like terms.
- 4 Use inspection or backtracking to solve for  $a$ .



## WRITE

$$\angle NMP = \angle NPM = a$$

$$\angle NMP + \angle NPM + \angle MNP = 180^\circ$$

$$a + a + 40^\circ = 180^\circ$$

$$2a + 40^\circ = 180^\circ$$

$$2a = 180^\circ - 40^\circ$$

$$2a = 140^\circ$$

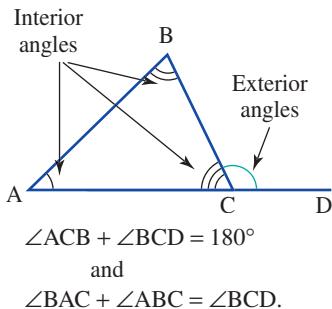
$$a = \frac{140^\circ}{2}$$

$$a = 70^\circ$$

## Interior and exterior angles of a triangle

- The angles inside a triangle are called **interior angles**.
- If any side of a triangle is extended outwards, the angle formed is called an **exterior angle**.
- The exterior angle and the interior angle adjacent (next) to it add up to  $180^\circ$ .

- The sum of the opposite interior angles is equal to the exterior angle.



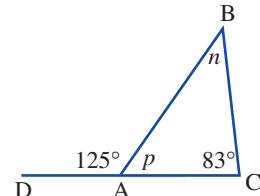
### WORKED EXAMPLE 14

Find the value of the pronumerals in the diagram at right.

#### THINK

- $\angle BAC$  (angle  $p$ ) together with its adjacent exterior angle ( $\angle DAB$ ) add up to  $180^\circ$ . Furthermore,  $\angle DAB = 125^\circ$ . So, form an equation.
- Solve for  $p$ , that is subtract  $125^\circ$  from  $180^\circ$ .
- The interior angles of  $\triangle ABC$  add up to  $180^\circ$ . Identify the values of the angles and form an equation.
- Simplify by adding  $83^\circ$  and  $55^\circ$ .
- Solve for  $n$ , that is subtract  $42^\circ$  from  $180^\circ$ .

#### WRITE



$$\angle BAC = p; \quad \angle DAB = 125^\circ$$

$$\angle BAC + \angle DAB = 180^\circ$$

$$p + 125^\circ = 180^\circ$$

$$p = 180^\circ - 125^\circ$$

$$p = 55^\circ$$

$$\angle BCA + \angle BAC + \angle ABC = 180^\circ$$

$$\angle BCA = 83^\circ; \quad \angle BAC = p = 55^\circ; \quad \angle ABC = n$$

$$83^\circ + 55^\circ + n = 180^\circ$$

$$n + 138^\circ = 180^\circ$$

$$n = 180^\circ - 138^\circ$$

$$n = 42^\circ$$

#### REMEMBER

- According to the lengths of the sides, a triangle can be classified as being:
  - equilateral* (three equal sides)
  - isosceles* (exactly two equal sides)
  - scalene* (no equal sides).
- A triangle can be classified according to the angle size, as being:
  - acute-angled* (all three angles are acute)
  - right-angled* (one angle is a right angle)
  - obtuse-angled* (one angle is obtuse).
- The sum of the interior angles in any triangle is equal to  $180^\circ$ .
- The angles at the base of an isosceles triangle are equal in size.
- An exterior angle of a triangle, and an interior angle adjacent (next) to it, are supplementary (that is, add up to  $180^\circ$ ).
- The sum of the two opposite interior angles is equal to the exterior angle.

## EXERCISE

## 5D

## Triangles

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 5-D-1

Triangles  
doc-1746

## Activity 5-D-2

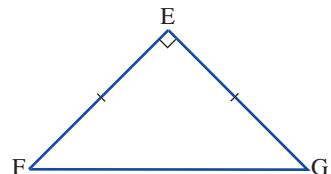
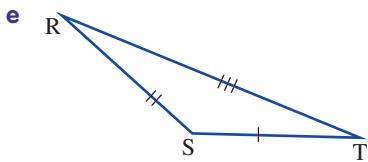
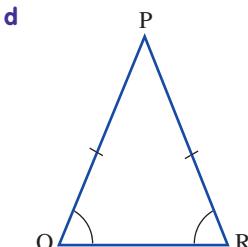
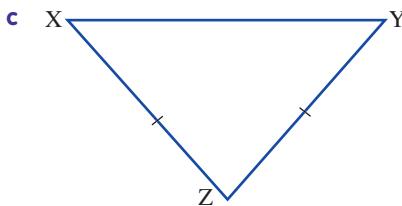
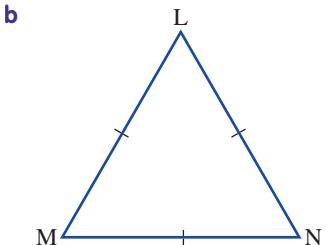
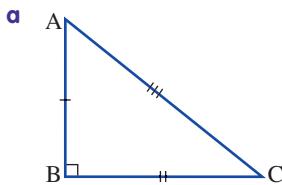
More triangles  
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## Activity 5-D-3

Advanced triangles  
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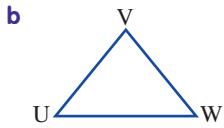
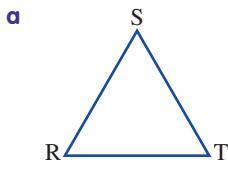
## FLUENCY

- 1 WE9** Name each of the following triangles using the capital letters, then classify each triangle according to the lengths of its sides.



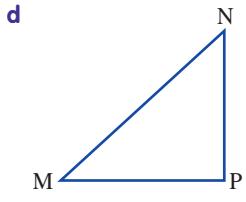
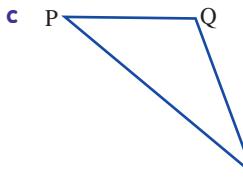
- 2** Classify each of the triangles in question 1 according to the size of its angles.

- 3 WE10** Add side and angle markings to these diagrams to show that:



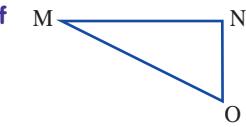
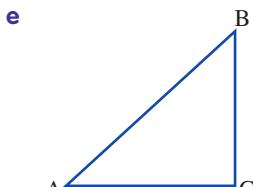
$\triangle RST$  is an equilateral triangle

$\triangle UVW$  is an isosceles triangle



$\triangle PQR$  is a scalene triangle

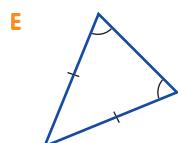
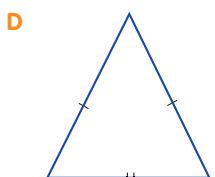
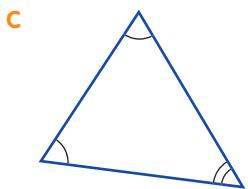
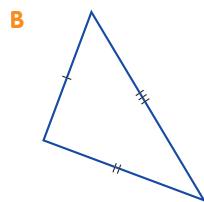
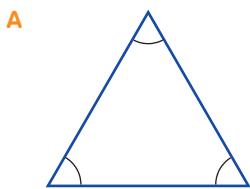
$\triangle MNP$  is a right-angled triangle



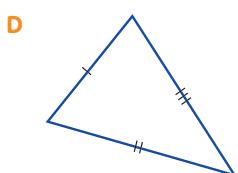
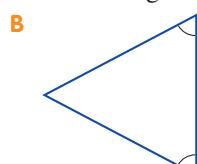
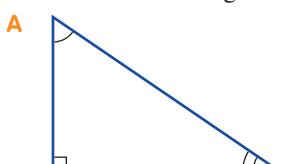
$\triangle ABC$  is a right-angled and isosceles triangle

$\triangle MNO$  is a right-angled and scalene triangle.

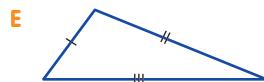
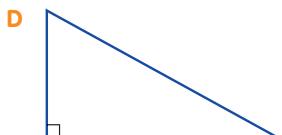
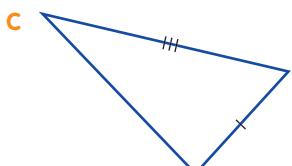
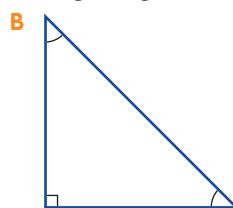
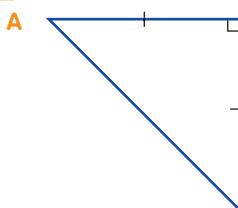
- 4 MC a Which of these triangles is an equilateral triangle?



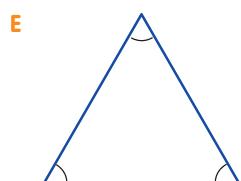
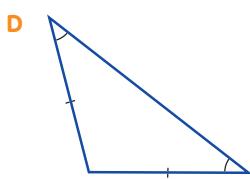
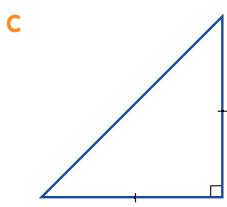
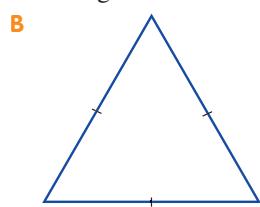
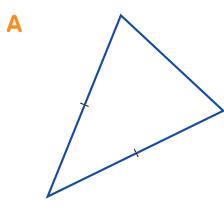
- b Which of these triangles is not a scalene triangle?



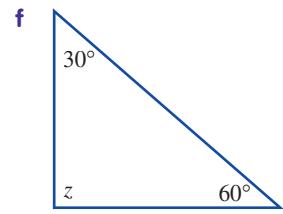
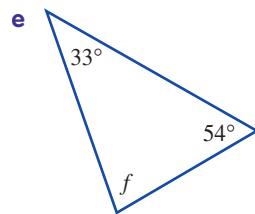
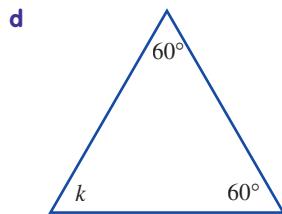
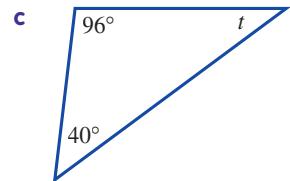
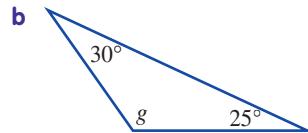
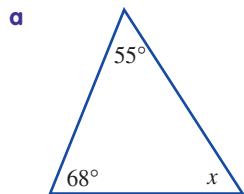
- 5 MC a Which of these triangles is both right-angled and scalene?



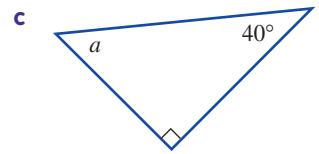
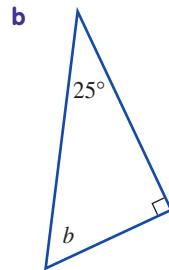
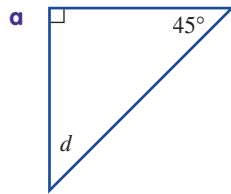
- b Which of these triangles is both acute-angled *and* isosceles?



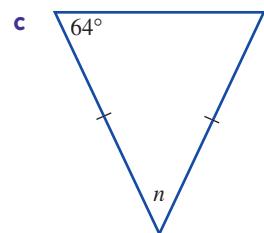
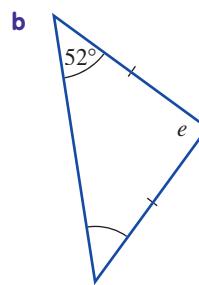
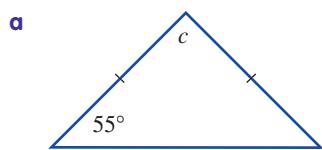
- 6 WE11** Find the value of the pronumeral in each of the following triangles, giving coded reasons.



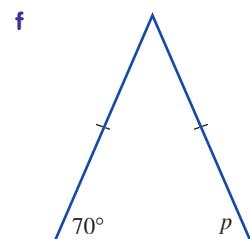
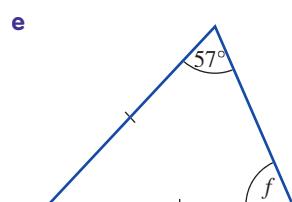
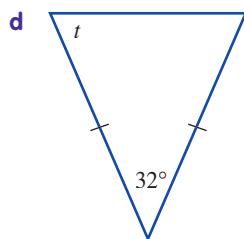
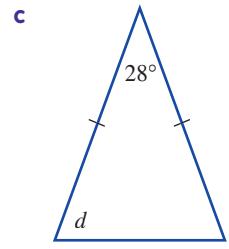
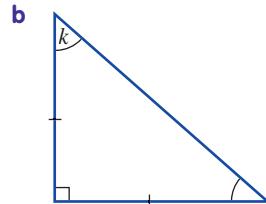
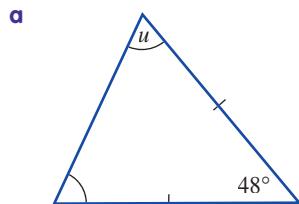
- 7** Find the value of the pronumeral in each of the following right-angled triangles, giving reasons.



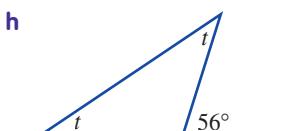
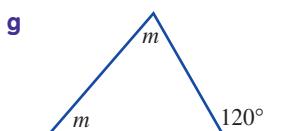
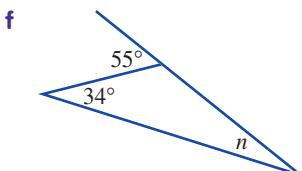
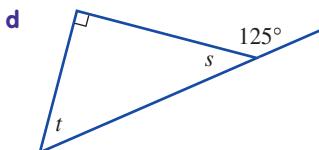
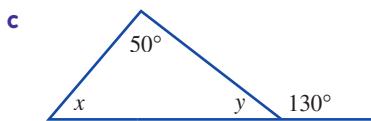
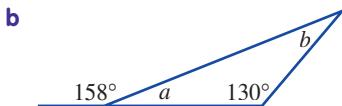
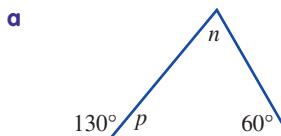
- 8 WE12** Find the value of the pronumeral in each of the following triangles, giving reasons.



- 9 WE13** Find the value of the pronumeral in each of the following triangles, providing reasons.



- 10 WE14** Find the value of the pronumerals in each of the following diagrams.



### UNDERSTANDING

- 11** What types of triangles can you see in the sandwiches below?



- 12** Write down three acute triangles you can see around you.

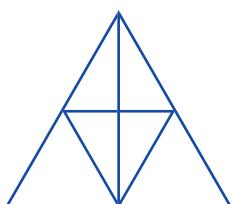
- 13** Find one example in your classroom or home of each of the six types of triangles described in this chapter. Describe clearly where the triangle occurs, draw the triangle and classify it according to both side and angle types.

- 14** In the picture at right:

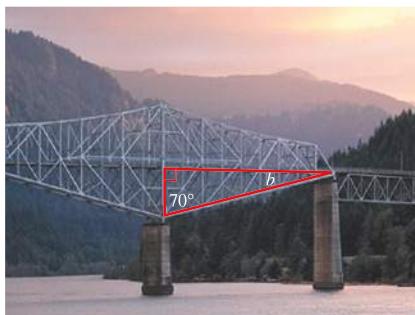
- a how many equilateral triangles can you find?
- b how many right-angled triangles can you find?
- c how many isosceles triangles can you find?

- 15** Use your ruler, pencil and protractor to accurately draw:

- a an equilateral triangle with side lengths 6 cm and all angles  $60^\circ$ .
- b an isosceles triangle with two sides which are 6 cm each with a  $40^\circ$  angle between them.
- c a right-angled triangle whose two short sides are 6 cm and 8 cm.  
(How long is the longest side?)
- d a scalene triangle with two of the sides measuring 4 cm and 5 cm and an angle of  $70^\circ$  between the two sides.



- 16** Find the missing angle in each of the triangles marked on the following photographs.

**a****b****c****d**

- 17** **a** An isosceles triangle has 2 angles of  $55^\circ$  each. Find the size of the third angle.  
**b** An isosceles triangle has 2 angles of  $12^\circ$  each. Find the size of the third angle.  
**c** Two angles of a triangle are  $55^\circ$  and  $75^\circ$  respectively. Find the third angle.  
**d** Two angles of a triangle are  $48^\circ$  and  $68^\circ$  respectively. Find the third angle.
- 18** **a** Use a ruler and a protractor to construct each of the following triangles.  
**i** An isosceles triangle with a base of 4 cm and equal angles of  $50^\circ$  each.  
**ii** An isosceles triangle with two sides which are 5 cm each and two equal angles which are  $45^\circ$  each.  
**b** On your diagrams label the size of each angle. Classify the triangles according to the size of their angles.
- 19** Below are sets of 3 angles. For each set state whether or not it is possible to construct a triangle with these angles. Give a reason for your answer.  
**a**  $40^\circ, 40^\circ, 100^\circ$   
**b**  $45^\circ, 60^\circ, 70^\circ$   
**c**  $45^\circ, 55^\circ, 85^\circ$   
**d**  $111^\circ, 34.5^\circ, 34.5^\circ$
- 20** Explain in your own words why it is impossible to construct a triangle with 2 obtuse angles.

### REASONING

- 21** Cut eighteen strips of paper — three each of the following six lengths: 3 cm, 4 cm, 5 cm, 6 cm, 8 cm and 10 cm. Use these strips to help you answer the following questions.
- a** Using a combination of three of these strips (without bending them), how many different triangles can you make?  
**b** What type of triangle is each one? How many are there of each kind?  
**c** Are there any similar triangles (same shape, but different size)?  
**d** Measure the size of the angles. What general relationship can you observe between the lengths of the sides of a triangle and the size of the angles?

- e Formulate a general statement which would enable you to determine whether it is possible to form a triangle from three given side lengths.  
Why is it not possible in some cases?

**REFLECTION**

How can you prove the statement:  
'The sum of the opposite interior angles is equal to the exterior angle'?

## 5E Quadrilaterals and their properties

### Types of quadrilaterals

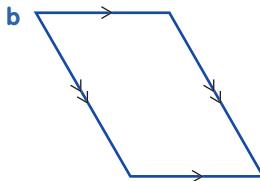
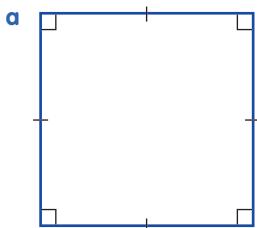
- A **quadrilateral** is a 2-dimensional closed shape with four straight sides.
- Quadrilaterals can be divided into two major groups: **parallelograms** and other quadrilaterals.
- Parallelograms are quadrilaterals with both pairs of opposite sides being parallel to each other.
- All **rectangles**, **squares** and **rhombi** (diamonds) are parallelograms.
- Other quadrilaterals (non-parallelograms) include **trapeziums**, **kites** and **irregular quadrilaterals**.

Parallelograms	Shape	Properties
Parallelogram		Opposite sides are equal in length. Opposite angles are equal in size.
Rectangle		Opposite sides are equal in length. All angles are the same and equal 90°.
Rhombus		All sides are equal in length. Opposite angles are equal in size.
Square		All sides are equal in length. All angles are the same and equal 90°.

Other quadrilaterals	Shape	Properties
Trapezium		One pair of opposite sides is parallel.
Kite		Two pairs of adjacent (next to each other) sides are equal in length. One pair of opposite angles (the ones that are between the sides of unequal length) are equal.
Irregular quadrilateral		This shape does not have any special properties.

## WORKED EXAMPLE 15

Name the following quadrilaterals, giving reasons for your answers.



## THINK

- a The markings on this quadrilateral indicate that all sides are equal in length and all angles equal  $90^\circ$ . Classify the quadrilateral by finding the matching description in the table.
- b The arrows on the sides of this quadrilateral indicate that there are two pairs of parallel sides and hence it is a parallelogram. Check the descriptions in the table to see if it is a particular type of parallelogram.

## WRITE

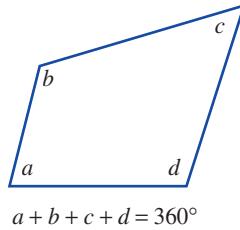
- a The given quadrilateral is a square, since all sides are equal and all angles are  $90^\circ$ .
- b The given quadrilateral is a parallelogram, since it has two pairs of parallel sides.

## Angles in a quadrilateral

- The sum of angles in any quadrilateral is  $360^\circ$ .

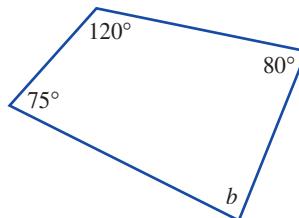
## eBook plus

**Interactivity**  
Angles in triangles and quadrilaterals  
int-2739



## WORKED EXAMPLE 16

Find the value of the pronumeral in the diagram.



## THINK

- 1 The sum of the angles in a quadrilateral is  $360^\circ$ . So, express this as an equation.
- 2 Simplify by adding  $120^\circ$ ,  $80^\circ$  and  $75^\circ$ .
- 3 Solve to find the value of  $b$ , that is, subtract  $275^\circ$  from  $360^\circ$ .

## WRITE

$$b + 80^\circ + 75^\circ + 120^\circ = 360^\circ$$

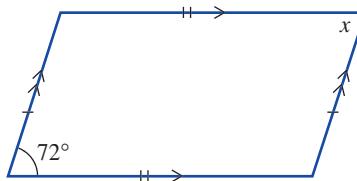
$$b + 275^\circ = 360^\circ$$

$$b = 360^\circ - 275^\circ$$

$$b = 85^\circ$$

**WORKED EXAMPLE 17**

**Find the value of the pronumeral in the diagram, giving a reason for your answer.**

**THINK**

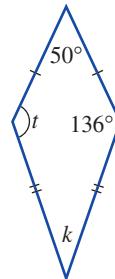
According to the markings, the opposite sides of the given quadrilateral are parallel and equal in length. Therefore, this quadrilateral is a parallelogram. In a parallelogram opposite angles are equal. So, state the value of the pronumeral.

**WRITE**

Opposite angles in a parallelogram are equal in size. Therefore,  $x = 72^\circ$ .

**WORKED EXAMPLE 18**

**Find the value of the pronumerals in the diagram at right.**

**THINK**

- 1 Form an equation by writing the sum of the angles on one side and  $360^\circ$  on the other side of an equals sign.
- 2 The quadrilateral shown in the diagram is a kite. Angle  $t$  and angle  $136^\circ$  are the angles between unequal sides and therefore must be equal in size.
- 3 Replace  $t$  in the equation with  $136^\circ$ .
- 4 Simplify.
- 5 Solve to find the value of  $k$ , that is, subtract  $322^\circ$  from  $360^\circ$ .

**WRITE**

$$k + t + 50^\circ + 136^\circ = 360^\circ$$

$$t = 136^\circ$$

(The figure is a kite.)

$$k + 136^\circ + 50^\circ + 136^\circ = 360^\circ$$

$$k + 322^\circ = 360^\circ$$

$$k = 360^\circ - 322^\circ$$

$$k = 38^\circ$$

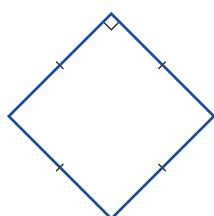
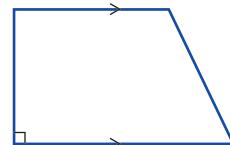
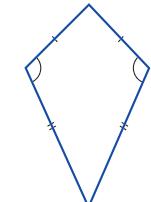
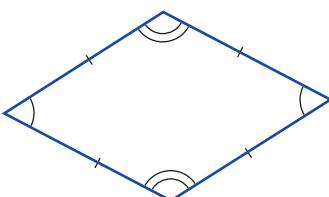
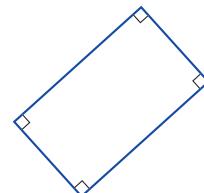
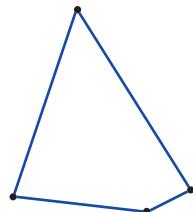
**REMEMBER**

1. A *quadrilateral* is a 2-dimensional closed shape with four straight sides.
2. All quadrilaterals can be divided into two major groups: parallelograms and other quadrilaterals.
3. *Parallelograms* have two pairs of parallel sides and include rectangles, squares and rhombuses.
4. Other quadrilaterals include trapeziums, kites and irregular quadrilaterals.
5. The sum of angles in any quadrilateral is equal to  $360^\circ$ .

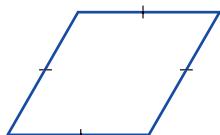
## EXERCISE

**5E****Quadrilaterals and their properties****INDIVIDUAL PATHWAYS****eBook plus****Activity 5-E-1**Quadrilaterals  
doc-1749**Activity 5-E-2**More quadrilaterals  
doc-1750**Activity 5-E-3**Advanced  
quadrilaterals  
doc-1751**FLUENCY**

- 1 WE15** Name the following quadrilaterals, giving reasons for your answers.

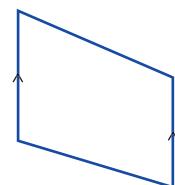
**a****b****c****d****e****f**

- 2 MC** a



This quadrilateral is a:

- A square
- B rectangle
- C kite
- D parallelogram
- E rhombus

**b**

This quadrilateral is a:

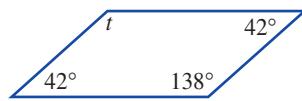
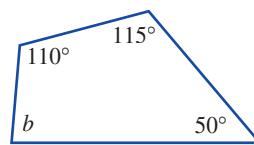
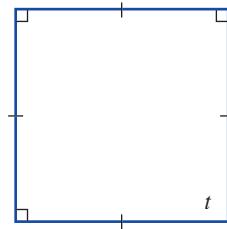
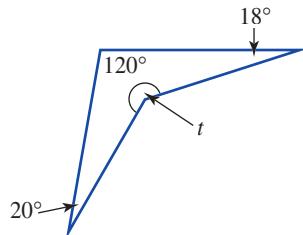
- A trapezium
- B parallelogram
- C rhombus
- D kite
- E square

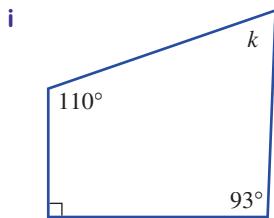
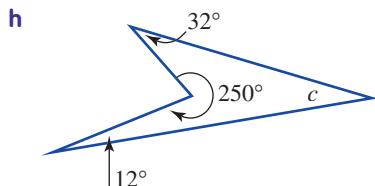
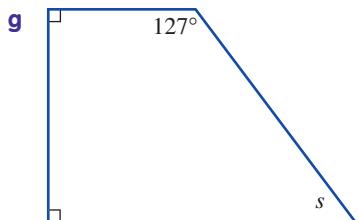
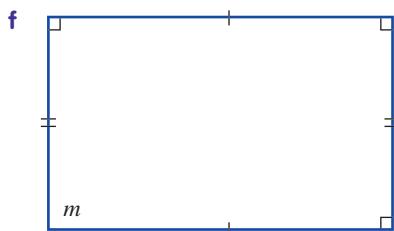
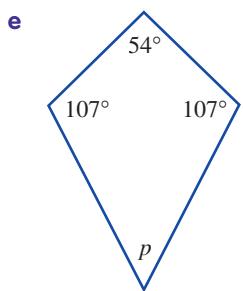
**c**

This quadrilateral is a:

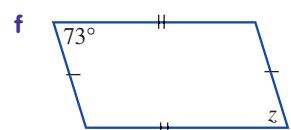
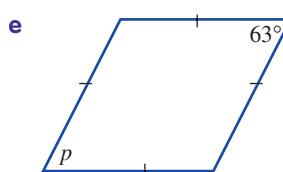
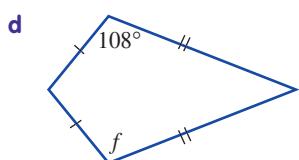
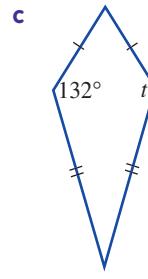
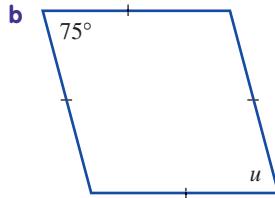
- A trapezium
- B square
- C kite
- D rhombus
- E parallelogram

- 3 WE16** Find the value of the pronumeral in each of the following diagrams.

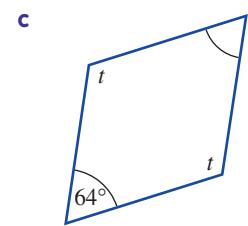
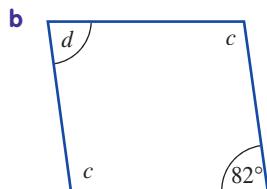
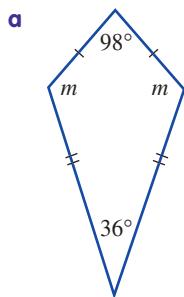
**a****b****c****d**

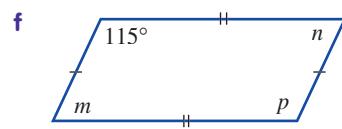
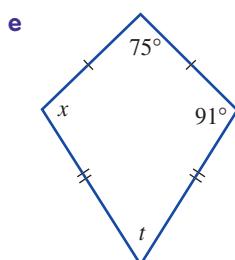
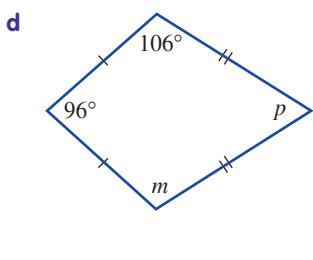


- 4 **WE17** Find the value of the pronumeral in each of the following diagrams, giving reasons for your answers.



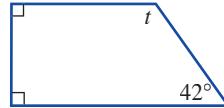
- 5 **WE18** Find the value of the pronumerals in each of the following diagrams.





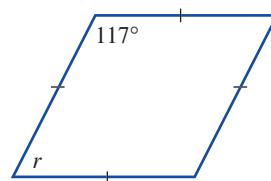
- 6 MC** The value of  $t$  in the diagram at right is:

A  $360^\circ$       B  $112^\circ$   
C  $222^\circ$       D  $138^\circ$   
E  $180^\circ$



- 7 MC** The value of  $r$  in the diagram at right is:

A  $117^\circ$       B  $180^\circ$   
C  $234^\circ$       D  $126^\circ$   
E  $63^\circ$



### UNDERSTANDING

- 8** State whether each of the following statements is true or false.

a All squares are rectangles.  
b All squares are rhombuses.  
c All rectangles are squares.  
d Any rhombus with at least one right angle is a square.  
e A rectangle is a parallelogram with at least one angle equal to  $90^\circ$ .

- 9 MC** A rectangle is a quadrilateral because:

A it has 4 right angles	B it has 2 pairs of parallel sides
C its opposite sides are equal in length	D it has 4 straight sides
E it has 2 pairs of parallel sides and 4 right angles	

- 10** Draw 4 equilateral triangles with side lengths 4 cm and cut them out.

a Use 2 of these triangles to make a rhombus. Draw your solution.  
b Use 3 of these triangles to make a trapezium. Draw your solution.  
c Use all 4 triangles to make a parallelogram.

Draw your solution.

- 11** Copy and cut out the set of shapes shown at right.

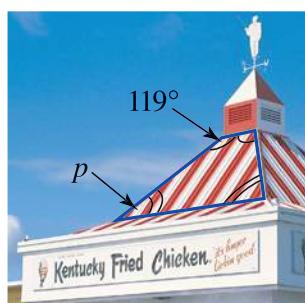
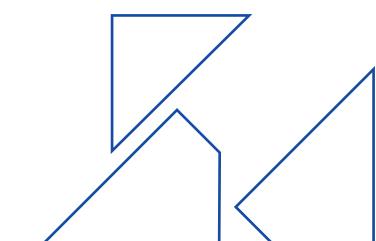
Arrange the shapes to form a square. Draw your solution.

- 12** In your house, find an example of each type of quadrilateral discussed in this section.

Write down the type of quadrilateral and where you found it.

In the following questions, remember to provide coded reasons for any relationships you form.

- 13** This photograph shows the roof of a fast food restaurant. Calculate the value of  $p$ .



- 14** Find the size of the obtuse angle in the kite shown at right.

- 15** Two angles in a parallelogram are  $45^\circ$  and  $135^\circ$ . Find the other 2 angles.

- 16** Tom measures 2 angles of a kite at  $60^\circ$  and  $110^\circ$ , but forgets which angle is which. Draw 3 different kites that Tom may have measured, showing the size of all angles in each diagram.

- 17** Below are sets of 4 angles. For each of the sets decide whether it is possible to construct a quadrilateral. Explain your answer.

a  $25^\circ, 95^\circ, 140^\circ, 100^\circ$

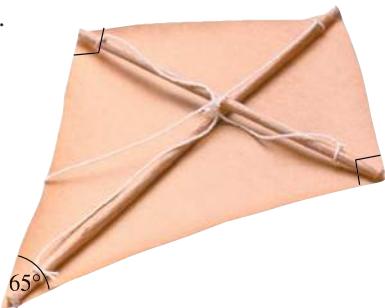
b  $40^\circ, 80^\circ, 99^\circ, 51^\circ$

- 18** Three angles of a quadrilateral are  $60^\circ, 70^\circ$  and  $100^\circ$ .

a What is the size of the fourth angle of this quadrilateral?

b How many quadrilaterals with this set of angles are possible?

c Construct one quadrilateral with the given angle sizes in your book. (The choice of the length of the sides is yours.)



### REASONING

- 19** Val and Peter want to replace their front gate with another of the same design. To have this gate made, they need to supply a diagram of it with all measurements and angles shown. Study the photograph of Val and Peter's gate and use it to help you answer the questions below.



- a There are 4 different shapes formed by the metal bars of the gate. How many different types of triangles are there? Can you name them?
- b How many types of quadrilaterals are there? Name them.
- c Draw a diagram of the gate showing the length measurements and the one angle that is given.
- d Use this angle to calculate all the remaining angles in the diagram.
- e Using a ruler and protractor, design a fence that is to be constructed using metal bars. Include different triangles and quadrilaterals to make your design as interesting as possible. Write a short report describing the shapes you have used and important angles which need to be marked on your design to assist in the construction of the fence.

### REFLECTION

What would be an easy way to remember the sum of angles in quadrilaterals?

# 5F Parallel and perpendicular lines

## eBook plus

**Interactivities**

Vertically opposite angles

int-0814

**Corresponding angles**

int-0815

**Alternate angles**

int-0816

**Co-interior angles**

int-0817

## Vertically opposite and adjacent angles

- When two straight lines intersect, they form four angles with a common vertex.
- The angles that are opposite each other are called **vertically opposite angles**.
- The angles next to each other are called **adjacent angles**. (Adjacent angles share an ‘arm’.)
- Vertically opposite angles are equal in size.

In the diagram at right there are two pairs of vertically opposite angles.

$$\angle AOB = \angle DOC \text{ and } \angle BOC = \angle AOD$$

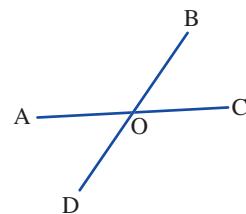
There are 4 pairs of adjacent angles.

$\angle AOB$  is adjacent to  $\angle BOC$ .

$\angle BOC$  is adjacent to  $\angle COD$ .

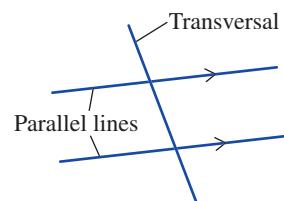
$\angle COD$  is adjacent to  $\angle AOD$ .

$\angle AOD$  is adjacent to  $\angle AOB$ .



## Parallel lines

- Parallel lines are indicated in a diagram by lines with arrows on them.
- A line intersecting a pair (or set) of parallel lines is called a **transversal**.
- Cutting parallel lines by a transversal creates a series of angles that are related in a number of ways.



## Corresponding angles

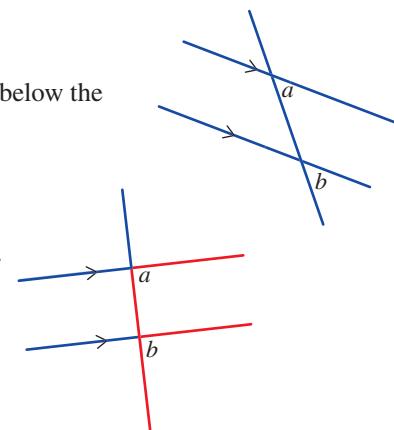
The diagram at right shows two angles,  $a$  and  $b$ , positioned below the parallel lines to the right of a transversal.

When both angles are on the same side of the transversal (both to the left, or both to the right of it) and are either both above, or both below the parallel lines, such angles are called **corresponding angles**.

Corresponding angles are equal in size.

Thus, in the diagram at right  $\angle a = \angle b$ .

The position of corresponding angles is easy to remember by associating it with the ‘F’ shape.



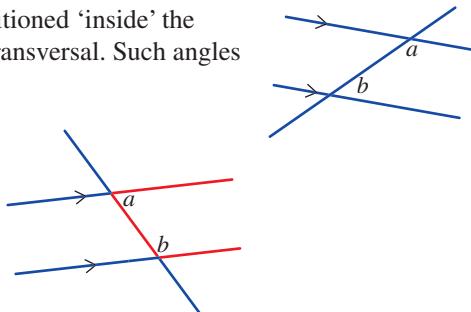
## Co-interior angles (or allied angles)

The diagram at right shows two angles,  $a$  and  $b$ , positioned ‘inside’ the parallel lines, on the same side (to the right) of the transversal. Such angles are called **co-interior angles** or **allied angles**.

Co-interior angles are supplementary; that is, they add up to  $180^\circ$ .

Thus, in the diagram at right  $\angle a + \angle b = 180^\circ$ .

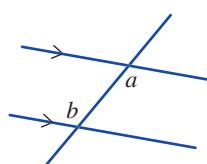
The position of the co-interior angles is easy to remember by associating it with the ‘C’ shape.



## Alternate angles

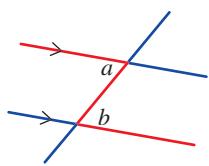
The diagram at right shows two angles,  $a$  and  $b$ , positioned ‘inside’ the parallel lines on alternate sides of the transversal. Such angles are called **alternate angles**.

Alternate angles are equal in size.



Thus, in the diagram at right,  $\angle a = \angle b$ .

The position of alternate angles is easy to remember by associating it with the ‘Z’ shape.



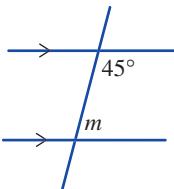
## Calculating angles associated with parallel lines

Angle relationships associated with parallel lines can be used to find the size of missing angles, as shown in the following worked examples. It is important to remember that the Z, F or C shapes may be upside-down or backwards in some diagrams.

### WORKED EXAMPLE 19

For the diagram at right:

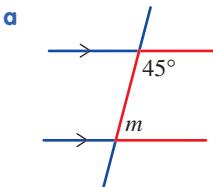
- state the type of angle relationship
- find the value of the pronumeral.



#### THINK

- Study the diagram: which shape — Z, F or C — would include both angles that are shown? Copy the diagram into your workbook and highlight the appropriate shape.
- State the name of the angles suggested by the ‘C’ shape.
- Co-interior angles add to  $180^\circ$ . Write this as an equation.
- Solve for  $m$ .

#### WRITE



Shown angles are co-interior.

b  $m + 45^\circ = 180^\circ$

$$\begin{aligned}m &= 180^\circ - 45^\circ \\&= 135^\circ\end{aligned}$$

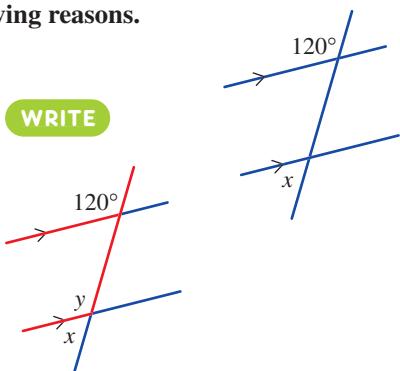
### WORKED EXAMPLE 20

Find the value of the pronumeral in the diagram shown, giving reasons.

#### THINK

- The two angles shown are neither C, Z nor F angles. So we must find some other angle first, that will enable us to find the size of angle  $x$ . (This other angle must be in a certain relation with *both* given angles.) Draw an F-shape that includes a  $120^\circ$  angle. The other angle in the ‘F’ shape is related to both given angles: it is corresponding to the  $120^\circ$  angle and it is supplementary to angle  $x$ . Call this angle  $y$ .
- State the size of angle  $y$ , specifying the reason (the angle relation).
- The angles  $x$  and  $y$  are supplementary (add to  $180^\circ$ ). State this as an equation.

#### WRITE



$$\begin{aligned}y &= 120^\circ && \text{(as corresponding angles are equal)} \\x + y &= 180^\circ && \text{(as angle sum of straight line is } 180^\circ)\end{aligned}$$



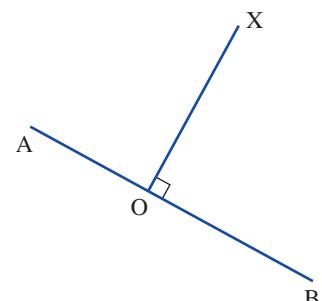
- 4 Substitute the value of  $y$  into the equation.  
5 Solve for  $x$ .

$$x + 120^\circ = 180^\circ$$

$$\begin{aligned} x &= 180^\circ - 120^\circ \\ &= 60^\circ \end{aligned}$$

## Perpendicular lines

- **Perpendicular** means ‘at right angles’.
- When two line segments meet at right angles they are said to be perpendicular. That is, the angle between them is  $90^\circ$ . In the diagram at right, the angles  $\angle AOX$  and  $\angle XOB$  are each right angled which means that the line segments AB and OX are perpendicular.
- To write ‘the line segments AB and OX are perpendicular’ using mathematical symbols, simply write  $AB \perp OX$ . The symbol  $\perp$  means ‘is perpendicular to’.



### REMEMBER

1. (a) Vertically opposite angles are equal in size.  
(b) Adjacent angles are next to each other.
2. Corresponding (F) angles are equal in size.
3. Co-interior (C) angles add to  $180^\circ$ .
4. Alternate (Z) angles are equal in size.
5. When looking for the F, C or Z shape, remember that they may be backwards or upside down.
6. The symbol for parallel is  $\parallel$ .
7. Supplementary angles add to  $180^\circ$ .
8. Complementary angles add to  $90^\circ$ .
9. If the given angles are in none of the above relations, we might need to find some other angle first. This other angle must be related to both given angles.
10. Perpendicular means ‘at right angles’.

### EXERCISE

## 5F

## Parallel and perpendicular lines

### INDIVIDUAL PATHWAYS

#### eBook plus

**Activity 5-F-1**  
Parallel and  
perpendicular lines  
doc-1752

**Activity 5-F-2**  
More parallel and  
perpendicular lines  
doc-1753

**Activity 5-F-3**  
Advanced parallel and  
perpendicular lines  
doc-1754

### FLUENCY

- 1 Copy and complete this table.

Diagram	Type of angle relation	Associated shape	Rule
	Corresponding		Are equal in size.
	Alternate	Z	
			Add up to $180^\circ$ .

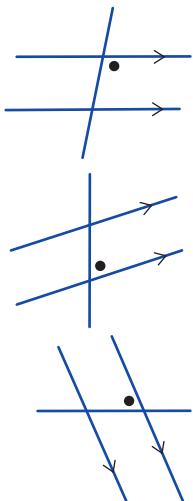
#### eBook plus

**Digital doc**  
Investigate parallel and  
perpendicular lines  
doc-6456

- 2 a** Copy the diagram into your workbook. Clearly draw the 'F' shape on your diagram and label the angle corresponding to the one that is marked.

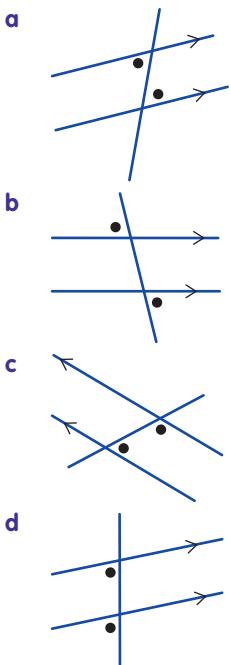
- b** Copy the diagram into your workbook. Clearly draw the 'Z' shape on your diagram and label the angle alternate to the marked angle.

- c** Copy the diagram and label the angle co-interior to the marked angle. Clearly draw the 'C' shape on your diagram.



- 3** Match each diagram with the appropriate name from the four options listed.

Diagram



Name

**A** Co-interior angles (C)

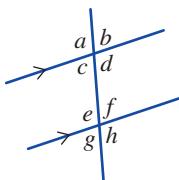
**B** Corresponding angles (F)

**C** Alternate angles (Z)

**D** None of the above

- 4** In the diagram at right, list all pairs of:

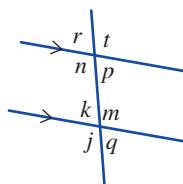
- a** vertically opposite angles
- b** corresponding angles
- c** co-interior angles
- d** alternate angles.



- 5 MC** In the diagram below:

- a** Which angle is vertically opposite to angle  $p$ ?

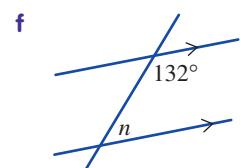
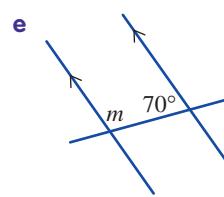
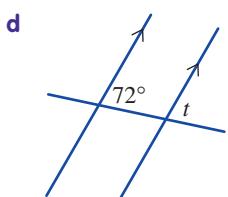
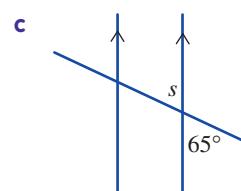
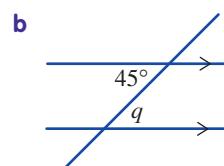
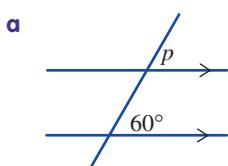
- A**  $k$
- B**  $m$
- C**  $r$
- D**  $q$
- E**  $t$



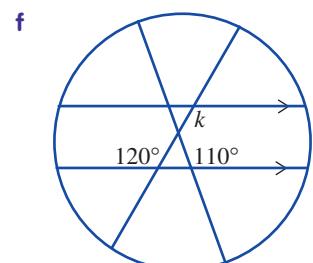
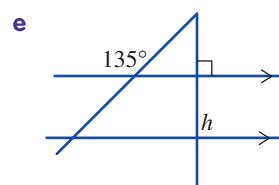
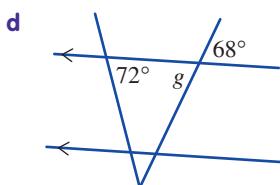
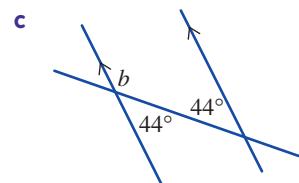
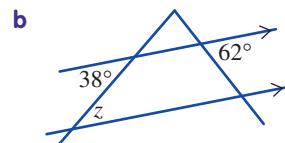
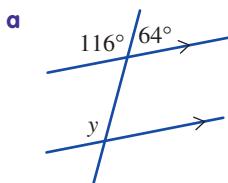
- b Which angle is corresponding to angle  $p$ ?  
**A**  $k$       **B**  $m$       **C**  $r$       **D**  $q$       **E**  $t$
- c Which angle is co-interior to angle  $p$ ?  
**A**  $k$       **B**  $m$       **C**  $r$       **D**  $q$       **E**  $n$
- d Which angle is alternate to angle  $p$ ?  
**A**  $k$       **B**  $m$       **C**  $r$       **D**  $q$       **E**  $n$
- e Name two angles adjacent to angle  $k$ .

**6 WE19** For each of the following diagrams:

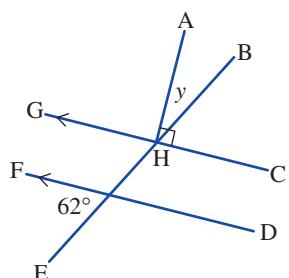
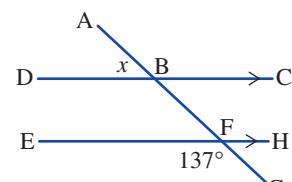
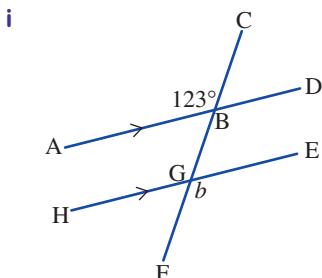
- i state the type of angle relationship  
ii find the value of the pronumeral.

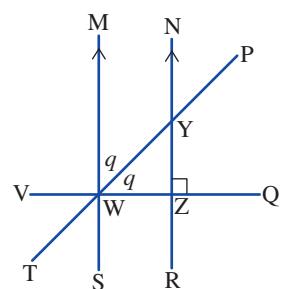
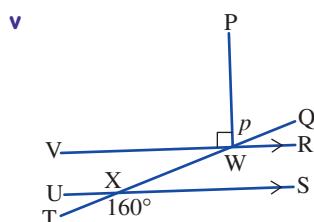
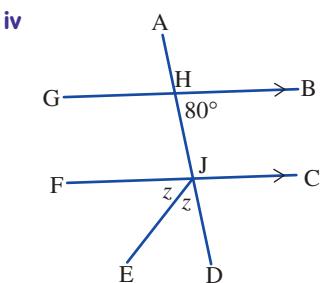


**7** Find the value of the pronumerals in each of the following diagrams, giving reasons.



**8 a WE20** Find the value of the pronumerals in each of the following diagrams.

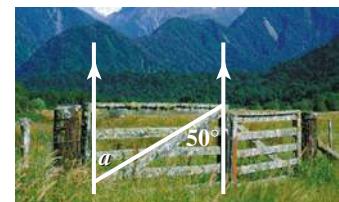
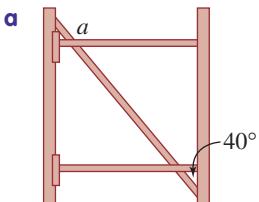
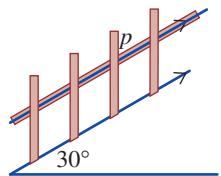




- b Name all the perpendicular lines in the diagrams above.

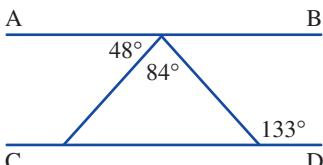
### UNDERSTANDING

- 9 If the angle allied to  $x$  is  $135^\circ$ , find the size of angle  $x$ .
- 10 If the angle corresponding to  $y$  is  $55^\circ$ , find the size of angle  $y$ .
- 11 A hill is at an angle of  $30^\circ$  to the horizontal. A fence is put in, consisting of a railing parallel to the ground and vertical fence posts. Find the angle between the top of the fence post and the rail.
- 12 Two gates consist of vertical posts, horizontal struts and diagonal beams. Find the angle,  $a$ , as shown in the gates below.



### REASONING

- 13 Is the line AB parallel to line CD? Explain your answer.



- 14 a What shapes can you identify in this picture?  
Make a list.

- b Name three places in the picture where you see parallel lines.

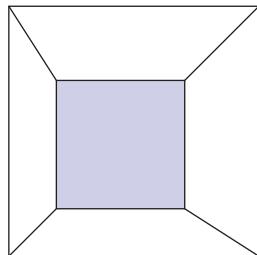
When examining a photograph of a 3-dimensional object, you notice that some sets of parallel lines (such as the wires in the railing) appear to meet in the distance. This point is called the *vanishing point* and is used by architects, engineers and artists to create diagrams and artworks in *perspective*.

- c Identify the vanishing point in the photograph.

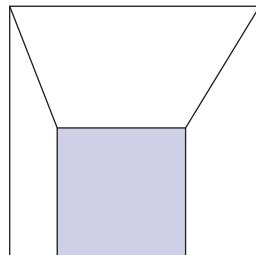


- 15** Perspective drawing also allows us to create the illusion of looking at the same object from a number of different angles. For example, consider a simple box as shown at right.

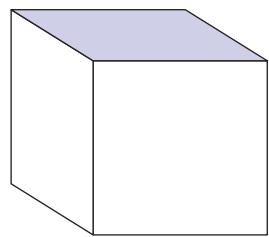
The two figures below are views of the cube from different angles. In all cases the top of the box has been shaded.



**Figure A**



**Figure B**



- a** Find the vanishing point in each diagram, by extending the edges in the diagram until they meet.
- b** True or false?
  - i** The viewer is above the box in Figure A.
  - ii** The viewer is behind the box in Figure B.
  - iii** The viewer of Figure A is probably taller than the viewer of Figure B.

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**REFLECTION**

Can you think of a way to remember the different types of angles associated with parallel lines and their special properties?

# Summary

## Measuring angles

- When two lines meet at a point, an *angle* is formed.
- The point where the lines meet is called a *vertex* and the lines are called the *arms* of the angle.
- All angles are measured in degrees using a protractor.
- Circular protractors are marked from  $0^\circ$  to  $360^\circ$ . If the centre of the protractor is placed on the vertex and one of the arms points at  $0^\circ$ , the other arm will point to the angle being measured.
- Semicircular protractors have two scales (inner and outer) which are marked from  $0^\circ$  to  $180^\circ$ . Either of these two scales can be used for measuring angles. Angles that are less than  $180^\circ$  can be measured in the same way as with circular protractors. To measure an angle that is larger than  $180^\circ$ , the angle enclosed by the arms (that is, the one that is less than  $180^\circ$ ) is measured first and is then subtracted from  $360^\circ$ .
- It is a good idea to estimate the size of an angle before measuring it.

## Constructing angles with a protractor

- To construct an angle, follow the steps given below.
  - Rule a straight line.
  - Put a dot at one end of the line. (The line represents one of the arms and the dot represents the vertex of the angle.)
  - Place the protractor so that its centre is on the vertex and  $0^\circ$  is on the baseline.
  - Find the required angle on the scale and mark a small dot at the edge of the protractor.
  - Join the small dot with the vertex to form the second arm of the angle.
  - Label the angle.
- To construct an angle that is larger than  $180^\circ$ , using a semicircular protractor:
  - subtract the given angle from  $360^\circ$
  - construct the resulting angle (it will be less than  $180^\circ$ )
  - label the required angle: it is ‘outside’ the one that has been constructed.

## Types of angles and naming angles

- Angles can be classified according to their size.

Angle size	Diagram	Name of angle
Between $0^\circ$ and $90^\circ$		Acute
$90^\circ$		Right
Between $90^\circ$ and $180^\circ$		Obtuse
$180^\circ$		Straight
Between $180^\circ$ and $360^\circ$		Reflex
$360^\circ$		Revolution

- An angle can be named using three capital letters from the English alphabet: two for the arms and one for the vertex of the angle.
- The letter representing the vertex of the angle is always placed in the middle. Use the symbol  $\angle$  instead of the word ‘angle’ before the three letters.

**Triangles**

- According to the lengths of the sides, a triangle can be classified as being:
  - *equilateral* (three equal sides)
  - *isosceles* (exactly two equal sides)
  - *scalene* (no equal sides).
- A triangle can be classified according to the angle size, as being:
  - *acute-angled* (all three angles are acute)
  - *right-angled* (one angle is a right angle)
  - *obtuse-angled* (one angle is obtuse).
- The sum of the interior angles in any triangle is equal to  $180^\circ$ .
- The angles at the base of an isosceles triangle are equal in size.
- An exterior angle of a triangle, and an interior angle adjacent (next) to it, are supplementary (that is, add up to  $180^\circ$ ).
- The sum of the two opposite interior angles is equal to the exterior angle.

**Quadrilaterals and their properties**

- A *quadrilateral* is a 2-dimensional closed shape with four straight sides.
- All quadrilaterals can be divided into two major groups: parallelograms and other quadrilaterals.
- *Parallelograms* have two pairs of parallel sides and include rectangles, squares and rhombuses.
- Other quadrilaterals include trapeziums, kites and irregular quadrilaterals.
- The sum of angles in any quadrilateral is equal to  $360^\circ$ .

**Parallel and perpendicular lines**

- (a) Vertically opposite angles are equal in size.
- (b) Adjacent angles are next to each other.
- Corresponding (F) angles are equal in size.
- Co-interior (C) angles add to  $180^\circ$ .
- Alternate (Z) angles are equal in size.
- When looking for the F, C or Z shape, remember that they may be backwards or upside down.
- The symbol for parallel is  $\parallel$ .
- Supplementary angles add to  $180^\circ$ .
- Complementary angles add to  $90^\circ$ .
- If the given angles are in none of the above relations, we might need to find some other angle first. This other angle must be related to both given angles.
- Perpendicular means ‘at right angles’.

**MAPPING YOUR UNDERSTANDING**

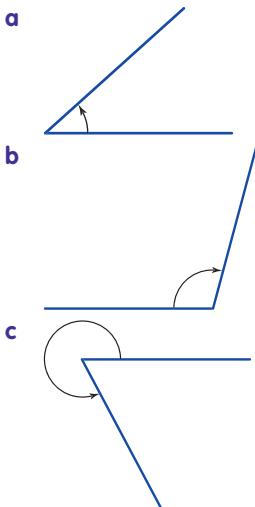
Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter’s opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

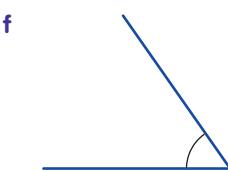
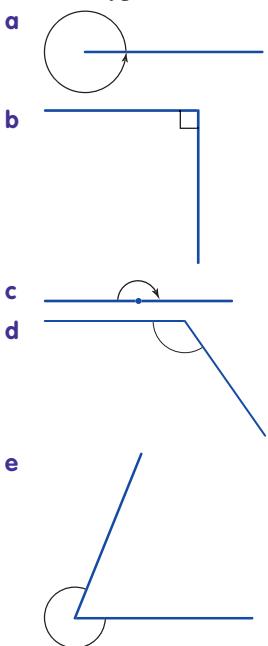
- 1 First estimate, then use a protractor to measure the magnitude of each of the following angles. Comment on the accuracy of your estimate.



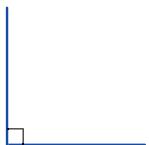
- 2 Use a protractor to construct each of the following angles.

- |               |               |
|---------------|---------------|
| a $43^\circ$  | b $68^\circ$  |
| c $90^\circ$  | d $128^\circ$ |
| e $170^\circ$ | f $226^\circ$ |
| g $270^\circ$ | h $285^\circ$ |
| i $300^\circ$ | j $310^\circ$ |

- 3 State the type of each of the angles shown below.



Questions 4 and 5 refer to the diagram at right:



- 4 MC The angle shown in the diagram has a magnitude of:

- A  $50^\circ$
- B  $40^\circ$
- C  $90^\circ$
- D  $100^\circ$
- E  $190^\circ$

- 5 MC The angle shown is:

- A a right angle
- B a straight angle
- C an acute angle
- D an obtuse angle
- E a reflex angle

- 6 MC An acute angle has a magnitude that is:

- A more than  $90^\circ$
- B less than  $90^\circ$
- C exactly  $90^\circ$
- D more than  $90^\circ$  but less than  $180^\circ$
- E less than  $180^\circ$

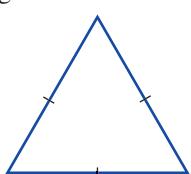
- 7 MC A straight angle has a magnitude of:

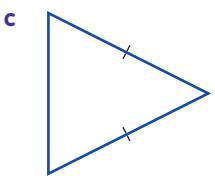
- A  $90^\circ$
- B  $270^\circ$
- C  $360^\circ$
- D  $180^\circ$
- E  $60^\circ$

- 8 Name the angle shown.

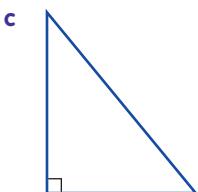
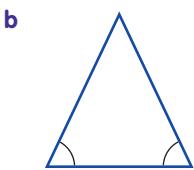
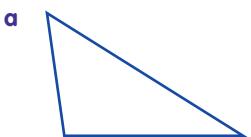


- 9 Name the following triangles according to the length of their sides.

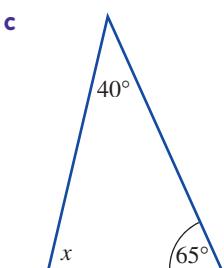
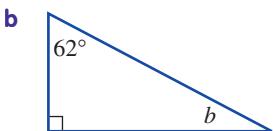
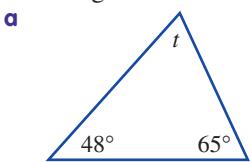




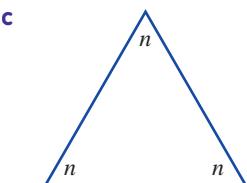
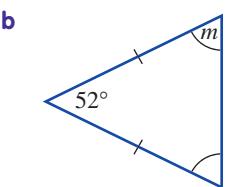
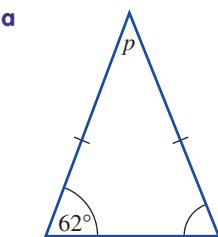
- 10 Name the following triangles according to the size of their angles.



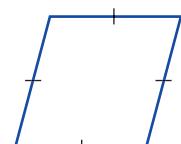
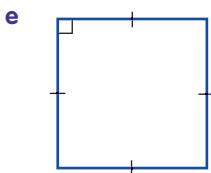
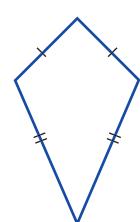
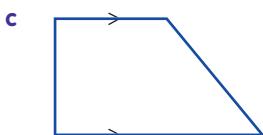
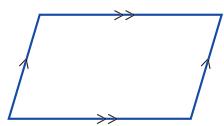
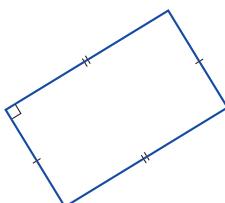
- 11 Find the value of the pronumeral in each of the following.



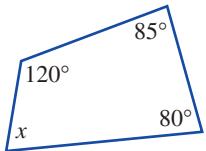
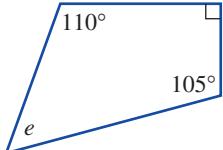
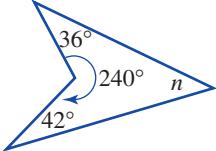
- 12 Find the value of the pronumeral in each of the following.



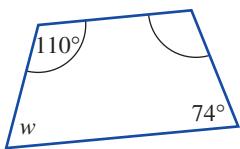
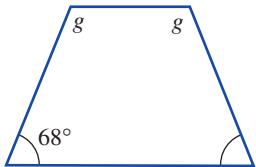
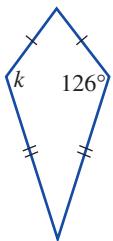
- 13 Name the following quadrilaterals, giving reasons for your answers.



- 14** Find the value of the pronumeral in each of the following.

**a****b****c**

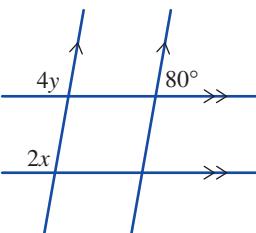
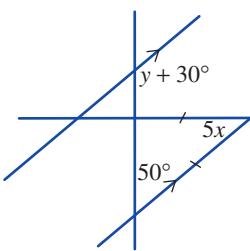
- 15** Find the value of the pronumeral in each of the following.

**a****b****c**

- 16** Find the value of each of the pronumerals, giving reasons for your answer.

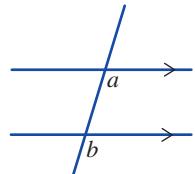
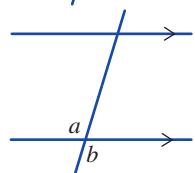
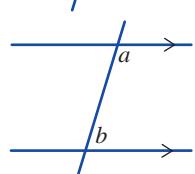
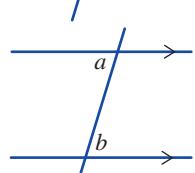
**a**

$$5x - 20^\circ \quad 3x + 10^\circ$$

**b****c**

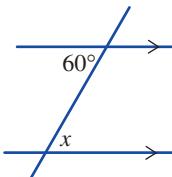
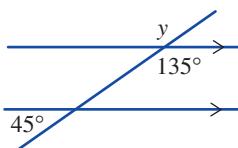
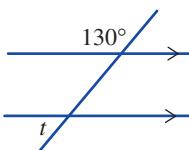
- 17** **MC** Note: There may be more than one correct answer.

In the diagrams at right, angles  $a$  and  $b$  are:

**a** **A** equal**B** corresponding**C** allied**D** alternate**E** supplementary**b** **A** vertically opposite**B** corresponding**C** co-interior**D** alternate**E** equal**c** **A** corresponding**B** co-interior**C** alternate**D** supplementary**E** equal**d** **A** vertically opposite**B** corresponding**C** allied**D** alternate**E** supplementary

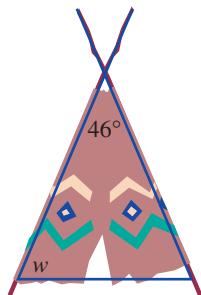
- 18** For each diagram:

- i state the type of angle relation that you use  
ii calculate the value of the pronumeral.

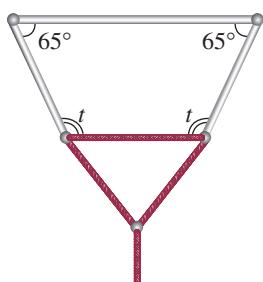
**a****b****c**

## PROBLEM SOLVING

- 1 The Indian teepee shown has an angle of  $46^\circ$  at its peak. What angle,  $w$ , does the wall make with the floor?



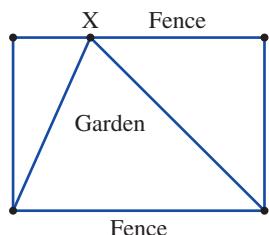
- 2 A circus trapeze attached to a rope is shown. Find the size of angle  $t$ .



- 3 Triangles can be classified according to the number of congruent sides (scalene, isosceles or equilateral) and by the type of angles (acute, obtuse or right). Decide which of the following combinations are possible, and draw an example.

Right-scalene, right-isosceles, right-equilateral  
Acute-scalene, acute-isosceles, acute-equilateral  
Obtuse-scalene, obtuse-isosceles, obtuse-equilateral.

- 4 Mark is building a triangular garden in his rectangular yard. He wants one side of the garden to lie along his fence line. The other corner of the garden (X) is to touch the opposite fence. He's trying to work out how much soil to order. Does his positioning of X change the amount of soil required?

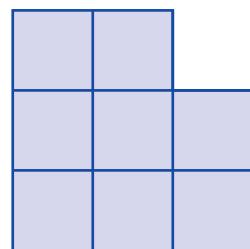


Mark's garden plan.

- 5 Jack wants to lay a rectangular concrete slab  $1.8 \text{ m} \times 1.2 \text{ m}$ . He has 2 pieces of timber  $1.8 \text{ m}$  long, and 2 pieces  $1.2 \text{ m}$  long. He positions

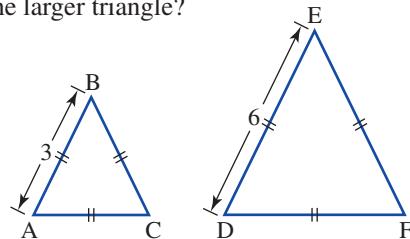
the timber so that they look like a rectangle, and he measures the diagonals to check. Explain why it is necessary to measure the diagonals if Jack wants to be sure that his slab is a rectangle.

- 6 Digital numbers are drawn using a number of horizontal and vertical strokes. Which digital numbers have the same number of strokes as the number itself?
- 7 What is the total number of right angles on the faces of a normal 6-sided die?
- 8 Show how you can use 9 matchsticks to construct a figure showing 5 equilateral triangles.
- 9 This figure has a small square removed from the right-hand corner.

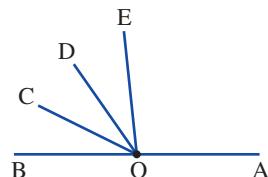


Make a copy. Draw two straight lines on the figure, cut along the lines, then rearrange the pieces to form a square.

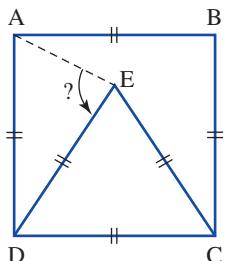
- 10 At midday, the hour hand, the minute hand and the second hand are all pointing to the 12. What is the angle between the hour hand and the minute hand after the second hand has completed 30 revolutions?
- 11 The triangles below are both equilateral triangles, but the side length of triangle ABC is 3 units and the side length of triangle DEF is 6 units.
- How many of the smaller triangles are required to fill the larger triangle?



- 12 If angle AOE is obtuse, name all the acute angles in the diagram below.

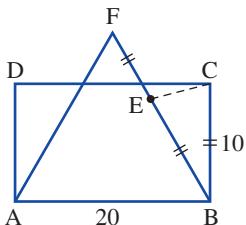


- 13** An equilateral triangle EDC is drawn inside a square ABCD as shown below. Determine the magnitude of the unknown angle below:



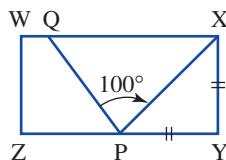
- 14** ABCD is a rectangle with a length of 20 and width of 10; that is,  $AB = 20$  and  $BC = 10$ .

ABF is an equilateral triangle that overlays the rectangle as shown. E is the midpoint of the side BF.



- a Determine the magnitude of  $\angle ECD$ .  
b Name all pairs of perpendicular line segments.

- 15** WXYZ is a rectangle as shown. Points P and Q are drawn so that  $XY = PY$  and  $\angle XPQ = 100^\circ$ .



Find the size of  $\angle PQX$

### eBook plus

#### Interactivities

- Test yourself
- Chapter 5
- int-1817
- Word search
- Chapter 5
- int-2591
- Crossword
- Chapter 5
- int-2592

### eBook plus

- #### Weblinks
- Guess the angle
  - Play pool Geometry puzzle
  - Angles.

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 5 (doc-6446) (*page 133*)

**Are you ready?****Digital docs (page 134)**

- SkillsHEET 5.1 (doc-6447) Reading the scale on a protractor
- SkillsHEET 5.2 (doc-6448) Comparing the size of angles
- SkillsHEET 5.3 (doc-6449) Estimating the size of an angle
- SkillsHEET 5.4 (doc-6450) Measuring angles with a protractor
- SkillsHEET 5.5 (doc-6451) Constructing angles with a protractor
- SkillsHEET 5.6 (doc-6452) Classifying angles
- SkillsHEET 5.7 (doc-6453) Naming angles
- SkillsHEET 5.8 (doc-6454) Classifying triangles according to the lengths of their sides
- SkillsHEET 5.9 (doc-6455) Classifying triangles according to the size of their angles

**5A Measuring angles****Interactivity**

- Digital protractor (int-2348) (*page 139*)

**Digital docs (page 139)**

- Activity 5-A-1 (doc-1737) Measuring angles
- Activity 5-A-2 (doc-1738) More measuring angles
- Activity 5-A-3 (doc-1739) Advanced measuring angles

**5B Constructing angles with a protractor****Interactivity**

- Bisectors (int-2347) (*page 144*)

**Digital docs (page 144)**

- Activity 5-B-1 (doc-1740) Constructing angles
- Activity 5-B-2 (doc-1741) More constructing angles
- Activity 5-B-3 (doc-1742) Advanced constructing angles

**5C Types of angles and naming angles****Digital docs**

- Activity 5-C-1 (doc-1743) Types of angles and naming angles (*page 146*)
- Activity 5-C-2 (doc-1744) More types of angles and naming angles (*page 146*)

- Activity 5-C-3 (doc-1745) Advanced types of angles and naming angles (*page 146*)
- WorkSHEET 5.1 (doc-1755) (*page 149*)

**5D Triangles****Digital docs (page 154)**

- Activity 5-D-1 (doc-1746) Triangles
- Activity 5-D-2 (doc-1747) More triangles
- Activity 5-D-3 (doc-1748) Advanced triangles

**5E Quadrilaterals and their properties****Interactivity**

- Angles in triangles and quadrilaterals (int-2739) (*page 160*)

**Digital docs (page 162)**

- Activity 5-E-1 (doc-1749) Quadrilaterals
- Activity 5-E-2 (doc-1750) More quadrilaterals
- Activity 5-E-3 (doc-1751) Advanced quadrilaterals

**5F Parallel and perpendicular lines****Interactivities (page 166)**

- Vertically opposite angles (int-0814)
- Corresponding angles (int-0815)
- Alternate angles (int-0816)
- Co-interior angles (int-0817)

**Digital docs**

- Activity 5-F-1 (doc-1752) Parallel and perpendicular lines (*page 168*)
- Activity 5-F-2 (doc-1753) More parallel and perpendicular lines (*page 168*)
- Activity 5-F-3 (doc-1754) Advanced parallel and perpendicular lines (*page 168*)
- WorkSHEET 5.2 (doc-6618) (*page 172*)
- Investigation (doc-6456) Parallel and perpendicular lines (*page 168*)

**Chapter review****Interactivities (page 179)**

- Test yourself Chapter 5 (int-1817) Take the end-of-chapter test to test your progress.
- Word search Chapter 5 (int-2591)
- Crossword Chapter 5 (int-2592)

**Weblinks (page 179)**

- Guess the angle
- Play pool
- Geometry puzzle
- Angles

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

## 6

# Decimals

STORE IN A COOL DRY PLACE. MADE AND PACKED IN AUSTRALIA.

(AVERAGE)			
Serving Size: 30g (2 biscuits) Servings per pack: 12	PER SERVE	TOTAL DAILY INTAKE (DI#)	PER 100g
<b>ENERGY</b>	447kJ 107Cal	8700kJ 2080Cal	1490kJ 356Cal
<b>PROTEIN</b>	3.7g	50g	12.4g
<b>FAT - TOTAL</b>	0.4g	70g	1.4g
- SATURATED FAT	0.1g	24g	0.3g
<b>CARBOHYDRATE - TOTAL</b>	20.1g	310g	67g
- SUGARS	1.0g	90g	3.3g
<b>DIETARY FIBRE</b>	3.3g	30g	11.0g
<b>SODIUM</b>	87mg	2300mg	290mg
<b>POTASSIUM</b>	102mg	% RDI*	PER 100g
<b>ZINC</b>	1.8mg	15% RDI*	340mg
<b>IRON</b>	3.0mg	25% RDI*	6.0mg
<b>MAGNESIUM</b>	32mg		10.0mg
<b>THIAMIN (Vitamin B1)</b>	0.55mg		
<b>RIBOFLAVIN (Vitamin B2)</b>	0.43mg		
<b>NIACIN (Vitamin B3)</b>	2.5mg		
<b>FOLATE</b>	100µg		

- 6A Place value and comparing decimals
- 6B Converting decimals to fractions and fractions to decimals
- 6C Rounding and repeating decimals
- 6D Adding and subtracting decimals
- 6E Multiplying decimals (including by multiples of 10)
- 6F Dividing decimals (including by multiples of 10)

**WHAT DO YOU KNOW?**

- 1 List what you know about decimals.  
Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of decimals.

**eBookplus**

Digital docs  
Hungry brain activity  
Chapter 6  
doc-6457

back for more nutrition information

**OPENING QUESTION**

In a 30-g serve, what is the total weight of the B group (B1, B2 and B3) vitamins received?

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can find extra help by completing the matching SkillsHEET located on your eBookPLUS.

- eBookplus**
- Digital docs**  
SkillsHEET 6.1  
doc-6458
- eBookplus**
- Digital docs**  
SkillsHEET 6.2  
doc-6459
- eBookplus**
- Digital docs**  
SkillsHEET 6.3  
doc-6460
- eBookplus**
- Digital docs**  
SkillsHEET 6.4  
doc-6461
- eBookplus**
- Digital docs**  
SkillsHEET 6.5  
doc-6462
- eBookplus**
- Digital docs**  
SkillsHEET 6.6  
doc-6463
- eBookplus**
- Digital docs**  
SkillsHEET 6.7  
doc-6464
- eBookplus**
- Digital docs**  
SkillsHEET 6.8  
doc-6465
- eBookplus**
- Digital docs**  
SkillsHEET 6.9  
doc-6466
- eBookplus**
- Digital docs**  
SkillsHEET 6.10  
doc-6467
- eBookplus**
- Digital docs**  
SkillsHEET 6.11  
doc-6468
- Writing decimals**
- 1 Write each of the following as a decimal.
- a 3 tenths      b 5 hundredths      c 7 thousandths
- Comparing decimals**
- 2 Find the largest number in each of the following.
- a 1.25, 1.35, 1.32      b 0.438, 0.483, 0.484
- Rounding to the nearest whole number**
- 3 Round each of the following to the nearest whole number.
- a 4.7      b 18.29      c 56.51
- Adding decimals (same number of decimal places)**
- 4 Rewrite the following in columns, then add.
- a  $4.3 + 2.4$       b  $0.52 + 0.85$       c  $7.43 + 6.59$
- Adding decimals (different number of decimal places)**
- 5 Rewrite the following in columns, then add.
- a  $2.76 + 1.2$       b  $5.2 + 0.376$       c  $15.37 + 11.4$
- Subtracting decimals (same number of decimal places)**
- 6 Rewrite the following in columns, then subtract.
- a  $9.6 - 4.3$       b  $0.87 - 0.55$       c  $12.31 - 7.14$
- Subtracting decimals (different number of decimal places)**
- 7 Rewrite the following in columns, then subtract.
- a  $8.25 - 2.7$       b  $1.3 - 0.85$       c  $26.796 - 19.3$
- Multiplying decimals by a single-digit number**
- 8 Calculate the following.
- a  $2.5 \times 2$       b  $3.68 \times 3$       c  $12.71 \times 5$
- Multiplying decimals by 10, 100, 1000**
- 9 Calculate the following.
- a  $1.456 \times 10$       b  $0.351 \times 100$       c  $16.039 \times 1000$
- Dividing decimals by a single-digit number**
- 10 Calculate the following.
- a  $6.9 \div 3$       b  $8.32 \div 2$       c  $15.735 \div 5$
- Dividing decimals by 10, 100, 1000**
- 11 Calculate the following.
- a  $14.35 \div 10$       b  $27.1 \div 100$       c  $38.664 \div 1000$

# 6A Place value and comparing decimals

## Whole numbers

eBook plus

eLesson

Place value  
eles-0004

- The position of a digit within a number indicates the value of the digit.

- The further the digit is to the left of the number, the larger the place value.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
100 000	10 000	1000	100	10	1

- Each place to the left of another has a value which is 10 times larger.
- Each place to the right of another has a value which is  $\frac{1}{10}$  of the previous position.
- In the following numbers, look at the value of the 3.

Number	Value of 3 in number
132	3 tens or 30
3217	3 thousands or 3000
4103	3 units (ones) or 3

## Decimal parts

- Whole numbers have units as their smallest place value.
- To show values smaller than units, a decimal is placed after the units.
- The value of the positions to the left and right of the decimal point are shown in the table below.

Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandsths	Ten thousandths
1000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$

- In the following numbers, look at the value of the 3.

Number	Value of 3 in number
14.32	the 3 is 3 tenths or $\frac{3}{10}$
106.013	the 3 is 3 thousandths or $\frac{3}{1000}$
0.000 03	the 3 is 3 hundred thousandths or $\frac{3}{100000}$ .

### WORKED EXAMPLE 1

Find the value of the 7 in each of the following.

- a 10.74      b 0.173      c 321.037

#### THINK

- a The value of the first place to the right of the decimal point is tenths, so the digit is tenths.
- b The second place after the decimal point is hundredths, so the digit is hundredths.
- c The digit is in the thousandths place, so the digit is thousandths.

#### WRITE

a  $\frac{7}{10}$

b  $\frac{7}{100}$

c  $\frac{7}{1000}$

**WORKED EXAMPLE 2**

For the number 76.204 write the value of each digit in words and numbers.

**THINK**

- 1 7 is in the tens position.
- 2 6 is in the units position.
- 3 2 is in the first position after the decimal point, so it is tenths.
- 4 0 is in the hundredths position.
- 5 4 is in the thousandths position.

**WRITE**

- seventy, 70  
six, 6  
two tenths,  $\frac{2}{10}$   
zero hundredths,  $\frac{0}{100}$   
four thousandths,  $\frac{4}{1000}$

- The number 76.204 can be written in expanded notation as:

$$(7 \times 10) + (6 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(4 \times \frac{1}{1000}\right)$$

**WORKED EXAMPLE 3**

Write 3.4501 in expanded notation.

**THINK**

- 1 Write the decimal.
- 2 Find the place value of each digit.  
3: 3 units = 3, 4: 4 tenths =  $\frac{4}{10}$ ,  
5: 5 hundredths =  $\frac{5}{100}$ , 0: 0 thousandths,  
1: 1 ten thousandth =  $\frac{1}{10000}$ .
- 3 Write the number in expanded notation.

**WRITE**

$$\begin{aligned} 3.4501 &= 3 \text{ units} + 4 \text{ tenths} + 5 \text{ hundredths} \\ &\quad + 0 \text{ thousandths} + 1 \text{ ten thousandth} \\ &= 3 + \frac{4}{10} + \frac{5}{100} + \frac{0}{1000} + \frac{1}{10000} \\ &= (3 \times 1) + (4 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (1 \times \frac{1}{10000}) \end{aligned}$$

**Comparing decimals**

- To compare the size of numbers that include decimal digits ('decimal numbers' or 'decimals'), it is necessary to compare the size of the digits with the same place value.
- First, compare the whole number part (digits to the left of the decimal point). If they are the same for each number, move to the first digit after the decimal point, then the second, the third and so on until the digits are different. The larger digit will belong to the larger number.

**WORKED EXAMPLE 4**

Find the largest number in each of the following.

- a 0.126, 0.216, 0.122    b 2.384, 2.388, 2.138    c 0.506, 0.605, 0.612

**THINK**

- a 1 As the units digit is 0 in each number, compare the tenths. The number 0.216 has 2 tenths, the others have 1 tenth so 0.216 is the largest number.
- 2 Answer the question.

**WRITE**

- a 0.216 is larger than 0.126 and 0.122.  
The largest number is 0.216.

- b** ① As the units digits are the same, compare the tenths and eliminate the smallest number.  
 ② The hundredths are the same so compare the thousandths and decide which number is bigger.  
 ③ Answer the question.
- c** ① As the unit digit is 0 compare the tenths and eliminate the smallest number.  
 ② Compare the hundredths and find the biggest number.  
 ③ Answer the question.

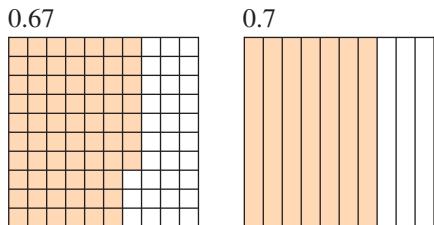
- b** 2.384 and 2.388 are both larger than 2.138.  
 2.388 is larger than 2.384 and 2.138.  
 The largest number is 2.388.
- c** 0.605 and 0.612 are larger than 0.506.  
 0.612 is larger than 0.605 and 0.506.  
 The largest number is 0.612.

■ Decimals can also be compared using the *area model*.

For example, we could use the area model to determine whether 0.67 or 0.7 is larger.

From the area model at right, it can be seen that 0.7 covers a greater area than 0.67 and is therefore larger.

- When comparing two numbers, it is easier to use symbols instead of words, as shown in Chapter 4 on fractions.
- In the worked example, the result ‘0.216 is larger than 0.126’ could be written as  $0.216 > 0.126$ . We could also say that 0.126 is less than 0.216, or write it as  $0.126 < 0.216$ .



### WORKED EXAMPLE 5

Insert the appropriate < or > sign between the following pairs of numbers to make true statements.

**a** 0.312    0.318      **b** 0.0246    0.0168

#### THINK

- a** Compare the numbers. Both numbers have the same number of tenths and the same number of hundredths, so compare thousandths and insert the correct sign.
- b** Compare the numbers. Both numbers have no tenths, so compare hundredths and insert the correct sign.

#### WRITE

**a**  $0.312 < 0.318$

**b**  $0.0246 > 0.0168$

#### REMEMBER

- The position of a digit within a number indicates the value of the digit. Each place is 10 times larger than the one immediately to the right. The positions are shown in the table below.

Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousands	Ten thousands
-----------	----------	------	-------	---	--------	------------	-----------	---------------

- The number of decimal places is the number of digits after the decimal point.
- Zeros at the end of a decimal do not change the size of the number.
- To compare the size of decimals, it is necessary to compare the size of the digits with the same place value.
- The symbol ‘<’ means ‘is less than’.
- The symbol ‘>’ means ‘is greater than’.

## EXERCISE

## 6A Place value and comparing decimals

## INDIVIDUAL PATHWAYS

**eBookplus****Activity 6-A-1**Place value and  
comparing decimals  
doc-1771**Activity 6-A-2**More place value and  
comparing decimals  
doc-1772**Activity 6-A-3**Advanced place  
value and comparing  
decimals  
doc-1773

## FLUENCY

- 1 WE1** Find the value of the 2 in each of the following.
- |             |            |          |              |
|-------------|------------|----------|--------------|
| a 5.2       | b 19.12    | c 0.02   | d 100.29     |
| e 0.982 047 | f 491.7521 | g 6.1342 | h 90.0002    |
| i 27.003    | j 12.14    | k 1.8902 | l 897.014 12 |
- 2** Find the value of the 9 in each of the following.
- |          |            |          |            |
|----------|------------|----------|------------|
| a 0.9    | b 14.98    | c 6.1937 | d 18.89    |
| e 12.090 | f 0.009 14 | g 3.4629 | h 1.276 89 |
| i 39.214 | j 9        | k 900.76 | l 90.612   |
- 3 WE2** For the following numbers write the value of each digit in *words* and *numbers*.
- |          |            |         |         |
|----------|------------|---------|---------|
| a 4.1    | b 1.85     | c 0.271 | d 9.020 |
| e 16.001 | f 3.402 07 |         |         |
- 4** For each of the following numbers write the value of each digit in *numbers*.
- |          |            |          |              |
|----------|------------|----------|--------------|
| a 0.4    | b 2.7      | c 6.80   | d 5.23       |
| e 0.763  | f 2.108    | g 19.910 | h 0.1101     |
| i 7.2964 | j 0.330 24 | k 300.03 | l 12.276 403 |
- 5 WE3** Write the following numbers in expanded notation.
- |          |          |          |          |
|----------|----------|----------|----------|
| a 2.47   | b 3.69   | c 1.25   | d 56.01  |
| e 39.01  | f 16.07  | g 7.123  | h 5.987  |
| i 13.482 | j 0.3062 | k 0.1305 | l 0.5002 |
- 6 MC**
- a Four tenths, 1 hundredth and 3 thousandths equals:  
A 4.13      B 3.14      C 0.413      D 0.314      E 314
  - b Five hundredths, 2 thousandths and 7 ten thousandths equals:  
A 527      B 52.7      C 5.27      D 0.0527      E 0.527
- 7** Copy and complete the table by putting only one digit in each box.
- |         | Tens   | Units | .<br>Tenths | Hundredths | .<br>Thousands |
|---------|--------|-------|-------------|------------|----------------|
| Example | 37.684 | 3     | 7           | 6          | 8              |
| a       | 0.205  |       | .           |            |                |
| b       | 1.06   |       | .           |            |                |
| c       | 74.108 |       | .           |            |                |
| d       | 0.108  |       | .           |            |                |
| e       | 50.080 |       | .           |            |                |
- 8 WE4** Find the largest number in each of the following.
- |                                |                                |
|--------------------------------|--------------------------------|
| a 0.24, 0.32, 0.12             | b 0.76, 0.68, 0.91             |
| c 0.57, 0.51, 0.59             | d 0.92, 0.99, 0.93             |
| e 0.192, 0.191, 0.901          | f 0.660, 0.666, 0.662          |
| g 0.1440, 0.1436, 0.1433       | h 0.0392, 0.039 90, 0.0039     |
| i 2.506, 2.305, 2.559          | j 10.0023, 10.0231, 10.0233    |
| k 0.110 43, 0.110 49, 0.110 40 | l 0.102 36, 0.100 23, 0.101 09 |
- 9 WES** Insert the appropriate < or > sign between each of the following pairs of numbers to make true statements.
- |                  |                  |                  |
|------------------|------------------|------------------|
| a 3.2      2.9   | b 8.6      8.9   | c 1.27      1.28 |
| d 0.64      0.67 | e 1.37      1.41 | f 0.29      0.39 |

<b>g</b>	13.103	13.112	<b>h</b>	0.427	0.424	<b>i</b>	0.580	0.508
<b>j</b>	0.0101	0.0120	<b>k</b>	0.04801	0.4801	<b>l</b>	1.3830	1.3824

**10** Write the following in order from smallest to largest (ascending order).

- a** 0.21, 0.39, 0.17, 0.45, 0.33
- b** 0.16, 0.19, 0.27, 0.12, 0.22
- c** 0.314, 0.413, 0.420, 0.391, 0.502
- d** 0.613, 0.624, 0.690, 0.710, 0.677
- e** 0.821, 0.803, 0.811, 0.807, 0.902
- f** 0.1164, 0.1245, 0.1033, 0.1002, 0.1196
- g** 0.9864, 0.9812, 0.9943, 0.9087, 0.9189
- h** 0.4004, 0.4139, 0.4826, 0.4100, 0.4076
- i** 4.6249, 4.5097, 4.802, 4.6031, 4.0292
- j** 13.0294, 13.0291, 13.0229, 13.0299, 13.0929
- k** 0.00465, 0.00502, 0.003, 0.0056, 0.009
- l** 0.507, 0.61, 0.595, 0.5079, 0.617

**11** Write the following in order from largest to smallest (descending order).

- a** 0.36, 0.31, 0.39, 0.48, 0.19
- b** 0.27, 0.38, 0.16, 0.02, 0.35
- c** 0.91, 0.97, 0.90, 0.95, 0.99
- d** 0.02, 0.29, 0.07, 0.13, 0.09
- e** 1.264, 1.279, 1.273, 1.291, 1.288
- f** 0.442, 0.437, 0.491, 0.406, 0.433
- g** 0.372, 0.318, 0.390, 0.309, 0.317
- h** 0.502, 0.556, 0.573, 0.602, 0.591
- i** 0.8207, 0.8889, 0.8823, 0.8217, 0.8448
- j** 0.7657, 0.6024, 0.0307, 0.1079, 0.7695
- k** 1.349 54, 1.486 59, 1.702 96, 1.843 21, 1.486 13
- l** 12.289 50, 12.208 64, 12.392, 12.002 36, 12.9092

**12 MC** The largest number in the following list; 0.4261, 0.4265, 0.4273, 0.4199, 0.3999 is:

- A** 0.4261      **B** 0.4199      **C** 0.4265      **D** 0.3999      **E** 0.4273

**13 MC** The smallest number in the following list; 0.4261, 0.4265, 0.4273, 0.4199, 0.3999 is:

- A** 0.4261      **B** 0.4199      **C** 0.4265      **D** 0.3999      **E** 0.4273

**14 MC** The following list; 0.4261, 0.4265, 0.4273, 0.4199, 0.3999 when arranged from smallest to largest is:

- A** 0.4273, 0.4265, 0.4261, 0.4199, 0.3999
- B** 0.4273, 0.4261, 0.4265, 0.4199, 0.3999
- C** 0.3999, 0.4199, 0.4265, 0.4261, 0.4273
- D** 0.3999, 0.4199, 0.4261, 0.4273, 0.4265
- E** 0.3999, 0.4199, 0.4261, 0.4265, 0.4273

### UNDERSTANDING

**15** For each of the following numbers:

- i** State the place value of the zero.
  - ii** Would the value of the number change if the zero wasn't there? (Write yes or no.)
- |               |                  |                |                 |
|---------------|------------------|----------------|-----------------|
| <b>a</b> 6.02 | <b>b</b> 10.49   | <b>c</b> 7.360 | <b>d</b> 13.10  |
| <b>e</b> 4.0  | <b>f</b> 133.027 | <b>g</b> 0.65  | <b>h</b> 17.809 |
| <b>i</b> 20   | <b>j</b> 108.62  |                |                 |

**16** Write true (T) or false (F) for each of the following.

- a** 76.34 has 4 decimal places.
- b**  $\frac{6}{10} + \frac{3}{100} + \frac{4}{10000}$  is the same as 0.6304.
- c** 4.03 has the same value as 4.3.

- d 29.60 has the same value as 29.6.
- e 1.2804 could be written as  $1 + \frac{3}{10} + \frac{8}{100} + \frac{4}{1000}$ .
- f 1090.26451 has 5 decimal places.

**REASONING**

- 17 Year 7 girls competing in their school swimming sports recorded the following times in the 50-metre freestyle, backstroke and breaststroke events.

Event	Time (seconds) recorded by contestants							
	Carolyn	Jessica	Mara	Jenika	Robyn	Shelley	Kyah	
Freestyle	37.23	39.04	40.90	38.91	37.45	37.02	37.89	
Backstroke	40.23	43.87	44.08	42.65	41.98	40.29	41.05	
Breaststroke	41.63	42.7	41.10	41.21	42.66	41.33	41.49	

- a Who won the freestyle event? How much did she win it by?
- b Who won the backstroke event? How much did she win it by?
- c Who won the breaststroke event? How much did she win it by?
- d List the first 3 placings of the freestyle event.
- e List the first 3 placings of the backstroke event.
- f List the first 3 placings of the breaststroke event.
- g Did any students obtain a placing in all three events?

**REFLECTION**

What strategies will you use to find the smallest decimal?

## 6B Converting decimals to fractions and fractions to decimals

### Converting decimals to fractions

- Decimals can be written as single fractions by using place values.

Number	Ones	Tenths	Hundredths
0.3	0.	3	0
1.25	1.	2	5

- The decimal 0.3 can be written as  $\frac{3}{10}$ .
- The decimal 1.25 can be thought of as  $1 + \frac{2}{10} + \frac{5}{100} = 1 + \frac{20}{100} + \frac{5}{100} = 1\frac{25}{100}$ .
- In practice, the digits after the decimal point become the numerator and the place value of the last digit gives the denominator.
- Once decimals have been written as fractions or mixed numbers, they can be simplified by dividing the denominator and numerator by the highest common factor.

- Sometimes the highest common factor may not be obvious, and extra steps may be required to obtain the simplest form. For example, when simplifying  $1\frac{25}{100}$  the following steps could be used:  $1\frac{25}{100} = 1\frac{5}{20} = 1\frac{1}{4}$ .

**WORKED EXAMPLE 6**

Write the following decimals as fractions, then simplify where appropriate.

- a** 0.2      **b** 0.86      **c** 0.6021

**THINK**

- a**
- 1 Write the decimal.
  - 2 The numerator is 2 and the last decimal place is tenths so the denominator is 10.
  - 3 Divide the numerator and the denominator by the highest common factor.
  - 4 Simplify the fraction.
- b**
- 1 Write the decimal.
  - 2 The numerator is 86. The last decimal place is hundredths so the denominator is 100.
  - 3 Repeat steps 3 and 4 of part **a**.
- c**
- 1 Write the decimal.
  - 2 The numerator is 6021. The last place is tens of thousandths so the denominator is 10 000.

**WRITE**

**a** 0.2  
 $= \frac{2}{10}$   
 $= \frac{1}{5}$

**b** 0.86  
 $= \frac{86}{100}$   
 $= \frac{43}{50}$

**c** 0.6021  
 $= \frac{6021}{10000}$

**WORKED EXAMPLE 7**

Write each of the following as a mixed number in its simplest form.

- a** 3.041      **b** 7.264

**THINK**

- a**
- 1 Write the decimal.
  - 2 Write the whole number part and change the decimal part to a fraction. The numerator is 41. The last decimal place is thousandths so the denominator is 1000.
- b**
- 1 Write the decimal.
  - 2 Write the whole number part and change the decimal part to a fraction. The numerator is 264 and the denominator is 1000.
  - 3 Divide the numerator and the denominator by the highest common factor.
  - 4 Simplify the fraction.

**WRITE**

**a** 3.041  
 $= 3\frac{41}{1000}$

**b** 7.264  
 $= 7\frac{264}{1000}$   
 $= 7\frac{264}{1000} - \frac{8}{8}$   
 $= 7\frac{33}{125}$

## Converting fractions to decimals

■ To change any fraction into a decimal, divide the denominator into the numerator. For example, to change  $\frac{1}{4}$  into a decimal, divide 4 into 1:

- $\bullet \quad \begin{array}{r} 0.25 \\ 4 \overline{) 1.00} \end{array}$  add a decimal point and as many zeros as required.

### WORKED EXAMPLE 8

Change the following fractions into finite decimals.

a  $\frac{2}{5}$       b  $\frac{1}{8}$

#### THINK

- a 1 Set out the question as for division of whole numbers, adding a decimal point and the required number of zeros. Note:  $\frac{2}{5} = 2 \div 5$ .
- a 2 Divide, writing the answer with the decimal point exactly in line with the decimal point in the question.
- b 1 Set out the question as for division of whole numbers, adding a decimal point and the required number of zeros. Note:  $\frac{1}{8} = 1 \div 8$ .
- b 2 Divide, writing the answer with the decimal point exactly in line with the decimal point in the question.

#### WRITE

a  $\begin{array}{r} 0.4 \\ 5 \overline{) 2.0} \end{array}$

$\frac{2}{5} = 0.4$

b  $\begin{array}{r} 0.125 \\ 8 \overline{) 1.000} \end{array}$

$\frac{1}{8} = 0.125$

■ By knowing the decimal equivalent of any fraction, it is possible to determine the equivalent of any multiple of that fraction. The following worked example illustrates this.

### WORKED EXAMPLE 9

Use the results of Worked example 8 to find decimal equivalents for:

a  $\frac{3}{8}$       b  $4\frac{5}{8}$

#### THINK

- a 1 Write the decimal equivalent for the fraction with 1 as the numerator.
- a 2 Multiply both sides of this equation by the appropriate multiple (3 in this case).
- a 3 Write the answer.
- b 1 Consider only the fraction part of the mixed number. Write the decimal equivalent of this fraction with 1 as the numerator.
- b 2 Multiply both sides of this equation by the appropriate multiple (5 in this case).
- b 3 Simplify both sides.
- b 4 Combine with the whole number and write the answer.

#### WRITE

a  $\frac{1}{8} = 0.125$

$\frac{1}{8} \times 3 = 0.125 \times 3$

$\frac{3}{8} = 0.375$

b  $\frac{1}{8} = 0.125$

$\frac{1}{8} \times 5 = 0.125 \times 5$

$\frac{5}{8} = 0.625$

$4\frac{5}{8} = 4.625$

**REMEMBER**

To write decimals as single fractions:

1. Write the whole number part first.
2. Write all decimal places as the numerator of the fraction.
3. Write the denominator as the place value of the last digit.
4. Simplify the fraction if required.
5. To change any fraction into a decimal, divide the denominator into the numerator.

**EXERCISE****6B**

## Converting decimals to fractions and fractions to decimals

**INDIVIDUAL PATHWAYS****eBook plus****Activity 6-B-1**

Toucan colouring  
puzzle  
doc-1774

**Activity 6-B-2**

Tiger colouring  
puzzle  
doc-1775

**Activity 6-B-3**

Soccer colouring  
puzzle  
doc-1776

**FLUENCY**

- 1 WE6** Write the following decimals as fractions, then simplify where appropriate.

- |          |          |          |
|----------|----------|----------|
| a 0.3    | b 0.5    | c 0.9    |
| d 0.21   | e 0.4    | f 0.8    |
| g 0.24   | h 0.44   | i 0.49   |
| j 0.63   | k 0.502  | l 0.617  |
| m 0.12   | n 0.30   | o 0.64   |
| p 0.28   | q 0.882  | r 0.9456 |
| s 0.9209 | t 0.4621 | u 0.75   |
| v 0.120  | w 0.286  | x 0.468  |

- 2 WE7** Write the following decimals as mixed numbers in simplest form.

- |          |          |          |            |
|----------|----------|----------|------------|
| a 1.3    | b 1.6    | c 2.7    | d 9.4      |
| e 1.2    | f 2.8    | g 4.2    | h 8.5      |
| i 2.13   | j 6.48   | k 5.27   | l 19.182   |
| m 12.42  | n 3.15   | o 6.25   | p 9.140    |
| q 12.843 | r 16.682 | s 2.4917 | t 4.3386   |
| u 37.205 | v 18.645 | w 24.345 | x 100.0048 |

- 3 MC** a 0.13 as a fraction is:

- A  $\frac{13}{10}$       B  $\frac{13}{100}$       C  $\frac{13}{1000}$       D  $1\frac{3}{10}$       E  $1\frac{3}{10}$

- b 0.207 as a fraction is:

- A  $\frac{207}{1000}$       B  $\frac{207}{100}$       C  $2\frac{7}{10}$       D  $20\frac{7}{10}$       E 207

- c 0.52 as a fraction in simplest form is:

- A  $\frac{52}{100}$       B  $\frac{26}{50}$       C  $\frac{13}{25}$       D  $\frac{26}{100}$       E  $\frac{13}{50}$

- d 0.716 as a fraction in simplest form is:

- A  $\frac{716}{10\ 000}$       B  $\frac{368}{500}$       C  $\frac{716}{1000}$       D  $\frac{179}{250}$       E  $\frac{358}{1000}$

- e 5.325 as a fraction in simplest form is:

- A  $\frac{5325}{1000}$       B  $\frac{325}{1000}$       C  $5\frac{325}{1000}$       D  $5\frac{65}{200}$       E  $5\frac{13}{40}$

- 4 WE8, 9** Change the following fractions to decimals.

- |                  |                 |                  |                  |                  |
|------------------|-----------------|------------------|------------------|------------------|
| a $\frac{3}{4}$  | b $\frac{1}{2}$ | c $\frac{4}{5}$  | d $\frac{1}{20}$ | e $\frac{3}{15}$ |
| f $\frac{3}{12}$ | g $\frac{3}{8}$ | h $\frac{1}{50}$ | i $\frac{8}{25}$ |                  |

## UNDERSTANDING

- 5 Write  $\frac{1}{8}$  as a decimal. Using this value, find:

- a  $\frac{3}{8}$  as a decimal.
- b  $\frac{7}{8}$  as a decimal.
- c  $\frac{1}{16}$  as a decimal.

## REASONING

- 6 Consider these fractions:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$$

Without performing any division, you can see that  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  and  $\frac{1}{8}$  will produce terminating decimals, while  $\frac{1}{3}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$  and  $\frac{1}{9}$  will produce repeating decimals. Explain how this can be seen, and write a general statement to determine whether a fraction will produce a terminating or repeating decimal.

## REFLECTION

If you need to choose between a fraction and a decimal, when is a fraction a better choice and when is a decimal a better choice?

## 6C Rounding and repeating decimals

### Rounding

- When **rounding** decimals, look at the first digit past the number of decimal places required.
- If this digit is *less than 5*, write the number with the required number of decimal places.
- If this digit is *5 or more*, add one to the digit in the last decimal place being kept.

#### WORKED EXAMPLE 10

Round the following to 2 decimal places.

a 3.641 883      b 18.965 402 0

#### THINK

- a 1 Write the number and underline the required decimal place.
- 2 Circle the next digit and round according to the rule.  
*Note:* Since the circled digit is less than 5, we leave the number as it is.
- b 1 Write the number and underline the required decimal place.
- 2 Circle the next digit and round according to the rule.  
*Note:* Since the circled digit is greater than or equal to 5, add 1 to the last decimal place that is being kept.

#### WRITE

a 3.641 883  
 $= 3.6\textcolor{red}{4}\textcolor{blue}{1} 883$   
 $\approx 3.64$

b 18.965 402 0  
 $= 18.9\textcolor{red}{6}\textcolor{blue}{5} 402 0$   
 $\approx 18.97$

- If you need to add 1 to the last decimal place and the digit in this position is a 9, the result is 10. The 0 is put in the last required place and the 1 is added to the digit in the next place to the left.
- 0.298 rounded to 2 decimal places is 0.30.

**WORKED EXAMPLE 11**

Round the following to the number of decimal places shown in the brackets.

- a 27.462 973 (4)      b 0.009 94 (3)

**THINK**

- a 1 Write the number and underline the required decimal place.
- 2 Circle the next digit and round according to the rule.

*Note:* Since the circled digit is greater than 5, add 1 to the last decimal place that is being kept. As 1 is being added to 9, write 0 in the last place and add 1 to the previous digit.

- b Repeat steps 1 and 2 of part a.

**WRITE**

a 27.462 973

$$\begin{aligned} &= 27.462 \underline{9} \textcircled{7} 3 \\ &\approx 27.4630 \end{aligned}$$

b 0.009 94

$$\begin{aligned} &= 0.009 \underline{9} \textcircled{4} \\ &\approx 0.010 \end{aligned}$$

**WORKED EXAMPLE 12**

Round 8.672 to the nearest unit.

**THINK**

- 1 Write the decimal and think of the number with the whole number part only.
- 2 Look at the first digit after the decimal point and, if it is greater than or equal to 5, add 1 to the whole number.

**WRITE**

8.672

≈ 9

- When trying to answer Worked example 12, you can think of the question as: ‘Is 8.672 closer to 8 or 9?’

**WORKED EXAMPLE 13**

Melinda had \$51.67 in her bank account. She wanted to withdraw all her money so the bank rounded the amount to the nearest 5 cents. How much money did the teller give to Melinda?

**THINK**

- 1 Write the actual amount she had in her account.
- 2 Determine whether the last digit is closer to 5 or closer to 10, then rewrite the approximate value.  
*Note:* Alternatively it can be seen that 67 cents is closer to 65 cents than 70 cents.
- 3 Write a sentence.

**WRITE**

\$51.67

≈ \$51.65

Melinda will receive \$51.65 from the bank.

## Repeating decimals

- When a decimal number ends (terminates) after any number of decimal places, it is said to be a **finite decimal**. The decimal 0.124 is an example of a finite decimal.
- Sometimes when we divide the denominator into the numerator, the answer keeps repeating and the amount left over each time keeps repeating too. When this happens, the answer is called an **infinite recurring decimal**.

### WORKED EXAMPLE 14

Convert  $\frac{1}{11}$  to a decimal. Continue dividing until a pattern emerges, then round the answer to 2 decimal places.

#### THINK

- Set out the question as for division of whole numbers, adding a decimal point and enough zeros to see a pattern emerging.
- Divide, writing the answer with the decimal point exactly in line with the decimal point in the question. (The amount left over each time is 10 then 1 then 10 then 1 again. The decimal answer is also repeating.)
- Write the approximate answer rounded to 2 decimal places.

#### WRITE

$$\begin{array}{r} 0.09090909\dots \\ 11 \overline{)1.000000000000000} \\ \end{array}$$

$$\frac{1}{11} \approx 0.09$$

- Recurring decimals** can be written in one of the following shorter ways for an exact answer:
  - 4.6666... could be written as 4. $\dot{6}$  (with a dot above the repeating part of the decimal).
  - 3.512512... could be written as 3. $\dot{5}1\dot{2}$  (with a dot above the first and last digits of the repeating part).
  - 6.121212... could be written as 6. $\overline{12}$  (with a line above the repeating part of the decimal).
  - Like finite decimals, the decimal equivalent of a fraction can be used to determine the decimal equivalent of any multiple of that fraction.

### WORKED EXAMPLE 15

Use the result from Worked example 12 to find the decimal equivalent for  $\frac{6}{11}$ .

#### THINK

- Write the decimal equivalent for the fraction with 1 as the numerator. In this case it is an infinite recurring decimal therefore 0.090909... can be written as 0. $\overline{09}$ .
- Multiply both sides of this equation by the appropriate multiple (6 in this case).
- Simplify and write the answer.

#### WRITE

$$\frac{1}{11} = 0.090909\dots$$

$$\begin{aligned} \frac{1}{11} \times 6 &= 0.090909\dots \times 6 \\ &= 0.545454\dots \end{aligned}$$

$$\frac{6}{11} = 0.\overline{54}$$

**REMEMBER**

- To round a decimal:  
Look at the first digit past the number of decimal places required.  
 (a) If this number is *less than 5*, write the number with the number of decimal places required.  
 (b) If this number is *5 or more*, add 1 to the last decimal place being kept.  
 If you add 1 to the last decimal place and the number in this position is a 9, the result is 10. The 0 is put in the last required place and the 1 is added to the digit in the next place to the left.
- Rounding an answer gives a useful approximation of the original decimal. Use the symbol  $\approx$  instead of  $=$  to show that the answer is an approximation.
- A *finite decimal* terminates after a particular number of decimal places.
- An *infinite recurring decimal* is obtained when the decimal places in the answer keep repeating and the amount left over each time also keeps repeating. An approximate answer can be found by rounding so that the answer includes one complete cycle of the pattern.
- An *infinite recurring decimal* can be written exactly by placing a dot above the first and last digits of the repeating part or by drawing a line above the repeating part.

**EXERCISE****6C****Rounding and repeating decimals****INDIVIDUAL PATHWAYS****eBookplus**

**Activity 6-C-1**  
Rounding and  
repeating decimals  
doc-1777

**Activity 6-C-2**  
More rounding and  
repeating decimals  
doc-1778

**Activity 6-C-3**  
Advanced rounding  
and repeating  
decimals  
doc-1779

**FLUENCY**

- WE10** Round the following to 2 decimal places.
 

<b>a</b> 0.3241	<b>b</b> 0.863	<b>c</b> 1.246 10
<b>d</b> 13.049 92	<b>e</b> 7.128 63	<b>f</b> 100.813 82
<b>g</b> 71.260 39	<b>h</b> 0.0092	<b>i</b> 0.185 00
<b>j</b> 19.6979	<b>k</b> 0.3957	<b>l</b> 0.999
- Round the following to 1 decimal place.
 

<b>a</b> 0.410	<b>b</b> 0.87	<b>c</b> 9.27
<b>d</b> 25.25	<b>e</b> 300.06	<b>f</b> 12.82
<b>g</b> 99.91	<b>h</b> 8.88	<b>i</b> 17.610 27
<b>j</b> 0.8989	<b>k</b> 93.994	<b>l</b> 0.959 027
- WE11** Round the following to the number of decimal places shown in the brackets.
 

<b>a</b> 2.386 214 (2)	<b>b</b> 14.034 59 (1)
<b>c</b> 0.027 135 (2)	<b>d</b> 0.876 4903 (4)
<b>e</b> 64.295 18 (4)	<b>f</b> 0.382 04 (3)
<b>g</b> 96.28 049 (1)	<b>h</b> 3.0409 (2)
<b>i</b> 8.902 (2)	<b>j</b> 47.879 69 (3)
<b>k</b> 0.099 498 632 (2)	<b>l</b> 0.486 2590 (2)
- MC**
  - 13.179 rounded to 2 decimal places is equal to:  
 A 13.17      B 13.20      C 13.18      D 13.27      E 13.19
  - 0.2465 rounded to 1 decimal place is equal to:  
 A 0.3      B 0.25      C 1.2      D 0.2      E 0.5
  - 1.7688 rounded to 3 decimal places is equal to:  
 A 1.768      B 1.770      C 1.778      D 1.769      E 1.800
  - 2.998 rounded to 1 decimal place is equal to:  
 A 3.0      B 2.9      C 2.8      D 3.1      E 3.9

**5 WE12** Round the following to the nearest unit.

- |                |                |                  |                  |
|----------------|----------------|------------------|------------------|
| <b>a</b> 10.7  | <b>b</b> 8.2   | <b>c</b> 3.6     | <b>d</b> 92.7    |
| <b>e</b> 112.1 | <b>f</b> 21.76 | <b>g</b> 42.0379 | <b>h</b> 2137.50 |
| <b>i</b> 0.12  | <b>j</b> 0.513 | <b>k</b> 0.99    | <b>l</b> 40.987  |

**6** Write the following infinite recurring decimals using one of the short forms.

- |                           |                           |                           |
|---------------------------|---------------------------|---------------------------|
| <b>a</b> 2.555 ...        | <b>b</b> 0.666 ...        | <b>c</b> 12.888 88 ...    |
| <b>d</b> 49.111 11 ...    | <b>e</b> 0.262 626 ...    | <b>f</b> 0.414 141 ...    |
| <b>g</b> 0.913 913 ...    | <b>h</b> 8.641 864 18 ... | <b>i</b> 0.040 121 21 ... |
| <b>j</b> 133.946 2462 ... | <b>k</b> 1.833 333 ...    | <b>l</b> 0.127 7777 ...   |

**7** Convert each of the following to a decimal. Continue dividing until a pattern emerges, then round the answer to the number of decimal places indicated in the brackets.

- |                             |                             |                             |
|-----------------------------|-----------------------------|-----------------------------|
| <b>a</b> $\frac{1}{6}$ (2)  | <b>b</b> $\frac{1}{3}$ (1)  | <b>c</b> $\frac{1}{9}$ (1)  |
| <b>d</b> $\frac{2}{15}$ (2) | <b>e</b> $\frac{2}{11}$ (2) | <b>f</b> $\frac{4}{9}$ (1)  |
| <b>g</b> $\frac{5}{12}$ (3) | <b>h</b> $\frac{1}{7}$ (6)  | <b>i</b> $\frac{7}{15}$ (2) |

**8 MC** **a**  $\frac{3}{5}$  as a decimal is:

- |  |                    |                 |                     |                      |                 |
|--|--------------------|-----------------|---------------------|----------------------|-----------------|
| <b>A</b> 0.3   | <b>B</b> 0.6       | <b>C</b> 0.2    | <b>D</b> 0.9        | <b>E</b> 0.5         |                 |
| <b>b</b> 1.8888 ... written as an exact answer is:     | <b>A</b> 1.8       | <b>B</b> 1.888  | <b>C</b> 1.88       | <b>D</b> 1.9         | <b>E</b> 1.889  |
| <b>c</b> 12.412 412 ... written as an exact answer is: | <b>A</b> 12.412    | <b>B</b> 12.412 | <b>C</b> 12.412 412 | <b>D</b> 12.412      | <b>E</b> 12.412 |
| <b>d</b> $\frac{3}{7}$ as a decimal in exact form is:  | <b>A</b> 0.428 571 | <b>B</b> 0.428  | <b>C</b> 0.4        | <b>D</b> 0.428 517 4 | <b>E</b> 2.3    |

**9** Find decimal equivalents for the following fractions. Give your answer in exact form as either a finite decimal or an infinite recurring decimal.

- |                        |                        |                        |                        |                        |
|------------------------|------------------------|------------------------|------------------------|------------------------|
| <b>a</b> $\frac{1}{2}$ | <b>b</b> $\frac{1}{3}$ | <b>c</b> $\frac{1}{4}$ | <b>d</b> $\frac{1}{5}$ | <b>e</b> $\frac{1}{9}$ |
|------------------------|------------------------|------------------------|------------------------|------------------------|

**10** Using the answers from question 5, find decimal equivalents for the following fractions.

- |                        |                         |                        |                         |                        |
|------------------------|-------------------------|------------------------|-------------------------|------------------------|
| <b>a</b> $\frac{7}{2}$ | <b>b</b> $4\frac{2}{3}$ | <b>c</b> $\frac{3}{4}$ | <b>d</b> $3\frac{4}{5}$ | <b>e</b> $\frac{7}{9}$ |
|------------------------|-------------------------|------------------------|-------------------------|------------------------|

### UNDERSTANDING

**11** Round the following to the nearest ten.

- |                 |                    |                |               |
|-----------------|--------------------|----------------|---------------|
| <b>a</b> 13     | <b>b</b> 76        | <b>c</b> 47    | <b>d</b> 138  |
| <b>e</b> 262    | <b>f</b> 175       | <b>g</b> 306.2 | <b>h</b> 1484 |
| <b>i</b> 10 024 | <b>j</b> 209 718.5 | <b>k</b> 18.6  | <b>l</b> 5.92 |

**12** Round the following to the nearest hundred.

- |               |                      |                |                   |
|---------------|----------------------|----------------|-------------------|
| <b>a</b> 320  | <b>b</b> 190         | <b>c</b> 894   | <b>d</b> 138      |
| <b>e</b> 125  | <b>f</b> 6751        | <b>g</b> 875.2 | <b>h</b> 9750.051 |
| <b>i</b> 1724 | <b>j</b> 1 462 836.5 | <b>k</b> 71    | <b>l</b> 47       |

**13** Round the following to the nearest thousand.

- |                 |               |                  |                  |
|-----------------|---------------|------------------|------------------|
| <b>a</b> 3426   | <b>b</b> 5890 | <b>c</b> 12 300  | <b>d</b> 18 640  |
| <b>e</b> 28 000 | <b>f</b> 9462 | <b>g</b> 1098    | <b>h</b> 496 830 |
| <b>i</b> 12 780 | <b>j</b> 862  | <b>k</b> 129 980 | <b>l</b> 49 899  |

**14 WE13** In the supermarket Christine's shopping bill came to \$27.68. As there are no 1- or 2-cent pieces, this amount must be rounded to the nearest 5 cents. How much will Christine pay for her shopping?

- 15** Using a calculator, Greg worked out that the piece of timber required to finish making a support for a gate should be 3.567 82 metres. Realistically, the timber can be measured only to the nearest millimetre (nearest thousandth of a metre). What measurement should be used for the length of the timber? Explain why 3.567 82 m is unreasonable as a measurement for timber.

- 16** Rank the following decimals from smallest to largest:

0.295, 0.29̄5, 0.2̄95, 0.2̄9̄5

- 17** Find a decimal that is greater than 0.5432̄ and less than 0.54322.

- 18** Barney rounds a decimal to 0.6 correct to one decimal place. Fred then says that the number before being rounded off could not have been bigger than 0.64.

Is Fred correct? Explain your answer.

### REASONING

- 19** The maximum temperature was recorded as 24.7 °C. In the news broadcast, the presenter quoted this to the nearest degree. What temperature was quoted?

### REFLECTION

Which number is bigger, 0.3 or 0.̄3? How do you know this?

## 6D Adding and subtracting decimals

### Adding decimals

- Decimals can be added using a method similar to that for whole numbers.
- Set out the addition in vertical columns and line up the decimal points so that the digits with the same place value are underneath each other.
- If the question is not written in columns, it is necessary to rewrite it with the decimal points lined up.

### Checking by estimating

- Answers to decimal addition and subtraction can be checked by estimating. Round each decimal to the nearest whole number or to a similar number of decimal places and then add or subtract them.

### WORKED EXAMPLE 16

Calculate:

<b>a</b>	<b>b</b>	<b>c</b>
<b>1.3</b>	<b>12.84</b>	<b>1.25</b>
<b>+0.5</b>	<b>+2.33</b>	<b>3.146</b>
—	—	—
		<b>+7.0</b>
		—

### THINK

- a** Copy the question exactly and add the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.
- b** Copy the question exactly and add the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.

### WRITE

<b>a</b>	<b>1.3</b>	<b>1.25</b>
	<b>+0.5</b>	<b>3.146</b>
	—	—
	<b>1.8</b>	<b>+7.0</b>
	—	—

<b>b</b>	<b>12.84</b>	<b>1.25</b>
	<b>+ 2.33</b>	<b>3.146</b>
	—	—
	<b>15.17</b>	<b>+7.0</b>
	—	—

- c 1 Write the question, replacing the spaces with zeros.
- 2 Add the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.
- c 1.250  
3.146  
+7.000  
11.396

**WORKED EXAMPLE 17****Rewrite in columns, then add  $0.26 + 1.8 + 12.214$ .****THINK**

- 1 Write the question in columns with the decimal points directly beneath each other with the zeros included.
- 2 Add the digits as for whole numbers. Write the decimal point directly below the decimal points in the question.
- 3 Check the answer by rounding to get an estimate and then adding  $0.3 + 2 + 12 = 14.3$  which is close to 14.274.

**WRITE**

$$\begin{array}{r} 0.260 \\ 1.800 \\ +12.214 \\ \hline 14.274 \end{array}$$

**Subtracting decimals**

- Decimals can be subtracted using a method similar to that for whole numbers.
- Set out the subtraction in vertical columns and line up the decimal points so that the digits with the same place value are underneath each other.
- If the question is not written in columns, it is necessary to rewrite it with the decimal points lined up.

**WORKED EXAMPLE 18****Calculate:**  $0.56 - 0.14$ **THINK**

- 1 Copy the question exactly and subtract the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.
- 2 Check the answer by rounding to get an estimate;  $0.6 - 0.1 = 0.5$  which is close to 0.42. (This step can be done in your head.)

**WRITE**

$$\begin{array}{r} 0.56 \\ -0.14 \\ \hline 0.42 \end{array}$$

**WORKED EXAMPLE 19****Rewrite the following in columns, then subtract.**

**a**  $1.82 - 0.57$

**b**  $2.641 - 0.85$

**THINK**

- a 1 Write in columns with the decimal points directly under each other. Subtract, and insert the decimal point directly below the other decimal points in the question.
- 2 Check the answer by rounding to get an estimate;  $2 - 1 = 1$  which is close to 1.25.

**WRITE**

$$\begin{array}{r} 1.82 \\ -0.57 \\ \hline 1.25 \end{array}$$

- b** 1 Write in columns with the decimal points directly under each other, adding zeros as appropriate. Subtract as for whole numbers and insert the decimal point directly below the other decimal points.

$$\begin{array}{r} \overset{5}{2.6} \\ -0.850 \\ \hline 1.791 \end{array}$$

- 2** Check the answer by rounding to get an estimate;  
 $3 - 1 = 2$  which is close to 1.791.

**REMEMBER**

- To add numbers containing decimals, write the numbers in columns, making sure that the decimal points are directly underneath each other. If there are blank places, replace them with zeros.
- Add numbers with decimal points as you would for whole numbers and insert the decimal point directly below the decimal points in the question.
- To subtract numbers containing decimals, write the numbers in columns, making sure that the decimal points are directly underneath each other. If there are blank places, replace them with zeros.
- Subtract as you would for whole numbers and insert the decimal point directly below the decimal points in the question.

**EXERCISE****6D Adding and subtracting decimals****INDIVIDUAL PATHWAYS****eBook plus**

**Activity 6-D-1**  
Adding and  
subtracting decimals  
doc-1780

**Activity 6-D-2**  
More adding and  
subtracting decimals  
doc-1781

**Activity 6-D-3**  
Advanced adding and  
subtracting decimals  
doc-1782

**eBook plus**

**Interactivity**  
Decimal target  
shoot plus  
worksheet  
int-0003

**FLUENCY**

- 1 WE16a, b** Calculate the following.

a  $1.2$   
 $\underline{+2.3}$   
 $\underline{\quad}$

b  $1.67$   
 $\underline{+1.02}$   
 $\underline{\quad}$

c  $8.062$   
 $\underline{+5.177}$   
 $\underline{\quad}$

d  $10.0364$   
 $\underline{+92.1494}$   
 $\underline{\quad}$

- 2 WE16c** Calculate the following, after filling the blank spaces.

a  $6.27$   
 $\underline{+0.5}$   
 $\underline{\quad}$

b  $3.26$   
 $\underline{+18.6460}$   
 $\underline{\quad}$

c  $4.2$   
 $62.013$   
 $\underline{+1946.12}$   
 $\underline{\quad}$

d  $48.12906$   
 $9$   
 $\underline{+204.32}$   
 $\underline{\quad}$

- 3 WE17** Rewrite the following in columns, then add. Check your answer by rounding to get an estimate.

a  $1.4 + 3.2$   
 $\underline{+0.21}$   
 $\underline{\quad}$

b  $6.5 + 0.4$   
 $\underline{+15.987 + 1.293}$   
 $\underline{\quad}$

c  $0.22 + 1.37$   
 $\underline{+8.027 + 0.9415}$   
 $\underline{\quad}$

g  $10.8271 + 6.5$

h  $1.8 + 18.6329$

i  $26.29 + 1030.4963$

- 4** Rewrite the following sums, then add. Check your answer by rounding to get an estimate.

a  $0.24 + 3.16 + 8.29$   
**b**  $14.23 + 1.06 + 86.29 + 3.64$   
**c**  $40.271 + 0.36 + 1.4$   
**d**  $5.27 + 1.381 + 12.3$   
**e**  $100 + 4.3 + 0.298 + 1.36$   
**f**  $82.3 + 100.6 + 0.9949 + 9$   
**g**  $3.026 + 5.9938 + 8.7718 + 3.2$   
**h**  $126 + 372.8 + 100.0264 + 2020.13$

- 5 MC** a 1.6 + 4.8 equals:  
**A** 5.4      **B** 6.4      **C** 0.54      **D** 0.64      **E** 64
- b 3.26 + 0.458 equals:  
**A** 3.718      **B** 0.784      **C** 0.037 18      **D** 3.484      **E** 7.84
- c 1.84 + 0.61 + 4.07 equals:  
**A** 6.52      **B** 6.42      **C** 5.42      **D** 5.52      **E** 0.652
- d 216 + 1.38 + 0.002 64 equals:  
**A** 217.4064      **B** 0.618      **C** 217.644      **D** 217.382 64      **E** 21.7644

- 6 WE18** Calculate the following, filling the spaces with zeros as required. Check your answer by rounding to get an estimate.

<b>a</b>	$\begin{array}{r} 6.87 \\ -6.27 \\ \hline \end{array}$	<b>b</b>	$\begin{array}{r} 12.231 \\ -8.026 \\ \hline \end{array}$	<b>c</b>	$\begin{array}{r} 0.6301 \\ -0.5495 \\ \hline \end{array}$
<b>d</b>	$\begin{array}{r} 3.0091 \\ -1.6723 \\ \hline \end{array}$	<b>e</b>	$\begin{array}{r} 31.02 \\ -26 \\ \hline \end{array}$	<b>f</b>	$\begin{array}{r} 98.26 \\ -9.07 \\ \hline \end{array}$
<b>g</b>	$\begin{array}{r} 146 \\ -58.91 \\ \hline \end{array}$	<b>h</b>	$\begin{array}{r} 3.2 \\ -0.467 \\ \hline \end{array}$		

- 7 WE19** Rewrite the following in columns, then subtract. Check your answer by rounding to get an estimate.

<b>a</b> 5.64 – 2.3	<b>b</b> 12.07 – 6.14
<b>c</b> 13.869 – 10.02	<b>d</b> 0.687 – 0.36
<b>e</b> 15.226 – 11.08	<b>f</b> 42.83 – 15
<b>g</b> 6.734 – 4.8	<b>h</b> 12.2 – 8.911
<b>i</b> 13.9009 – 12.65	<b>j</b> 100.562 – 86.0294
<b>k</b> 38 – 21.234	<b>l</b> 47 – 8.762
<b>m</b> 5 – 0.8864	<b>n</b> 0.2 – 0.0049
<b>o</b> 3.279 – 2.506 84	

- 8 MC** a 0.39 – 0.15 equals:  
**A** 0.0024      **B** 0.024      **C** 0.24      **D** 2.4      **E** 24
- b 1.4 – 0.147 would be rewritten as:  
**A** 
$$\begin{array}{r} 1.4 \\ -0.417 \\ \hline \end{array}$$
      **B** 
$$\begin{array}{r} -1.400 \\ -0.147 \\ \hline \end{array}$$
      **C** 
$$\begin{array}{r} 1.40 \\ -1.47 \\ \hline \end{array}$$
      **D** 
$$\begin{array}{r} 1.004 \\ -0.147 \\ \hline \end{array}$$
      **E** 
$$\begin{array}{r} 1.040 \\ -0.147 \\ \hline \end{array}$$
- c 0.3 – 0.024 equals:  
**A** 0.06      **B** 0.276      **C** 0.7      **D** 0.76      **E** 76
- d 150.278 – 0.99 equals:  
**A** 150.728      **B** 149.288      **C** 1.492 88      **D** 159.388      **E** 1.593 88

### UNDERSTANDING

- 9** Josh deposited \$27.60 into his bank account. If his balance before the deposit was \$139.40, what is Josh's new bank balance?
- 10** Jessica bought the following items at the school canteen: 1 can of Coke for \$1.60, 1 sausage roll for \$1.20, 1 packet of chips for \$1.50 and 2 Redskins for \$0.40 (Redskins cost 20 cents each). How much did Jessica spend?
- 11** A triathlon consists of a 0.5-kilometre swim, a 15.35-kilometre ride and a 4.2-kilometre run. How far do the competitors have to travel altogether?

- 12** In one day Amy walked 3.6 kilometres to school, 0.8 kilometres from school to the shops, 1.2 kilometres from the shops to a friend's house and finally 2.5 kilometres from her friend's house to her home. How far did Amy walk?
- 13** For lunch Paula ordered 1 potato cake, 1 dim sim, the minimum of chips and a milkshake from the menu shown below. How much did Paula spend on her lunch?

MENU			
Flake	\$3.50	Coffee	\$2.20
Whiting	\$3.50	Tea	\$2.20
Dim sims	\$0.60	Soft drinks	\$1.80
Potato cakes	\$0.50	Milkshakes	\$3.00
Minimum chips	\$2.50	Water	\$1.80

- 14** Ryan works in a newsagency. A customer buys \$9.65 worth of goods and gives Ryan a \$20 note. How much change should Ryan give the customer?
- 15** A jockey has a mass of 52.3 kilograms. After exercising and training for 2 days and spending time in a sauna, the jockey has lost 1.82 kilograms. What is the jockey's mass now?
- 16** If 1.27 metres is cut from a piece of material that is 13 metres long, how much material is left?

### REASONING

- 17** Cathy Freeman won a particular 400 metres race in 51.35 seconds. In her next race, her time was 2.97 seconds faster than this. What was Cathy's time for this race?
- 18** Gary and Liz are replacing the skirting boards in their lounge room. They know the perimeter of the room is 34.28 metres. If there is a door 0.82 metres wide and a fireplace 2.18 metres wide that do not require skirting boards, how much wood will they need to buy for their lounge room?
- 19** The following table shows the times recorded for each swimmer in the under-13, 50-metre freestyle relay for 6 teams.

Times for each swimmer (seconds)				
Team	Swimmer 1	Swimmer 2	Swimmer 3	Swimmer 4
1	36.7	41.3	39.2	35.8
2	38.1	46.5	38.8	35.9
3	34.6	39.2	39.9	35.2
4	41.6	40.8	43.7	40.5
5	37.9	40.2	38.6	39.2
6	38.3	39.1	40.8	37.6

- eBook plus**
- Digital docs**  
Worksheet 6.1  
doc-1769
- a** Find the total time for each team. Put your results in a table.
  - b** Which team won the relay?
  - c** What was the difference in time between the first and second placed teams?

### REFLECTION

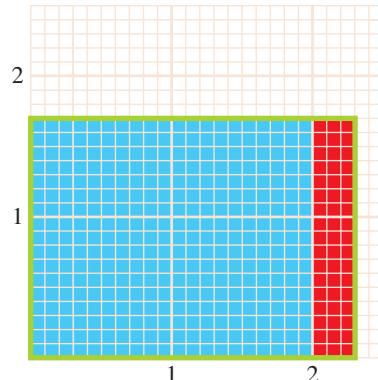
How is rounding used in estimation.

# 6E Multiplying decimals (including by multiples of 10)

## Multiplying decimals

- The calculation at right shows the multiplication  $1.7 \times 2.3$ . The diagram below right is a visual representation of each step in the calculation. There are 1.7 rows of 2.3, or 1.7 groups of 2.3.
- The smallest place value in the answer is determined by multiplying the smallest place values from each of the decimal numbers. In the calculation at right, the first multiplication is  $\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$ , so the smallest place value in the answer will be hundredths.
- A quicker method of determining the number of decimal places in the answer is to count the total number of decimal places in the decimal numbers being multiplied.

$$\begin{array}{r}
 1.7 \\
 \times 2.3 \\
 \hline
 51 \quad \leftarrow 0.3 \times 1.7 = 0.51 \\
 340 \quad \leftarrow 2.0 \times 1.7 = 3.40 \\
 \hline
 3.91 \quad \leftarrow \text{Total}
 \end{array}$$



### WORKED EXAMPLE 20

Calculate the following:

a  $12.6 \times 7$

b  $3.26$

$$\begin{array}{r}
 \times 0.4 \\
 \hline
 \end{array}$$

c  $0.4629$

$$\begin{array}{r}
 \times 2.6 \\
 \hline
 \end{array}$$

#### THINK

- Rewrite and multiply digits as for whole numbers, ignoring the decimal point. Count the number of decimal places altogether (1) and put in the decimal point.
- Check the answer by rounding;  $10 \times 7 = 70$  which is close to 88.2.
- Multiply, ignoring the decimal places.
- Count the number of digits after the point in both the decimals being multiplied and insert the decimal point in the answer. There are 2 decimal places in 3.26 and 1 in 0.4 so there will be 3 decimal places in the answer.
- Check the answer by rounding;  $3 \times 0.4 = 1.2$  which is close to 1.304.

#### WRITE

a  $12.6$

$$\begin{array}{r}
 \times 7 \\
 \hline
 88.2
 \end{array}$$

b  $326$

$$\begin{array}{r}
 \times 4 \\
 \hline
 1304
 \end{array}$$

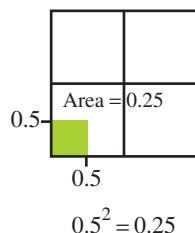
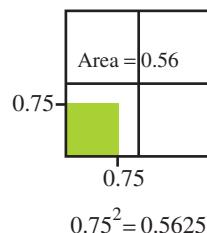
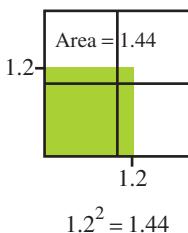
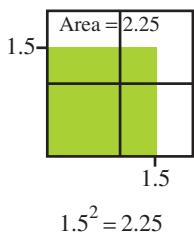
$3.26 \times 0.4 = 1.304$

- c 1 Multiply, ignoring the decimal places.

c 
$$\begin{array}{r} 4629 \\ \times 26 \\ \hline 27774 \\ 92580 \\ \hline 120354 \end{array}$$

## Squaring decimals

- To square a decimal, multiply the number by itself. The number of decimal places in the square is twice the number of decimal places in the original number.
- The diagrams below shows how squaring decimal numbers can be represented visually.



### WORKED EXAMPLE 21

Calculate the following.

a  $0.5^2$       b  $1.2^2$

#### THINK

- a 1 Multiply the number by itself, ignoring the decimal places.
- 2 Count the number of digits after the point in both the decimals being multiplied and insert the decimal point in the answer. There will be 2 decimal places in the answer.
- 3 Write the answer.
- b 1 Multiply the number by itself, ignoring the decimal places.
- 2 Count the number of digits after the point in both the decimals being multiplied and insert the decimal point in the answer. There will be 2 decimal places in the answer.
- 3 Write the answer.

#### WRITE

a  $5 \times 5 = 25$

$0.5 \times 0.5 = 0.25$

$0.5^2 = 0.25$

b  $12 \times 12 = 144$

$1.2 \times 1.2 = 1.44$

$1.2^2 = 1.44$

## Finding square roots of decimals

- To find the square root of a decimal, ignore the decimal and find the square root of the number. The number of decimal places in the answer will be half the number of decimal places in the number whose square root you are trying to find.

**WORKED EXAMPLE 22**

**Calculate the following.**

a  $\sqrt{0.81}$

b  $\sqrt{0.0121}$

**THINK**

- a 1 Take the number under the square root symbol, ignoring the decimal places. Find the square root of this number.
- 2 Count the number of decimal places in the original number (2 in this case). The number which needs to be squared to form this decimal will have half the number of decimal places. So, the final answer will have 1 decimal place. Write the answer.
- 3 Check the result using a calculator or by squaring the answer. ( $0.9^2 = 0.81$ )
  
- b 1 Take the number under the square root symbol, ignoring the decimal places. Find the square root of this number.
- 2 Count the number of decimal places in the original number (4 in this case). The number, which needs to be squared to form this decimal, will have half this number of decimal places. So, the final answer will have 2 decimal places. Write the answer.
- 3 Check the result using a calculator or by squaring the answer. ( $0.11^2 = 0.0121$ )

**WRITE**

a  $\sqrt{81} = 9$

$\sqrt{0.81} = 0.9$

b  $\sqrt{121} = 11$

$\sqrt{0.0121} = 0.11$

**Multiplying by multiples of 10**

- The multiples of 10 are 10, 20, 30, 40, ... 120, ... 1000 ...
- When you are multiplying a number by a multiple of 10, factorise the multiple to give a power of 10 and the other factor. Multiply the number by the other factor first, and then by the power of 10. For example, if you are multiplying a number by 1200, first write  $1200 = 12 \times 100$ , multiply by 12 then by 100.

**WORKED EXAMPLE 23**

**Calculate:**

a  $5.1 \times 600$

b  $0.0364 \times 24\,000$ .

**THINK**

- a 1 Multiplying by 600 is the same as first multiplying by 6 then multiplying by 100. Calculate  $5.1 \times 6$ .
- 2 Multiply the result by 100. Move the position of the decimal point 2 places to the right.
- 3 Write the final answer.

**WRITE**

a

5.1	×	6	<hr/>	30.6
-----	---	---	-------	------

$30.6 \times 100 = 3060$

$5.6 \times 600 = 3060$

- b** 1 Multiplying by 24 000 is the same as first multiplying by 24 then multiplying by 1000. Calculate  $0.0364 \times 24$ .

**b** 0.0364

$$\begin{array}{r} \times 24 \\ \hline 1456 \\ 7280 \\ \hline 0.8736 \end{array}$$

$$0.0364 \times 24 = 0.8736$$

- 2 Multiply the result by 1000. Move the position of the decimal point 3 places to the right.
- 3 Write the answer.

$$0.8736 \times 1000 = 873.6$$

$$0.0364 \times 24 000 = 873.6$$

### REMEMBER

- To multiply numbers containing decimals, ignore the decimal points and multiply them in the same way as for whole numbers. Count the number of decimal places in all parts of the question. Place a decimal point in the answer so that the number of decimal places is the same as the total number in the question.
- Check calculations by rounding.
- To square a decimal number, multiply the number by itself, ignoring the decimal places. Count the number of digits after the point in both the decimal numbers being multiplied and insert the decimal point in the answer. The number of decimal places in the answer must be the same as the total number in both parts of the question.
- The opposite of squaring a number is taking the square root.
- When taking the square root of a decimal, ignore the decimal point and solve as for whole numbers. The number of decimal places in the answer will be half the number of decimal places in the original number.
- When multiplying by a multiple of 10, factorise the multiple to give a power of 10 and the other factor. Multiply by the other factor first, and then by the power of 10.

### EXERCISE

## 6E

### Multiplying decimals (including by multiples of 10)

#### INDIVIDUAL PATHWAYS

##### eBook plus

##### Activity 6-E-1

Multiplying decimals  
doc-1783

##### Activity 6-E-2

More multiplying  
decimals  
doc-1784

##### Activity 6-E-3

Advanced multiplying  
decimals  
doc-1785

#### FLUENCY

- 1 Calculate the following.

a  $3.5 \times 4$

b  $15.7 \times 8$

c  $16.3 \times 9$

d  $10.2 \times 6$

e  $22.34 \times 5$

f  $47.63 \times 9$

g  $27.18 \times 7$

h  $64.87 \times 8$

i  $3.724 \times 7$

j  $1.064 \times 6$

k  $0.264\bar{8}1 \times 3$

l  $14.192\bar{6}83 \times 8$

- 2 WE2Oa Calculate the following.

a  $1.4 \times 0.6$

b  $4.2 \times 0.7$

c  $0.8 \times 0.4$

d  $9.7 \times 0.8$

e  $0.35 \times 0.4$

f  $0.64 \times 0.3$

g  $0.77 \times 0.5$

h  $0.49 \times 0.9$

i  $1.63 \times 0.2$

j  $5.38 \times 0.8$

k  $0.347 \times 0.6$

l  $0.498 \times 0.7$

m  $4.832 \times 0.6$

n  $12.2641 \times 0.4$

o  $20.032\bar{7}9 \times 0.5$

- 3 Calculate the following.

a  $0.002 \times 0.05$

b  $0.003 \times 0.004$

c  $0.7 \times 0.09$

d  $0.037 \times 0.006$

e  $0.000\bar{0}61 \times 0.04$

f  $0.004 \times 0.09$

g  $0.56 \times 0.7$

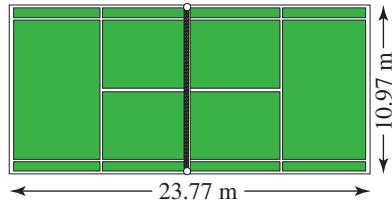
h  $0.030\bar{3}1 \times 0.02$

i  $0.0006 \times 0.007$

- 4 WE2Ob** Calculate the following. (You may use a calculator for parts **g** to **o**.)
- |                              |                              |                              |
|------------------------------|------------------------------|------------------------------|
| <b>a</b> $0.25 \times 1.2$   | <b>b</b> $0.37 \times 2.3$   | <b>c</b> $0.47 \times 5.4$   |
| <b>d</b> $0.79 \times 8.3$   | <b>e</b> $4.68 \times 3.6$   | <b>f</b> $8.04 \times 7.5$   |
| <b>g</b> $11.64 \times 4.8$  | <b>h</b> $15.08 \times 1.9$  | <b>i</b> $35.17 \times 0.35$ |
| <b>j</b> $67.73 \times 0.28$ | <b>k</b> $90.65 \times 0.88$ | <b>l</b> $46.96 \times 0.76$ |
| <b>m</b> $13.56 \times 0.31$ | <b>n</b> $3.694 \times 0.46$ | <b>o</b> $12.41 \times 1.2$  |
- 5 MC** a When calculating  $8.32 \times 0.64$ , the number of decimal places in the answer is:
- |            |            |            |            |            |
|------------|------------|------------|------------|------------|
| <b>A</b> 0 | <b>B</b> 1 | <b>C</b> 2 | <b>D</b> 3 | <b>E</b> 4 |
|------------|------------|------------|------------|------------|
- b  $0.2 \times 0.2$  equals:
- |                |               |              |            |             |
|----------------|---------------|--------------|------------|-------------|
| <b>A</b> 0.004 | <b>B</b> 0.04 | <b>C</b> 0.4 | <b>D</b> 4 | <b>E</b> 40 |
|----------------|---------------|--------------|------------|-------------|
- c  $1.4 \times 0.8$  equals:
- |               |              |             |               |              |
|---------------|--------------|-------------|---------------|--------------|
| <b>A</b> 1.12 | <b>B</b> 8.2 | <b>C</b> 82 | <b>D</b> 11.2 | <b>E</b> 112 |
|---------------|--------------|-------------|---------------|--------------|
- d  $0.0312 \times 0.51$  equals:
- |                     |                    |                    |                   |                 |
|---------------------|--------------------|--------------------|-------------------|-----------------|
| <b>A</b> 0.001 5912 | <b>B</b> 0.015 912 | <b>C</b> 0.156 312 | <b>D</b> 0.159 12 | <b>E</b> 1.5912 |
|---------------------|--------------------|--------------------|-------------------|-----------------|
- 6 WE21, 22** Calculate the following.
- |                        |                        |                          |
|------------------------|------------------------|--------------------------|
| <b>a</b> $0.02^2$      | <b>b</b> $1.3^2$       | <b>c</b> $2.05^2$        |
| <b>d</b> $\sqrt{0.09}$ | <b>e</b> $\sqrt{0.16}$ | <b>f</b> $\sqrt{0.0049}$ |
- 7** Calculate the following by moving the position of the decimal point.
- |   |  |
|---|--|
| <b>a</b> $6.48 \times 10$                         | <b>b</b> $13.896 \times 100$                   |
| <b>c</b> $589.0643 \times 100$                    | <b>d</b> $0.2708 \times 1000$                  |
| <b>e</b> $217.148\ 96 \times 1000$                | <b>f</b> $8.426\ 19 \times 100\ 000$           |
| <b>g</b> $0.820\ 496\ 783 \times 100\ 000$        | <b>h</b> $32.689\ 043\ 267 \times 100\ 000$    |
| <b>i</b> $0.984\ 326\ 641 \times 1\ 000\ 000$     | <b>j</b> $59.027\ 683\ 017 \times 1\ 000\ 000$ |
| <b>k</b> $0.000\ 278\ 498\ 32 \times 1\ 000\ 000$ | <b>l</b> $0.46 \times 1000$                    |
| <b>m</b> $529 \times 10\ 000$                     | <b>n</b> $39.486 \times 1\ 000\ 000$           |
- 8 WE23** Calculate the following.
- |                              |                                 |                               |                               |
|------------------------------|---------------------------------|-------------------------------|-------------------------------|
| <b>a</b> $3.64 \times 300$   | <b>b</b> $7.58 \times 600$      | <b>c</b> $26.9 \times 500$    | <b>d</b> $42.6 \times 900$    |
| <b>e</b> $0.127 \times 8000$ | <b>f</b> $0.543 \times 11\ 000$ | <b>g</b> $4.6 \times 32\ 000$ | <b>h</b> $8.1 \times 54\ 000$ |

**UNDERSTANDING**

- 9** Change the following amounts of money to cents. (*Hint:* There are 100 cents in one dollar.)
- |                  |                  |                   |
|------------------|------------------|-------------------|
| <b>a</b> \$35    | <b>b</b> \$127   | <b>c</b> \$11     |
| <b>d</b> \$25.35 | <b>e</b> \$58.20 | <b>f</b> \$110.15 |
- 10** One thousand Year 7 students contributed 75 cents each to the bushfire appeal. How many dollars did they contribute altogether?
- 11** Benjamin and Robyn were providing ice-cream for 600 children. How much ice-cream would be needed if each child was expected to eat 0.18 litres?
- 12** Find the area of the tennis court shown if  
area = length  $\times$  width.

**REASONING**

- 13** Michael bought 0.65 kilograms of cubed steak at this butcher's shop. This meat costs \$8.50 a kilogram. How much did Michael pay for the steak?  
*Note:* Round to the nearest 5c.

- 14** Judy bought 34.5 litres of petrol at 92.9 cents per litre. How much did she pay for her petrol:  
**a** in cents? **b** in dollars?
- 15** James is using the recipe for chocolate chip muffins to make 1.5 times the given amount. If the recipe lists 0.25 litres of milk in the ingredients, how much milk should James use for his muffins?
- 16** A ball is dropped from a height of 1 metre. If it reaches 0.7 times the height of the previous bounce at each bounce, work out how many times the ball bounces until the height is less than 1 centimetre.

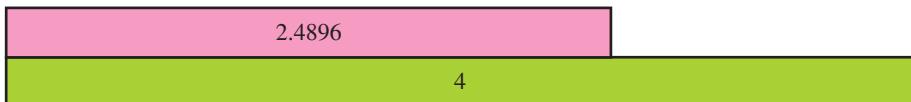
**REFLECTION**

What strategy could you use to demonstrate that  $0.5 \times 0.5 = 0.25$ ?

## 6F Dividing decimals (including by multiples of 10)

### Dividing a decimal by a whole number

- The method for dividing a decimal number by a whole number is the same as the method used for dividing whole numbers.
- A visual representation of  $2.4896 \div 4$  is shown below.



$2.4896 \div 4$  can be interpreted as ‘how many times does 4 divide into 2.4896?’

As shown in the diagram above, 4 divides into 2.4896 less than once.

#### WORKED EXAMPLE 24

Find the value of  $2.4896 \div 4$ .

**THINK**

- Set out the question as you would for whole numbers.
- Divide 4 into the first digit of 2.4896 ( $2 \div 4 = 0$  remainder 2). Write the 0 above the 2, and write the remainder beside the next digit, as shown in black.
- The second digit in the number being divided is to the right of the decimal point, so write the decimal point in the answer directly above the decimal point in the question, as shown in red.
- Divide 4 into the second digit, which include the carried 2 ( $24 \div 4 = 6$  remainder 0). Write the 6 above the 4, as shown in green.
- Divide 4 into the third digit ( $8 \div 4 = 2$ ). Write the 2 above the 8, as shown in pink.
- Divide 4 into the fourth digit ( $9 \div 4 = 2$  remainder 1). Write the 2 above the 9, and write the remainder beside the next digit, as shown in purple.

**WRITE**

$$\begin{array}{r} 4 \\ \overline{)2.4896} \end{array}$$

$$\begin{array}{r} 0 \\ 4 \\ \overline{)2.4896} \end{array}$$

$$\begin{array}{r} 0. \\ 4 \\ \overline{)2.4896} \end{array}$$

$$\begin{array}{r} 0.6 \\ 4 \\ \overline{)2.4896} \end{array}$$

$$\begin{array}{r} 0.62 \\ 4 \\ \overline{)2.4896} \end{array}$$

$$\begin{array}{r} 0.622 \\ 4 \\ \overline{)2.4896} \end{array}$$



- 7 Divide 4 into the fifth digit, which include the carried 1 ( $16 \div 4 = 4$ ). Write the 4 above the 6, as shown in orange.
- 8 Write the answer.

$$\begin{array}{r} 0.622\ 4 \\ 4 \overline{)2.489\ 6} \end{array}$$

$$2.4896 \div 4 = 0.6224$$

■ Sometimes, when you are dividing numbers, you will find that there is a remainder.

For example  $15.3 \div 4$ :  $\begin{array}{r} 3.8 \\ 4 \overline{)15.3} \end{array}$  remainder 1

Instead of leaving a remainder, you can sometimes add zeros to the end of the decimal and keep dividing until there is no remainder.

$$\begin{array}{r} 3.825 \\ 4 \overline{)15.300} \end{array}$$

### WORKED EXAMPLE 25

**Calculate  $21.76 \div 5$ . Add zeros and keep dividing until there is no remainder.**

#### THINK

- Set up the division. Write the decimal point in the answer directly above the decimal point in the question and divide as for short division adding zeros as required.
- Check the answer by rounding;  
 $20 \div 5 = 4$  which is close to 4.352.

#### WRITE

$$\begin{array}{r} 4.352 \\ 5 \overline{)21.17610} \end{array}$$

## Dividing a decimal number by a multiple of 10

■ When dividing by a multiple of 10, factorise the multiple to give a power of 10 and its other factor. Divide by the other factor first, and then by the power of 10.

### WORKED EXAMPLE 26

**Calculate:** a  $4.8 \div 40$       b  $19.2 \div 6000$ .

#### THINK

- a 1 Dividing by 40 is the same as first dividing by 4 then dividing by 10.
- 2 To divide by 10, move the position of the decimal point 1 place to the left.
- 3 Write your final answer.
- b 1 Dividing by 6000 is the same as dividing by 6 then dividing by 1000.
- 2 To divide by 1000, move the position of the decimal point 3 places to the left.
- 3 Write your final answer.

#### WRITE

a  $\begin{array}{r} 1.2 \\ 4 \overline{)4.8} \end{array}$

$$1.2 \div 10 = 0.12$$

$$4.8 \div 40 = 0.12$$

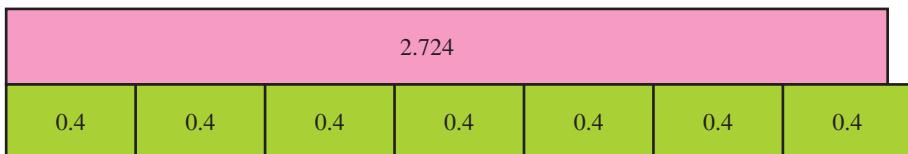
b  $\begin{array}{r} 3.2 \\ 6 \overline{)19.2} \end{array}$

$$\begin{array}{r} 3.2 \\ 1000 \end{array} 3.2 \div 1000 = 0.0032$$

$$19.2 \div 6000 = 0.0032$$

## Dividing a decimal number by another decimal number

- A visual representation of  $2.724 \div 0.4$  is shown below.



$2.724 \div 0.4$  can be interpreted as 'how many times does 0.4 divide into 2.724?'

$$2.724 \div 0.4 = 6.81$$

- When dividing one decimal by another, multiply the decimal you are dividing by (divisor) by a power of 10 to make it a whole number. Multiply the other decimal by the same power of 10, as shown at right. This is the same as writing an equivalent fraction with a whole number as the denominator:  $\frac{2.724}{0.4} \times \frac{10}{10} = \frac{27.24}{4}$ .

First multiply this number by 10 to make a whole number.

$$\textcolor{red}{2.724} \div \textcolor{blue}{0.4}$$

Then multiply this number by 10 also.

### WORKED EXAMPLE 27

Calculate:

a  $26.724 \div 0.4$

b  $3.0276 \div 0.12$

#### THINK

a 1 Rewrite the question as a fraction.

2 Multiply both the numerator and the denominator by the appropriate multiple of 10.

3 Divide the decimal by the whole number.

4 Write the answer.

b 1 Rewrite the question as a fraction.

2 Multiply both the numerator and the denominator by the appropriate multiple of 10.

3 Divide the decimal by the whole number.

4 Write the answer.

#### WRITE

$$\begin{aligned} \mathbf{a} \quad 26.724 \div 0.4 &= \frac{26.724}{0.4} \\ &= \frac{26.724}{0.4} \quad \frac{10}{10} \\ &= \frac{267.24}{4} \end{aligned}$$

$$\begin{array}{r} 6 \ 6.81 \\ 4 \overline{)26^27.324} \end{array}$$

$$26.724 \div 0.4 = 66.81$$

$$\begin{aligned} \mathbf{b} \quad 3.0276 \div 0.12 &= \frac{3.0276}{0.12} \\ &= \frac{3.0276}{0.12} \quad \frac{100}{100} \\ &= \frac{302.76}{12} \end{aligned}$$

$$\begin{array}{r} 2 \ 5.23 \\ 12 \overline{)30^62.2736} \end{array}$$

$$3.0276 \div 0.12 = 25.23$$

**WORKED EXAMPLE 28**

How many litres of petrol could be purchased for \$50.88 if 1 litre costs \$1.06?

**THINK**

- 1 Write the problem.
- 2 Rewrite the problem as a fraction.
- 3 Multiply the numerator and denominator by the appropriate multiple of 10, in this case 100. Alternatively, the decimal point could be moved twice to the right in both numbers so that the divisor is a whole number (that is,  $50.88 \div 1.06 = 5088 \div 106$ ).
- 4 Divide the decimal by the whole number.
- 5 Answer the question.

**WRITE**

$$\begin{aligned} & 50.88 \div 1.06 \\ &= \frac{50.88}{1.06} \\ &= \frac{50.88}{1.06} \quad \frac{100}{100} \\ &= \frac{5088}{106} \end{aligned}$$

$$= 106 \overline{)508^{84}8}$$

$$50.88 \div 1.06 = 48$$

Forty-eight litres of petrol could be purchased for \$50.88.

**REMEMBER**

1. To divide a decimal by a whole number, set the question out as for division of whole numbers and divide as for whole numbers.
2. The decimal point in the answer must be exactly in line with the decimal point in the question.
3. When there is a remainder, add zeros to the end of the decimal and keep dividing until there is no remainder.
4. When dividing a number by a multiple of 10, factorise the multiple to give a power of 10 and its other factor. Divide by the other factor first, and then by the power of 10.
5. To divide a decimal by a decimal, change the second decimal (the divisor) to a whole number. This can be done by rewriting the question as a fraction, then multiplying both numerator and denominator by whichever power of 10 makes the denominator a whole number.

**EXERCISE****6F****Dividing decimals (including by multiples of 10)****INDIVIDUAL PATHWAYS****eBook plus**

**Activity 6-F-1**  
Dividing decimals  
doc-1786

**FLUENCY**

- 1 **WE24** Calculate:

<b>a</b> $3.6 \div 6$	<b>b</b> $21.7 \div 7$	<b>c</b> $17.4 \div 6$	<b>d</b> $4.86 \div 9$
<b>e</b> $8.05 \div 5$	<b>f</b> $14.13 \div 3$	<b>g</b> $9.68 \div 4$	<b>h</b> $1.576 \div 2$
<b>i</b> $17.847 \div 9$	<b>j</b> $8.029 \div 7$	<b>k</b> $32.5608 \div 8$	<b>l</b> $41.8645 \div 5$
<b>m</b> $20.5782 \div 3$	<b>n</b> $126.4704 \div 4$	<b>o</b> $37.56 \div 12$	<b>p</b> $46.80 \div 15$
<b>q</b> $24.541 \div 11$	<b>r</b> $17.108 \div 14$	<b>s</b> $77.052 \div 12$	<b>t</b> $121.3421 \div 11$

- 2 **WE25** Calculate the following. In each case, add zeros and keep dividing until there is no remainder.

$$\text{a } 3.7 \div 2 \quad \text{b } 9.5 \div 2 \quad \text{c } 7.3 \div 5 \quad \text{d } 9.8 \div 4 \quad \text{e } 7.5 \div 6 \quad \text{f } 55.6 \div 8$$

INDIVIDUAL  
PATHWAYS

## eBookplus

## Activity 6-F-2

More dividing decimals  
doc-1787

## Activity 6-F-3

Advanced dividing decimals  
doc-1788

- 3 Calculate the following by changing the position of the decimal point.

- |                                 |                             |                                  |
|---------------------------------|-----------------------------|----------------------------------|
| a $14.07 \div 10$               | b $968.13 \div 100$         | c $985.06 \div 100$              |
| d $620.8 \div 1000$             | e $3592.87 \div 1000$       | f $2349.78 \div 100\,000$        |
| g $5332.0667 \div 100\,000$     | h $9.0769 \div 100\,000$    | i $103\,454.97 \div 1\,000\,000$ |
| j $802\,405.6 \div 1\,000\,000$ | k $152.70 \div 1\,000\,000$ | l $0.583 \div 1000$              |
| m $0.7205 \div 10\,000$         | n $0.0032 \div 1\,000\,000$ | o $0.0487 \div 1\,000\,000$      |

- 4 WE26 Calculate the following.

- |                  |                    |                    |
|------------------|--------------------|--------------------|
| a $15.9 \div 60$ | b $23.7 \div 30$   | c $164.5 \div 700$ |
| d $238 \div 400$ | e $8.79 \div 6000$ | f $5.22 \div 3000$ |

- 5 WE27a Calculate each of the following.

- |                     |                         |                     |
|---------------------|-------------------------|---------------------|
| a $2.5 \div 0.5$    | b $4.2 \div 0.6$        | c $6.4 \div 0.8$    |
| d $8.1 \div 0.9$    | e $2.8 \div 0.7$        | f $6.66 \div 0.6$   |
| g $0.248 \div 0.8$  | h $3.57 \div 0.7$       | i $1.32 \div 0.6$   |
| j $39.6 \div 0.6$   | k $57.68 \div 0.8$      | l $4.924 \div 0.4$  |
| m $0.2556 \div 0.3$ | n $0.067\,354 \div 0.2$ | o $0.5468 \div 0.4$ |

- 6 WE27b Calculate:

- |                          |                           |                       |
|--------------------------|---------------------------|-----------------------|
| a $172.0488 \div 0.11$   | b $0.510\,48 \div 0.12$   | c $6.4032 \div 0.32$  |
| d $2.5473 \div 0.21$     | e $21.470\,10 \div 0.15$  | f $142.888 \div 0.08$ |
| g $0.028\,692 \div 0.06$ | h $473.159\,61 \div 0.03$ | i $32.619 \div 0.02$  |

- 7 MC a To calculate  $9.84 \div 0.8$ , rewrite it as:

- A  $9.84 \div 8$       B  $0.984 \div 0.8$       C  $98.4 \div 0.8$       D  $98.4 \div 8$       E  $984 \div 4$

- b To calculate  $151.368 \div 1.32$ , rewrite it as:

- A  $151.368 \div 132$       B  $151.368 \div 13.2$       C  $1513.68 \div 132$   
D  $15\,136.8 \div 132$       E  $151\,368 \div 132$

- c  $0.294 \div 0.7$  equals:

- A 0.042      B 0.42      C 4.2      D 42      E 420

- d  $21.195 \div 0.15$  equals:

- A 0.1413      B 1.413      C 14.13      D 141.3      E 1413

## UNDERSTANDING

- 8 Change the following to dollars (\$) by dividing by 100.

- |             |             |              |
|-------------|-------------|--------------|
| a 365 cents | b 170 cents | c 5685 cents |
| d 75 cents  | e 90 cents  | f 6350 cents |

- 9  $100\,987.5412 \div 10^4 = ?$

Round the answer to the nearest hundredth.

## REASONING

- 10 Stephanie spent \$6.95 on these chocolates from The Chocolate Box. What was the cost of each chocolate? Give your answer to the nearest 5 cents.



- 11 If you have \$22.50 for bus fares to school for the week, how much would you spend on each of the 5 days?

- 12 Emily wants to make 10 cushions from 6.75 metres of material that she found on a table of remnants at Costlight Fabrics. How much material would she have for each cushion?

- 13 WE28 How many 1.25-litre bottles of water could be poured into a 25-litre drink dispenser?

- 14** The area of Tanya's lounge room floor is 85.8 square metres. How many people could she fit in the lounge room if each person takes up 1.2 square metres?



- 15** How many compact discs can be stacked on a shelf that is 28.6 centimetres high if each compact disc case is 1.1 centimetres high?  
**16** How many Big Burgers could be bought for \$562.80 if each Big Burger costs \$2.80?

eBookplus

Digital docs  
WorkSHEET 6.2  
doc-1770

**REFLECTION**

Compare the processes involved when dividing by 0.1 and dividing by  $\frac{1}{10}$ .

# Summary

## Place value and comparing decimals

- The position of a digit within a number indicates the value of the digit. Each place is 10 times larger than the one immediately to the right. The positions are shown in the table below.

				.	Tenths	Hundredths	Thousandsths	Ten thousandths
Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandsths	Ten thousandths

- The number of decimal places is the number of digits after the decimal point.
- Zeros at the end of a decimal do not change the size of the number.
- To compare the size of decimals, it is necessary to compare the size of the digits with the same place value.
- The symbol ' $<$ ' means 'is less than'.
- The symbol ' $>$ ' means 'is greater than'.

## Converting decimals to fractions and fractions to decimals

To write decimals as single fractions:

- write the whole number part first
- write all decimal places as the numerator of the fraction
- write the denominator as the place value of the last digit
- simplify the fraction if required.
- To change any fraction into a decimal, divide the denominator into the numerator.

## Rounding and repeating decimals

- To round a decimal:

Look at the first digit past the number of decimal places required.

- If this number is *less than 5*, write the number with the number of decimal places required.
- If this number is *5 or more*, add 1 to the last decimal place being kept.

If you add 1 to the last decimal place and the number in this position is a 9, the result is 10.

The 0 is put in the last required place and the 1 is added to the digit in the next place to the left.

- Rounding an answer gives a useful approximation of the original decimal. Use the symbol  $\approx$  instead of  $=$  to show that the answer is an approximation.
- A *finite decimal* terminates after a particular number of decimal places.
- An *infinite recurring decimal* is obtained when the decimal places in the answer keep repeating and the amount left over each time also keeps repeating. An approximate answer can be found by rounding so that the answer includes one complete cycle of the pattern.
- An *infinite recurring decimal* can be written exactly by placing a dot above the first and last digits of the repeating part or by drawing a line above the repeating part.

## Adding and subtracting decimals

- To add numbers containing decimals, write the numbers in columns, making sure that the decimal points are directly underneath each other. If there are blank places, replace them with zeros.
- Add numbers with decimal points as you would for whole numbers and insert the decimal point directly below the decimal points in the question.
- To subtract numbers containing decimals, write the numbers in columns, making sure that the decimal points are directly underneath each other. If there are blank places, replace them with zeros.
- Subtract as you would for whole numbers and insert the decimal point directly below the decimal points in the question.

**Multiplying decimals (including by multiples of 10)**

- To multiply numbers containing decimals, ignore the decimal points and multiply them in the same way as for whole numbers. Count the number of decimal places in all parts of the question. Place a decimal point in the answer so that the number of decimal places is the same as the total number in the question.
- Check calculations by rounding.
- To square a decimal, multiply the number by itself, ignoring the decimal places. Count the number of digits after the point in both the decimal numbers being multiplied and insert the decimal point in the answer. The number of decimal places in the answer must be the same as the total number in both parts of the question.
- The opposite of squaring a number is taking the square root.
- When taking the square root of a decimal, ignore the decimal point and solve as for whole numbers. The number of decimal places in the answer will be half the number of decimal places in the original number.
- When multiplying by a multiple of 10, factorise the multiple to give a power of 10 and the other factor. Multiply by the other factor first, and then by the power of 10.

**Dividing decimals (including by multiples of 10)**

- To divide a decimal by a whole number, set the question out as for division of whole numbers and divide as for whole numbers.
- The decimal point in the answer must be exactly in line with the decimal point in the question.
- When there is a remainder, add zeros to the end of the decimal and keep dividing until there is no remainder.
- When dividing a number by a multiple of 10, factorise the multiple to give a power of 10 and its other factor. Divide by the other factor first, and then by the power of 10.
- To divide a decimal by a decimal, change the second decimal (the divisor) to a whole number. This can be done by rewriting the question as a fraction, then multiplying both numerator and denominator by whichever power of 10 makes the denominator a whole number.

**MAPPING YOUR UNDERSTANDING**

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter.

Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 Give the value of the 7 in each of the following.
 

<b>a</b> 1.719	<b>b</b> 3.0726	<b>c</b> 4.7218
<b>d</b> 0.2078	<b>e</b> 23.1487	<b>f</b> 0.00257
<b>g</b> 17.592	<b>h</b> 50.007	
- 2 Write the following numbers in expanded notation.
 

<b>a</b> 2.64	<b>b</b> 0.369	<b>c</b> 18.406	<b>d</b> 96.3428
---------------	----------------	-----------------	------------------
- 3 Add 2 tenths to the following.
 

<b>a</b> 6.2	<b>b</b> 0.743	<b>c</b> 12.06	<b>d</b> 3.91
--------------	----------------	----------------	---------------
- 4 Add 3 thousandths to the following.
 

<b>a</b> 0.456	<b>b</b> 12.803	<b>c</b> 1.6	<b>d</b> 2.79
----------------	-----------------	--------------	---------------
- 5 Put < or > between the following.
 

<b>a</b> 8.72 _____ 8.27
<b>b</b> 0.35 _____ 0.37
<b>c</b> 1.06 _____ 1.27
<b>d</b> 10.214 _____ 10.219
<b>e</b> 0.021 _____ 0.018
<b>f</b> 13.0496 _____ 13.149
<b>g</b> 0.804 06 _____ 0.804 17
<b>h</b> 0.000 879 _____ 0.000 876
- 6 Write the following decimals in order from smallest to largest.
 

<b>a</b> 0.13, 0.86, 0.34, 0.71, 0.22
<b>b</b> 0.247, 0.274, 0.124, 0.258, 0.285
<b>c</b> 0.834, 0.826, 0.859, 0.888, 0.891
<b>d</b> 0.356, 0.358, 0.365, 0.385, 0.217
- 7 Write the following decimals as fractions in simplest form.
 

<b>a</b> 0.8	<b>b</b> 0.17	<b>c</b> 0.36	<b>d</b> 0.187
<b>e</b> 0.125	<b>f</b> 0.568	<b>g</b> 0.205	<b>h</b> 0.950
- 8 Write the following decimals as mixed numerals in simplest form.
 

<b>a</b> 1.5	<b>b</b> 4.60	<b>c</b> 3.48	<b>d</b> 5.25
<b>e</b> 2.75	<b>f</b> 2.625	<b>g</b> 1.56	<b>h</b> 8.32
- 9 Round the following to the number of decimal places shown in the brackets.
 

<b>a</b> 1.29 (1)	<b>b</b> 2.047 (2)
<b>c</b> 13.8649 (2)	<b>d</b> 0.0482 (3)
<b>e</b> 1.925 96 (4)	<b>f</b> 17.898 193 (2)
- 10 Round the following to the nearest unit.
 

<b>a</b> 13.6	<b>b</b> 29.02	<b>c</b> 86.99	<b>d</b> 100.09
---------------	----------------	----------------	-----------------
- 11 Calculate the following.
 

<b>a</b> $1.8 + 7.3$	<b>b</b> $4.21 + 5.88$
<b>c</b> $6.75 + 0.243$	<b>d</b> $12.047 + 3.6$
<b>e</b> $194 + 18.62 + 3.1$	<b>f</b> $34.1 + 7.629 + 0.008\ 45$

- 12 Calculate the following.
 

<b>a</b> $9.6 - 4.3$	<b>b</b> $18.25 - 9.18$
<b>c</b> $3.92 - 1.88$	<b>d</b> $100 - 9.341$
<b>e</b> $4.876 - 3.927$	<b>f</b> $1.6 - 0.025$
- 13 Calculate the following.
 

<b>a</b> $6.2 \times 3$	<b>b</b> $4.67 \times 9$
<b>c</b> $13.2036 \times 5$	<b>d</b> $0.7642 \times 7$
- 14 Calculate the following.
 

<b>a</b> $0.23 \times 11$
<b>b</b> $16.28 \times 41$
<b>c</b> $182.94 \times 28$
<b>d</b> $0.028\ 94 \times 32$
- 15 Calculate.
 

<b>a</b> $0.26 \times 10$	<b>b</b> $1.345 \times 10$
<b>c</b> $0.0645 \times 100$	<b>d</b> $1.8294 \times 100$
<b>e</b> $146.6281 \times 100$	<b>f</b> $0.048\ 0643 \times 1000$
<b>g</b> $0.839\ 204 \times 1000$	<b>h</b> $0.368 \times 1000$
- 16 Calculate.
 

<b>a</b> $3.2 \times 0.41$	<b>b</b> $1.72 \times 0.3$
<b>c</b> $0.87 \times 0.9$	<b>d</b> $0.03 \times 0.006$
<b>e</b> $0.58 \times 1.5$	<b>f</b> $2.83 \times 0.96$
<b>g</b> $11.468 \times 1.3$	<b>h</b> $1.248 \times 0.82$
- 17 Calculate each of the following.
 

<b>a</b> $2.5^2$	<b>b</b> $0.03^2$
<b>c</b> $\sqrt{0.64}$	<b>d</b> $\sqrt{0.0025}$
- 18 Calculate each of the following.
 

<b>a</b> $2.4 \div 8$	<b>b</b> $1.64 \div 4$
<b>c</b> $12.48 \div 6$	<b>d</b> $147.24 \div 2$
<b>e</b> $1.76 \div 11$	<b>f</b> $264.88 \div 8$
- 19 Calculate each of the following.
 

<b>a</b> $14.623 \div 10$	<b>b</b> $102.36 \div 10$
<b>c</b> $9612.347 \div 1000$	<b>d</b> $20.032 \div 100$
<b>e</b> $264\ 983.0026 \div 1000$	<b>f</b> $3462.94 \div 100$
- 20 Write as a finite decimal or as an infinite recurring decimal.
 

<b>a</b> $\frac{1}{20}$	<b>b</b> $\frac{5}{8}$
<b>c</b> $\frac{5}{16}$	<b>d</b> $\frac{2}{16}$
- 21 Write these infinite recurring decimals using a short form.
 

<b>a</b> 4.555 ...
<b>b</b> 0.8282 ...
<b>c</b> 19.278 127 81 ...
<b>d</b> 83.016 262 62 ...

**22** Calculate.

- a  $4.8 \div 0.6$   
 c  $12.1 \div 1.1$   
 e  $17.8946 \div 0.02$   
 g  $0.289\ 56 \div 0.12$

- b  $35.7 \div 0.7$   
 d  $13.72 \div 0.4$   
 f  $372.045\ 72 \div 0.06$   
 h  $3214.0170 \div 0.15$

**PROBLEM SOLVING**

- 1 Jim saved the following amounts of pocket money to take away on holidays: \$12.50, \$15.00, \$9.30, \$5.70, \$10.80. How much money did Jim have to spend on holidays?



- 2 Mandie poured 0.375 litres from a 1.5-litre bottle of juice. How much juice was left in the bottle?



- 3 Tara bought 0.350 kilogram of shaved ham at \$10.50 per kilogram. How much did Tara pay for the ham?  
 4 The decimal equivalent of  $\frac{3}{13}$  is  $0.\overline{230\ 769}$ .  
   a What is the decimal equivalent of  $\frac{1}{13}$ ?  
   b Find the decimal equivalent of  $\frac{5}{13}$ .

- 5 If a bottle and its cork cost \$1.10 and the bottle costs \$1.00 more than the cork, how much does the cork cost? (*Hint:* The answer is not 10 cents.)



- 6 What decimal gives the same result when multiplied by 5 as it does when 5 is added to it?

- 7 Identify the pattern in this sequence of numbers:

0.1, 0.2, 0.3, 0.6, 1.1, 2.0.

What are the next three numbers in the sequence?

- 8 Michael added the following on his calculator:  
 22.5, 0.678, 138.4 and 23.1. He estimated the answer to be about 184 but the calculator display showed 60.118. If his error was putting a decimal point in the wrong place, which number did he enter incorrectly?



Michael also performs the following calculation:  
 $5.24 + 23.87 - 2.092$ .

He compares his answer to the answers of three of his friends and they are all different. The answers are: 5.535, 26.19, 27.018 and 74.178. Determine the correct answer and discuss the key-stroke errors made in the other calculations.

- 9 Leesa has 3 m of ribbon. It takes 0.15 m of ribbon to make a bow. How many bows can she make with the ribbon?



- 10 Steven lives  $\frac{3}{4}$  km from school. He can walk at 6.25 km/h. How long does it take to walk to school?



### eBook plus

#### Interactivities

- Test yourself
- Chapter 6
- int-1818
- Word search
- Chapter 6
- int-2593
- Crossword
- Chapter 6
- int-2594

### eBook plus

#### Weblinks

- [Decimal game 1](#)
- [Decimal game 2](#)
- [Decimal game 3](#)

**Chapter Opener****Digital docs (page 181)**

- Hungry brain activity Chapter 6 (doc-6457)

**Are you ready?****Digital docs (page 182)**

- SkillsHEET 6.1 (doc-6458) Writing decimals
- SkillsHEET 6.2 (doc-6459) Comparing decimals
- SkillsHEET 6.3 (doc-6460) Rounding to the nearest whole number
- SkillsHEET 6.4 (doc-6461) Adding decimals (same number of decimal places)
- SkillsHEET 6.5 (doc-6462) Adding decimals (different number of decimal places)
- SkillsHEET 6.6 (doc-6463) Subtracting decimals (same number of decimal places)
- SkillsHEET 6.7 (doc-6464) Subtracting decimals (different number of decimal places)
- SkillsHEET 6.8 (doc-6465) Multiplying decimals by a single-digit number
- SkillsHEET 6.9 (doc-6466) Multiplying decimals by 10, 100, 1000
- SkillsHEET 6.10 (doc-6467) Dividing decimals by a single-digit number
- SkillsHEET 6.11 (doc-6468) Dividing decimals by 10, 100, 1000

**6A Place value and comparing decimals****eLesson**

- Place value (eles-0004) (page 183)

**Digital docs (page 186)**

- Activity 6-A-1 (doc-1771) Place value and comparing decimals
- Activity 6-A-2 (doc-1772) More place value and comparing decimals
- Activity 6-A-3 (doc-1773) Advanced place value and comparing decimals

**6B Converting decimals to fractions and fractions to decimals****Digital docs (page 191)**

- Activity 6-B-1 (doc-1774) Toucan colouring puzzle
- Activity 6-B-2 (doc-1775) Tiger colouring puzzle
- Activity 6-B-3 (doc-1776) Soccer colouring puzzle

**6C Rounding and repeating decimals****Digital docs (page 195)**

- Activity 6-C-1 (doc-1777) Rounding and repeating decimals
- Activity 6-C-2 (doc-1778) More rounding and repeating decimals
- Activity 6-C-3 (doc-1779) Advanced rounding and repeating decimals

**6D Adding and subtracting decimals****Digital docs**

- Activity 6-D-1 (doc-1780) Adding and subtracting decimals (page 199)
- Activity 6-D-2 (doc-1781) More adding and subtracting decimals (page 199)
- Activity 6-D-3 (doc-1782) Advanced adding and subtracting decimals (page 199)
- Worksheet 6.1 (doc-1769) (page 201)

**Interactivity**

- Decimal target shoot plus worksheet (int-0003) (page 199)

**6E Multiplying decimals  
(including by multiples of 10)****Digital docs (page 205)**

- Activity 6-E-1 (doc-1783) Multiplying decimals
- Activity 6-E-2 (doc-1784) More multiplying decimals
- Activity 6-E-3 (doc-1785) Advanced multiplying decimals

**6F Dividing decimals  
(including by multiples of 10)****Digital docs**

- Activity 6-F-1 (doc-1786) Dividing decimals (page 210)
- Activity 6-F-2 (doc-1787) More dividing decimals (page 211)
- Activity 6-F-3 (doc-1788) Advanced dividing decimals (page 211)
- Worksheet 6.2 (doc-1770) (page 212)

**Chapter review****Interactivities (page 217)**

- Test yourself Chapter 6 (int-1818) Take the end-of-chapter test to test your progress.
- Word search Chapter 6 (int-2593)
- Crossword Chapter 6 (int-2594)

**Weblinks (page 217)**

- Decimal game 1
- Decimal game 2
- Decimal game 3

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

## 7

# Percentages



- 7A Percentages as fractions
- 7B Percentages as decimals
- 7C Fractions and decimals to percentages
- 7D Finding percentages of an amount
- 7E One amount as a percentage of another
- 7F Common percentages and shortcuts

## WHAT DO YOU KNOW?

- 1 List what you know about percentages. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of percentages.

eBook plus

Digital docs

Hungry brain activity

Chapter 7

doc-6469

## OPENING QUESTION

All scooters are currently discounted by 20%. With \$200 birthday money, which models can you afford to buy?

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can get extra help by completing the matching SkillsHEET located on your eBookPLUS.



## Simplifying fractions with a denominator of 100

- 1 Simplify the following fractions.

a  $\frac{25}{100}$

b  $\frac{32}{100}$

Digital docs

SkillsHEET 7.1  
doc-6470



## Converting a mixed number into an improper fraction

- 2 Express these mixed numbers as improper fractions.

a  $2\frac{7}{10}$

b  $8\frac{7}{20}$

Digital docs

SkillsHEET 7.2  
doc-6471



## Changing fractions to equivalent fractions with a denominator of 100

- 3 Write an equivalent fraction with a denominator of 100 for each of the following.

a  $\frac{16}{20}$

b  $\frac{12}{25}$

Digital docs

SkillsHEET 7.3  
doc-6472



## Multiplying fractions by 100

- 4 Evaluate the following.

a  $\frac{3}{4} \times 100$

b  $\frac{7}{8} \times 100$

Digital docs

SkillsHEET 7.4  
doc-6473



## Multiplying decimals by 100

- 5 Calculate the following.

a  $1.375 \times 100$

b  $0.02 \times 100$

Digital docs

SkillsHEET 7.5  
doc-6474



## Multiplying fractions by a whole number

- 6 Evaluate the following.

a  $\frac{35}{100} \quad \frac{50}{1}$

b  $\frac{30}{100} \quad \frac{610}{1}$

Digital docs

SkillsHEET 7.6  
doc-6475



## Rounding to the nearest whole number

- 7 Round the following decimals to the nearest whole number.

a 1.67

b 6.053

Digital docs

SkillsHEET 7.7  
doc-6476



## Rounding money to the nearest 5 cents

- 8 Round the following amounts to the nearest 5 cents.

a \$23.48

b \$207.91

Digital docs

SkillsHEET 7.8  
doc-6477

## 7A Percentages as fractions

- The term **per cent** means ‘per hundred’. The symbol for per cent is %. For example, 7% means 7 out of 100.
- Whether a percentage is a whole number, a mixed number, a proper fraction, or a decimal, it can be expressed as a fraction.
- To convert a whole number percentage into a fraction, write it over 100 and simplify.
- To convert a mixed number percentage into a fraction:
  - change the mixed number into an improper fraction first
  - multiply the denominator by 100
  - simplify if possible.
- If the percentage is a proper fraction, multiply the denominator by 100 and simplify.
- To convert a percentage containing a decimal number into a fraction:
  - place the decimal number over 100
  - change the numerator to a whole number by multiplying both the numerator and the denominator by a required multiple of 10
  - simplify if possible.

### WORKED EXAMPLE 1

Write 47% as a fraction.

**THINK**

Write the percentage and then change it to a fraction with a denominator of 100.

**WRITE**

$$47\% = \frac{47}{100}$$

### WORKED EXAMPLE 2

Write 20% as a fraction in simplest form.

**THINK**

- 1 Write the percentage, change it to a fraction with a denominator of 100 and then cancel by dividing numerator and denominator by the same number.
- 2 Write the answer and check that the fraction cannot be simplified further.

**WRITE**

$$\begin{aligned} 20\% &= \frac{20^1}{100^5} \\ &= \frac{1}{5} \end{aligned}$$

### WORKED EXAMPLE 3

Write the following percentages as fractions in simplest form.

a  $\frac{1}{4}\%$

b  $15\frac{1}{3}\%$

**THINK**

- a 1 Write the percentage and then multiply the denominator by 100.
- 2 Simplify.

**WRITE**

$$\begin{aligned} a \quad \frac{1}{4}\% &= \frac{1}{4} \cdot \frac{1}{100} \\ &= \frac{1}{400} \end{aligned}$$





- b 1 Write the percentage and then change the mixed number to an improper fraction.
- 2 Multiply the denominator by 100 and simplify by cancelling. Check to make sure that the fraction cannot be simplified further.

$$\begin{aligned} \mathbf{b} \quad 15\frac{1}{3}\% &= \frac{46}{3}\% \\ &= \frac{46}{3 \cdot 100} \\ &= \frac{46^{23}}{300^{150}} \\ &= \frac{23}{150} \end{aligned}$$

**WORKED EXAMPLE 4****Write 36.4% as a fraction in simplest form.****THINK**

- 1 Write the percentage and change it to a fraction out of 100.
- 2 Change the numerator to a whole number by multiplying it by an appropriate multiple of 10, multiply the denominator by the same multiple of 10 and simplify the fraction.
- 3 Write the answer.

**WRITE**

$$\begin{aligned} 36.4\% &= \frac{36.4}{100} \\ &= \frac{36.4 \cdot 10}{100 \cdot 10} \\ &= \frac{364^{91}}{1000^{250}} \\ &= \frac{91}{250} \end{aligned}$$

**REMEMBER**

1. *Per cent* means ‘out of 100’.
2. The symbol for per cent is %.
3. To convert a percentage to a fraction, write it as a fraction with a denominator of 100 and simplify.
4. If the percentage is a fraction, multiply the denominator by 100 and simplify.
5. If the percentage is a mixed number, either:
  - (a) change the mixed number to an improper fraction, write it as a fraction out of 100 and simplify the fraction or
  - (b) change the mixed number to an improper fraction, multiply the denominator by 100 and simplify.
6. If the percentage contains a decimal number, place it over 100 and then multiply the numerator and the denominator by an appropriate multiple of 10 to make the numerator a whole number. Simplify where appropriate.

## EXERCISE

## 7A

## Percentages as fractions

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 7-A-1

Percentages and fractions  
doc-6478

## Activity 7-A-2

More percentages and fractions  
doc-6479

## Activity 7-A-3

Advanced percentages and fractions  
doc-6480

## FLUENCY

- 1 **WE1** Write the following percentages as fractions. (Leave the answer as an improper fraction where appropriate.)

- |               |               |               |               |
|---------------|---------------|---------------|---------------|
| <b>a</b> 17%  | <b>b</b> 29%  | <b>c</b> 81%  | <b>d</b> 79%  |
| <b>e</b> 99%  | <b>f</b> 43%  | <b>g</b> 3%   | <b>h</b> 19%  |
| <b>i</b> 67%  | <b>j</b> 33%  | <b>k</b> 9%   | <b>l</b> 189% |
| <b>m</b> 243% | <b>n</b> 127% | <b>o</b> 353% |               |

- 2 **WE2** Write the following percentages as fractions in simplest form. (Change the answer to a mixed number where appropriate.)

- |               |               |               |               |
|---------------|---------------|---------------|---------------|
| <b>a</b> 50%  | <b>b</b> 80%  | <b>c</b> 25%  | <b>d</b> 35%  |
| <b>e</b> 60%  | <b>f</b> 85%  | <b>g</b> 10%  | <b>h</b> 45%  |
| <b>i</b> 98%  | <b>j</b> 12%  | <b>k</b> 5%   | <b>l</b> 56%  |
| <b>m</b> 74%  | <b>n</b> 2%   | <b>o</b> 110% | <b>p</b> 150% |
| <b>q</b> 90%  | <b>r</b> 180% | <b>s</b> 200% | <b>t</b> 500% |
| <b>u</b> 112% |               |               |               |

- 3 **WE3** Write the following percentages as fractions in simplest form. (Leave answers as mixed numbers where appropriate.)

- |                             |                             |                            |
|-----------------------------|-----------------------------|----------------------------|
| <b>a</b> $\frac{1}{2}\%$    | <b>b</b> $\frac{1}{5}\%$    | <b>c</b> $\frac{3}{4}\%$   |
| <b>d</b> $\frac{2}{3}\%$    | <b>e</b> $\frac{1}{10}\%$   | <b>f</b> $\frac{5}{8}\%$   |
| <b>g</b> $\frac{6}{11}\%$   | <b>h</b> $8\frac{1}{4}\%$   | <b>i</b> $3\frac{3}{4}\%$  |
| <b>j</b> $20\frac{2}{3}\%$  | <b>k</b> $9\frac{2}{3}\%$   | <b>l</b> $14\frac{1}{4}\%$ |
| <b>m</b> $60\frac{1}{4}\%$  | <b>n</b> $15\frac{1}{2}\%$  | <b>o</b> $22\frac{1}{2}\%$ |
| <b>p</b> $11\frac{1}{5}\%$  | <b>q</b> $10\frac{3}{8}\%$  | <b>r</b> $11\frac{2}{3}\%$ |
| <b>s</b> $150\frac{1}{2}\%$ | <b>t</b> $120\frac{1}{2}\%$ | <b>u</b> $33\frac{1}{3}\%$ |

- 4 **WE4** Write the following percentages as fractions in simplest form.

- |                 |                 |                 |
|-----------------|-----------------|-----------------|
| <b>a</b> 3.5%   | <b>b</b> 7.2%   | <b>c</b> 11.8%  |
| <b>d</b> 19.7%  | <b>e</b> 32.4%  | <b>f</b> 71.5%  |
| <b>g</b> 62.9%  | <b>h</b> 15.5%  | <b>i</b> 8.7%   |
| <b>j</b> 16.2%  | <b>k</b> 28.3%  | <b>l</b> 16.25% |
| <b>m</b> 41.38% | <b>n</b> 57.99% | <b>o</b> 86.32% |
| <b>p</b> 18.14% | <b>q</b> 12.15% | <b>r</b> 0.05%  |

- 5 **MC** a 40% as a fraction in simplest form is:

- |                        |                        |                          |                           |                             |
|------------------------|------------------------|--------------------------|---------------------------|-----------------------------|
| <b>A</b> $\frac{1}{4}$ | <b>B</b> $\frac{2}{5}$ | <b>C</b> $\frac{4}{100}$ | <b>D</b> $\frac{4000}{1}$ | <b>E</b> $\frac{0.4}{1000}$ |
|------------------------|------------------------|--------------------------|---------------------------|-----------------------------|

- b  $10\frac{1}{2}\%$  as a fraction is:

- |                         |                             |                           |                           |                           |
|-------------------------|-----------------------------|---------------------------|---------------------------|---------------------------|
| <b>A</b> $\frac{21}{2}$ | <b>B</b> $\frac{10.5}{100}$ | <b>C</b> $\frac{2100}{2}$ | <b>D</b> $\frac{42}{100}$ | <b>E</b> $\frac{21}{200}$ |
|-------------------------|-----------------------------|---------------------------|---------------------------|---------------------------|

- c 1.5% as a fraction in simplest form is:

- |                            |                          |                          |                           |                            |
|----------------------------|--------------------------|--------------------------|---------------------------|----------------------------|
| <b>A</b> $\frac{1.5}{100}$ | <b>B</b> $\frac{3}{100}$ | <b>C</b> $\frac{3}{200}$ | <b>D</b> $\frac{15}{100}$ | <b>E</b> $\frac{15}{1000}$ |
|----------------------------|--------------------------|--------------------------|---------------------------|----------------------------|

- d 138% as an improper fraction in simplest form is:

- |              |                            |                             |                          |                             |
|--------------|----------------------------|-----------------------------|--------------------------|-----------------------------|
| <b>A</b> 138 | <b>B</b> $\frac{138}{100}$ | <b>C</b> $\frac{13.8}{100}$ | <b>D</b> $\frac{69}{50}$ | <b>E</b> $\frac{138}{1000}$ |
|--------------|----------------------------|-----------------------------|--------------------------|-----------------------------|

## eBookplus

Digital docs  
Spreadsheet  
Percentages as fractions  
doc-1909

## UNDERSTANDING

- 6 Imran saves 20% of his pocket money each week. What fraction does he save?
- 7 In one game, 35% of a football team is injured. What fraction of the team is injured?
- 8 Each week Jodie spends 45% of her wages at the supermarket. What fraction of Jodie's wages is spent at the supermarket?
- 9 What fraction of a class of students are boys if 68% are girls?
- 10 If the unemployment rate in Australia is 8%:
  - a what fraction of the population is unemployed?
  - b what fraction of the population is employed?
  - c out of 100 people, how many would you expect to be unemployed?
- 11 Seventeen per cent of visitors to Australia in 2005 were from New Zealand.
  - a What fraction of visitors to Australia was from New Zealand?
  - b What fraction of visitors to Australia were not from New Zealand?
  - c Out of 100 visitors to Australia, how many would you expect to be from New Zealand?
- 12 Less than 50% of the estimated 200 000 Australian invertebrate species have been described. What fraction is this?
- 13 What fraction remains if:
  - a 65% of the winnings have been spent?
  - b 19% of the audience hated the movie?
  - c all stock was discounted by 15%?
  - d 93.5% of the school population supports the uniform policy?



## REFLECTION

Percentages are used everywhere in our daily lives. Why is that so?

## 7B Percentages as decimals

- Percentages can be expressed as decimals.
- To convert a percentage to a decimal, divide the percentage by 100 by moving the decimal point two spaces to the left.

## WORKED EXAMPLE 5

Write the following percentages as decimals.

- a** 81%      **b** 16.8%

## THINK

- a**
- 1 Per cent means 'out of 100', so write the percentage as a fraction out of 100.
  - 2 To write as a decimal, move the position of the decimal point 2 places to the left and place a zero in the units column.
- b**
- 1 Write the percentage as a fraction out of 100.
  - 2 To write as a decimal, move the position of the decimal point 2 places to the left and place a zero in the units column.

## WRITE

$$\begin{aligned} \text{a } 81\% &= \frac{81}{100} \\ &= 81 \div 100 \\ &= 0.81 \end{aligned}$$

$$\begin{aligned} \text{b } 16.8\% &= \frac{16.8}{100} \\ &= 16.8 \div 100 \\ &= 0.168 \end{aligned}$$

**REMEMBER**

- To convert a percentage to a decimal, first write the percentage as a fraction out of 100.
- Change the fraction to a decimal by moving the position of the decimal point 2 places to the left. After moving the decimal point, fill any gaps with zeros.

**EXERCISE****7B****Percentages as decimals****INDIVIDUAL PATHWAYS****eBookplus****Activity 7-B-1**Percentages and decimals  
doc-6481**Activity 7-B-2**More percentages and decimals  
doc-6482**Activity 7-B-3**Advanced percentages and decimals  
doc-6483**eBookplus****Digital docs**  
Spreadsheet  
Percentages as decimals  
doc-1910**FLUENCY**

- 1 WE5a** Write the following percentages as decimals.

**a** 36%**b** 14%      **c** 19%      **d** 28%      **e** 73%      **f** 92%**g** 66%**h** 59%**i** 11%**j** 99%**k** 9%**l** 7%**m** 4%**n** 1%**o** 25%**p** 200%**q** 150%**r** 360%

- 2 WE5b** Write the following percentages as decimals.

**a** 12.3%**b** 31.6%**c** 59.2%**d** 84.9%**e** 37.6%**f** 42.1%**g** 21.9%**h** 16.9%**i** 10.7%**j** 11.1%**k** 3.1%**l** 4.6%**m** 9.2%**n** 5.9%**o** 6.8%**p** 8.8%**q** 14.25%**r** 31.75%**s** 23.55%**t** 45.75%**u** 0.05%**v** 1.02%**w** 4.01%**x** 0.02%

- 3 MC** **a** 41% as a decimal is:

**A** 41**B** 0.41**C** 4.100**D** 4.1**E** 0.041**b** 8% as a decimal is:**A** 8**B** 0.008**C** 0.08**D** 0.8**E** 800**c** 43.64% as a decimal is:**A** 0.4364**B** 4.364**C** 43.6400**D** 436.4**E** 4364.0**d** 110% as a decimal is:**A**  $\frac{110}{100}$ **B** 1100**C** 0.11**D** 1.10**E** 11.0**UNDERSTANDING**

- 4** Car prices have dropped by 17% over the past 8 years.

What is this percentage as a:

**a** fraction?**b** decimal?

- 5** The maximum legal blood alcohol concentration (BAC) for drivers in Victoria is 0.05%. What is the BAC as a:

**a** fraction?**b** decimal?

- 6** In 2006, the number of visitors to Australia during February was 2.85% lower than in January. Write this percentage as a decimal.

- 7** Over the past decade, insurance premiums have risen by 218%. Write this percentage as a decimal.

- 8** In the 3 months following a drought, the price of vegetables fell by 13.8%. Write this percentage as a decimal.

- 9** Government funding to state schools rose by 8.35%. Write this percentage as a decimal.

**REFLECTION**

When is it more convenient to express a percentage as a decimal, rather than as a fraction?

# 7C Fractions and decimals to percentages

eBook plus

**Interactivity**  
Converting  
percentages,  
decimals, fractions  
and ratios  
int-2740

## Changing fractions to percentages

- To change a fraction to a percentage, multiply by  $\frac{100}{1}$ , simplify if possible and add the percentage (%) sign.

### WORKED EXAMPLE 6

Change each of the following fractions to a percentage, giving the answer as a mixed number where appropriate.

a  $\frac{1}{10}$

b  $\frac{5}{8}$

c  $\frac{1}{7}$

#### THINK

- a 1 Write the fraction.
- 2 Multiply by  $\frac{100}{1}$  and include the % sign.
- 3 Cancel or simplify as appropriate.
- 4 Multiply the numerators and then multiply the denominators.
- 5 Simplify.

- b 1 Write the fraction.

- 2 Multiply by  $\frac{100}{1}$  and include the % sign.

- 3 Cancel or simplify as appropriate.

- 4 Multiply the numerators and then multiply the denominators.

- 5 Simplify by writing as a mixed number.

- c 1 Write the fraction.

- 2 Multiply by  $\frac{100}{1}$  and include the % sign.

- 3 Multiply the numerators and then multiply the denominators.

- 4 Divide the numerator by the denominator.

- 5 Simplify by writing as a mixed number.

#### WRITE

$$\text{a } \frac{1}{10} = \frac{1}{10} \cdot \frac{100}{1} \% = \frac{10}{1} \% = 10\%$$

$$\text{b } \frac{5}{8} = \frac{5}{8} \cdot \frac{100}{1} \% = \frac{125}{2} \% = 62\frac{1}{2}\%$$

$$\text{c } \frac{1}{7} = \frac{1}{7} \cdot \frac{100}{1} \% = \frac{100}{7} \% = 14\frac{2}{7}\%$$

**WORKED EXAMPLE 7**

A survey showed that 3 out of 15 locals rated the Portsea Pier to Pub as their favourite spectator sport. What percentage is this?

**THINK**

- 1 Write the information as a fraction by making the numerator the number of people who rated the Portsea Pier to Pub as their favourite spectator sport and the denominator the total number of people.
- 2 Change to a percentage by multiplying by  $\frac{100}{1}$  and including the % sign.
- 3 Cancel or simplify as appropriate.
- 4 Multiply the numerators and then multiply the denominators.
- 5 Simplify.
- 6 Answer the question by writing a sentence.

**WRITE**

$$\frac{3}{15} = \frac{3}{15^3} \quad \frac{100^{20}}{1}\%$$

$$= \frac{3^1}{1^1} \quad \frac{20}{1}\% \\ = \frac{20}{1}\% \\ = 20\%$$

20% of the locals surveyed rated the Portsea Pier to Pub as their favourite spectator sport.

**Changing decimals to percentages**

- To change a decimal to a percentage, multiply the decimal by 100 by moving the decimal point two spaces to the right, and add a % sign.

**WORKED EXAMPLE 8**

Change each of the following decimals to percentages.

- a 0.38      b 0.275      c 4.2

**THINK**

- a 1 Write the decimal, multiply it by 100 (that is, move the decimal point two places to the right) and include the % sign.
- b 1 Write the decimal, multiply it by 100 (that is, move the decimal point two places to the right) and include the % sign.
- c 1 Write the decimal, multiply it by 100, fill the blanks with zeros and include the % sign.
- 2 Write the answer.

**WRITE**

$$\begin{aligned} a & 0.38 = 0.38 \times 100\% \\ & = 38\% \\ b & 0.275 = 0.275 \times 100\% \\ & = 27.5\% \\ c & 4.2 = 4.2 \times 100\% \\ & = 420\% \end{aligned}$$

## REMEMBER

- To change a fraction to a percentage, multiply by  $\frac{100}{1}$ , include the percentage sign (%) and simplify the fraction by cancelling.
- To change a decimal to a percentage, multiply the decimal by 100, include a % sign and simplify.

## EXERCISE

## 7C

## Fractions and decimals to percentages

## eBookplus

eLesson  
Converting  
percentages  
eles-0005

## FLUENCY

- 1 WE6a** Change each of the following fractions to a percentage.

a  $\frac{21}{100}$

b  $\frac{48}{100}$

c  $\frac{9}{100}$

d  $\frac{93}{100}$

e  $\frac{14}{50}$

f  $\frac{36}{50}$

g  $\frac{8}{50}$

h  $\frac{40}{50}$

i  $\frac{3}{25}$

j  $\frac{15}{25}$

k  $\frac{10}{25}$

l  $\frac{6}{25}$

m  $\frac{2}{20}$

n  $\frac{7}{20}$

o  $\frac{1}{20}$

p  $\frac{2}{10}$

q  $\frac{8}{10}$

r  $\frac{1}{5}$

s  $\frac{2}{5}$

t  $\frac{4}{5}$

u  $\frac{7}{10}$

v  $\frac{13}{20}$

w  $\frac{3}{10}$

x  $\frac{3}{5}$

- 2 WE6b** Change each of the following fractions to a percentage.

a  $\frac{1}{2}$

b  $\frac{1}{4}$

c  $\frac{3}{5}$

d  $\frac{3}{12}$

e  $\frac{20}{50}$

f  $\frac{10}{50}$

g  $\frac{45}{90}$

h  $\frac{15}{30}$

i  $\frac{3}{4}$

j  $\frac{12}{15}$

k  $\frac{15}{150}$

l  $\frac{4}{20}$

- 3 WE6c** Change each of the following fractions to percentages, giving the answer as a mixed number in simplest form (if appropriate).

a  $\frac{1}{3}$

b  $\frac{1}{6}$

c  $\frac{2}{9}$

d  $\frac{8}{18}$

e  $\frac{3}{11}$

g  $\frac{5}{9}$

h  $\frac{9}{16}$

i  $\frac{11}{12}$

j  $\frac{7}{8}$

k  $\frac{1}{13}$

m  $\frac{1}{7}$

n  $\frac{5}{6}$

o  $\frac{2}{3}$

- 4 WE7 MC a** To change a fraction to a percentage:

A divide by 100      B multiply by  $\frac{100}{1}$       C divide by  $\frac{1}{100}$

D multiply by 100      E write a percentage sign (%)

b Before multiplying,  $\frac{3}{10} \times \frac{100}{1}$  would cancel to:

A  $\frac{300}{10}$

B  $\frac{3}{1} \times \frac{10}{1}$

C  $\frac{3}{1000}$

D  $\frac{3}{10} \times \frac{10}{1}$

E  $\frac{3}{10} \times \frac{1}{100}$

c The fraction  $\frac{7}{50}$  as a percentage is:

A 14%

B 7%

C  $3\frac{1}{2}$

D 700%

E 70%

d The fraction  $\frac{4}{9}$  as a percentage is:

A  $2\frac{1}{4}\%$

B 40%

C  $54\frac{5}{9}\%$

D 4%

E  $44\frac{4}{9}\%$

## INDIVIDUAL PATHWAYS

## eBookplus

**Activity 7-C-1**  
Converting to  
percentage  
doc-6484

**Activity 7-C-2**  
More converting to  
percentage  
doc-6485

**Activity 7-C-3**  
Advanced  
conversions  
doc-6486

**eBook plus**

**Digital docs**  
Spreadsheet  
Fractions/  
decimals to  
percentages  
doc-1911

- 5 WE8a** Change each of the following decimals to percentages.

a 0.45	b 0.32	c 0.56	d 0.68	e 0.90	f 0.84
g 0.12	h 0.08	i 0.02	j 0.10	k 0.99	l 0.05
m 0.29	n 0.09	o 0.19	p 0.105	q 0.001	r 0.067

- 6 WE8b, c** Write the following decimals as percentages.

a 0.3	b 0.8	c 0.9	d 0.1	e 0.002	f 0.007
g 0.005	h 0.009	i 1.32	j 1.50	k 8.65	l 2.05
m 4.50	n 0.00015	o 2.00	p 10	q 5	r 100

- 7 MC** a 0.14 is the same as:

A 1.4%      B 140%      C 0.0014%      D 14%      E 0.145

- b When multiplying a decimal by 100, move the position of the decimal point:

A two places right      B two places left      C one place left  
D one place right      E not at all

- c 0.73 as a percentage is:

A 7.3%      B 730%      C 0.073%      D 0.73%      E 73%

- d 6.1 as a percentage is:

A 6.1%      B 61%      C 610%      D 0.61%      E 0.061%

**eBook plus**

**Weblink**  
Converting  
percentages

**UNDERSTANDING**

- 8** Put the following in order from smallest to largest by converting the fractions to percentages first.

$$15\%, \frac{1}{4}, \frac{125}{1000}, \frac{85}{100}, \frac{3}{4}\%, \frac{1}{2}, \frac{94}{100}$$

- 9** One-fifth of the people in the snowfields preferred snowboarding to skiing. What percentage preferred snowboarding?

- 10** Eight twenty-fifths of people participate in sport or physical activity. What percentage is this?

- 11** Three-fortieths of the Australian population attended at least one cricket match last year. What percentage attended a cricket match?

- 12** Twenty-nine out of every 50 kilograms of waste at the tip is household waste. What percentage is household waste?

- 13** Three out of 10 people surveyed preferred water to soft drink. What percentage preferred water?

- 14** Sixty students in Year 8 have seen all of the Harry Potter films. If there are 150 students in Year 8, what percentage of them have seen all of the Harry Potter films?

- 15** Eight out of 22 houses in one street get the newspaper delivered. What percentage get the newspaper delivered?

- 16** In 2010, 23 out of every 44 Year 12 students were female. What percentage were female?

- 17** In Australia, 0.7 of all students attend government schools. What percentage of students attend government schools?

- 18** Sabrine is a secondary-school teacher. She works 0.4 of the week. What percentage of the week does Sabrine work?

- 19** Sales of notebooks have increased by 0.43 over the past 3 years. By what percentage have notebook sales increased?



- 20** The last census showed that 0.031 of the Australian population does not speak any English. What percentage does not speak any English?
- 21** A teacher recorded her class test results as decimals (decimal amount = mark student achieved ÷ total possible mark). The table below shows some of the class marks. Copy the table and convert each mark to a percentage.

<b>Student name</b>	<b>Directed number test</b>	
	<b>Mark</b>	<b>Percentage</b>
Mandy Adams	0.86	
Sandra Bazumik	0.72	
Malcolm Boncev	0.64	
James Callan	0.91	
Kate D'Arpa	0.79	
Louise Edmonds	0.92	
Chris Edwards	0.95	
Thomas Evancik	0.10	
Jessie Farmer	0.88	
Mia Yazzett	0.46	

**REFLECTION**

Think of a situation related to your school life where it would be convenient to express fractions as percentages.

## 7D Finding percentages of an amount

### Finding percentages of an amount using fractions

- To find a percentage of an amount using fractions, follow these steps.
  - Convert the percentage into a fraction.
  - Multiply by the amount.
- A percentage of something means a part of it; therefore, the percentage of an amount will be measured in the same units as the amount itself. For example, the percentage of an amount of dollars will be a certain number of dollars; the percentage of an amount of kilograms will be a certain number of kilograms and so on.

#### WORKED EXAMPLE 9

Find 40% of 135.

**THINK**

- Write the question.
- Write the percentage as a fraction with a denominator of 100, change *of* to  $\times$  and write the amount as a fraction over 1.
- Cancel and simplify as appropriate.
- Multiply the numerators and then multiply the denominators.
- Simplify by dividing the numerator by the denominator.

**WRITE**

$$40\% \text{ of } \$135$$

$$= \frac{40}{100} \times \frac{135}{1}$$

$$= \frac{40^2}{20^1} \times \frac{27}{1}$$

$$= \frac{54}{1}$$

$$= \$54$$

**WORKED EXAMPLE 10**

Find 26% of 75 and write the answer as a mixed number.

**THINK**

- 1 Write the question.
- 2 Write the percentage as a fraction, change *of* to  $\times$  and write the number as a fraction over 1.
- 3 Cancel and simplify as appropriate.
- 4 Multiply the numerators and then multiply the denominators.
- 5 Write the answer as a mixed number by dividing the denominator into the numerator.

**WRITE**

$$\begin{aligned} \text{26\% of } 75 \\ = \frac{26}{100} \times \frac{75}{1} \\ = \frac{26^{13}}{4^2} \times \frac{3}{1} \\ = \frac{39}{2} \\ = 19\frac{1}{2} \end{aligned}$$

**WORKED EXAMPLE 11**

Of the 250 students selected at random to complete a survey, 18% were in Year 11.  
How many students were in Year 11?

**THINK**

- 1 Decide what percentage of the total is required and write an expression to find the percentage of the total.
- 2 Write the percentage as a fraction, change *of* to  $\times$  and write the total as a fraction.
- 3 Cancel and simplify as appropriate.
- 4 Multiply the numerators and then multiply the denominators.
- 5 Simplify by dividing the numerator by the denominator.
- 6 Answer the question by writing a sentence.

**WRITE**

$$\begin{aligned} \text{18\% of } 250 \\ = \frac{18}{100} \times \frac{250}{1} \\ = \frac{18^9}{2^1} \times \frac{5}{1} \\ = \frac{45}{1} \\ = 45 \\ \text{45 of the 250 students were in} \\ \text{Year 11.} \end{aligned}$$

### Finding percentages of an amount using decimals

- To find the percentage of an amount using decimals, convert the percentage to a decimal and multiply by the amount.
- Remember: when you are multiplying decimal numbers, the total number of decimal places in the question gives the number of decimal places in the answer.

## WORKED EXAMPLE 12

Evaluate each of the following by converting the percentage to a decimal.

- a 20% of 50    b 34% of 15    c 4.3% of 12

## THINK

- a 1 Write the question.  
 2 Write the percentage as a number out of 100 and change *of* to  $\times$ .  
 3 Write the fraction as a decimal.  
 4 Multiply the numbers, inserting the decimal point in the answer.  
*Note:* There are 2 decimal places in 0.20, so there will be 2 decimal places in the answer.

- b 1 Write the question.  
 2 Write the percentage as a number out of 100 and change *of* to  $\times$ .  
 3 Write the fraction as a decimal.  
 4 Multiply the numbers, ignoring the decimal point.

- 5 Write the answer with the decimal point in the correct position.  
*Note:* There are 2 decimal places in 0.34, so there will be 2 decimal places in the answer.

- c 1 Write the question.  
 2 Write the percentage as a number out of 100 and change *of* to  $\times$ .  
 3 Write the fraction as a decimal.  
 4 Multiply the numbers, ignoring the decimal point.

## WRITE

a 20% of 50  
 $= \frac{20}{100} \times 50$   
 $= 0.20 \times 50$   
 $= 10.00$   
 $= 10$

b 34% of 15  
 $= \frac{34}{100} \times 15$   
 $= 0.34 \times 15$   

$$\begin{array}{r} 1215 \\ \times 34 \\ \hline 60 \\ 450 \\ \hline 510 \end{array}$$

34% of 15 = 5.1

c 4.3% of 12  
 $= \frac{4.3}{100} \times 12$   
 $= 0.043 \times 12$   

$$\begin{array}{r} 12 \\ \times 43 \\ \hline 36 \\ 480 \\ \hline 516 \end{array}$$

4.3% of 12 = 0.516

## WORKED EXAMPLE 13

Only 2.3% of Zambians have a television set. In a class of 32 Zambian Year 8 students, how many would be expected to have a television set? Write the answer to the nearest whole number.

## THINK

- 1 Write the question.  
 2 Write the percentage as a number out of 100 and change *of* to  $\times$ .  
 3 Write the fraction as a decimal.

## WRITE

2.3% of 32  
 $= \frac{2.3}{100} \times 32$   
 $= 0.023 \times 32$

- 4 Multiply the numbers, ignoring the decimal point.

$$\begin{array}{r} 32 \\ \times 23 \\ \hline 96 \\ 640 \\ \hline 736 \end{array}$$

- 5 Write the answer with the decimal point in the correct position.

$$2.3\% \text{ of } 32 = 0.736$$

- 6 Round to the nearest whole number because we are referring to whole students.

 $\approx 1$ 

- 7 Write the answer as a sentence.

One student in a class of 32 would be expected to have a television set.

### REMEMBER

- To find the percentage of an amount using fractions, change the percentage into a fraction and multiply by the amount.
- To find the percentage of an amount using decimals, change the percentage into a decimal and multiply by the amount.

### EXERCISE

## 7D Finding percentages of an amount

eBookplus

**Interactivity**  
Percentages  
int-0004

**INDIVIDUAL PATHWAYS**

eBookplus

**Activity 7-D-1**  
Calculating percentages  
doc-6487

**Activity 7-D-2**  
More calculating of percentages  
doc-6488

**Activity 7-D-3**  
Advanced calculations of percentages  
doc-6489

### FLUENCY

- 1 Copy each of the following problems and then find the answers by completing the working.

a 90% of 200 =  $\frac{90}{100} \times \frac{200}{1} =$

b 8% of 50 =  $\frac{8}{100} \times \frac{50}{1} =$

c 50% of 120 =  $\frac{50}{100} \times \frac{120}{1} =$

d 20% of 90 =  $\frac{20}{100} \times \frac{90}{1} =$

e 30% of 150 =  $\frac{30}{100} \times \frac{150}{1} =$

f 75% of 16 =  $\frac{75}{100} \times \frac{16}{1} =$

g 5% of 30 =  $\frac{5}{100} \times \frac{30}{1} =$

h 80% of 55 =  $\frac{80}{100} \times \frac{55}{1} =$

i 15% of 70 =  $\frac{15}{100} \times \frac{70}{1} =$

j 65% of 120 =  $\frac{65}{100} \times \frac{120}{1} =$

- 2 **WE9** Find the following.

a 50% of 20

b 20% of 80

c 5% of 60

d 10% of 30

e 9% of 200

f 31% of 300

g 40% of 15

h 12% of 50

i 35% of 80

j 70% of 110

k 52% of 75

l 90% of 70

m 80% of 5000

n 44% of 150

o 68% of 25

p 24% of 175

q 38% of 250

r 95% of 200

s 110% of 50

t 150% of 8

u 125% of 20

- 3 **WE10** Find the following and write the answer as a mixed number.

a 18% of 20

b 16% of 30

c 11% of 70

d 8% of 120

e 74% of 25

f 66% of 20

g 2% of 95

h 55% of 45

i 15% of 74

j 32% of 220

k 95% of 62

l 32% of 65

m 18% of 80

n 82% of 120

o 27% of 60

- 4 MC** a 45% written as a fraction is:

A  $\frac{45}{100}$       B  $\frac{45}{1}$       C  $\frac{450}{1}$       D  $\frac{1}{45}$       E  $\frac{90}{45}$

- b When finding 17% of 22, the ‘of’ will be changed to:

A  $\div$       B of      C +      D  $\times$       E –

- c Which of the following would find 15% of 33?

A 15 of 33      B  $\frac{15}{1} \times 33$       C  $\frac{15}{100} \times \frac{33}{100}$       D  $\frac{15}{1} \times \frac{33}{100}$       E  $\frac{15}{100} \times \frac{33}{1}$

- d 60% of 30 is:

A  $19\frac{4}{5}$       B  $\frac{31}{5}$       C 186      D 19      E 18

- 5 WE11** Of the 300 Year 7 students selected to complete a questionnaire on ice-cream, 70% said that their favourite flavour was chocolate chip. How many favoured chocolate chip ice-cream?

- 6 WE12a,b** Evaluate each of the following by converting the percentage to a decimal.

a 10% of 40	b 20% of 90	c 5% of 80
d 10% of 50	e 20% of 35	f 50% of 82
g 50% of 74	h 5% of 100	i 80% of 180
j 90% of 45	k 90% of 56	l 60% of 620
m 30% of 15	n 80% of 318	o 110% of 20
p 70% of 5	q 40% of 590	r 4% of 312

- 7** Find the following by converting the percentage to a decimal.

a 24% of 30	b 12% of 14.5	c 33% of 71.3
d 18% of 12.2	e 67% of 18.1	f 28% of 39.6
g 19% of 61.7	h 42% of 42.3	i 8% of 13.4
j 63% of 105.6	k 17% of 192.4	l 33% of 982.3
m 53% of 507.1	n 75% of 146.85	o $12\frac{1}{2}\%$ of 9.95

- 8 WE12c** Evaluate each of the following by converting the percentage to a decimal.

a 3.2% of 14.5	b 12.8% of 22	c 9.4% of 56
d 23.1% of 97	e 1.7% of 160	f 19.8% of 88
g 14.1% of 27.5	h 17.3% of 176.9	i 4.6% of 257.25
j 57.2% of 500	k 98.3% of 4500	l 33.3% of 2800
m 84.1% of 45.50	n 68.5% of 19.95	o 24.5% of 360

- 9 MC** a 7% of 20 equals:

A 35      B 1.4      C 3.5      D 14      E 2.86

- b 4.2% of 160 equals:

A 67.2      B 67 200      C 672      D 0.672      E 6.72

- c 63.5% of 12 equals:

A 76.20      B 1.905      C 7.660      D 7.620      E 19.05

- d 13% of 51.4 equals:

A 2.056      B 66.82      C 6.682      D 20.56      E 668.2

**eBookplus**

Digital docs  
Spreadsheet  
Calculating  
percentages  
doc-1912

- 10 WE13** A thunder day at a given location is a calendar day on which thunder is heard at least once. About 20% of days near Darwin are thunder days. How many days in one year are thunder days?



**UNDERSTANDING**

- 11** Two per cent of Australians play lawn bowls. In a group of 50 people, how many would you expect to play lawn bowls?
- 12** Two thousand people entered a marathon. Some walked and the rest jogged. If 20% walked:
- what percentage jogged?
  - how many people jogged?



- 13** William earns \$570 per week. He has just received a pay rise of 3%.
- How much more will William earn per week?
  - How much in total does William earn after his pay rise?
- 14** The Tattslotto jackpot for Saturday night is \$20 million. If you win 8% of the jackpot, how much money will you win?
- 15** The Australian cricket team was fined 15% of their match payments for a slow over rate. If the team was paid \$80 000 for the match, how much was the team fined?
- 16** Australia has 315 species of mammals. Of these, 15% are threatened. How many threatened species of mammals does Australia have? (Round your answer to the nearest whole number.)
- 17** Two million people attended at least one game of Australian Rules Football last year. Of these people, 30% attended 10 games or more. How many people attended 10 or more games of Australian Rules Football?
- 18** The water content of a particular brand of shampoo is 35%. After performing a calculation Peter claims that an 800 mL bottle of this particular shampoo contains 28 mL of water.
- Why is Peter's calculation incorrect?
  - How much of the shampoo in the 800 mL bottle is actually water?
- 19** In Perth, 36% of adults use a telephone to pay bills and 2% use the internet. If there are 50 adults who need to pay a bill, how many will use the:
- telephone?
  - internet?
- 20** In Weburbia, 24% of all households have access to the internet. If there are 34 houses in Website Street, how many would you expect to have access to the internet? Write the answer to the nearest whole number.

- 21** Terry runs a factory that makes parts for trucks. He has increased his staff by 12%. If Terry had 34 workers, how many workers does he have in the factory after the increase? Write the answer to the nearest whole number.
- 22** In the year 2006, 60% of households in Melbourne had a smoke alarm; of these, 94% worked. A street in Melbourne has 20 houses.
- How many houses would you expect to have a smoke alarm?
  - How many would have a smoke alarm that works? Write the answer to the nearest whole number.

**REASONING**

- 23** During pregnancy or childbirth 6.25% of African women die. Of 30 African women who are pregnant, how many are likely to die during pregnancy or childbirth? Because the answer is people, write it to the nearest whole number. Justify your answer.

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WorkSHEET 7.1  
doc-6490

**REFLECTION**

When would it be easier to find percentage of an amount using fractions, and when — using decimals?



## 7E One amount as a percentage of another

- To express one amount as a percentage of another, write the amounts as a fraction and change to a percentage. (Remember: to change a fraction into a percentage, multiply by 100 and add the % sign.)
- When expressing one amount as a percentage of another, make sure that both amounts are in the same units.

**WORKED EXAMPLE 14****Express:**

- 15 as a percentage of 20
- 9 as a percentage of 33 (write the answer as a mixed number).

**THINK**

- 1 Write the amount as a fraction of the total:  $\frac{\text{amount}}{\text{total}}$ .
- 2 Multiply by  $\frac{100}{1}$ , include the % sign and cancel.
- 3 Multiply the numerators and then multiply the denominators.
- 4 Simplify.

**WRITE**

$$\begin{aligned} \text{a} \quad & \frac{15}{20} \\ &= \frac{15}{20} \cdot \frac{100}{1}\% \\ &= \frac{75}{1}\% \\ &= 75\% \end{aligned}$$

b 1 Write the amount as a fraction of the total:  $\frac{\text{amount}}{\text{total}}$ .

b  $\frac{9}{33}$

2 Multiply by  $\frac{100}{1}$ , include the % sign and cancel.

$$= \frac{9^3}{33^{11}} \quad \frac{100\%}{1}$$

3 Multiply the numerators and then multiply the denominators.

$$= \frac{300}{11}\%$$

4 Write as a mixed number by dividing the denominator into the numerator.

$$= 27\frac{3}{11}\%$$

### WORKED EXAMPLE 15

Express the number 8 as a percentage of 26. Round the answer to the nearest whole number.

#### THINK

1 Write the amount as a fraction of the total:  $\frac{\text{amount}}{\text{total}}$ .

$$\frac{8}{26}$$

2 Multiply by  $\frac{100}{1}$ , include the % sign and cancel.

$$= \frac{8^4}{26^{13}} \quad \frac{100\%}{1}$$

3 Multiply the numerators and then multiply the denominators.

$$= \frac{400}{13}\%$$

4 Write as a mixed number by dividing the denominator into the numerator.

$$= 30\frac{10}{13}\%$$

5 Round to the nearest whole number.

$$\approx 31\%$$

#### WRITE

### WORKED EXAMPLE 16

Write 45c as a percentage of \$2.

#### THINK

1 Write the larger amount using the smaller unit.

$$\$2 = 200 \text{ cents}$$

2 Write the first amount as a fraction of the second amount:  $\frac{\text{amount}}{\text{total}}$ .

$$\frac{45}{200}$$

3 Multiply by  $\frac{100}{1}$ , include the % sign and cancel.

$$= \frac{45}{200^2} \quad \frac{100^1}{1}\%$$

4 Multiply the numerators and then multiply the denominators.

$$= \frac{45}{2}\%$$

5 Write as a mixed number by dividing the denominator into the numerator.

$$= 22\frac{1}{2}\%$$

This means that 45c is  $22\frac{1}{2}\%$  of \$2.

## WORKED EXAMPLE 17

Kye obtained 17 out of 30 on his Science test. What percentage did he score?  
(Round your answer to the nearest whole number.)

## THINK

1 Write the amount as a fraction of the total:  $\frac{\text{amount}}{\text{total}}$ .

2 Multiply by  $\frac{100}{1}$ , include the % sign and cancel.

3 Multiply the numerators and then multiply the denominators.

4 Write as a mixed number by dividing the denominator into the numerator.

5 Round to the nearest whole number. (If the fraction is a half or more, add one to the number.)

6 Write the answer as a sentence.

## WRITE

$$\frac{17}{30}$$

$$= \frac{17}{30} \times \frac{100^{\cancel{10}}}{\cancel{1}^3} \%$$

$$= \frac{170}{3}\%$$

$$= 56\frac{2}{3}\%$$

$$\approx 57\%$$

Kye obtained 57% for his Science test.

## REMEMBER

- To express one amount as a percentage of another, write it as a fraction, multiply by  $\frac{100}{1}$  and include the % sign. Then cancel and simplify.
- When expressing one amount as a percentage of another, make sure that both amounts are in the same units.

## EXERCISE

## 7E

## One amount as a percentage of another

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 7-E-1

What percentage is it?  
doc-6491

## Activity 7-E-2

Make it a percentage  
doc-6492

## Activity 7-E-3

Advanced percentages  
doc-6493

## FLUENCY

- 1 WE14a Express:

- a 2 as a percentage of 10
- c 6 as a percentage of 12
- e 8 as a percentage of 20
- g 15 as a percentage of 60
- i 9 as a percentage of 15
- k 12 as a percentage of 30
- m 40 as a percentage of 200
- o 9 as a percentage of 45

- b 13 as a percentage of 52
- d 8 as a percentage of 80
- f 35 as a percentage of 700
- h 120 as a percentage of 150
- j 8 as a percentage of 25
- l 30 as a percentage of 150
- n 10 as a percentage of 200
- p 180 as a percentage of 720.

- 2 WE14b Express (write the answers as mixed numbers):

- a 63 as a percentage of 200
- c 9 as a percentage of 120
- e 620 as a percentage of 3000
- g 70 as a percentage of 80
- i 9 as a percentage of 66
- 3 Express (write the answers as mixed numbers):
- a 11 as a percentage of 30
- c 8 as a percentage of 60

- b 25 as a percentage of 400
- d 32 as a percentage of 500
- f 30 as a percentage of 45
- h 12 as a percentage of 42
- j 52 as a percentage of 78.
- b 4 as a percentage of 15
- d 5 as a percentage of 70

- e 4 as a percentage of 18  
g 3 as a percentage of 7  
i 9 as a percentage of 35

- f 55 as a percentage of 66  
h 64 as a percentage of 72  
j 15 as a percentage of 21.

- 4 **WE15** Express (round all answers to the nearest whole number):  
 a 7 as a percentage of 42  
 c 9 as a percentage of 16  
 e 7 as a percentage of 65  
 g 30 as a percentage of 36  
 i 80 as a percentage of 450  
 b 18 as a percentage of 54  
 d 95 as a percentage of 150  
 f 2 as a percentage of 22  
 h 14 as a percentage of 18  
 j 8 as a percentage of 68.

- 5 **WE16** Express the first amount as a percentage of the second amount.  
 a 30c and \$3  
 c 5 mm and 4 cm  
 e 15 minutes and 1 hour  
 b 200 m and 6 km  
 d 6 days and 3 weeks  
 f 25 mL and 2 L
- 6 **MC** a 4 as a percentage of 50 is:  
 A 2%      B 12.5%      C 46%      D 8%      E 4%  
 b 9 out of 36 as a percentage is:  
 A 25%      B 9%      C 400%      D 27%      E 36%  
 c 14 as a percentage of 35 is:  
 A 14%      B 40%      C 20%      D 35%      E  $2\frac{1}{2}\%$   
 d Alice got 42 out of 60 for her mathematics test. Her mark as a percentage is:  
 A 58%      B 60%      C 30%      D 42%      E 70%

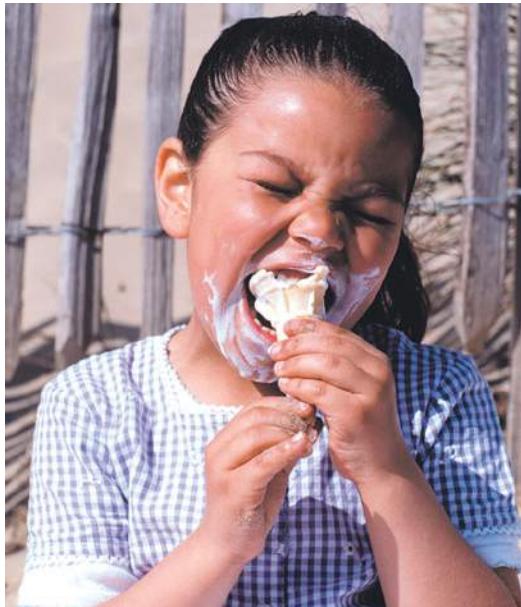
- 7 **WE17** Alicia Molik has won 7 out of her last 10 singles matches. What percentage of her last 10 matches has Alicia won?

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Digital docs  
Spreadsheet  
One amount  
as a  
percentage  
of another  
doc-1913

**UNDERSTANDING**

- 8 Three out of five people prefer chocolate ice-cream to vanilla. What percentage prefer chocolate ice-cream to vanilla?  
 9 Caillan's pocket money increased from \$15 per week to \$20 per week.  
 a By how much has Caillan's pocket money increased?  
 b What percentage increase is this?  
 10 Daniel earns \$500 per week. He spends \$50 on petrol, \$70 on rent and \$60 on food.  
 a What percentage does Daniel spend on:  
 i petrol?  
 ii rent?  
 iii food?  
 b What percentage of his total wage does Daniel spend on petrol, rent and food combined?  
 c What percentage does Daniel have left?  
 11 David shot 40 baskets from the free throw line. Twenty-five of the 40 were goals. What percentage of David's shots were goals?  
 12 Jordan had a mass of 60 kg. After one month of not eating McDonald's, he lost 4 kg. What percentage of his mass did Jordan lose?



- 13** After practising for one month, Tayla increased the speed of her tennis serve from 120 km/h to 140 km/h. Give exact answers.
- By how much has the speed of Tayla's serve increased?
  - By what percentage has the speed of Tayla's serve increased?
- 14** There are 28 students in a class. A survey showed that 12 walk home, 8 catch a bus and 8 ride in a car.
- Calculate the percentage of students who walk.
  - Calculate the percentage of students who catch a bus.
  - Calculate the total percentage who walk or catch a bus.
  - Calculate the percentage of students who ride in a car.
- 15** During one NRL season, the Brisbane Broncos won 16 out of 22 games. What percentage of games did the Broncos lose?
- 16** The teacher's mark book shows Karina has achieved the following test results in Maths.

Topic	Probability	Measurement	Algebra	Equations	Geometry
<b>Score</b>	$\frac{15}{20}$	$\frac{13}{15}$	$\frac{27}{30}$	$\frac{70}{80}$	$\frac{95}{100}$
<b>Percentage</b>					

- Copy and complete the table to show Karina's percentage for each test (round answers to the nearest whole number).
- In which test did Karina achieve her best result?
- Find Karina's average percentage mark by adding up the percentages and dividing by 5, the number of tests.

#### REFLECTION

Why do we need to be able to express one quantity as a percentage of another quantity?

## 7F Common percentages and shortcuts

- Common percentages that are often used in everyday life include 10%, 20%, 25%, 50%, 75% and so on.
- It is useful to remember the relationship between some commonly used percentages and their related fractions, as it may help to calculate those percentages mentally. For example,  $10\% = \frac{1}{10}$ , so to find 10% of any amount we need to find  $\frac{1}{10}$  of that amount; this means that we simply need to divide the amount by 10. Likewise,  $25\% = \frac{1}{4}$ , so to find 25% of any amount we need to find  $\frac{1}{4}$  of that amount; this means we simply need to divide the amount by 4.

Percentage	Fraction	Percentage	Fraction
1%	$\frac{1}{100}$	25%	$\frac{1}{4}$
5%	$\frac{1}{20}$	$33\frac{1}{3}\%$	$\frac{1}{3}$
10%	$\frac{1}{10}$	50%	$\frac{1}{2}$
$12\frac{1}{2}\%$	$\frac{1}{8}$	$66\frac{2}{3}\%$	$\frac{2}{3}$
20%	$\frac{1}{5}$	75%	$\frac{3}{4}$

- Finding 10% without a calculator is easy, as it involves dividing the amount by 10; this is done just by moving decimal point 1 space to the left. Once you have found 10% of the amount, it's easy to find some other percentages. For example:
  - to find 5%, halve 10%
  - to find 20%, double 10%
  - to find 15%, add 10% and 5% together . . . and so on.
- $1\% = \frac{1}{100}$ , so to find 1% of the amount, divide it by 100 by moving the decimal point two spaces to the left. Once you have found 1% of the amount, it's easy to find some other percentages. For example, to find 2%, double 1%.

**WORKED EXAMPLE 18**

**Find 10% of each of the following, rounding the answer to the nearest 5 cents.**

- a \$37      b \$12.95

**THINK**

- a Write the question and move the position of the decimal point one place to the left for the answer. Remember that if there is no decimal point, put it at the end of the number.  
(\$17 = \$17.00)
- b 1 Write the question and move the position of the decimal point one place to the left for the answer.  
2 Round to the nearest 5 cents.

**WRITE**

- a  $10\% \text{ of } \$37 = \$3.70$
- b  $10\% \text{ of } \$12.95 = \$1.295$   
 $\approx \$1.30$

**WORKED EXAMPLE 19**

**Find:**    a 5% of \$180                          b 20% of \$7                                  c 15% of \$52                                  d 25% of \$46.

**THINK**

- a 1 Find 10% of the amount.  
2 Find 5% by halving the amount obtained in step 1.
- b 1 Find 10% of the amount.  
2 Find 20% by doubling the amount obtained in step 1.
- c 1 Find 10% of the amount.  
2 Find 5% of the amount by halving 10%, that is, halving \$5.20.  
3 Find 15% of the amount by adding 10% and 5%, that is, \$5.20 and \$2.60.
- d 1 Find 10% of the amount.  
2 Find 20% by doubling 10%, that is, doubling \$4.60.  
3 Find 5% of the original amount by halving 10%, that is, halving \$4.60.  
4 Find 25% by adding the 20% and the 5%, that is, \$9.20 and \$2.30.

**WRITE**

- a  $10\% \text{ of } \$180 = \$18$   
 $5\% \text{ of } \$180 = \$9$
- b  $10\% \text{ of } \$7 = \$0.70$   
 $20\% \text{ of } \$7 = 2 \times \$0.70 = \$1.40$
- c  $10\% \text{ of } \$52 = \$5.20$   
 $5\% \text{ of } \$52 = \$5.20 \div 2 = \$2.60$   
 $15\% \text{ of } \$52 = \$5.20 + \$2.60 = \$7.80$
- d  $10\% \text{ of } \$46 = \$4.60$   
 $20\% = 2 \times \$4.60 = \$9.20$   
 $5\% = \$4.60 \div 2 = \$2.30$   
 $25\% = \$9.20 + \$2.30 = \$11.50$

## WORKED EXAMPLE 20

Find: **a** 12% of \$53      **b** 43% of \$120.      Round answers to the nearest 5 cents.

## THINK

- a**
- 1 Break up the percentage into lots of 10% and 1%.
  - 2 Find 10% of the amount.
  - 3 Find 1% of the amount and then double it.
  - 4 Find 12% of the amount by adding 10% and 2% of the amount.
  - 5 Round to the nearest 5 cents.
- b** To find 43%, find  $4 \times 10\% + 3 \times 1\%$ .

## WRITE

**a**  $12\% = 10\% + 2 \times 1\%$

$10\% \text{ of } \$53 = \$5.30$

$1\% \text{ of } \$53 = 0.53$   
 $2\% \text{ of } \$53 = 2 \times 0.53$   
 $= \$1.06$

$12\% \text{ of } \$53 = \$5.30 + \$1.06$   
 $= \$6.36$

Another way to write this solution would be:

$$\begin{aligned} 12\% \text{ of } \$53 &= 10\% \text{ of } \$53 + 2 \times 1\% \text{ of } \$53 \\ &= \$5.30 + 2 \times \$0.53 \\ &= \$6.36 \\ &\approx \$6.35 \end{aligned}$$

**b**  $43\% \text{ of } \$120 = 4 \times 10\% \text{ of } \$120 + 3 \times 1\% \text{ of } \$120$   
 $= 4 \times \$12 + 3 \times \$1.20$   
 $= \$48 + \$3.60$   
 $= \$51.60$

## REMEMBER

1. To find 10% of an amount, move the position of the decimal point one place to the left.
2. To find 1% of an amount, move the position of the decimal point two places to the left.

## EXERCISE

## 7F

## Common percentages and shortcuts

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 7-F-1

Shortcuts  
doc-6494

## Activity 7-F-2

More shortcuts  
doc-6495

## Activity 7-F-3

Advanced shortcuts  
doc-6496

## FLUENCY

- 1 WE18** Find 10% of each of the following, rounding the answer to the nearest 5 cents.

<b>a</b> \$10.00	<b>b</b> \$18.00	<b>c</b> \$45.00	<b>d</b> \$81.00
<b>e</b> \$150.00	<b>f</b> \$112.00	<b>g</b> \$93.00	<b>h</b> \$79.00
<b>i</b> \$47.00	<b>j</b> \$22.00	<b>k</b> \$16.50	<b>l</b> \$17.20
<b>m</b> \$12.60	<b>n</b> \$1.50	<b>o</b> \$32.90	<b>p</b> \$47.80
<b>q</b> \$81.40	<b>r</b> \$192.40	<b>s</b> \$507.00	<b>t</b> \$4620.00
<b>u</b> \$1926.00	<b>v</b> \$3041.50	<b>w</b> \$7219.60	<b>x</b> \$1999.90

- 2** Find 10% of the following. Round your answers to the nearest 5 cents.

<b>a</b> \$15	<b>b</b> \$51	<b>c</b> \$17	<b>d</b> \$9	<b>e</b> \$137
<b>f</b> \$172	<b>g</b> \$4.29	<b>h</b> \$6.37	<b>i</b> \$8.12	<b>j</b> \$39.17
<b>k</b> \$74.81	<b>l</b> \$13.95	<b>m</b> \$102.75	<b>n</b> \$67.87	<b>o</b> \$42.96
<b>p</b> \$517.83	<b>q</b> \$304.77	<b>r</b> \$628.41	<b>s</b> \$100.37	<b>t</b> \$207.08

- 3 WE19a** Find 5% of each of the following. Round the answers to the nearest 5 cents.

<b>a</b> \$8.20	<b>b</b> \$6.40	<b>c</b> \$1.60	<b>d</b> \$2.20	<b>e</b> \$140.20
<b>f</b> \$81.00	<b>g</b> \$42.40	<b>h</b> \$10.60	<b>i</b> \$242.60	<b>j</b> \$304.80
<b>k</b> \$1000	<b>l</b> \$642.75	<b>m</b> \$103.27	<b>n</b> \$31.70	<b>o</b> \$5.90

- 4 WE19b** Find 20% of the following. Round the answers to the nearest 5 cents.

- |            |            |          |           |
|------------|------------|----------|-----------|
| a \$21.50  | b \$42.30  | c \$8.20 | d \$3.30  |
| e \$74.10  | f \$0.90   | g \$0.79 | h \$16.40 |
| i \$135.80 | j \$261.70 | k \$1237 | l \$5069  |

- 5 WE19c,d** Find the following. Round the answers to the nearest 5 cents.

- |                  |                   |                   |
|------------------|-------------------|-------------------|
| a 15% of \$12    | b 15% of \$8.00   | c 15% of \$20.00  |
| d 15% of \$60.00 | e 25% of \$30.00  | f 25% of \$45.00  |
| g 25% of \$90.00 | h 25% of \$220.00 | i 30% of \$15.00  |
| j 30% of \$25.00 | k 30% of \$47.50  | l 30% of \$102.20 |

- 6** Find 1% of the following. Round the answers to the nearest 5 cents.

- |           |             |           |            |
|-----------|-------------|-----------|------------|
| a \$268   | b \$713     | c \$573   | d \$604    |
| e \$5.60  | f \$12      | g \$13    | h \$14.80  |
| i \$21.70 | j \$81.75   | k \$19.89 | l \$429.50 |
| m \$4.25  | n \$6.49    | o \$9.99  | p \$0.24   |
| q \$0.77  | r \$1264.37 |           |            |

- 7 WE20** Find the following. Round the answers to the nearest 5 cents.

- |                  |                  |                 |
|------------------|------------------|-----------------|
| a 12% of \$11    | b 21% of \$50    | c 11% of \$30   |
| d 3% of \$22     | e 6% of \$40     | f 22% of \$10   |
| g 13% of \$14    | h 35% of \$210   | i 12% of \$150  |
| j 9% of \$17     | k 2% of \$53     | l 7% of \$29    |
| m 45% of \$71.50 | n 33% of \$14.50 | o 42% of \$3.80 |
| p 31% of \$1.45  | q 64% of \$22.50 | r 41% of \$1200 |

- 8 MC** a 10% of \$7.25 equals:

- |                         |           |           |           |           |
|-------------------------|-----------|-----------|-----------|-----------|
| A \$725                 | B \$7.30  | C \$72.50 | D \$0.73  | E \$0.72  |
| b 1% of \$31.48 equals: |           |           |           |           |
| A \$3.14                | B \$0.31  | C \$0.32  | D \$31.50 | E \$3.15  |
| c 15% of \$124 equals:  |           |           |           |           |
| A \$12.40               | B \$1.24  | C \$6.20  | D \$13.64 | E \$18.60 |
| d 22% of \$5050 equals: |           |           |           |           |
| A \$60.60               | B \$50.50 | C \$1111  | D \$43.56 | E \$55.55 |

### UNDERSTANDING

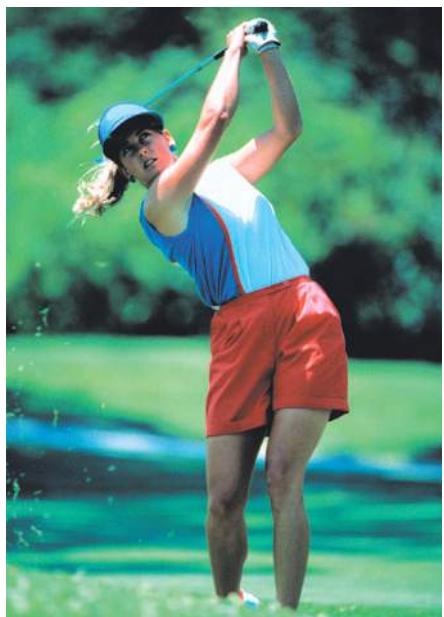
- 9** Maria is buying a new set of golf clubs. The clubs are marked at \$950, but if Maria pays cash, the shop will take 10% off the marked price. How much will the clubs cost if Maria pays cash?

- 10** Thirty per cent of residents in the shire of Sutherland are over the age of 65. If there are 180 000 residents, how many are over the age of 65?

- 11** Jay is buying a new lounge suite worth \$2150. Jay has to leave a 15% deposit and then pay the balance in monthly instalments.

- a How much deposit does Jay have to pay?
- b How much will Jay have to pay each month if he plans to pay the balance off in one year?

- 12** Ninety per cent of students at a school were present for school photographs. If the school has 1100 students, how many were absent on the day the photographs were taken?



- 13** Jim can swim 50 m in 31 seconds. If he improves his time by 10%, what will Jim's time for 50 m be?
- 14** In a survey, 40 people were asked if they liked or disliked Vegemite. Of the people surveyed, 5% said they disliked Vegemite. How many people:
- disliked Vegemite?
  - liked Vegemite?
- 15** Thirty-two thousand four hundred people went to the MCG to watch a Richmond versus Port Adelaide football match. Of the crowd, 42% went to the game by car and 55% caught public transport. How many people:
- arrived by car?
  - caught public transport?



### REASONING

- 16** When I am 5% older than I am now, I will be 21 years old. How old am I now?
- 17** The price of bread has increased by 250% in the past 20 years. If a loaf of bread costs \$2.00 now, how much would it have cost 20 years ago?
- 18** I am six months old. If I gain 10% of my current mass I will be three times my birth mass. If my birth mass was 3 kg, what is my mass now? Round your answer to 1 decimal place.
- 19** I am 33 years old. I have lived in England for 8 years. If I stay in England, how old will I be when the number of years I have lived there is 50% of my age?
- 20** My mother is four times older than I am. My sister is 75% of my age, and 10% of my grandfather's age. My father is 50, 2 years older than my mother. How old are my sister and grandfather?

### REFLECTION

Why is it important to be able to quickly evaluate common percentages mentally?

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# Summary

## Percentages as fractions

- *Per cent* means ‘out of 100’.
- The symbol for per cent is %.
- To convert a percentage to a fraction, write it as a fraction with a denominator of 100 and simplify.
- If the percentage is a mixed number, either:
  - (a) change the mixed number to an improper fraction, write it as a fraction out of 100 and simplify the fraction or
  - (b) change the mixed number to an improper fraction, multiply the denominator by 100 and simplify.
- If the percentage contains a decimal number, place it over 100 and then multiply the numerator and the denominator by an appropriate multiple of 10 to make the numerator a whole number. Simplify where appropriate.

## Percentages as decimals

- To convert a percentage to a decimal, first write the percentage as a fraction out of 100.
- Change the fraction to a decimal by moving the position of the decimal point 2 places to the left. After moving the decimal point, fill any gaps with zeros.

## Fractions and decimals to percentages

- To change a fraction to a percentage, multiply by  $\frac{100}{1}$ , include the percentage sign (%) and simplify the fraction by cancelling.
- To change a decimal number to a percentage, multiply the decimal number by 100, include a % sign and simplify.

## Finding percentages of an amount

- To find the percentage of an amount using fractions, change the percentage into a fraction and multiply by the amount.
- To find the percentage of an amount using decimals, change the percentage into a decimal and multiply by the amount.

## One amount as a percentage of another

- To express one amount as a percentage of another, write it as a fraction, multiply by  $\frac{100}{1}$  and include the % sign. Then cancel and simplify.
- When expressing one amount as a percentage of another, make sure that both amounts are in the same units.

## Common percentages and shortcuts

- To find 10% of an amount, move the position of the decimal point one place to the left.
- To find 1% of an amount, move the position of the decimal point two places to the left.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter.

Compare your concept map with the one that you created in *What do you know?* on this chapter’s opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 Write the following percentages as fractions in simplest form.

- a 13%                              b 70%
- c 26%                              d 132%
- e  $\frac{1}{4}\%$                             f  $2\frac{2}{5}\%$
- g 9.7%                              h 73.17%

- 2 Write the following percentages as decimals.

- a 42%                              b 5%
- c 94%                              d 139%
- e 6.7%                              f 19.7%
- g 58.03%                            h 0.8%

- 3 Change the following fractions to percentages.

- a  $\frac{15}{100}$                             b  $\frac{11}{20}$
- c  $\frac{3}{5}$                                     d  $\frac{18}{36}$
- e  $\frac{30}{80}$                                     f  $\frac{8}{15}$
- g  $\frac{5}{6}$                                       h  $\frac{4}{11}$

- 4 Write the following decimals as percentages.

- a 0.71
- b 0.84
- c 0.03
- d 0.2
- e 0.5
- f 0.008
- g 1.64
- h 3.8

- 5 Find the following percentages using fractions.

- a 5% of 120
- b 60% of 20
- c 75% of 52
- d 130% of 30
- e 14% of 45
- f 7% of 530
- g 32% of 15
- h 17% of 80

- 6 Express:

- a 30 as a percentage of 50
- b 18 as a percentage of 60
- c 32 as a percentage of 80
- d 28 as a percentage of 70
- e 44 as a percentage of 132
- f 6 as a percentage of 48
- g 7 as a percentage of 15
- h 35 as a percentage of 40

- i 4 days as a percentage of 8 weeks
- j 20 minutes as a percentage of 3 hours.

- 7 Find the following percentages by converting the percentage to a decimal.

- a 20% of 25
- b 12% of 31
- c 4.5% of 50
- d 9.2% of 75
- e 21.4% of 90
- f 32.3% of 120
- g 76.5% of 8
- h 42.3% of 96.2

- 8 Find 10% of each of the following by moving the position of the decimal point. Round your answer to the nearest 5 cents.

- a \$63.00
- b \$42.00
- c \$105.00
- d \$216
- e \$3.45
- f \$42.68
- g \$118.55
- h \$2125.85

- 9 Find 5% of the following by finding 10% and halving your answer. Round your answer to the nearest 5 cents.

- a \$8.00
- b \$21.00
- c \$64.00
- d \$104.00
- e \$35.00
- f \$52.00
- g \$205.50
- h \$77.30

- 10 Calculate the following using 'shortcuts'. Round your answer to the nearest 5 cents.

- a 1% of \$16.00
- b 1% of \$28.00
- c 12% of \$42.00
- d 30% of \$90.00
- e 22% of \$220.00
- f 43% of \$27
- g 15% of \$19.50
- h 8% of \$37

## PROBLEM SOLVING

- 1 Ninety-five per cent of Year 8 students participated in the school's athletics day. What fraction of the Year 8 students participated?



- 2** Twenty-four per cent of visitors to Australia in 2005 were from Germany.
- What fraction of visitors to Australia was from Germany?
  - What fraction of visitors to Australia was not from Germany?
  - Out of 400 visitors to Australia, how many would you expect to be from Germany?
- 3** What fraction remains if:
- 38% of the winnings have been spent?
  - all stock was discounted 25%?
- 4** Five-eighths of the plants in my garden are Australian natives. What percentage are Australian natives?
- 5** Of the Australian population, 0.0041 speak Polish at home. What percentage of Australians speak Polish at home?
- 6** Eighty per cent of a Year 8 Maths class got question 1 correct on their percentages test. How many got the question correct if there are 30 students in the class?
- 7** Forty per cent of primary-school children can sing the national anthem. In a group of 675 primary-school children, how many would you expect to be able to sing the national anthem?
- 8** Nineteen per cent of Australia is forest and woodland. If Australia's area is approximately  $7\ 500\ 000\text{ km}^2$ , how many  $\text{km}^2$  is forest and woodland?



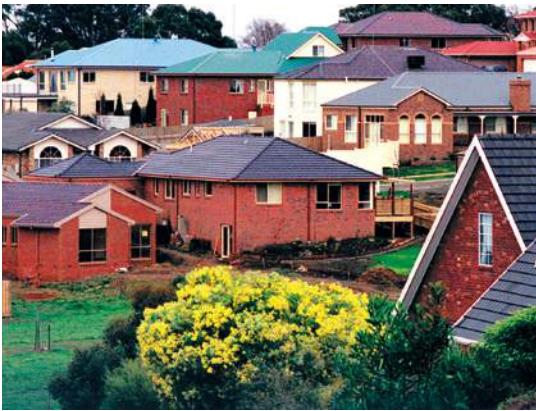
- 9** Ninety-nine of Canada's 3300 plant species are under threat of extinction. What percentage is threatened?



- 10** Lake Eyre in South Australia has an annual rainfall of around 100 mm. If 8 mm fell in one week, what percentage of the annual rainfall is this?
- 11** If a 4-litre can of paint, normally costing \$53.50, is discounted by 5%, how much will it cost?
- 12** A tennis player agreed to pay her coach 8% of her winnings from tournaments. If she wins \$145 000 in a tournament, how much does her coach get?
- 13** A bronze propeller is 75% copper. If there are 300 g of copper in the propeller, what is the mass of the propeller?
- 14** Alex cuts his fish into 3 pieces — head, body and tail. The head is 30% of the fish and the tail is 10% of the fish. If the body weighs 120 g, how much did the fish weigh?
- 15** At Queens High School, there are 900 students. 45% are girls  
55% are boys  
12% of students wear glasses.  
Which of the following statements is definitely false? (There may be more than one answer.)
- There are 100 boys with glasses.
  - 10% of the boys and 2% of the girls wear glasses.
  - 20% of girls wear glasses.
  - More than 22% of boys wear glasses.



- 16** The label on a 500-gram pack of butter states that it contains 81% fat. On a milk bottle it says that the milk contains 3 grams of fat per 100 mL. How many litres of this milk will it take to contain the same amount of fat as there is in the butter?
- 17** Dragoncity has a population of 85 400. Of these, 74% are sports fans. Of the fans, 90% are followers of some kind of football (soccer, Australian rules and rugby), but only 2.1% of football fans follow all three codes.
- How many people are interested in soccer, Australian rules and rugby?
  - What percentage of the Dragoncity population are fans of all three codes?
- 18** After having 24% of her pay taken out as taxes, Maya had \$646 left. What was her gross pay?
- 19** If you travel 50% faster, would you get to your destination 50% sooner? Explain.
- 20** Two triangles have the same area. Show that if one triangle has a 25% wider base than the other, then it must be 20% shorter too (that is, its height must be 20% less).
- 21** In a recent survey of house occupancy in a town, it was discovered that 40% of the houses contained 2 or more people. Of those houses that contain only 1 person, 25% of them contained a male. Of all the houses in a town, what percentage of them contains only 1 female and no males?



- 22** In the Student Council elections at a school, there were 3 candidates — Ann, Bob and Charlie. Ann received 227 votes, Bob 182 and Charlie 239 votes. If 90% of the school population voted, what is the total number of students in the school?
- 23** For the last two years, the price of entry to a theme park has increased 12.5% each year. The latest price rise increased a ticket to \$20.25. What was the entry price 2 years ago?

**24** A store prices its T-shirts so that it makes 25% profit. At a sale, the T-shirts were reduced to cost price. By what percentage were the T-shirts reduced from their normal selling price?

- 25** Michael arrived home from shopping and realised that he had lost his receipt. He needed to document the GST he paid for tax purposes. Michael remembered that the total amount he paid at the store was \$110. He did the following calculation and discovered that he paid \$10 GST:  $\frac{\$110}{11} = \$10$

To answer the questions below, consider the following information.

- When calculating the total price of an item in a store, the vendor adds a 10% Goods and Service Tax (GST) to the cost of the item.
  - $\text{GST} = 10\% \times \text{original cost of the item.}$
  - $\text{Total cost of item} = (\text{Original cost of item}) + \text{GST.}$
  - $\text{Total cost of item} = 110\% \times (\text{Original cost of item}).$
- Does Michael's calculation work in this instance? Explain.
  - What does the number 11 represent in this calculation? Explain.
  - If the GST % changes from 10% to 12%, would dividing by 11 give the correct amount of GST? Explain.

- 26** A television set has a ticketed price of \$2500. A 10% discount is allowed on the set.



- What is the negotiated price of the television set?
- If a further 10% is negotiated off the price of the set, what is the price paid?
- Calculate the total discount as a percentage of the original ticketed price.
- Explain why two successive 10% discounts do not equal a 20% discount.

- 27** Lena saved \$32 when she purchased a pair of jeans at a sale. If the sale had 40% off any item, what was the original price for the pair of jeans?



- 28** Two different items of clothing costing \$45 and \$50 are on sale with 10% off. The shop assistant in the store claims that if you buy both items, you will be saving a total of 20%. Is the shop assistant correct?

- 29** You are given two options at work:

OPTION 1: Your salary can be raised by 10% this month and then reduced by 10% next month, or  
 OPTION 2: Your salary will be reduced by 10% this month and increased by 10% next month.

At the end of the two months, will your salary be greater than your current salary?

Which is the better option?

- 30** Joseph wants to spend \$2500 on a new BBQ. Since he works at Harvey Norman he can get a staff discount of 10%. He can also get a discount of 30% in the stocktaking sale. Joseph wants to know whether he should ask the salesman to apply his staff discount first, or the stock-take sale discount first, to achieve the cheapest price.



- a How much will the BBQ cost if the staff discount is taken off first and then the stock-take sale discount is taken off?
- b How much will it cost if done the other way around?
- c Which way should Joseph select to get the best price?

- 31** a When Nathan will be 10% older than he is now, he will be 33 years old. How old is Nathan now?  
 b The price of milk has increased by 300% in the past 15 years. If a litre of milk costs \$2.50 now, how much would it have cost 15 years ago?

#### eBookplus

##### Interactivities

Test yourself

Chapter 7

int-1822

Word search

Chapter 7

int-2601

Crossword

Chapter 7

int-2602

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 7 (doc-6469) (*page 219*)

**Are you ready?****Digital docs** (*page 220*)

- SkillsSHEET 7.1 (doc-6470) Simplifying fractions with a denominator of 100
- SkillsSHEET 7.2 (doc-6471) Converting a mixed number into an improper fraction
- SkillsSHEET 7.3 (doc-6472) Changing fractions to equivalent fractions with a denominator of 100
- SkillsSHEET 7.4 (doc-6473) Multiplying fractions by 100
- SkillsSHEET 7.5 (doc-6474) Multiplying decimals by 100
- SkillsSHEET 7.6 (doc-6475) Multiplying fractions by a whole number
- SkillsSHEET 7.7 (doc-6476) Rounding to the nearest whole number
- SkillsSHEET 7.8 (doc-6477) Rounding money to the nearest 5 cents

**7A Percentages as fractions****Digital docs** (*page 223*)

- Activity 7-A-1 (doc-6478) Percentages and fractions
- Activity 7-A-2 (doc-6479) More percentages and fractions
- Activity 7-A-3 (doc-6480) Advanced percentages and fractions
- Spreadsheet Percentages as fractions (doc-1909)

**7B Percentages as decimals****Digital docs** (*page 225*)

- Activity 7-B-1 (doc-6481) Percentages and decimals
- Activity 7-B-2 (doc-6482) More percentages and decimals
- Activity 7-B-3 (doc-6483) Advanced percentages and decimals
- Spreadsheet Percentages as decimals (doc-1910)

**7C Fractions and decimals to percentages****Digital docs**

- Activity 7-C-1 (doc-6484) Converting to percentage (*page 228*)
- Activity 7-C-2 (doc-6485) More converting to percentage (*page 228*)
- Activity 7-C-3 (doc-6486) Advanced conversions (*page 228*)
- Spreadsheet Fractions/decimals to percentages (doc-1911) (*page 229*)

**eLesson** (*page 228*)

- Converting percentages (eles-0005)

**Weblink** (*page 228*)

- Converting percentages

**Interactivity** (*page 226*)

- Converting percentages, decimals, fractions and ratios (int-2740)

**7D Finding percentages of an amount****Digital docs**

- Activity 7-D-1 (doc-6487) Calculating percentages (*page 233*)
- Activity 7-D-2 (doc-6488) More calculating of percentages (*page 233*)
- Activity 7-D-3 (doc-6489) Advanced calculations of percentages (*page 233*)
- Spreadsheet Calculating percentages (doc-1912) (*page 234*)
- WorkSHEET 7.1 (doc-6490) (*page 236*)

**Interactivity** (*page 233*)

- Percentages (int-0004)

**7E One amount as percentages of another****Digital docs**

- Activity 7-E-1 (doc-6491) What percentage is it? (*page 238*)
- Activity 7-E-2 (doc-6492) Make it a percentage (*page 238*)
- Activity 7-E-3 (doc-6493) Advanced percentages (*page 238*)
- Spreadsheet One amount as a percentage of another (doc-1913) (*page 239*)

**7F Common percentages and shortcuts****Digital docs**

- Activity 7-F-1 (doc-6494) Shortcuts (*page 242*)
- Activity 7-F-2 (doc-6495) More shortcuts (*page 242*)
- Activity 7-F-3 (doc-6496) Advanced shortcuts (*page 242*)
- WorkSHEET 7.2 (doc-6497) (*page 244*)

**Chapter review****Interactivities** (*page 249*)

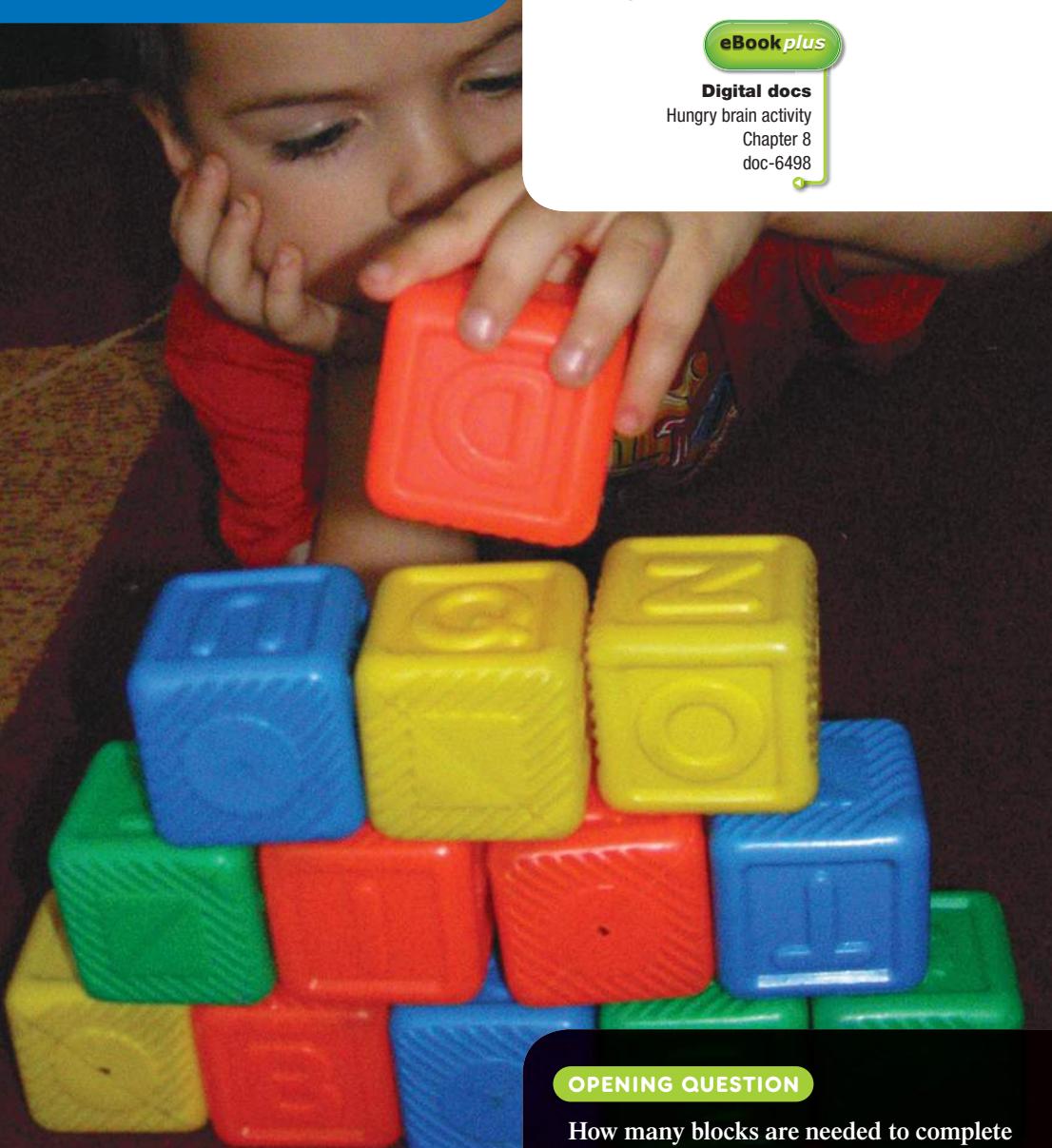
- Test Yourself Chapter 7 (int-1822) Take the end-of-chapter test to test your progress.
- Word search Chapter 7 (int-2601)
- Crossword Chapter 7 (int-2602)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

## 8

# Algebra



- 8A Using rules
- 8B Finding a formula
- 8C Substitution
- 8D Terms, expressions and equations
- 8E Simplifying and the distributive law
- 8F The associative law

**WHAT DO YOU KNOW?**

- 1 List what you know about algebra.  
Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of algebra.

**eBookplus****Digital docs**

Hungry brain activity

Chapter 8

doc-6498

**OPENING QUESTION**

How many blocks are needed to complete the tower?

# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.



**Digital docs**  
SkillsHEET 8.1  
doc-6499

## Number patterns

- For each of the following sequences of numbers, describe the pattern in words and then write down the next three numbers in the pattern.
  - 7, 9, 11, 13, ...
  - 28, 24, 20, 16, ...
  - 3, 6, 12, 24, ...
  - 100 000, 10 000, 1000, ...



**Digital docs**  
SkillsHEET 8.2  
doc-6500

## Worded operations

- Use 8 as your starting number in each case and find the result when you follow each instruction.
  - Multiply by 2
  - Add 15
  - Subtract 6
  - Divide by 4



**Digital docs**  
SkillsHEET 8.3  
doc-6501

## Describing a number pattern from a table

- Describe each number pattern shown in the tables.

a	First number	1	2	3	4	5
	Second number	7	8	9	10	11

b	First number	1	3	5	7	9
	Second number	3	9	15	21	27



**Digital docs**  
SkillsHEET 8.4  
doc-6502

## Alternative expressions used to describe the four operations

- Rewrite each of the following as a mathematical sentence (that is, rewrite each using numbers and one of the four operations +, -, × or ÷).
  - The sum of 3 and 5
  - The product of 7 and 8
  - The difference between 6 and 2
  - 9 more than 5



**Digital docs**  
SkillsHEET 8.5  
doc-6503

## Using tables to show number patterns

- Complete the table shown, using the diagrams below as a guide.



1 square



2 squares



3 squares

Number of squares	1	2	3	4	5	6
Number of sides	4	8				

- How many sides are there for 10 squares?

## 8A Using rules

- Mathematics can be used to describe relationships in the world around us.

### Patterns and rules

- A number pattern can be called a **sequence**.
- Each number in the sequence is called a **term**.
- A sequence has a **rule** that describes the pattern.

#### WORKED EXAMPLE 1

Consider the number pattern 1, 4, 7, 10, ...

Describe the pattern in words and then write the next three numbers in the pattern.

##### THINK

- The sequence starts with 1 and increases by 3 to get the second term. Check to see if this works for the remaining terms.
- We get the next term by adding 3.

##### WRITE

Add 3 to each term to find the next term.

The next 3 terms are 13, 16 and 19.

#### WORKED EXAMPLE 2

For the set of shapes below



- Construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.
- Devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.
- Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.

##### THINK

- Three matches were needed for 1 triangle, 6 matches for 2 triangles and 9 matches for 3 triangles. Construct a table. If there was a fourth triangle, it would need 12 matches.

##### WRITE

<b>Number of triangles</b>	1	2	3	4
<b>Number of matchsticks</b>	3	6	9	12

- Look for a pattern between the number of triangles and number of matchsticks.
- Apply the rule ‘the number of matchsticks equals 3 times the number of triangles’ for 20 triangles.

The number of matchsticks equals 3 times the number of triangles.

$$20 \times 3 = 60$$

To construct 20 triangles, 60 matchsticks would be required.

## Input and output tables



- A rule can be used to produce a set of output numbers from a set of input numbers.
- We can think of this as a machine that takes any input number and changes it according to a given rule. In this case, 5 is added to each input number. Any number can be used as an input number. The machine might print out a table like this:

<b>Input</b>	3	5	89	222
<b>Output</b>	8	10	94	227

- The rule can involve more than one step; for example: ‘multiply each input number by 1 and then add 5’.

### WORKED EXAMPLE 3

Complete the table at right using the given rules to work out the correct output numbers.

- a Rule: Subtract 3 from each input number.  
b Rule: Divide each input number by 2.

#### THINK

- a 1 Take the first input number (4) and apply the rule; that is, subtract 3.  
2 Repeat for the other input numbers.  
3 Enter these output values in the table.
- b 1 Take the first input number (4) and apply the rule; that is, divide by 2.  
2 Repeat for the other input numbers.  
3 Enter these output values in the table.

<b>Input</b>	4	10	38	144
<b>Output</b>				

#### WRITE

a  $4 - 3 = 1$

$10 - 3 = 7$

$38 - 3 = 35$

$144 - 3 = 141$

<b>Input</b>	4	10	38	144
<b>Output</b>	1	7	35	141

b  $4 \div 2 = 2$

$10 \div 2 = 5$

$38 \div 2 = 19$

$144 \div 2 = 72$

<b>Input</b>	4	10	38	144
<b>Output</b>	2	5	19	72

### WORKED EXAMPLE 4

Complete the table at right by using the following rule to work out the correct output numbers in each case.

Rule: Multiply each input number by 8, then subtract 2.

<b>Input</b>	1	3	5	24
<b>Output</b>				

#### THINK

- 1 Take the first input number (1) and apply the rule.  
*Note:* First multiply the input number by 8 and then subtract 2 from the answer.

#### WRITE

$1 \times 8 = 8$  then  $8 - 2 = 6$

- 2 Repeat for the other input numbers.

$$3 \times 8 = 24 \text{ then } 24 - 2 = 22$$

$$5 \times 8 = 40 \text{ then } 40 - 2 = 38$$

$$24 \times 8 = 192 \text{ then } 192 - 2 = 190$$

- 3 Enter the output values in the table.

Input	1	3	5	24
Output	6	22	38	190

### WORKED EXAMPLE 5

Use the rule given below to work out the missing input and output numbers.

Rule: Add 4 to each input number.

Input	1	5	25			
Output				6	38	190

#### THINK

- Consider the input numbers that are given and apply the rule; that is, add 4.
- Consider the output numbers that are given and perform the opposite operation; that is, subtract 4.
- Enter the new output and input values in the table.

#### WRITE

$$\begin{aligned}1 + 4 &= 5 \\5 + 4 &= 9 \\25 + 4 &= 29\end{aligned}$$

$$\begin{aligned}6 - 4 &= 2 \\38 - 4 &= 34 \\190 - 4 &= 186\end{aligned}$$

Input	1	5	25	2	34	186
Output	5	9	29	6	38	190

#### REMEMBER

- By observing number patterns, we can discover the rules by which a pattern is produced.
- In algebra, a rule can be used to produce a set of output numbers from a set of input numbers.

### EXERCISE

#### 8A

## Using rules

### INDIVIDUAL PATHWAYS

#### eBookplus

#### Activity 8-A-1

Using rules  
doc-1821

#### Activity 8-A-2

More using rules  
doc-1822

#### Activity 8-A-3

Advanced using rules  
doc-1823

#### FLUENCY

- 1 **WE1** Copy the patterns below, describe the pattern in words and then write the next three numbers in the pattern.

a 2, 4, 6, 8, ...

b 3, 8, 13, 18, ...

c 27, 24, 21, 18, ...

d 1, 3, 9, 27, ...

e 128, 64, 32, 16, ...

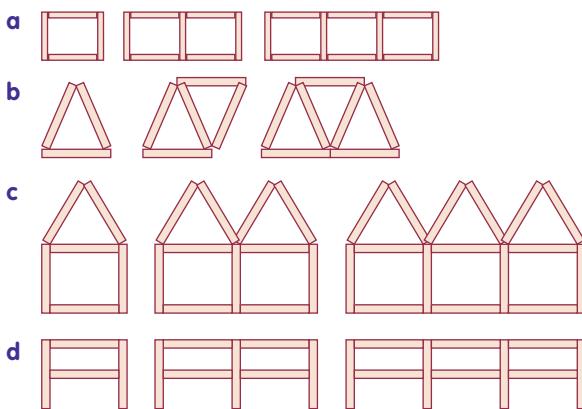
f 1, 4, 9, 16, ...

- 2 **WE2** For each of the sets of shapes at the top of the next page:

i Construct a table, as shown on the previous page, to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.

ii Devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.

iii Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.



- 3 WE3** Copy and complete the following tables. Use the rule given in each case to work out the correct output numbers.

a Subtract 4 from each input number.

Input	4	5	6	10	14
Output					

c Multiply each input number by 2.

Input	1	10	51	60	144
Output					

b Add 12 to each input number.

Input	1	2	6	13	107
Output					

d Divide each input number by 3.

Input	3	12	21	66	141
Output					

- 4 WE4** Copy and complete the following tables. Use the rule given in each case to work out the correct output numbers.

a Multiply each input number by 2, then add 5.

Input	2	3	4	10	17
Output					

c Add 3 to each input number, then multiply by 5.

Input	2	3	5	12	43
Output					

b Multiply each input number by 8, then subtract 4.

Input	2	4	5	20	100
Output					

e Multiply each input number by itself.

Input	0	1	5	8	10
Output					

d Subtract 3 from each input number, then multiply by 11.

Input	3	4	5	8	25
Output					

f Multiply each input number by itself, then add 4.

Input	0	1	4	7	12
Output					

- 5 WES** Copy and complete the following tables. Use the rule given in each case to work out the missing input and output numbers.

a Add 2 to each input number.

Input	1	3	27			
Output				4	55	193

b Add 20 to each input number.

Input	3	25	56			
Output				20	94	1773

- c Subtract 5 from each input number.

<b>Input</b>	7	15	96			
<b>Output</b>				4	12	104

- d Multiply each input number by 4.

<b>Input</b>	1	6	321			
<b>Output</b>				8	28	412

### UNDERSTANDING

- 6 a Multiply each input number by 2, then add 2.

<b>Input</b>	3	13	21			
<b>Output</b>				4	22	102

- c Divide each input number by 3.

<b>Input</b>	3	15	273			
<b>Output</b>				2	21	54

- e Multiply each input number by 20, then subtract 6.

<b>Input</b>	2	7	15			
<b>Output</b>				54	174	214

- b Multiply each input number by 5, then subtract 7.

<b>Input</b>	2	5	16			
<b>Output</b>				8	18	93

- d Multiply each input number by itself.

<b>Input</b>	1	5	17			
<b>Output</b>				4	49	144

- f Subtract 3 from each input number, then divide by 4.

<b>Input</b>	7	19	31			
<b>Output</b>				5	12	15

### REFLECTION

What strategies can you use to work out the input rule from a table of input and output values?

## 8B Finding a formula

- **Algebra** can be used to write a rule in a simple and concise manner.
- By using algebra, we can use a letter or symbol in place of a number.
- When a rule is written using a letter or a symbol, it is called a **formula**.
- A letter or symbol in a formula is called a **variable** because its value can change. (It is also called a **pronumeral**.)
- If a letter is used in place of a number, the value is not determined by its position in the alphabet. If we use  $b$  for a number, this does not mean that it has to be a smaller number than  $x$  or  $c$ .
- When multiplying numbers and variables, we do not need to show the multiplication sign, and the number is written in front of the variable, so  $h = k \times 5$  is written as  $h = 5k$ .

### WORKED EXAMPLE 6

Rewrite the following formulas, leaving out the multiplication ( $\times$ ) sign.

a  $m = q \times 4 + 3$       b  $b = (m + 2) \times 5$       c  $g = (2 \times w - 6) \times 3$

#### THINK

- a 1 Write the formula.  
2 Leave out the multiplication sign. (Remember to write the number being multiplied with the variable, in front of the variable.)

#### WRITE

a  $m = q \times 4 + 3$   
 $m = 4q + 3$





- b**
- 1 Write the formula.
  - 2 Leave out the multiplication sign. (Remember to write the number being multiplied in front of the bracket.)
- c**
- 1 Write the formula.
  - 2 Leave out the multiplication signs. (Remember to write the number being multiplied in front of the respective variable and bracket.)

**b**  $b = (m + 2) \times 5$

$$b = 5(m + 2)$$

**c**  $g = (2 \times w - 6) \times 3$

$$g = 3(2w - 6)$$

### WORKED EXAMPLE 7

Look at the table at right and complete the formula by inserting a number in the gap.

$$g = f + \underline{\quad}$$

<i>f</i>	3	6	8	13	20
<i>g</i>	5	8	10	15	22

#### THINK

- 1 Look at the first pair of numbers, 3 and 5. Look at the formula and try to guess the number which must be added to 3 to get 5.
- 2 Look at the next pair of numbers to see if adding 2 works again.
- 3 Check the other number pairs to see if this works every time.
- 4 Write the formula.

#### WRITE

$$5 = 3 + 2$$

$$8 = 6 + 2$$

$$10 = 8 + 2$$

$$15 = 13 + 2$$

$$22 = 20 + 2$$

$$g = f + 2$$

### WORKED EXAMPLE 8

Use the variables given to write a formula for the table at right.

<i>a</i>	4	5	6	7	13
<i>b</i>	3	5	7	9	21

#### THINK

- 1 Look for two consecutive numbers in the top row, for example,  $a = 4$  and  $a = 5$ , and compare the difference between their corresponding  $b$  values.
- 2 Repeat step 1 for a different pair of consecutive numbers in the top row, for example,  $a = 6$  and  $a = 7$ .
- 3 If the value obtained in step 1 is the same as the one in step 2, use it along with the basic operations ( $+, -, \times, \div$ ) or a combination of operations to obtain the correct result.

*Note:* The  $b$ -value increases by 2 for every increase of 1 in the  $a$ -value. This means the formula will contain  $a \times 2$ . Therefore possible rules may be ‘Multiply  $a$  by 2 and add a number to obtain  $b$ ’ or ‘Multiply  $a$  by 2 and subtract a number to obtain  $b$ ’.

#### WRITE

$$5 - 3 = 2$$

$$9 - 7 = 2$$

Attempt 1

$$a = 4 \text{ and } b = 3$$

Multiply  $a$  by 2 and add a number

$$4 \times 2 = 8, 8 + \text{any number does not equal } 3$$

Attempt 2

$$a = 4 \text{ and } b = 3$$

Multiply  $a$  by 2 and subtract a number

$$4 \times 2 = 8, 8 - 5 = 3 \checkmark$$

- 4** Check that the combination of operations (rule) works for the other number pairs.  
*Note:* If the rule does not work for all pairs of values in the table further attempts must be used.
- 5** Write the formula carefully, being sure to put the correct variables in the right places. Remember when the rule is written using variables, it is called a formula.

$$\begin{array}{ll} a = 5 \text{ and } b = 5 & 5 \times 2 = 10, 10 - 5 = 5 \\ a = 6 \text{ and } b = 7 & 6 \times 2 = 12, 12 - 5 = 7 \\ a = 7 \text{ and } b = 9 & 7 \times 2 = 14, 14 - 5 = 9 \\ a = 13 \text{ and } b = 21 & 2 \times 13 = 26, 26 - 5 = 21 \\ b = 2a - 5 & \end{array}$$

**REMEMBER**

1. A *variable* (or pronumeral) is a letter or symbol that is used in place of a number.
2. When a rule is written using variables it is called a *formula*.
3. When multiplying numbers and variables we don't need to show the multiplication sign, and we place numbers being multiplied in front of variables.

**EXERCISE****8B****Finding a formula****INDIVIDUAL PATHWAYS****eBookplus****Activity 8-B-1**  
Finding a formula  
doc-1824**Activity 8-B-2**  
More finding a formula  
doc-1825**Activity 8-B-3**  
Advanced formulae  
doc-1826**FLUENCY**

- 1 WE6** Rewrite the following formulas, leaving out the multiplication ( $\times$ ) sign.

<b>a</b> $b = 4 \times h$	<b>b</b> $m = f \times 4$	<b>c</b> $r = a \times 5$
<b>d</b> $m = t \times 4$	<b>e</b> $x = (k + 4) \times 5$	<b>f</b> $k = 6 \times w - 2$
<b>g</b> $t = 4 \times (20 - g)$	<b>h</b> $b = 10 \times a - 5$	<b>i</b> $d = 6 \times f + 7$
<b>j</b> $h = (x + 5) \times 9$	<b>k</b> $y = (b \times 3 + 6) \times 8$	<b>l</b> $y = 8 \times p - 6$
<b>m</b> $g = 2 \times (3 \times r + 17)$	<b>n</b> $j = (h \times 5 - 4) \times 18$	

- 2 WE7** Look at the following tables and complete the formula for each table by inserting a number in the gap.

**a**

<i>f</i>	0	1	3	6	8
<i>g</i>	9	10	12	15	17

$$g = f + \underline{\hspace{2cm}}$$

**c**

<i>k</i>	1	3	5	6	12
<i>t</i>	3	7	11	13	25

$$t = 2k + \underline{\hspace{2cm}}$$

**e**

<i>a</i>	1	4	6	9	12
<i>g</i>	6	18	26	38	50

$$g = 4a + \underline{\hspace{2cm}}$$

**g**

<i>p</i>	0	4	5	6	13
<i>w</i>	2	14	17	20	41

$$w = \underline{\hspace{2cm}} \times p + 2$$

**i**

<i>m</i>	1	2	4	7	15
<i>p</i>	2	5	11	20	44

$$p = 3m - \underline{\hspace{2cm}}$$

**b**

<i>a</i>	3	4	7	8	11
<i>b</i>	0	1	4	5	8

$$b = a - \underline{\hspace{2cm}}$$

**d**

<i>x</i>	1	3	5	8	11
<i>y</i>	2	16	30	51	72

$$y = 7x - \underline{\hspace{2cm}}$$

**f**

<i>m</i>	1	2	3	4	11
<i>t</i>	20	29	38	47	110

$$t = 9m + \underline{\hspace{2cm}}$$

**h**

<i>t</i>	2	3	7	9	12
<i>x</i>	2	7	27	37	52

$$x = \underline{\hspace{2cm}} \times t - 8$$

**j**

<i>s</i>	1	5	6	7	12
<i>b</i>	3	43	53	63	113

$$b = s \times \underline{\hspace{2cm}} - 7$$

**3 WE8** Look at the following tables and use the variables given to write a formula for each table.

**a**

<b>a</b>	2	4	5	8	12
<b>b</b>	5	7	8	11	15

**c**

<b>m</b>	4	5	7	11	12
<b>a</b>	0	1	3	7	8

**e**

<b>f</b>	1	3	4	6	11
<b>g</b>	1	5	7	11	21

**g**

<b>d</b>	2	4	5	7	12
<b>a</b>	5	13	17	25	45

**i**

<b>f</b>	2	3	4	8	11
<b>e</b>	2	13	24	68	101

**k**

<b>b</b>	20	55	60	75	100
<b>t</b>	80	220	240	300	400

**b**

<b>t</b>	1	2	3	8	15
<b>w</b>	6	7	8	13	20

**d**

<b>s</b>	0	1	2	5	12
<b>t</b>	3	5	7	13	27

**f**

<b>s</b>	1	3	7	9	12
<b>c</b>	4	10	22	28	37

**h**

<b>s</b>	1	2	5	6	10
<b>g</b>	1	6	21	26	46

**j**

<b>p</b>	1	2	5	6	12
<b>q</b>	24	44	104	124	244

**l**

<b>u</b>	5	10	20	100	7000
<b>p</b>	14	29	59	299	20999

### UNDERSTANDING

For questions **4** to **12**, a formula has been used by an algebra machine to produce each table.

**4 MC**

<b>a</b>	0	1	3	6	36
<b>b</b>	2	3	5	8	38

The formula used is:

- A  $b = a + 5$       B  $b = 3a$       C  $b = a + 2$   
 D  $b = a - 2$       E  $b = a - 5$

**5 MC**

<b>g</b>	1	2	4	18	56
<b>k</b>	0	1	3	17	55

The formula used is:

- A  $k = g + 1$       B  $k = 2g$       C  $k = g - 1$   
 D  $k = 2g - 2$       E  $k = 2g + 2$

**6 MC**

<b>r</b>	5	6	8	9	17
<b>w</b>	0	1	3	4	12

The formula used is:

- A  $w = r + 5$       B  $w = 5r$       C  $w = 2r - 5$   
 D  $w = r - 5$       E  $w = r + 6$

**7 MC**

<b>p</b>	0	6	7	9	21
<b>m</b>	0	12	14	18	42

The formula used is:

- A  $m = p$       B  $m = p + 6$       C  $m = p + 2$   
 D  $m = 12p$       E  $m = 2p$

**8 MC**

<b>m</b>	1	2	5	7	98
<b>w</b>	8	9	12	14	105

The formula used is:

- A  $w = 8m$       B  $w = 2m + 6$       C  $w = m + 7$   
 D  $w = 2m + 5$       E  $w = 2m$

**9 MC**

<b>p</b>	2	3	4	11	78
<b>t</b>	8	11	14	35	236

The formula used is:

- A  $t = p + 6$       B  $t = 2p + 3$       C  $t = 4p$   
 D  $t = 3p + 2$       E  $t = 3p + 1$

**10 MC**

<b>g</b>	2	3	4	11	78
<b>k</b>	10	15	20	55	390

The formula used is:

- A  $k = 5g$       B  $k = 4g + 2$       C  $k = g + 8$   
 D  $k = 2g + 6$       E  $k = 4g - 2$

**11 MC**

<b>b</b>	1	2	9	14	65
<b>a</b>	9	11	25	35	137

The formula used is:

- A  $a = 8b + 1$       B  $a = 3b + 5$       C  $a = b + 8$   
 D  $a = 2b + 7$       E  $a = b - 8$

**12 MC**

<b>m</b>	2	3	5	7	11
<b>e</b>	25	37	61	85	133

The formula used is:

- A  $e = m + 78$       B  $e = 10m + 11$   
 C  $e = 12m + 1$       D  $e = 6m + 43$   
 E  $e = 10m - 11$

- 13 Breanna was having some friends over after school. She needed enough biscuits so that they could have 3 biscuits each. Her sisters would also be home. She needed 9 extra biscuits for herself and her sisters. If she wants to have  $n$  friends over, write a formula to decide how many biscuits she would need.

**REASONING**

- 14 Look at the following tables and use the variables given to write a formula for each table.

<b>a</b>	$a$	1	2	3	5	10
	$t$	0	3	8	24	99

<b>b</b>	$b$	1	3	7	11	15
	$x$	1	2	4	6	8

<b>c</b>	$w$	0	1	2	4	10
	$p$	0	2	8	32	200

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**REFLECTION**

What strategies can you use to help you find a formula?

## 8C Substitution

- If we know the rule or formula for a particular table, we can take on the role of an ‘algebra machine’ and find the numbers that are needed to fill any gap.
- When a variable in a formula is replaced by a number, we say that the number is *substituted* into the formula.

**WORKED EXAMPLE 9**

If  $x = 3t - 6$ , substitute the given values of  $t$  into the formula to find the value of  $x$  in each case.

- a  $t = 5$       b  $t = 12$

**THINK**

- a 1 Write the formula.  
 2 Substitute 5 for  $t$ .  
 3 Perform the multiplication.  
 4 Subtract 6 from the above result.  
 5 Write the answer.  
 b 1 Write the formula.  
 2 Substitute 12 for  $t$ .  
 3 Perform the multiplication.  
 4 Subtract 6 from the above result.  
 5 Write the answer.

**WRITE**

- a  $x = 3t - 6$   
 If  $t = 5$   
 $x = 3 \times 5 - 6$   
 $x = 15 - 6$   
 $x = 9$   
 b  $x = 3t - 6$   
 If  $t = 12$   
 $x = 3 \times 12 - 6$   
 $x = 36 - 6$   
 $x = 30$

## WORKED EXAMPLE 10

Find the value of  $m$  by substituting the given value of  $x$  into the formula below.

$$m = 3(2x + 3), x = 4$$

## THINK

- 1 Write the formula.
- 2 Substitute 4 for  $x$ .
- 3 Work inside the brackets first:
  - (a) Multiply 2 by 4 (remember that  $2x$  means  $2 \times x$ ).
  - (b) Add 3 to the above result.
- 4 Multiply by the number outside the brackets.
- 5 Write the answer.

## WRITE

$$m = 3(2x + 3)$$

If  $x = 4$

$$m = 3(2 \times 4 + 3)$$

$$m = 3(8 + 3)$$

$$m = 3(11)$$

$$m = 3 \times 11$$

$$= 33$$

## REMEMBER

When a variable in a formula is replaced by a number, we say that the number is *substituted* into the formula.

## EXERCISE

## 8C Substitution

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 8-C-1

Substitution  
doc-1827

## Activity 8-C-2

More substitution  
doc-1828

## Activity 8-C-3

Advanced  
substitution  
doc-1829

## FLUENCY

- 1 **WE9** Substitute the given values into each formula to find the value of  $m$  in each case.

- |                        |                  |                   |                      |                     |
|------------------------|------------------|-------------------|----------------------|---------------------|
| <b>a</b> $m = g - 2$   | <b>i</b> $g = 4$ | <b>ii</b> $g = 5$ | <b>iii</b> $g = 2$   | <b>iv</b> $g = 102$ |
| <b>b</b> $m = 2t - 3$  | <b>i</b> $t = 7$ | <b>ii</b> $t = 2$ | <b>iii</b> $t = 100$ | <b>iv</b> $t = 8$   |
| <b>c</b> $m = 12h + 7$ | <b>i</b> $h = 1$ | <b>ii</b> $h = 0$ | <b>iii</b> $h = 5$   | <b>iv</b> $h = 20$  |
| <b>d</b> $m = 25 - 4w$ | <b>i</b> $w = 1$ | <b>ii</b> $w = 3$ | <b>iii</b> $w = 6$   | <b>iv</b> $w = 0$   |

- 2 **WE10** Find the value of  $m$  by substituting the given value of the variable into the formula.

- |                             |                  |                    |                     |                    |
|-----------------------------|------------------|--------------------|---------------------|--------------------|
| <b>a</b> $m = 2(g + 1)$     | <b>i</b> $g = 1$ | <b>ii</b> $g = 0$  | <b>iii</b> $g = 12$ | <b>iv</b> $g = 75$ |
| <b>b</b> $m = 5(x - 2)$     | <b>i</b> $x = 6$ | <b>ii</b> $x = 10$ | <b>iii</b> $x = 11$ | <b>iv</b> $x = 2$  |
| <b>c</b> $m = 4(12 - p)$    | <b>i</b> $p = 2$ | <b>ii</b> $p = 3$  | <b>iii</b> $p = 12$ | <b>iv</b> $p = 11$ |
| <b>d</b> $m = 5(2g - 3)$    | <b>i</b> $g = 2$ | <b>ii</b> $g = 14$ | <b>iii</b> $g = 5$  | <b>iv</b> $g = 9$  |
| <b>e</b> $m = 2(d + 2) - 3$ | <b>i</b> $d = 3$ | <b>ii</b> $d = 0$  | <b>iii</b> $d = 7$  | <b>iv</b> $d = 31$ |

- f**  $m = 3(f - 1) + 17$
- i**  $f = 1$       **ii**  $f = 3$       **iii**  $f = 6$       **iv**  $f = 21$
- g**  $m = 4s - s$
- i**  $s = 3$       **ii**  $s = 1$       **iii**  $s = 101$       **iv**  $s = 72$
- h**  $m = 3(y + 5) - 2$
- i**  $y = 1$       **ii**  $y = 5$       **iii**  $y = 0$       **iv**  $y = 12$
- i**  $m = 50 - 6v$
- i**  $v = 4$       **ii**  $v = 7$       **iii**  $v = 1$       **iv**  $v = 8$

**UNDERSTANDING**

- 3** Most tinned and packaged foods that we buy have labels which list the nutritional content of the food. Most of the energy (measured in calories or kilojoules) in food comes from fat, protein and carbohydrates.

It is estimated that there are 9 calories in each gram of fat and 4 calories in each gram of protein and each gram of carbohydrate. Hence, if a particular food contained 1.6 grams of fat, 2.1 grams of protein and 16.3 grams of carbohydrate, it would be equal to  $1.6 \times 9 = 14.4$  calories of fat,  $2.1 \times 4 = 8.4$  calories of protein and  $16.3 \times 4 = 65.2$  calories of carbohydrate. The total number of calories would be 88 (that is,  $14.4 + 8.4 + 65.2$ ).

- a** From the label shown, write the number of grams of fat, protein and carbohydrate in 100 grams of baked beans.



- b** Estimate how many calories there would be in 100 grams of baked beans using the information supplied above.

If we wanted to look at a wide variety of foods, it would be more convenient to have a rule or formula into which we could substitute numbers to easily obtain our answer.

- c** Clearly explain how you obtained your answer in question **b**.

- d** Choose a variable for each of the following quantities:

- i** number of grams of fat
- ii** number of grams of protein
- iii** number of grams of carbohydrate
- iv** number of calories.

- e** Using the variables you have chosen, write a formula to calculate the number of calories.

- f** The table below contains data on the fat, protein and carbohydrate content of selected foods. Copy this table into your workbook and write in your chosen variables in the first row within the brackets. Use your formula to work out the calorie content of each of these foods. (The last column will be completed later on.)

Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrate	Number of calories	Number of kilojoules
1 hot cross bun	3	3	19		
100 grams of chocolate cake	16	4	56		
100 grams of roast chicken	14	26	0		
70 grams of bacon	8	21	0		
2 grilled sausages	17	13	15		
1 piece of fish (flake), no batter	1	21	0		
10 grams of sultanas	0	0	6		
25 grams of dried apricots	0	0	17		
1 banana	0	1	20		
1 apple	0	0	17		
1 serve of carrots	0	0	5		
1 serve of potatoes	0	0	17		
250 millilitres of milk	10	8	12		
210 grams of tinned tomato noodle soup	0.8	2.6	14.9		

- g** Which quantity of food in this list has the highest number of calories? Is this what you expected?  
**h** What types of food in this list generally have a lower number of calories?

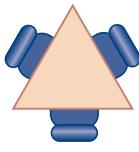
A calorie is an energy measurement unit in the imperial system. Nowadays we mostly use the metric system unit of kilojoules to measure energy. It is estimated that the number of kilojoules is equivalent to the number of calories multiplied by 4.2.

- i** Use a variable to represent each of the quantities ‘number of calories’ and ‘number of kilojoules’ and write a formula connecting them.  
**j** Convert the calories you calculated in **b** to kilojoules (rounding to the nearest kilojoule). Compare this to the value on the label.  
**k** Complete the final column of your table by using the formula to find the number of kilojoules for each listed food. (Round your answers to the nearest kilojoule.)

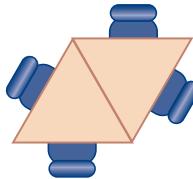


### REASONING

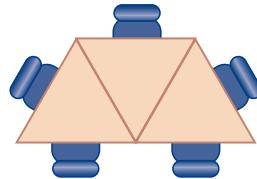
- 4** The dining tables at the ‘Trio’ theatre restaurant are triangular in shape. Diners are seated at the tables in the arrangements shown below.



One table, 3 diners



Two tables, 4 diners

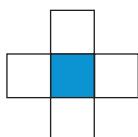


Three tables, 5 diners

The manager of the restaurant has received a booking from a large party. She sets out a row of 24 tables. How many diners can be seated at this row of tables?

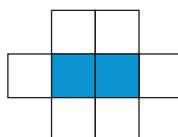
Follow the steps below to solve this problem.

- a Draw a table with a row for the number of tables ( $T$ ) and another row for the number of diners ( $D$ ). Using the diagrams above, write in the pairs of numbers.
  - b Draw a diagram to work out the number of diners who could sit at a row of 4 tables. Now draw another diagram to show the number of diners who could sit at a row of 5 tables. Write these pairs of values in your table.
  - c Work out the formula which connects the number of diners and the number of tables. Write the formula in the form  $D = \dots$
  - d Substitute  $T = 24$  into the formula to find out how many guests could be seated at the row of 24 tables.
- 5 Jane is tiling the floor in her bathroom. She has decided to use blue and white tiles in the following pattern:



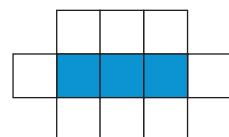
Number of blue tiles = 1

Number of white tiles = 4



Number of blue tiles = 2

Number of white tiles = 6



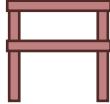
Number of blue tiles = 3

Number of white tiles = 8

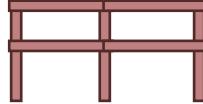
Find out how many white tiles Jane would need to complete the pattern for a row of 10 blue tiles by following the steps below:

- a Draw a table with two rows; one headed  $B$  showing the number of blue tiles and one headed  $W$  showing the number of white tiles required. Count the numbers of blue and white tiles from each of the tiling patterns above and put them into the table in the correct row.
- b Draw a diagram of the tiling pattern for a row of 4 blue tiles and work out the number of white tiles needed to complete the pattern. Then draw another pattern for a row of 5 blue tiles. Add these two sets of values for  $B$  and  $W$  into the table.
- c Look carefully at the table and work out the formula that connects  $W$  and  $B$ . Write the formula in the form  $W = \dots$
- d Substitute  $B = 10$  into the formula to find out how many white tiles would be needed for a row of 10 blue tiles, using Jane's tiling pattern.

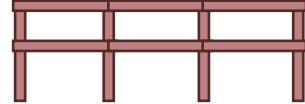
- 6 Michael is constructing a timber fence at his stud farm. The sections of fence are shown below:



1 section, 4 pieces



2 sections, 7 pieces



3 sections, 10 pieces

Michael calculates that he will need 220 sections to fence off his first paddock.

How many pieces of timber will Michael need? Find the cost of the fence if each piece of timber costs \$3.85.

Follow the steps below.

- a Draw a table with 2 rows, one headed  $S$  showing the number of sections and one headed  $P$  showing the number of pieces of timber required. Write in the numbers from the diagrams above.
- b Draw a sketch of the fence with 4 sections, following the same pattern. Count the number of pieces of timber required. Draw a sketch for 5 sections and count the pieces required. Enter both pairs of numbers into your table in the correct columns.
- c Use trial and error to find the formula connecting  $P$  and  $S$ . Write the formula in the form  $P = \dots$
- d Substitute  $S = 220$  into the formula to find out how many pieces of timber Michael needs.
- e Find the cost of the fence.

- 7 A T-shirt shop uses the letter T as its logo. The company wishes to make up a large advertising sign from square lights. Several small T-shirt shop signs are shown below:



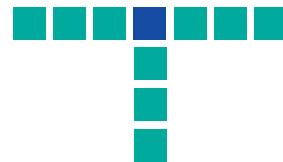
Arm length = 1

Number of lights = 4



Arm length = 2

Number of lights = 7



Arm length = 3

Number of lights = 10

Find out how many square light panels would be needed to make up a large sign with an arm length of 75. Follow the steps below.

- Draw a table with rows headed  $A$  (arm length), and  $L$  (number of lights needed).
- Write in the values of  $A$  and  $L$  from the diagrams above.
- Complete the diagrams showing the number of lights needed for a sign with an arm length of 4, and a sign with an arm length of 5. Add these values to your table.
- Look at the table to determine the formula which connects  $L$  and  $A$ . Write the formula in the form  $L = \dots$
- Substitute  $A = 75$  into the formula to find the number of lights needed for the large display sign.

- 8 Karl Frederick Gauss, when he was a young man, discovered the sum  $S$  of the first  $n$  natural numbers is given by the formula  $S = \frac{n(n+1)}{2}$ .

Using this formula, find the answer to

- $1 + 2 + 3 + 4 + \dots + 46 + 47 + 48 + 49$
- $54 + 55 + 56 + 57 + \dots + 85 + 86 + 87 + 88$ .

#### REFLECTION

What strategies could you use to check that your answer is reasonable?

## 8D Terms, expressions and equations

- A **variable** (or pronumeral) is a letter or symbol that is used in place of a number.
- The number in front of a variable is called the **coefficient**. The coefficient of  $7x$  is 7. The coefficient of  $6xyz$  is 6. The coefficient of  $p$  is 1.
- **Expressions** are made up of variables and numbers. Some expressions are  $3k - 8$ ,  $3t + 4$ ,  $6(g - 9)$ ,  $9p - 3y + 5$ .

### WORKED EXAMPLE 11

Write expressions to represent the total number of coins in each of the following situations where



represents a full moneybox and represents one coin. Use  $c$  to represent the number of coins in a moneybox.

- Mary fills one moneybox and has two coins left over.
- Valentino and Loris decide to combine their coins. Valentino has three moneyboxes with four coins left over and Loris has two moneyboxes with six coins left over. That is,

Valentino has



Loris has



#### THINK

- The total number of coins will be found by adding the number of coins in one moneybox plus two coins left over.
- Remove the multiplication sign to write the expression.

#### WRITE

$$\begin{aligned} a & 1 \times c + 2 \\ & = c + 2 \end{aligned}$$

- b** 1 Add the number of coins Valentino and Loris have.



- 2 Count the moneyboxes first, and then the coins.



- 3 Simplify.

$$\mathbf{b} \quad (3c + 4) + (2c + 6)$$

$$= (3c + 2c) + (4 + 6)$$

$$= 5c + 10$$

### WORKED EXAMPLE 12

Write an expression for the sum of  $T$  and  $G$ .

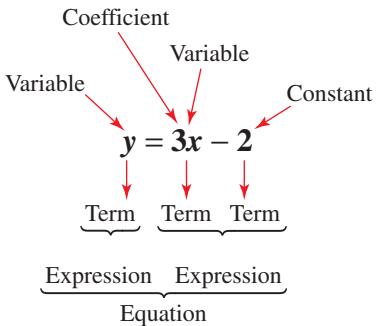
#### THINK

The word ‘sum’ means to add together.

#### WRITE

The expression is  $T + G$  or  $G + T$ .

- Equations always contain an equality sign, whereas expressions do not. Examples of equations are:  $y = 4t - 6$ ,  $m + 3 = y - 8$ ,  $2(g + 5) = 5$ ,  $y = 6t$ ,  $w = e - 87$ ,  $9 = 7 + 4r$ .
- Expressions and equations are made up of **terms**. Terms are separated by + or – signs.
- Terms can contain a combination of numbers and one or more variables, or may consist of a number only. Examples of terms are  $3t$ ,  $2y$ ,  $7$ ,  $5gh$ ,  $m$ ,  $brt$ .
- The letter, or symbol, is called a **variable** because its value can change.
- Because the value of a number cannot change, if a term contains only a number, it is called a **constant**.
- The definitions for this section can be summarised as shown below.



### WORKED EXAMPLE 13

If  $Y$  represents any number, write expressions for:

- 5 times that number
- 2 less than that number
- 8 more than that number
- the number divided by 4 (or the quotient of  $Y$  and 4)
- the next consecutive number (that is, the counting number which comes after  $Y$ ).

#### THINK

In each case think about which operations are being used and the order in which they occur.

- When multiplying we don't show the multiplication sign. Remember to put the number first.

#### WRITE

$$\mathbf{a} \quad 5 \times Y \text{ or } 5Y$$



- ◀**
- b An expression with 2 less means ‘subtract 2’.
  - c An expression with 8 more means ‘add 8’.
  - d A number divided by 4 means ‘write as a quotient (fraction)’.
  - e The next consecutive number means ‘add 1 to the number’.
  - b  $Y - 2$
  - c  $Y + 8$
  - d  $\frac{Y}{4}$  or  $Y \div 4$
  - e  $Y + 1$

**WORKED EXAMPLE 14**

Write expressions for the following rules.

- a Take a number and add another number.
- b Multiply 2 numbers.
- c Add 2 numbers and multiply the answer by 6.
- d Take a number and multiply it by 4, then subtract 8 from that answer.

**THINK**

- 1 In each case choose any variable to represent an unknown number.
  - 2 In each case think about which operations are being used and the order in which they occur.
- a Add two variables.
  - b Multiply two variables but don’t write a multiplication sign ( $\times$ ).
  - c Add two variables and place the expression in brackets. Multiply by 6.  
(Remember to write the numbers first.)
  - d First multiply a variable by 4, writing the number first ( $4b$ ). Then subtract 8.

**WRITE**

- a  $B + G$  or  $G + B$
- b  $xy$
- c  $6(t + f)$
- d  $4b - 8$

**WORKED EXAMPLE 15**

Write equations for the following rules.

- a If 15 is added to a number the result is 21.
- b The quotient of a number and 7 is equal to 5.

**THINK**

- a 1 Choose a variable to represent the number.
- 2 Add 15 to the variable then place an equals sign followed by 21.
- b 1 Choose a variable to represent the number.
- 2 Divide the variable by 7 (that is, write it as a fraction) then place an equals sign followed by 5.

**WRITE**

- a Let  $x =$  the number  
 $x + 15 = 21$
- b Let  $z =$  the number  
 $\frac{z}{7} = 5$

**REMEMBER**

1. A variable (or pronumeral) is a letter used in place of a number.
2. A number in front of a variable is known as a *coefficient*.
3. *Expressions* are parts of a rule made up of variables and numbers.
4. *Equations* always contain an equals sign whereas expressions do not.
5. Expressions and equations are made up of *terms*.
6. When writing expressions, think about which operations are being used, and the order in which they occur.
7. If variables are not given in a question, you can choose any letters you like.

**EXERCISE****8D****Terms, expressions and equations****INDIVIDUAL PATHWAYS****eBookplus****Activity 8-D-1**

Terms and expressions  
doc-1830

**Activity 8-D-2**

More terms and expressions  
doc-1831

**Activity 8-D-3**

Advanced terms and expressions  
doc-1832

**FLUENCY**

- 1** **WE11a, b** Write an expression for the total number of coins in each of the following, using  $c$  to represent the number of coins in a moneybox.



- 2** Christie and Jane both had two full moneyboxes and seventeen coins each. They combined their money and spent every cent on a day out in town.

$$2 \times (\text{piggy bank} + \text{coins})$$

- a Write an expression to represent the number of coins they had in total.  
 b Christie opened the first moneybox and counted 52 twenty-cent pieces inside. Use your expression to calculate how many coins the girls had in total, assuming that all the moneyboxes held the same number of coins.  
 c What is the total amount that the girls spent on their day out?  
 d Luke has three macadamia nut trees in his backyard. He saves takeaway containers to store the nuts in. He has two types of containers, rectangular  and round . Using  $m$  to represent the number of nuts in a rectangular container and  $n$  to represent the number of nuts in a round container, write expressions for the following.



- 4** Luke found that a rectangular container holds 17 nuts and a round container holds 12 nuts. Calculate how many nuts Luke would have for each part in question 3 by substituting the appropriate values into each expression.

**5 WE12** Write an expression for each of the following.

- a The sum of  $B$  and 2
- b 3 less than  $T$
- c 6 added to  $D$
- d 5 taken away from  $K$
- e The sum of  $G$ ,  $N$  and  $W$
- f  $D$  increased by  $H$
- g  $N$  increased by  $N$
- h  $H$  added to  $C$
- i  $G$  subtracted from 12
- j  $D$  multiplied by 4
- k 6 added to  $H$
- l The difference between  $Z$  and  $G$
- m  $B$  multiplied by  $F$
- n  $Y$  added to the product of 3 and  $M$

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Algebraic  
expressions  
game

**6 WE13** If  $A$ ,  $B$  and  $C$  represent any 3 numbers, write an expression for each of the following.

- a The sum of all 3 numbers
- b The difference between  $A$  and  $C$
- c The product of  $A$  and  $B$
- d The product of all 3 numbers
- e The quotient of  $B$  and  $C$  (that is,  $B$  divided by  $C$ )
- f The sum of  $A$  and  $C$ , divided by  $B$
- g 3 more than  $A$

**7 WE14** Write expressions for the following rules.

- a The number of students left in the class if  $X$  students leave for the canteen out of a total group of  $T$  students
- b The amount of money earned by selling  $B$  cakes where each cake is sold for \$4.00
- c The total number of sweets if there are  $G$  bags of sweets with 45 sweets in each bag
- d The cost of one concert ticket if 5 tickets cost \$ $T$
- e The number of students in a class that contains  $R$  boys and  $M$  girls
- f The number of insects if there are  $Y$  legs altogether
- g The cost of  $M$  movie tickets if each ticket costs \$11.00
- h The total amount of fruit sold if  $A$  apples and  $H$  bananas are sold
- i The number of cards left in a pack of 52 cards if  $J$  cards are removed
- j The total number of floor tiles if there are  $B$  boxes of tiles with 12 tiles in each box
- k The average of  $B$  and  $G$
- l The total runs scored by 3 batsmen if they have scores of  $A$ ,  $H$  and  $K$  respectively
- m The total number of legs if there are  $R$  chairs

**8 WE15** Write equations for the following rules.

- a If 8 is added to a number the result is 11.
- b If 11 is subtracted from a number the result is 63.
- c The product of a number and 8 is equal to 128.
- d The quotient of a number and 7 is equal to 5.
- e The total cost of  $A$  apples and  $H$  bananas is \$6.49.
- f When  $Q$  cards are removed from a pack of 52 cards only 19 remain.
- g The total cost of  $M$  concert tickets at \$99 each was \$792.
- h The total number of legs if there are  $R$  chairs and  $D$  tables is 164.



## UNDERSTANDING

- 9** Answer True or False for each of the statements below.
- a  $3x$  is a term.
  - b  $3mn$  is a term.
  - c  $g = 23 - t$  is an expression.
  - d  $g = 5t - 6$  is an equation.
  - e  $rt = r \times t$
  - f  $5 + A = A + 5$
  - g  $3d + 5$  is an expression.
  - h  $7 - B = B - 7$
  - i  $3x = 9$  is an expression.
  - j The expression  $g + 2t$  has two terms.
  - k  $2f + 4 = 4 + 2f$
  - l  $5a - 7t + h$  is an equation.
- 10 MC** In each of the following, the letter  $N$  has been used to represent a number. For each expression written in words, choose the answer that you think matches:
- a five times the number  
A  $N + 5$       B  $5N$       C  $N - 5$       D  $N \div 5$       E  $5 - N$
  - b the sum of the number and 52  
A  $52 - N$       B  $52N$       C  $N + 52$       D  $N - 52$       E  $N \div 52$
  - c the next consecutive number  
A  $O$       B  $2N$       C  $N - 1$       D  $N + 1$       E  $12N$
  - d the number multiplied by another number  
A  $N + T$       B  $N \div S$       C  $N + 4$       D  $NS$       E  $N - 5$
  - e half the number  
A  $2N$       B  $N \div 2$       C  $N + \frac{1}{2}$       D  $N - 2$       E  $2 + N$
  - f four more than the number  
A  $N + 4$       B  $4N$       C  $N - 4$       D  $4 - N$       E  $N \div 4$
  - g the number plus the product of another number and 2  
A  $N + 2D$       B  $D(N + 2)$       C  $2N + 2B$       D  $2N + D$       E  $N + T$
- 11** If  $x$  is a whole number, write down the value of:
- a the number 5 more than  $x$
  - b the number 8 less than  $x$
  - c the number twice as large as  $x$
  - d the number one quarter of  $x$
  - e the next largest number after  $x$
  - f the number of cents in  $x$  dollars
  - g the number of metres in  $x$  centimetres
  - h the number of days in  $x$  weeks
  - i the (mean) average of  $x$  and 20
  - j how much it costs to rent a tennis court for  $x$  hours at \$10 per hour
  - k the average of 5, 18 and  $k$ .
- 12 MC**
- a John is  $x$  years old. In 5 years his age will be:  
A  $5x$       B  $17$       C  $x - 5$       D  $x + 5$       E  $10$
  - b Chris is 2 years older than Jenny. If Jenny is  $x$  years old then Chris's age is:  
A  $x - 2$       B  $2x$       C  $x + 2$       D  $2$       E  $\frac{x}{2}$
  - c Duy earns \$5 per week more than Ben. If Duy earns \$ $x$ , then Ben earns:  
A  $x + 5$       B  $5x$       C  $x - 5$       D  $5 + x$       E  $5 - x$
  - d Assume  $x$  is an even number. The next largest even number is:  
A  $x + 1$       B  $2x + 2$       C  $4$       D  $2x$       E  $x + 2$
  - e Assume  $x$  is an odd number. The next largest odd number is:  
A  $x + 1$       B  $x + 2$       C  $3$       D  $2x + 1$       E  $2x + 2$
  - f Gian is half Carla's age. If Gian is  $x$  years old, then Carla's age is:  
A  $\frac{x}{2}$       B  $x + \frac{1}{2}$       C  $2x$       D  $\frac{1}{2}x$       E  $x - \frac{1}{2}$
- 13**
- a Ali earns \$7 more per week than Halit. If Halit earns \$ $x$ , find their total earnings per week.
  - b Sasha is twice as old as Kapila. If Kapila is  $x$  years old, find their total age.
  - c Frank has had 3 more birthdays than James. If James is  $2x$  years old, find their total age.

- d If  $x$  is the smaller of 3 consecutive whole numbers, find the sum of the 3 numbers.
- e If  $x$  is the smallest of 3 consecutive odd numbers, find the sum of the 3 numbers.
- f If  $x$  is the smallest of 3 consecutive even numbers, find the sum of the 3 numbers.

**REASONING**

- 14** a Write a rule to describe how you would calculate the total cost of 3 items, two hamburgers and one drink.  
 b Let  $h$  represent the cost of a hamburger. If the drink costs \$3.75 and the total cost is \$7.25, write an equation which links all this information.  
 c To hire a windsurfer, there is a \$50 deposit, and you are charged \$12 per hour.



- a How much will it cost to hire a windsurfer for:
  - i 1 hour?
  - ii 2 hours?
  - iii 3 hours?
- b Write a rule that could be used to calculate the cost of hiring a windsurfer for  $n$  hours.
- c Use the rule to calculate the cost of hiring a windsurfer for 8 hours.
- d You have \$100 to spend. Write an equation to help you work out the number of hours for which you could hire a windsurfer.

**REFLECTION**

What strategies can you use to check that your formula is correct?

## 8E Simplifying and the distributive law

### Simplifying expressions containing like terms

- Terms containing exactly the same variables, regardless of order, are called **like terms**.
- Examples of like and non-like terms are:

**Like terms**

- $5e$  and  $6e$
- $x$  and  $13x$
- $2bc$  and  $4cb$
- $3g^2$  and  $45g^2$

**Non-like terms**

- $5x$  and  $5y$
- $12xy$  and  $21x$
- $15abc$  and  $4bcd$
- $16g$  and  $45g^2$

- When simplifying expressions, we can add or subtract like terms only.

- Some examples of simplifying are:

$$6b + 4b = 10b$$

$$5xy + 8xy = 13xy$$

$$12y + 3y - 5y = 10y$$

$$4t - 3t = t$$

$$7t - 5t = 2t$$

- Some expressions that cannot be simplified are:

$5g + 5c$  cannot be simplified

$4t - 4$  cannot be simplified

$3xy + 4x$  cannot be simplified

$14abc - 6bc$  cannot be simplified

$45b - 43$  cannot be simplified

$4g - 4n$  cannot be simplified.

### WORKED EXAMPLE 16

Where possible, simplify the following expressions by adding or subtracting like terms.

- a**  $4g + 6g$       **b**  $11ab - ab$       **c**  $6ad + 5da$   
**d**  $4t + 7t - 5$       **e**  $8x + 3y$

#### THINK

- a** The variables are the same. The terms can be added.
- b** The variables are the same. The terms can be subtracted. Note that  $ab$  is the same as  $1ab$ .
- c** Although the order of the variables is different, the terms are like terms and can be added.  
*Note:* Variables may be rearranged in alphabetical order.
- d** The variables are the same and the first two terms can be added. Do not add the other term.
- e** The variables are not the same. These are not like terms and therefore cannot be simplified.

#### WRITE

- a**  $4g + 6g = 10g$   
**b**  $11ab - ab = 10ab$   
**c**  $6ad + 5da = 6ad + 5ad$   
 $= 11ad$   
**d**  $4t + 7t - 5 = 11t - 5$   
**e**  $8x + 3y$  cannot be simplified.

### WORKED EXAMPLE 17

Simplify the following expressions first, then find the value of the expression if  $a = 4$ .

- a**  $5a + 2a$   
**b**  $7a - a + 5$

#### THINK

- a**
  - 1 Add the like terms.
  - 2 Substitute 4 for  $a$  in the simplified expression.
  - 3 Evaluate.
- b**
  - 1 Subtract the like terms.
  - 2 Substitute 4 for  $a$  in the simplified expression.
  - 3 Evaluate.

#### WRITE

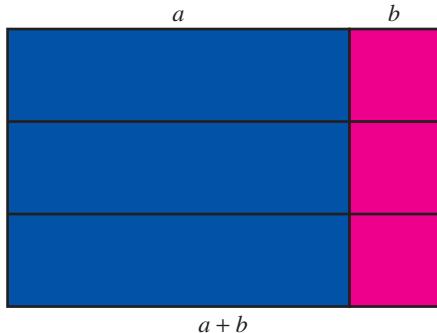
- a**  $5a + 2a = 7a$   
If  $a = 4$   
 $7a = 7 \times a$   
 $= 7 \times 4$   
 $= 28$
- b**  $7a - a + 5 = 6a + 5$   
If  $a = 4$   
 $6a + 5 = 6 \times a + 5$   
 $= 6 \times 4 + 5$   
 $= 24 + 5$   
 $= 29$

## The distributive law

- As can be seen in the diagram,  $3(a + b) = 3a + 3b$ .
- We can think of  $3(a + b) = (a + b) + (a + b) + (a + b)$ .  
Collecting like terms,  
 $3(a + b) = a + a + a + b + b + b = 3a + 3b$ .
- An expression containing a bracket multiplied by a number can be written in expanded or factorised form.

$$\text{factorised form} \quad \text{expanded form}$$

$$3(a + b) = 3a + 3b$$



- The **distributive law** is the name given to this process because the number out in front is distributed to each of the terms in the brackets.

$$a(b + c) = ab + ac$$

- The distributive law can be used when the terms inside the brackets are either added or subtracted.

$$a(b - c) = ab - ac$$

- The distributive law is not used when the terms inside the brackets are multiplied or divided. You can see this with numbers  $2(4 \times 5) = 2 \times 4 \times 5$  not  $(2 \times 4) \times (2 \times 5)$ .
- When simplifying expressions, you can leave the result in either factorised form or expanded form, but not a combination of both.

### WORKED EXAMPLE 18

Write the following expressions in expanded form, then simplify.

- a**  $4(2a + d)$       **b**  $a(3x + 2y)$       **c**  $3(k + 4a - m)$

#### THINK

- a** 1 The number 4 is distributed to both the  $2a$  and the  $d$ .  
2 Simplify the terms.
- b** 1 The number represented by the variable  $a$  is distributed to both the  $3x$  and the  $2y$ .  
2 Simplify the terms.
- c** 1 The number 3 is distributed to  $k$ ,  $4a$  and  $m$ .  
2 Simplify the terms.

#### WRITE

$$\begin{aligned} \text{a } 4(2a + d) &= 4 \times 2a + 4 \times d \\ &= 8a + 4d \\ \text{b } a(3x + 2y) &= a \times 3x + a \times 2y \\ &= 3ax + 2ay \\ \text{c } 3(k + 4a - m) &= 3 \times k + 3 \times 4a - 3 \times m \\ &= 3k + 12a - 3m \end{aligned}$$

### WORKED EXAMPLE 19

Use the distributive law to expand and simplify the following.

- a**  $4(2s + 3k) - 2k$       **b**  $a(3 - c) + a$

#### THINK

- a** 1 Use the distributive law to expand  $4(2s + 3k)$ .  
2 Simplify the terms.  
3 Collect like terms.
- b** 1 Use the distributive law to expand  $a(3 - c)$ .  
2 Simplify the terms.  
3 Collect like terms.

#### WRITE

$$\begin{aligned} \text{a } 4(2s + 3k) - 2k &= 4 \times 2s + 4 \times 3k - 2k \\ &= 8s + 12k - 2k \\ &= 8s + 10k \\ \text{b } a(3 - c) + a &= a \times 3 - a \times c + a \\ &= 3a - ac + a \\ &= 4a - ac \end{aligned}$$

**REMEMBER**

1. Terms containing the same variables are called *like terms*.
2. The process of adding or subtracting like terms is called *simplifying*.
3. Substituting known values for variables in an expression allows us to evaluate the expression.
4. Using the distributive law, expressions in factorised form can be written in expanded form.
5.  $a(b + c) = ab + ac$ .
6.  $a(b - c) = ab - ac$ .

**EXERCISE****8E****Simplifying and the distributive law****INDIVIDUAL PATHWAYS****eBook plus****Activity 8-E-1**The distributive law  
doc-1833**Activity 8-E-2**More of the  
distributive law  
doc-1834**Activity 8-E-3**Advanced use of  
the distributive law  
doc-1835**FLUENCY**

- 1 WE16** Where possible, simplify the following expressions by adding or subtracting like terms.

- |                       |                          |                         |
|-----------------------|--------------------------|-------------------------|
| <b>a</b> $3a + 2a$    | <b>b</b> $9y + 5y$       | <b>c</b> $3c + 12c$     |
| <b>d</b> $4u - 2u$    | <b>e</b> $7e + 13e$      | <b>f</b> $7t - 2t$      |
| <b>g</b> $12ab + 2ab$ | <b>h</b> $8fg - 2fg$     | <b>i</b> $4e - e$       |
| <b>j</b> $6t + t$     | <b>k</b> $f + 4f$        | <b>l</b> $6y - 6y$      |
| <b>m</b> $4x + 14x$   | <b>n</b> $3m + 16m - 7m$ | <b>o</b> $6a + 4a - 2a$ |

- 2** Simplify the following expressions if possible.

- |                          |                             |                             |
|--------------------------|-----------------------------|-----------------------------|
| <b>a</b> $24ab + ab - 7$ | <b>b</b> $5y + y - 3y$      | <b>c</b> $5t + 5s$          |
| <b>d</b> $18i + 12i - 2$ | <b>e</b> $4t + 8t - 3 + 2t$ | <b>f</b> $7r + 2r + 5r - r$ |
| <b>g</b> $2x + 2y$       | <b>h</b> $7y + 6$           | <b>i</b> $18f - 2f + 5$     |
| <b>j</b> $4 + 3g - g$    | <b>k</b> $6t - 5t$          | <b>l</b> $18bg - 18bg$      |
| <b>m</b> $13mno - 11mno$ | <b>n</b> $11pq + 3qp$       | <b>o</b> $6pr + 2 + 5rp$    |
| <b>p</b> $9e - 9t - 1$   | <b>q</b> $7t + 4t - 5$      | <b>r</b> $32t - 31t$        |
| <b>s</b> $11aw - 3aw$    | <b>t</b> $7xy - 7x$         | <b>u</b> $5t + 6t - 8$      |
| <b>v</b> $5t - t + 3$    | <b>w</b> $3g + 7g - 2$      | <b>x</b> $5r + 17r + 4 + 2$ |

- 3** Simplify each of the following.

- |                          |                        |                        |
|--------------------------|------------------------|------------------------|
| <b>a</b> $3t - 3t$       | <b>b</b> $18r - 18r$   | <b>c</b> $12ab - 12ab$ |
| <b>d</b> $5x - 5x + 8$   | <b>e</b> $6t + 7 - 6t$ | <b>f</b> $9g - 9g + 2$ |
| <b>g</b> $13xyz - 13xyz$ | <b>h</b> $5x + 7 - 5x$ | <b>i</b> $5y + 2y - y$ |

- 4 WE17** Simplify the following expressions first, then find the value of the expression if  $a = 7$ .

- |                         |                          |                          |
|-------------------------|--------------------------|--------------------------|
| <b>a</b> $3a + 2a$      | <b>b</b> $7a + 2a$       | <b>c</b> $6a - 2a$       |
| <b>d</b> $9a + a$       | <b>e</b> $13a + 2a - 5a$ | <b>f</b> $3a + 7a$       |
| <b>g</b> $17 + 5a + 3a$ | <b>h</b> $6a - a + 2$    | <b>i</b> $a + a$         |
| <b>j</b> $7a - 6a$      | <b>k</b> $7a - 7a$       | <b>l</b> $12a + 5a - 16$ |

- 5 WE18** Use the distributive law to write the following expressions in expanded form, then simplify.

- |                        |                        |
|------------------------|------------------------|
| <b>a</b> $3(a + 2b)$   | <b>b</b> $5(x - 4z)$   |
| <b>c</b> $10(2g + 3h)$ | <b>d</b> $b(2c - 3a)$  |
| <b>e</b> $2c(8d - q)$  | <b>f</b> $3b(3b - 2g)$ |
| <b>g</b> $x(9a + 7b)$  | <b>h</b> $7k(t + 3m)$  |

- 6 WE19** Write the following expressions in expanded form, then simplify if possible.

- |                             |                              |
|-----------------------------|------------------------------|
| <b>a</b> $5(a + 2b) - 3a$   | <b>b</b> $4(n - 2c) + n$     |
| <b>c</b> $9(b + 2c) - 9b$   | <b>d</b> $x(a - b) + x$      |
| <b>e</b> $y(3 + 2z) + 2y$   | <b>f</b> $3x(d - 3) + 4x$    |
| <b>g</b> $6u(2f - 1) + 20u$ | <b>h</b> $7n(10 + 3h) - 50n$ |

## UNDERSTANDING

**7** Answer True or False to each of the following statements.

- a**  $4t$  and  $6t$  are like terms.
- c**  $5g$  and  $5t$  are like terms.
- e**  $5a$  and  $6a$  are like terms.
- g**  $9fg$  and  $4gf$  are like terms.
- i**  $6r$  and  $5r$  are like terms.
- k**  $3acd$  and  $6cda$  are like terms.
- m**  $8gef h$  and  $3efgh$  are like terms.
- o**  $8xy$  and  $5xy$  are like terms.
- q**  $7eg$  and  $7g$  are like terms.
- b**  $3x$  and  $x$  are like terms.
- d**  $7t$  and  $5i$  are like terms.
- f**  $4a$  and  $4ab$  are like terms.
- h**  $6gh$  and  $7gk$  are like terms.
- j**  $yz$  and  $45zy$  are like terms.
- l**  $5g$  and  $5fg$  are like terms.
- n**  $6ab$  and  $3ba$  are like terms.
- p**  $12ep$  and  $4pe$  are like terms.
- r**  $7y$  and  $18yz$  are like terms.

**8** **MC** **a** Which one of the following is a like term for  $7t$ ?

- |   |                   |                 |                 |                  |                      |
|---|-------------------|-----------------|-----------------|------------------|----------------------|
| <b>A</b> $7g$   | <b>B</b> $5tu$    | <b>C</b> $2$    | <b>D</b> $6f$   | <b>E</b> $5t$    |                      |
| <b>b</b> Which one of the following is a like term for $3a$ ?   | <b>A</b> $5ab$    | <b>B</b> $6a$   | <b>C</b> $3w$   | <b>D</b> $a + 2$ | <b>E</b> $6a$        |
| <b>c</b> Which one of the following is a like term for $5ab$ ?  | <b>A</b> $6abe$   | <b>B</b> $4b$   | <b>C</b> $2ba$  | <b>D</b> $5$     | <b>E</b> $5abe$      |
| <b>d</b> Which one of the following is a like term for $7bgt$ ? | <b>A</b> $7t$     | <b>B</b> $4gt$  | <b>C</b> $2gb$  | <b>D</b> $btg$   | <b>E</b> $b + g + t$ |
| <b>e</b> Which one of the following is a like term for $2fgk$ ? | <b>A</b> $fg + k$ | <b>B</b> $5fg$  | <b>C</b> $2fgk$ | <b>D</b> $12f$   | <b>E</b> $12p$       |
| <b>f</b> Which one of the following is a like term for $20m$ ?  | <b>A</b> $11m$    | <b>B</b> $20mn$ | <b>C</b> $20f$  | <b>D</b> $m + 2$ | <b>E</b> $20$        |
| <b>g</b> Which one of the following is a like term for $9xyz$ ? | <b>A</b> $9x$     | <b>B</b> $2yz$  | <b>C</b> $8xz$  | <b>D</b> $xzy$   | <b>E</b> $9z$        |

**9** Answer True or False to each of the following statements.

- a** The equation  $y = 4x + 3x$  can be simplified to  $y = 7x$ .
- b** The equation  $k = 8y + 4y$  can be simplified to  $k = 10y$ .
- c** The equation  $y = 4x + 3$  can be simplified to  $y = 7x$ .
- d** The equation  $b = 3a - a$  can be simplified to  $b = 2a$ .
- e** The equation  $k = 7a + 4d$  can be simplified to  $k = 11ad$ .
- f** The equation  $y = 5x - 3x$  can be simplified to  $y = 2x$ .
- g** The equation  $m = 7 + 2x$  can be simplified to  $m = 9x$ .
- h** The equation  $p = 3x - 2x$  can be simplified to  $p = x$ .
- i** The equation  $t = 3h + 12h + 7$  can be simplified to  $t = 15h + 7$ .
- j** The equation  $y = 16g + 6g - 7g$  can be simplified to  $y = 15g$ .

**10** Expand each of the following and then simplify.

- a**  $3(x - y) + 5(y + 2x)$
- b**  $5(a - 2y) + a(7 + 6y)$
- c**  $6c(g + 3d) + g(3c + d)$
- d**  $y(6d - r) + 2y(3d + 2r)$

## REASONING

**11** Write the following in factored form:

- a**  $4x + 2$
- b**  $ab + 3a$
- c**  $12ay + 9ab$
- d**  $3bd - 6bc + 9ab$

## REFLECTION

What strategies will you use to identify like terms?

## 8F The associative law

- The **associative law for addition** is:  $(a + b) + c = a + (b + c) = a + b + c$ . Using numbers we can see that  $(9 + 3) + 7 = 12 + 7 = 19$  and  $9 + (3 + 7) = 9 + 10 = 19$ .
- The **associative law for multiplication** is:  $(a \times b) \times c = a \times (b \times c) = a \times b \times c$ . Using numbers we can see that  $(2 \times 3) \times 5 = 6 \times 5 = 30$  and  $2 \times (3 \times 5) = 2 \times 15 = 30$ .
- The associative law does not hold for subtraction and division. Using numbers for subtraction,  $(8 - 3) - 2 = 5 - 2 = 3$ , but  $8 - (3 - 2) = 8 - 1 = 7$ . Using numbers for division,  $(24 \div 6) \div 2 = 4 \div 2 = 2$ , but  $24 \div (6 \div 2) = 24 \div 3 = 8$ .

### WORKED EXAMPLE 20

Use the associative law to complete the following.

a  $2w + (8d + \square) = (\square + 8d) + 5h$

b  $(x \quad \square) \cdot 4g = \square(d \cdot 4g)$

#### THINK

- The associative law says that the numbers can be regrouped.  $2w$ ,  $8d$  and  $5h$  are numbers.  $5h$  is missing from the left-hand side and  $2w$  is missing from the right-hand side.
- The associative law says that the numbers can be regrouped.  $d$  is missing from the left-hand side and  $x$  is missing from the right-hand side.

#### WRITE

a  $2w + (8d + \boxed{5h}) = (\boxed{2w} + 8d) + 5h$

b  $(x \quad \boxed{d}) \cdot 4g = \boxed{x} \cdot (d \cdot 4g)$

- The **commutative law for addition** is:  $a + b = b + a$ . Using numbers we can see that  $5 + 3 = 8$  and  $3 + 5 = 8$ .
- The **commutative law for multiplication** is:  $a \times b = b \times a$ . Using numbers we can see that  $2 \times 3 = 6$  and  $3 \times 2 = 6$ .
- The commutative law does not hold for subtraction and division. Using numbers, we can see that  $5 - 3 \neq 3 - 5$  and  $6 \div 2 \neq 2 \div 6$ .

### WORKED EXAMPLE 21

Use the associative and commutative laws to complete the following.

a  $4r + (6t + \square) = 5g + (\square + \square)$

b  $a \cdot (3t - 2w) = (2w \quad \square) \square a$

#### THINK

- The associative and commutative laws say that the numbers can be regrouped and rearranged.  $5g$  is missing from the left-hand side. Both  $4r$  and  $6t$  are missing from the right-hand side.
- The associative and commutative laws say that the numbers can be regrouped and rearranged.  $3t$  and a multiplication sign are missing from the right-hand side.

#### WRITE

a  $4r + (6t + \boxed{5g}) = 5g + (\boxed{4r} + \boxed{6t})$

b  $a \cdot (3t - 2w) = (2w \quad \boxed{3t}) \square a$

**REMEMBER**

- The associative law for addition is:  $(a + b) + c = a + (b + c) = a + b + c$ .
- The associative law for multiplication is:  $(ab)c = a(bc) = abc$ .
- The associative law does not hold for subtraction or division.
- The commutative law for addition is:  $a + b = b + a$ .
- The commutative law for multiplication is:  $ab = ba$ .
- The commutative law does not hold for subtraction or division.

**EXERCISE****8F****The associative law****INDIVIDUAL PATHWAYS****eBookplus****Activity 8-F-1**The associative law  
doc-1836**Activity 8-F-2**More of the  
associative law  
doc-1837**Activity 8-F-3**Advanced use of  
the associative law  
doc-1838**FLUENCY**

- 1 WE20** Use the associative law to complete the following.

a  $w + (r \square 6y) = (\square + \square) + 6y$       b  $6t (4r \square) = (6\square - 4r) \square 3s$   
 c  $(9y + \square) + 3w = \square + (2r + 3w)$       d  $(z + 2p) + \square = z \square (\square + 6t)$

- 2 WE21** Use the associative and commutative laws to complete the following.

a  $6t + (3w + \square) = \square + (6t + 7v)$       b  $s (9r \square) = 2c (\square \square)$   
 c  $(3c + w) + \square = d + (\square + \square)$       d  $(g \square) 2y = 2y (3b \square)$

**UNDERSTANDING**

- 3** State whether the following are True or False. (*Hint:* If you're not sure, try substituting values for the variables to help you decide.)

a  $2s + (3w + 5z) = (2s + 3w) + 5z$       b  $x \times (d + y) = (x \times d) + y$   
 c  $g(jk) = (gj)k$       d  $4 \div (a \div c) = (4 \div a) \div c$   
 e  $3g + (k \div m) = (3g + k) \div m$       f  $4t - (p + 2b) = (4t - p) + 2b$   
 g  $3r + (a + 4c) = a + (3r + 4c)$       h  $a + (b \times c) = (a \times c) + b$

- 4** Find the values of the following expressions and comment on the results if  $a = 3$ ,  $b = 8$ ,  $c = 2$ .

a i  $3a + (2b + 4c)$       ii  $(3a + 2b) + 4c$   
 b i  $9a - (2b - 5c)$       ii  $(9a - 2b) - 5c$   
 c i  $a \times (b \times c)$       ii  $(a \times b) \times c$   
 d i  $4a \div (3b \div 2c)$       ii  $(4a \div 3b) \div 2c$

**eBookplus****Interactivity**Associative and  
commutative  
laws  
int-2349**REASONING**

- 5** Janet and Judy want to buy a set of DVDs. Janet has \$47 less than the purchase price, and Judy has \$2 less. If they pool their money, they still do not have enough to buy the DVDs. If the set costs a whole number of dollars, what is its price?

- 6** The associative law refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time. It always holds true for addition and multiplication. So:  $12 + (6 + 2) = (12 + 6) + 2$  and  $12 \times (6 \times 2) = (12 \times 6) \times 2$ .

Generally it does not hold true for subtraction and division. So:

$$a - (b - c) \neq (a - b) - c \text{ and}$$

$$a \div (b \div c) \neq (a \div b) \div c.$$

Investigate to find values for  $a$ ,  $b$  and  $c$  that would make the associative law true for subtraction and division.

**REFLECTION**

Does the associative law hold when you are adding fractions? How do you know?

# Summary

## Using rules

- By observing number patterns, we can discover the rules by which a pattern is produced.
- In algebra, a rule can be used to produce a set of output numbers from a set of input numbers.

## Finding a formula

- A *variable* (or *pronominal*) is a letter or symbol that is used in place of a number.
- When a rule is written using variables it is called a *formula*.
- When multiplying numbers and variables we don't need to show the multiplication sign, and we place numbers being multiplied in front of variables.

## Substitution

- When a variable in a formula is replaced by a number, we say that the number is *substituted* into the formula.

## Terms, expressions and equations

- A variable (pronominal) is a letter used in place of a number.
- A number in front of a variable is known as a *coefficient*.
- *Expressions* are parts of a rule made up of variables and numbers.
- *Equations* always contain an equals sign whereas expressions do not.
- Expressions and equations are made up of *terms*.
- When writing expressions, think about which operations are being used, and the order in which they occur.
- If variables are not given in a question, you can choose any letters you like.

## Simplifying and the distributive law

- Terms containing the same variables are called *like terms*.
- The process of adding or subtracting like terms is called *simplifying*.
- Substituting known values for variables in an expression allows us to evaluate the expression.
- Using the distributive law, expressions in factorised form can be written in expanded form.
- $a(b + c) = ab + ac$ .
- $a(b - c) = ab - ac$ .

## The associative law

- The associative law for addition is:  $(a + b) + c = a + (b + c) = a + b + c$ .
- The associative law for multiplication is:  $(ab)c = a(bc) = abc$ .
- The associative law does not hold for subtraction or division.
- The commutative law for addition is:  $a + b = b + a$ .
- The commutative law for multiplication is:  $ab = ba$ .
- The commutative law does not hold for subtraction or division.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter.

Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 Copy and complete the tables below. For each table, use the rule to work out the correct output numbers.

a Multiply each input number by 4.

Input	1	3	4	7	13
Output					

b Add 7 to each input number.

Input	1	4	6	7	15
Output					

c Multiply each input number by 2, then subtract 3.

Input	3	4	7	10	13
Output					

d Divide each input number by 4.

Input	0	4	8	16	20
Output					

- 2 Copy and complete the tables below. Use the rule given in each case to work out the missing input and output numbers.

a Subtract 7 from each input number.

Input	15	16	33		
Output				3	7

b Multiply each input number by 3, then add 7.

Input	1	4	15		
Output				22	106

c Add 5 to each input number, then multiply by 2.

Input	1	3	6		
Output				18	96

d Multiply each input number by 2, then subtract 5.

Input	8	11	15		
Output				21	71

- 3 MC Which formula has the algebra machine used in each of the following tables?

a

<i>a</i>	1	4	5	6	23
<i>g</i>	11	14	15	16	33

- A  $g = 5a + 6$   
B  $g = a - 10$   
C  $g = a + 10$

- D  $g = 10a$   
E  $g = 5a - 10$

b	<i>m</i>	1	2	3	4	11
	<i>t</i>	2	5	8	11	32

- A  $t = 3m$   
B  $t = 3m - 1$   
C  $t = m + 5$   
D  $t = 3m + 1$   
E  $t = 3m - 2$

- 4 Look carefully at the pattern in each of the following tables. Then complete the formula for each table by inserting a number into the gap.

a	<i>x</i>	1	4	7	8	23
	<i>y</i>	5	32	59	68	203

$$y = 9x - \underline{\quad}$$

b	<i>p</i>	0	3	5	6	12
	<i>q</i>	6	15	21	24	42

$$q = \underline{\quad} p + 6$$

- 5 Use the variables given to write a formula for each table.

a	<i>x</i>	0	2	3	4	7
	<i>y</i>	6	8	9	10	13

b	<i>c</i>	5	6	8	12	13
	<i>d</i>	0	1	3	7	8

c	<i>g</i>	3	4	6	9	23
	<i>h</i>	17	22	32	47	117

d	<i>m</i>	1	4	5	6	11
	<i>n</i>	3	15	19	23	43

- 6 Complete the tables below by substituting each of the input numbers (*x*) into the rule.

a  $y = x - 1$

	<i>x</i>	1	3	4	7	105
	<i>y</i>					

b  $d = 3x$

	<i>x</i>	1	2	6	7	12
	<i>d</i>					

c  $h = 6 - x$

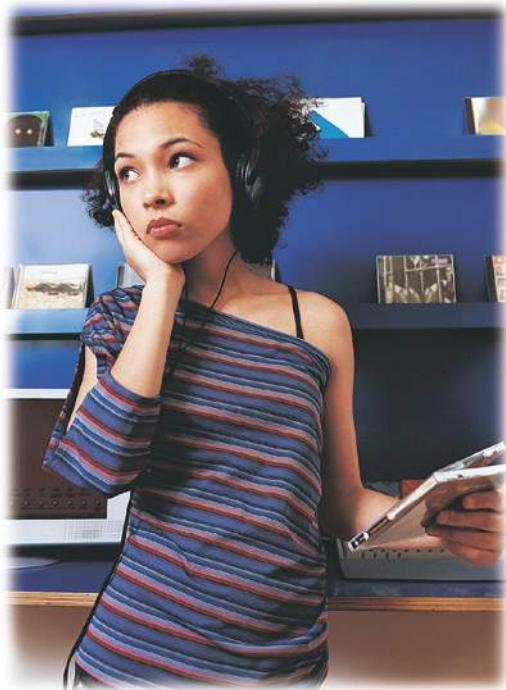
	<i>x</i>	0	2	4	5	6
	<i>h</i>					

d  $n = 11x + 3$

$x$	0	1	3	4	7
$n$					

- 7 Otto works in a warehouse, packing boxes of CDs for distribution to music stores. An order from Sanity's city store fills 35 boxes with 18 CDs left over.

- a Write an expression to represent the Sanity order, using the variable  $x$  to represent a full box of CDs.  
 b If each full box contains 30 CDs, use your expression to calculate how many CDs the store ordered.



- 8 Write expressions for the following:

- a the difference between  $M$  and  $C$   
 b money earned by selling  $B$  cakes for \$3 each  
 c the product of  $X$  and  $Y$   
 d 15 more than  $G$   
 e 1 more than  $D$   
 f the cost of 12 bananas at  $H$  cents each  
 g  $T$  multiplied by 5.

- 9 Simplify the following expressions by adding or subtracting like terms.

- a  $3g + 4g$   
 b  $8y - 2y$   
 c  $4h + 5h$   
 d  $7ag - 2ag$   
 e  $6gy - 3yg$   
 f  $8r - 8r$

g  $6y - 2y + y$

h  $4t + 6 + 3t$

i  $12gh + 6hg$

j  $8t - 2m + 3t$

k  $3m + m$

l  $7g + 8g + 8 + 4$

m  $7h + 4t - 3h$

n  $2b + 7c + 8b$

o  $11axy - 3axy$

- 10 Simplify the following expressions first, then find the value of the expression if  $x = 5$ .

a  $7x + 3x$

b  $2x + 3x - 4$

c  $11x + 12x$

d  $x + 2x$

e  $4x - x$

f  $3x - 2x + 16$

g  $21x - 13x + 7$

h  $11 + 2x + 5x$

i  $7x - 4x + 3x$

- 11 Use the distributive law to expand the following.

a  $7(m - 3k)$

b  $w(g + 9a)$

c  $2y(8h - 7)$

d  $10m(7r - 2p)$

e  $3g(2a + 2c)$

f  $4j(6x - 3y)$

- 12 Expand the following and simplify if possible.

a  $5(t - 2s) - 3t$

b  $4(a + 3g) - 2g$

c  $d(3f + 9) - 2d$

d  $4z(2 - a) + 7z$

e  $3p(f - 2q) + 7p$

f  $10b(a + 6) + 5b$

- 13 Use the associative law to complete the following.

a  $a + (3b + \square) = (\square + \square) + 5c$

b  $\frac{1}{a} + (2b + \square) = (\square + 2b) \square 7g$

c  $(d - 3y) z = \square (\square \square)$

d  $\left(\frac{a}{b} \times c\right) \times \square = \frac{a}{\square} \times (\square \times f)$

- 14 Use the associative and commutative laws to complete the following.

a  $g + (\square + 2k) = (2k + \square) + 3m$

b  $(w + \square) + 4r = \square + (5g + w)$

c  $\frac{2}{3} (2d \square) = (r - 2d) \square$

d  $(3z - 5b) \square = \square (6m - 5b)$

**PROBLEM SOLVING**

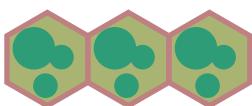
- 1** Johanna (a landscape architect) is installing hexagonal flowerbeds in a park. The flowerbeds, which are surrounded by wooden sleepers, are arranged in rows as shown below:



1 bed, 6 sleepers required



2 beds, 11 sleepers required



3 beds, 16 sleepers required

How many wooden sleepers will Johanna need to install a row of 12 flowerbeds? Follow the steps below:

- Draw a table with columns headed  $B$  (the number of flowerbeds), and  $S$  (the number of sleepers needed). Write in the numbers from the diagrams above.
- Complete a diagram showing the number of sleepers needed for a row of 4 flowerbeds, and a row of 5 flowerbeds. Add these values to your table.
- Look at the table to determine the formula which connects  $S$  and  $B$ . Write the formula in the form  $S = \dots$
- Substitute  $B = 12$  into the formula to find the number of sleepers needed for the row of flowerbeds.



- 2** The metal ceiling rafters in a school classroom consist of a series of triangles connected end to end as shown. The number of triangle side lengths required in each rafter depends on the total number

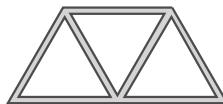
of triangles. However, adjacent triangles share a common side length.



1 triangle, 3 side lengths



2 triangles, 5 side lengths



3 triangles, 7 side lengths

How many side lengths will be needed to build a single rafter if each rafter contains 25 triangles? Follow the steps below:

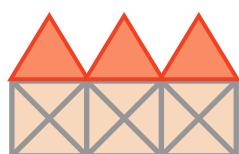
- Draw up a table with columns headed  $T$  (the number of triangles), and  $S$  (the number of side lengths needed). Write in the numbers from the diagrams above.
  - Complete a diagram showing the number of side lengths needed for a rafter containing 4 triangles. Add these values to your table.
  - Look at the table to determine the formula which connects  $S$  and  $T$ . Write the formula in the form  $S = \dots$
  - Substitute  $T = 25$  into the formula to find the number of side lengths needed for each ceiling rafter.
- 3** The diagrams below show the number of struts planned for the back walls of the Surfside beach huts, which are to be built in rows.



1 hut, 6 struts



2 huts, 11 struts



3 huts, 16 struts

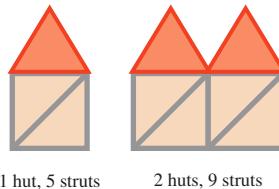
Surfside Constructions has a contract to build a row of 34 beach huts at Golden Beach. Based on the plans above, how many struts will be required for the back walls of this row of beach huts?

Follow the steps below to solve this problem.

- Draw a table with rows headed  $H$  (the number of huts), and  $S$  (the number of struts needed). Write in the numbers from the diagrams above.

- b** Following the same pattern, complete a diagram showing the number of struts needed for a row of 4 huts and a row of 5 huts. Add these values to your table.
- c** Look at the table to determine the formula that connects  $S$  and  $H$ . Write the formula in the form  $S = \dots$
- d** Substitute  $H = 34$  into the formula to find the number of metal struts needed for the row of huts.

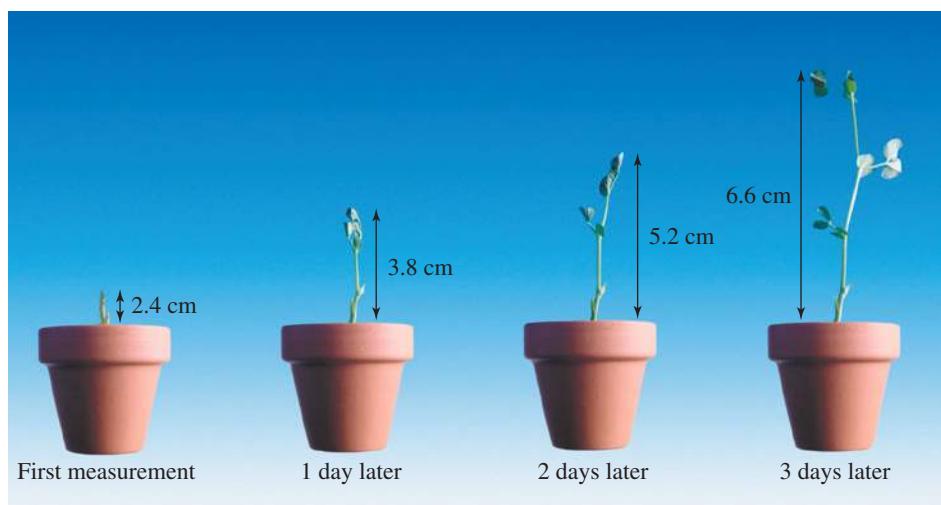
The engineers at Surfside Constructions have advised builders that only one of the metal strut cross bracings is required. Only 5 struts are required for the back wall of each hut. The diagram below is the amended plan for the huts' struts.



- e** Draw a diagram showing the number of struts needed for a row of 3 huts.
- f** Repeat steps **a** to **c** for a beach hut containing 5 struts as illustrated in the diagram above.
- g** How many metal struts will now be required for the back wall of a row of 34 beach huts?



- 4** A seed you have planted in a pot is starting to grow. When you first notice the seedling, you measure it to be 2.4 centimetres high. Each day after this you continue to measure its height.



- a If the seedling continues to grow at the same rate, how tall will it be 4 days later?
- b How tall will it be 5 days later?
- c Write a formula connecting the height of the plant with the number of days after the plant height was first measured. Remember to identify what each variable represents in your formula.
- d Check your formula by calculating the height of the plant after 7 days.
- e What is the height of the plant after 18 days?
- f What would be the height of the plant after 25 days?
- g Challenge: Use your formula to estimate when the plant would be approximately 32 centimetres high.
- h Discuss and give reasons why this formula may not be accurate to predict the plant height after a period of time.
- 5 It's Bianca's birthday today. She is  $m$  years old. She asks the ages of people at her party. Her cousin Paul is 8 years older than Bianca. His age today is  $m + 8$  years old.



- a Write the ages of the following using algebra as for Paul's age.
- i Bianca's sister Kate is 5 years younger than Bianca.
- ii Bianca's aunt Theresa is three times Bianca's age.
- b If Bianca is 7 years old today. Work out the ages of her relatives and write their names in order from youngest to eldest.
- c If Bianca is 13 years old today. Work out the ages of her relatives and write their names in order from youngest to eldest.
- 6 Use the variable  $x$  to make a general statement of the form  $\square \quad \square = 1$  from the following clues.

$$4 - \frac{1}{4} = 1, 30 - \frac{1}{30} = 1, 1.2 - \frac{1}{1.2} = 1$$

- 7 You work at a restaurant. The manager asks you to push tables together until you have seats for 20 people. You have tables that seat 4 people. You notice that when you push two tables together you can seat 6 people. Do you see a pattern that will help you to work out how many tables will be needed to seat 20 people?



- 8 Mary has  $y$  books. Tom has 4 more books than Mary has. Cindy has 5 more times as many books as Tom has. Write down an expression, in terms of  $y$ , for the number of books that Cindy has.



- 9 If we count by 7s, starting with 3, we get the numbers:  
3, 10, 17, 24, 31, 38, ...  
What is the 50th number in this set?

- 10** A couple spends \$ $x$  on developing eighty small wedding photos and \$ $y$  for sixty large wedding photos. How much do they need to spend if they want an extra dozen photos of each size?



- 13** A bird's head is  $\frac{1}{3}$  the length of its body. Its tail is as long as its head and body combined. If the total length of the bird is 24 cm, how long is its head?
- A detailed illustration of a blue jay perched on a thin brown branch. The bird has a bright blue crest and wings, with white wing bars and a black tail with white spots. It is facing slightly to the left.
- 14** Tom's father is 3 times as old as Tom, and his mother is  $\frac{5}{6}$  his father's age. His sister is  $\frac{3}{4}$  Tom's age. Tom's father is 6 years older than his mother. How old is Tom's sister?
- 15** Imagine that a new mathematical operation is being used. Its symbol is  $\#$ . See the following equations.  
 $1 \# 1 = 2$   
 $3 \# 5 = 34$   
 $6 \# 9 = 117$   
 $10 \# 14 = 296$   
 Find the value of  $15 \# 19$ , and explain your reasoning.

eBook plus

Weblink

Algebra puzzle

eBook plus

**Interactivities**

Test yourself

Chapter 8

int-1820

Word search

int-2597

Crossword

int-2598

- 11** It costs \$ $x$  for four adults' tickets. A child's ticket costs \$3 less than an adult's ticket. On family night, all the tickets go for half price. Develop a formula in terms of  $x$  to calculate how much it will cost for a family of 2 adults and 3 children on family night.

- 12** You have four friends to whom you gave some money. To the first friend, you gave  $\frac{1}{3}$  of your coins. You then gave  $\frac{1}{2}$  of the coins that you had left to your second friend.

Then you gave your third friend  $\frac{1}{4}$  of the coins that you had left. Finally you gave your fourth friend  $\frac{2}{3}$  of your remaining coins. At this point you had 7 coins remaining.

How many coins did you begin with?

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 8 (doc-6498) (*page 251*)

**Are you ready?****Digital docs (page 252)**

- SkillsHEET 8.1 (doc-6499) Number patterns
- SkillsHEET 8.2 (doc-6500) Worded operations
- SkillsHEET 8.3 (doc-6501) Describing a number pattern from a table
- SkillsHEET 8.4 (doc-6502) Alternative expressions used to describe the four operations
- SkillsHEET 8.5 (doc-6503) Using tables to show number patterns

**8A Using rules****Digital docs (page 255)**

- Activity 8-A-1 (doc-1821) Percentages and fractions
- Activity 8-A-2 (doc-1822) More percentages and fractions
- Activity 8-A-3 (doc-1823) Advanced percentages and fractions

**8B Finding a formula****Digital docs**

- Activity 8-B-1 (doc-1824) Finding a formula (*page 259*)
- Activity 8-B-2 (doc-1825) More finding a formula (*page 259*)
- Activity 8-B-3 (doc-1826) Advanced formulae (*page 259*)
- WorkSHEET 8.1 (doc-1819) (*page 261*)

**8C Substitution****Digital docs (page 262)**

- Activity 8-C-1 (doc-1827) Substitution
- Activity 8-C-2 (doc-1828) More substitution
- Activity 8-C-3 (doc-1829) Advanced substitution

**8D Terms, expressions and equations****Digital docs (page 269)**

- Activity 8-D-1 (doc-1830) Terms and expressions
- Activity 8-D-2 (doc-1831) More terms and expressions
- Activity 8-D-3 (doc-1832) Advanced terms and expressions

**Weblink (page 270)**

- Algebraic expressions game

**8E Simplifying and the distributive law****Digital docs**

- Activity 8-E-1 (doc-1833) The distributive law (*page 275*)
- Activity 8-E-2 (doc-1834) More of the distributive law (*page 275*)
- Activity 8-E-3 (doc-1835) Advanced use of the distributive law (*page 275*)
- WorkSHEET 8.2 (doc-1820) (*page 276*)

**8F The associative law****Digital docs (page 278)**

- Activity 8-F-1 (doc-1836) The associative law
- Activity 8-F-2 (doc-1837) More of the associative law
- Activity 8-F-3 (doc-1838) Advanced use of the associative law

**Interactivity**

- Associative and commutative laws (int-2349) (*page 278*)

**Chapter review****Weblink**

- Algebra puzzle (*page 285*)

**Interactivities (page 285)**

- Test yourself Chapter 8: (int-1820) Take the end-of-chapter test to test your progress.
- Word search (int-2597)
- Crossword (int-2598)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

## 9

# Measurement



- 9A** Units of measurement and converting units of measurement
- 9B** Reading scales and measuring length
- 9C** Perimeter
- 9D** Area
- 9E** Area of composite shapes, using addition and subtraction
- 9F** Volume
- 9G** Capacity

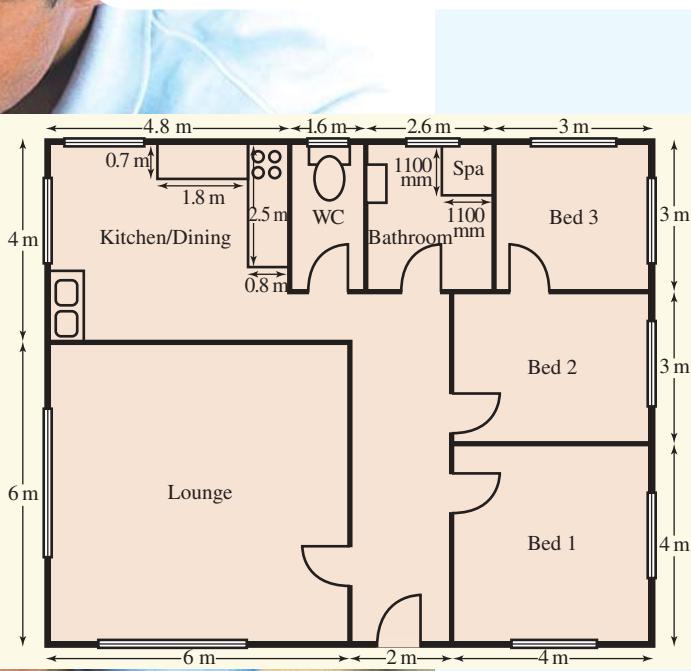
## WHAT DO YOU KNOW?

- 1 List what you know about length, area and volume. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of length, area and volume.

### eBook plus

#### Digital docs

Hungry brain activity  
Chapter 9  
doc-6504



## OPENING QUESTION

In the floor plan above, estimate the cost of putting new carpet in the lounge and the bedrooms. (Allow \$60 per square metre of carpet, fully laid.)

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can find extra help by completing the matching SkillsHEET located on your eBookPLUS.

## Metric units of length



**Digital docs**

SkillsHEET 9.1  
doc-6505

- 1 a For each metric unit of length, match the name with the correct abbreviation.

- |               |    |
|---------------|----|
| i centimetre  | m  |
| ii kilometre  | mm |
| iii metre     | cm |
| iv millimetre | km |

- b Give an example to describe the length of each unit in part a.

## Reading scales (how much is each interval worth?)

- 2 For the scales shown, how much is each interval worth?



## Measuring the length of a line



**Digital docs**

SkillsHEET 9.3  
doc-6507

- 3 a Measure the length of this line to the nearest mm. \_\_\_\_\_

- b Measure the length of this line in cm. \_\_\_\_\_

## Relationship between unit size and the number of units used

- 4 The distance from your home to school is measured in both metres and kilometres. Which unit would you have more of for this measurement?



**Digital docs**

SkillsHEET 9.4  
doc-6508

## Converting units (metres and kilometres)



**Digital docs**

SkillsHEET 9.5  
doc-6509

- 5 a Convert 18 km to m.

- b Convert 6000 m to km.

## Converting units to compare lengths or distances

- 6 Arrange the following in order from smallest to largest.

- a 25 mm, 2.3 cm, 0.2 m

- b 1.8 m, 1700 mm, 185 cm



**Digital docs**

SkillsHEET 9.6  
doc-6510

## Perimeter

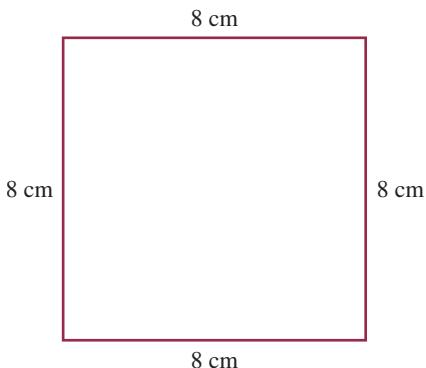
- 7 Calculate the perimeter of each shape shown below. (Note: The figures are not drawn to scale.)



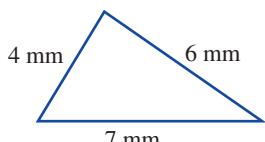
**Digital docs**

SkillsHEET 9.7  
doc-6511

a



b



**Area units**

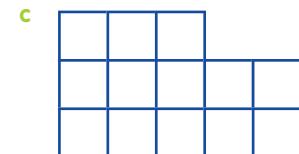
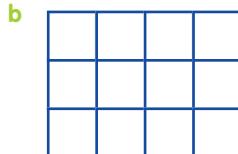
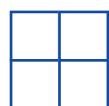
**eBook plus**  
**Digital docs**  
SkillsHEET 9.8  
doc-6512

- 8 a Write the abbreviation used for each of the following area units.
- square metre
  - square centimetre
  - square kilometre
  - hectare
- b Give an example to indicate the size of each area unit.

**Area of figures drawn on one-centimetre grid paper**

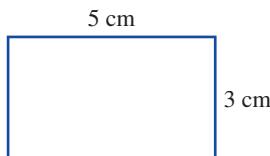
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**Digital docs**  
SkillsHEET 9.9  
doc-6513

- 9 The figures below are drawn on centimetre grid paper. What is the area of each one?

**Area of rectangles**

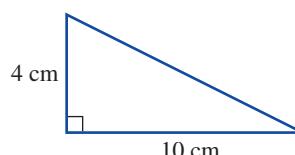
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**Digital docs**  
SkillsHEET 9.10  
doc-6514

- 10 a Explain how to find the area of a rectangle using its length and width.  
b Find the area of this rectangle.

**Area of triangles**

**eBook plus**  
**Digital docs**  
SkillsHEET 9.11  
doc-6515

- 11 a Explain how to find the area of a triangle using its base and height.  
b Find the area of this triangle.

**Volume units**

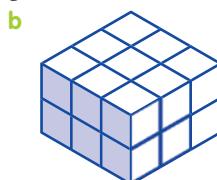
**eBook plus**  
**Digital docs**  
SkillsHEET 9.12  
doc-6516

- 12 a Write the abbreviation used for each of the following volume units.
- cubic centimetre
  - litre
  - millilitre
  - cubic metre
- b Give an example to indicate the size of each volume unit.

**Volume of a solid constructed from cubic-centimetre blocks**

**eBook plus**  
**Digital docs**  
SkillsHEET 9.13  
doc-6517

- 13 The following objects have been constructed using cubic centimetre blocks. How many cubic centimetres are in each solid shape?

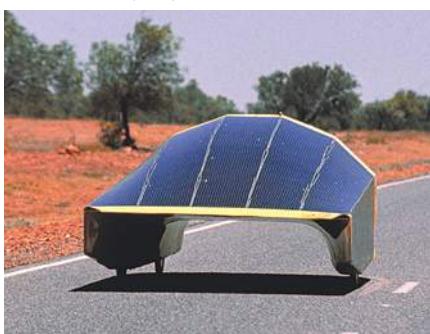


# 9A Units of measurement and converting units of measurement

## Metric units of length

- The metric system is based on number 10.
- The base unit of the metric system is the metre.
- All other units of length in the metric system are related to the metre by powers of 10.
- The most commonly used units of length together with their abbreviation and approximate examples are shown below.

### 1. Kilometre (km)



The distance travelled by a car travelling at the speed limit of 60 kilometres per hour in one minute

### 2. Metre (m)



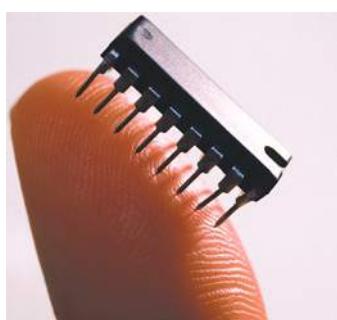
The length of an adult's stride

### 3. Centimetre (cm)



The width of each of your fingers

### 4. Millimetre (mm)



The width of a wire in this computer chip

### WORKED EXAMPLE 1

You have been given the task of measuring the length of a piece of rope to be used around the boundary of a cricket ground. Which metric units of length would you use?

#### THINK

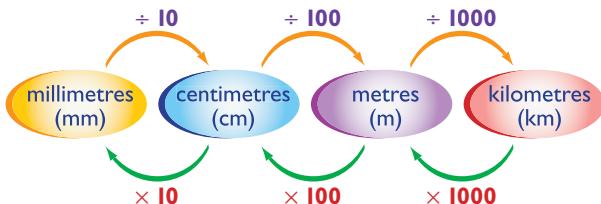
Estimate the length involved. In this case, it is in the order of 100s of metres. Would other units give a more manageable figure? (Would using centimetres, for example, 10 000 cm, be more convenient? No! What about kilometres? No — the length would be less than 1 kilometre.)

#### WRITE

Metres (or m)

## Converting units of length

- The main metric units of length are related as follows:
- $1 \text{ km} = 1000 \text{ m}$
- $1 \text{ m} = 100 \text{ cm}$
- $1 \text{ cm} = 10 \text{ mm}$
- Units of length can be converted as shown in the diagram below.



- In the diagram above, the numbers next to each arrow are called **conversion factors**.
- When converting from a larger unit to a smaller unit, we multiply by the conversion factor; when converting from a smaller unit to a larger unit, we divide by the conversion factor.

### WORKED EXAMPLE 2

Complete each of the following metric conversions.

a  $0.3285 \text{ km} = \underline{\hspace{2cm}} \text{ m}$

b  $560 \text{ m} = \underline{\hspace{2cm}} \text{ mm}$

c  $480 \text{ cm} = \underline{\hspace{2cm}} \text{ km}$

d  $2\frac{3}{5} \text{ m} = \underline{\hspace{2cm}} \text{ cm}$

#### THINK

- a Refer to the metric conversion chart above. To convert kilometres to metres, multiply by 1000. (Move the decimal point 3 places to the right.)
- b This conversion involves two steps in the chart. First convert metres to centimetres by multiplying by 100 (since there is no decimal point place two zeros after the final digit). Then convert centimetres to millimetres by multiplying by 10 (place one zero after the final digit).

Note: Overall, we need to multiply by  $100 \times 10$  or 1000.

- c To convert 480 centimetres to kilometres involves two division steps. From centimetres to metres, divide by 100 (move the decimal point 2 places to the left).

From metres to kilometres, divide by 1000 (move the decimal point 3 places to the left).

- d 1 Convert  $2\frac{3}{5} \text{ m}$  to an improper fraction.

#### WRITE

a  $0.\overline{3285} \text{ km} = 0.3285 \times 1000 \text{ m}$   
 $= 328.5 \text{ km}$

b  $560 \text{ m} = 560 \times 100 \text{ cm}$   
 $= 56000 \text{ cm}$   
 $= 56000 \times 10 \text{ mm}$   
 $= 560000 \text{ mm}$

c  $480 \text{ cm} = \frac{480}{100} \text{ m}$   
 $= \frac{4.8}{10} \text{ m}$   
 $= \frac{48}{1000} \text{ km}$   
 $= 0.0048 \text{ km}$

d  $2\frac{3}{5} \text{ m} = \frac{2 \times 5 + 3}{5} \text{ m}$   
 $= \frac{13}{5} \text{ m}$

$$\begin{aligned} &= \frac{13}{5} \times 100 \text{ cm} \\ &= \frac{1300}{5} \text{ cm} \end{aligned}$$

- 2 Convert metres to centimetres by multiplying by 100.

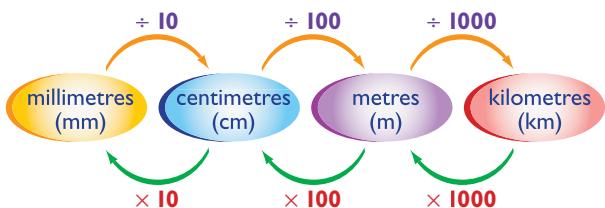
Note: Remember when multiplying fractions by a whole number, the number multiplies the numerator.

- 3 Simplify the fraction.

$$= 260 \text{ cm}$$

**REMEMBER**

- The most commonly used units to measure length are kilometres (km), metres (m), centimetres (cm) and millimetres (mm).
- When measuring length, it is easiest to use units that give us convenient values (not too large and not too small).
- When converting units of length, it is useful to note the points below.
- Imagine a decimal point at the right-hand end of any whole numbers; for example, the number 35 can be thought of as having a decimal point after the 5 (35).
- Look at or draw a conversion chart.



- When converting to a smaller unit, multiply by the conversion factor.
- When converting to a larger unit, divide by the conversion factor.

**EXERCISE****9A****Units of measurement and converting units of measurement****INDIVIDUAL PATHWAYS****eBook plus****Activity 9-A-1**

Units of measurement  
doc-1855

**Activity 9-A-2**

More units of measurement  
doc-1856

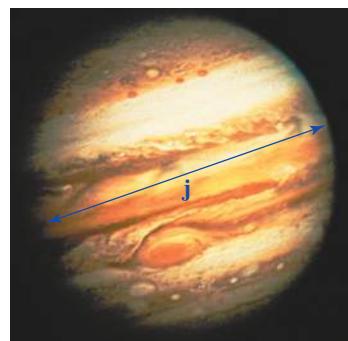
**Activity 9-A-3**

Advanced units of measurement  
doc-1857

**FLUENCY**

- 1 (WEI)** Which units (mm, cm, m or km) would be most suitable for measuring the real lengths marked in each photograph? (You may give two different units if you think both would be suitable.)





- 2** Which metric units of length would you use for measuring the following (you may give two different units if you think both could be used)?
- a** The length of a netball court
  - b** The diameter of a netball
  - c** A photo frame's width
  - d** The thickness of a slice of bread
  - e** The length of a worm
  - f** The distance between Melbourne and Sydney
  - g** The depth of an Olympic swimming pool
  - h** The depth of a bowl of soup
  - i** The length of a mathematics textbook
  - j** The thickness of a magazine
  - k** The distance around the shores of a reservoir
  - l** The length of a cut-out shape on a GeoMat mathematics template
- 3 MC** The thicknesses of two different brands of chocolate biscuit would be best compared by measuring in:
- A** millimetres
  - B** kilometres
  - C** metres
  - D** centimetres
  - E** kilometres or metres
- 4 MC** Girouk wishes to measure the length of his mathematics folder to see what size paper he could fit into it. Which of the following would be the best measuring instrument to use?
- A** A science department metre ruler
  - B** A 10-cm mini ruler
  - C** A 30-cm plastic ruler
  - D** A 20-m flexible tape measure
  - E** A one-metre trundle wheel
- 5** Give three examples (not mentioned earlier in this exercise) of something that would be best measured in:
- a** metres
  - b** millimetres
  - c** kilometres
  - d** centimetres.
- 6** Give an example of a length which each of the following people might measure in their jobs. (For example, a carpet layer would measure the length of a room.)
- a** Veterinary surgeon
  - b** Cartographer (map maker)
  - c** Costume designer
  - d** Carpenter
  - e** Doctor
  - f** Landscape gardener
  - g** Plumber
  - h** Computer technician
  - i** Track and field athlete
  - j** Weather reporter
- 7 WE2a** Complete each of the following metric conversions.
- a**  $2.0 \text{ km} = \underline{\hspace{2cm}}$  m
  - b**  $7.0 \text{ km} = \underline{\hspace{2cm}}$  m
  - c**  $65.0 \text{ km} = \underline{\hspace{2cm}}$  m
  - d**  $5.3 \text{ km} = \underline{\hspace{2cm}}$  m
  - e**  $0.66 \text{ km} = \underline{\hspace{2cm}}$  m
  - f**  $9.0 \text{ m} = \underline{\hspace{2cm}}$  cm
  - g**  $0.25 \text{ m} = \underline{\hspace{2cm}}$  cm
  - h**  $28.0 \text{ cm} = \underline{\hspace{2cm}}$  mm
  - i**  $200.0 \text{ cm} = \underline{\hspace{2cm}}$  mm
  - j**  $700.0 \text{ m} = \underline{\hspace{2cm}}$  cm
- 8** Convert to the units indicated.
- a**  $8000 \text{ m} = \underline{\hspace{2cm}}$  km
  - b**  $6500 \text{ m} = \underline{\hspace{2cm}}$  km
  - c**  $700 \text{ m} = \underline{\hspace{2cm}}$  km
  - d**  $50 \text{ m} = \underline{\hspace{2cm}}$  km

- e  $10.5 \text{ m} = \underline{\hspace{2cm}}$  km  
 g  $300 \text{ cm} = \underline{\hspace{2cm}}$  m  
 i  $9 \text{ cm} = \underline{\hspace{2cm}}$  m  
 k  $835 \text{ mm} = \underline{\hspace{2cm}}$  cm

- f  $6000 \text{ cm} = \underline{\hspace{2cm}}$  m  
 h  $57 \text{ cm} = \underline{\hspace{2cm}}$  m  
 j  $45 \text{ mm} = \underline{\hspace{2cm}}$  cm  
 l  $25600 \text{ mm} = \underline{\hspace{2cm}}$  cm

9 **WE2b,c,d** Copy and complete.

- a  $8 \text{ km} = \underline{\hspace{2cm}}$  cm  
 c  $101 \text{ m} = \underline{\hspace{2cm}}$  mm  
 e  $60\frac{1}{4} \text{ km} = \underline{\hspace{2cm}}$  mm  
 g  $72.33 \text{ m} = \underline{\hspace{2cm}}$  mm  
 i  $5.05 \text{ km} = \underline{\hspace{2cm}}$  m

- b  $5400 \text{ mm} = \underline{\hspace{2cm}}$  m  
 d  $7800 \text{ cm} = \underline{\hspace{2cm}}$  km  
 f  $112\frac{4}{5} \text{ cm} = \underline{\hspace{2cm}}$  m  
 h  $40.5 \text{ mm} = \underline{\hspace{2cm}}$  cm  
 j  $30\frac{7}{20} \text{ mm} = \underline{\hspace{2cm}}$  m

10 **MC** A distance of 6.25 km is the same as:

- A  $625 \text{ m}$       B  $0.00625 \text{ m}$   
 D  $625000 \text{ cm}$       E  $62.50 \text{ cm}$

- C  $625000 \text{ mm}$

11 **MC** A distance of 7 860 000 cm is equal to:

- A  $78.6 \text{ km}$       B  $786 \text{ m}$   
 D  $786000 \text{ mm}$       E  $78.6 \text{ km}$

- C  $786 \text{ km}$

12 Convert to the units indicated.

- a  $67\frac{7}{10} \text{ km}$  to m      b  $45\frac{1}{5} \text{ m}$  to km  
 d  $450000 \text{ cm}$  to km      e  $8\frac{3}{4} \text{ cm}$  to mm  
 g  $7.21 \text{ m}$  to mm      h  $3\frac{9}{100} \text{ km}$  to cm  
 j  $11.655 \text{ m}$  to mm

- c  $560 \text{ mm}$  to m  
 f  $0.0006 \text{ km}$  to cm  
 i  $48 \text{ mm}$  to cm

### UNDERSTANDING

13 The longest snake ever held in captivity was a female reticulated python named ‘Colossus’. She was measured to be 8.68 m long. Convert this to centimetres and compare it to the adult length of the shortest species of snake, the West Indian *Leptophylops bilineata*, which grows to only 108 mm.



14 The world’s largest mountains are Everest (8863 m) and K2 in Pakistan (8607 m). Convert these heights to kilometres.

15 The tallest tomato plant on record reached 16.3 m. How many centimetres is this?

16 The tallest Rialto Tower (shown to the left of the photograph of the Melbourne skyline at right) stands 0.448 km high. How many metres is this?

17 Arrange the following in order from smallest to largest.

- a  $12.5 \text{ m}$ ,  $150 \text{ cm}$ ,  $0.02 \text{ km}$   
 b  $350 \text{ cm}$ ,  $0.445 \text{ m}$ ,  $3000 \text{ mm}$   
 c  $50 \text{ km}$ ,  $500 \text{ m}$ ,  $50000 \text{ mm}$   
 d  $1700 \text{ cm}$ ,  $1.7 \text{ m}$ ,  $0.17 \text{ km}$   
 e  $0.052 \text{ cm}$ ,  $0.0052 \text{ mm}$ ,  $0.000052 \text{ m}$   
 f  $990 \text{ cm}$ ,  $0.909 \text{ m}$ ,  $9000 \text{ mm}$



- 18** Add the following lengths, giving your answer in either unit.
- a 75 cm and 3 m
  - b 2700 m and 7.5 km
  - c 3800 mm and 52 cm
  - d 1.66 m and 58.2 cm
  - e 208 cm and 1.83 m
  - f 5500 mm and 20.1 m
  - g 40 km and 7800 m
  - h 0.000 675 km and 87.8 cm
- 19** Find the difference between each of the following lengths, giving your answer in either unit.
- a 72 km and 5600 m
  - b 418 000 mm and 7.6 m
  - c  $34\frac{3}{5}$  cm and  $\frac{9}{20}$  m
  - d  $2\frac{4}{5}$  km and 450 000 cm
- 20** A particular brand of computer CD is 1.2 mm thick. How high (in centimetres) would a pile of 23 such CDs be?
- 21** A builder needs to build a wall 3.5 m high. If each layer of bricks adds 8 cm of height, how many layers high will the wall be?
- 22** Deanne buys a length of rope and cuts it into three smaller sections, each of length 7200 cm. How many metres long was the original piece of rope?
- 23** Norbert is 1.53 m tall in bare feet. If he wears shoes with 6.3 cm thick soles, how high would he stand then? Give your answer in metres.
- 24** Adrian is driving a truck with a rooftop 3.4 m above road level, when he approaches an overpass bridge which has a clearance of 376 cm. Will Adrian's truck get under the bridge? If so, how much room does he have to spare? Give your answer in centimetres.
- 25** A childcare centre has three large cardboard boxes which the children stack up in various combinations. What stack heights are possible for a stack if the boxes' individual heights are 600 mm, 45 cm and 1.1 m? Give your answer in centimetres.
- 26** Finita attaches a trellis that is 0.6 m high to the top of her 180 cm high fence. How high is it to the top of the new trellis? Give your answer in metres.
- 27** Zvenglo is stacking identical boxes of height 330 mm. How high would a stack be if it contained six boxes? Give your answer in centimetres.
- 28** Waldo's noticeboard is 1.5 m long and 1.2 m wide. If he pins a calendar of length 70 cm and width 60 cm exactly in the middle of the board, what is the width of the space above the calendar?  
(Hint: Draw a diagram of the situation.)
- 29** A licorice strap machine takes 3.75 m lengths of licorice, and chops them into 10 cm long pieces. How many pieces does each 3.75 m length produce?



#### REFLECTION

Why is it important to be able to convert from one metric unit to the other?

## 9B Reading scales and measuring length

- A scale is a series of marks used to measure length, mass, temperature or any other quantity.
- When reading scales and measuring lengths:
  - check that the scale starts with zero
  - check the value of each small division by counting along the scale to the next major mark
  - always give units (for example, centimetres) with your answer.

## WORKED EXAMPLE 3

State the reading indicated by the arrow in each case below giving answers in:

- decimal form
- fractional form where appropriate.

**a****b****THINK**

- Check that the line starts at 0. It does.
- Note the units printed on the ruler (cm).
- Read the last centimetre mark (5).
- Does the line go past the last centimetre mark? No.
- Write the value with units.

*Note:* There is no need to go any further as the value is a whole number.

- b i**
  - Check that the line starts at 0. It does.

- Note the units printed on the ruler (cm).

- Read the last centimetre mark (7).

- Consider how many smaller intervals there are between the numbered divisions. There are 10, so each smaller division represents 0.1 of a unit.

- Count the number of small divisions past the last centimetre mark (3).

- Write the value with units.

- ii** Express the answer obtained in **b i** as a fraction.

*Note:* The value of the first place to the right of the decimal point is tenths, so the digit 3 is 3 tenths.

**WRITE****a** **5 cm****b i****7.3 cm****ii**  $7\frac{3}{10}$  cm

## WORKED EXAMPLE 4

Use the given length of the climber's lower leg to estimate the labelled length of the rope on the diagram at right.



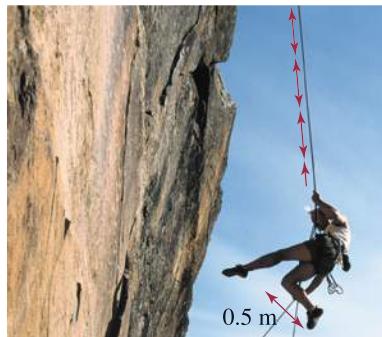
## THINK

- 1 Refer to the 0.5-m lengths indicated on the diagram and determine how many of these lengths correspond to the unknown length of the rope above the climber.

*Note:* There are approximately  $3\frac{1}{2}$  or 3.5 of these lengths in the rope.

- 2 Multiply the number of lengths by 0.5 m.  
3 Write the answer.

## WRITE



From the above diagram there are approximately  $3\frac{1}{2}$  (or 3.5) of these lengths in the rope.

$$\begin{aligned}\text{Length} &= 3.5 \times 0.5 \\ &= 1.75\end{aligned}$$

The length of the rope is approximately 1.75 m.

## REMEMBER

1. The most commonly used units to measure length are kilometres (km), metres (m), centimetres (cm) and millimetres (mm).
2. When measuring length, it is easiest to use units that give us convenient values (not too large and not too small)
3. When reading scales and measuring length:
  - (a) check that the scale starts at zero
  - (b) check the value of each small division by counting along the scale to the next major mark
  - (c) always give units (for example, centimetres) with your answer.

## EXERCISE

## 9B

## Reading scales and measuring length

## INDIVIDUAL PATHWAYS

## eBook plus

**Activity 9-B-1**  
Reading scales  
doc-1858

**Activity 9-B-2**  
More scales  
doc-1859

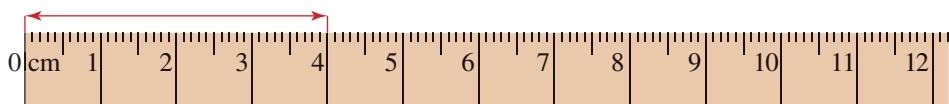
**Activity 9-B-3**  
Advanced scales  
doc-1860

## FLUENCY

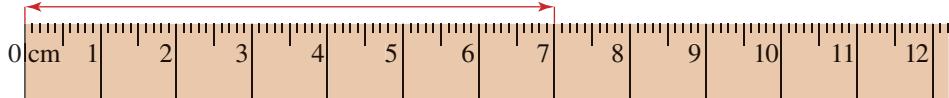
- 1 **WE3a** State the reading indicated by the arrow in each case below giving answers in:

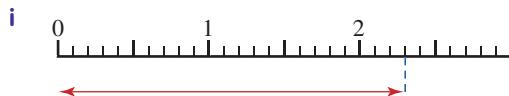
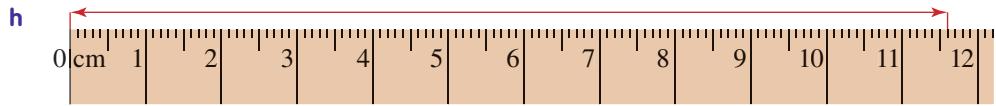
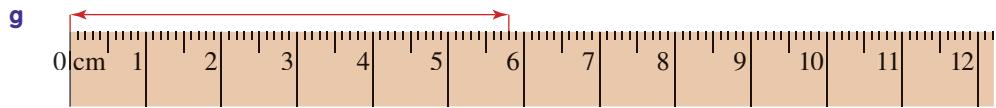
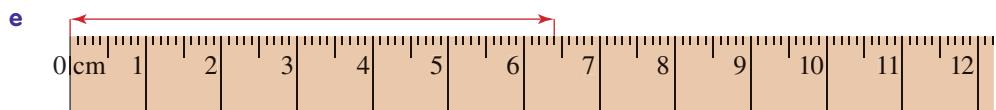
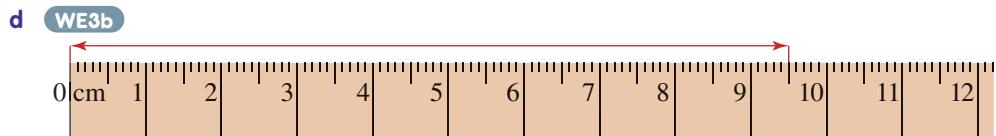
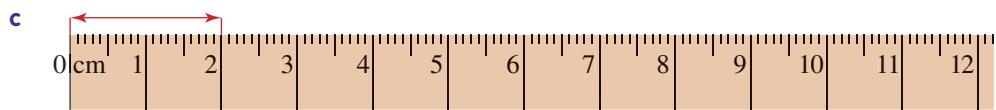
- i decimal form
- ii fractional form where appropriate.

a

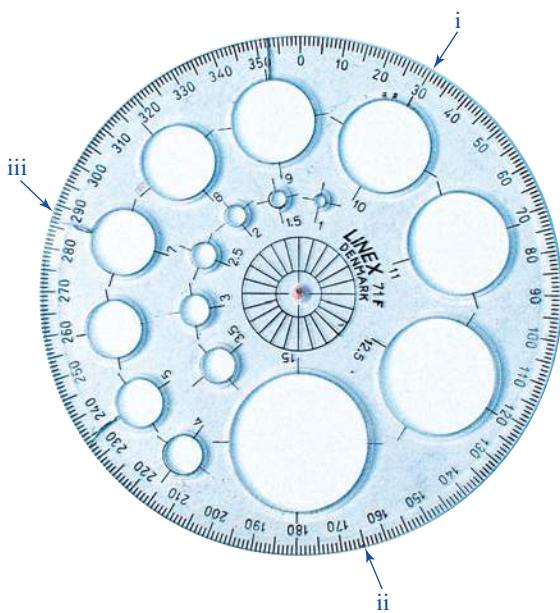
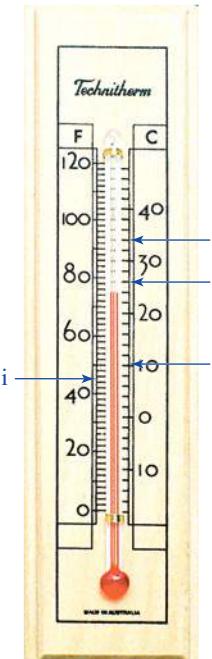


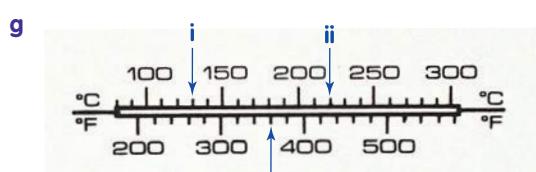
b



**eBookplus****Weblink**  
**Measure it!**

2 Read the values shown on the following scales. For parts **a** to **d** and **g** read from the arrow.

**a****b**



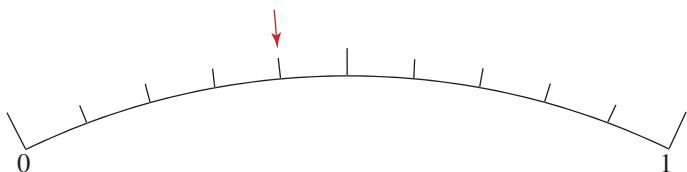
- 3 MC The reading on the scale below is:



- A 2.2  
D 2.8

- B 2.4  
E 3.3

- 4 MC The reading on the scale below is:



A  $\frac{4}{5}$   
D  $\frac{2}{10}$

B 4  
E  $\frac{1}{2}$

C  $\frac{2}{5}$

- 5 Measure each of the following lines in centimetres.

a

b

c

d

e

f

g

h

- 6 Measure each of the following lines to the nearest millimetre.

a

b

c

d

e

f

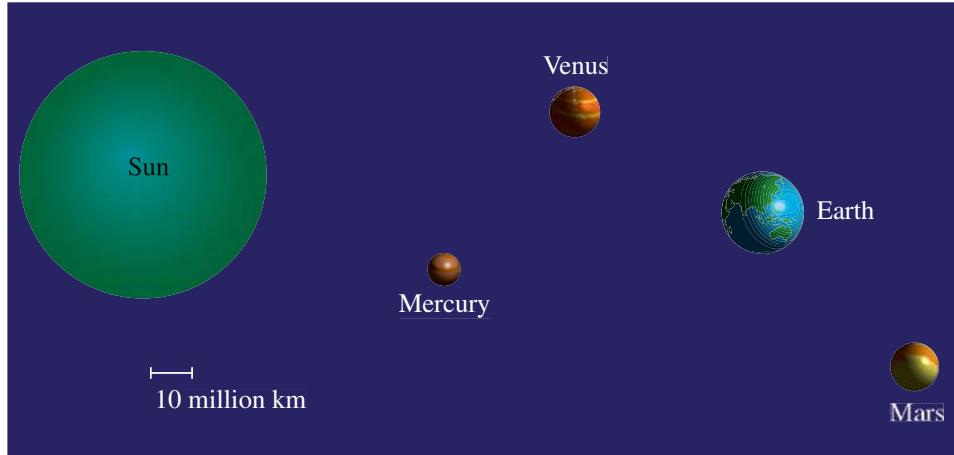
g

h

- 7 Convert each of the lengths obtained in question 6 to centimetres in fraction form.

- 8 WE4 In each of the following illustrations, use the given length to estimate the other lengths mentioned.

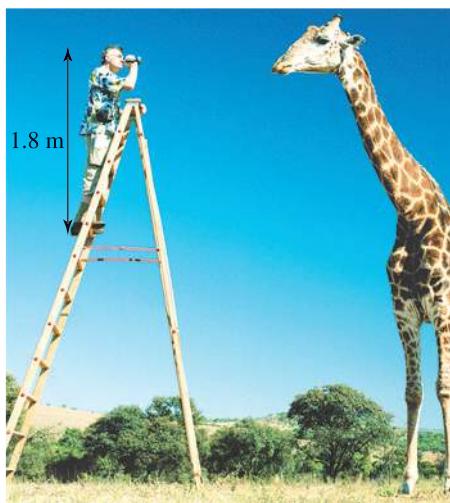
- a The distance from the centre of the Earth to the centre of the Sun



- b The length of the train if it has four carriages



- c The height of the giraffe

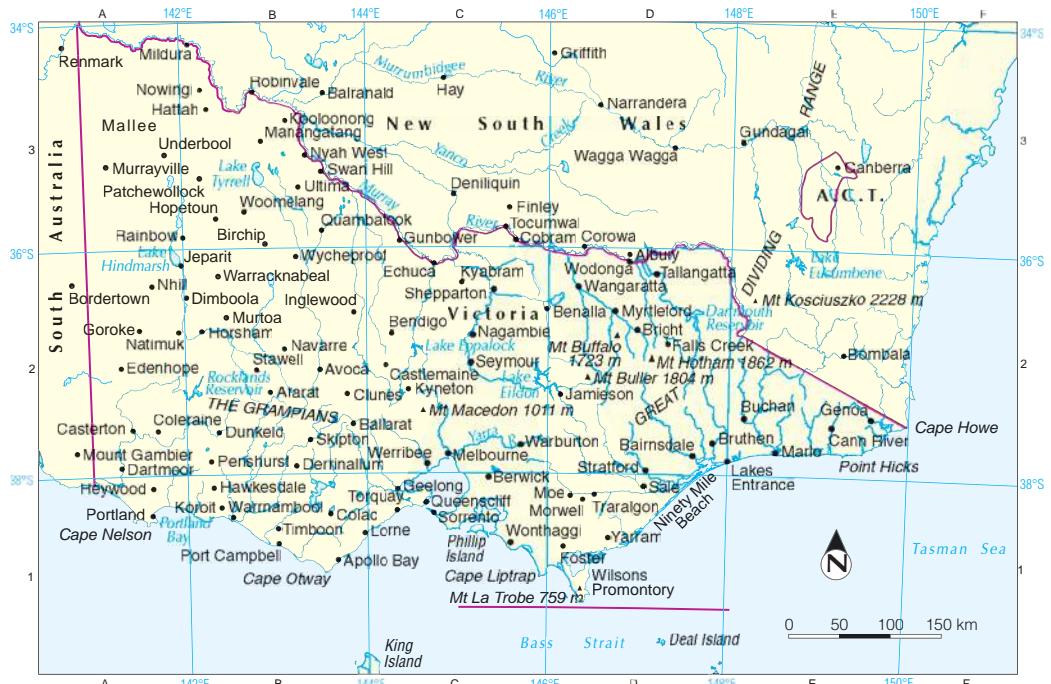


- d The height of the window (to the inside centre of the frame)



- e The distance in a straight line from:

- i Cann River to Falls Creek    ii Melbourne to Underbooi    iii Geelong to Canberra



- f The height of the building in the foreground



- g The height ( $x$ ) of the pole near the utility



### REASONING

- 9 Explain how you could use a normal ruler to find the thickness of a page in this textbook. Calculate the thickness of the page in your Maths textbook and Science textbook.
- 10 Devise a way of measuring the length of a student's pace during her walking, jogging and running. (Hint: you may use a long tape measure — preferably 30 m — or a trundle wheel.)

### REFLECTION

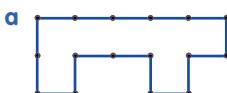
Think of a situation from your daily life in which you would need to estimate some lengths.

# 9C Perimeter

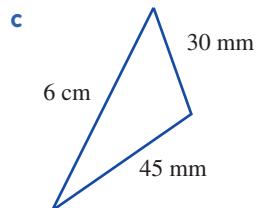
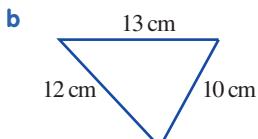
- A **perimeter** is the distance around the outside (border) of a shape.
- To find the perimeter of the shape, change all lengths to the same units and add them up.

## WORKED EXAMPLE 5

Find the perimeter of each shape below.

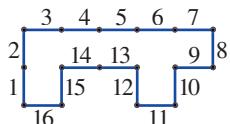


Note: The dots are 1 unit apart.



### THINK

- a 1 Count the number of unit intervals around the outside of the shape (16).



- 2 Write the number.

### WRITE

16 units

- b 1 Check that the measurements are in the same units. They are.

$$\begin{array}{r} 12 \\ 13 \\ + 10 \\ \hline 35 \text{ cm} \end{array}$$

- 2 Add the measurements.

- 3 Write the total, with units.

- c 1 Notice the measurements are not all the same. Convert to the smaller unit ( $6 \text{ cm} = 60 \text{ mm}$ ).

c  $6 \text{ cm} = 60 \text{ mm}$

- 2 Add the measurements that now have the same unit (mm).

$$\begin{array}{r} 60 \\ 30 \\ + 45 \\ \hline 135 \text{ mm} \end{array}$$

- 3 Write the total using the smaller unit.

## WORKED EXAMPLE 6

Find the perimeter of:

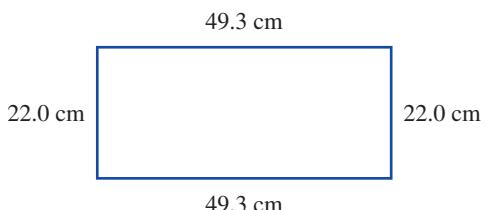
- a a rectangular print which is 49.3 cm long and 22.0 cm wide  
b a square print whose side length is 28 cm.

### THINK

- a 1 Draw a diagram of the print and write in the measurements.

### WRITE

a



- 2 The perimeter is the distance around the print, so add all the distances together.

$$\begin{aligned} P &= 49.3 + 22.0 + 49.3 + 22.0 \\ &= 142.6 \end{aligned}$$

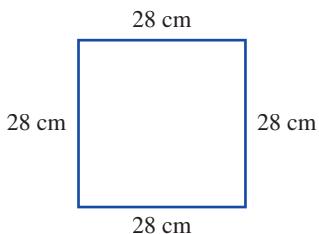




- 3** Write the worded answer, with the correct unit.
- b** **1** Draw a diagram of the print and write in the measurements.

The perimeter of the print is 142.6 cm.

**b**



$$\begin{aligned}P &= 28 + 28 + 28 + 28 \\&= 112\end{aligned}$$

The perimeter of the print is 112 cm.

- 2** The perimeter is the distance around the print, so add all the distances together.
- 3** Write the worded answer, with the correct unit.

## The perimeter of a rectangle and a square

- The perimeter,  $P$ , of a rectangle can be found using the short-cut formula  $P = 2(l + w)$ , where  $l$  is the length of the rectangle and  $w$  is its width.
- The perimeter,  $P$ , of a square can be found using a short-cut formula  $P = 4l$ , where  $l$  is the side length of the square.

### REMEMBER

- The perimeter is the distance around the outside of a shape.
- To calculate the perimeter, change all lengths to the same unit before adding them.
- The perimeter of a rectangle is given by the rule  $P = 2(l + w)$ .
- The perimeter of a square is given by the rule  $P = 4l$ .

### EXERCISE

## 9C

## Perimeter

### INDIVIDUAL PATHWAYS

#### eBook plus

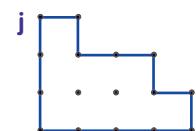
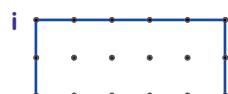
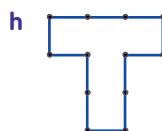
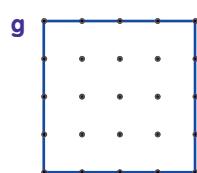
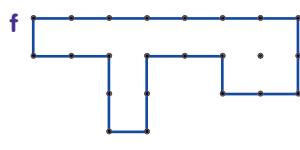
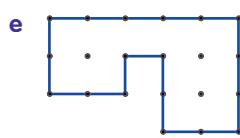
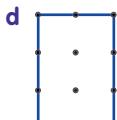
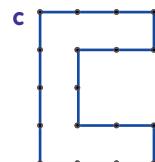
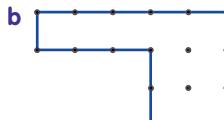
**Activity 9-C-1**  
Perimeter  
doc-1861

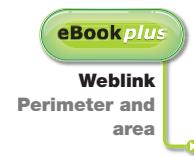
**Activity 9-C-2**  
More perimeter  
doc-1862

**Activity 9-C-3**  
Advanced perimeter  
doc-1863

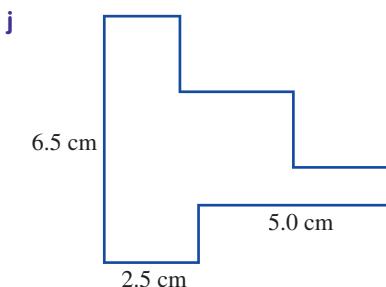
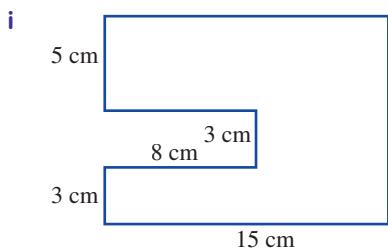
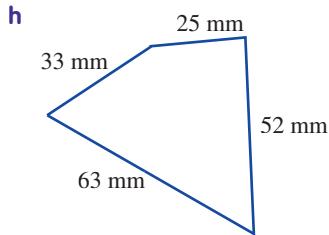
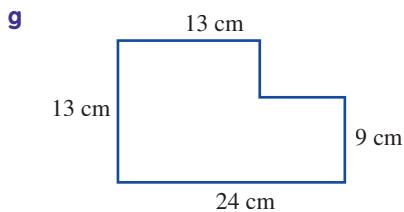
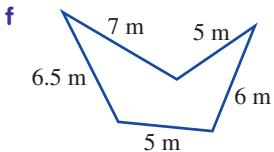
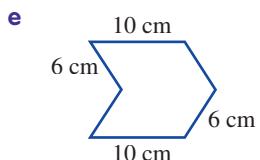
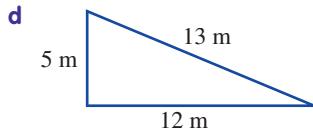
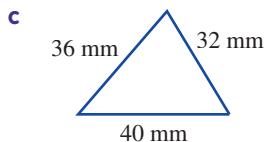
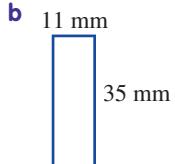
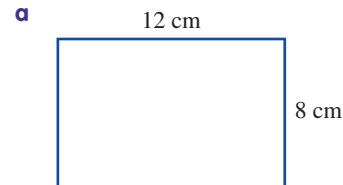
### FLUENCY

- 1 WE5a** Find the perimeter of each shape below. The dots are 1 unit apart.

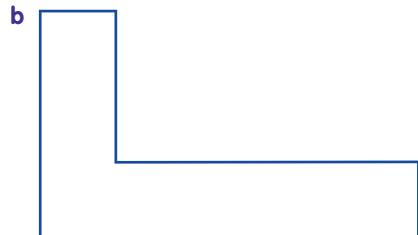


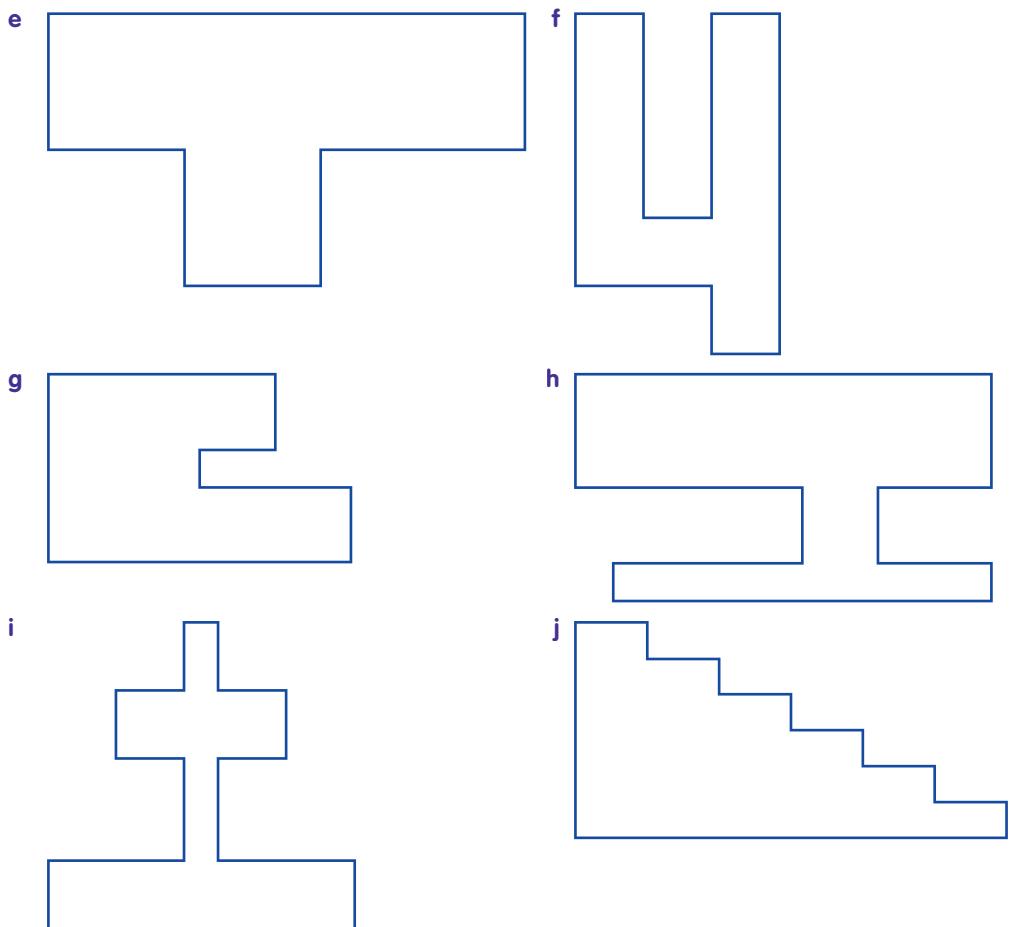


**2 WE5b** Find the perimeter of each of the following.

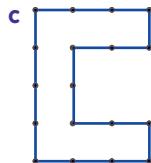
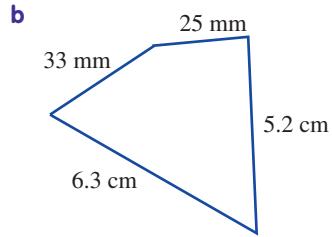
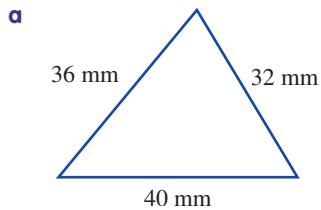


**3** Measure the perimeter of each of the following, giving your answers in centimetres.

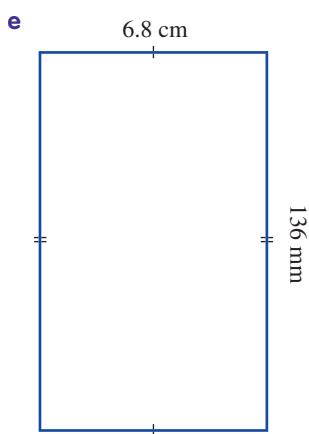
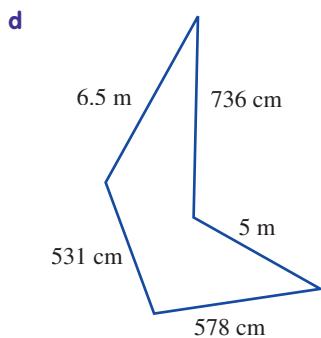


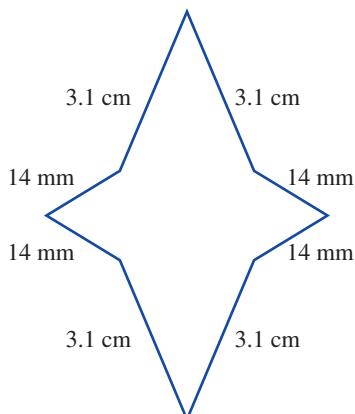
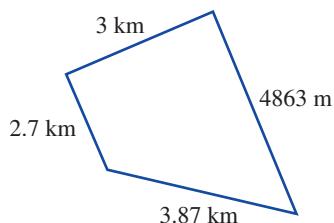
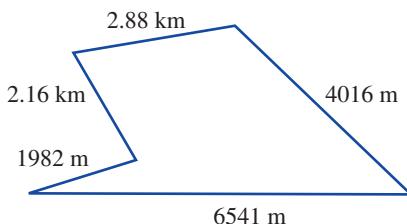


**4** **WESc** Find the perimeter of each shape below, giving answers in the smaller unit in each case.



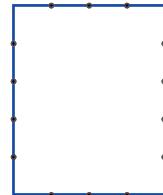
*Note:* The dots are 1 unit apart.



**f****g****h**

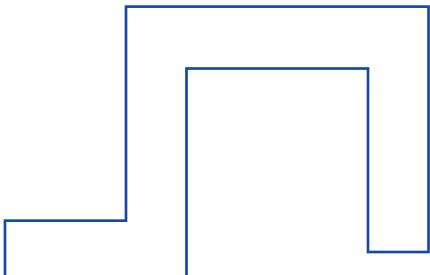
- 5 MC** The perimeter of the shape at right is:
- A 9 units      B 18 units  
 C 16 units      D 14 units  
 E 12 units

Note: The dots are 1 unit apart.



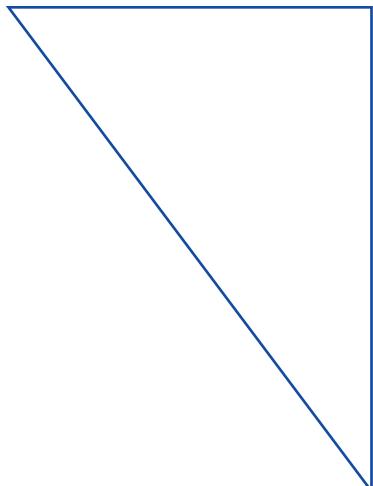
- 6 MC** The shape at right has a perimeter of:
- A 12 cm      B 4 cm  
 C 30 cm      D 29 cm  
 E 60 cm

Note: Use your ruler to measure.



- 7 MC** Measuring to the nearest half centimetre, the perimeter of the shape at right is:
- A 24 cm      B 24.5 cm  
 C 26 cm      D 27 cm  
 E 28 cm

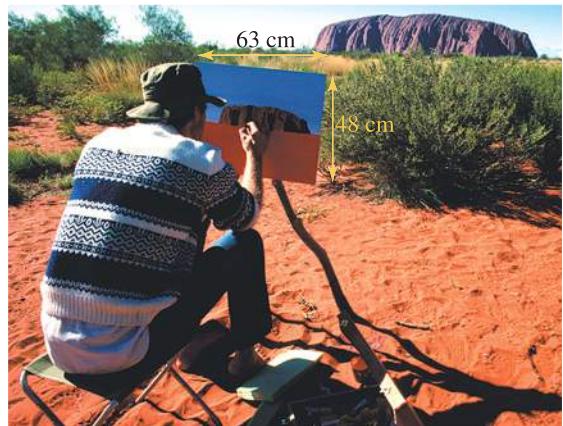
- 8 WE6a** a Calculate the perimeter of a rectangle of length 45 cm and width 33 cm.  
 b Calculate the perimeter of a rectangle of length 2.8 m and width 52.1 cm.  
 c Calculate the perimeter of a rectangle of length  $4\frac{1}{4}$  m and width  $2\frac{1}{5}$  m.



- 9 a** Calculate the perimeter of a square of side length 3.7 cm.  
**b WE6b** Calculate the perimeter of a square of side length 2.14 m.  
**c** Calculate the perimeter of a square of side length  $8\frac{1}{5}$  mm.

**UNDERSTANDING**

- 10** What length of party lights is needed to decorate the perimeter of a rectangular tent with dimensions 15.5 m by 8.75 m?
- 11** Allowing an extra 30 cm for waste, what length of picture frame is required to frame the artwork shown at right?



- 12** Phang's property boundary dimensions are shown on the right.

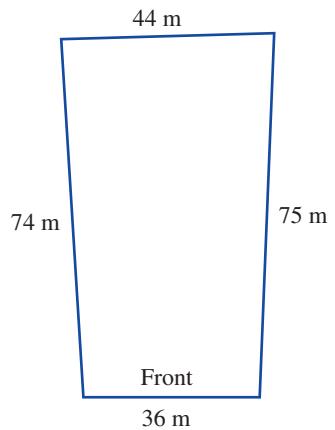
- a** How many metres of fencing will he need to fence all but the front boundary?  
**b** If the fencing costs \$19 per metre, what will the total cost be?

- 13** Lucille has a quote of \$37 per metre for new tennis court fencing. A tennis court is 23.77 m long and 10.97 m wide. There should be a 3.6-m space between the sides of the court and the fence, and a 6.4-m gap between the ends of the court and the fence.

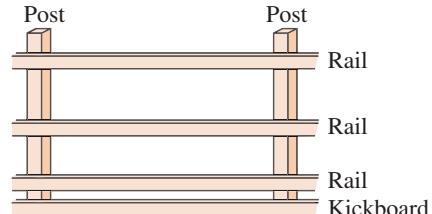
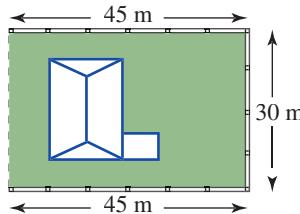
- a** Draw a diagram showing all given measurements.  
**b** How many metres of fencing are needed?  
**c** What will the total cost of the quote be?

- 14** Zedken wishes to install three strands of barbed wire at the top of the fences around a rectangular work site. The length of the site is 34.5 m, and its width is 19.8 m. What length of wire will Zedken need?

- 15** A new game, ‘Bop-ball’ is played on a triangular field, where each side of the triangle measures 46.6 m. A greenkeeper is marking the field’s perimeter using a chalk dispensing trundle wheel. How far will the greenkeeper walk in marking the field?

**REASONING**

- 16** Marc and Cathy are seeking quotes on the cost of building a fence on three sides of their property. They want to calculate approximate costs for each item to decide whether the quotes supplied sound reasonable.

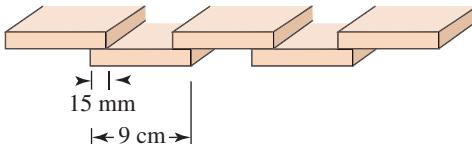


The new fence requires three rails, a kickboard, posts and palings.

- a What length of timber will be needed for the kickboard?
- b Calculate the cost of the kickboard if the timber required for this costs \$1.90 per metre.
- c If the timber for the rails costs \$2.25 per metre, what will be the total cost of the timber for the railings?
- d How many posts will be needed for the new fence if each post is to be 5 metres apart and there needs to be a post at the end of each straight section of fence?
- e Calculate the cost of the posts if the price of each post is \$13.65.

### Challenge

Palings are 9 cm wide and are nailed so they overlap each other by 15 mm on each side.



- f Calculate the approximate number of palings needed for the fence.
- g Palings cost \$1.05 each. How much money should be allowed for the total cost of the palings?
- h Write an itemised list of all the costs involved. Include an amount to cover labour charges and miscellaneous items like the cost of nails.

This amount is around \$1000 (two people for two days at approximately \$30.00 per hour for an eight-hour day). Estimate the cost of the new fence. This will provide Marc and Cathy with information to use when comparing builders' quotes.

### REFLECTION

In which professions will one need to calculate perimeters on a daily basis?

**eBook plus**

Digital docs  
WorkSHEET 9.1  
doc-1853

## 9D Area

- The **area** of a shape is the amount of flat surface enclosed by the shape. For example, if we consider a window, then the frame would represent its perimeter, while the glass inside the frame, its area.

### Metric units of area

- Area is measured in square units such as square centimetres, square metres, square kilometres etc. Commonly used metric units of area, with their abbreviations and examples, are shown below.

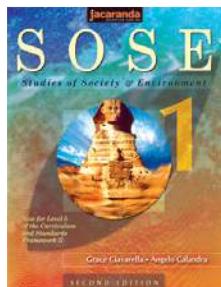
1. Square kilometre ( $\text{km}^2$ )



The area of a country or a large city like Sydney is given in square kilometres.

2. Square metre ( $\text{m}^2$ )

Square metres are used to measure the area of a classroom floor, blackboard or this window.

3. Square centimetre ( $\text{cm}^2$ )

Small areas, such as the area of a sheet of A4 paper or a book cover, are measured in square centimetres.

4. Square millimetre ( $\text{mm}^2$ )

Very small areas, such as the area of a button or a postage stamp, are measured in square millimetres.

- Large areas of land are often measured in hectares (ha).  $1 \text{ ha} = 10\,000 \text{ m}^2$ .



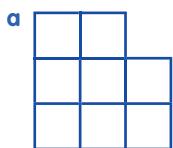
The area of a farm or a large city park is measured in hectares.

- A square grid paper is made up of identical squares with sides that are 1 cm long. The area of each such square is  $1 \text{ cm}^2$ .
- If a shape is drawn on 1-centimetre grid paper, its area can be found by counting the number of squares that the shape covers.

**WORKED EXAMPLE 7**

The figures below are drawn on centimetre grid paper. Find the area of each one.

(Note: The figures are not drawn to scale.)

**THINK**

- a Count the squares. Remember to include the correct unit ( $\text{cm}^2$ ) in the answer.
- b Some of the squares are cut in half by the diagonal line. It takes two of these to make up one square centimetre. Count the squares. Remember to include the correct unit ( $\text{cm}^2$ ) in the answer.

**WRITE**

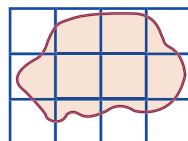
a  $8 \text{ cm}^2$

b  $8 \text{ cm}^2$

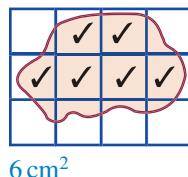
- If a square of the grid paper is not completely covered by the shape, use the following rule to obtain an estimate of the area:
  - if more than half the square is covered, count it as a full square
  - if less than half the square is covered, do not count it at all.

**WORKED EXAMPLE 8**

Estimate the shaded area in the diagram at right, if the figure is drawn on centimetre grid paper.

**THINK**

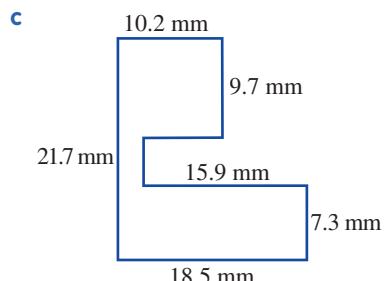
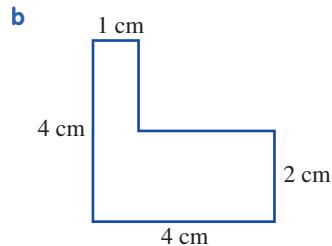
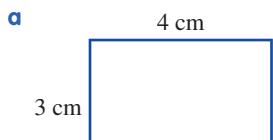
- Tick the squares that are more than half covered and count them.
- State the answer.

**WRITE****Finding the area of a rectangle**

- The area,  $A$ , of a rectangle is given by the formula  $A = lw$ , where  $l$  is the rectangle's length and  $w$  is its width.
- The area,  $A$ , of a square is given by the formula  $A = l^2$ , where  $l$  is the length of the square's side.

## WORKED EXAMPLE 9

Find the area of the following shapes.



## THINK

- a 1 Write the formula for the area of a rectangle.

## WRITE

$$\text{a } A = lw$$

- 2 Substitute the value 4 for  $l$  and 3 for  $w$ .

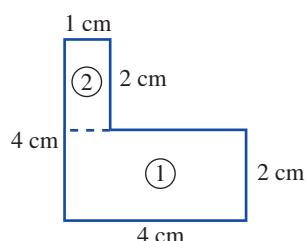
$$A = 4 \times 3$$

- 3 Calculate the result. Remember to answer in the correct unit ( $\text{cm}^2$ ).

$$= 12 \text{ cm}^2$$

- b 1 Divide the shape into two rectangles.

## b



- 2 Calculate the area of each rectangle separately by substituting the correct values of  $l$  and  $w$  into the formula  $A = lw$ .

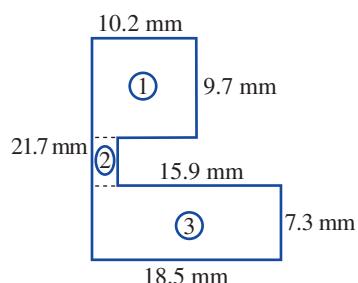
$$\begin{aligned} \text{Area of rectangle 1} &= l \times w \\ &= 4 \times 2 \\ &= 8 \text{ cm}^2 \end{aligned}$$

- 3 Add the two areas. Remember to answer in the correct unit ( $\text{cm}^2$ ).

$$\begin{aligned} \text{Area of rectangle 2} &= l \times w \\ &= 2 \times 1 \\ &= 2 \text{ cm}^2 \end{aligned}$$

- c 1 Divide the shape into three rectangles.

## c



- 2** Calculate the area of each rectangle separately by substituting the correct values of  $l$  and  $w$  into the formula  $A = lw$ .

$$\begin{aligned}\text{Area of rectangle 1} &= l \times w \\ &= 10.2 \times 9.7 \\ &= 98.94 \text{ mm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle 2} &= l \times w \\ &= (21.7 - 9.7 - 7.3) \\ &\quad \times (18.5 - 15.9) \\ &= 4.7 \times 2.6 \\ &= 12.22 \text{ mm}^2\end{aligned}$$

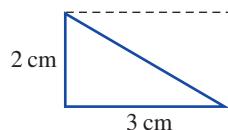
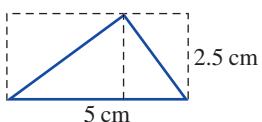
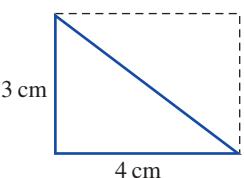
$$\begin{aligned}\text{Area of rectangle 3} &= l \times w \\ &= 18.5 \times 7.3 \\ &= 135.05 \text{ mm}^2\end{aligned}$$

- 3** Add the three areas. Remember to answer in the correct unit ( $\text{mm}^2$ ).

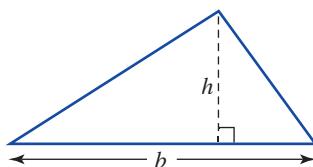
$$\begin{aligned}\text{Area of shape} &= \text{Area 1} + \text{Area 2} + \text{Area 3} \\ &= 98.94 \text{ mm}^2 + 12.22 \text{ mm}^2 \\ &\quad + 135.05 \text{ mm}^2 \\ &= 246.21 \text{ mm}^2\end{aligned}$$

**eBook plus****Interactivity**Area of a triangle  
int-2350**Finding the area of a triangle**

Triangles can be formed by cutting rectangles in half.



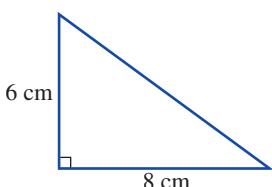
- The area of a triangle is equal to half the area of a rectangle of the same base length and height.
- The area,  $A$ , of a triangle is given by the formula  $A = \frac{1}{2}bh$ , where  $b$  is the length of its base and  $h$  is its height.



- The base and the height of a triangle are perpendicular (at right angles) to each other.
- The height of a triangle could be located inside the triangle, outside the triangle, or as one of the sides of the triangle (if the triangle is right-angled).

**WORKED EXAMPLE 10**

Find the area of the shape at right.



## THINK

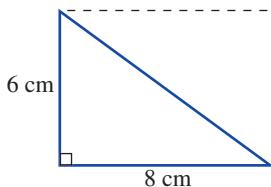
- 1 Draw an imaginary rectangle that contains the triangle. The rectangle should have the same base length and height as the triangle. Notice that the triangle forms half of the rectangle.

- 2 Use the formula  $A = lw$  to find the area of this rectangle.

- 3 Halve the area of the rectangle to get the area of the triangle. Remember to answer in the correct unit ( $\text{cm}^2$ ).

Note: Formula  $A = \frac{1}{2}bh$  could be used.

## WRITE



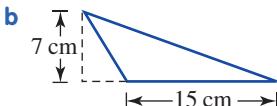
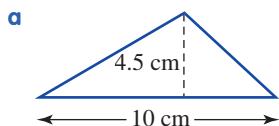
$$\begin{aligned} A &= lw \\ &= 8 \text{ cm} \times 6 \text{ cm} \\ &= 48 \text{ cm}^2 \end{aligned}$$

Area of triangle =  $\frac{1}{2}$  of area of rectangle

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times 48 \\ &= 24 \text{ cm}^2 \end{aligned}$$

## WORKED EXAMPLE 11

Find the area of each of these triangles.



## THINK

- a 1 Write the formula for the area of a triangle.  
2 Identify the values of  $b$  and  $h$ .  
3 Substitute the values of  $b$  and  $h$  into the formula.  
4 Evaluate. (Since one of the values is even, halve it first to make calculations easier.) Remember to include the correct unit ( $\text{cm}^2$ ).

- b 1 Write the formula for the area of a triangle.  
2 Identify the values of  $b$  and  $h$ .  
3 Substitute the values of  $b$  and  $h$  into the formula.  
4 Evaluate. (Since neither value is even, multiply 15 and 7 first, and then divide by 2.) Remember to include the correct unit ( $\text{cm}^2$ ).

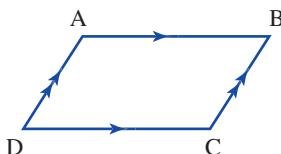
## WRITE

$$\begin{aligned} a \quad A &= \frac{1}{2}bh \\ b &= 10, h = 4.5 \\ A &= \frac{1}{2} \times 10 \times 4.5 \\ &= 5 \times 4.5 \\ &= 22.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} b \quad A &= \frac{1}{2}bh \\ b &= 15, h = 7 \\ A &= \frac{1}{2} \times 15 \times 7 \\ &= \frac{1}{2} \times 105 \\ &= 52.5 \text{ cm}^2 \end{aligned}$$

## Area of a parallelogram

- A **parallelogram** is a quadrilateral such that each pair of opposite sides is parallel.

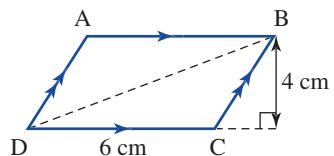
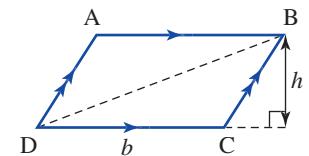


- To calculate the area of the parallelogram, first draw a diagonal from B to D to form two triangles, ABD and BDC. Label the base  $b$ , and the perpendicular height,  $h$ .

Then find the area of the two triangles. The area of triangle  $BCD = \frac{1}{2} b \times h$  and the area of triangle  $ABD = \frac{1}{2} b \times h$ .

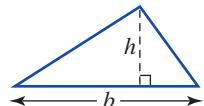
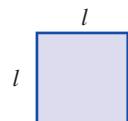
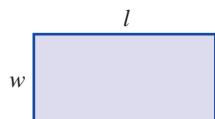
$$\text{So, the area of the parallelogram } ABCD = \frac{1}{2} b \times h + \frac{1}{2} b \times h \\ = b \times h$$

For example, the area of the parallelogram shown at right is:  
Area  $ABCD = b \times h = 6 \times 4 = 24 \text{ cm}^2$



### REMEMBER

- The area of a shape is the amount of flat surface enclosed by the shape.
- Area is measured in square units such as  $\text{mm}^2$ ,  $\text{cm}^2$ ,  $\text{m}^2$  and  $\text{km}^2$ .
- Large areas of land are often measured in hectares.  $1 \text{ ha} = 10000 \text{ m}^2$ .
- Area may be found by counting the number of squares needed to cover a surface.
- A square with a side measure of 1 cm has an area of 1 square centimetre ( $1 \text{ cm}^2$ ).
- The area of a rectangle is found by using the formula  $A = lw$ , where  $A$  is the area of the rectangle,  $l$  is its length and  $w$  is its width.
- A square is a rectangle whose length and width are equal.  
The area of a square is found by using the formula  $A = l^2$ , where  $l$  is its side length.
- The area of a triangle is equal to half that of a rectangle with the same base and height.
- The area of a triangle may be found using the formula  $A = \frac{1}{2}bh$ , where  $b$  is the base and  $h$  is the height of the triangle.
- The area of a parallelogram is:  $A = bh$  where  $b$  is the length of the base and  $h$  is the perpendicular length of the parallelogram.



### EXERCISE

## 9D Area

### INDIVIDUAL PATHWAYS

#### eBook plus

##### Activity 9-D-1

Area  
doc-1864

##### Activity 9-D-2

More area  
doc-1865

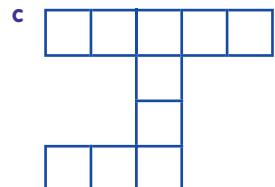
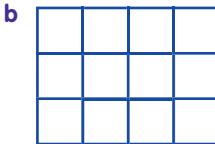
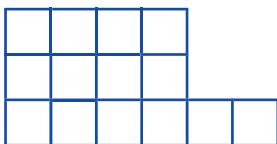
##### Activity 9-D-3

Advanced area  
doc-1866

### FLUENCY

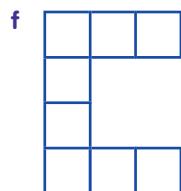
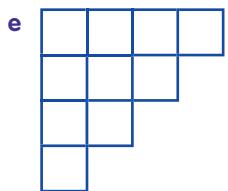
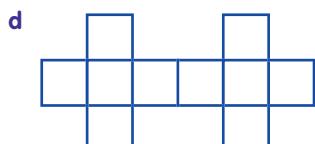
- Which unit would be most suitable to measure the following areas? Choose from  $\text{mm}^2$ ,  $\text{cm}^2$ ,  $\text{m}^2$ , ha or  $\text{km}^2$ .
 

<b>a</b> A computer screen	<b>b</b> A sheet of A3 paper
<b>c</b> Melbourne Cricket Ground	<b>d</b> A shirt button
<b>e</b> The Melbourne metropolitan area	<b>f</b> A compact disc
<b>g</b> A house block	<b>h</b> A room
<b>i</b> Alaska	<b>j</b> A basketball court
<b>k</b> A dairy farm	<b>l</b> The area inside the letter D
- WE7a** The figures below are drawn on centimetre grid paper. Find the area of each one.
  - 
  - 
  -

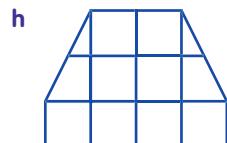
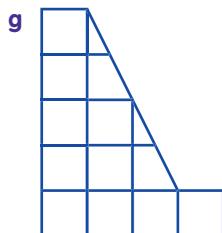
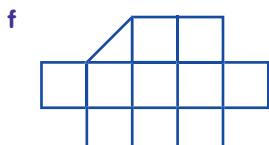
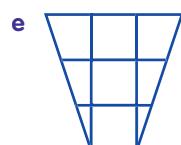
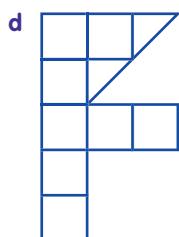
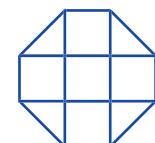
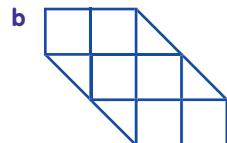
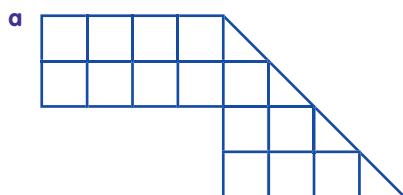


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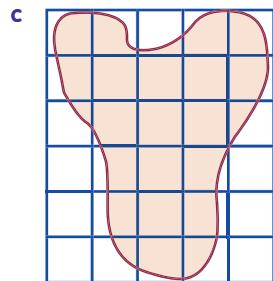
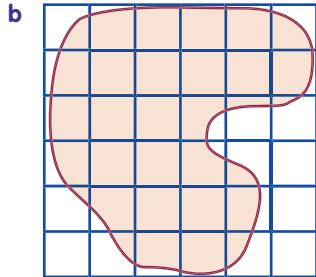
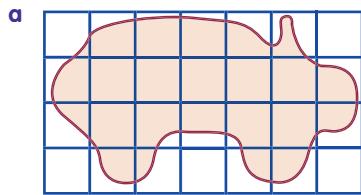
**Interactivity**  
Area plus  
worksheet  
int-0005



- 3 **WE7b** Find the area of the figures below, which are drawn on centimetre grid paper.

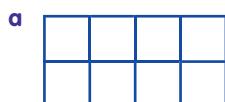


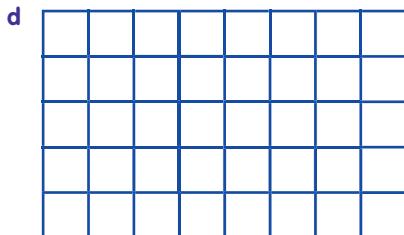
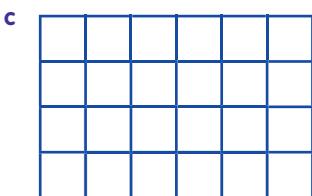
- 4 **WE8** Estimate the shaded area of each of the following figures if each is drawn on centimetre grid paper.



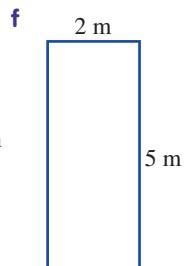
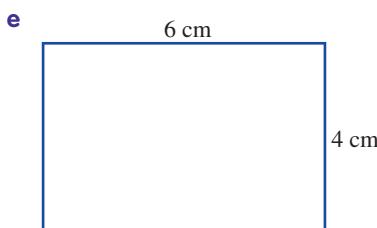
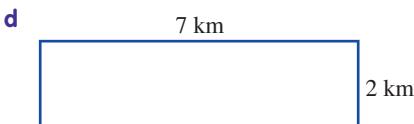
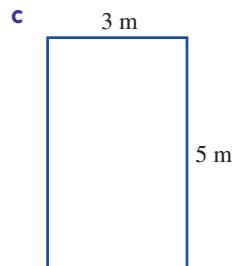
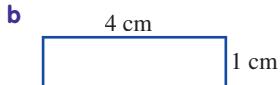
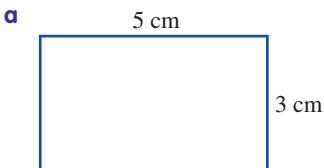
- 5 Four rectangles are drawn on centimetre grids that follow.

- State the length and width of each rectangle.
- Use your answers to part i to calculate the area of each rectangle.
- Count the number of  $1\text{-cm}^2$  squares to find the area of each rectangle.
- Compare your answers to parts ii and iii.





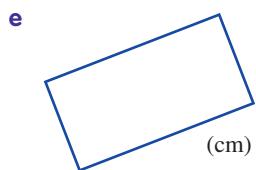
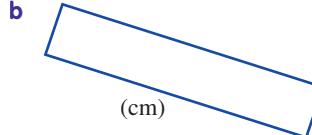
- 6 WE9a** Find the area of the following rectangles. (Hint: Use the formula  $A = lw$ .)



- 7** Accurately measure the length and width of each rectangle in the units specified by the bracket. Hence, calculate the area of the rectangle.

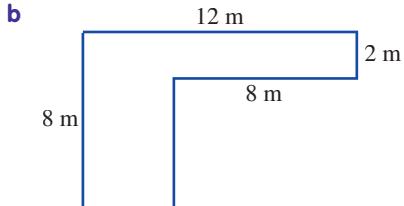
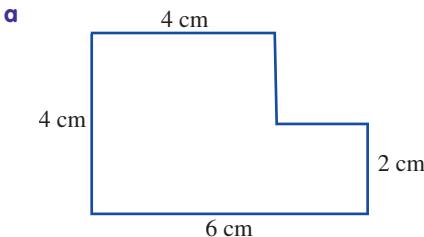
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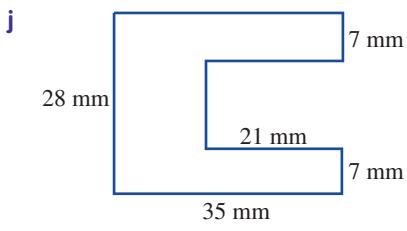
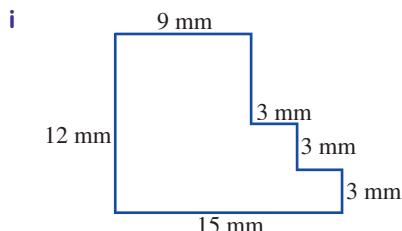
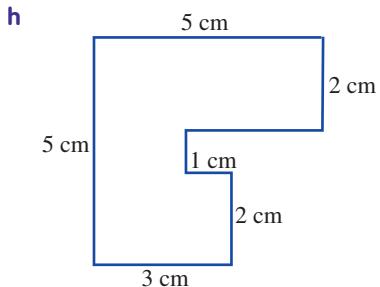
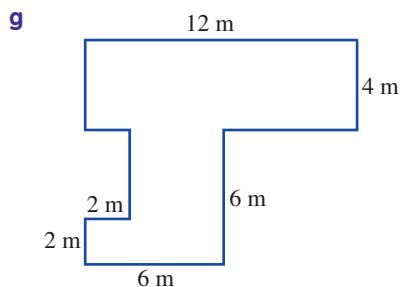
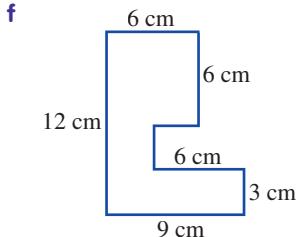
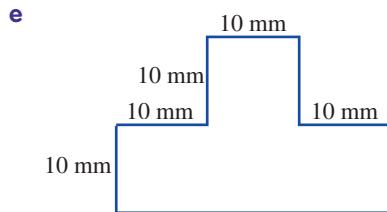
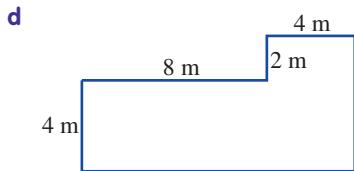
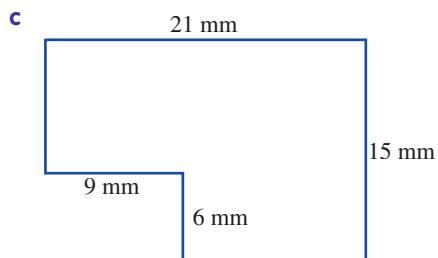
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Spreadsheet  
Area of a rectangle  
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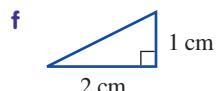
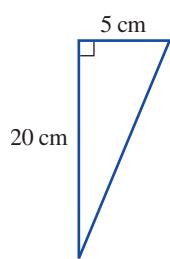
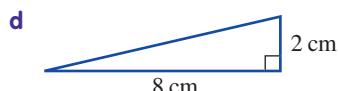
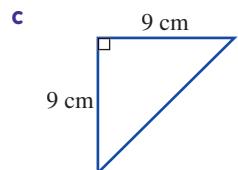
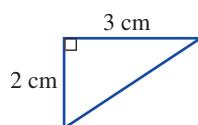
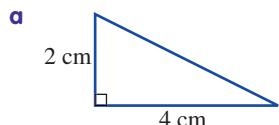
- 8 WE9b,c** Find the area of the following shapes.

(Hint: Divide the shapes into rectangles and squares before using the formula  $A = lw$ .)



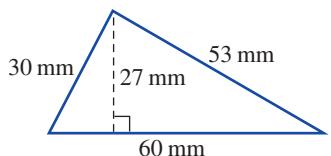


**9** **(WEIO)** Find the area of the following triangles.



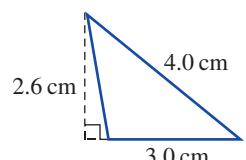
- 10 MC a** In the diagram shown at right, the height of the triangle is:

A 60 mm      B 30 mm      C 27 mm  
D 53 mm      E 49 mm



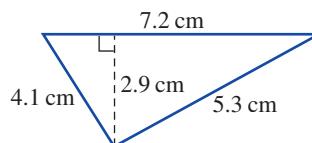
- b** In the diagram shown at right, the height of the triangle is:

A 2.6 cm      B 4.0 cm      C 3.0 cm  
D 4.2 cm      E 3.8 cm



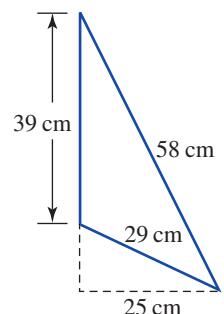
- c** In the diagram shown at right, the base length of the triangle is:

A 4.1 cm      B 2.9 cm  
C 5.3 cm      D 7.2 cm  
E 7.0 cm



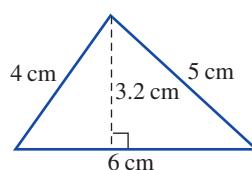
- d** In the diagram shown at right, the base length of the triangle is:

A 39 cm      B 58 cm      C 29 cm  
D 25 cm      E 43 cm



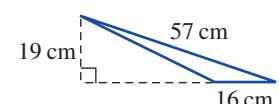
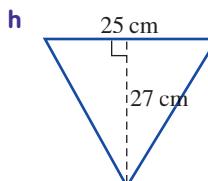
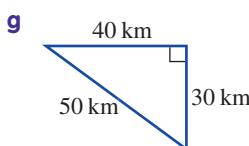
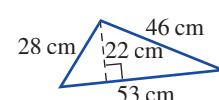
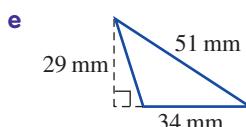
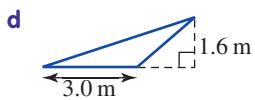
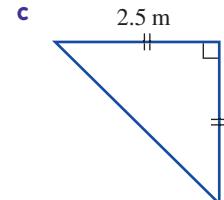
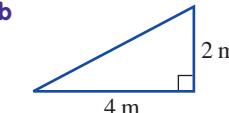
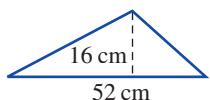
- e** The area of the triangle shown at right is:

A  $19.2 \text{ cm}^2$       B  $9.6 \text{ cm}^2$   
C  $12 \text{ cm}^2$       D  $15 \text{ cm}^2$   
E  $24 \text{ cm}^2$



- 11 WEI** Find the area of each of these triangles.

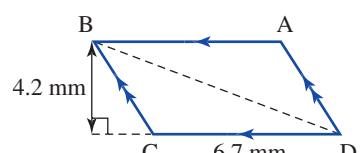
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Spreadsheet  
Area of a  
triangle  
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- 12** Determine the area of the parallelogram shown at right.

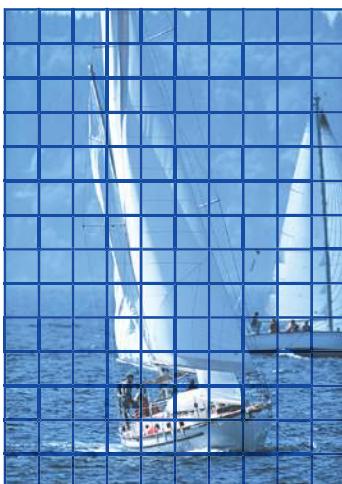
- 13** The table below gives the bases, heights and areas of various parallelograms. Complete the table.

Base	Height	Area
a 5 mm	3 mm	
b 8 cm		$56 \text{ cm}^2$
c	5.2 m	$52 \text{ m}^2$
d 6 km	2 km	
e 4 cm		$1200 \text{ mm}^2$
f 4.9 m	8.7 m	

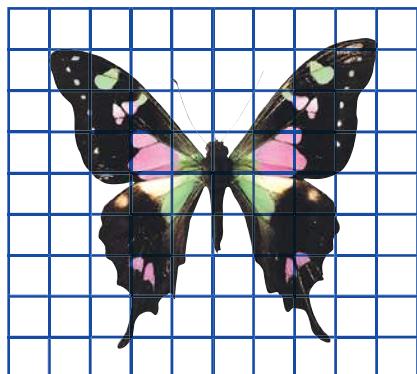


## UNDERSTANDING

- 14 a** Estimate the area of the sails of this yacht if a metre grid is over it as shown.

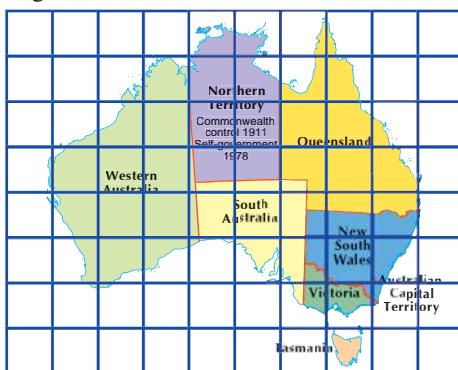


- b** Estimate the area of this butterfly if a centimetre grid is over it as shown.

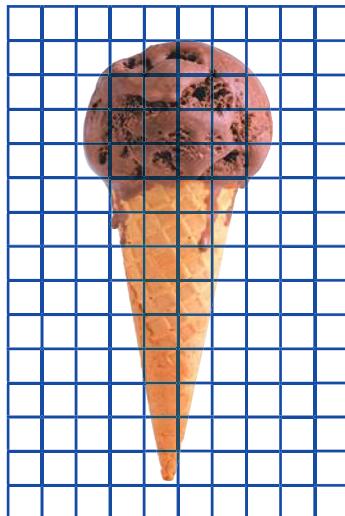


- c** Estimate the area of Australia if a 470 km grid is over it as shown.

*Note:* The area of each square of the grid is  $470 \text{ km} \times 470 \text{ km}$  or  $220\ 900 \text{ km}^2$ .



- d** Estimate the area of the chocolate scoop in this picture of an ice-cream if a centimetre grid is over it as shown.



- 15** One of the largest open-air shopping centres is the Ala Moana centre in Honolulu, Hawaii, USA with more than 200 shops covering an area of 20 hectares. How many square metres is this?

- 16** The Taupo volcanic eruption, in New Zealand nearly 2000 years ago, is estimated to have flattened an area of  $16\ 000 \text{ km}^2$ .

- a** How many square metres is this?  
**b** How many hectares is this?

- 17** How many square metres of carpet are needed to carpet a rectangular room of length 5 m and width 3.5 m?

- 18** Find the area of material ( $\text{cm}^2$ ) needed to make a rectangular table napkin 32 cm long and 25 cm wide.

- 19** Find the area of material needed to make a rectangular rug which is 4.2 m long and 230 cm wide. (Give your answer in square metres.)

(*Hint:* Convert the width measurement into metres first.)



- 20** Toby is tiling his bathroom with ceramic tiles that cost \$35.20 per box.

- a How many square metres of tiles will he need, if the rectangular room has a width of 2.5 m and a length of 3 m?
- b How many boxes of tiles should he order if each box contains enough tiles to cover  $0.5 \text{ m}^2$ ?
- c What will be the cost of the tiles?



- 21** Jane is a landscape gardener who is laying a new lawn. The rectangular lawn is 13 m long and 8 m wide. How many square metres of turf should Jane order? What is the total cost of the turf if it costs \$12.50 per square metre?

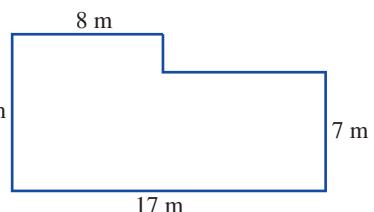
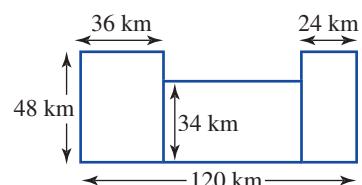
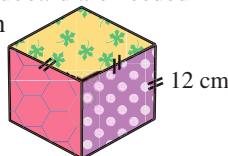
- 22** Alana, who works for Fast Glass Replacements, has been asked for a quote to replace three windows. Each window is 1.8 m long and 0.8 m wide. What price should Alana quote if the glass costs \$27 per square metre? (Include a delivery cost of \$25 in the quote.)

- 23** Find the total area of the cattle station shown at right, which has 3 large paddocks. (Give your answer in square kilometres.)

- 24** Rectangular laminate sheets are 1.8 m long and 0.9 m wide. How many sheets would be needed to cover  $4.8 \text{ m}^2$  of bench space in a kitchen?

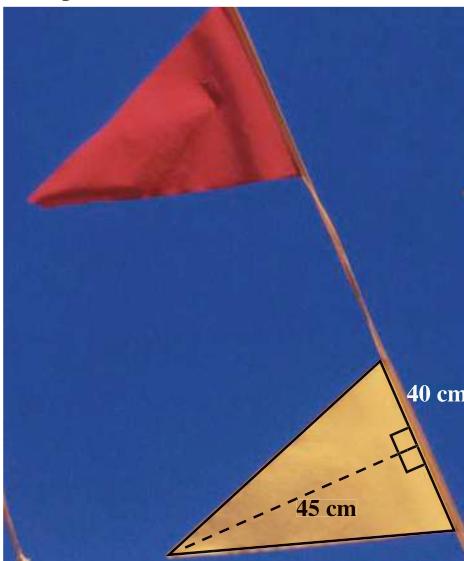
- 25** Calculate the total floor area of a concrete slab for a house as shown at right.

- 26** How many square centimetres of cardboard are needed to make the cube with an edge length of 12 cm shown at right.

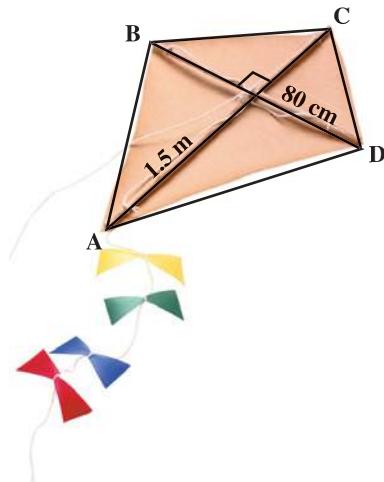


- 27** Calculate the approximate area of the triangular mainsail when the sail is flat. Use the dimensions shown on the yacht at right.

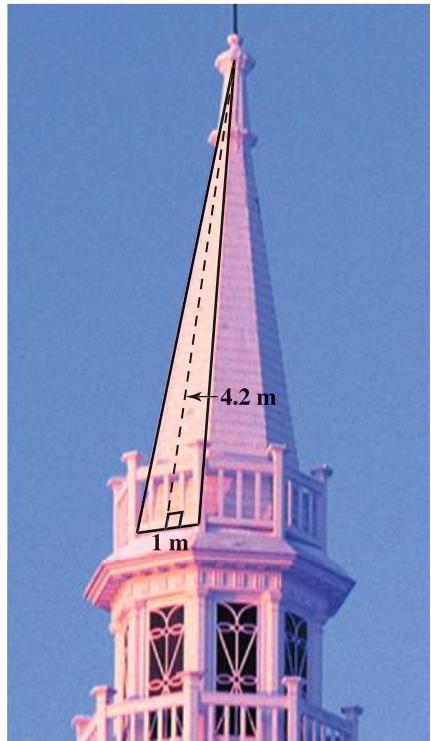
- 28** What area of cloth would be required to make one of the pennants shown below?



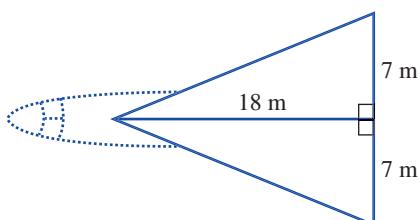
- 29** Find the area of cloth required to make the kite shown, given that the length BD is 80 cm and the length AC is 1.5 m.



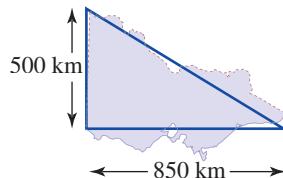
- 30** A church spire has six identical triangular faces that have the dimensions shown at right. What area of copper roofing would be required to cover all six faces of the spire?



- 31** Find the total wing area of the delta winged jet aircraft shown below.



- 32** The triangle shown below can be used to make an estimate of the area of the state of Victoria.



- a Find the area of the triangle shown.  
b Compare your estimate with the correct value from your atlas.

#### REASONING

- 33** A floor tiler charged \$640.00 to tile a rectangular room. Her next job is to tile the floor of a rectangular room twice as long and twice as wide. How much should she charge for the larger room? (The answer is not \$1280.00.)

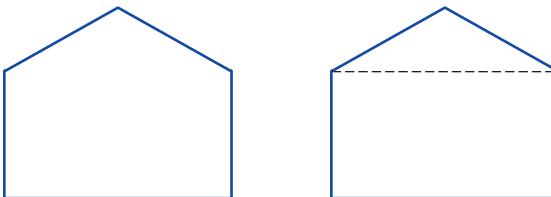
- 34 A rectangle has an area of  $36 \text{ cm}^2$  and a perimeter of  $26 \text{ cm}$ . Each side of the rectangle is a whole number of centimetres. What is the length and width of this rectangle?
- 35 What is the perimeter of a square that has the same area as a rectangle with length  $18 \text{ mm}$  and width  $8 \text{ mm}$ ?

**REFLECTION**

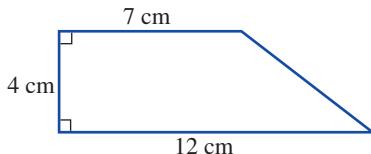
Name some objects from your immediate surroundings whose area you can find using the formulae for the areas of rectangles, squares and triangles.

## 9E Area of composite shapes, using addition and subtraction

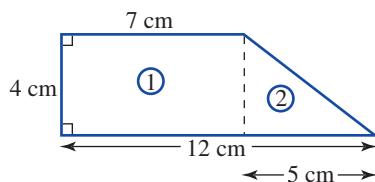
- **Composite shapes** can be divided into parts, each of which is a simple shape (such as, for example, a rectangle or a triangle), whose areas are easy to find using familiar formulae.
- The area of a composite shape can be found by adding the areas of each part of the divided shape. For example, the area of the pentagon shown below can be found by finding and then adding the areas of the triangle and rectangle.

**WORKED EXAMPLE 12**

Find the area of the shape at right.

**THINK**

- 1 Divide the shape into a rectangle and a triangle.
- 2 Use the formula  $A = lw$  to find the area of the rectangle.
- 3 Use the formula  $A = \frac{1}{2}bh$  to find the area of the triangle.
- 4 Add the two areas to get the total area.

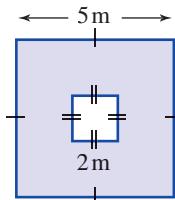
**WRITE**

$$\begin{aligned}\text{Area 1} &= lw \\ &= 7 \times 4 \\ &= 28 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area 2} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 5 \times 4 \\ &= 10 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total area} &= A_1 + A_2 \\ &= 28 + 10 \\ &= 38 \text{ cm}^2\end{aligned}$$

- In many situations the area of a given shape can be found by subtracting individual areas from each other.
- If a shaded area is required, the total area is found first and then the area of the ‘unshaded’ part(s) is subtracted from it.

**WORKED EXAMPLE 13****Find the shaded area in the figure below.****THINK**

- The figure shown is a large square from which a small square has been removed. Write the formula for the area of the square.
- Calculate the area of the large square first.
- Calculate the area of the small square.
- Now find the shaded area by subtracting the area of the small square from that of the large one.

$$\text{Area of a square} = l^2$$

$$\begin{aligned}\text{Large square: } A &= 5^2 \\ &= 25 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Small square: } A &= 2^2 \\ &= 4 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Shaded area} &= 25 - 4 \\ &= 21 \text{ m}^2\end{aligned}$$

**WRITE****REMEMBER**

- Many composite shapes can be divided into two or more ‘pieces’ which have a simple area formula.
- The area of a composite shape can be found by adding together the areas of each part the shape is divided into.
- Many problems such as finding a shaded area can be solved by subtracting one area from another.

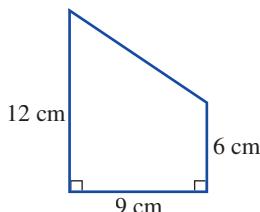
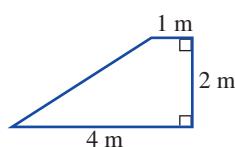
**EXERCISE****9E****Area of composite shapes, using addition and subtraction****INDIVIDUAL PATHWAYS****eBookplus**

**Activity 9-E-1**  
Composite shapes  
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**Activity 9-E-2**  
More composite shapes  
doc-1868

**FLUENCY**

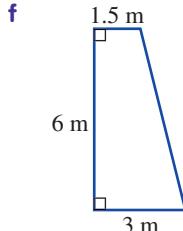
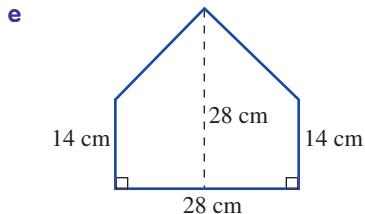
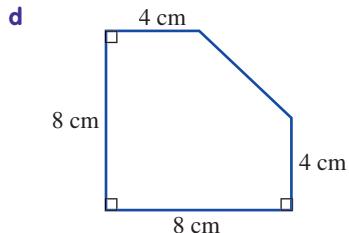
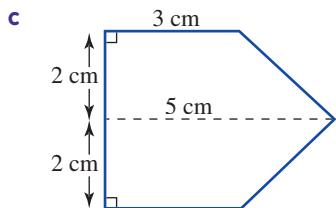
- 1 WEI2** Find the area of the following composite shapes.

**a****b**

## INDIVIDUAL PATHWAYS

## eBook plus

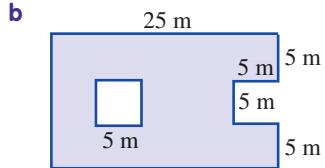
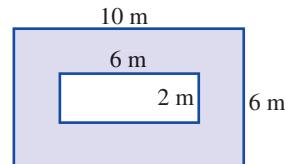
**Activity 9-E-3**  
Advanced composite shapes  
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- 2 WE13 Find the area of the following shapes.

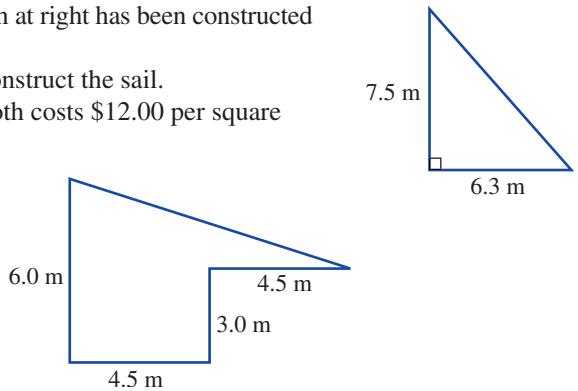
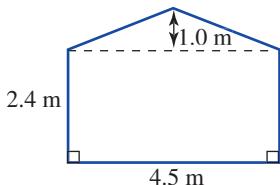
## eBook plus

Digital docs  
Spreadsheet  
**Area**  
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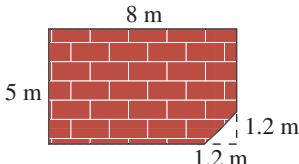


## UNDERSTANDING

- 3 The sail whose measurements are shown at right has been constructed for the sailing vessel *Nautilus*.
- Find the area of cloth required to construct the sail.
  - What is the cost of the sail if the cloth costs \$12.00 per square metre?
- 4 a Find the area of carpet needed for the room at right.  
b What would the carpet for the room cost if it is sold for \$25 per square metre?
- 5 Find the area of the garage wall shown below.

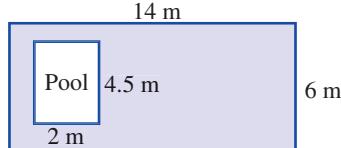


- 6 Determine the area of bricks needed to cover the courtyard shown below.

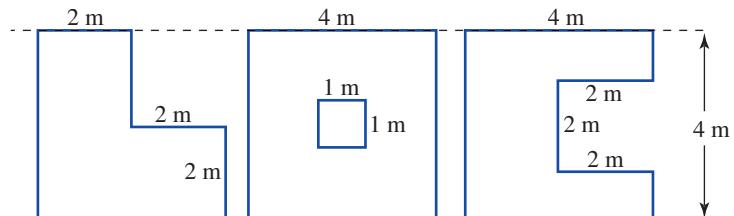


- 7 A sheet of A4 paper is approximately 30 cm long and 21 cm wide. Find the area of the triangle formed by cutting an A4 sheet in half diagonally.

- 8 Find the area of the entrance foyer shown on the right.
- 9 Michael is paving a rectangular yard, which is 15.5 m long and 8.7 m wide. A square fishpond with side lengths of 3.4 m is in the centre of the yard. What will be the cost of paving Michael's yard, if the paving material costs \$17.50 per square metre?
- 10 Members of the Lee family want to pave the area around their new swimming pool. The pool is set into the corner of the yard as shown at right. Find the area of paving (in square metres) required to cover the yard around the pool (shaded in the diagram).



- 11 In order to construct an advertising sign, the letters below are cut from plastic sheets. Find the total area of plastic required to make all 3 letters.



#### REFLECTION

How will you decide whether to find the area of a given composite shape by addition or by subtraction?

## 9F Volume

- The **volume** of a 3-dimensional object is the amount of space it occupies.
- Volume is measured in cubic units such as  $\text{mm}^3$ ,  $\text{cm}^3$ ,  $\text{m}^3$ , etc.
- A cubic centimetre ( $\text{cm}^3$ ) is the space occupied by a cube with sides of 1 cm.

A sugar cube has a volume of about  $1 \text{ cm}^3$ .



The volume of a discman is about  $30 \text{ cm}^3$ .



- A cubic metre ( $m^3$ ) is a space occupied by a cube with sides of 1 m.

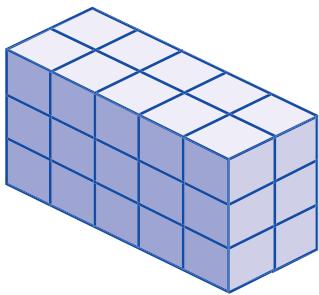
A bathtub can hold about  $\frac{1}{2} m^3$  of water and each of these recycling bins would hold about  $1 m^3$ .



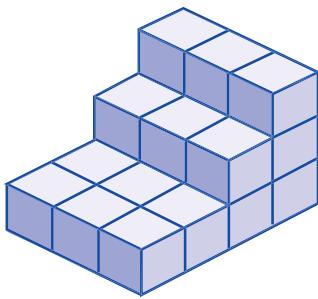
#### WORKED EXAMPLE 14

How many cubic centimetres are in each solid shape? (Each small cube represents  $1 \text{ cm}^3$ .)

a



b



##### THINK

- a 1 Count the cubes.

There are 10 cubes in each layer.

There are three layers altogether.

- 2 Give the answer in cubic centimetres.

##### WRITE

$$\text{a } \text{Volume} = 10 \text{ cm}^3 \times 3$$

$$= 30 \text{ cm}^3$$

- b 1 Count the cubes.

There are 12 cubes in the first layer.

There are 6 cubes in the second layer.

There are 3 cubes in the third layer.

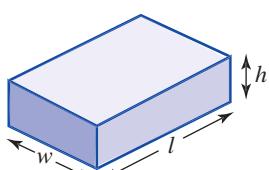
- 2 Give the answer in cubic centimetres.

$$\text{b } \text{Volume} = 12 \text{ cm}^3 + 6 \text{ cm}^3 + 3 \text{ cm}^3$$

$$= 21 \text{ cm}^3$$

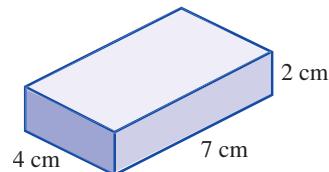
## Finding the volume of a rectangular prism

- The volume of a rectangular prism is given by the formula  $V = lwh$ , where  $l$  is the prism's length,  $w$  is its width and  $h$  is its height.
- The volume of a rectangular prism can also be written as  $V = Ah$ , where  $A$  is the area of the prism's base and  $h$  is its height.



**WORKED EXAMPLE 15**

Use the formula  $V = lwh$  to calculate the volume of the rectangular prism shown at right.

**THINK**

- 1 Write the formula for volume.
- 2 Identify the length ( $l = 7 \text{ cm}$ ), width ( $w = 4 \text{ cm}$ ), and height ( $h = 2 \text{ cm}$ ) of the prism and substitute the values into the formula.
- 3 Calculate the result. Remember to write the answer using the correct unit ( $\text{cm}^3$ ).

**WRITE**

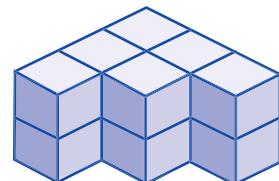
$$\begin{aligned} V &= lwh \\ &= 7 \text{ cm} \times 4 \text{ cm} \times 2 \text{ cm} \\ &= 56 \text{ cm}^3 \end{aligned}$$

**Finding the volume of other types of prisms and shapes**

- The volume of any prism is given by the formula  $V = Ah$ , where  $A$  is the area of its base and  $h$  is its height.
- The base of the prism could be of any shape (a rectangle, a square, a triangle etc.)
- The height of the prism is always perpendicular (at right angles) to its base.

**WORKED EXAMPLE 16**

Find the volume of the prism shown at right if each cube has a volume of  $1 \text{ cm}^3$ .

**THINK**

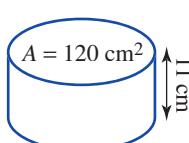
- 1 Write the formula for the volume of a prism.
- 2 The area of the base ( $A$ ) is the same as the area of the top layer which is  $6 \text{ cm}^2$ . The height is 2 cm.
- 3 Substitute the values of  $A$  and  $h$  into the formula  $V = Ah$ .
- 4 Calculate the answer and give the units in cubic centimetres.

**WRITE**

$$\begin{aligned} V &= Ah \\ A &= 6 \text{ cm}^2, h = 2 \text{ cm} \\ V &= 6 \text{ cm}^2 \times 2 \text{ cm} \\ &= 12 \text{ cm}^3 \end{aligned}$$

**WORKED EXAMPLE 17**

Calculate the volume of this shape.



**THINK**

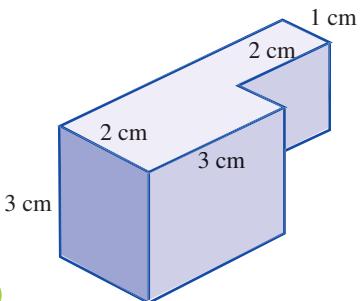
- 1 Write the formula for the volume of the shape.
- 2 The area of the base,  $A$ , is given ( $120 \text{ cm}^2$ ), the height is  $11 \text{ cm}$ .
- 3 Substitute the values of  $A$  and  $h$  into the formula  $V = Ah$ .
- 4 Calculate the answer and give the units in cubic centimetres.

**WRITE**

$$\begin{aligned}V &= Ah \\A &= 120 \text{ cm}^2 \text{ and } h = 11 \text{ cm} \\V &= 120 \text{ cm}^2 \times 11 \text{ cm} \\&= 1320 \text{ cm}^3\end{aligned}$$

**WORKED EXAMPLE 18**

Use the information given in the diagram shown on the right to calculate the volume of the prism.

**THINK**

- 1 Write the formula for the volume of a prism.
- 2 The area of the base ( $A$ ) can be found by adding the areas of the 2 rectangles on the top surface. The height is  $3 \text{ cm}$ .
- 3 Substitute the values of  $A$  and  $h$  into the formula  $V = Ah$ .
- 4 Calculate the answer and give the units in cubic centimetres.

**WRITE**

$$\begin{aligned}V &= Ah \\ \text{Area of large rectangle} &= lw \\ &= 3 \text{ cm} \times 2 \text{ cm} \\ &= 6 \text{ cm}^2 \\ \text{Area of small rectangle} &= lw \\ &= 2 \text{ cm} \times 1 \text{ cm} \\ &= 2 \text{ cm}^2 \\ \text{Total area of base} &= 6 \text{ cm}^2 + 2 \text{ cm}^2 \\ &= 8 \text{ cm}^2 \\ \text{So } A &= 8 \text{ cm}^2 \text{ and } h = 3 \text{ cm} \\ V &= 8 \text{ cm}^2 \times 3 \text{ cm} \\ &= 24 \text{ cm}^3\end{aligned}$$

**REMEMBER**

1. The volume of a 3-dimensional object is the amount of space it occupies.
2. Cubic centimetres and cubic metres are units that are commonly used to measure volume.
3. Volume ( $V$ ) of a rectangular prism:  $V = lwh$ , where  $l$  is the length,  $w$  is the width,  $h$  is the height and  $A$  is the area of the base of the prism.
4. Volume of any prism ( $V$ ) = Area of base ( $A$ )  $\times$  height ( $h$ ) or  $V = Ah$ .

## EXERCISE

## 9F Volume

## INDIVIDUAL PATHWAYS

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## Activity 9-F-1

Volume  
doc-1870

## Activity 9-F-2

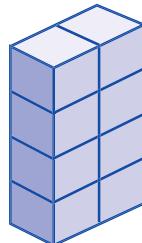
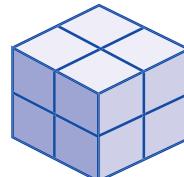
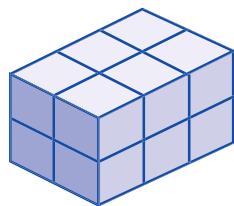
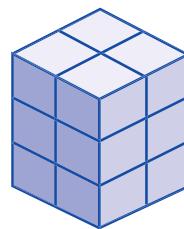
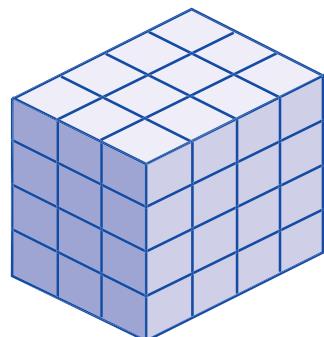
More volume  
doc-1871

## Activity 9-F-3

Advanced volume  
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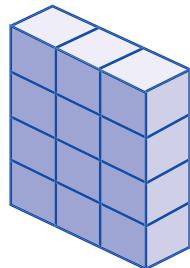
## FLUENCY

- 1 **WE14a** How many cubic centimetres are in each solid shape? (Each small cube represents  $1 \text{ cm}^3$ .)

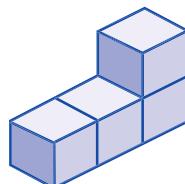
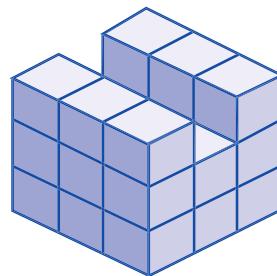
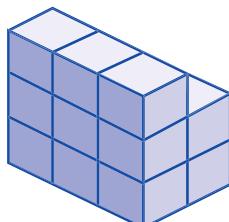
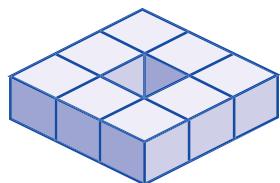
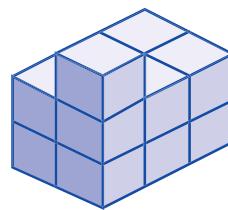
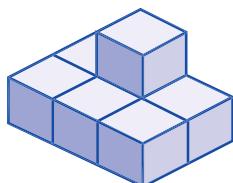
**a****b****c****d****e****f**

- 2 **MC** The volume of the prism shown at right is:

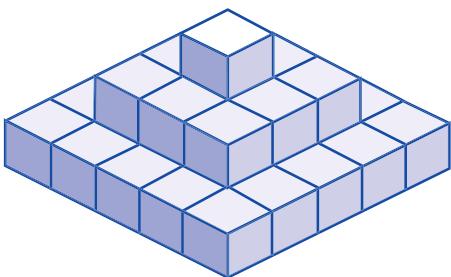
- A  $4 \text{ cm}^3$
- B  $12 \text{ cm}^3$
- C  $1 \text{ cm}^3$
- D  $8 \text{ cm}^3$
- E  $19 \text{ cm}^3$



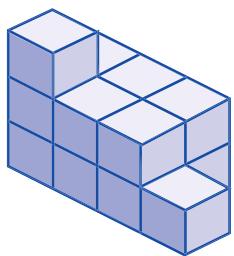
- 3 **WE14b** Find the volume of the following solids. (Each small cube represents  $1 \text{ cm}^3$ .)

**a****b****c****d****e****f**

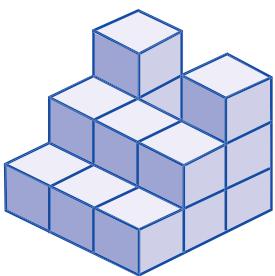
g



h



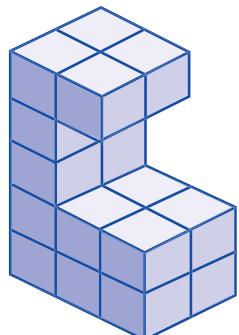
i



- 4 MC The volume of the prism on the right is:

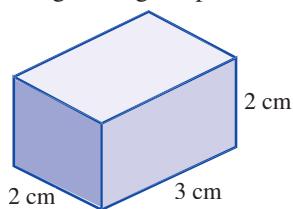
- A  $20 \text{ cm}^3$
- B  $16 \text{ cm}^3$
- C  $10 \text{ cm}^3$
- D  $22 \text{ cm}^3$
- E  $3 \text{ cm}^3$

Note: Each cube has a volume of  $1 \text{ cm}^3$ .

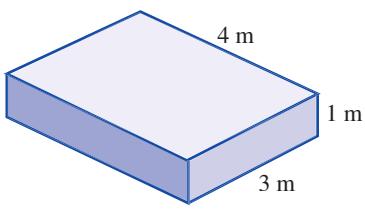


- 5 WE15 Use the formula  $V = lwh$  to calculate the volume of the following rectangular prisms.

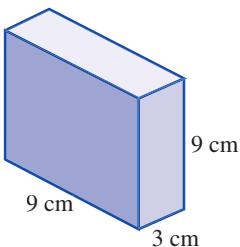
a



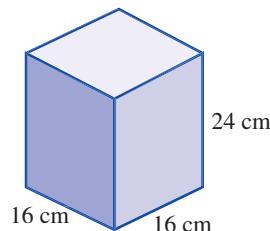
b



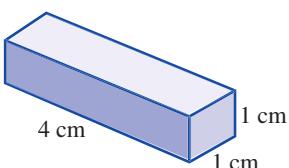
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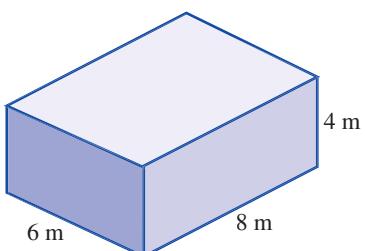
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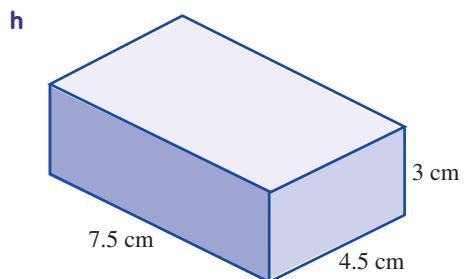
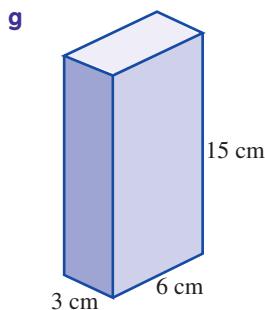


e

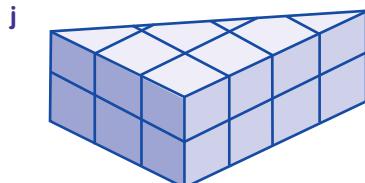
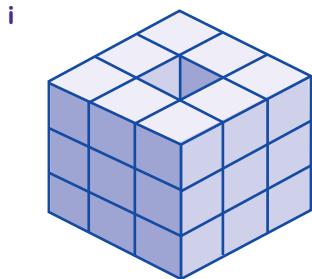
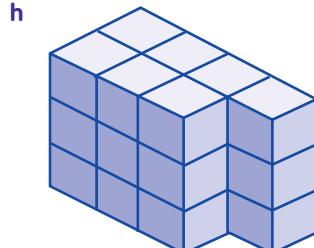
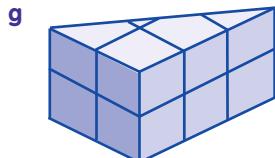
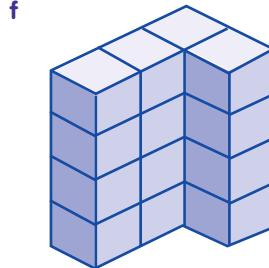
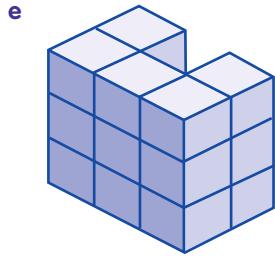
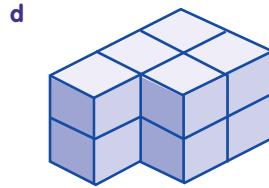
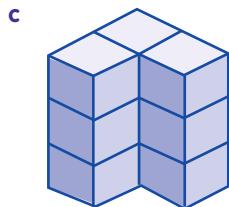
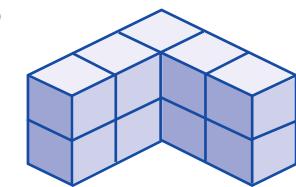
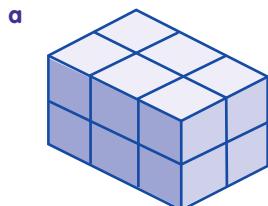


f





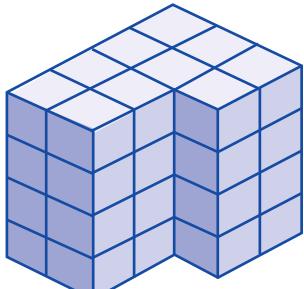
- 6 WE16** Find the volume of each of the following prisms by copying and completing the following table. (Each cube has a volume of  $1 \text{ cm}^3$ .)



Prism	Area of base ( $\text{cm}^2$ )	Height (cm)	Volume ( $\text{cm}^3$ )
a			
b			
c			
d			
e			
f			
g			
h			
i			
j			

- 7 **MC** The volume of the prism shown at right (assuming each cube has a volume of  $1 \text{ cm}^3$ ) is:

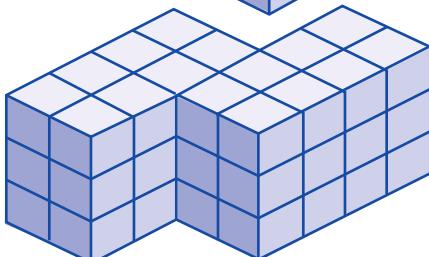
A  $10 \text{ cm}^3$   
 B  $20 \text{ cm}^3$   
 C  $30 \text{ cm}^3$   
 D  $40 \text{ cm}^3$   
 E  $80 \text{ cm}^3$



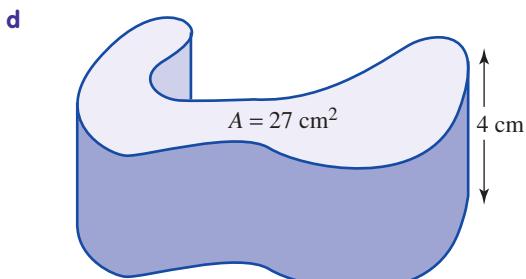
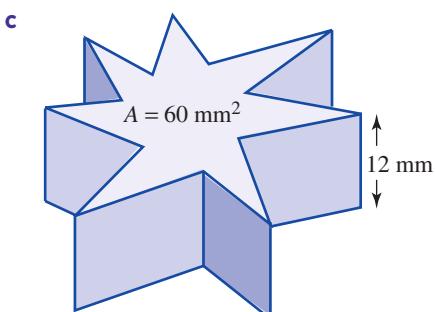
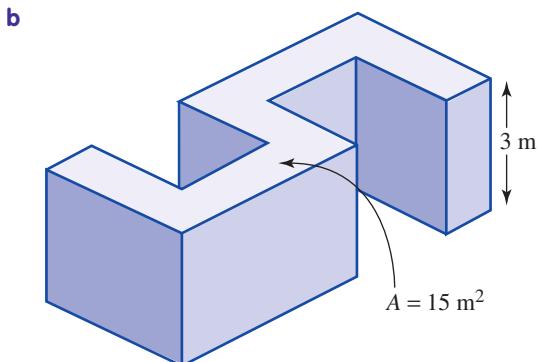
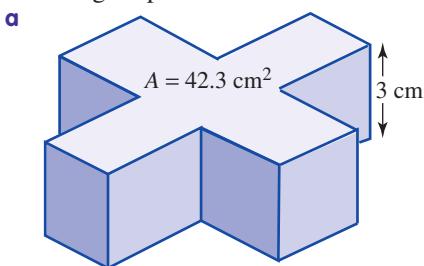
- 8 **MC** The volume of the prism shown at right is:

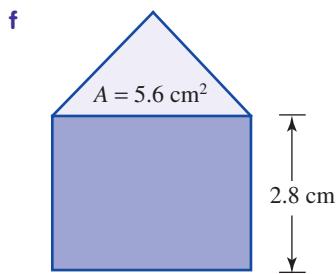
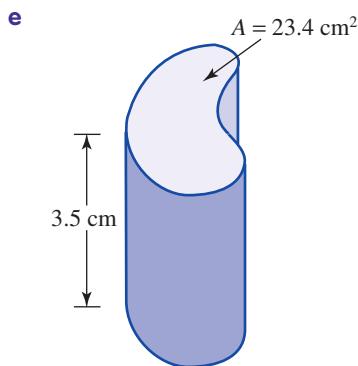
A  $16 \text{ cm}^3$   
 B  $48 \text{ cm}^3$   
 C  $32 \text{ cm}^3$   
 D  $40 \text{ cm}^3$   
 E  $160 \text{ cm}^3$

Each cube has a volume of  $1 \text{ cm}^3$ .

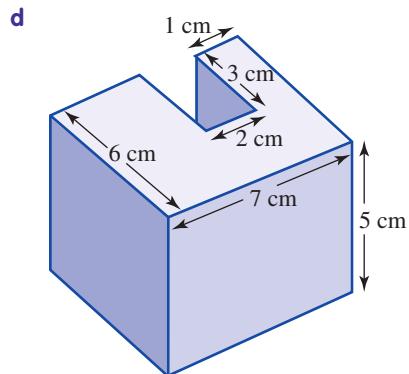
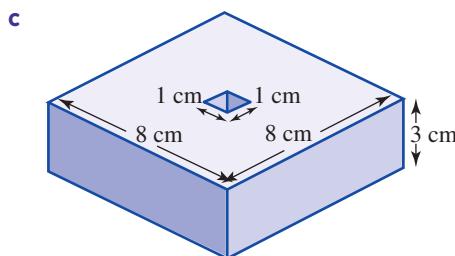
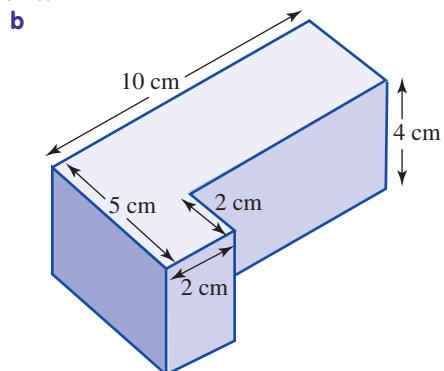
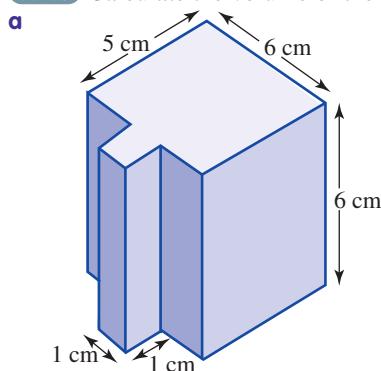


- 9 **WE17** Calculate the volume of each of the following shapes.



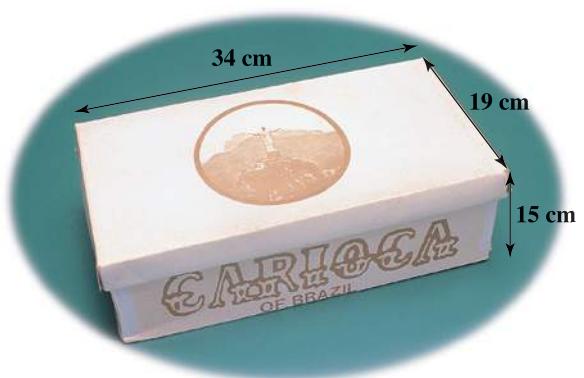


- 10 WE18** Calculate the volume of the following prisms.



### UNDERSTANDING

- 11** Find the volume of the shoe box shown right. (Give your answer in cubic centimetres,  $\text{cm}^3$ .)

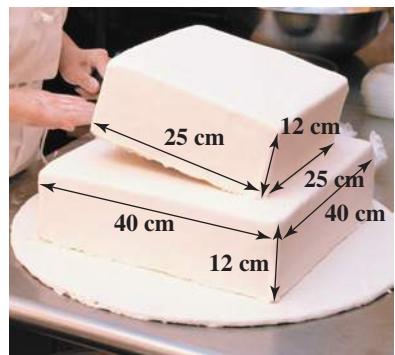




- 12** The inside dimensions of the ‘Workinghorse’ frost-free refrigerator are shown on the left. Find the volume available for food storage inside the refrigerator.
- 13** A rectangular prism has a length of 30 cm and a width of 15 cm. If its volume is  $9000 \text{ cm}^3$ , find the height of the rectangular prism.
- 14** A rectangular prism has a length of 13 cm and a width of 17 cm. If its volume is  $1989 \text{ cm}^3$ , find the height of the rectangular prism.
- 15** A rectangular prism has a length of 10 cm and a width of 15 cm. If its volume is  $1800 \text{ cm}^3$ , find the height of the rectangular prism.



- 16** The lunch box at right is a rectangular prism.
- Find the volume of the lunch box in cubic centimetres.
  - Change each measurement to millimetres and hence find the volume of the lunch box in cubic millimetres.
- 17** Find the volume of concrete (in cubic metres) which would be needed to make the base for a garage (at right) that is 6.5 m wide and 3 m long. The concrete base is 0.25 m deep.
- 18**
  - Find the volume of the matchbox that has been converted to a cockroach hotel, shown at right (in cubic centimetres).
  - Change each measurement to millimetres and hence find the volume of the matchbox in cubic millimetres.
  - Matches are rectangular prisms of length 44 mm, width 2 mm and height 2 mm. Find the volume of a match (in cubic millimetres). (Ignore the red substance on the end of each match.)
  - If a matchbox contains 50 matches, how much space is left in a full box?
- 19** How many cubic metres of water would be needed to fill a diving pool which has a length of 16 m, a width of 12 m and a depth of 4 m?
- 20** Heather wishes to cover a rectangular lawn with topsoil to a depth of 0.1 m. If the lawn is 24 m long and 17 m wide, what volume of soil (in cubic metres) should she order?
- 21** The fruitcake shown at right is to be divided equally among 100 people at a wedding reception. What volume of cake (in cubic centimetres) will each guest receive?
- 22** Find the volume of clay in a house brick which has a length of 23 cm, a width of 11 cm and a height of 9 cm.



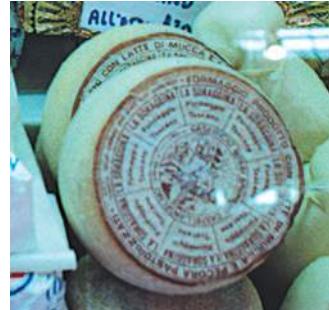
- 23** A swimming pool is rectangular and has a width exactly half its length. What volume of water would be needed to fill it if the swimming pool is 50 metres long and has a constant depth of 2 metres?



- 24** The photograph at right shows a ‘round’ of cheese that has a base of area  $320 \text{ cm}^2$  when whole. If the height of the cheese is 6 cm, find the following:
- the volume of the whole cheese round
  - the number of equal segments that could be obtained if the round is cut to produce segments of  $160 \text{ cm}^3$  in volume.

#### REFLECTION

What shapes can be classified as prisms?

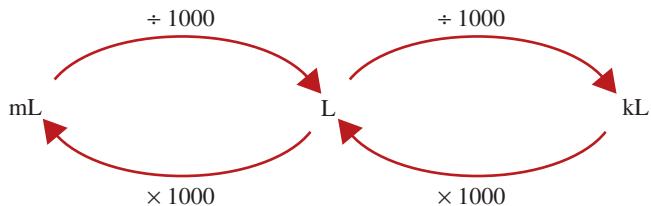


## 9G Capacity

- The **capacity** of a container is the volume of liquid that it can hold.
- Capacity can be measured in the same units as volume (e.g.  $\text{cm}^3$  or  $\text{m}^3$ ).
- Capacity is more commonly measured in millilitres (mL), litres (L) and kilolitres (kL).
- Cubic units can be converted to millilitres, litres and kilolitres as follows:

$$1 \text{ cm}^3 = 1 \text{ mL} \quad 1000 \text{ cm}^3 = 1 \text{ L} \quad 1 \text{ m}^3 = 1 \text{ kL}$$

- It may be useful to know that  $1\ 000\ 000 \text{ cm}^3 = 1 \text{ m}^3$
- The following chart can be used to convert between millilitres, litres and kilolitres.



#### WORKED EXAMPLE 19

Copy and complete the following unit conversions.

a  $6 \text{ L} = \underline{\hspace{2cm}} \text{ mL}$       b  $700 \text{ mL} = \underline{\hspace{2cm}} \text{ L}$       c  $0.45 \text{ L} = \underline{\hspace{2cm}} \text{ cm}^3$

## THINK

- a Check the conversion chart.

To convert litres to millilitres, multiply by 1000.

Since 6 is a whole number, we do this by adding three zeros to the end of the number.

- b Check the conversion chart.

To convert millilitres to litres, divide by 1000. To do this, move the decimal point three places to the left.

- c Check the conversion chart.

To convert litres to millilitres, multiply by 1000. To do this, move the decimal point three places to the right.

Note that  $1 \text{ mL} = 1 \text{ cm}^3$ .

## WRITE

$$\begin{aligned}\text{a } 6 \text{ L} &= 6 \times 1000 \text{ mL} \\ &= 6000 \text{ mL}\end{aligned}$$

$$\begin{aligned}\text{b } 700 \text{ mL} &= 700 \div 1000 \text{ L} \\ &= 0.7 \text{ L}\end{aligned}$$

$$\begin{aligned}\text{c } 0.45 \text{ L} &= 0.45 \times 1000 \text{ mL} \\ &= 450 \text{ mL} \\ &= 450 \text{ cm}^3\end{aligned}$$

## REMEMBER

1. The capacity of a container is the volume of liquid that it can hold.
2. A millilitre is equivalent to  $1 \text{ cm}^3$ .
3.  $1 \text{ litre (L)} = 1000 \text{ millilitres (mL)}$ .  $1 \text{ kilolitre (kL)} = 1000 \text{ litres (L)}$ .

## EXERCISE

## 9G

## Capacity

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 9-G-1

Capacity  
doc-1873

## Activity 9-G-2

More capacity  
doc-1874

## Activity 9-G-3

Advanced capacity  
doc-1875

## eBookplus

## Weblink

Jugs

## FLUENCY

- 1 WE19a, b Copy and complete the following unit conversions.

$$\begin{array}{ll} \text{a } 2 \text{ L} = \text{_____ mL} & \text{b } 3000 \text{ mL} = \text{_____ L} \\ \text{c } 13 \text{ kL} = \text{_____ L} & \text{d } 7000 \text{ mL} = \text{_____ L} \\ \text{e } 5500 \text{ mL} = \text{_____ L} & \text{f } 260 \text{ L} = \text{_____ kL} \\ \text{g } 2\frac{1}{2} \text{ L} = \text{_____ mL} & \text{h } 32000 \text{ mL} = \text{_____ L} \\ \text{i } 55 \text{ mL} = \text{_____ L} & \text{j } 0.035 \text{ L} = \text{_____ mL} \\ \text{k } 420 \text{ L} = \text{_____ mL} & \text{l } 0.99 \text{ kL} = \text{_____ L} \\ \text{m } 1\frac{87}{100} \text{ L} = \text{_____ mL} & \text{n } 22500 \text{ mL} = \text{_____ L} \\ \text{o } 350 \text{ mL} = \text{_____ L} & \text{p } \frac{1}{10} \text{ L} = \text{_____ mL} \\ \text{q } 25 \text{ L} = \text{_____ kL} & \text{r } 0.007 \text{ L} = \text{_____ mL} \end{array}$$

- 2 WE19c Copy and complete the following unit conversions.

$$\begin{array}{ll} \text{a } 750 \text{ cm}^3 = \text{_____ mL} & \text{b } 2500 \text{ m}^3 = \text{_____ kL} \\ \text{c } 800 \text{ cm}^3 = \text{_____ mL} = \text{_____ L} & \text{d } 40000 \text{ cm}^3 = \text{_____ mL} = \text{_____ L} \\ \text{e } 6 \text{ L} = \text{_____ mL} = \text{_____ cm}^3 & \text{f } 5200 \text{ L} = \text{_____ kL} = \text{_____ m}^3 \\ \text{g } 2.45 \text{ L} = \text{_____ cm}^3 & \text{h } 78000 \text{ cm}^3 = \text{_____ L} \end{array}$$

- 3 MC A capacity of 25 L is equal to:

A 0.025 mL      B 250 mL      C 0.25 mL      D 25 000 mL      E 2500 mL

- 4 MC A capacity of 35 400 mL is equal to:

A 35 400 000 L      B 0.354 L      C 3.5400 L      D 35.4 L      E 35 L

- 5 Arrange in order from smallest to largest:

$$\begin{array}{ll} \text{a } 2\frac{1}{2} \text{ L}, 25000 \text{ mL}, \frac{1}{4} \text{ L}, 2.45 \text{ L} & \text{b } 760 \text{ mL}, 0.765 \text{ L}, 7.65 \text{ mL}, 7.60 \text{ L} \\ \text{c } 110 \text{ mL}, 0.1 \text{ L}, 0.011 \text{ L}, 1.1 \text{ L} & \end{array}$$

eBookplus

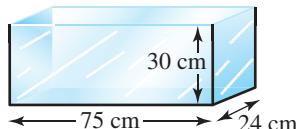
**Digital docs**  
Spreadsheet  
**Capacity unit conversions**  
doc-1879

**UNDERSTANDING**

- 6 A water bottle has a capacity of 2 litres. How many 125-mL bottlefuls are required to fill it?
- 7 A bottle contains 250 mL of orange juice concentrate. How much water should be added to make up 2 L of juice from the concentrate?
- 8 A scientist dilutes ('waters down') an acid solution by adding 120 mL of acid to 1.5 L of water. How much of the diluted solution will this make?
- 9 Most sports drinks are sold in 500-mL bottles. How many litres of sports drink are there in one dozen such bottles?
- 10 A medicine bottle contains 125 mL of cough syrup. How many 2.5-mL doses could be administered from this bottle, assuming that none is spilt?
- 11 Anthea runs a market stall selling detergent. How many 200-mL bottles of detergent could she fill from a 45-L bulk container?
- 12 A 185-mL container of 'Shine' hair conditioner is sold at the special price of \$3.70. A 0.5-L container of the same conditioner costs \$11.00. Which is the better buy?  
(Hint: Find the cost of 1 mL of hair conditioner in each case.)



- 13 A milk bar sells 55 small bottles of lemon drinks in one week. How many litres of drink is sold if each bottle contains 180 mL?
- 14 Petrov is working as a school laboratory technician. How many litres of salt solution should Petrov prepare for an experiment in which there are 12 groups of students if each group requires 400 mL of solution?
- 15 How many millilitres of milk does the container on the left hold?
- 16 Claire has made her favourite green cordial with 0.25 L of water and 30 mL of cordial. Will a glass with a capacity of 250 mL hold this volume?
- 17 Sharmila has made a fresh juice with 0.3 L of apple juice and 40 mL of carrot juice. Will a glass with a capacity of 350 mL hold this volume?
- 18 Water is to be poured into this fish tank. How many litres of water are needed to fill the tank to a depth of 28 cm?



- 19** Estimate how many litres of water each sink will hold if filled to the top. Use the measurements shown. (*Hint:* First convert the measurements to centimetres.)



- 20** To achieve a world record, a 16.4 kL strawberry milkshake was made in the UK in 1996. To understand how large this is, consider the size of a rectangular prism that would have this capacity. Suggest three possible sets of measurements for this container.
- 21** One litre of paint covers an area of 20 square metres. How many millilitres of paint are required to cover an area of 6 square metres?

eBook plus

Digital docs  
WorkSHEET 9.2  
doc-1854

#### REFLECTION

How will you remember conversion between units for volume and units for capacity?

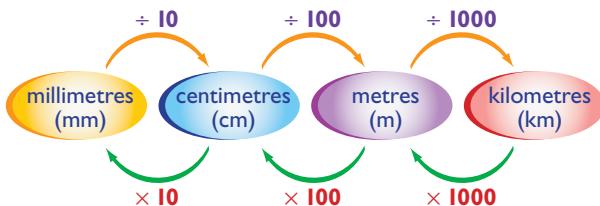
# Summary

## Units of measurement and converting units of measurement

- The most commonly used units to measure length are kilometres (km), metres (m), centimetres (cm) and millimetres (mm).
- When measuring length, it is easiest to use units that give us convenient values (not too large and not too small).

When converting units of length, it is useful to note the points below.

- Imagine a decimal point at the right-hand end of any whole numbers; for example, the number 35 can be thought of as having a decimal point after the 5 (35).
- Look at or draw a conversion chart.



- When converting to a smaller unit, multiply by the conversion factor.
- When converting to a larger unit, divide by the conversion factor.

## Reading scales and measuring length

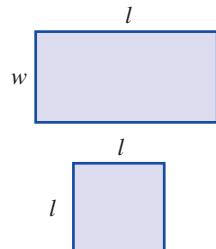
- The most commonly used units to measure length are kilometres (km), metres (m), centimetres (cm) and millimetres (mm).
- When measuring length, it is easiest to use units that give us convenient values (not too large and not too small).
- When reading scales and measuring length:
  - check that the scale starts at zero
  - check the value of each small division by counting along the scale to the next major mark
  - always give units (for example, centimetres) with your answer.

## Perimeter

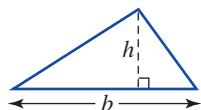
- The perimeter is the distance around the outside of a shape.
- To calculate the perimeter, change all lengths to the same unit before adding them.
- The perimeter of a rectangle is given by the rule  $P = 2(l + w)$ .
- The perimeter of a square is given by the rule  $P = 4l$ .

## Area

- The area of a shape is the amount of flat surface enclosed by the shape.
- Area is measured in square units such as  $\text{mm}^2$ ,  $\text{cm}^2$ ,  $\text{m}^2$  and  $\text{km}^2$ .
- Large areas of land are often measured in hectares.  $1 \text{ ha} = 10\,000 \text{ m}^2$ .
- Area may be found by counting the number of squares needed to cover a surface.
- A square with a side measure of 1 cm has an area of 1 square centimetre ( $1 \text{ cm}^2$ ).
- The area of a rectangle is found by using the formula  $A = lw$ , where  $A$  is the area of the rectangle,  $l$  is its length and  $w$  is its width.
- A square is a rectangle whose length and width are equal. The area of a square is found by using the formula  $A = l^2$ , where  $l$  is its side length.



- The area of a triangle is equal to half that of a rectangle with the same base and height.
- The area of a triangle may be found using the formula  $A = \frac{1}{2}bh$ , where  $b$  is the base and  $h$  is the height of the triangle.
- The area of a parallelogram is:  $A = bh$  where  $b$  is the length of the base and the  $h$  is the perpendicular height of the parallelogram.



### Area of composite shapes, using addition and subtraction

- Many composite shapes can be divided into two or more ‘pieces’ which have a simple area formula.
- The area of a composite shape can be found by adding together the areas of each part the shape is divided into.
- Many problems such as finding a shaded area can be solved by subtracting one area from another.

### Volume

- The volume of a 3-dimensional object is the amount of space it occupies.
- Cubic centimetres and cubic metres are units that are commonly used to measure volume.
- Volume ( $V$ ) of a rectangular prism:  $V = lwh$ , where  $l$  is the length,  $w$  is the width,  $h$  is the height and  $A$  is the area of the base of the prism.
- Volume of any prism ( $V$ ) = Area of base ( $A$ )  $\times$  height ( $h$ ) or  $V = Ah$ .

### Capacity

- The capacity of a container is the volume of liquid that it can hold.
- A millilitre is equivalent to  $1 \text{ cm}^3$ .
- 1 litre (L) = 1000 millilitres (mL). 1 kilolitre (kL) = 1000 litres (L).

### MAPPING YOUR UNDERSTANDING

Using terms from the summary, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter’s opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 Which metric unit would be most suitable for measuring:
- the distance walked by a teacher at school during a week?
  - the length of a piece of spaghetti?
  - the width of a pencil?

- 2 Copy and complete the following conversions.

- $560 \text{ mm} = \underline{\hspace{2cm}}$  m
- $2300 \text{ cm} = \underline{\hspace{2cm}}$  km
- $17 \text{ m} = \underline{\hspace{2cm}}$  cm
- $\frac{3}{4} \text{ km} = \underline{\hspace{2cm}}$  mm
- $2.09 \text{ m} = \underline{\hspace{2cm}}$  mm
- $6\frac{4}{5} \text{ cm} = \underline{\hspace{2cm}}$  m
- $22.5 \text{ mm} = \underline{\hspace{2cm}}$  cm
- $\frac{63}{1000} \text{ km} = \underline{\hspace{2cm}}$  m
- $82 000 000 \text{ m} = \underline{\hspace{2cm}}$  km
- $5\frac{9}{10} \text{ mm} = \underline{\hspace{2cm}}$  cm

- 3 Arrange from smallest to largest: 44.5 m, 455 cm, 455 000 mm, 0.004 45 km.

- 4 a Add 45.6 km to 5600 m.  
 b Find the difference between 80 m and 4300 cm.
- 5 What is the reading in kilograms on these bathroom scales? Note: The scale for kilograms is the inner circle

a



b



c



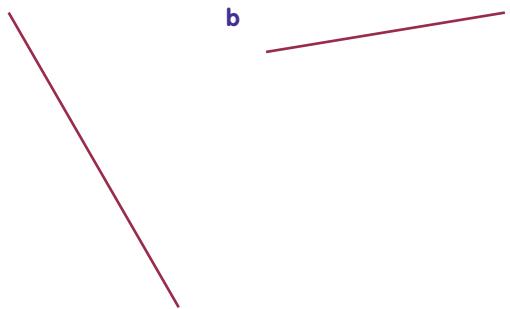
d



- 6 Measure each of the following line segments in centimetres, giving the answers in:

- decimal form
- fractional form.

a



b

c



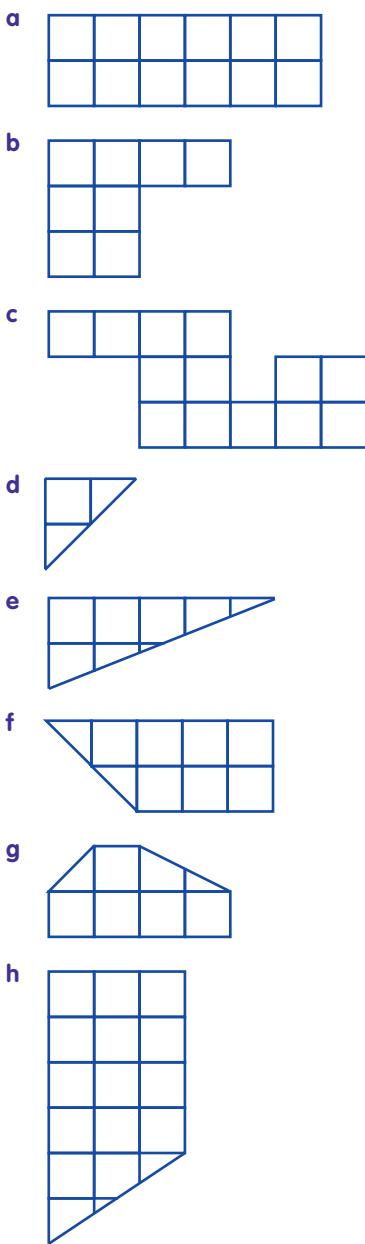
- 7 Estimate the height of the tree below, given that the person in the photograph at right is 1.7 m tall.



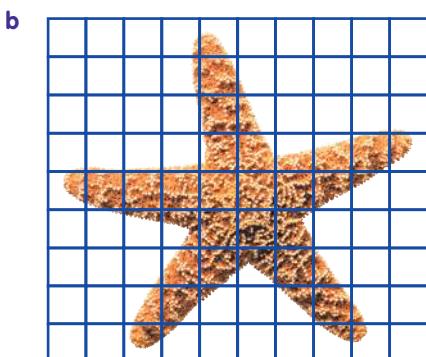
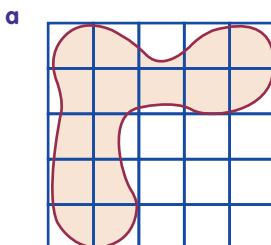
8 Which unit of area would be most appropriate for measuring the following? (Choose from  $\text{mm}^2$ ,  $\text{cm}^2$ ,  $\text{m}^2$ , ha or  $\text{km}^2$ .)

- a The floor area of your classroom
- b The area of the state of Victoria
- c A pin head
- d A market garden
- e A floor tile
- f A bench top
- g The floor area in a house
- h The area of the continent of Antarctica

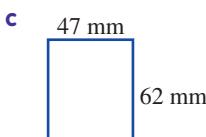
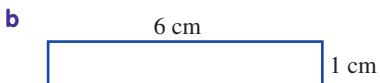
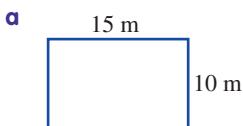
9 Find the area of the following figures, which are drawn on centimetre grid paper.



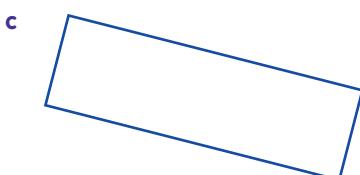
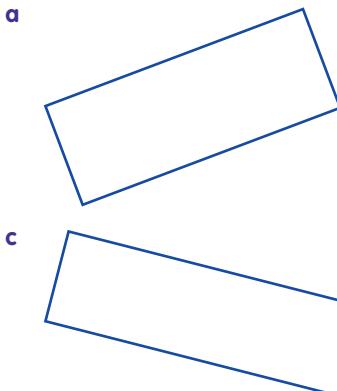
10 Estimate the shaded area in each of the figures below if each is drawn on centimetre grid paper.  
Note: Each square on the grid has an area of  $1 \text{ cm}^2$ .



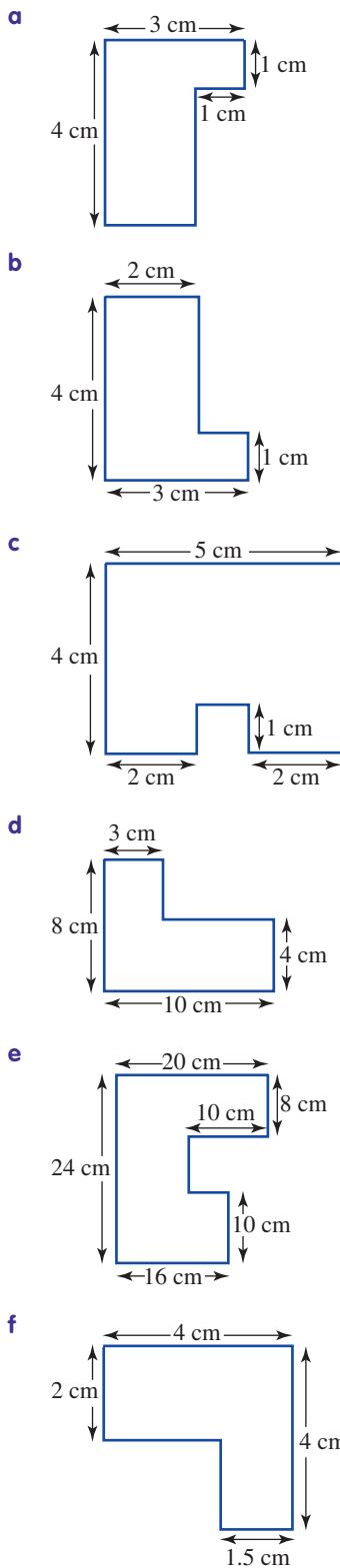
11 Use the formula  $A = lw$  to find the area of the following rectangles.



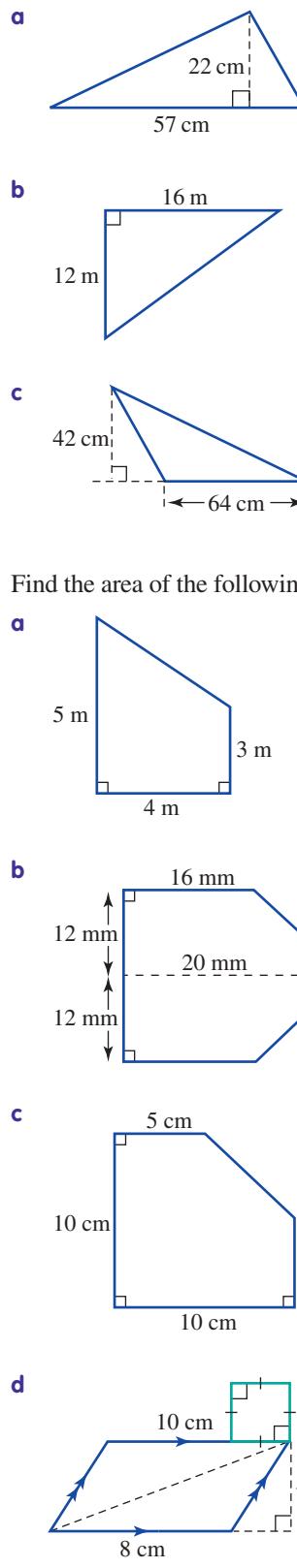
12 Measure the lengths of the sides in centimetres and hence calculate the area of each rectangle.



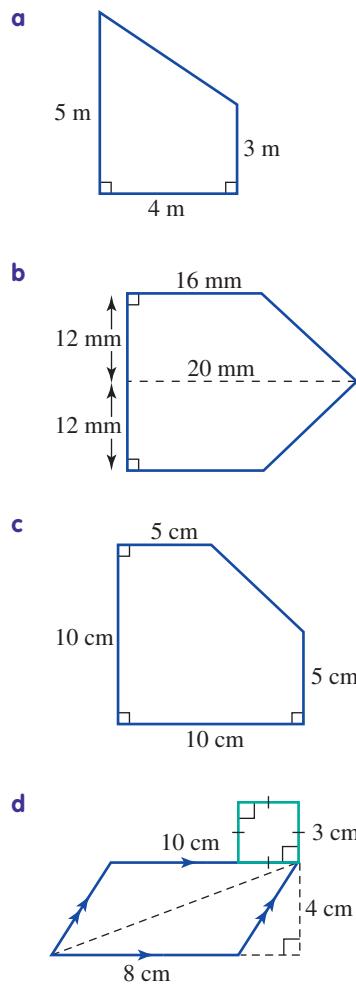
- 13** Find the area of the following shapes by first dividing them into rectangles.



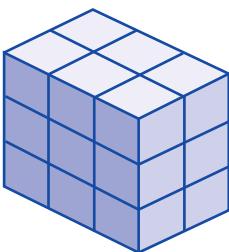
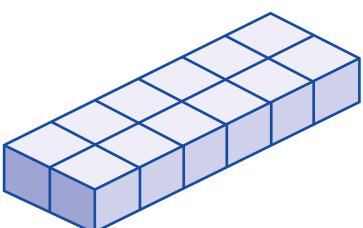
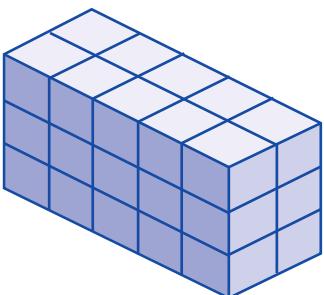
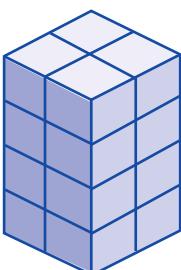
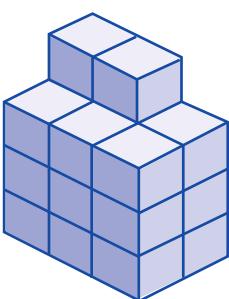
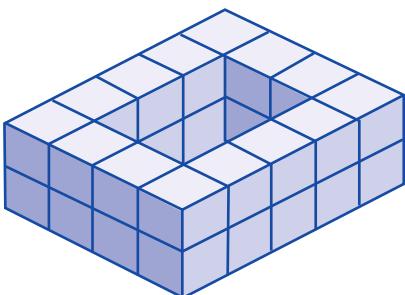
- 14** Find the area of the following triangles.



- 15** Find the area of the following shapes.



- 16** How many cubic centimetres ( $\text{cm}^3$ ) are in each shape below? Each cube has a volume of  $1 \text{ cm}^3$ .

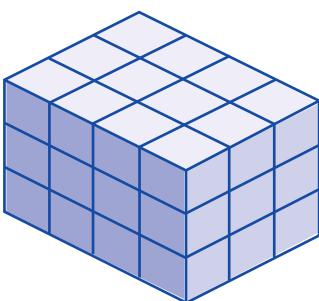
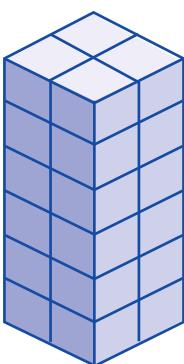
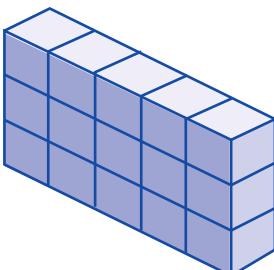
**a****b****c****d****e****f**

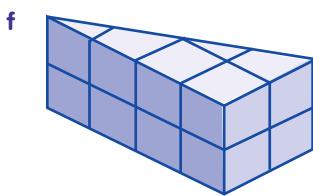
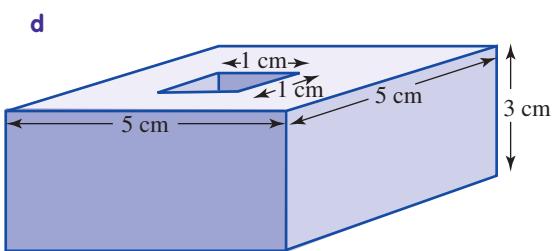
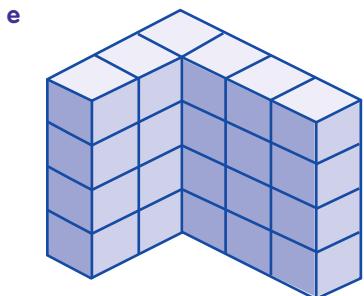
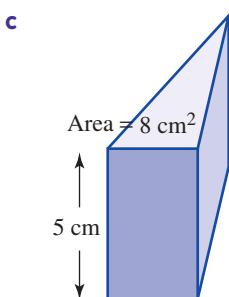
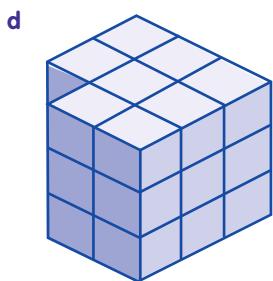
- 17** Use the rule:

'Volume of a prism = area of base  $\times$  height'

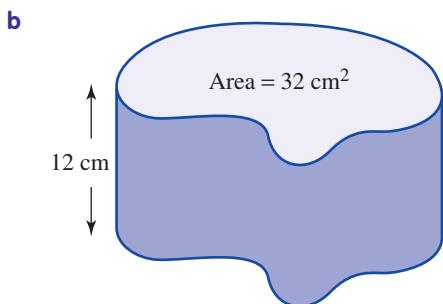
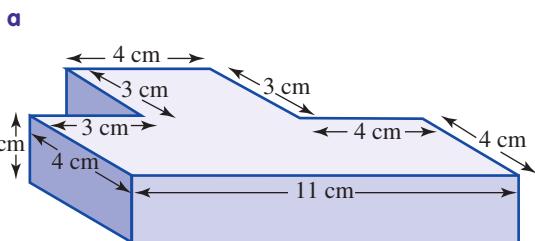
to find the volume of the prisms shown below.

Each cube has a volume of  $1 \text{ cm}^3$ .

**a****b****c**



- 18 Use the information given in the diagrams below to find the volume of each of the following.



- 19 Copy and complete the following conversions.

a  $8 \text{ L} = \underline{\hspace{2cm}}$  mL

b  $\frac{21}{50} \text{ L} = \underline{\hspace{2cm}}$  mL

c  $3300 \text{ mL} = \underline{\hspace{2cm}}$  L

d  $1\frac{3}{250} \text{ L} = \underline{\hspace{2cm}}$  mL

e  $4\frac{3}{10} \text{ kL} = \underline{\hspace{2cm}}$  L

f  $0.0034 \text{ kL} = \underline{\hspace{2cm}}$  mL

g  $4755 \text{ L} = \underline{\hspace{2cm}}$  kL

h  $432 \text{ mL} = \underline{\hspace{2cm}}$  L.

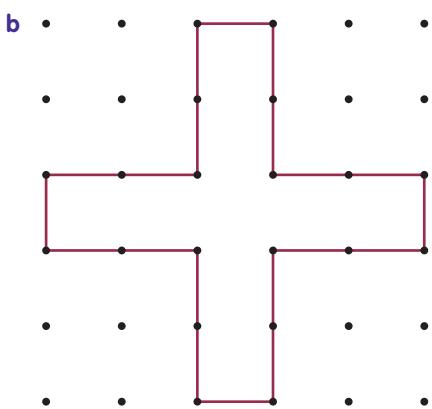
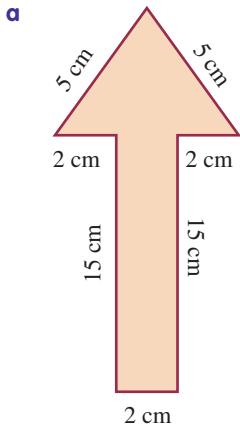
### PROBLEM SOLVING

- 1 During a rescue operation in calm seas, a 16.5 m rope is dangled from a helicopter hovering 20 m above sea level. A 175 cm tall man standing on the deck of a boat reaches 50 cm above his head for the rope. By how much does he fail to reach the rope if the deck is 1 m above sea level?

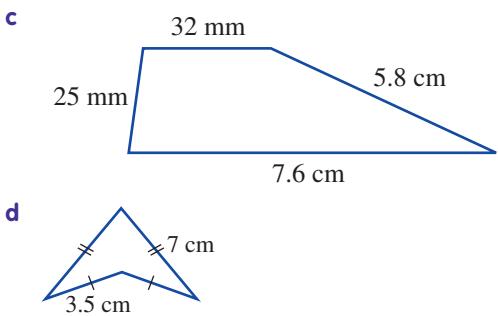
- 2 The Amazing Ape-Impersonating Gymnastic Troupe consists of two people who are 150 cm tall, and three people who are 1.8 m tall to their shoulders. When these people form a tower five people high by standing on each other's shoulders, how high is the tower? Note: The head of the top person is 30 cm high.



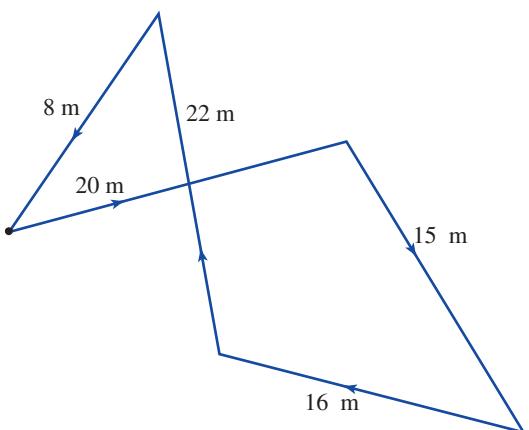
- 3 Find the perimeter of each shape below.



Note: The dots at left are 1 cm apart.

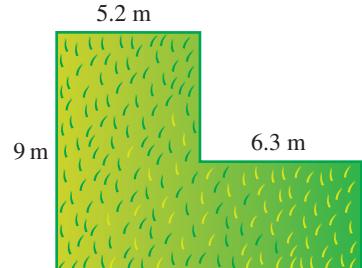


- 4 Michelle rides three laps of the dirt-bike track shown below. How far does she ride altogether?



- 5 A woven rectangular rug is 160 cm wide and 230 cm long. Find the perimeter of the rug.

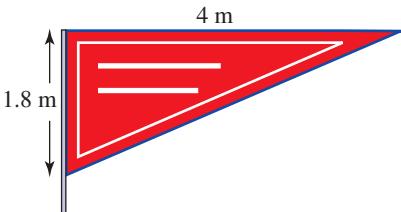
- 6 a What length of concrete edging will be required to form an edge around the lawn area shown below?  
 b What will be the cost of the job if edging costs \$8.45 per metre?



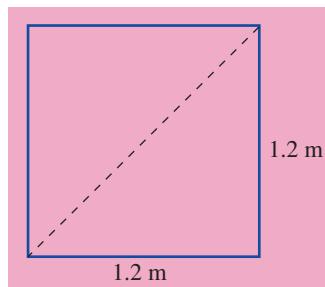
- 7 What length of satin ribbon is required to edge a rectangular blanket on all four sides if the blanket is 240 cm long and 195 cm wide? (Assume there is no overlap.)
- 8 Flavio runs five times around the baseball field shown below. What distance has he run in total?



- 9 On a sheet of graph paper, draw five different figures each having an area of  $12 \text{ cm}^2$ .
- 10 Find the area of a rectangular newspaper sheet which is 41 cm long and 30 cm wide.
- 11 Find the cost of paving a rectangular courtyard which is 6.5 m long and 3.2 m wide. The courtyard is to be paved with concrete paving blocks, which cost \$28 per square metre.
- 12 Find the area of card needed to make the triangular display sign shown below.



- 13 When making a box kite, Katie cuts a square piece of material diagonally, as shown by the dotted line. What is the area of each triangular piece of material?



- 14 The tip truck below is filled with sand, level with the top of the tray. How many cubic metres of sand does the truck hold?



- 15 Sandro wishes to cover a rectangular lawn with topsoil to a depth of 0.2 m. If the lawn is 30 m long and 24 m wide, what volume of soil (in cubic metres) should he order?
- 16 A rectangular prism has a length of 40 cm and a width of 26 cm. If its volume is  $29\,120 \text{ cm}^3$ , find the height of the rectangular prism.
- 17 A rectangular prism has a length of 30 cm and a width of 15 cm. If its volume is  $3825 \text{ cm}^3$ , find the height of the rectangular prism.

- 18** If the base area of a compact disc case is  $168 \text{ cm}^2$  and its height is 1 cm, find the volume of a stack of 95 compact disc cases.



- 19** In a food technology class, each of 14 groups of students uses 350 mL of fresh milk to make pancakes. How many litres of milk should be ordered to provide for the whole class?



- 20** Mario makes up raspberry cordial by pouring 275 mL of concentrate into a 2-litre container and filling the container with cold water.
- How much cold water does Mario add?
  - How many 250-mL glasses of cordial will Mario be able to pour from the large container?

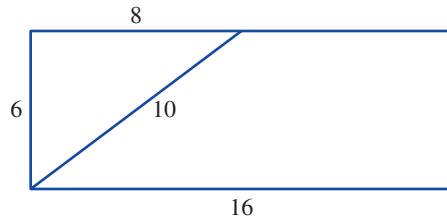
- 21** Estimate the number of litres of water that a goldfish bowl, similar to the one shown at right, would hold.



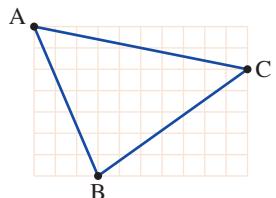
- 22** Farmer Brown uses his roof to collect rain water which he stores in a covered rainwater tank. His roof covers a horizontal area of  $280 \text{ m}^2$  and the tank has a constant cross-sectional

area of  $60 \text{ m}^2$ . If 11 mm of rain falls overnight, by how much will the depth of water in the tank increase?

- 23** A  $6 \times 16$  rectangle is cut into two pieces as indicated. The two pieces are rearranged to form a right triangle. What is the perimeter of the resulting triangle?



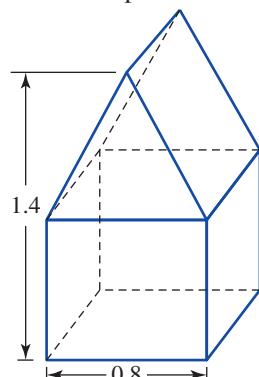
- 24** Each small square in the diagram at right measures  $1 \text{ cm} \times 1 \text{ cm}$ . What is the area of triangle ABC (in square cm)?



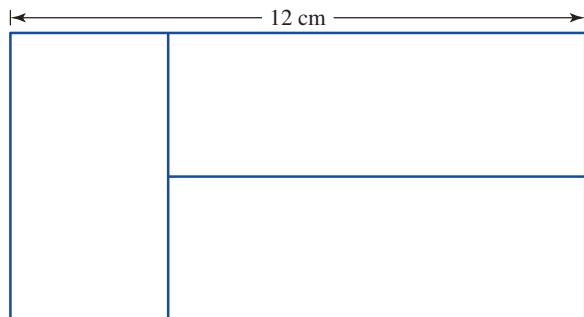
- 25** Bert the beetle is crawling around the perimeter of a regular pentagonal tile of side length 10 cm. He has a rest when he is  $\frac{5}{8}$  of the way around. How much further does he need to travel (to the nearest millimetre) to circumnavigate the tile?

- 26** Stewart is having a swimming pool built in his back yard. The hole for the pool is 6 metres long, 4 metres wide and 1.5 metres deep. The excavated soil becomes aerated as it is dug out, and its volume is increased by  $\frac{1}{10}$ . Determine the volume of the excavated soil.

- 27** A dog kennel is designed in the shape of a cube with a triangular prism as the roof (as shown below). The width of the kennel is 0.8 metres and the overall height is 1.4 metres. The ceiling is removed to allow more room inside. Determine the volume of space inside the dog kennel.



- 28** A rectangular cake is three times as long as it is wide. A layer of icing that is 1 cm thick is spread on top of the cake. If the total volume of icing used is  $300 \text{ cm}^3$ , determine the dimensions of the top of the cake.
- 29** A gardener is employed to pave a 2-meter-wide path around a  $10 \text{ m} \times 5.5 \text{ m}$  rectangular swimming pool. Find the area of the paved path. What is the minimum number of bricks that should be ordered if there are 60 bricks per square metre?
- 30** The perimeter of a rectangle is 20 cm. Investigate the shape of all such rectangles if the sides are whole numbers. Find the dimensions of the rectangle with the largest area.
- 31** A display tank at the entrance to Water World is 2.5 m long, 1.4 m wide and 1.3 m high, and has water to a depth of 1 m. A solid cubic decoration with an edge of 70 cm is placed at the bottom of the tank. Explain what will happen to the level of the water in the tank. What is the largest sized cube that can be placed in the tank without the water overflowing?
- 32** The Reserve Bank of Australia is responsible for printing and issuing Australia's currency notes. The money notes in circulation at the moment (\$5, \$10, \$20, \$50 and \$100) are all the same width (65 mm), but increase in length as their value increases. The lengths of the five notes (not in order) are: 15.1 cm, 0.158 m, 130 mm, 14.4 cm and 0.137 m. Match each of the notes with its correct length.
- 33** Lake Eyre is in South Australia, but its basin extends into the Northern Territory and Queensland, and covers one-sixth of our continent. The base of the lake itself varies from 15 m below sea level to 10 m above sea level. The volume of the lake varies, but an average estimate is said to be about 30 million mL. If an Olympic swimming pool holds 2500 kL, how many of these pools could an 'average' Lake Eyre fill?
- 34** This rectangle is divided into three identical smaller rectangles. The length of the large rectangle is 12 cm.



Determine the width of the large rectangle.

**eBook plus**
**Interactivities**

Test yourself

Chapter 9

int-1821

Word search

int-2599

Crossword

int-2600

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 9 (doc-6504) (*page 287*)

**Are you ready?****Digital docs (pages 288–9)**

- SkillSHEET 9.1 (doc-6505) Metric units of length
- SkillSHEET 9.2 (doc-6506) Reading scales (how much is each interval worth?)
- SkillSHEET 9.3 (doc-6507) Measuring the length of a line
- SkillSHEET 9.4 (doc-6508) Relationship between unit size and the number of units used
- SkillSHEET 9.5 (doc-6509) Converting units (metres and kilometres)
- SkillSHEET 9.6 (doc-6510) Converting units to compare lengths and distances
- SkillSHEET 9.7 (doc-6511) Perimeter
- SkillSHEET 9.8 (doc-6512) Area units
- SkillSHEET 9.9 (doc-6513) Area of figures drawn on one-centimetre grid paper
- SkillSHEET 9.10 (doc-6514) Area of rectangles
- SkillSHEET 9.11 (doc-6515) Area of triangles
- SkillSHEET 9.12 (doc-6516) Volume units
- SkillSHEET 9.13 (doc-6517) Volume of a solid constructed from cubic-centimetre blocks

**9A Units of measurement and converting units of measurement****Digital docs (page 292)**

- Activity 9-A-1 (doc-1855) Units of measurement
- Activity 9-A-2 (doc-1856) More units of measurement
- Activity 9-A-3 (doc-1857) Advanced units of measurement

**9B Reading scales and measuring length****Digital docs (page 297)**

- Activity 9-B-1 (doc-1858) Reading scales
- Activity 9-B-2 (doc-1859) More scales
- Activity 9-B-3 (doc-1860) Advanced scales

**Weblink (page 298)**

- Measure it!

**9C Perimeter****Digital docs**

- Activity 9-C-1 (doc-1861) Perimeter (*page 304*)
- Activity 9-C-2 (doc-1862) More perimeter (*page 304*)
- Activity 9-C-3 (doc-1863) Advanced perimeter (*page 304*)
- WorkSHEET 9.1 (doc-1853) (*page 309*)

**Weblink (page 305)**

- Perimeter and area

**9D Area****Digital docs**

- Activity 9-D-1 (doc-1864) Area (*page 315*)
- Activity 9-D-2 (doc-1865) More area (*page 315*)
- Activity 9-D-3 (doc-1866) Advanced area (*page 315*)
- Spreadsheet Area of a rectangle (doc-1876) (*page 317*)
- Spreadsheet Area of a triangle (doc-1877) (*page 319*)

**Interactivities**

- Area of a triangle (int-2350) (*page 313*)
- Area plus worksheet (int-0005) (*page 316*)

**9E Area of composite shapes, using addition and subtraction****Digital docs (pages 324–5)**

- Activity 9-E-1 (doc-1867) Composite shapes
- Activity 9-E-2 (doc-1868) More composite shapes
- Activity 9-E-3 (doc-1869) Advanced composite shapes
- Spreadsheet Area (doc-1878)

**9F Volume****Digital docs**

- Activity 9-F-1 (doc-1870) Volume (*page 330*)
- Activity 9-F-2 (doc-1871) More volume (*page 330*)
- Activity 9-F-3 (doc-1872) Advanced volume (*page 330*)

**9G Capacity****Digital docs**

- Activity 9-G-1 (doc-1873) Capacity (*page 337*)
- Activity 9-G-2 (doc-1874) More capacity (*page 337*)
- Activity 9-G-3 (doc-1875) Advanced capacity (*page 337*)
- Spreadsheet Capacity unit conversions (doc-1879) (*page 338*)
- WorkSHEET 9.2 (doc-1854) (*page 339*)

**Weblink**

- Jugs (*page 337*)

**Chapter review****Interactivities (page 350)**

- Test yourself Chapter 9 (int-1821) Take the end-of-chapter test to test your progress.
- Word search (int-2599)
- Crossword (int-2600)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# ICT ACTIVITY

## projects plus

### Sustainable by design

SEARCHLIGHT ID: PRO-0094

#### Scenario

As part of the EnviroWorks Design architect team, you have been asked to design a house for the McMahon family. The house is to accommodate Mr McMahon, Mrs McMahon and their two children.

As part of their brief to the architect team, the family has stated that the house must meet the following requirements:

- Have at least three bedrooms
- Have a floor plan area of exactly 250 m<sup>2</sup> (square metres)
- Include a minimum of three environmentally sustainable features.

The McMahons have allocated \$25 000 to equip their house with environmentally sustainable features. They are interested to see what options are available to them to help make their house more environmentally friendly.

Some of the options they are already aware of are:

- solar hot water system
- water tanks to collect rain water and connected to laundry or toilets

- solar energy
- insulation
- double glazed windows
- recycled materials/furnishing
- aspect (e.g. north/south orientation)

They are also open to other suggestions that your team are able to find, and would like to use the entire \$25 000 if possible.

The family has also requested that the team research and price the cost of furnishing the house (not including permanent fixtures such as ovens, bathtubs, taps etc.). They have allowed a budget of \$30 000 for furnishings.

The McMahons are very excited about building their dream home and are ready to start construction straight away. They can't wait to see the fantastic ideas your team comes up with for their new home.

#### Task

You will produce a presentation for the McMahon family showing the floor plan you have designed, the environmentally friendly features included and the furnishings you have selected for their house. Your presentation should explain why you have made the decisions you have in each of the above categories and why you think they are the best choices for the McMahon family. A budget estimate must also be submitted for reference.



## Process

- Open the ProjectsPLUS application for this chapter in your eBookPLUS. Watch the introductory video lesson, click the ‘Start Project’ button and then set up your project group. You can complete this project individually or invite other members of your class to form a group. Save your settings and the project will be launched.
- Navigate to your **Research forum**. Here you will find a series of topics that will help you to complete the task. Select the sustainable features you wish to research and delete the additional topics. Alternatively, you may add new topics if your team finds other environmentally friendly initiatives they wish to include.
- Research. Make notes of important facts, and ideas that you discover during your research. Enter your findings as articles under your topics in the **Research forum**.

You should each find at least three sources of information (including offline resources, such as books, newspapers etc.). You can view and comment on other group members' articles and rate the information they have entered. When your research is complete, print your Research Report to hand in to your teacher.

### SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft Word
- Microsoft PowerPoint, Keynote or other presentation software
- Microsoft Excel
- Google SketchUp

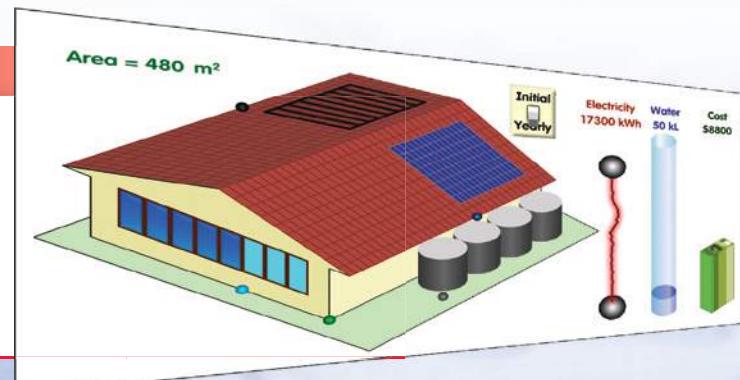
- Visit the **Media centre** and download the PowerPoint sample, the Floor plan template, the Budget template and the PowerPoint planning template to help you prepare your presentation. Your **Media centre** also includes images that can help to liven up your presentations.
- Use the Floor plan template to start designing your house layout by printing the template and completing your design by hand, or by using autoshapes in Microsoft Word to complete your design. Make sure you remember the requirements that the McMahon family have requested in their house design. Once you have settled on a design you may want to use Google SketchUp or Floor planner at [www.floorplan.com](http://www.floorplan.com) to present your floor plan in more detail.
- Use the PowerPoint template to develop your presentation. Remember that you are trying to convince the McMahon family that your design and proposal will result in their building the house of their dreams. Make sure you include all the information they requested, and that your presentation will grab their attention.

## Interactivity

### A SUSTAINABLE HOUSE

SEARCHLIGHT ID: int-2356

There are many factors that influence the planning and construction of a house. Use the interactivity *A sustainable house* to investigate the effect of changing some of these factors while continuing to monitor cost.





# 10

# Probability



- 10A The language of chance
- 10B The sample space
- 10C Simple probability
- 10D Using a table to show sample spaces
- 10E Experimenting with chance

## WHAT DO YOU KNOW?

- 1 List what you know about probability.  
Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of probability.

eBook plus

Digital doc

Hungry brain activity  
Chapter 10  
doc-6521

## OPENING QUESTION

If Cameron shuts his eyes and selects a token, which colour is he most likely to end up with?

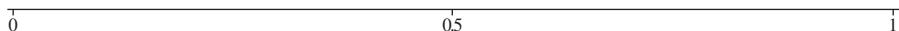
# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.

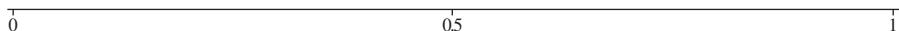

**Digital docs**

SkillsHEET 10.1

doc-6522


**Understanding 'chance' words**

- 1 a On a number line, order the following commonly used 'chance words' from zero (impossible) to one (certain): even chance, unlikely, likely, impossible, certain.



- b Give an example of an event which describes each.


**Digital docs**

SkillsHEET 10.2

doc-6523

**Comparing the likelihood of different events occurring**

- 2 Compare the given events i, ii and iii and order them from least likely to most likely.
- The captain of the Australian cricket team will win the coin toss at the start of the match.
  - There will be snow falling on Christmas day in Darwin.
  - You will have at least one mathematics test this term.


**Digital docs**

SkillsHEET 10.3

doc-6524

**Describing the chance of an event occurring as more or less than half**

- 3 Describe the chance of each of the following events occurring as being more or less than a half.
- It will be sunny in Melbourne for most of the summer months.
  - You will win a prize in your school raffle after buying one ticket.
  - You will use a textbook for one of your classes today.


**Digital docs**

SkillsHEET 10.4

doc-6525

**Listing all possible outcomes for an event**

- 4 List all the possible outcomes for each of the following.
- Tossing a coin
  - Rolling a six-sided die
  - Selecting a day of the week


**Digital docs**

SkillsHEET 10.5

doc-6526

**Understanding a standard deck of playing cards**

- 5 Consider the cards in a standard deck of playing cards.
- How many cards are there in the deck?
  - Name the four suits in the deck.
  - How many aces are there in the deck?


**Digital docs**

SkillsHEET 10.6

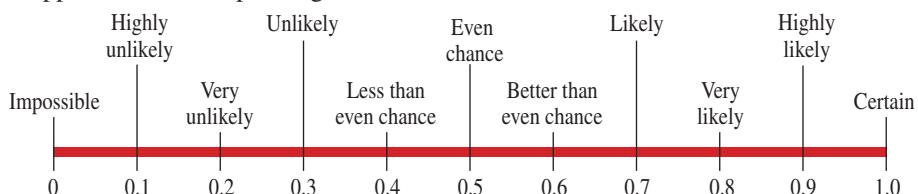
doc-6527

**Writing a simple probability as a fraction**

- 6 a If there are three red pegs in a bag containing five pegs, write a fraction which describes the chance of picking a red peg from the bag.
- b For a six-sided die, write a fraction which describes the chance of rolling a five.

# 10A The language of chance

- An **event** is a result or outcome that may occur.
- When rating the chance of a certain event occurring, we use words like *certain*, *likely*, *even chance*, *unlikely* and *impossible*.
- The **probability** of an event occurring is given a value between 0 and 1 inclusive.
- If an event is **certain**, like an Australian Cricket team being named this summer, the probability of it occurring is 1.
- If an event is **impossible**, such as Christmas Day falling in June this year, the probability of it occurring is 0.
- If an event has an **even chance** of occurring, such as the probability of tossing a head with a fair coin, the probability of it occurring is 0.5. Probabilities can be written as fractions and percentages. For example,  $0.5 = \frac{1}{2} = 50\%$ .
- The probability scale, below, associates important words used in describing probability with their approximate corresponding numerical values.



## WORKED EXAMPLE 1

For each of the given statements, specify whether the chance of the following events occurring is **certain**, **likely**, **even chance**, **unlikely** or **impossible**.

- You will compete in the next Olympics.
- Every student in Year 7 will obtain 100% in their next mathematics test.
- Each person in your class has been to the zoo.
- You flip a coin and Tails comes up.
- March is followed by April.

### THINK

- 1 Read the given statement and associate the chance of the event occurring with one of the given words from the list. Provide reasons.

- 2 Answer the question.

- Repeat steps 1 and 2 of part a.

- Repeat steps 1 and 2 of part a.

- Repeat steps 1 and 2 of part a.

- Repeat steps 1 and 2 of part a.

### WRITE

- The chance of a person competing in the next Olympics is very small; however, it could happen.

*It is unlikely that this event will occur.*

- Due to each student having different capabilities and the number of students involved, this situation could never happen. It is *impossible* for this event to occur.
- The chance that each student in your class has been to the zoo, either with their family or primary school, is very probable. However, there may be a few students who missed out.

*It is likely this event will occur.*

- When you flip a coin there are only two possibilities, a Head or a Tail. So there is a 50% chance of Tails coming up.

*There is an even chance this event will occur.*

- This is a true statement. April always follows the month of March.

*It is certain this event will occur.*

**WORKED EXAMPLE 2**

Assign a number between and including 0 and 1 to represent the estimated probability of the following events, depending on how likely they are.

- One of two evenly matched tennis players will win the next game.
- You will guess the correct answer on a multiple choice question with four options.
- Rolling a fair die and obtaining a number less than 6.

**THINK**

- a 1 Determine the likelihood of an event occurring, with reasoning.

- 2 Express the answer as a decimal.

- b 1 Determine the likelihood of an event occurring, with reasoning.

- 2 Express the answer as a decimal.

- c 1 Determine the likelihood of an event occurring, with reasoning.

- 2 Express the answer as a decimal, correct to 2 decimal places.

**WRITE**

- a Since the two players are evenly matched, one does not have an advantage over the other. Therefore, they each have an equal chance of winning the next game.

The probability that one player wins the game is  $\frac{1}{2}$  or 0.5.

- b When guessing an answer on a multiple choice question with 4 options, 1 out of the 4 possibilities will be correct. One out of 4 may be expressed as a fraction.

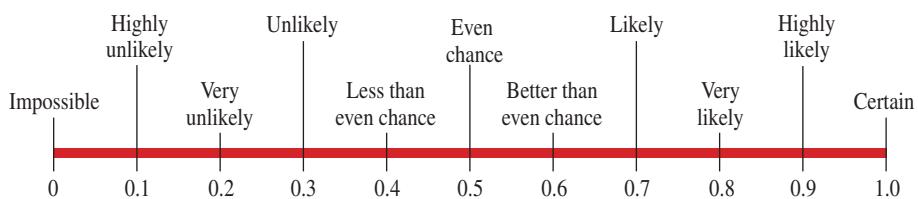
The probability of guessing the correct answer is  $\frac{1}{4}$  or 0.25.

- c When rolling a die there are six possibilities. They are 1, 2, 3, 4, 5, 6. A number less than 6 includes 1, 2, 3, 4, 5. Therefore, five out of the six possibilities may be rolled. Five out of six may be expressed as a fraction.

The probability of obtaining a number less than six is  $\frac{5}{6}$  or approximately 0.83.

**REMEMBER**

- An *event* is a result that may occur.
- Probability* is the chance of a particular event (result) occurring.
- The probability of an event occurring ranges from and includes 0 (impossible) to 1 (certain).
- A probability scale associates important words used in describing probabilities with their approximate corresponding numerical values.



**EXERCISE****10A****The language of chance****INDIVIDUAL PATHWAYS****eBook plus****Activity 10-A-1**The language  
of chance  
doc-6528**Activity 10-A-2**More chance  
doc-6529**Activity 10-A-3**Advanced chance  
doc-6530**FLUENCY**

- 1 WE1** For each of the given statements, specify whether the chance of the following events occurring is certain, likely, even chance, unlikely or impossible.
- New Year's Day will be on 1 January next year.
  - Australia will experience at least one earth tremor this year.
  - Water will boil in the fridge.
  - There will be at least one day with a maximum temperature under 5 °C in Cairns in January.
  - A horse will win the Melbourne Cup.
  - There will be snow at Mt Buller this winter.
  - You will grow 18 cm taller this year.
  - You will win first prize in Tattslotto.
  - You choose a blue ball from a bag which contains only white balls.
  - You roll a fair die and obtain an odd number.
  - The year 2020 will be a leap year.
  - You choose a white ball from a bag which contains only white balls.
  - You roll a fair die and obtain a number greater than 6.
  - You choose a yellow ball from a bag containing 4 red balls and 4 yellow balls.
  - You roll a fair die and obtain a number less than 5.
- 2** Compare the given events A, B, C, D and order them from least to most likely.
- It will be sunny in Queensland most of the time when you visit.
  - Melbourne Cup Day will be on the first Tuesday in November next year.
  - You will be the next Australian Idol.
  - Saturn will be populated next year.
- 3 WE2** Assign a number between and including 0 and 1 to represent the estimated probability of the following events, depending on how likely they are.
- You flip a coin and obtain a Tail.
  - You choose a red ball from a bag containing only 8 white balls.
  - You guess the correct answer in a multiple choice question with 5 options.
  - You roll a die and obtain an even number on a fair die.
  - You flip a coin and obtain a Head.
  - Your class elects one class captain from four candidates.
  - You guess the incorrect answer in a multiple-choice question with five options.
  - You choose a green ball from a bag containing only four green balls.
  - You have science classes this year.
  - Australia will win the Boxing Day cricket test.



## UNDERSTANDING

- 4 List five events that are:
- impossible
  - unlikely to happen
  - likely to happen
  - sure to happen.
- 5 **MC** The word which has the same meaning as *improbable* is the word:
- unlikely
  - impossible
  - uncertain
  - certain
  - likely
- 6 **MC** The word which has the same meaning as *certain* is the word:
- definite
  - possible
  - sure
  - probable
  - unlikely
- 7 Match the words below with one of the numbers between 0 and 1 that are given. Choose the number depending on what sort of chance the word means, between *impossible* and *certain*. You may use a number more than once. If you are unsure, discuss your choice with another class member.
- Numbers to choose from: 1, 0.75, 0.25, 0, 0.5.
- |               |                     |                    |
|---------------|---------------------|--------------------|
| a Certain     | b Likely            | c Unlikely         |
| d Probable    | e Improbable        | f Definite         |
| g Impossible  | h Slim chance       | i Sure             |
| j Doubtful    | k Not able to occur | l More than likely |
| m Fifty-fifty | n Fair chance       |                    |

## REFLECTION

Give an example of an event that is certain.

## 10B The sample space

- The **sample space** refers to the list of all possible outcomes to a probability experiment. If we think of tossing a coin, the possible outcomes are either ‘Heads’ or ‘Tails’. We write this as  $S = \{\text{Heads, Tails}\}$
- Sometimes we don’t need to list the elements in the sample space, but how many elements are in the sample space. For tossing a coin, we can say  $n(S) = 2$ .

## WORKED EXAMPLE 3

A card is drawn from a standard deck. The suit of the card is then noted.  
List the sample space for this experiment.

## THINK

Although there are 52 cards in the deck we are concerned only with the suit. List each of the four suits as the sample space for this experiment.

## WRITE

$$S = \{\text{clubs, spades, diamonds, hearts}\}$$



## WORKED EXAMPLE 4

A die is rolled and the number on the uppermost face is noted. How many elements are in the sample space?

## THINK

The die can show the numbers 1, 2, 3, 4, 5 or 6.

## WRITE

$$n(S) = 6$$

**REMEMBER**

1. The sample space represents a list of all possible outcomes to a probability experiment.
2. The sample space can be denoted by the symbol  $S$  and all outcomes are then listed within braces {}.
3.  $n(S)$  represents the number of elements to the sample space.

**EXERCISE****10B****The sample space****INDIVIDUAL PATHWAYS****eBookplus****Activity 10-B-1**Sample spaces  
doc-6531**Activity 10-B-2**More sample spaces  
doc-6532**Activity 10-B-3**Advanced sample  
spaces  
doc-6533**FLUENCY**

- 1 WE3** A spinner with 10 equal sectors labelled 1 to 10 is spun. List the sample space.
- 2** For each of the following probability experiments list the sample space.
- a A coin is tossed.
  - b A multiple-choice question has five alternative answers, A, B, C, D and E.
  - c A soccer team plays a match and the result is noted.
  - d A card is selected from the four aces in a deck.
  - e An exam paper is given the grade A to F.
- 3** A card is selected from a standard deck. List the sample space if we are interested in:
- a the suit of the card chosen
  - b the face of the card chosen
  - c the colour of the card chosen.
- 4** A bag contains 8 red marbles, 9 green marbles and 2 orange marbles. A marble is selected from the bag. List the sample space.
- 5 WE4** A coin is tossed. How many elements are in the sample space?
- 6** In each of the following, state the number of elements in the sample space.
- a A card is selected from a standard deck.
  - b The first ball drawn in the Tattslotto draw. (Balls are numbered from 1 to 45.)
  - c The winner of the AFL premiership. (There are 16 teams.)
  - d A day of the year is selected.
  - e A letter from the alphabet is selected at random.
  - f The first prize in the lottery is chosen from tickets numbered 1 to 180 000.
  - g A term is selected from a school year.
  - h You win a medal at your chosen event at the world swimming championships.

**UNDERSTANDING**

- 7 MC** From the list below select the event that has the most elements in the sample space.
- A Selecting a card from a standard deck
  - B Selecting a page at random from this book
  - C Selecting an exercise book from your school bag
  - D Selecting a student at random from your class
  - E Selecting a page at random from the phone directory

**REASONING**

- 8** In how many different ways can change be given for a 50 cent coin using only 20 cent, 10 cent and 5 cent coins?
- 9** Sabita remembered that her mother's car registration plate had 2 letters followed by 3 digits. She recalled that the letters were S and C and that the digits were 8, 3 and 7 but couldn't remember the order. What combination of letters and numbers could her mother's car registration plate have? Make a list of the possibilities.

**REFLECTION**

Are all elements in a sample space equally likely to occur?

## 1OC Simple probability

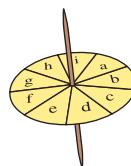
- An **outcome** is a particular result of an experiment.
- A **favourable outcome** is one that we are looking for.
- The **theoretical probability** of a particular result or event is defined as

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

**WORKED EXAMPLE 5**

State how many possible outcomes there are for each of the following experiments and specify what they are:

- a** tossing a coin
- b** spinning a circular spinner with 9 equal sectors labelled from a to i as shown at right
- c** drawing a picture card (jack, king, queen) from a standard pack of cards.

**THINK**

- a**
  - 1** Make a note of how many sides the coin has and what each side represents.
  - 2** Answer the question.
- b**
  - 1** Make a note of how many sectors the circular spinner has and what each one represents.
  - 2** Answer the question.
- c**
  - 1** Make a note of how many picture cards there are and what they are.
  - 2** Answer the question.

**WRITE**

- a** The coin has 2 sides, a Head and a Tail.  
When tossing a coin there are two possible outcomes; they are Head or Tail.
- b** The circular spinner has 9 sectors labelled a to i.  
When spinning the circular spinner, there are 9 possible outcomes; they are a, b, c, d, e, f, g, h or i.
- c** There are 3 picture cards in each of the four suits.  
When drawing a picture card there are 12 possible results; they are: the jack, king and queen of clubs, the jack, king and queen of diamonds, the jack, king and queen of hearts and the jack, king and queen of spades.

**WORKED EXAMPLE 6**

Christopher rolls a fair 6-sided die.

- What are all the possible results that could be obtained?
- What is the probability of obtaining:
  - a 4?
  - a number greater than 2?
  - an odd number?

**THINK**

- Write all the possible outcomes and answer the question.
- 1** Write the number of favourable outcomes. A 4 occurs once. Write the number of possible outcomes.
  - 2** Write the rule for probability.
  - 3** Substitute the known values into the rule and evaluate.
  - 4** Answer the question.
- 1** Write the number of favourable outcomes and the number of possible outcomes.  
*Note:* ‘Greater than 2’ implies 3, 4, 5, 6.
  - 2** Substitute the known values into the rule for probability and evaluate.
  - 3** Simplify the fraction.
  - 4** Answer the question.
- iii** Repeat steps 1 to 4 of part **b ii**.  
*Note:* ‘An odd number’ implies 1, 3, 5.

**WRITE**

- There are six possible outcomes; they are 1, 2, 3, 4, 5, 6.

- i** Number of favourable outcomes = 1

$$\text{Number of possible outcomes} = 6$$

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$P(4) = \frac{1}{6}$$

The probability of obtaining a 4 is  $\frac{1}{6}$ .

- Number of favourable outcomes = 4  
Number of possible outcomes = 6

$$P(\text{greater than } 2) = \frac{4}{6}$$

$$= \frac{2}{3}$$

The probability of obtaining a number greater than two is  $\frac{2}{3}$ .

- Number of favourable outcomes = 3  
Number of possible outcomes = 6

$$P(\text{an odd number}) = \frac{3}{6} \\ = \frac{1}{2}$$

The probability of obtaining an odd number is  $\frac{1}{2}$  or 50%.

**REMEMBER**

The probability of a particular result or event is defined as the number of favourable outcomes divided by the number of possible outcomes. This is written as:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

## EXERCISE

## 10C Simple probability

## INDIVIDUAL PATHWAYS

## eBook plus

**Activity 10-C-1**

Pick that card A

doc-6534

**Activity 10-C-2**

Pick that card B

doc-6535

**Activity 10-C-3**

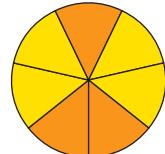
Pick that card C

doc-6536

## FLUENCY

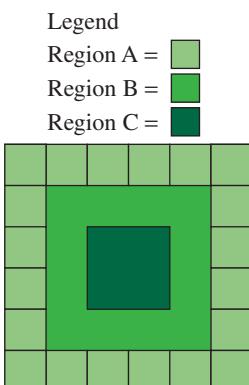
- 1 WE5** State how many possible outcomes there are for each of the following experiments and specify what they are.
- Rolling a 12-sided die, numbered 1 to 12 inclusive
  - Spinning a spinner for a game that has 5 equal-sized sections, numbered 1 to 5 inclusive
  - Choosing a consonant from the word ‘cool’
  - Choosing a sock out of a drawer containing 3 different socks coloured red, blue and black
  - Picking a marble out of a bag containing 5 different marbles coloured black, blue, green, red and yellow
  - Rolling an even number on a fair 6-sided die
  - Rolling an even number greater than 2 on a fair 6-sided die
  - Choosing an odd number from the first 20 counting numbers
- 2** List all the possible results in the following experiments. Comment on whether all results in each case are equally likely. Explain your answer.
- Rolling a fair 6-sided die
  - Tossing a normal coin
  - Spinning a spinner where half is white and half is black
  - Spinning a spinner where half is white, a quarter is blue and a quarter is red
  - Rolling a 6-sided die that has the numbers 1, 2, 3, 4, 5, 5 on it
  - Shooting at a target where  $\frac{1}{3}$  of the area is blue,  $\frac{1}{3}$  green and  $\frac{1}{3}$  red
  - Choosing a vowel in the word ‘mathematics’
- 3 WE6** Christina rolls a fair 10-sided die with faces numbered from 1 to 10.
- What are all the possible results that could be obtained?
  - What is the probability of obtaining:
 

i a 9?	ii a number less than 7?
iii a prime number?	iv a number greater than 3?
v a multiple of 3?	vi a number greater than 10?
vii an even number greater than 4?	viii an odd number divisible by 3?
- 4** Leo has been given a bag of marbles to play with. Inside the bag there are 3 blue, 6 red, 4 green and 7 black marbles.
- How many marbles are in the bag?
- 
- b** If Leo takes out one marble from the bag what is the:
- $P(\text{getting a red marble})$ ?
  - $P(\text{getting a green marble})$ ?
  - $P(\text{getting a black marble})$ ?
  - $P(\text{getting a blue marble})$ ?

- c How many marbles in the bag are either blue or black?  
 d Find P(getting a blue or a black marble).  
 e Find P(getting a green or red marble).  
 f Find P(getting a green, red or blue marble).  
 g Find P(getting a green, red, blue or black marble).  
 h Explain your answer to part g.
- 5 There is a valuable prize behind 2 of the 5 doors in the game show ‘Pick Your Door’. What is the probability that a player choosing any door will win a valuable prize?
- 6 **MC** A circular spinner is shown at right. When it is spun, the probability of obtaining an orange sector is:
- A  $\frac{4}{7}$       B  $\frac{1}{7}$       C 75%      D  $\frac{1}{2}$       E  $\frac{3}{7}$
- 
- 7 **MC** For an octagonal spinner with equal sectors numbered 1 to 8, the chance of getting a number *between* 2 and 7 is:
- A  $\frac{5}{8}$       B  $\frac{3}{8}$       C  $\frac{3}{4}$       D 0.5      E 25%

**UNDERSTANDING**

- 8 A pack of playing cards is shuffled and a card is chosen at random (in no particular order or pattern).  
 Find the probability that the card chosen is:
- |   |   |
|---|---|
| a a black card (that is, spades or clubs) | b an ace  |
| c a diamond                               | d a picture card (that is, a jack, king, queen) |
| e the queen of hearts                     | f a diamond or a black card                     |
| g not a king                              | h a club, diamond, heart or spade               |
| i not a spade                             | j red and a ten.                                |
- 9 Jim operates a parachute school. Being a man who is interested in statistics, he keeps a record of the landing position of each jump from first-time parachutists. With experience, parachutists are able to land on a particular spot with great accuracy. However, first-time parachutists do not possess this ability. He has marked out his landing field with squares, as shown below.



We are going to look at the areas of each of the regions A, B and C. To do this, we will determine each of the areas in terms of one of the small squares in region C. We will say that each small square has an area of 1 square unit.

- a What is the area of Jim’s whole paddock (in square units)?  
 b Determine the areas of regions A, B and C (in square units).  
 c Assuming that the parachutist lands in the field, calculate the probability that the landing will occur in:
- i region A                          ii region B                          iii region C.

These represent theoretical probabilities.

- d Jim's records indicate that, from 5616 jumps of first-time parachutists, the landing positions were:
- 592 in region C
  - 1893 in region B
  - 3131 in region A.

Comment on these results in comparison with the probabilities you calculated in question c.

### REASONING

- 10 In a raffle where there is only 1 prize (a car), 100 000 tickets have been sold, at a cost of \$5.00 each. What is the chance of winning the prize for a buyer who:
- purchases only 1 ticket?
  - purchases 20 tickets?
  - purchases 50 tickets?
  - purchases all the tickets?
  - Would the buyer who buys all the tickets have made a wise purchase? Explain.



### REFLECTION

If we know the probability of an event occurring, how can we work out the probability of it not occurring?

## 10D Using a table to show sample spaces

- Some experiments have two steps or stages that give a pair of results, such as when we toss 2 coins, or toss a coin and roll a die, or roll 2 dice.
- When the result is a combined one, we usually write the outcome as an **ordered pair**, in brackets, separated by a comma. The ordered pair (H, 6) would correspond to obtaining a Head on the coin and a 6 on the die.
- The sample space of two-step experiments may be displayed in a table.

### WORKED EXAMPLE 7

- Draw a two-way table and list the sample space for the experiment ‘tossing a coin and rolling a die’.
- State how many different outcomes or results are possible.
- Determine the probability of obtaining:
  - a Head
  - a Tail and an even number
  - a 5
  - a Tail and a number greater than 2.

### THINK

- Rule a table consisting of 7 rows and 3 columns. Leave the first cell blank.
- Label the second and third cells of the first row as H and T respectively.
- Label cells 2 to 7 of the first column as 1, 2, 3, 4, 5, 6 respectively.

### WRITE

	H	T
1	H 1	T 1
2	H 2	T 2
3	H 3	T 3
4	H 4	T 4
5	H 5	T 5
6	H 6	T 6

- 4** Combine the outcome pairs in the order in which they occur in each of the remaining cells; that is, the first event result followed by the second event result.
- 5** Answer the question.
- b** Count the number of different outcomes and answer the question.
- c**
- i**
    - 1** Write the number of favourable outcomes.  
Write the number of possible outcomes.  
*Note:* The favourable outcomes featuring a Head are (H, 1), (H, 2), (H, 3), (H, 4), (H, 5) and (H, 6).
    - 2** Write the rule for probability.
    - 3** Substitute the known values into the rule and evaluate.
    - 4** Simplify the fraction.
    - 5** Answer the question.

- ii** Repeat steps 1 to 5 of part **c i**.

*Note:* The favourable outcomes featuring a Tail and an even number are (T, 2), (T, 4) and (T, 6).

- iii** Repeat steps 1 to 5 of part **c i**.

*Note:* The favourable outcomes featuring a five are (H, 5) and (T, 5).

- iv** Repeat steps 1 to 5 of part **c i**.

*Note:* The favourable outcomes featuring a Tail and a number greater than 2 are (T, 3), (T, 4), (T, 5) and (T, 6).

The sample space for the experiment ‘tossing a coin and rolling a die’ is {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}.

- b** There are 12 different outcomes.

**c**

- i** Number of favourable outcomes = 6  
Number of possible outcomes = 12

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\begin{aligned} P(\text{Head}) &= \frac{6}{12} \\ &= \frac{1}{2} \end{aligned}$$

The probability of obtaining a Head is  $\frac{1}{2}$  on 50%.

**ii** Number of favourable outcomes = 3  
Number of possible outcomes = 12

$$\begin{aligned} P(\text{event}) &= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} \\ P(\text{Tail and even number}) &= \frac{3}{12} \\ &= \frac{1}{4} \end{aligned}$$

The probability of obtaining a Tail and an even number is  $\frac{1}{4}$  on 25%.

**iii** Number of favourable outcomes = 2  
Number of possible outcomes = 12

$$\begin{aligned} P(\text{event}) &= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} \\ P(\text{five}) &= \frac{2}{12} \\ &= \frac{1}{6} \end{aligned}$$

The probability of obtaining a five is  $\frac{1}{6}$ .

**iv** Number of favourable outcomes = 4  
Number of possible outcomes = 12

$$\begin{aligned} P(\text{event}) &= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} \\ P(\text{Tail and number greater than } 2) &= \frac{4}{12} \\ &= \frac{1}{3} \end{aligned}$$

The probability of obtaining a Tail and a number greater than 2 is  $\frac{1}{3}$ .

**REMEMBER**

1. A *sample space* is a list of all the possible outcomes obtained from an experiment and it is shown as a list enclosed in a pair of curled brackets { }.
2. An *ordered pair*  $(a, b)$  displays the result of a two-step experiment.
3. A *two-way table* allows us to display and obtain the sample space in a logical manner.

**EXERCISE****10D****Using a table to show sample spaces****INDIVIDUAL PATHWAYS****eBook plus****Activity 10-D-1**Using tables  
doc-6537**Activity 10-D-2**More using tables  
doc-6538**Activity 10-D-3**Advanced using  
tables  
doc-6539**FLUENCY**

- 1 Write down the sample space as an ordered list for each of the following simple or one-step experiments:
    - a rolling a 6-sided die
    - b spinning a spinner which can land on any of the numbers from 1 to 10
    - c choosing an item from a menu that contains fruit salad, cheesecake, mudcake and cheese platter
    - d choosing a number which is a multiple of 5 in the first 50 counting numbers
    - e choosing an Australian state or territory for a holiday destination
    - f picking the correct answer in a true/false question
    - g choosing a king from a pack of standard cards
    - h choosing an instrument from the following list: guitar, drum, saxophone, piano and trumpet.
  - 2 **WE7**
    - a Draw a two-way table and list the sample space for the experiment ‘spinning a circular spinner divided into 3 equal sectors labelled A, B, C and rolling a die’.
    - b State the number of different outcomes or results.
    - c Determine the probability of obtaining:
      - i the letter A
      - ii the number 4
      - iii a number greater than 2
      - iv a number which is a multiple of 3
      - v an odd number
      - vi the letter C and a prime number
      - vii the letter A, B or C
      - viii any number except the number 6
      - ix the letter B and a number less than 3
      - x a number greater than 6.
  - 3 **a** Draw a table to show the sample space for the experiment ‘tossing 2 coins at once’. (*Hint:* Call the first coin ‘Coin 1’ and the other ‘Coin 2’.)
  - b** How many possible results are there for this experiment?
  - c** How many times does the result  $(H, H)$  appear?
  - d** How many times does the result  $(T, T)$  appear?
  - e** How many times does a result with a Tail and a Head in any order appear?
  - f** What is  $P(H, H)$ ?
  - g** What is  $P(T, T)$ ?
  - h** What is  $P(\text{getting a Tail and a Head in any order})$ ?
  - 4 **a** Draw a table to show the sample space for the experiment ‘tossing a 5c coin and tossing a 10c coin’.
- Note:* In this case order will matter.



- b** How many possible results are there?
- c** What is  $P(\text{getting a H on the } 5\text{c coin and getting a T on the } 10\text{c coin})$ ?
- d** What is  $P(\text{getting a H on the } 5\text{c coin and a H on the } 10\text{c coin})$ ?
- e** What is  $P(\text{getting a T on the } 5\text{c coin and a H on the } 10\text{c coin})$ ?
- f** What is  $P(\text{getting a T on the } 5\text{c coin and a T on the } 10\text{c coin})$ ?
- 5 a** Draw a table to show the sample space for the experiment ‘tossing a coin and rolling a 10-sided die’.
- b** How many possible results are there?
- c** What is  $P(\text{getting a H on the coin and a 6 on the die})$ ?
- d** What is  $P(\text{getting a H on the coin and an even number on the die})$ ?
- e** What is  $P(\text{getting either a H or a T on the coin and an even number on the die})$ ?
- f** What is  $P(\text{getting a T on the coin and a number divisible by 3 on the die})$ ?
- g** What is  $P(\text{getting a number less than } 3)$ ?
- h** What is  $P(\text{getting a number greater than } 5)$ ?
- i** What is  $P(\text{getting a H on the coin and a number less than } 6 \text{ on the die})$ ?
- j** What is  $P(\text{getting either a H or a T on the coin})$ ?
- 6 a** Draw a table to show the sample space for the experiment ‘rolling a red die and a blue die’.
- b** How many possible results are there?  
(Note: There is a difference between a 5 on the red and a 5 on the blue die.)
- c** What is  $P(\text{getting 1 on both dice})$ ?
- d** What is  $P(\text{getting 1 on the red die and 6 on the blue die})$ ?
- e** What is  $P(\text{getting 1 on any die})$ ?
- f** What is  $P(\text{getting an even number on the red die and an odd number on the blue die})$ ?
- g** What is  $P(\text{getting an even number on both dice})$ ?
- h** What is  $P(\text{getting an odd number on both dice})$ ?
- i** What is  $P(\text{getting 2 numbers whose sum is } 6)$ ?
- j** What is  $P(\text{getting 2 numbers whose sum is } 1)$ ?
- k** What is  $P(\text{getting 2 numbers whose sum is } 13)$ ?
- 7 a** Draw a table to show the sample space for the following experiment ‘picking a marble out of a bag containing a red, a blue and a green marble and tossing a coin’.
- b** How many possible results are there?
- c** What is  $P(\text{picking out a green marble and getting a Head})$ ?
- d** What is  $P(\text{picking out a red marble and getting a Head})$ ?
- e** What is  $P(\text{picking out a blue marble and getting a Head})$ ?
- f** What is  $P(\text{picking out a blue or green marble and getting a Head})$ ?
- g** What is  $P(\text{picking out a blue or red marble and getting a Head})$ ?
- h** What is  $P(\text{picking out a blue or red or green marble and getting a Head})$ ?
- i** How is your answer to part **h** related to the probability of getting a Head if you were tossing only a coin? Can you explain why this is the case?

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**UNDERSTANDING**

- 8 MC** Two dice are rolled simultaneously. The probability of obtaining the sum of 7 (by adding the results of the two dice being rolled simultaneously) is:
- A  $\frac{1}{12}$       B  $\frac{7}{36}$       C  $\frac{1}{6}$       D  $\frac{1}{4}$       E 0
- 9** For the events given below, determine the following, without listing the sample space.
- i** State how many rows and columns would be needed to draw up a table representing the sample space.
- ii** State the number of possible outcomes in the sample space.
- a** Picking a day in January from a calendar and tossing a coin
- b** Tossing a coin and shooting a dart at a board with 3 zones
- c** Choosing a pencil from a set of 72 and rolling a 6-sided die

- d Rolling a 10-sided die and rolling a 6-sided die
- e Choosing a member from a class of 30 students and rolling a 6-sided die
- f Choosing a politician from a list of 100 and tossing a coin

**REASONING**

- 10** Within the Australian states, a common number plate system for cars is 3 numbers combined with 3 letters e.g., 123 ABC. How many more number plates does this allow each state to issue than would be the case if the plates were simply 6 numbers (for example, 123 456)?



- 11** Callum has a regular die and a strange die that has sides numbered 2, 3, 4, 5, 6, 6. If he rolls these two dice together, what is the probability that he gets a double six?

**REFLECTION**

When we toss 2 coins, which result (if any) is more likely: the coins match, or the coins are different?

## 10E Experimenting with chance

- The **theoretical probability** of a particular result or event is defined as  

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$
- In real life, the chance of something occurring may be based on factors other than the number of favourable and possible outcomes. For example, the chances of you beating your friend in a game of tennis could theoretically be  $\frac{1}{2}$  as you are one of 2 possible winners. In practice there are other factors (like experience and skill) that would influence your chance of winning.
- A **trial** is one performance of an experiment to collect a result.
- An **experiment** is a process that allows us to collect data by performing trials. In experiments with repeated trials, it is important to keep the conditions for each trial the same.
- A **successful trial** is one that results in the desired outcome.
- The experimental probability of an event is found by conducting an experiment and counting the number of times the event occurs.
- The **experimental probability** of a particular result or event is defined as  

$$P(\text{event}) = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

**WORKED EXAMPLE 8**

A coin is flipped 10 times and the results are seven Heads and three Tails. Find the experimental probability of obtaining a Tail.

**THINK**

- 1 Obtaining a Tail is considered a success. Each flip of the coin is a trial.
- 2 Tails was flipped three times, so there were three successful trials out of a total of 10 trials.

**WRITE**

$$P(\text{success}) = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

$$\begin{aligned} P(\text{Tail}) &= \frac{3}{10} \\ &= 0.3 \end{aligned}$$

**Experimental probability versus actual probability**

- The more times an experiment is performed, the closer the average of the measured results should be to the theoretically expected answer for it.
- The *long-term trend* (that is the trend observed for results from a very large number of trials) shows that results obtained through experimental probability will match those of theoretical probability.

**WORKED EXAMPLE 9**

- a Copy the table below. Toss a coin 10 times and record the results in row 1 of the table.

Experiment number	Heads		Tails	
	Tally	Count	Tally	Count
1				
2				
3				
4				
5				
6				
	Total		Total	

- b What is the probability of obtaining a Head from your experiment?  
 c What is the probability of obtaining a Tail from your experiment?  
 d How do these values compare with the theoretical results?  
 e Repeat step a another 5 times and combine all of your results.  
 f How does the combined result compare with the theoretical results?

**THINK**

- a Toss a coin 10 times and record the results in the first row of the table.

Notes: (a) Place a stroke in the appropriate tally column each time an outcome is obtained. Five is denoted by a ‘gatepost’; that is, 4 vertical strokes and 1 diagonal stroke (||||).

(b) The same coin must be used throughout the experiment. The style of the toss and the surface the coin lands on must be the same.

**WRITE**

Exp. No.	Heads		Tails	
	Tally	Count	Tally	Count
1		4		6
2		4		6
3		7		3
4		3		7
5		4		6
6		8		2
	Total	30	Total	30





- b** 1 Calculate the probability of obtaining a Head for this experiment using the rule.

2 Substitute the given values into the rule.

3 Evaluate and simplify.

4 Convert the fraction to a percentage by multiplying by 100%.

5 Answer the question.

- c** 1 Calculate the probability of obtaining a Tail for this experiment.

2 Substitute the given values into the rule and simplify.

3 Convert the fraction to a percentage by multiplying by 100%.

- d** Compare the results obtained in parts **b** and **c** with the theoretical results.

- e** 1 Repeat the procedure of part **a** 5 times.

2 Calculate the total number of Heads and Tails obtained and enter the results in the table.

- f** 1 Calculate the probability of obtaining a Head for this experiment.

2 Substitute the given values into the rule and simplify.

3 Convert the fraction to a percentage by multiplying by 100%.

**b**  $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

$$P(\text{Heads}) = \frac{\text{number of Heads obtained}}{\text{total number of tosses}}$$

$$P(\text{Heads}) = \frac{4}{10}$$

$$= \frac{2}{5}$$

$$\text{As a percentage } \frac{2}{5} = \frac{2}{5} \times 100\%$$

$$= \frac{200}{5}\%$$

$$= 40\%$$

The probability of obtaining a Head in this experiment is  $\frac{2}{5}$  or 40%.

**c**  $P(\text{Tails}) = \frac{\text{number of Tails obtained}}{\text{total number of tosses}}$

$$P(\text{Tails}) = \frac{6}{10}$$

$$= \frac{3}{5}$$

$$\text{As a percentage } \frac{3}{5} = \frac{3}{5} \times 100\%$$

$$= \frac{300}{5}\%$$

$$= 60\%$$

The probability of obtaining a Tail in this experiment is  $\frac{3}{5}$  or 60%.

- d** The experimental value obtained for the  $P(\text{Head})$  is  $\frac{2}{5}$  (or 40%) and the  $P(\text{Tail})$  is  $\frac{3}{5}$  (or 60%). The theoretical value of these probabilities is  $\frac{1}{2}$  (or 50%). Therefore, the experimental probabilities differ from the theoretical probabilities by 10%.

- e** Refer to results in table in part **a**.

**f**  $P(\text{Heads}) = \frac{\text{number of Heads obtained}}{\text{total number of tosses}}$

$$P(\text{Heads}) = \frac{30}{60}$$

$$= \frac{1}{2}$$

$$\text{As a percentage } \frac{1}{2} = \frac{1}{2} \times 100\%$$

$$= \frac{100}{2}\%$$

$$= 50\%$$

- 4 Answer the question.

The probability of obtaining a Head in this experiment is  $\frac{1}{2}$  or 50%.

$$P(\text{Tails}) = \frac{\text{number of Tails obtained}}{\text{total number of tosses}}$$

$$\begin{aligned} P(\text{Tails}) &= \frac{30}{60} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{As a percentage } \frac{1}{2} &= \frac{1}{2} \times 100\% \\ &= \frac{100}{2}\% \\ &= 50\% \end{aligned}$$

The probability of obtaining a Tail in this experiment is  $\frac{1}{2}$  or 50%.

- 5 Compare the combined result obtained with the theoretical results.

The combined results in this experiment produced probability values that were equal to the theoretical probability values. Therefore, the long-term trend of obtaining a Head or Tail when tossing a coin is equal to  $\frac{1}{2}$ .

## Simulations

- A **simulation** is the process of imitating or modelling a real life situation using simple devices or technology.
- Instead of physically tossing a coin, we can use technology to simulate or copy this process. Because of this, simulations are often quicker, more convenient and safer than carrying out a real life experiment.
- **Random numbers** are sets of numbers that are generated in such a way that each number has an equal chance of occurring each time.
- Random numbers can be used to simulate experiments by assigning a unique number value to each unique outcome in the sample space.

### WORKED EXAMPLE 10

Alex wants to know how many packets of cereal she must purchase in order to collect 4 different types of plastic toys during a promotion.

- Design an experiment which will simulate the given situation, providing details of the equipment required and procedure involved.
- Discuss the fairness of the experiment and findings.



#### THINK

- Determine the sample space.  
*Note:* There are 4 toys, each of which is equally likely to be found in a packet of cereal.

#### WRITE

- Sample space = {toy 1, toy 2, toy 3, toy 4}  
The 4 events are equally likely.



- 2** Decide on the equipment required for this experiment.  
*Note:* Choose an item which produces 4 possibilities; that is, a spinner or 4 different coloured balls in a box, etc.

- 3** Give details of how the experiment will be conducted.

- 4** Record the results in a table comprising 3 columns, headed: ‘Experiment number’, ‘Results’ and ‘Number of packets’.

A circular spinner divided into 4 equal sectors and labelled 1, 2, 3, 4 will be used to simulate the outcomes of obtaining a toy from the cereal packet. Sector 1 represents toy 1 (T1). Sector 2 represents toy 2 (T2). Sector 3 represents toy 3 (T3). Sector 4 represents toy 4 (T4).

Spin the pointer on the circular spinner until all 4 toys are represented; that is, until T1, T2, T3 and T4 are obtained.

Repeat this procedure another 19 times.

Experiment number	Results	Number of packets
1	T4 T3 T4 T3 T2 T4 T4 T1	8
2	T3 T4 T4 T3 T4 T1 T2	7
3	T1 T4 T4 T2 T1 T4 T4 T3	8
4	T1 T4 T4 T4 T4 T4 T2 T3	8
5	T1 T4 T3 T4 T1 T2	6
6	T1 T4 T1 T2 T1 T2 T4 T2 T4 T4 T1 T1 T4 T3	14
7	T1 T4 T4 T2 T2 T4 T4 T3	8
8	T4 T2 T2 T3 T2 T4 T3 T2 T4 T4 T1	11
9	T3 T4 T2 T4 T2 T2 T4 T2 T1	9
10	T4 T3 T3 T1 T1 T2	6
11	T3 T1 T4 T3 T1 T1 T1 T3 T1 T4 T1 T3 T1 T1 T3 T2	16
12	T3 T2 T1 T1 T4	5
13	T3 T4 T1 T2	4
14	T4 T2 T3 T3 T4 T1	6
15	T4 T1 T4 T3 T3 T4 T4 T3 T1 T2	10
16	T1 T1 T2 T1 T1 T3 T4	7
17	T4 T2 T3 T2 T4 T2 T4 T4 T4 T4 T3 T4 T4 T3 T4 T3 T4 T4 T3 T4 T2 T1	22
18	T1 T2 T1 T1 T4 T3	6
19	T2 T4 T2 T4 T1 T4 T2 T4 T1 T1 T2 T2 T1 T3	14
20	T1 T1 T1 T1 T4 T3 T2	7
Total		182

- b** 1 Comment on the fairness of the experiment.

- 2 Answer the original question.  
Write the total number of packets and total number of experiments.

- 3 Write the rule for the average number of packets.  
4 Substitute the known values into the rule.  
5 Evaluate.  
6 Round the value to the nearest whole number.  
*Note:* Since we are dealing with packets we must work with whole numbers.  
7 Summarise your findings.  
*Note:* Comment on points of interest; that is, the maximum and minimum number of packets that need to be purchased.

- b** In order to ensure fairness throughout the experiment, the pointer of the spinner was spun from the wider end by the same person each time. Twenty experiments were performed in total; however, this amount could be increased.

Total number of packets

$$= 8 + 7 + \dots + 14 + 7$$

$$= 182$$

Total number of experiments = 20

$$\text{Average} = \frac{\text{total number of packets}}{\text{total number of experiments}}$$

$$\text{Average number of packets} = \frac{182}{20}$$

$$= 9.1$$

$$= 9$$

The average number of packets of cereal Alex must purchase to obtain each of the 4 plastic toys is 9.

From this experiment, the minimum number of packets needed to obtain the 4 toys is 4 while the largest number of packets needed is 22.

## Generating random numbers on an Excel spreadsheet

The random number generator [=rand()] on the spreadsheet generates a decimal number between 0 and 1. We need to convert this decimal number to a whole number between 1 and 5.

Enter the formula  
**=INT(RAND()\*5)+1** into cell A1.

Multiplying the random decimal number by 5 produces a random decimal number between 0 and 5. The **INT** function will change the decimal to a whole number. We finally add 1 to make it a whole number between 1 and 5.

Use the **Fill Down** function to fill this formula down to cell A20. You should now have 20 random numbers as shown in this screen dump.

Note that every time you perform an action on this spreadsheet, the random numbers will change. We can stop this by following these instructions.

	A	B	C	D	E
1	2				
2	3				
3	4				
4	2				
5	2				
6	2				
7	1				
8	3				
9	2				
10	1				
11	4				
12	5				
13	5				
14	2				
15	5				
16	4				
17	4				
18	3				
19	5				
20	3				

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**Interactivity**  
Random  
number  
generators  
int-0089

- Choose **Options** from the **Tools** menu.
- Select the calculation tab and click the manual calculation radio button.
- You can obtain a recalculation of your random numbers by pressing **(F9)**.

**REMEMBER**

- The *long-term trend* (that is, the trend observed for results from a very large number of trials) shows that results obtained through experimental probability will match those of theoretical probability.
- A *simulation* is the use of simple devices such as coins, dice, a pack of cards and spinners to model or mimic a real life situation.

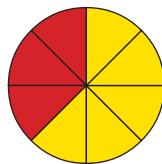
**EXERCISE****10E****Experimenting with chance****INDIVIDUAL PATHWAYS**

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**Activity 10-E-1**Coloured marbles  
doc-6541**Activity 10-E-2**Numbered disks  
doc-6542**Activity 10-E-3**Lettered disks  
doc-6543**FLUENCY**

- WEB** Teagan was playing Trouble and kept count of how many times she rolled a 6. During the game, she was successful 5 times out of the 25 times she tried. What was the experimental probability of rolling a 6 in the game?
- WE9** **a** Copy the table at right. Toss a coin 10 times and record the results in the first row of the table.  
**b** What is the probability of obtaining a Head from your experiment?  
**c** What is the probability of obtaining a Tail from your experiment?  
**d** How do these values compare with the theoretical results?  
**e** Repeat step **a** another 5 times and combine all your results.  
**f** How does the combined result compare with the theoretical results?
- If you wanted to create a device that would give a theoretical probability of achieving a particular result as  $\frac{1}{4}$ , how many sections would a spinner such as this need to be divided into?
- How would you divide or colour a spinner if you wanted to achieve the probability of a success equal to  $\frac{3}{10}$ ?
- For the spinner at right, what would be the probability of getting the red section?
- WE10** Repeat the experiment described in Worked example 10.

Experiment number	Heads		Tails	
	Tally	Count	Tally	Count
1				
2				
3				
4				
5				
6				
	Total		Total	

**UNDERSTANDING**

- Use your results from question **2** to answer the following.
  - The long-term trend of the probability of obtaining a Head on the toss of a coin is the P(Heads) from your experiment.  
What is the long-term trend of the probability after:
    - 10 tosses of the coin?
    - 20 tosses of the coin?
    - 30 tosses of the coin?
    - 60 tosses of the coin?
  - Obtain a classmate's 60 results. Combine these with yours. State the long-term trend of P(Heads) obtained.
  - Combine your pair's 120 results with another pair's. State the long-term trend of P(Heads) obtained.

- d Finally, obtain the results obtained by whole class for this experiment. (Make sure nobody's results are counted twice.) You should have 60 tosses per person. State the long term trend of  $P(\text{Heads})$  obtained.
- e Copy and complete the table below.

Number of tosses	Heads		Tails	
	P(Heads)	P(Heads) as percentage	P(Tails)	P(Tails) as percentage
10				
20				
30				
60				
120				
240				
Whole class (specify number of tosses)				

- f Comment on the changes of the long-term trend value of  $P(\text{Heads})$  as you toss the coin more times.
- 8 Use the weblink for the Adjustable Spinner in your ebookPLUS to simulate a 5-colour spinner.
- a What is the chance of getting any one of the 5 colours when you spin the spinner (theoretically)?
  - b Spin the spinner 10 times and, using a table such as the one below, record your results.
- | Colour                    | 1 | 2 | 3 | 4 | 5 |
|---------------------------|---|---|---|---|---|
| Number of times it occurs |   |   |   |   |   |
- c From your results, list the probabilities of obtaining each colour. For example, divide the number of times a particular colour was obtained by the total number of spins (that is, 10).
  - d Why might these probabilities not be the same as the theoretical probability would suggest?
  - e Spin your spinner and record the results for another 10 spins.
  - f Spin your spinner so that you have 100 results. Is the experimental probability closer to the pure probability? Why might this be? Discuss.
- 9 Inside a bag are 36 shapes which are either squares or triangles. One shape is taken out at random, its shape noted and put back in the bag. After this is repeated 72 times, it is found that a triangle was taken out 24 times. Estimate how many triangles and how many squares there are in the bag.

### REASONING

- 10 You have calculated previously that the chance of getting any particular number on a 6-sided die is  $\frac{1}{6}$ . You have 2 different coloured dice. Is there any difference in your dice apart from colour? Could one be biased (more likely to give a particular result than theory says it should)?
- a Design a test to determine whether the dice you have are fair. Write down what you are going to do.
  - b Perform your test, and record your results.



- c Calculate the probability of getting each of the numbers on each of your dice, based on your tests and on the long-term trend you have observed.
- d What does your test say about your 2 dice? Are there any things that need to be considered before giving your answer? (Perhaps your dice have slightly uneven shapes or something that might cause them to lean towards one result more than others.)
- 11 Use a box of Smarties or marbles, or technology such as a graphics calculator or an Excel computer program, for this question. Count the number of items of each colour in the box before you start.
- a Calculate the theoretical probability of getting a particular colour if you pick 1 out of the box without looking.
- b Design an experiment to determine the probability of getting a particular colour out of the box, using the long-term trend.
- c Why is it important that if you take Smarties (marbles or a colour in the simulation) out of the box for this experiment, you must put them back each time?
- d Could you use something other than your box of Smarties, marbles or technology to determine this probability? What other things could you use to simulate this experiment?

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**Weblinks**  
**Probability crossword**  
**Experimental probability**

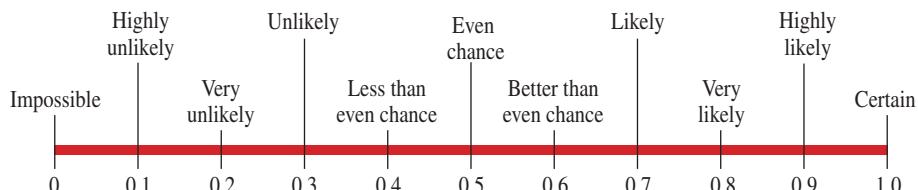
**REFLECTION**

How is the experimental probability of an event related to its theoretical probability?

# Summary

## The language of chance

- An event is a result that may occur.
- Probability is the chance of a particular event (result) occurring.
- The probability of an event occurring ranges from and includes 0 (impossible) to 1 (certain).
- A probability scale associates important words used in describing probabilities with their approximate corresponding numerical values.



## The sample space

- The sample space represents a list of all possible outcomes to a probability experiment.
- The sample space can be denoted by the symbol  $S$  and all outcomes are then listed within braces {}.
- $n(S)$  represents the number of elements in the sample space.

## Simple probability

- The probability of a particular result or event is defined as the number of favourable outcomes divided by the number of possible outcomes. This is written as:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

## Using a table to show sample spaces

- A sample space is a list of all the possible outcomes obtained from an experiment and it is shown as a list enclosed in a pair of curled brackets {}.
- An ordered pair  $(a, b)$  displays the result of a two-step experiment.
- A two-way table allows us to display and obtain the sample space in a logical manner.

## Experimenting with chance

- The long-term trend (that is, the trend observed for results from a very large number of trials) shows that results obtained through experimental probability will match those of theoretical probability.
- A simulation is the use of simple devices such as coins, dice, a pack of cards and spinners to model or mimic a real life situation.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter.

Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 For each of the given statements, specify whether the chance of the following events occurring is certain, likely, even chance, unlikely or impossible.



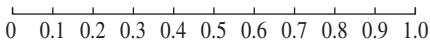
- a Australia is in the southern hemisphere.
- b You will still be alive in the next century.
- c Obtaining an even number on a circular spinner numbered from 1 to 16.
- d England is in the southern hemisphere.
- e You roll a fair die and obtain a number less than or equal to 4.
- f Humans can survive without water indefinitely.

- 2 List three events where the chance of each occurring would be:

- a impossible
- b certain
- c improbable.

- 3 Copy the number line below and place the list of words on it to indicate what sort of chance each number would represent.

Words to place: certain, very unlikely, unlikely, likely, highly likely, highly unlikely, even chance, impossible, very likely.

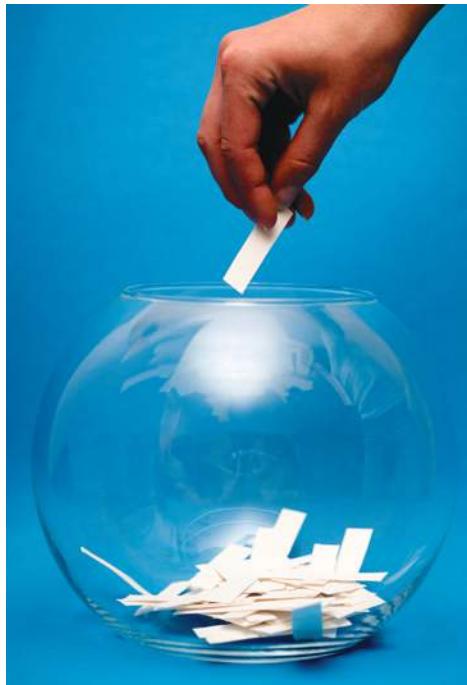


- 4 List the sample space for the following experiments:
- a tossing a coin
  - b drawing a marble out of a bag containing red, green and blue marbles

- c rolling a 6-sided die
- d spinning a circular spinner numbered from 1 to 10
- e drawing a 4 card from a pack of playing cards
- f answering a true/false question in a test
- g competing in a race in which there are 9 runners
- h choosing a year level to represent the secondary school at a public event.

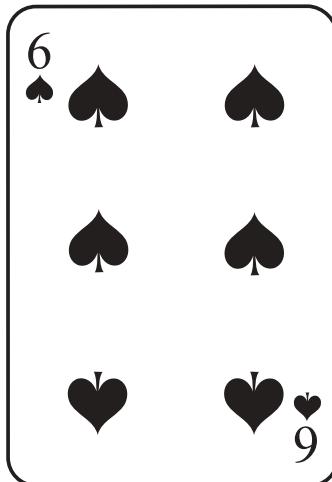
- 5 For each of the following, state the number of elements in the sample space.

- a The first prize in a raffle is chosen from tickets numbered from 1 to 5000.



- b A card selected from the red cards in a standard deck.
  - c A circular spinner numbered 1 to 5 is spun.
  - d A day is selected in the month of July.
- 6 A six-sided die, labelled 1, 2, 3, 3, 4, 5 is rolled. Find:
- a  $P(\text{getting a } 5)$
  - b  $P(\text{getting a } 3)$
  - c  $P(\text{getting an even number})$
  - d  $P(\text{getting an odd number})$
  - e  $P(\text{getting a number greater than or equal to } 3)$
  - f  $P(\text{getting a number less than } 3)$
  - g  $P(\text{getting a } 0)$
  - h  $P(\text{getting a } 6)$ .

- 7 A person has a normal pack of cards and draws one out. Find:



- a P(picking a heart)
  - b P(picking a spade)
  - c P(picking a picture card)
  - d P(picking a card with a number less than 5 on it). Do not count aces as a number.
- 8 Ten cards are numbered from 1 to 10, shuffled and placed face down on a table. If a card is selected at random, find the probability that the card selected is:
- a 4
  - b an even number
  - c divisible by 3
  - d an even number and divisible by 3
  - e an even number or divisible by 3
  - f not divisible by 3
  - g greater than 8
  - h 12.
- 9 Use a table to show the sample space for the experiment ‘tossing a coin and rolling a die’. Find:
- a P(a tail and a number less than 3)
  - b P(a head and a number greater than 3)
  - c P(a tail and an even number)
  - d P(a head and a prime number).
- 10 Use a table to show the sample space for the experiment ‘spinning a spinner with 10 sectors and rolling a die’. Find:
- a P(the sum totals to an even number)
  - b P(odd number on the spinner and an even number on the die)
  - c P(both digits having a value less than 5)
  - d P(the sum totals to a value between 7 and 10)
  - e P(the sum totals to a value greater than or equal to 13 but less than 16)
  - f P(the sum totals to a prime number).

- 11 A sample of 250 students at a particular school found that 225 of them had access to the internet at home. Given this sample is a good representation for the entire school, what is the probability that a student selected at random in the school will have internet access at home?



- 12 Jane and her father have decided to play a game of chess against each other. Neither has played before but each has been given the same explanation of the rules of the game.
- a What would be the probability of either of them winning a game?
  - b Explain your answer.



Since her first game with her father, Jane has joined a chess club and competes regularly against various players. Her father, on the other hand, plays occasionally. Over a number of years Jane and her father have played 340 games. Jane has won 255 of these games.

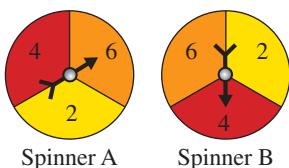
- c Given this long-term trend, what would be the probability that the father wins the next game?
- d Does the probability remain the same over a period of time? Explain your answer.

**PROBLEM SOLVING**

- 1** In your sock drawer there are only 4 identical blue socks and 5 identical black socks. What is the smallest number of socks you can remove from the drawer in the dark so that you will be absolutely sure of having:



- a** a pair of black socks?  
**b** a pair of blue socks?  
**c** a black pair and a blue pair of socks?
- 2** Danielle wants to know how many packets of cereal she must purchase in order to collect 6 different limited edition Disney cards. Design an experiment that will simulate the given situation. Provide clear instructions and details of 2 different devices that may be used for the simulation.

**3**

The arrows on Spinner A and Spinner B are spun. (If an arrow lands on a line, the spinner is spun again.) The two numbers are added to get a score. For example, in the diagram above, the score is 10.  
**a** What is the highest total score possible?  
**b** What are the possible total scores?  
**c** List all the ways to get a total score of 8.

- 8** Consider the following game. You have a board (see diagram), a token to move and a fair coin to flip. Each turn consists of the following two steps.  
 1. Flip the coin.  
 2. Move two squares to the right for Heads, and one square to the right for Tails.

What is the probability that you will land on Square 5 in exactly three turns?

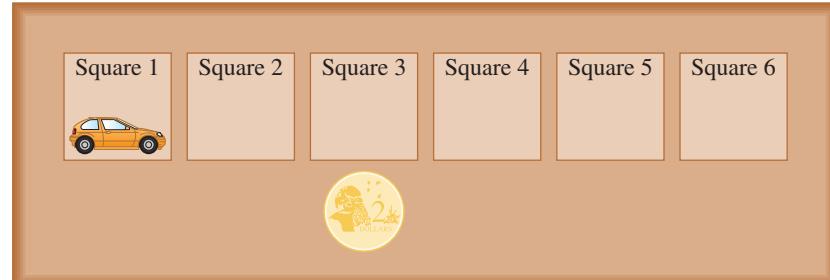
- d** What is the chance of getting a total of 9?  
**e** What is the chance of getting a total score of 10?  
**f** Draw a grid showing the probabilities of getting all possible totals.

- 4** Mark and Cameron play a game in which they toss two coins. Mark wins if two Heads turn up, and Cameron wins if two Tails turn up. If both a Head and a Tail turn up, they throw one of the coins again. If this comes up Heads, Mark wins; if it comes up Tails, Cameron wins. Do you think this is a fair game? Discuss with a classmate and explain your reasoning.

- 5** Charlotte and Rhianna have a flower garden. They have 17 red flowers, 12 pink flowers and 13 yellow flowers. Charlotte picks some flowers for her vase. She selects 6 red flowers, 3 pink and 3 yellow flowers. If Rhianna picks a flower at random for her hair, what is the probability that it is yellow?



- 6** A bowl contains blue marbles and white marbles. If there are twice as many blue marbles as white marbles, what is the probability that a blue marble is selected?  
**7** Mrs Prince gives Charles 2 containers, 10 green marbles and 10 gold marbles. She tells him to arrange the marbles any way he likes (all of the marbles must go into the containers). She chooses 1 marble from 1 container. If she picks a gold one, she'll give Charles \$100. How should Charles arrange the marbles to have the best chance of getting \$100?



- 9 You have a spinner. Draw a circle graph to represent the following.

Colour	Degree
Red	20°
Orange	40°
Yellow	60°
Blue	100°
Green	20°
Purple	120°

Discuss the chances of landing on each of the colours. List the likelihoods from smallest to largest.

- 10 Chloë is a contestant on a game show. There are five sealed cases on the podium and each of the cases contains one of the following amounts: \$1, \$50, \$500, \$1000 and \$5000. The game-show host offers her a deal of \$400, or she can choose a case and keep the amount of money in it, instead. What is the probability she will win more than the \$400 the host is offering?
- 11 To ‘get out of jail’ when playing a board game, the player must throw a double from two dice that are rolled three times. What is the probability of getting at least one double in these three throws?



- 12 Rebecca plays a sideshow game where she puts four balls into a clown’s mouth, and the balls then fall into slots numbered 1 to 6. To win, one of her balls must go into Slot 6. She has noticed that the numbers 2 and 5 come up 4 times as often as 1 and 6, and the numbers 3 and 4 come up 5 times as often as 1 and 6. What is the probability that she will get a 6?


**eBook plus**
**Interactivities**

- Test yourself Chapter 10 int-2744
- Word search Chapter 10 int-2595
- Crossword Chapter 10 int-2596

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 10 (doc-6521) (*page 355*)

**Are you ready?****Digital docs (page 356)**

- SkillsHEET 10.1 (doc-6522) Understanding ‘chance’ words
- SkillsHEET 10.2 (doc-6523) Comparing the likelihood of different events occurring
- SkillsHEET 10.3 (doc-6524) Describing the chance of an event occurring as more or less than half
- SkillsHEET 10.4 (doc-6525) Listing all possible outcomes for an event
- SkillsHEET 10.5 (doc-6526) Understanding a standard deck of playing cards
- SkillsHEET 10.6 (doc-6527) Writing a simple probability as a fraction

**10A The language of chance****Digital docs (page 359)**

- Activity 10-A-1 (doc-6528) The language of chance
- Activity 10-A-2 (doc-6529) More chance
- Activity 10-A-3 (doc-6530) Advanced chance

**10B The sample space****Digital docs (page 361)**

- Activity 10-B-1 (doc-6531) Sample space
- Activity 10-B-2 (doc-6532) More sample spaces
- Activity 10-B-3 (doc-6533) Advanced sample spaces

**10C Simple probability****Digital docs (page 364)**

- Activity 10-C-1 (doc-6534) Pick that card A
- Activity 10-C-2 (doc-6535) Pick that card B
- Activity 10-C-3 (doc-6536) Pick that card C

**10D Using a table to show sample spaces****Digital docs**

- Activity 10-D-1 (doc-6537) Using tables (*page 368*)
- Activity 10-D-2 (doc-6538) More using tables (*page 368*)
- Activity 10-D-3 (doc-6539) Advanced using tables (*page 368*)
- WorkSHEET 10.1 (doc-6540) (*page 369*)

**10E Experimenting with chance****Digital docs (page 376)**

- Activity 10-E-1 (doc-6541) Coloured marbles
- Activity 10-E-2 (doc-6542) Numbered disks
- Activity 10-E-3 (doc-6543) Lettered disks
- WorkSHEET 10.2 (doc-6544)

**Interactivity**

- Random number generators (int-0089) (*page 376*)

**Weblinks (page 378)**

- Probability crossword
- Experimental probability

**Chapter review****Interactivities (page 383)**

- Test yourself Chapter 10 (int-2744) Take the end-of-chapter test to test your progress.
- Word search Chapter 10 (int-2595)
- Crossword Chapter 10 (int-2596)

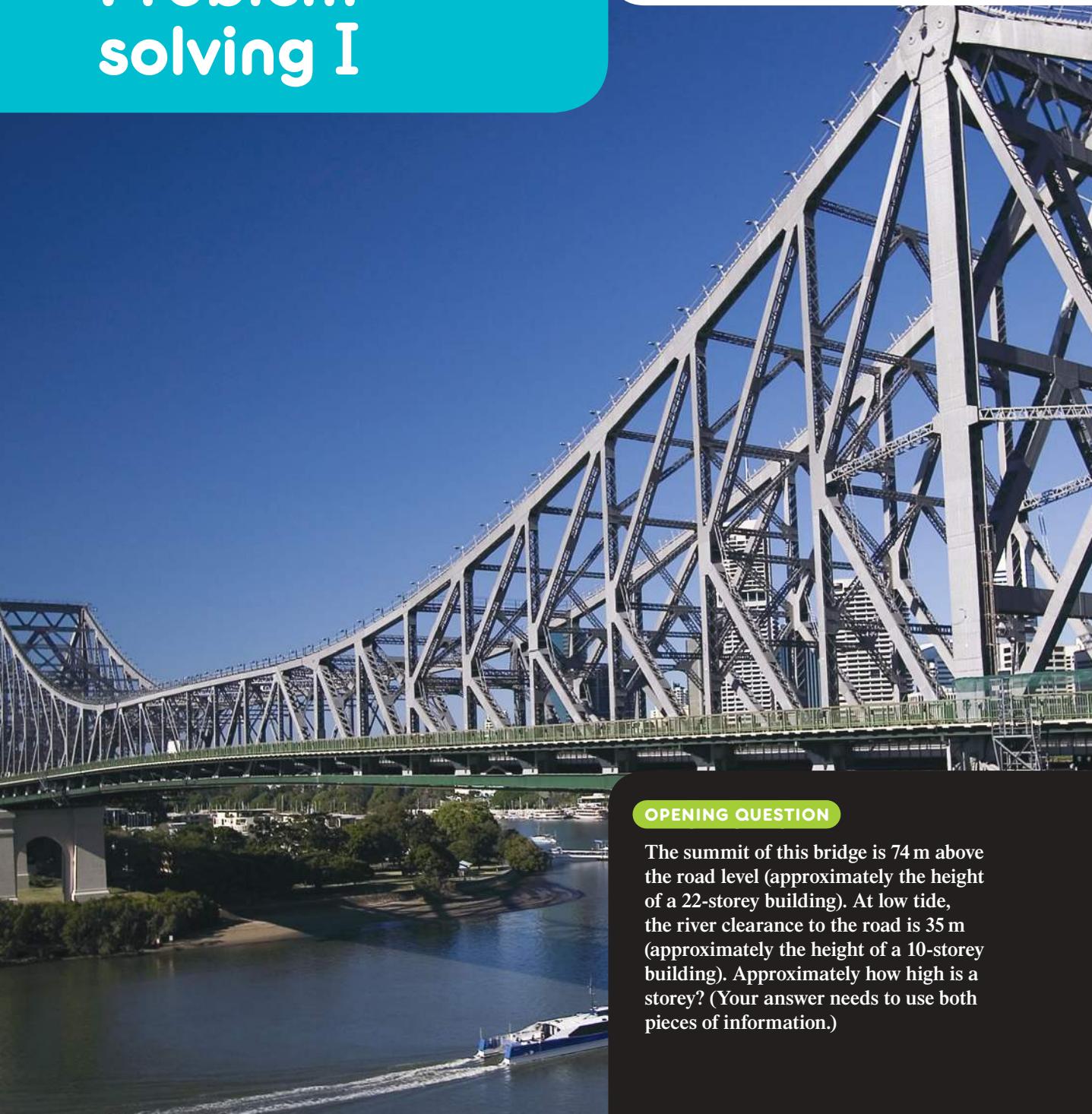
To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# 11

## Problem solving I

- 11A Introduction to problem solving — create a table or diagram
- 11B Draw a diagram
- 11C Look for a pattern — using technology
- 11D Work backwards from the answer
- 11E Elimination
- 11F Simplify the problem
- 11G Guess and check
- 11H Mixed problems 1
- 11I Mixed problems 2
- 11J Mixed problems 3



### OPENING QUESTION

The summit of this bridge is 74 m above the road level (approximately the height of a 22-storey building). At low tide, the river clearance to the road is 35 m (approximately the height of a 10-storey building). Approximately how high is a storey? (Your answer needs to use both pieces of information.)

## 11A

# Introduction to problem solving — create a table or diagram

eBookplus

Weblinks  
Problem solving 1  
Problem solving 2

## Introduction to problem solving

- When solving problems, the main processes that we can use are:
  1. Read the question at least twice and take note of all the important facts.
  2. Identify the solution required.
  3. Solve the problem using an appropriate strategy.
  4. Communicate the solution using appropriate language and mathematical terms.
  5. Support the solution with mathematical reasoning.
  6. Reflect on the solution. Does it answer the question and does it make sense? Could it have been solved a better way?
- The problem solving processes are interrelated. By practising the skills involved in using all processes, you will learn to tackle new mathematics problems with confidence and arrive at the correct and complete solution using the most appropriate methods.
- **READ THE QUESTION** at least twice. Make sure you know what the question is asking you to do. Do you have enough information to solve the problem?
- **IDENTIFY THE SOLUTION REQUIRED.** What is the question asking you to do?
- **SOLVE THE PROBLEM** using an appropriate strategy. Decide on a suitable strategy to solve the problem. Examples of strategies that could be used are:
  1. Create a table.
  2. Draw a diagram.
  3. Look for a pattern, using technology.
  4. Work backwards from the answer.
  5. Elimination.
  6. Simplify the problem.
  7. Guess and check.
- **COMMUNICATE THE SOLUTION.** Another person reading your work needs to be able to follow your method or strategy. You need to present your data, explanation and solutions in a clear and concise form, using correct mathematical terms and appropriate diagrams.
- **SUPPORT THE SOLUTION** with mathematical reasoning. When you think you've solved the problem, use mathematical reasoning to verify that your answer is correct and your method is justified.
- **REFLECT ON THE SOLUTION.** Have you answered the question? Think back over how you solved the problem. Could it have been solved in a different or better way? Learn from the experience, and use this knowledge to solve problems in the future.

## Create a table

- A table is a way of organising or grouping numbers.
- A table can help you see patterns in the numbers you have organised.
- A table can demonstrate to others how you arrived at your solution.

### WORKED EXAMPLE 1

Amad operates a petrol station on a busy highway. He is investigating the purchasing habits of his customers. From the sample of purchases below, calculate the most frequently purchased amount of petrol. First, round each amount to the nearest 5 litres.

14.7 L	22.0 L	32.2 L	8.1 L	23.5 L	15.5 L	27.9 L	44.0 L
27.2 L	32.9 L	25.5 L	42.4 L	23.7 L	31.7 L	39.7 L	

**THINK**

- 1 Read the question at least twice and take note of all the important facts.
- 2 Identify the solution required.
- 3 Decide how the ‘amounts’ of petrol can be arranged in a table.
- 4 State what the smallest and largest amounts of petrol are.  
*Note:* The smallest and largest amounts give us an indication of the range of values required for the table.
- 5 Create a table showing each amount as it appears in the given list and place a tick in the column representing the amount rounded to the nearest 5 litres.
- 6 Total the ticks in each column and look for the multiple of 5 L with the most ticks.

**WRITE**

There are 15 pieces of data showing the individual quantities of petrol purchased.

The question asks for the most frequently purchased amount of petrol.

Each ‘amount’ of petrol can be organised into multiples of 5 L.

The smallest amount is 8.1 L, which rounds to 10 L; the largest amount is 44.0 L, which rounds to 45 L.

Actual amount	Rounded amount							
	10 L	15 L	20 L	25 L	30 L	35 L	40 L	45 L
14.7 L		✓						
22.0 L				✓				
32.2 L						✓		
8.1 L	✓							
23.5 L					✓			
15.5 L		✓						
27.9 L						✓		
44.0 L								✓
27.2 L						✓		
32.9 L						✓		
25.5 L				✓				
42.4 L							✓	
23.7 L				✓				
31.7 L					✓			
39.7 L							✓	
<b>TOTAL</b>	1	2	1	3	5	0	2	1

- 7 Answer the question.

To the closest multiple of 5 L, the amount of petrol that is purchased the most often is 30 L.

**REMEMBER**

1. The five main processes used to solve problems are:
  - (a) questioning
  - (b) applying strategies
  - (c) communicating
  - (d) reasoning
  - (e) reflecting.
2. One strategy that can be used is to create a table. This strategy allows you to organise or group numbers. It can help you see patterns in the numbers and can demonstrate to others how you arrived at your solution.

## 11A

# Introduction to problem solving — create a table or diagram

## PROBLEM SOLVING

- 1 A new mobile phone company is investigating the habits of telephone users. From the sample of telephone calls below, find out the most popular length of phone call. First, round each call time to the nearest half minute.

132 s	10 s	43 s	71 s	243 s	52 s
142 s	332 s	6 s	38 s	63 s	32 s
132 s	32 s	43 s	52 s	352 s	101 s
124 s	28 s	153 s	10 s	243 s	34 s

- 2 Sharyn opened a bank account when she started working at her local fish and chip shop. Over the four weeks of one month she made deposits of \$56.00, \$44.00, \$52.80 and \$39.20 and withdrawals of \$45.00, \$75.00, \$22.00 and \$50.00. If her bank account was opened with a \$10 deposit, how much was in her account at the end of the month? Was there a time that the balance of her account was negative?
- 3 A hospital building has 6 above-ground-level floors and 4 floors below ground level. Over a 10-minute period one busy morning, the lift started from ground level (Reception) and made the following moves: up 2 floors, up 1 floor, down 4 floors, down 2 floors, up 3 floors, down 4 floors and up 6 floors. Where did the lift finish and how far did it travel in the 10 minutes?
- 4 With a certain pre-paid mobile phone deal, there is a 20-cent flag-fall that is charged for every call made, then peak hour calls are charged at a rate of 1 cent per second.
- a Set up a table to show the cost of peak calls of 1, 2, 3, ..., 8 seconds duration.
  - b What is the rule that shows the relationship between the length of the call and its cost?
  - c Use the rule to predict the cost of a 2-minute call with this deal.
  - d If the cost of a call was \$2.00, how long was the call?
- 5 You received a SMS from your friend. After 5 minutes, you forward the SMS to two of your other friends. After 5 more minutes, those two friends forward it to two more friends. If the SMS is sent every five minutes in this way, how many people have received the SMS in 30 minutes?
- 6 Angus is the youngest in his family and today he and his Dad share a birthday. Both their ages are prime numbers. Angus's age has the same two digits as his Dad's but in reverse order. In 10 years' time, Dad will be three times as old as Angus. How old will each person be when this happens?
- 7 Use the following clues to find the values of  $A$ ,  $B$ ,  $C$  and  $D$ .
- $A$  is a factor of  $B$ .
  - $C$  is the second multiple of  $B$  and the fourth multiple of  $A$ .
  - $D$  is a prime number between 10 and 20.
  - All numbers are different, and add up to 41.
- 8 Sam accidentally divided two numbers on his calculator and got 0.6, when he should have added the two numbers and got 16. What are the two numbers?
- 9 Australian coins, which are minted by the Royal Australian Mint, have the following masses:

5c	2.83 g
10c	5.65 g
20c	11.3 g
50c	15.55 g
\$1	9.0 g
\$2	6.6 g

I have 6 coins in my pocket, with a total value of \$2.45. What could the total mass of these coins be?

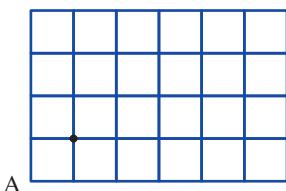


- 10** William and Olivia make pool tables with a difference. A diagram of one of their tables is shown. This one is 6 squares long (the base) and 4 squares wide (the height). There are only 4 corner pockets. William and Olivia use their tables to play a game called *Pool multiply*. The rules of the game are simple.

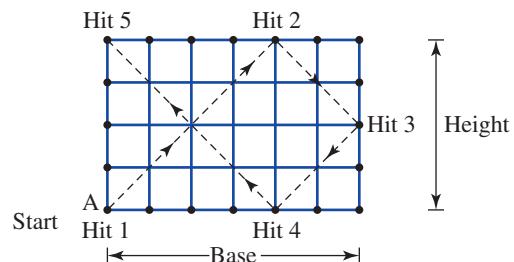
1. Place a ball in front of the corner pocket A (Table 1).
2. Shoot the ball through the dot shown on the table. This will make the ball rebound at  $45^\circ$  when it hits a side cushion (Table 2).
3. Count the number of squares that the ball travels through.
4. Count the number of times the ball hits the sides of the table. Include the initial hit of the cue on the ball as 1 hit and the ball falling into the corner pocket as one hit.

William and Olivia's other pool tables have different numbers of squares along the bases and heights (see some of them in Table 3). Explore these pool tables and complete Table 3.

**Table 1**



**Table 2**



**Table 3**

Base	Height	Number of squares	Number of hits
6	4	24	5
3	4		
3	5		
3	6		
4	8		
$b$	$h$		

Develop a rule for the number of squares and number of hits if you know the base and height of the table. You may need to try extra tables to help you find the rule.

## 11B Draw a diagram

- When information is represented as a diagram, it can be seen all at once, and is therefore easier to study.
- There are many different types of diagram, so no single diagram is necessarily the best.

### WORKED EXAMPLE 2

The Davies family have a hobby-farm, which they use on weekends and school holidays. Their home is on one side of a large freshwater lake and their farm is on the other. It takes about 3 hours to drive around the lake to the farm. Alternatively, there is a train that goes about halfway around the lake; the rest of the trip requires a bus. There are still other options: the Davies could drive to the lake, where there is a ferry service that will take them across; from the other side they could catch a bus, a taxi or just walk to the farm. How many different ways can the Davies travel from their home to their hobby-farm?



## THINK

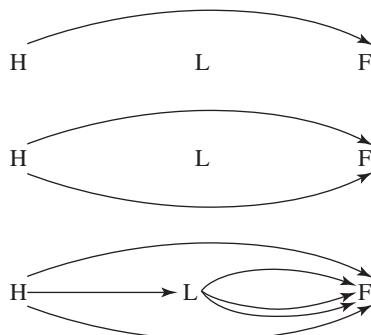
- 1 Read the question at least twice and take note of all the important facts.  
*Note:* The information regarding the time it takes to drive to the farm is not relevant.
- 2 Identify the solution required.
- 3 Start a diagram by showing the position of their home H, the farm F and the lake L.
- 4 Draw an arrow from H to F around L to represent the car trip.
- 5 Draw a different arrow to represent the train and bus combination.
- 6 An arrow from H through L could represent the car trip to the lake followed by the ferry ride. Three arrows from L could represent travelling by bus, travelling by taxi or just walking.
- 7 Each different path from H to F needs to be counted.
- 8 Answer the question.

## WRITE

The Davies' home and farm are on opposite sides of the lake. They are able to travel from their home to the farm in a number of ways: drive by car; take the train, then the bus; drive to the lake, take the ferry across the lake, then choose to take either the bus or taxi or to walk.

The question asks to find the number of ways to travel from the home to the farm.

H                            L                            F



There are 5 different ways for the Davies family to travel from their home to their hobby-farm.

## REMEMBER

- 1 Drawing a diagram can help to solve the problem.

## EXERCISE

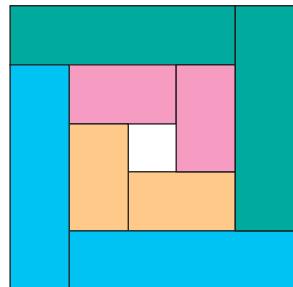
### 11B Draw a diagram

#### PROBLEM SOLVING

- 1 Nadia's family live on a property in outback Victoria. Nadia's Aunt Emily lives in the closest town, which is 45 km from the property. Nadia goes to a university in Melbourne, which is 570 km away. From university, Nadia communicates with her parents in many ways: she can write a letter; send an email; send a fax; telephone her family at home; or telephone her Aunt, who may decide either to drive over to the property with the message or telephone Nadia's parents later that night. How many ways are there for Nadia to communicate with her parents from university?
- 2 A rectangular chart can be attached to a wall with a drawing pin in each corner. You have 8 rectangular charts all the same size. What is the minimum number of pins you would need to attach all your charts to the wall if you must have a pin in each corner, and if overlapping is allowed at the margins? (Each chart must be visible.)
- 3 Five termites start munching into a log. After an hour:  
Pixie is 19 mm ahead of Bitsie.  
Trixie is 5 mm ahead of Mixie.  
Mixie is twice as far as Itsie.  
Itsie is 10 mm behind Bitsie.  
Mixie has eaten through 52 mm.  
Which termite is the fastest eater?



- 4 Stephanie puts four equally spaced points on a number line. The first point is at the  $\frac{3}{10}$  mark and the last point is at the  $\frac{7}{20}$  mark. Where has she placed the two points in between?
- 5 Anna has a wooden rod that is 2.4 metres long. She needs to cut 3 lengths, each of 0.4 metres, and 2 lengths, each of 0.2 metres. What fraction of the original rod will be left over after she has cut the required lengths?
- 6 Two candles of equal length are lit at the same time. The first candle takes 6 hours to burn out and the second takes 3 hours to burn out. After how many hours will the slower burning candle be twice as long as the faster burning candle?
- 7 Sophie, Hamish and Thomas shared a pizza and have eaten it all. Sophie ate 4 slices, Hamish ate 3 slices and Thomas ate 5 slices. If Thomas's share of the bill was \$4.50, calculate the amount Sophie and Hamish had to pay.
- 8 In a 'log cabin' quilt, a pattern is created from a series of squares and rectangles. From a centre square, 4 congruent rectangles are placed to build a larger square. This process is repeated a number of times to build larger and larger squares. Laurel starts with a centre square of side length 2 cm. If each rectangle has a width of 2 cm and a length that depends on its position in the block, give the dimensions of the rectangles that will need to be cut if the final block is to be 18 cm square. (Don't worry about seam allowance.)
- 9 Glen wants to pave his back courtyard. His courtyard is  $12 \text{ m} \times 12 \text{ m}$ . He has a choice of  $10 \text{ cm} \times 25 \text{ cm}$  clay pavers that cost \$2.30 each, or concrete pavers that are  $70 \text{ cm} \times 70 \text{ cm}$  and cost \$42 each. What would the cheaper option be?
- 10 Roseanne is planning a tour of Australia. She calculates that 70% of the distance will be on sealed roads and the remainder on dirt roads. If there are 1356 km of dirt roads, how long will Roseanne's journey be?



## 11C Look for a pattern — using technology

- Repetitive tasks are suited to a spreadsheet.
- A spreadsheet can list patterns of numbers from which a result can be found.

### WORKED EXAMPLE 3

Carla enjoys sport. She is going to invite those people in her class who play both tennis and netball to her home. In her class of 30 students, 11 play tennis and 22 play netball. How many play both sports? How many will be invited if Carla is one of the people who play both sports?

#### THINK

- 1 Read the question at least twice and take note of all the important facts.
- 2 Identify the solution required.

#### WRITE

Students who play both tennis and netball will be invited to Carla's house.  
In Carla's class there are 30 students of whom 11 play tennis and 22 play netball.  
Carla plays both sports.  
The question asks to find how many students play both sports and how many students will be invited to Carla's house (keeping in mind she plays both tennis and netball).



- 3** Set up a spreadsheet and label the first cell of each column. In cell A1 type ‘Tennis and netball’; in cell B1 type ‘Tennis only’; in cell C1 type ‘Netball only’; and in cell D1 type ‘Total’. (Do not include the quotation marks.)
- 4** The first column is the list of possible numbers of people who play both sports; it cannot be more than 11, since that is the total of all who play tennis. We can call this the *common amount*. In cells A2, A3, A4 …, enter 1, 2, 3 …, up to 11 in cell A12.
- 5** In the second column, we will calculate the numbers who play tennis only, by subtracting the common amount from 11. In cell B2, enter **=11-A2**. Highlight cells B2 to B12 and from the **Edit** menu select **Fill Down** to perform the same calculation with the remaining cells in this column.
- 6** Similarly, enter the calculation to obtain the C column for those who play netball only. In cell C2, enter **=22-A2**. Then highlight cells C2 to C12 and fill down the column.
- 7** Each cell in the D column is the sum of the preceding three cells in the same row. In cell D2, enter **=sum(A2:C2)**. Then highlight cells D2 to D12 and fill down the column.
- 8** Now look for the row with the set of values that fit the given information. We know that there should be 30 people altogether, so look for the answer 30 in the D column.

	A	B	C	D
1	Tennis and netball	Tennis only	Netball only	Total
2	1	10	21	32
3	2	9	20	31
4	3	8	19	30
5	4	7	18	29
6	5	6	17	28
7	6	5	16	27
8	7	4	15	26
9	8	3	14	25
10	9	2	13	24
11	10	1	12	23
12	11	0	11	22
13				
14				
15				
16				
17				
18				
19				

In Carla’s class of 30 students, 3 play both tennis and netball (she is one of these), 8 play tennis only and 19 play netball only. Carla will invite two people from her class.

#### REMEMBER

1. Repetitive tasks are suited to a spreadsheet.
2. If a pattern of numbers is listed in a spreadsheet, the result can be more readily found.

#### EXERCISE

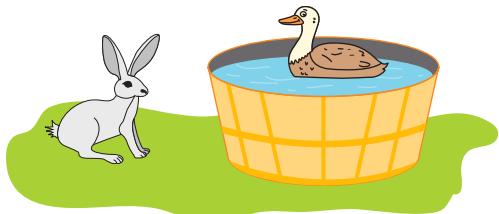
## 11C Look for a pattern — using technology

These problems can be solved with or without a spreadsheet.

#### PROBLEM SOLVING

- 1** In a class of 22 students, each attends swimming practice, or lifesaving, or both. If 16 attend swimming practice and 9 attend lifesaving, find the number of students who attend both.
- 2** There are 100 cows in McDonald’s meadow. Of these, 85 of them moo, 85 are motley and just 15 are mad. What is the minimum number of cows that are motley, moo but are not mad?
- 3** You drop a rubber ball from a height of 10 m. The ball rebounds to a height of 8 m and then to a height of 6.4 m. The ball continues to bounce in the same ratio. What is the height of the ball when the top of its bounce is less than 3 m, and on what bounce does this occur?

- 4 Lisa is 6 years older than Susan, Jessie is 5 years older than Lisa, and the total of their ages is 41. How old is Susan?
- 5 A duck in a bucket weighs 5 kg. A rabbit in the same bucket weighs 4 kg. The rabbit and duck together in the bucket weigh 6 kg. How much does the bucket weigh?
- 6 In a mathematics competition, a student scores three marks for every correct answer and loses one mark for every wrong answer. The student can choose from 4 options each time. If the student attempts 100 questions and scores 188 marks overall, find the number of questions she answers correctly.



- 7 Write the following as decimals:  $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}$ . Can you see the pattern forming?

Using this pattern, find the value of  $\frac{6}{11} \div 9$ .

- 8 Calculate values for:
- a i  $1.5^2$       ii  $2.5^2$       iii  $3.5^2$
  - b Describe the pattern.
  - c Explain how you could use this pattern to calculate  $8.5^2$  without the aid of a calculator.
- 9 Calculate values for:

- a  $11^2$       b  $101^2$       c  $1001^2$ .

Without using a calculator, write down values for:

- d  $1000001^2$       e  $22^2$       f  $2002^2$ .

- 10 The decimal equivalents of  $\frac{1}{7}, \frac{2}{7}, \frac{3}{7} \dots \frac{6}{7}$  can be found in the sequence 142857142857142857...  
If  $\frac{1}{7} = 0.\dot{1}4285\dot{7}$  and  $\frac{2}{7} = 0.\dot{2}8571\dot{4}$ , without using a calculator, write  $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$  and  $\frac{6}{7}$  as decimals.

## 11D

## Work backwards from the answer

- If there is a sequence of steps for which we know the final answer, then a useful strategy is to work backwards from this final result or answer.

### WORKED EXAMPLE 4

Luke was paid for mowing the lawn at his neighbour's home. However, for weeding and trimming the garden, his older brother Tom received \$4 more than Luke did. Luke's mother spent \$45 on groceries. If this is five times the amount that Tom is paid, how much did Luke receive?

#### THINK

- 1 Read the question at least twice and take note of all the important facts.
- 2 Identify the solution required.
- 3 From the given information, list the amount of money that is known. Use this to work out how much Tom received.
- 4 Work backwards from Tom's payment to calculate Luke's payment.
- 5 Answer the question.

#### WRITE

- Luke was paid for mowing the lawn.  
Tom was paid \$4 more than Luke for weeding and trimming.  
Their mother spent \$45 on groceries: this amount is 5 times Tom's pay.  
The question asks to find how much Luke was paid.  
Luke's mother has spent \$45. This is 5 times the amount Tom received. So Tom received  $\frac{1}{5}$  of \$45, that is,  $\frac{1}{5} \times \$45 = \$9$ .  
Luke received \$4 less than Tom.  
Luke received  $\$9 - \$4 = \$5$ .

**REMEMBER**

1. If there is a sequence of steps for which we know the final answer, then a useful strategy is to work backwards from this final result or answer.

**EXERCISE****11D****Work backwards from the answer****PROBLEM SOLVING**

- 1 Jo and Anton made a certain number of telephone calls over a 2-month period. There is a charge of 25 cents for each call, and the monthly rental of the phone line is \$13.60. The bill for the 2 months comes to \$39.45. How many calls did they make in the 2-month period?
- 2 Jane sold  $\frac{2}{3}$  of the cupcakes she made. She had only 45 cupcakes left. How many cupcakes did she make?
- 3 Jessica has a ‘clean-up’ button on her mobile phone that will clear half of the read messages in her inbox. She pressed the button six times and the result was just one message left in her inbox. How many messages were in her inbox before the ‘clean-up’?
- 4 A man and his grandson share the same birthday. For exactly six consecutive years the grandson’s age is a factor of his grandfather’s age. How old is each of them at the sixth of these birthdays, given that the grandson is no more than 10?
- 5 Peter is unaware that his computer has a virus. Every time he shuts down, the virus deletes half of his hard drive. Peter has also decided that he will have a ‘clean-up’ to free up some of his computer’s memory. Each time he starts up his computer, he transfers 2 GB of work onto a USB key (hence deleting this from his hard drive) then shuts down. Soon after, Peter gets a message: ‘Hard drive wiped’. He realises his computer must have a virus because he hasn’t saved or deleted any work since he started his clean-up, and he is certain he had much more work than the 10 GB that has been transferred to the USB drives in front of him. How much work (in GB) has Peter lost?
- 6 World-wide figures indicate that  $\frac{1}{10}$  of men are left-handed, while  $\frac{2}{25}$  of women are left-handed. If there are 400 men in a room, how many women would need to be there so that there are the same number of left-handed men as left-handed women in the room?
- 7 Aunt Mary was expecting a bumper crop from her lemon tree this season, but unfortunately the possums discovered it too. One night the possums ate half the fruit on the tree. The next night they ate one third of what was left on the tree, and the following night one quarter of the remainder. Aunt Mary managed to pick what was left, but unfortunately one quarter of those had been spoiled by wasps. She ended up with just nine lemons. How many lemons did the possums eat?
- 8 If  $a > b$ ,  $b > 2c$ ,  $c > 3d$ ,  $d > 4e$  and  $e > 5f$ , find the smallest possible value for  $a$  if all of the unknowns are positive integers.
- 9 If you multiply this number by 6 and subtract 3 the answer is 21. What is the number?
- 10 A *cryptarithm* is a puzzle where each digit of a number has been replaced by a prounomial or letter. Solve the following cryptarithms by determining the digit that corresponds to each letter to make the calculation true.



$$\begin{array}{r} \text{A} \ \text{B} \\ + \ \text{B} \ \text{A} \\ \hline \text{C} \ \text{A} \ \text{C} \end{array}$$

# 11E Elimination

- When using a process of elimination we remove or eliminate possible solutions that do not match the given information.
- We first write down all the possible combinations or solutions in a grid or table. From the information supplied, we cross out (eliminate) those combinations that do not match.

## WORKED EXAMPLE 5

Harriett is trying to find the house where she grew up 35 years ago. The house number is either 27a or 27 or 72a and the street name is either North Avenue or North Road or North Lane. Assuming the house is still there, help Harriett by matching the house numbers with the street names. Use each number only once.

North Avenue has no odd-numbered house numbers because there is a golf course on one side of the avenue. Number 27a shares a block with number 27 and number 72a shares a block with number 72. There are no shared blocks in North Lane.

### THINK

- Read the question at least twice and take note of all the important facts.
- Identify the solution required.
- Rule up a grid of possibilities.
- Go through each clue and begin the process of elimination.
  - Eliminate (cross out) 27a and 27 against North Avenue as there are no odd numbered houses in North Avenue.
  - Eliminate 27a and 72a against North Lane, as there are no shared blocks in North Lane.
  - Tick 72a against North Avenue and 27 against North Lane since there are no other options.
  - Eliminate 27 and 72a from North Road as these numbers have been allocated.
  - Tick 27a against North Road as there are no other options.
- List the resulting combinations from the grid.

### WRITE

The possible house numbers are 27a, 27 or 72a. The possible streets are North Avenue, North Road or North Lane.

North Avenue has no odd numbers.

Numbers 27a and 72a are on shared blocks. No shared blocks are in North Lane.

The question seeks to find options for the location of Harriett's house.

	27a	72a	27
North Avenue	✗		✗
North Road			
North Lane	✗	✗	

	27a	72a	27
North Avenue	✗	✓	✗
North Road	✓	✗	✗
North Lane	✗	✗	✓

The house numbers and matching street names are 72a North Avenue, 27a North Road and 27 North Lane.

### REMEMBER

- When using a process of elimination we remove or eliminate possible solutions that do not match the given information.
- We first write down all the possible combinations or solutions in a grid or table. From the information supplied, we cross out (eliminate) those combinations that do not match.

## PROBLEM SOLVING

- 1** Mrs Flushcombe needs to find Fernando before lunch. Fernando is in either a French, a German or a Visual Arts lesson. These lessons take place in rooms K1, L1 or L2. We know that the Kitchen, K1, is never used by a Visual Arts class. The French class has too many students to fit in K1 (Kitchen) or L1 (the smaller language room). Match each lesson with its room.
- 2** Brandon works after school delivering milk. In Bellevue Road he delivers two 2-litre containers of milk to one house, 1 litre of skim milk to another house, and three 1-litre containers of milk to another. The houses on his delivery run are numbers 5, 7 and 18. Brandon remembers that the houses next to each other both have multiple orders (more than one item), and the largest containers of milk are delivered to the house with the smallest house number. Match each house number with its milk order.
- 3** Four girls — Eugenie, Florence, Kim and Anthea — each prefer a different means of communication. Match the girls to one of the following: home phone, mobile with SMS messaging, email, mail. Use the following information: both Eugenie and Florence have no computer, though Florence is the only one with a mobile phone with SMS; also, Australia Post can collect mail from Kim and Florence.
- 4** Alan, Bob and Cam each have one of the following coins — a 50c piece, a \$1 coin or a \$2 coin. Alan and Cam's coins are circular and Alan's coin is smaller in size than Cam's. What coin does each of the boys have?
- 5** Penny, Teresa and Raelene are friends in high school and each has one brother. Their three brothers are Steve, Matt and Fred. Penny's brother is at primary school. Fred has finished school and is working as an apprentice carpenter. Steve attends the same school as his sister. Fred's sister is not Raelene. Match the girls with their brothers.
- 6** Find the number described by the following clues:
  - It is bigger than 251.
  - It is divisible by 4.
  - The sum of the digits is 11.
  - It is smaller than 523.
  - Its last digit is 6.
- 7** You are one of fifteen people playing a game being run by your friend, James. Across the room there are 15 chairs numbered 1 to 15. James secretly writes down a 'winning' number, then shouts 'Go!'. You and your 14 competitors run to claim a chair. James then declares that if your seat number is a factor of his winning number, you win a prize. Much to his astonishment, all except two of you win a prize, and those two people are sitting next to each other.
  - a** Which seat numbers missed out on a prize?
  - b** What was James' secret winning number (in prime factor form)?
- 8** Can you guess the number that matches the following?
 

It is odd, and it has an odd number of factors.  
The sum of the digits is a two-digit prime.  
The number is less than  $\sqrt{10\,000}$  but greater than  $\sqrt{100}$ .  
What number could it be?



**9** What is my number?

My number contains four different even digits and has three decimal places. The digit in the thousandths position is half the value of the digit in the units position. The sum of the digits in the units and tenths positions is the same as the sum of the digits in the hundredths and thousandths positions.

- 10** The first fleet to Australia consisted of 11 ships. From the information below, determine the skipper of each ship and its passenger details.

The largest ship carried 195 male convicts.

The smallest ship, skippered by Captain Henry Bull, led the way, carrying 50 people.

HMS *Sirius* and HMS *Supply* were both skippered by Captains.

Captain John Hunter later became Governor of Australia.

HMS *Sirius* was 370 tons heavier than HMS *Supply*.

The *Borrowdale*, the *Fishburn* and the *Golden Grove* did not carry convicts.

The *Scarborough* was not the largest ship, but carried 208 male convicts — 13 more than the largest ship.

Skippered by Master Duncan Sinclair, the *Alexander* carried male convicts.

Master Sharp skippered the ship transporting the Reverend Richard Johnson and his wife. They did not travel with convicts.

There were female convicts on the *Prince of Wales*, the *Lady Penrhyn*, the *Friendship* and the *Charlotte*.

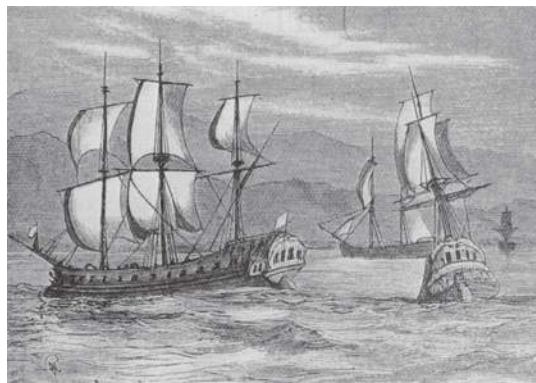
Two ships carrying convicts were skippered by people called John. Master John Marshall had 208 male convicts, and Master John Mason had 1 male and 49 female convicts.

The *Lady Penrhyn*, skippered by Master William Sever and carrying 101 female convicts, was the only convict ship without male convicts.

Master Francis Walton carried 76 male and 21 female convicts and did not skipper the *Charlotte* or the *Prince of Wales*.

The *Charlotte*, skippered by Master Thomas Gilbert, carried 12 more male and 1 fewer female convicts than the *Friendship*.

The *Fishburn* and the *Borrowdale* were skippered by Master Robert Brown and Master Readthorn Hobson respectively.



## 11F Simplify the problem

- If you are overwhelmed by the size of the numbers involved in a question, try to solve a similar but simpler question. This can be achieved by changing the numbers in the original question to smaller numbers.
- After finding the answer to the simpler question, the same method can be used to solve the original problem.

### WORKED EXAMPLE 6

The World Cup is a world championship soccer tournament that is held every 4 years and is played over several rounds. In the first round of finals, 24 teams compete against one another. The teams are divided into six groups of four teams. Within each group, each team plays the other three teams. Find the total number of games played in the first round of the World Cup finals.



## THINK

- 1 Read the question at least twice and take note of all the important facts.
  
- 2 Identify the solution required.
  
- 3 Consider a simpler competition with only one group of four teams.
  
- 4 Use a diagram to work out how many games are played within this group if each team plays every other team. Team 1 plays Teams 2, 3 and 4. Team 2 plays Teams 3 and 4. (Don't count playing Team 1 again.) Team 3 plays Team 4. (Don't count playing Teams 1 and 2 again.)
  
- 5 Consider how many games would be played if there were two groups of four teams.
  
- 6 Relate this result to the original problem where there are six of these groups.

## WRITE

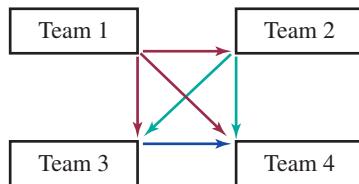
There are 24 soccer teams competing. The teams are divided into 6 groups; each group consisting of 4 teams.

In each group, each team plays 3 other teams.

The question asks to find the total number of games played.

Consider a simpler problem with only one group of 4 teams.

Each team is to play every other team in the group.



With 4 teams playing each other, there is a total of 6 games.

For two groups of four teams, there would be a total of  $2 \times 6$  (that is, 12) games.

For six groups of four teams, there would be a total of  $6 \times 6$  (that is, 36) games.

## REMEMBER

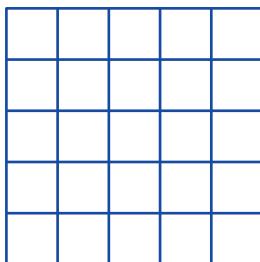
1. If you are overwhelmed by the size of the numbers involved in a question, try to solve a similar but simpler question. This can be achieved by changing the numbers in the original question to smaller numbers.
  
2. After finding the answer to the simpler question, the same method can be used to solve the original problem.

## EXERCISE

### 11F Simplify the problem

#### PROBLEM SOLVING

- 1 By joining all the vertices of a decagon, how many diagonals are formed?
  
- 2 Find the exact value of  $(111\ 111\ 111)^2$ .  
*Hint:*  $1^2 = 1$ ;  $11^2 = 121$ ,  $111^2 = 12\ 321$ .
  
- 3 How many squares of any size can be found in the following diagram?



*Hint:* Count the number of single squares, count the number of  $2 \times 2$  squares, then count the number of  $3 \times 3$  squares. Look for a pattern in these numbers, and use the pattern to help you.

- 4** The student lockers at Macleod College are to be numbered consecutively from 1 to 800 using plastic digits. Each digit costs 10c. What will be the total cost of all the digits?
- 5** Triangular numbers are those that can be represented by dots in the shape of an equilateral triangle. The first five triangular numbers are 1, 3, 6, 10 and 15. Investigate these numbers to find a pattern in them. If the 20th triangular number is 210, show how you could determine the 25th triangular number.
- 6** The streets in Metropolis run north–south and east–west (see the diagram below). Adam wants to visit Betty. If he can travel only south or east, how many paths can he follow?



- 7** Ten couples go out for dinner. If everyone shakes hands once with everyone else except their spouse, how many handshakes are there?
- 8** A librarian is using 5 books for a display. How many arrangements of the books are possible?
- 9** Calculate  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \dots + \frac{9}{10}$
- 10** A knock-out tennis competition ends with 2 players playing the grand final. If the grand final is the 8th round of the competition, and half the players are knocked out in each round, how many players were there at the start of the competition?



## 11G Guess and check

- Sometimes it may not be easy to solve a problem directly; in this case we can use a strategy where we guess at the solution. We test this guess using the available information supplied in the problem to check whether it is the solution. We continue to guess and check until the solution is found.
- Technology, such as a spreadsheet, can be used to give instant feedback about the guess.

### WORKED EXAMPLE 7

Jono is 12 years older than Jade. The sum of their ages is 48. How old is Jade?

#### THINK

- 1 Read the question at least twice and note all of the important facts.
- 2 Identify the solution required.
- 3 Since Jade's age is the unknown value, guess any number (say 20). On a spreadsheet, type the heading 'Jade's age' in cell A1. Enter 20 in cell A2.
- 4 Jono's age is 12 more than Jade's age, so type the heading 'Jono's age' in cell B1 and enter =A2+12 in cell B2.
- 5 As we are interested in finding the sum of their ages, type the heading 'Sum of ages' in cell C1 and enter =A2+B2 in cell C2.
- 6 Change the number in cell A2 to make a new guess and observe what happens to cell C2. This is our check. Alternatively, to keep a record of previous guesses, enter a new value in the next row of column A, and highlight and **Fill Down** in columns B and C.
- 7 When 48 appears in column C, we know that our guess must be correct! (You may need to use decimal numbers as well as whole numbers as your guess in some cases.)
- 8 Answer the question.

#### WRITE

Jono is 12 years older than Jade.  
The sum of Jono and Jade's ages is 48.

The question asks to find the age of Jade.

A	B	C	D	E
1 Jade's age	20	Jono's age	52	
2	32			
3	31	50		
4	30	48		
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				

Jade is 18 years old.

#### REMEMBER

1. Sometimes it may not be easy to solve a problem directly, in this case we can use a strategy where we guess at the solution. We test this guess using the available information supplied in the problem to check whether it is the solution. We continue to guess and check until the solution is found.
2. Technology, such as a spreadsheet, can be used to give instant feedback about the guess.

### EXERCISE

## 11G Guess and check

#### PROBLEM SOLVING

- 1 Rachel and Jackson are 7 years apart in age. Jackson is older than Rachel. The sum of their ages is 51. Find Rachel's age.
- 2 Eric and Colin are partners in owning a taxi. They share the driving so that the taxi is on the road 24 hours a day. Yesterday, Colin drove for  $2\frac{1}{2}$  hours longer than Eric. For how long did Eric drive yesterday?
- 3 Marcus and Melanie pooled their funds and purchased shares on the Stock Exchange. Melanie invested \$350 more than Marcus. Together they invested \$2780. Find how much Marcus invested.

- 4** The sum of two numbers is 32 and their product is 247. What are the two numbers?
- 5** At a take-away food stall Martin purchased 3 hamburgers and 2 drinks while Ken purchased 2 hamburgers and 3 drinks. If Martin paid \$16.15 and Ken paid \$14.85, what are the individual costs of a hamburger and a drink?
- 6** Using the six consecutive numbers from 4 to 9, complete the magic square at right so that each row, column and diagonal totals 15.
- 7** Using 20 cubes, make four piles so that the first pile contains 4 more cubes than the second pile, the second pile contains 1 cube less than the third pile, and the fourth pile contains twice as many cubes as the second pile. How many cubes are in each pile?
- 8** **a** Jack was organising a get-together of all his football mates from the last 3 years. He decided to phone 4 mates and ask them to also phone 4 footballers each, who in turn would also phone 4 more footballers. Each team member was given a list of 4 names so that no one received more than one call. How many footballers would receive a phone call about the party?
- b** Jack's friend Karin liked the idea, so decided to contact 218 people about a 10-year reunion using Jack's method. Assuming there were three rounds of calls, how many people should each person call?
- 9** The sum of 3 consecutive odd numbers is 39. What are the 3 numbers?
- 10** Hypatia was an Egyptian female mathematician born in AD 370. In her paper on Diophantus, Hypatia posed the following problem. Find a number that satisfies these two conditions:
1. it is the sum of two squares, and
  2. its square is also the sum of two squares.
- Can you find 4 numbers that satisfy Hypatia's conditions?

	1	
3		
		2



## 11H Mixed problems 1

### Communicating, reasoning and reflecting

- It is important to understand that solving problems involves much more than just writing numbers on a page. Words should accompany the mathematics, and these words should be in the form of appropriate English and correct mathematical terms and expressions.
- Care should be taken that when the equals (=) sign is used, the mathematics following is indeed equal.
- After you provide a solution to a question, it is good practice to review your solution to see whether another person could understand your work without first reading the question.



- Finally, if you take the time to reflect on your work, you may increase your understanding of the problem and the strategies you used to solve it. You may be able to connect this to previous experiences as well as to future problems you will have to tackle.

### WORKED EXAMPLE 8

**Tom makes house calls to fix electrical appliances. His charges involve a call-out fee of \$35 plus a cost of \$12.50 per quarter of an hour for service. Any replacement parts are added to the cost. The bill to fix the Adams family's washing machine amounted to \$145. If the replacement part cost \$72.50, how long did Tom spend fixing the machine?**

#### THINK

- Read the question at least twice and take note of all the important facts.
- Identify the solution required.
- Consider an appropriate strategy. In this case, working backwards from the total bill cost seems the best one. The service cost can be obtained by subtracting both the call-out fee and replacement parts from the total cost.
- Communicate the solution.
- Provide mathematical reasoning.
- Reflect on the solution.

#### WRITE

The call-out fee is \$35 and each  $\frac{1}{4}$  hour of service is \$12.50.

The total cost can be expressed as an equation:

$$\text{Total cost} = \text{call-out fee} + \text{service fee} + \text{replacement parts}$$

The bill totals \$145 and a replacement part was \$72.50.

The question asks to find the service time.

$$\begin{aligned}\text{Service cost} &= \text{total cost} - \text{call-out fee} - \text{replacement parts} \\ &= \$145 - \$35 - \$72.50 \\ &= \$37.50\end{aligned}$$

$$\frac{1}{4} \text{ hour rate} = \$12.50$$

$$\begin{aligned}\text{Number of } \frac{1}{4} \text{ hours} &= \$37.50 \div \$12.50 \\ &= 3\end{aligned}$$

$$\text{Therefore, service time} = \frac{3}{4} \text{ hours}$$

Tom spent  $\frac{3}{4}$  hours or 45 minutes repairing the washing machine.

Check the answer using the original equation:

$$\begin{aligned}\text{Total cost} &= \text{call-out fee} + \text{service fee} + \text{replacement parts} \\ &= \$35 + 3 \times \$12.50 + \$72.50 \\ &= \$145\end{aligned}$$

The solution seems reasonable and answers the question posed.

#### REMEMBER

- It is important to communicate your solution clearly.
- Take care with how equals (=) signs are used.
- After you have written your solution, review your work.
- Reflect on your work.

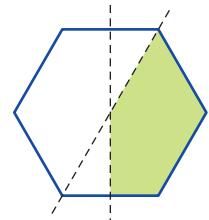
### EXERCISE

## 11H Mixed problems 1

#### PROBLEM SOLVING

- Minh was asked to buy enough ice-creams for everyone in his class. There are 31 people in the class. At the supermarket, identical ice-creams come in packets of 3 or packets of 5. Minh is told that the cost of one ice-cream is the same from either packet. What packets or combination of packets should Minh buy?

- 2** Bronwyn was born on a Wednesday in March. It was not a leap year, so it had 365 days. Investigate the days of the week on which Bronwyn's first birthday could have fallen the next year.
- 3** In a class of 30 students, 22 play basketball and 18 play volleyball. Some of the students play both sports. How many students play volleyball only?
- 4** The product of two consecutive numbers is 1190. What are the two numbers and what is their sum?
- 5** Each of the dotted lines drawn on this rectangular hexagon is an axis of symmetry. Determine the fraction of the hexagon that is shaded.
- 6** Given the digits 987654321 in descending order, place addition symbols (+) between any of the digits to produce an expression that is equivalent to 99. You may use as many addition symbols as you wish, but you may not use any other mathematical symbols or alter the order of the digits. One such example is  $9 + 8 + 7 + 6 + 5 + 43 + 21 = 99$ . Find another possibility.
- 7** Year 12 students at Minnicom College decided to have a 'pizza and pasta day' on the last day of term. Mrs Day decided to offer a limited choice of food and a drink. The restaurant charged her the following: marinara, \$6.00; lasagne, \$6.50; and any small pizza, \$5.00. The local supermarket sold drinks for \$6.40 per dozen.
- If 70 students chose pizzas, 15 chose lasagne and 8 chose marinara, and everyone gets a drink, what is the overall cost of the day?
  - If each student is to be charged the same amount, how much will that be? (Remember to round your answer to the nearest 5c.)
  - If there is any change left over, how much is there, and what would you do with it?
- 8** A skydiver opens his parachute at 846 m above ground level. He has already fallen five-sevenths of the distance to the ground. How high was the plane from which he jumped?
- 9** You have a length of ribbon which is 4.2 metres long. You cut off  $\frac{1}{3}$  of this ribbon. The remaining piece is cut into 14 pieces. What is the length of each piece?
- 10** Your friend is working on the top floor of a very tall office building. She walks up 726 steps from the ground floor to her office. Each step is 23 cm high. How high does she climb (in km) correct to 1 decimal place?



## 11I

# Mixed problems 2

### EXERCISE

## 11I

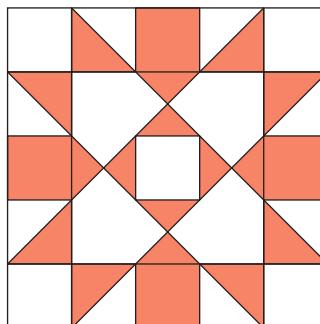
# Mixed problems 2

### PROBLEM SOLVING

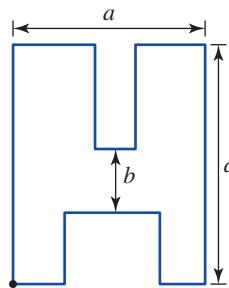
- 1** Two men and two boys wish to cross a river from one side to the other. Their small canoe will carry only one man or two boys. What is the minimum number of canoe trips needed to get everyone across? List the number of people transported during each canoe trip and the destination.
- 2** A *cryptarithm* is a puzzle where each digit of a number has been replaced by a prounumeral or letter. Solve the following cryptarithms by determining the digit that corresponds to each letter to make the calculation true.

$$\begin{array}{r} \text{D} \\ \times \quad \text{D} \\ \hline \text{E D} \end{array}$$

- 3** The carat is a unit of measure used to weigh precious stones. It equals 3.086 grains. How many grains does a 2.9-carat diamond weigh?
- 4** One of the designs for a patchwork motif is made up of squares and right-angled isosceles triangles as shown. What fraction of the whole has been shaded?



- 5** Use the numbers 1, 5, 6 and 7 with any arithmetic operations to result in the number 21.
- 6** Given the following diagram, what is the equation for the perimeter in terms of  $a$  and  $b$ ?



(Diagram not drawn to scale.)

- 7** You are competing in a long jump contest. Your first jump was 3.78 m, your second jump was  $3\frac{8}{9}$  m and your third jump is  $3\frac{4}{5}$  m. You are congratulated on your third jump, as being your best. Do you agree? Explain.
- 8** Find:
- the highest common factor of 36 and 60
  - the highest common factor of  $36xy$  and  $60x^2$
  - the highest common factor of  $x^2y$  and  $xy^2$ .
- 9** Two brothers are going on a trip. They are each carrying a backpack weighing 2 kg. In these backpacks they will carry 4 kg of books, 6 kg of clothes, 3 kg of toiletries, 3 kg of shoes, 2 kg of snacks and 2 kg of water. The older brother offers to carry double the weight carried by his younger brother. Can the older brother divide the luggage such that his backpack weighs double that of his younger brother's backpack?
- 10** By some accounts, the energy saved by recycling one aluminium can equals the amount of energy it takes to run a TV set for 4 hours. How many aluminium cans would have to be recycled to furnish enough energy to operate 68 television sets for 4.25 hours per day for one week?



# 11J Mixed problems 3

## EXERCISE

### 11J Mixed problems 3

#### PROBLEM SOLVING

- If a mother is 37 years old and her daughter is 9 years old, in how many years will the mother be three times as old as the daughter?
- A class of 25 students has 7 more boys than girls. How many boys are there?



- What is the largest five-digit number you can write if each digit must be different and no digit may be prime?
- You purchased some shares over a 4-month period. The price fell by \$4.23 in the first month, rose by \$6.67 in the second month, rose by \$1.35 in the third month and fell by \$3.28 in the fourth month. Did the shares increase or decrease in value over the four months?
- It is said that the average person walks the equivalent of four times around the Earth in a lifetime. The circumference of the Earth is about 40 000 km. If you lived to age 80, approximately how many kilometres per week would you walk to achieve this distance?
- Watchmakers take particular care that the angles between the hands of an analogue clock represent the time accurately. What is the smaller angle between the hour and minute hands when an analogue clock displays the time as ‘ten past nine’?



- 7** Twin primes are pairs of prime numbers that differ by 2. Except for the pair of primes 2 and 3, this is the smallest difference between two prime numbers. The first twin primes are 3 and 5, followed by 5 and 7, then 11 and 13. What other twin primes are there below 100?
- 8** There are five different whole numbers which add to 25. The product of these five numbers is 945. One of the numbers is 1 and another is 9. What are the other three numbers?
- 9** Take the numbers 1, 2, 3 and 4. Use these numbers once each, along with one multiplication sign ( $\times$ ) and one division sign ( $\div$ ) to write an expression which is equal to 2.
- 10** Tricia has 896 songs on her iPod. One-quarter of her music is Hip Hop. She wants to add some songs so that  $\frac{1}{3}$  of her music is Hip Hop. How many Hip Hop tunes does she need to add?



# 12

# Positive and negative integers

- 12A** Integers on the number line
- 12B** Integers on the number plane
- 12C** Addition and subtraction of integers
- 12D** Extension: Multiplication and division of integers
- 12E** Extension: Combined operations

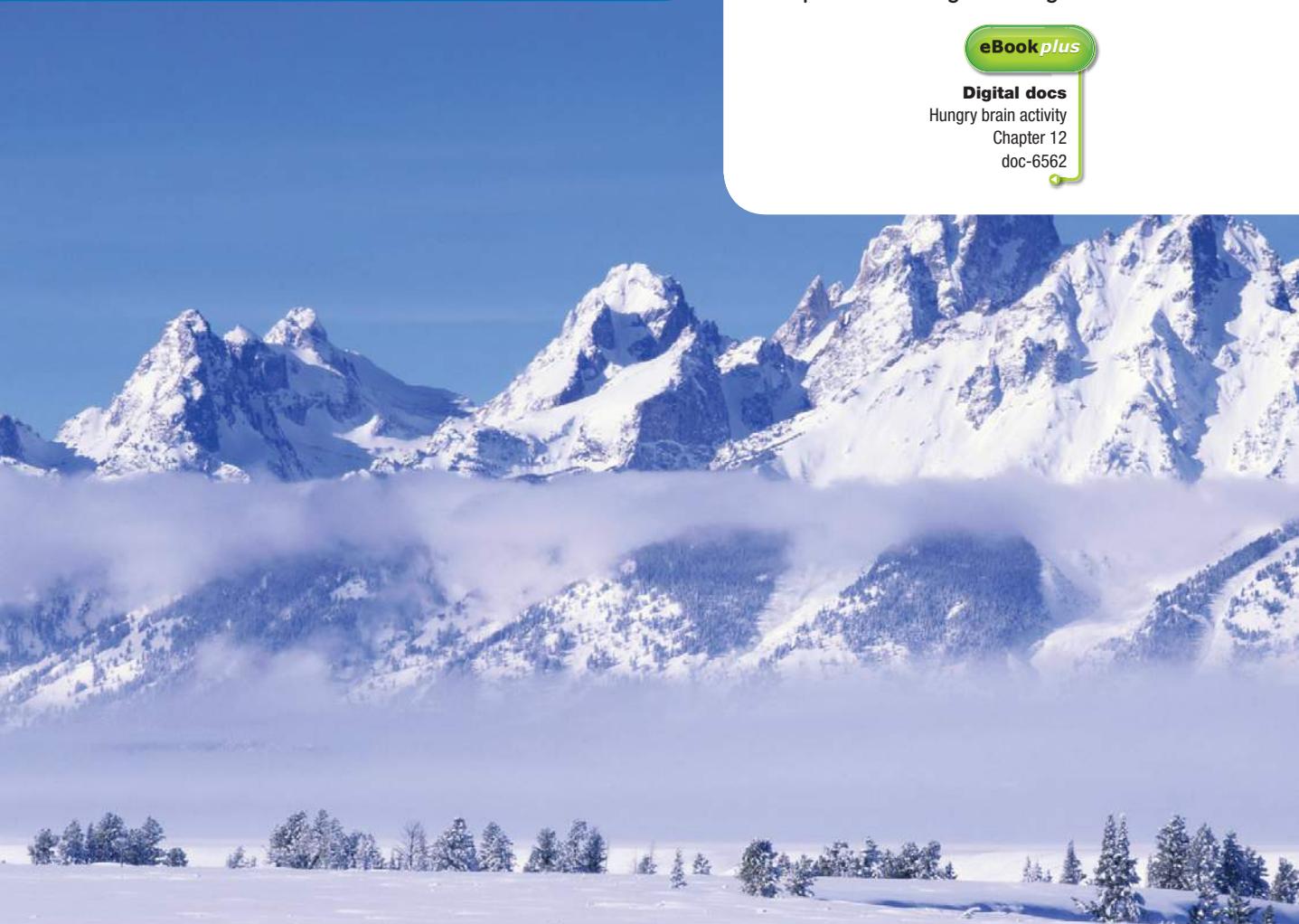
## WHAT DO YOU KNOW?

- 1 List what you know about positive and negative integers. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of positive and negative integers.

eBook plus

### Digital docs

Hungry brain activity  
Chapter 12  
doc-6562



Minimum temperature  $-4^{\circ}\text{C}$   
Maximum temperature  $3^{\circ}\text{C}$

## OPENING QUESTION

By how much is the temperature expected to rise?

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can get extra help by completing the matching SkillsHEET located on your eBookPLUS.



## Using < or > to compare the size of numbers

**Digital docs**

SkillsHEET 12.1

doc-6563

- 1 Complete each statement by inserting the correct symbol, < or >.

a  $5 \quad 1$

b  $17 \quad 71$

c  $0 \quad 10$



## Ascending and descending order

**Digital docs**

SkillsHEET 12.2

doc-6564

- 2 a Arrange the following in ascending order: 5, 120, 0, 3, 15.

- b Arrange the following in descending order: 25, 19, 42, 4, 24.



## Marking numbers on a number line

**Digital docs**

SkillsHEET 12.3

doc-6565

- 3 Draw a number line with 10 equal intervals marked from 0 to 10. Mark the following numbers on the number line with a dot.

a  $3$

b  $0$

c  $8$



## Working with numbers on a number line

**Digital docs**

SkillsHEET 12.4

doc-6566

- 4 Refer to the number line drawn for question 3. Which two numbers are:

a 1 unit away from 3?

b 2 units away from 8?



## Evaluating squares, cubes and cube roots

**Digital docs**

SkillsHEET 12.5

doc-6567

- 5 Evaluate the following.

a  $3^2$

b  $2^3$

c  $\sqrt[3]{27}$



## Order of operations II

**Digital docs**

SkillsHEET 12.6

doc-6568

- 6 Calculate the following using the correct order of operations.

a  $5 + 2 \times 7$

b  $18 \div 3 \times 2$

c  $3^2 - 4 \times 2$



## Operations with fractions

**Digital docs**

SkillsHEET 12.7

doc-6569

- 7 Calculate each of the following.

a  $\frac{1}{3} + \frac{1}{2}$

b  $\frac{3}{5} - \frac{2}{7}$

c  $2\frac{3}{4} + 1\frac{2}{3}$

d  $\frac{5}{6} \times \frac{3}{7}$

e  $\frac{3}{8} \div \frac{1}{4}$

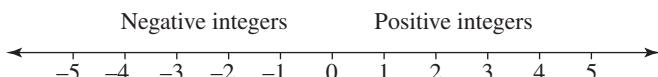
f  $3\frac{2}{5} \div 2\frac{1}{2}$

## 12A Integers on the number line

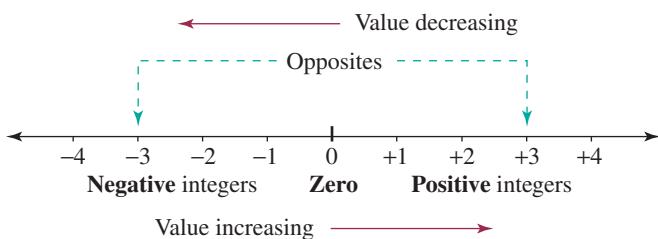
eBook plus

eLesson  
Integers on  
the number  
line  
eles-0040

- **Whole numbers** are the numbers  $0, 1, 2, 3, 4, \dots$
- Sometimes it is necessary to use numbers smaller than 0.
- **Integers** are the numbers  $\dots -3, -2, -1, 0, 1, 2, 3, \dots$
- The integers  $-1, -2, -3, \dots$  are called **negative integers**.
- The integers  $1, 2, 3, \dots$  are called **positive integers** and can be written as  $+1, +2, +3, \dots$
- 0 is neither positive nor negative.
- Integers can be represented on a number line. The positive integers are written to the right of 0 and the negative integers are written to the left of 0.



- As integers have both size (distance from 0) and direction (above or below 0), they can be called **directed numbers**.
- The further a number is to the right of any number on a number line, the larger it is;  $+5$  is bigger than  $+2$ ,  $+2$  is greater than  $0$ ,  $+2$  is greater than  $-5$ .



- The following symbols can be used when comparing integers.  
 $>$  means greater than       $\geq$  means greater than or equal to  
 $<$  means less than       $\leq$  means less than or equal to
- **Opposite integers** are equidistant from 0 but on opposite sides; 5 and  $-5$  are opposites.

### WORKED EXAMPLE 1

Write an integer suggested by the following descriptions.

- a My unit is on the 5th floor.      b A town is 20 m below sea level.

THINK

- a Numbers above 0 are positive.  
 b Numbers below 0 are negative.

WRITE

- a  $+5$  (or 5)  
 b  $-20$

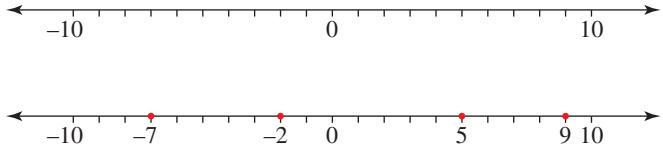
### WORKED EXAMPLE 2

Place the numbers  $9, -2, -7, 5$  on a number line.

THINK

- 1 Draw a number line from  $-10$  to  $10$  so that you can include all of the numbers.
- 2 Mark the values  $9, -2, -7, 5$  on the number line.

WRITE

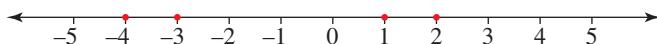
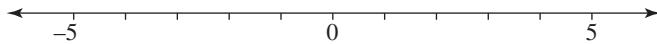


**WORKED EXAMPLE 3**

Use a number line to place the numbers  $-4, 2, -3, 1$  in numerical order.

**THINK**

- 1 A number line from  $-5$  to  $5$  will be long enough to include all of the numbers. Draw the number line.
- 2 Mark the values. As you move from left to right on the number line, the numbers become larger.

**WRITE**

In numerical order:  $-4, -3, 1, 2$

**WORKED EXAMPLE 4**

Write the opposites of:

a  $-2$

b  $3$

c  $4$ .

**THINK**

- a An opposite is the same distance from  $0$ , but has the opposite sign.
- b An opposite is the same distance from  $0$ , but has the opposite sign.
- c An opposite is the same distance from  $0$ , but has the opposite sign.

**WRITE**

a The opposite of  $-2$  is  $2$ .

b The opposite of  $3$  is  $-3$ .

c The opposite of  $4$  is  $-4$ .

**WORKED EXAMPLE 5**

Complete each statement by inserting the correct symbol:  $>$ ,  $<$  or  $=$ .

a  $2 \square 5$

b  $-4 \square -1$

c  $0 \square -3$

d  $6 \square -2$

**THINK**

- a  $2$  is to the left of  $5$  on the number line, so  $2$  is smaller.
- b  $-4$  is to the left of  $-1$  on the number line, so  $-4$  is smaller.
- c  $0$  is to the right of  $-3$  on the number line, so  $0$  is larger.
- d  $6$  is to the right of  $-2$  on the number line, so  $6$  is larger.

**WRITE**

a  $2 < 5$

b  $-4 < -1$

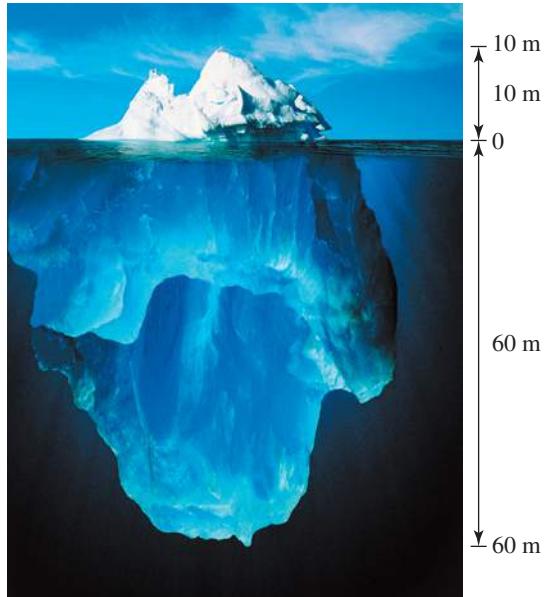
c  $0 > -3$

d  $6 > -2$

## Using positive and negative numbers in our daily lives

- Sometimes it is necessary to use numbers smaller than  $0$  in everyday situations.
- Directed numbers are often used to measure objects from a reference point; for example, temperature above or below zero, or the height above or below sea level, or archaeological measurements below the Earth's surface.

- Directed numbers are used in describing banking transactions. Withdrawals of money from a bank account are given negative signs while deposits are given positive signs.



### WORKED EXAMPLE 6

Write an integer suggested by each of the following examples.

- The temperature was 1 degree below zero.
- Kate has \$500 in her bank account.

#### THINK

- Below zero refers to negative.
- She actually has money so the integer is positive.

#### WRITE

- $-1$
- $+500$

#### REMEMBER

- Integers have both size (distance from 0) and direction (above or below 0).
- The integers  $1, 2, 3, \dots$  are called positive integers.
- The integers  $-1, -2, -3, \dots$  are called negative integers.
- The integer 0 is neither positive nor negative.
- Integers can be placed on a number line.
- On a number line, positive integers are to the right of zero and negative integers are to the left of zero.
- Symbols and their meaning:
 

$>$ means greater than	$\geq$ means greater than or equal to
$<$ means less than	$\leq$ means less than or equal to.

## EXERCISE

## 12A

## Integers on the number line

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 12-A-1

The number line  
doc-1924

## Activity 12-A-2

More number lines  
doc-1925

## Activity 12-A-3

Advanced number  
lines  
doc-1926

## eBook plus

Interactivity  
Directed  
number  
target  
int-0074

## FLUENCY

- 1 WE1 WE6** Write the integer suggested by the following description.

- a The temperature today is 7 degrees below zero.
- b The penthouse is on the 10th floor.
- c The Dead Sea is 422 m below sea level.
- d Mount Kosciuszko is 2228 m above sea level.
- e The Dow Jones dropped 3 points overnight.
- f Chloe borrowed \$30 from her mother.

- 2 WE2** Draw a number line from  $-10$  to  $10$ . Mark the following values on your number line.

A 2      B  $-4$       C  $-9$       D 7      E 5      F  $-1$       G 0      H  $-3$

- 3 WE3** Use a number line to place the following numbers in numerical order.

a  $5, -2, 3, -6, 7$       b  $0, -1, 3, -4, -6$       c  $-5, 7, -2, -3, 1$       d  $-4, 2, 1, -3, 5$

- 4 WE4** Write the opposite of:

a  $-6$       b 7      c 1      d  $-8$

## UNDERSTANDING

- 5 WES** Complete the following by inserting  $<$  or  $>$  in the box.

a  $2 \square -5$       b  $3 \square 7$       c  $-2 \square -6$       d  $6 \square -2$       e  $-1 \square 3$       f  $-10 \square -6$

- 6** From the following lists, select:

i the smallest number      ii the largest number.  
 a  $-3, 7, -5$       b  $2, -4, 5, 3, -2$       c  $7, -10, 5, -2, -4$       d  $-4, -1, 3, 0, -2$

## REASONING

- 7** Overnight, the temperature in Oslo dropped to  $-6^{\circ}\text{C}$ . During the day, the temperature rose by  $4^{\circ}\text{C}$ . What temperature was reached?

- 8** A new apartment building was to include a ground floor reception area. If there were 10 floors of apartments and 3 floors of underground parking, how many buttons were needed in the lift?

- 9 a** Historians use BC and AD when discussing dates.

Explain what is meant by 2000 AD

- b** If the millennium year AD 2000 (+2000) was considered to be zero:

i What year would 2010 be?

ii What year would 1894 be?

- c** Captain James Cook first sighted the east coast of Australia in 1770. In 1788 a fleet of British ships with 1530 passengers, including 736 convicts, landed at Botany Bay. If we consider 1788 as being zero, what directed number could be used to describe the following?

i The year that Captain Cook first sighted the east coast of Australia

ii The year the following settlements were established:

Hobart (Tasmania) 1803

Brisbane River (Queensland) 1824

Swan River (Western Australia) 1829.

- d** Draw a number line from  $-20$  to  $50$  to show your answers to part c.



Port Phillip Bay (Victoria) 1835

Gulf St Vincent (South Australia) 1836

- 10** Lake Mungo in Mungo National Park is a world heritage listed area. Archaeologists estimate that the lake dried out 18 000 years ago, and that before that, Aboriginal people lived on the shores of the lake. Lake Mungo became one of the world's most significant archaeological sites following the discovery there of some of the oldest remains of modern man on Earth.

Explain what  $-60\ 000$  could mean in terms of Australian settlement.



#### REFLECTION

Give an example of when a negative integer would be needed.

## 12B Integers on the number plane

### Positive integers and zero on the number plane

- The number plane, or Cartesian plane, has two axes: the **horizontal** or **x-axis** and the **vertical** or **y-axis**.
- Each point on the number plane is described by its position relative to the x- and y-axes.
- A pair of **coordinates** or an ordered pair  $(x, y)$  fix the position of the point, where  $x$  units is the distance along the x-axis and  $y$  units is the distance along the y-axis.

#### WORKED EXAMPLE 7

In the number plane at right, find the coordinates of:

- a A      b B.

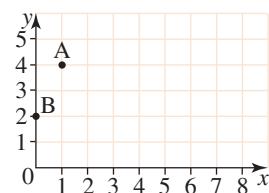
#### THINK

- a Point A is 1 unit to the right of zero along the x-axis and 4 units up the y-axis from zero.  
b Point B is on the y-axis, so is 0 units to the right of zero along the x-axis and is 2 units up the y-axis from zero.

#### WRITE

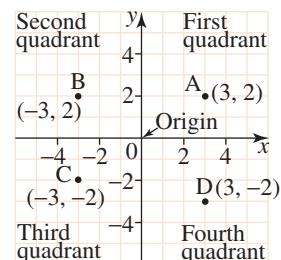
- a A (1, 4)

- b B (0, 2)



### Integers on the number plane

- The axes on the number plane can be extended to include negative integers as shown below.
- The **origin**  $(0, 0)$  is the point where the two axes intersect, dividing the plane into four **quadrants**.
- The quadrant with positive  $y$  and positive  $x$  (the quadrant that you have already been using for graphing) is called Quadrant 1.
- The quadrants are numbered in an anticlockwise direction.



**WORKED EXAMPLE 8**

Write the coordinates and state the quadrant or axis of each point on this number plane.

**THINK**

- a 1 A is 2 units to the right of the origin along the  $x$ -axis and is 3 units down from the origin along the  $y$ -axis.
- 2 It is in the lower right-hand corner.
- b 1 B is 1 unit to the left of the origin along the  $x$ -axis and is 0 units up or down from the origin along the  $y$ -axis.
- 2 It is on the  $x$ -axis.

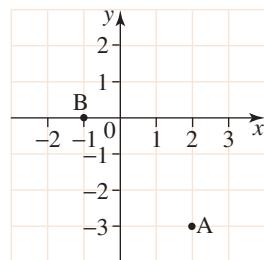
**WRITE**

- a A (2, -3)

A is in the fourth quadrant.

- b B (-1, 0)

B is on the  $x$ -axis.

**REMEMBER**

- 1 The horizontal number line is the  $x$ -axis, and the vertical number line is the  $y$ -axis.
- 2 Points are represented by ordered pairs of integers  $(x, y)$ .
- 3 The origin is the point  $(0, 0)$ , where the axes intersect.
- 4 Each quarter is called a quadrant and is numbered from one to four in an anticlockwise direction beginning with the top right-hand quadrant.

**EXERCISE****12B****Integers on the number plane****INDIVIDUAL PATHWAYS****eBookplus****Activity 12-B-1**

Number planes  
doc-1927

**Activity 12-B-2**

More number planes  
doc-1928

**Activity 12-B-3**

Advanced number  
planes  
doc-1929

**FLUENCY**

Questions 1 and 2 refer to the diagram shown at right.

- 1 **WE7** Find the coordinates of the following.

- |     |     |     |
|-----|-----|-----|
| a A | b B | c C |
| d D | e E |     |

- 2 Write a letter to name each of the points with the following coordinates.

- |          |          |          |
|----------|----------|----------|
| a (0, 0) | b (6, 6) | c (8, 1) |
| d (1, 8) |          |          |

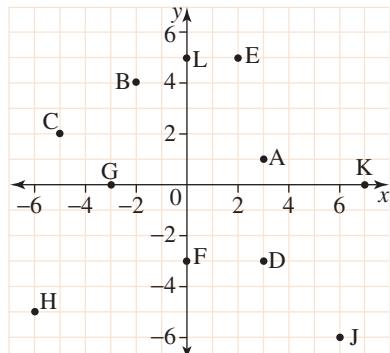
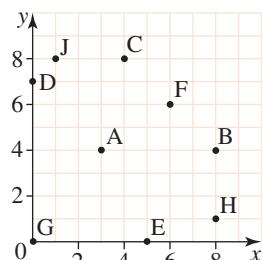
Use the diagram shown at right to find the answers to questions 3 to 11.

- 3 **WE8** Write the coordinates and state the quadrant or axis of each point.

- |     |     |     |
|-----|-----|-----|
| a A | b B | c H |
| d F | e J |     |

- 4 Name the point and give its quadrant or axis.

- |           |           |           |
|-----------|-----------|-----------|
| a (-5, 2) | b (0, 5)  | c (3, -3) |
| d (2, 5)  | e (-3, 0) |           |



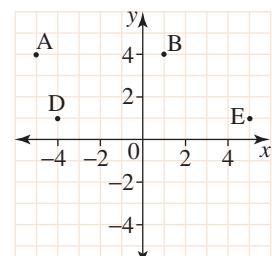
- 5 Give the  $x$ -coordinate of the following.
- a A      b D      c K      d L
- 6 Give the  $y$ -coordinate of the following.
- a C      b J      c G      d F
- 7 List all points lying in the third quadrant in the diagram.

**UNDERSTANDING**

- 8 What is the  $x$ -value of all points on the  $y$ -axis?
- 9 What is the  $y$ -value of all points on the  $x$ -axis?
- 10 In which quadrant do all points show the sign pattern  $(+, -)$ ?
- 11 True or false?
- a F and D have the same  $y$ -coordinate.  
 b A and D have the same  $x$ -coordinate.  
 c The origin has coordinates  $(0, 0)$ .  
 d The point at  $(3, 5)$  is the same point as  $(5, 3)$ .  
 e The point at  $(-5, 4)$  is in the third quadrant.  
 f The point at  $(0, 2)$  must lie on the  $y$ -axis.
- 12 **MC** a The point  $(5, -2)$  lies:
- |                          |                          |                         |
|--------------------------|--------------------------|-------------------------|
| A in the first quadrant  | B in the second quadrant | C in the third quadrant |
| D in the fourth quadrant | E on the $x$ -axis       |                         |
- b The point at  $(0, 4)$  lies:
- |                          |                    |                         |
|--------------------------|--------------------|-------------------------|
| A in the first quadrant  | B on the $y$ -axis | C in the third quadrant |
| D in the fourth quadrant | E on the $x$ -axis |                         |
- c The point  $(-4, 5)$  lies:
- |                          |                          |                         |
|--------------------------|--------------------------|-------------------------|
| A in the first quadrant  | B in the second quadrant | C in the third quadrant |
| D in the fourth quadrant | E on the $x$ -axis       |                         |
- 13 Draw up a number plane with both axes scaled from  $-6$  to  $6$ . Plot the points listed and join them with straight lines in the order given. Name the completed shape.
- a  $(5, 5), (3, 2), (-2, 2), (0, 5), (5, 5)$   
 b  $(4, -1), (4, -5), (-1, -3), (4, -1)$   
 c  $(-4, 4), (-2, 1), (-4, -5), (-6, 1), (-4, 4)$   
 d  $(-2, 1), (1, 1), (1, -2), (-2, -2), (-2, 1)$

**REASONING**

- 14 a Find the coordinates of a point, C, so that ABCD is a parallelogram.  
 b Find the coordinates of a point, F, so that DBEF is a kite shape.  
 c Show that the point  $(4, -1)$  lies on the line through D and the origin.  
 d List 2 points on the line joining D to E.  
 e Give the coordinates of a point, T, in the third quadrant which would complete the isosceles triangle ADT.

**REFLECTION**

If you swap the  $x$ - and  $y$ -coordinates of a point, it may change to another quadrant. What happens to points from each quadrant if you swap their coordinates?

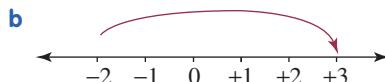
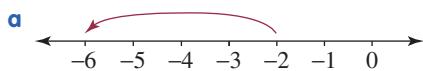
# 12C Addition and subtraction of integers

## Addition of integers

- A number line can be used to add integers, as demonstrated in Worked example 9. To add a positive integer, move to the right. To add a negative integer, move to the left.
- A sign model can also be used to add integers as demonstrated in Worked example 10. In the sign model, a '+' and a '-' cancel each other out.

### WORKED EXAMPLE 9

Write number sentences to show the addition problems suggested by the following diagrams:



#### THINK

- Write the addition statement suggested by the diagram. (The first integer is 2 units to the *left* of zero, so start at -2. The second integer is 4 units to the *left* of -2, finishing at -6.) Find the answer.
- Write the addition statement suggested by the diagram. (The first integer is 2 units to the *left* of zero so start at -2. The second integer is 5 units to the *right* of -2, finishing at +3.) Find the answer.

#### WRITE

$$\begin{aligned} \text{a } & -2 + -4 = -6 \\ \text{b } & -2 + +5 = +3 \end{aligned}$$

### WORKED EXAMPLE 10

Draw a sign model for each of the following and state the result.

$$\begin{aligned} \text{a } & -4 + +1 \\ \text{b } & +5 + -3 \end{aligned}$$

#### THINK

- Show  $-4$  as four negative signs.
  - Show  $+1$  as one positive sign.
  - Cancel out the opposite signs.
  - Write the answer.
- Show  $+5$  as five positive signs.
  - Show  $-3$  as three negative signs.
  - Cancel out the opposite signs.
  - Write the answer.

#### WRITE

$$\begin{aligned} \text{a } & \begin{array}{c} \cancel{-} \cancel{-} \cancel{-} \cancel{-} \\ + \\ \cancel{+} \cancel{+} \cancel{+} \\ -4 + +1 = -3 \end{array} \\ \text{b } & \begin{array}{c} + + + + + \\ \cancel{-} \cancel{-} \cancel{-} \\ + + + \\ + + + \\ +5 + -3 = +2 \end{array} \end{aligned}$$

## Subtraction of integers

- Subtracting a number gives the same result as *adding its opposite*.
- By developing and extending a pattern, we can show that subtracting negatives has the same effect as adding a positive; that is, subtracting a negative is the same as adding its inverse. Look at the following pattern.

$7 - 4 =$	3
$7 - 3 =$	4
$7 - 2 =$	5
$7 - 1 =$	6
$7 - 0 =$	7
$7 - (-1) =$	8
$7 - (-2) =$	9
$7 - (-3) =$	10
$7 - (-4) =$	11

It can be seen from the table that subtracting a negative is the same as adding its inverse; for example,  $7 - (-2) = 9 = 7 + 2$ .

### WORKED EXAMPLE 11

Calculate the following.

a  $2 - 5$

b  $-3 - 6$

c  $-5 - -3$

d  $-5 - -4$

#### THINK

- a 1 Write the question.  
 2 Rewrite, changing subtraction to addition of the opposite integer.  
 3 Add, using the addition rule for addition of integers.
- b 1 Write the question.  
 2 Rewrite, changing subtraction to addition of the opposite integer.  
 3 Add, using the addition rule for addition of integers.
- c 1 Write the question.  
 2 Rewrite, changing subtraction to addition of the opposite integer.  
 3 Add, using the addition rule for addition of integers.
- d 1 Write the question.  
 2 Rewrite, changing subtraction to addition of the opposite integer.  
 3 Add, using the addition rule for addition of integers.

a  $2 - 5 = 2 - +5$

$= 2 + -5$

$= -3$

b  $-3 - 6 = -3 - +6$

$= -3 + -6$

$= -9$

c  $5 - -3$

$= 5 + +3$

$= 5 + 3$

$= 8$

d  $-5 - -4 = -5 + +4$

$= -5 + 4$

$= -1$

### WORKED EXAMPLE 12

Evaluate the algebraic expression  $a + b - c$ , if  $a = -2$ ,  $b = 1$  and  $c = -5$ .

#### THINK

- 1 Write the expression.  
 2 Replace each pronumeral with the appropriate integer.  
 3 Evaluate the expression; that is:  
 (a) rewrite, changing subtraction to addition of the opposite integer  
 (b) add, using the addition rule for addition of integers.

#### WRITE

$$\begin{aligned} a + b - c \\ = -2 + 1 - -5 \\ = -1 - -5 \\ = -1 + +5 \\ = 4 \end{aligned}$$

## WORKED EXAMPLE 13

A news flash in Freezonia announced that there had been a record drop in temperature overnight. At 6 pm the temperature was  $10^{\circ}\text{C}$  and by 4 am it had fallen  $25^{\circ}\text{C}$ . What was the temperature at 4 am?

## THINK

- 1 Write the original temperature. Decide whether the temperature rose (addition) or fell (subtraction). Write the number sentence.
- 2 Find the answer.
- 3 Write the answer in a sentence.

## WRITE

$$10 - 25 \\ = 10 + -25$$

$$= -15$$

The temperature in Freezonia at 4 am was  $-15^{\circ}\text{C}$ .

## eBook plus

**Interactivity**  
Working with integers  
int-2351

## REMEMBER

1. To add integers, use a number line, number sentences or sign models.
2. Adding opposites always results in zero.
3. Assume that the number is positive if there is no sign.
4. To subtract an integer, *add its opposite*:  $a - b = a + -b$ .
5. Only the number *after* the subtraction symbol changes to its opposite.

$$3 - 5 = 3 + -5 \\ = -2$$

## EXERCISE

## 12C

## Addition and subtraction of integers

## INDIVIDUAL PATHWAYS

## eBook plus

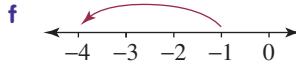
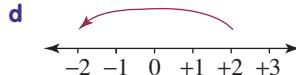
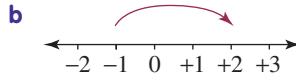
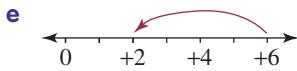
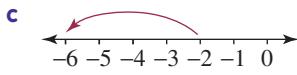
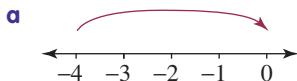
**Activity 12-C-1**  
Adding and subtracting integers  
doc-1930

**Activity 12-C-2**  
More addition and subtraction of integers  
doc-1931

**Activity 12-C-3**  
Advanced addition and subtraction of integers  
doc-1932

## FLUENCY

- 1 **WE9** Write number sentences to show the addition problems suggested by the following diagrams.



- 2 **WE10** Draw a sign model for each of the following and state the result.

a  $+2 + -3$

b  $+3 + -4$

c  $+4 + -4$

d  $+3 + -2$

e  $-4 + +2$

f  $-5 + +3$

- 3 Copy and complete these number sentences. (Draw a number line or other model if you wish.)

a  $5 + -2$

b  $-3 + -4$

c  $-2 + 2$

d  $6 + -5$

e  $-5 + 5$

f  $4 + -6$

g  $-5 + 7$

h  $6 + -9$

i  $-4 + 6$

j  $3 + -3$

k  $-8 + -2$

l  $0 + -6$

- 4 Write the answer for each of the following.

a  $-5 + -2$

b  $-6 + 4$

c  $-8 + 8$

d  $3 + -7$

e  $-3 + 7$

f  $-3 + -7$

g  $-8 + 12$

h  $19 + -22$

i  $-64 + -36$

j  $-80 + 90$

k  $-2 + 4$

l  $-15 + -7$

- 5 Copy and complete this addition table.

+	-13	5
21		
-18		

- 6 **WE11**) Calculate the following.

a) $7 - 5$	b) $8 - -2$	c) $-4 - 6$	d) $-6 - -8$
e) $1 - 10$	f) $-5 - 5$	g) $-8 - -8$	h) $0 - 4$
i) $0 - -3$	j) $-10 - -20$	k) $-11 - 3$	l) $-5 - -5$

- 7 Calculate the following mentally and write just the answer.

a) $-7 - 3$	b) $8 - -5$	c) $-6 - -9$	d) $0 - -12$
e) $-8 - 8$	f) $3 - 20$	g) $20 - -3$	h) $-4 - 8$

- 8 Find the missing number in these incomplete number sentences.

a) $8 + \underline{\hspace{1cm}} = 0$	b) $-2 + \underline{\hspace{1cm}} = -8$	c) $\underline{\hspace{1cm}} + -6 = 4$
d) $\underline{\hspace{1cm}} + 5 = -2$	e) $\underline{\hspace{1cm}} + -5 = -2$	f) $\underline{\hspace{1cm}} - 7 = -6$
g) $-8 - \underline{\hspace{1cm}} = -17$	h) $-8 - \underline{\hspace{1cm}} = 17$	i) $\underline{\hspace{1cm}} - -2 = 7$

- 9 **WE12**) Evaluate each algebraic expression, if  $a = 5$ ,  $b = -2$ ,  $c = -8$ .

a) $a + b$	b) $b + c$	c) $a - b$	d) $b - c$	e) $a + b + c$
f) $a - b - c$	g) $a - (b + c)$	h) $c - b - a$	i) $a - b + c$	

### UNDERSTANDING

- 10 a) Copy and complete the addition table shown at right.  
 b) What pattern is shown along the leading (dotted) diagonal?  
 c) What pattern is shown along the other (unmarked) diagonal?  
 d) Is the chart symmetrical about the leading diagonal?  
 e) Comment on any other number patterns you can see.

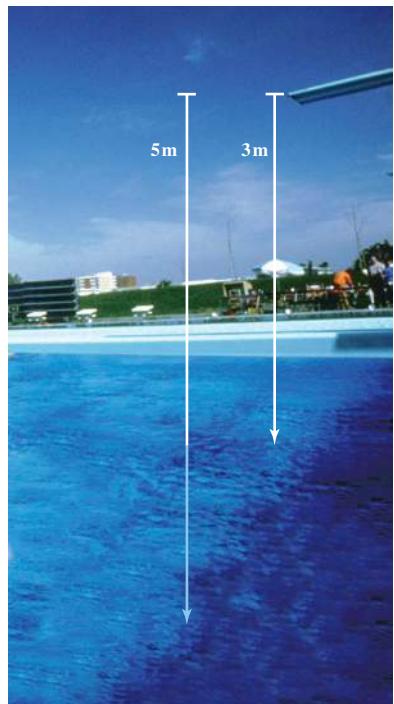
- 11 Write the number that is:

- a) 6 more than  $-2$
- b) 5 more than  $-8$
- c)  $8^\circ\text{C}$  above  $-1^\circ\text{C}$
- d)  $3^\circ\text{C}$  below  $2^\circ\text{C}$
- e) 3 to the right of  $-4$
- f) 4 to the left of  $-3$ .

- 12 Model each situation with an integer number sentence that shows the result of the following.

- a) From ground level, a lift went down 2 floors, then down another 3 floors.
- b) From ground level, a lift went down 3 floors, then up 5 floors.
- c) From ground level, a lift went up 5 floors, then down 6 floors.
- d) Australia was 50 runs behind, then made another 63 runs.
- e) An Olympian dived down 5 metres from a board 3 metres above water level.
- f) At 5.00 pm in Falls Creek the temperature was  $1^\circ\text{C}$ . It then fell 6 degrees by 11.00 pm.
- g) A submarine at sea level dived 50 metres, then rose 26 m.
- h) An account with a balance of \$200 had \$350 withdrawn from it.

+	-2	-1	0	1	2
-2	-	-3			
-1		-	0		
0	-2		-		
1				-	
2					-



- 13 MC** From ground level, a lift went down 2 floors, then down another 3 floors to a level 5 floors below the ground. The number sentence that describes this situation is:

A  $2 + 3 = 5$

B  $-2 + -3 = -5$

C  $-2 + 3 = 1$

D  $2 + -3 = -1$

E  $-3 + -2 = 5$

- 14** Describe a situation to fit each of the number sentences below.

a  $-3 + -2 = -5$

b  $-10 + -40 = -50$

c  $2 + -6 = -4$

d  $-20 + 20 = 0$

e  $-8 + 10 = 2$

- 15** Are these number sentences true or false?

a  $7 - 9 = 7 + -9$

b  $0 - 8 = -8 - 0$

c  $8 - 12 = -12 + 8$

d  $0 - p = -p$

- 16** Show working to calculate the following.

*Remember:* Order of operations: brackets,  $\times$  or  $\div$  moving from left to right,  $+$  or  $-$  moving from left to right.

a  $6 + -3 - 2$

b  $-9 + (5 - 7)$

c  $3 - (8 - -2)$

d  $6 + -8 + -5 - 2$

e  $4 + (-8 + 10)$

f  $16 - (-2 - -6)$

- 17 MC** a  $7 + -4 - -2$  is equal to:

A 9

B 1

C 13

D 5

E  $-5$

b  $6 - (2 + -3)$  is equal to:

A 1

B 7

C 11

D 5

E  $-7$

- 18 WE13** The temperature in the freezer was  $-20^\circ\text{C}$ . Just before he went to bed, Dennis had a spoonful of ice-cream and left the freezer door ajar all night. The temperature in the front of the freezer rose  $18^\circ\text{C}$  and the ice-cream melted. What was the temperature in the front of the freezer when his mother found the ice-cream in the morning?

### REASONING

- 19** Jill is climbing up a steep and slippery path to fetch a bucket of water. When she is 6 m above her starting point, she slips back 1 m, grasps some bushes by the side of the path and climbs 7 m more to a flat section. How far above her starting point is she when she reaches the resting place?

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WorkSHEET 12.1  
doc-1922



### REFLECTION

What effect does adding a number to its opposite have? What effect does subtracting the opposite have?

# 12D Extension: Multiplication and division of integers

## Multiplication of integers

- Consider the number pattern table below.

$\times$	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							

- Notice that multiplying by 0 always results in 0.
- Notice that multiplying by 1 does not change the value of the number.

$\times$	-3	-2	-1	0	1	2	3
3				0	3	6	9
2				0	2	4	6
1	-3	-2	-1	0	1	2	3
0	0	0	0	0	0	0	0
-1				0	-1		
-2				0	-2		
-3				0	-3		

- Notice that multiplying by -1 changes the sign of the number.

$\times$	-3	-2	-1	0	1	2	3
3	-9	-6	-3	0	3	6	9
2	-6	-4	-2	0	2	4	6
1	-3	-2	-1	0	1	2	3
0	0	0	0	0	0	0	0
-1	3	2	1	0	-1	-2	-3
-2			2	0	-2	-4	-6
-3			3	0	-3	-6	-9

- We already know how to complete part of the table.

$\times$	-3	-2	-1	0	1	2	3
3				0	3	6	9
2				0	2	4	6
1				0	1	2	3
0				0	0	0	0
-1				0	-1	-2	-3
-2				0	-2	-4	-6
-3				0	-3	-6	-9

- If we continue the pattern, we get the following.

$\times$	-3	-2	-1	0	1	2	3
3	-9	-6	-3	0	3	6	9
2	-6	-4	-2	0	2	4	6
1	-3	-2	-1	0	1	2	3
0	0	0	0	0	0	0	0
-1				0	-1	-2	-3
-2				0	-2	-4	-6
-3				0	-3	-6	-9

- Completing the pattern results in the following.

$\times$	-3	-2	-1	0	1	2	3
3	-9	-6	-3	0	3	6	9
2	-6	-4	-2	0	2	4	6
1	-3	-2	-1	0	1	2	3
0	0	0	0	0	0	0	0
-1	3	2	1	0	-1	-2	-3
-2	6	4	2	0	-2	-4	-6
-3	9	6	3	0	-3	-6	-9

- From this table, we can see the rules for multiplication:
  - When multiplying two integers with the *same sign*, the answer is *positive*.
  - When multiplying two integers with *different signs*, the answer is *negative*.

**WORKED EXAMPLE 14****Evaluate:**

a  $-5 \times +2$

b  $-4 \times -6$ .

**THINK**

- a 1 Write the question.  
   2 Negative  $\times$  positive = negative.
- b 1 Write the question.  
   2 Negative  $\times$  negative = positive.

**WRITE**

$$\begin{aligned} \text{a } -5 \times +2 \\ &= -10 \\ \text{b } -4 \times -6 \\ &= 24 \end{aligned}$$

**Division of integers**

- The division operation is the *inverse*, or opposite, of multiplication.

$$\begin{array}{lll} \text{Since } 3 \times 2 = 6 & \text{then } 6 \div 3 = 2 & \text{and } 6 \div 2 = 3. \\ \text{Since } 3 \times -2 = -6 & \text{then } -6 \div 3 = -2 & \text{and } -6 \div -2 = 3. \\ \text{Since } -2 \times -3 = 6 & \text{then } 6 \div -3 = -2 & \text{and } 6 \div -2 = -3. \end{array}$$

- The rules for division are:

- When dividing two integers with the *same sign*, the answer is *positive*.
- When dividing two integers with *different signs*, the answer is *negative*.

**WORKED EXAMPLE 15****Calculate:**

a  $10 \div -2$

b  $-12 \div 4$

c  $-20 \div -5$ .

**THINK**

- a 1 Write the question.  
   2 Look at the directions to decide whether the answer is positive or negative. If they are different, the answer is negative and if they are the same the answer is positive.  
     Divide.
- b 1 Write the question.  
   2 Different directions give a negative answer. Divide.
- c 1 Write the question.  
   2 Divide using the kind of directions to indicate the sign.

**WRITE**

$$\begin{aligned} \text{a } 10 \div -2 \\ &= -5 \\ \text{b } -12 \div 4 \\ &= -3 \\ \text{c } -20 \div -5 \\ &= 4 \end{aligned}$$

**WORKED EXAMPLE 16****Simplify:**

a  $\frac{-16}{+2}$

b  $-5 \times \frac{4}{20}$ .

## THINK

- a 1 Write the question.

Note:  $\frac{-16}{+2}$  is the same as  $-16 \div +2$ .

- 2 Evaluate the expression.

Note: negative  $\div$  positive = negative

- b 1 Write the question.

- 2 Write the integer as a fraction with a denominator of 1 and simplify by cancelling.

- 3 Multiply the numerators then multiply the denominators and simplify.

Note: negative  $\div$  positive = negative

- 4 Write the answer.

## WRITE

a  $\frac{-16}{+2}$

$= -8$

b  $-5 \times \frac{4}{20}$   
 $= \frac{-5}{1} \times \frac{4^1}{20^5}$

$= \frac{-5}{5}$

$= -1$

## REMEMBER

- When multiplying two integers with the *same sign*, the answer is *positive*.
- When multiplying two integers with *different signs*, the answer is *negative*.
- When dividing two integers with the *same sign*, the answer is *positive*.
- When dividing two integers with *different signs*, the answer is *negative*.

## EXERCISE

## 12D

## Extension: Multiplication and division of integers

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 12-D-1

Multiplying and dividing integers  
doc-1933

## Activity 12-D-2

More multiplication and division of integers  
doc-1934

## Activity 12-D-3

Advanced multiplication and division of integers  
doc-1935

## FLUENCY

- 1 WE14 Evaluate the following.

a  $2 \times -5$

d  $6 \times -5$

g  $-5 \times -5$

j  $-24 \times 1$

m  $-3 \times 18$

b  $-6 \times 3$

e  $-2 \times -3$

h  $0 \times -7$

k  $10 \times -1$

n  $-20 \times -10$

c  $-7 \times 9$

f  $-4 \times -5$

i  $-8 \times -1$

l  $-15 \times 2$

o  $-6 \times -6$

- 2 WE15 Calculate the following.

a  $-8 \div -2$

d  $12 \div -3$

g  $-90 \div -10$

j  $-6 \div -1$

m  $-184 \div 2$

p  $129 \div -3$

b  $8 \div -2$

e  $-15 \div 5$

h  $88 \div -11$

k  $0 \div -4$

n  $-125 \div -5$

q  $-284 \div 4$

c  $-8 \div 2$

f  $-16 \div -8$

i  $-6 \div 1$

l  $-84 \div 4$

o  $-67 \div -1$

r  $336 \div -6$

- 3 WE16 Simplify the following.

a  $\frac{-6}{2}$

b  $\frac{-24}{-8}$

c  $\frac{-8}{8}$

d  $3 \times \frac{-2}{-6}$

e  $4 \times \frac{-5}{10}$

f  $-9 \times \frac{-3}{18}$

- 4 Simplify the following.

a  $2 \times -3 \times 4$

b  $-4 \times -3 \times 3$

c  $-8 \times 9 \times -2$

- 5 Fill in the missing numbers.

a  $-6 \times -3 = \underline{\hspace{2cm}}$

b  $6 \times \underline{\hspace{2cm}} = -18$

c  $\underline{\hspace{2cm}} \times 3 = -18$

d  $-8 \times \underline{\hspace{2cm}} = -8$

e  $-8 \times \underline{\hspace{2cm}} = 8$

f  $-8 \times \underline{\hspace{2cm}} = 0$

g  $-1 \times \underline{\hspace{2cm}} = 1$

- 6** Fill in the missing numbers in these number sentences.

<b>a</b> $-21 \div \underline{\quad} = -7$	<b>b</b> $\underline{\quad} \div -8 = -4$	<b>c</b> $\underline{\quad} \div -9 = 8$
<b>d</b> $-11 \div \underline{\quad} = 1$	<b>e</b> $\underline{\quad} \div -7 = 0$	<b>f</b> $\underline{\quad} \div -4 = 4$
<b>g</b> $-42 \div \underline{\quad} = -6$	<b>h</b> $-96 \div \underline{\quad} = 2$	<b>i</b> $-150 \div -25 = \underline{\quad}$

- 7** Evaluate each algebraic expression if  $c = -2$ ,  $d = -5$ .

<b>a</b> $c + d$	<b>b</b> $c \times d$ (or $cd$ )	<b>c</b> $d - c$	<b>d</b> $dc$
<b>e</b> $3cd$	<b>f</b> $3c + d$	<b>g</b> $\frac{cd}{10}$	

### UNDERSTANDING

- 8** Copy and complete the following.

<b>a</b> $-4 + -4 =$	<b>b</b> $-2 + -2 + -2 =$
$2 \times -4 =$	$3 \times -2 =$
<b>c</b> $-3 + -3 =$	<b>d</b> $-5 + -5 + -5 + -5 =$
$2 \times -3 =$	$4 \times -5 =$

- 9 MC** The missing numbers in the following number sentences could be:

<b>a</b> $16 \div \underline{\quad} =$	<b>b</b> $\underline{\quad} \div \underline{\quad} = -5$	<b>c</b> $-4, 4$	<b>d</b> $-2, 8$	<b>e</b> $1, -16$
<b>A</b> $2, -8$	<b>B</b> $-2, -8$	<b>C</b> $-4, 4$	<b>D</b> $-2, 8$	<b>E</b> $1, -16$
<b>A</b> $-15, 3$	<b>B</b> $15, 5$	<b>C</b> $25, 5$	<b>D</b> $-30, -6$	<b>E</b> $-25, -5$

- 10** Simplify each algebraic expression.

<b>a</b> $3 \times 2 \times p$	<b>b</b> $-3 \times 4 \times t$	<b>c</b> $-2 \times -5 \times b$	<b>d</b> $2 \times a \times 4$
<b>e</b> $-3 \times c \times 5$	<b>f</b> $-2 \times d \times -7$	<b>g</b> $6 \times a \times -2 \times b$	<b>h</b> $-5x \times -2g$

- 11** If  $a = -6$ ,  $b = -3$ ,  $c = 2$ , evaluate the following.

<b>a</b> $a \div b$	<b>b</b> $\frac{a}{c}$	<b>c</b> $a \div b \div c$
<b>d</b> $\frac{bc}{a}$	<b>e</b> $\frac{ab}{c}$	<b>f</b> $\frac{ab}{bc}$
<b>g</b> $\frac{a}{b} + c$	<b>h</b> $\frac{a}{cb}$	<b>i</b> $\frac{(a + bc)}{b}$

- 12 MC** Six people each owe the bank \$50. The combined total of the six accounts is:

<b>A</b> \$300	<b>B</b> -\$50	<b>C</b> \$50	<b>D</b> $-\$ \frac{6}{5}$	<b>E</b> -\$300
----------------	----------------	---------------	----------------------------	-----------------

### REASONING

- 13** Dawn was taking her younger brother and sister to the local pool for a swim but she had spent all her money. It cost \$2 for each person so she borrowed the money from her parents. How much did she have if she swam too?

- 14** A spider is running down the stairs from the first floor of an old lady's house to the basement below. It stops every 5 steps to catch a fly. If there are 26 steps above ground and 14 below, how many flies does the spider catch?



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Digital docs

WorkSHEET 12.2

doc-1923

### REFLECTION

When you multiply two integers, is the result an integer? What about when you divide two integers?

## 12E Extension: Combined operations

- The mathematical rules about order of operations apply when we work with directed numbers.
- When we are simplifying numbers, the order of operations, BIDMAS, helps us to remember the correct order in which we should perform the various operations. This means we do brackets first; then powers or indices; then  $\times$  or  $\div$  (working from left to right); and finally,  $+$  and  $-$  (working from left to right).

### WORKED EXAMPLE 17

Calculate  $58 - (2 \times -8 + 3^2)$  using the correct order of operations.

#### THINK

- Write the question.
- Working inside grouping symbols, (Brackets) simplify the squared term (Exponent).
- Multiplication within the brackets. (Multiplication).
- Addition within the brackets. (Addition).
- When brackets have been removed work the subtraction outside the brackets. (Subtraction).

#### WRITE

$$\begin{aligned} & 58 - (2 \times -8 + 3^2) \\ & = 58 - (2 \times -8 + 9)* \\ & = 58 - (-16 + 9) \\ & = 58 - -7 \\ & = 58 + +7* \\ & = 65 \end{aligned}$$

\*These steps could be omitted with practice.

### WORKED EXAMPLE 18

Evaluate  $5a \div b$ , if  $a = -20$  and  $b = 4$ .

#### THINK

- Write the expression as given.
- Substitute the given value for each pronumeral, inserting operation signs as required.
- Perform the operations as they occur from left to right.

#### WRITE

$$\begin{aligned} & 5a \div b \\ & = 5 \times -20 \div 4 \\ & = -100 \div 4 \\ & = -25 \end{aligned}$$

#### REMEMBER

When we are simplifying numbers, the order of operations, BIDMAS, helps us to remember the correct order in which we should perform the various operations. This means we do brackets first; then powers or indices; then  $\times$  or  $\div$  (working from left to right); and finally,  $+$  and  $-$  (working from left to right).

## EXERCISE

12E

## Extension: Combined operations

## INDIVIDUAL PATHWAYS

eBook plus

**Activity 12-E-1**  
Combined operations  
with integers  
doc-1936

**Activity 12-E-2**  
More combined  
operations with  
integers  
doc-1937

**Activity 12-E-3**  
Advanced combined  
operations with  
integers  
doc-1938

## FLUENCY

- 1 **WE17** Calculate the following, using the correct order of operations.

- |                               |                        |                                |
|-------------------------------|------------------------|--------------------------------|
| a $6 + 3 \times -4$           | b $18 - 12 \div -3$    | c $8 + -4 - 10$                |
| d $17 - 3 + -8$               | e $6 \times -3 \div 9$ | f $72 \div 8 \times -3$        |
| g $7 + (-3 - 4)$              | h $(6 + 3) \div -9$    | i $-3 \times -2 + 3 \times -1$ |
| j $-6 \times 5 - 2 \times -6$ | k $-4 \times (6 - -4)$ | l $(-8 + 3) \times -7$         |

- 2 Evaluate each of the following.

- |                                      |  |
|--------------------------------------|--|
| a $4 + 7 \times -3 - 2$              | b $-6 - 4 + (-3)^2$                      |
| c $(-2)^3 - 3 \times -2$             | d $3 + (2 - 8) + -6$                     |
| e $-8 \div 2 + (-2)^2$               | f $-4 \times -8 - [2 + (-3)^2]$          |
| g $(-7 + 5) - -24 \div 6$            | h $-15 \div (2 - 5) - 10$                |
| i $54 \div -6 + 8 \times -9 \div -4$ | j $(9 - -6) \div -5 + -8 \times 0$       |
| k $-7 + -7 \div -7 \times -7 - -7$   | l $-9 \times -5 - (3 - -2) + -48 \div 6$ |

- 3 **WE18** Evaluate each of the following.

- |                                     |   |
|-------------------------------------|---|
| a $2x + 3x$ , if $x = -4$           | b $5 + 3d$ , if $d = -2$                        |
| c $-5b - 3$ , if $b = -7$           | d $a(b + c)$ , if $a = 6$ , $b = -2$ , $c = -4$ |
| e $x^3 - y$ , if $x = -4$ , $y = 4$ |   |

## UNDERSTANDING

- 4 **MC** The expression  $6 + 2 \times -5 - -10 \div 2$  is equal to:

- A  $-15$       B  $-35$       C  $-60$       D  $1$       E  $3$

- 5 Model each situation with integers, then find the result.

- a A submarine dives 100 m from sea level, rises 60 m then dives 25 m. What is its final position?
- b Jemma has \$274 in the bank, then she makes the following transactions: 2 withdrawals of \$68 each, and then 3 deposits of \$50 each.
- c If 200 boxes of apples were each 3 short of the stated number of 40 apples, what was the overall shortfall in the number of apples?
- d A person with a mass of 108 kg wants to reduce his mass to 84 kg in 3 months. What average mass reduction is needed per month?

## REASONING

- 6 Local time in Melbourne is 3 hours ahead of Singapore time, which is 5 hours behind Auckland (NZ) time. Auckland is 11 hours ahead of Berlin (Germany) time. What is the time difference between:
- a Melbourne and Berlin?
  - b Singapore and Berlin?
- 7 Merlin is riding his bike east at a steady 10 km/h, while Morgan is riding her bike west at a steady 8 km/h. They pass each other on Backpedal Bridge at 12 noon. (Assume that east is the positive direction and west is negative and that time *before* noon is negative and *after* noon is positive.)
- a What is the location of each person with respect to the bridge at 9 am?
  - b What are their locations with respect to the bridge at 2 pm?
  - c How far apart were they at 10 am?
  - d How far apart will they be at 4 pm?

## REFLECTION

What do you need to be careful about when you are applying operation order to positive and negative integers?

# Summary

## Integers on the number line

- Integers have both size (distance from 0) and direction (above or below 0).
- The integers 1, 2, 3, ... are called positive integers.
- The integers -1, -2, -3, ... are called negative integers.
- The integer 0 is neither positive nor negative.
- Integers can be placed on a number line.
- On a number line, positive integers are to the right of zero and negative integers are to the left of zero.
- Symbols and their meaning:
 

> means greater than	$\geq$ means greater than or equal to
< means less than	$\leq$ means less than or equal to.

## Integers on the number plane

- The horizontal number line is the  $x$ -axis, and the vertical number line is the  $y$ -axis.
- Points are represented by ordered pairs of integers  $(x, y)$ .
- The origin is the point  $(0, 0)$ , where the axes intersect.
- Each quarter is called a quadrant and is numbered from one to four in an anticlockwise direction beginning with the top right-hand quadrant.

## Addition and subtraction of integers

- To add integers, use a number line, number sentences or sign models.
- Adding opposites always results in zero.
- Assume that the number is positive if there is no sign.
- To subtract an integer, *add its opposite*:  $a - b = a + -b$
- Only the number *after* the subtraction symbol changes to its opposite.

$$\begin{aligned} 3 - 5 &= 3 + -5 \\ &= -2 \end{aligned}$$

## Extension: Multiplication and division of integers

- When we multiply two integers with the *same sign*, the answer is *positive*.
- When we multiply two integers with *different signs*, the answer is *negative*.
- When we divide two integers with the *same sign*, the answer is *positive*.
- When we divide two integers with *different signs*, the answer is *negative*.

## Extension: Combined operations

- When we are simplifying numbers, the order of operations, BIDMAS, helps us to remember the correct order in which we should perform the various operations. This means we do brackets first; then powers or indices; then  $\times$  or  $\div$  (working from left to right); and finally,  $+$  and  $-$  (working from left to right).

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?

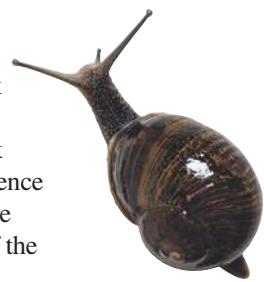


# Chapter review

## FLUENCY

- 1 Which of the following are integers?  
 a  $-2\frac{1}{2}$       b 0.45  
 c 0      d  $-201$
- 2 Complete each statement by inserting the correct symbol:  $>$ ,  $<$  or  $=$ .  
 a  $-6 \square -2$       b  $-7 \square 7$   
 c  $0 \square -5$       d  $-100 \square 9$
- 3 List the integers between  $-21$  and  $-15$ .
- 4 Arrange in *descending* order:  $-3, 2, 0, -15$ .
- 5 Describe the integers graphed on each number line.
  - a
  - b
  - c
- 6 Graph each set of integers on a number line:  
 a integers between  $-7$  and  $-2$   
 b integers  $> -3$   
 c integers  $\leq -4$ .
- 7 State whether the following points are on the  $x$ -axis, the  $y$ -axis, both axes or in the first quadrant.  
 a  $(0, 0)$       b  $(0, 5)$   
 c  $(3, 0)$       d  $(3, 2)$
- 8 Draw and label appropriately a set of axes. Plot the following points in the order given, joining each point to the next one. Name the shape that has been drawn.  
 $(-2, 3), (1, 3), (2, -2), (-1, -2), (-2, 3)$
- 9 In which quadrant or on which axes do the following points lie?  
 a  $(-2, 3)$       b  $(3, -1)$   
 c  $(-4, -1)$       d  $(0, 2)$   
 e  $(-1, 0)$       f  $(7, 9)$
- 10 Calculate the following.  
 a  $-12 + 7$       b  $-9 + -8$   
 c  $18 + -10$       d  $5 + 1$
- 11 Write the number that is 2 more than each of the following integers.  
 a  $-4$       b  $5$   
 c  $-1$       d  $0$   
 e  $-2$

- 12 A snail begins to climb up the side of a bucket. It climbs 3 cm and slips back 2 cm, then climbs a further 4 cm and slips back 1 cm. Write a number sentence to help you find how far the snail is from the bottom of the bucket.



- 13 Calculate the following.  
 a  $5 - 3$       b  $17 - -9$   
 c  $-6 - -9$       d  $6 - 8$   
 e  $12 - 20$       f  $-10 - -12$

- 14 Evaluate the following.  
 a  $-6 \times 7$       b  $4 \times -8$   
 c  $-2 \times -5$       d  $(-8)^2$   
 e  $-8^2$       f  $-2 \times (-8)^2$

- 15 Calculate the following.  
 a  $-36 \div 3$       b  $-21 \div -7$   
 c  $45 \div -9$       d  $\frac{-18}{-2}$   
 e  $64 \div -4$       f  $-100 \div 25$

- 16 Calculate the following.  
 a  $10 - 6 \times 2$   
 b  $-7 - -8 \div 2$   
 c  $-3 \times -5 - -6 \times 2$   
 d  $(-2)^3$   
 e  $(-3 - 12) \div (-10 + 7)$   
 f  $2c + 3c$ , if  $c = -4$   
 g  $-2x(x + 5)$ , if  $x = -2$   
 h  $2a^2 + a$  if  $a = -3$   
 i  $b^3$  if  $b = -5$

- 17 Replace the box with either  $=$ ,  $<$  or  $>$  to make the statement correct.

- a  $-5 \square -3$   
 b  $-22 \times -2 \square 44$   
 c  $4 \square 2$   
 d  $-5 \times -3 \square -15$   
 e  $0 \square -7$   
 f  $-2(-4 + 7) \square 6$   
 g  $-2 \times 5 \square -9$   
 h  $5 \times (-2 - 18) \div 4 \square 25$   
 i  $12 \div -4 \square 3$   
 j  $5 \times (-2 - 18) \div 4 \square -25$   
 k  $-10 \times -5 \square 50$   
 l  $5 \times -2 - 18 \div 4 \square 2.5$

- 18** Model this situation with integers, then find the result: A scuba diver at 52 metres below sea level made his ascent in 3 stages of 15 metres each. At what level was he then?



- 19** Some historians believe that the Roman era began in 146 BC and ended in 455 AD. For how many years did the Roman era last?
- 20** The lowest temperature recorded on Earth was  $-90^{\circ}\text{C}$  in Antarctica and the highest was  $58^{\circ}\text{C}$  in Africa. What is the difference between these two temperatures?
- 21** Rachel's bank account has a balance of  $-\$43$ . She pays a bill using direct debit of  $\$51$ . What is Rachel's bank balance now?

### PROBLEM SOLVING

- 1 a** Complete the addition table below.

+	6		3
	11	15	
12			
	15		

- b** Complete the multiplication table below.

$\times$	4		$-6$
	$-12$	24	18
5			
	8		

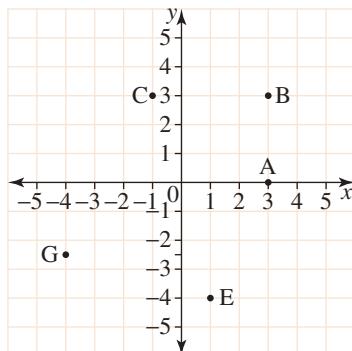
- 2** For the following expressions determine if the answer will be negative or positive. Explain your answers using mathematical reasoning.
- a**  $-17\ 489 - 25\ 636$
- b**  $-65\ 234 + 123\ 468$
- c**  $-44\ 322 + 31\ 212$

- 3** Examine the graph below.

- a** Find the coordinates of the point D, so that ABCD is a square.
- b** State the points that:
- i** make a right-angled triangle
  - ii** make an isosceles triangle that doesn't include a right angle.

(There may be more than one solution.)

- c** Write down the coordinates of the point F, so that ACFE is a parallelogram. Mark this on the grid and draw the parallelogram.
- d** State the coordinates of the midpoint of CA.
- e** State the coordinates of the midpoint of AE.
- f** Estimate the coordinates of G.

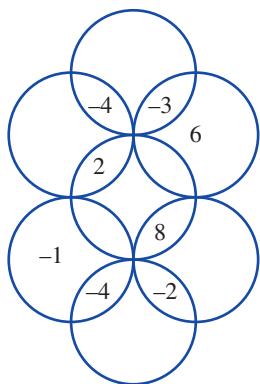


- 4** At many locations around the world, minimum and maximum temperatures are recorded.

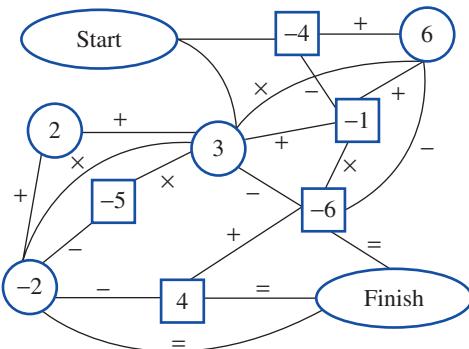
- a** On one particular day, the minimum temperature in the Arctic circle was recorded as  $-24^{\circ}\text{C}$  and the maximum temperature was recorded as  $-7^{\circ}\text{C}$ . In London, UK, the maximum temperature was  $32^{\circ}\text{C}$  and the minimum temperature was  $-2^{\circ}\text{C}$ . In Mexico, the maximum temperature was  $38^{\circ}\text{C}$  and the minimum temperature was  $16^{\circ}\text{C}$ . What was the difference in temperature at each of these three places?



- b** In Melbourne, the temperature often changes rapidly during the day. On one very hot day the temperature reached  $42^{\circ}\text{C}$ . A cool change arrived during the afternoon and the temperature dropped by  $18^{\circ}\text{C}$ . What was the temperature after the cool change?
- c** In Alaska, the weather often changes suddenly as storms sweep across the frozen plains. On one day, the temperature was  $3^{\circ}\text{C}$ , but a storm caused the temperature to drop by  $24^{\circ}\text{C}$ . What was the temperature during the storm?
- 5 a** Fill the gaps in this diagram so that the numbers in each circle add to zero.



- b** Now fill in the gaps so that the numbers in each circle sum to  $-2$ .
- 6** There are many ways to go through the following number network. Follow the addition, subtraction or multiplication signs as you progress from the Start to the Finish.
- a** Find a path (using each number only once) that gives a final answer of  $0$ .
- b** Find a path (using each number only once in the order that they come) that gives the highest final answer.
- c** Find a path (using each number only once in the order that they come) that gives the lowest final answer.



**7** You have 30 questions to answer on a multiple-choice test. Each correct answer scores one mark whilst each incorrect answer scores  $-2$  marks. Any unmarked questions will be counted as incorrect. What is the least number of questions that you can answer correctly and still obtain a score greater than  $0$ ? How many marks will you earn? Explain your reasoning.

- 8** If  $X - Y > X$ , and  $Y - X < Y$ , and  $X$  and  $Y$  are integers, can you determine whether  $X$  and  $Y$  are positive or negative values?
- 9** Place the digits 1–9 (no repeats) in the grid below so that the equations reading across and down are true.

	-		×		=	-16
-		×		-		
	-		+		=	7
×		÷		÷		
	-		-		=	4
=		=		=		
-13		10		-3		

- 10** The negative integers  $-2$  to  $-1000$  are arranged in five columns as shown.

Column 1	Column 2	Column 3	Column 4	Column 5
	-2	-3	-4	-5
-9	-8	-7	-6	
	-10	-11	-12	-13
-17	-16	-15	-14	

... and so on.

In which column will  $-1000$  lie?

- 11** The following table lists the profit and loss that has occurred for the past four years. Which two consecutive years had the maximum difference?

Year	2006	2007	2008	2009
\$Profit/loss	-6420	3808	-2651	8222

- 12 a** Copy and use your calculator to complete the following table.

Integer ( $x$ )	$x^2$	$x^3$	$x^4$	$x^5$
2				
-2				
3				
-3				
4				
-4				

- b** Look at your results in the  $x^2$  column. What is the sign of all the numbers?
- c** Consider the sign of the numbers in the  $x^3$  column. What do you notice?
- d** Describe the resulting sign of the numbers in the  $x^4$  and  $x^5$  columns.
- e** Is the resulting sign in the  $x^2$  column the same as that in the  $x^4$  column? What about the signs of the numbers in the  $x^3$  and  $x^5$  columns?
- f** Copy and complete the following statements.  
When a positive number is raised to any power, the sign of the answer is \_\_\_\_\_.  
When a negative number is raised to an even power, the sign of the answer is \_\_\_\_\_.  
When a negative number is raised to an odd power, the sign of the answer is \_\_\_\_\_.  
**g** Let us now look at the reverse of raising a number to a power — taking the root of a number. You will notice that  $2^2 = 4$  and  $(-2)^2 = 4$ . It follows that, if we take the square root of 4, we can get +2 or -2. (Your calculator will only give you the positive answer.) This has a shorthand way of being written as  $\sqrt{4} = \angle 2$ . Similarly, you will notice that  $\sqrt[4]{16} = \angle 2$ . This only applies to even roots. Write a statement showing the square root of 100.

- h** It is not possible to take the even root of a negative number (because no number raised to an even power will produce a negative number). What happens when you try to evaluate  $\sqrt{-144}$  on the calculator?
- i** Notice that with odd-numbered roots the sign of the answer is the same as the sign of the original number;  $\sqrt[3]{8} = 2$  but  $\sqrt[3]{-8} = -2$ . Calculate  $\sqrt[3]{-125}$ .
- j** Copy and complete the following table. (Fill in the blocks that are not blacked out.) Consider each answer carefully, as some are not possible.

Integer ( $x$ )	$\sqrt{x}$	$\sqrt[3]{x}$	$\sqrt[4]{x}$	$\sqrt[5]{x}$
16	= 4			
-16				
27				
-27				
32				
-32				
81				
-81				
64				
-64				

- k** In your own words, describe the sign resulting after taking odd and even roots of positive and negative numbers.

### eBook plus

#### Interactivities

Test yourself  
Chapter 12  
int-1823  
Word search  
Chapter 12  
int-2603  
Crossword  
Chapter 12  
int-2604

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 12 (doc-6562) (*page 407*)

**Are you ready?****Digital docs** (*page 408*)

- SkillsHEET 12.1 (doc-6563) Using < or > to compare the size of numbers
- SkillsHEET 12.2 (doc-6564) Ascending and descending order
- SkillsHEET 12.3 (doc-6565) Marking numbers on a number line
- SkillsHEET 12.4 (doc-6566) Working with numbers on a number line
- SkillsHEET 12.5 (doc-6567) Evaluating squares, cubes and cube roots
- SkillsHEET 12.6 (doc-6568) Order of operations II
- SkillsHEET 12.7 (doc-6569) Operations with fractions

**12A Integers on the number line****Digital docs** (*page 412*)

- Activity 12-A-1 (doc-1924) The number line
- Activity 12-A-2 (doc-1925) More number lines
- Activity 12-A-3 (doc-1926) Advanced number lines

**eLesson**

- Integers on the number line (eles-0040) (*page 409*)

**Interactivity**

- Directed number target (int-0074) (*page 412*)

**12B Integers on the number plane****Digital docs** (*page 414*)

- Activity 12-B-1 (doc-1927) Number planes
- Activity 12-B-2 (doc-1928) More number planes
- Activity 12-B-3 (doc-1929) Advanced number planes

**12C Addition and subtraction of integers****Digital docs**

- Activity 12-C-1 (doc-1930) Adding and subtracting integers (*page 418*)

- Activity 12-C-2 (doc-1931) More addition and subtraction of integers (*page 418*)
- Activity 12-C-3 (doc-1932) Advanced addition and subtraction of integers (*page 418*)
- WorkSHEET 12.1 (doc-1922) (*page 420*)

**Interactivity**

- Working with integers (int-2351) (*page 418*)

**12D Extension: Multiplication and division of integers****Digital docs**

- Activity 12-D-1 (doc-1933) Multiplying and dividing integers (*page 423*)
- Activity 12-D-2 (doc-1934) More multiplication and division of integers (*page 423*)
- Activity 12-D-3 (doc-1935) Advanced multiplication and division of integers (*page 423*)
- WorkSHEET 12.2 (doc-1923) (*page 424*)

**12E Extension: Combined operations****Digital docs** (*page 426*)

- Activity 12-E-1 (doc-1936) Combined operations with integers
- Activity 12-E-2 (doc-1937) More combined operations with integers
- Activity 12-E-3 (doc-1938) Advanced combined operations with integers

**Chapter review****interactivities** (*page 431*)

- Test Yourself Chapter 12 (int-1823) Take the end-of-chapter test to test your progress.
- Word search Chapter 12 (int-2603)
- Crossword Chapter 12 (int-2604)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# 13

# Linear equations

- 13A Solving equations using trial and error
- 13B Using inverse operations
- 13C Building up expressions
- 13D Solving equations using backtracking
- 13E Checking solutions
- 13F Keeping equations balanced

## WHAT DO YOU KNOW?

- 1 List what you know about equations.  
Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of equations.

eBook plus

Digital docs

Hungry brain activity

Chapter 13

doc-6570



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**Stop! Look no further...** this is the only way you'll want to experience the famous harbour in the world. Book now for one of the many packages available and you'll never forget it or you'll ever take another which you'll be talking about for years. Downunder Jet is the newest custom built jet boat on the harbour and is rated the best valued jet-boating experience in Sydney.

On boarding this newly custom-built vessel, your pulse will race as your adrenaline builds in anticipation for the ride to come. You'll experience the power and speed of the Downunder Jet as well as take you on an action packed tour of Sydney's icons. And just for fun he'll take the boat to speeds where you'll experience a speed boat G Force while reaching speeds of up to 35mph. The skipper will thrill you with awesome stunts, mixed in with some power lock-in 270° spins, wave surfing, wave jumps & power braking nose dives.

And finally as an added bonus, your skipper will provide you with some intimate local knowledge and what only can be described as some lively and light hearted commentary. All this whilst you're experiencing some of the most amazing sights which will add the final dimension to your tour of this great harbour city.

## OPENING QUESTION

Which harbour tour would be the best value? Does your answer change depending on the number of people and their ages in your group?

# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.


**Digital docs**

SkillsHEET 13.1

doc-6571

## Completing number sentences

- 1 Complete the following number sentences by calculating the missing values.

a  $5 + \square = 8$

b  $\square - 7 = 4$

c  $\square \times 2 = 30$

d  $\square \div 3 = 9$


**Digital docs**

SkillsHEET 13.2

doc-6572

## Writing number sentences from written information

- 2 Construct a number sentence, using  $\square$  for the unknown number, to match each problem described below.

a I am thinking of a number, so that when I add 12, the answer is 30.

b I am thinking of a number, so that when I subtract 9, the answer is 6.

c I am thinking of a number, so that when I multiply by 4, the answer is 48.

d I am thinking of a number, so that when I divide by 8, the answer is 7.


**Digital docs**

SkillsHEET 13.3

doc-6573

## Applying the four operations

- 3 Use 8 as your starting number and find the result after applying the following operations.

a  $+3, \times 2$

b  $\times 2, -7$

c  $-6, \div 2$

d  $\div 4, +5$

e  $+7, \div 3$

f  $-4, \times 3$


**Digital docs**

SkillsHEET 13.4

doc-6574

## Inverse operations

- 4 Write the inverse operation for each of the following.

a  $+2$

b  $-5$

c  $\times 7$

d  $\div 3$


**Digital docs**

SkillsHEET 13.5

doc-6575

## Combining like terms

- 5 Simplify each of the following expressions by combining like terms.

a  $2x + 3x + 6y$

b  $3x + 2y + 4x + 7y$

c  $x + 7 + 9x - 3$

d  $8x - 5x + 6 + x$


**Digital docs**

SkillsHEET 13.6

doc-6576

## Checking solutions by substitution

- 6 Answer True or False for each of the following.

a The solution to the number sentence  $\square + 3 = 8$  is 5.

b The solution to the number sentence  $\square \times 4 = 12$  is 8.

c The solution to the number sentence  $\square - 1 = 4$  is 3.

d The solution to the number sentence  $\square \div 10 = 2$  is 20.

e The solution to the number sentence  $\square \times 2 + 4 = 18$  is 6.

f The solution to the number sentence  $\square \div 6 - 6 = 0$  is 36.

# 13A Solving equations using trial and error

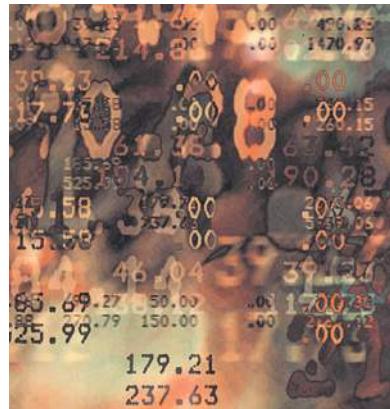
## Writing equations

- By setting an expression equal to a constant or another expression, we create an **equation**; for example,  $2x + 5 = 7$ ,  $3y - 4 = y + 6$  are equations.
- We can use equations to describe a problem that needs solving.
- To write a problem as an equation, use a variable to stand for the unknown number.

### WORKED EXAMPLE 1

Write an equation to represent each of these puzzles. I am thinking of a number.

- When I multiply the number by 8 the answer is 24.
- When I divide the number by 5 the answer is 7.
- When I divide 60 by the number, the answer is 10.
- When I subtract 7 from the number, the answer is 25.
- When I subtract the number from 72, the answer is 52.
- When I square the number, the result is 36.



### THINK

- 1 Use a variable to describe the number.  
2 Multiply the number by 8.  
3 Write the equation.
- 1 Use a variable to describe the number.  
2 Divide the number by 5.  
3 Write the equation.
- 1 Use a variable to describe the number.  
2 Divide 60 by the number.  
3 Write the equation.
- 1 Use a variable to describe the number.  
2 Subtract 7 from the number (that is, take 7 away from the number).  
3 Write the equation.
- 1 Use a variable to describe the number.  
2 Subtract the number from 72 (that is, take the number away from 72).  
3 Write the equation.

### WRITE

- Let  $m$  be the number.  
 $m \times 8 = 8m$   
 $8m = 24$
- Let  $t$  be the number.  
 $t \div 5 = \frac{t}{5}$   
 $\frac{t}{5} = 7$
- Let  $s$  be the number.  
 $60 \div s = \frac{60}{s}$   
 $\frac{60}{s} = 10$
- Let  $l$  be the number.  
 $l - 7$   
 $l - 7 = 25$
- Let  $a$  be the number.  
 $72 - a$   
 $72 - a = 52$

- f**
- 1 Use a variable to describe the number.
  - 2 Square the number (that is, multiply the number by itself).
  - 3 Write the equation.

**f** Let  $z$  be the number.

$$z \times z = z^2$$

$$z^2 = 36$$

- Sometimes the solution to an equation is obvious after close inspection.

### WORKED EXAMPLE 2

Solve the following equations by inspection.

a  $\frac{w}{3} = 4$

b  $h - 9 = 10$

#### THINK

- a 1 Write the equation.  
 2 Think of a number which when divided by 3 gives 4. Try 12.  
 3 So  $w$  must be 12.
- b 1 Write the equation.  
 2 Think of a number which equals 10 when 9 is subtracted from it. Try 19.  
 3 So  $h$  must be 19.

#### WRITE

a  $\frac{w}{3} = 4$

$$12 \div 3 = 4$$

$$w = 12$$

b  $h - 9 = 10$

$$19 - 9 = 10$$

$$h = 19$$

## Guess, check and improve

- We can try to guess the solution to an equation.
- By noting the result when we substitute each guess, we can improve the guess and come closer to the solution.

### WORKED EXAMPLE 3

Use guess, check and improve to solve the equation  $2x + 21 = 4x - 1$ .

#### THINK

- 1 Set up a table with four columns displaying the value of  $x$ , the value of the left-hand and right-hand side equations (after substitution) and a comment on how these two values compare.
- 2 Substitute the first guess, say  $x = 1$ , into the LHS and RHS equations and comment on results.
- 3 Repeat step 2 for  $x = 6, x = 10$  until the correct answer is obtained.
- 4 State the solution.

#### WRITE

Guess $x$	Check		Comment
	$2x + 21$	$4x - 1$	
1	23	3	$4x - 1$ is too small.
6	33	23	This is closer.
10	41	39	Very close
11	43	43	That's it!

The solution is  $x = 11$ .

- This method also works when we need to find two variables.

## WORKED EXAMPLE 4

Find two numbers whose sum is 31 and whose product is 150.

## THINK

- 1 The numbers add up to 31 so guess two numbers that do this. Then check by finding their product.
- 2 Guess 1 and 30.
- 3 Guess 10 and 21.  
Try a number between 1 and 10 for the first number.  
Try a number between 5 and 10 for the first number.  
Try a number between 5 and 8 for the first number.
- 4 State the solution.

## WRITE

Guess	Check	
Sum (small number first)	Product (P)	Comment
1, 30	$1 \times 30 = 30$	P is too low.
10, 21	$10 \times 21 = 210$	P is too high.
5, 26	$5 \times 26 = 130$	P is too low.
8, 23	$8 \times 23 = 184$	P is too high.
6, 25	$6 \times 25 = 150$	That's it!

The numbers are 6 and 25.

## REMEMBER

1. An equation can be used to describe a problem.
2. Sometimes equations can be solved by guessing the solution.
3. Guess, check and improve is a method where the guess is noted, so that the next guess is closer to the solution.

## EXERCISE

## 13A

## Solving equations using trial and error

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 13-A-1

Solving equations  
doc-1948

## Activity 13-A-2

More solving  
equations  
doc-1949

## Activity 13-A-3

Advanced equations  
doc-1950

## FLUENCY

- 1 **WEI** Write an equation to represent each of these puzzles.

I am thinking of a number:

- a when I add 7, the answer is 11
- b when I add 3, the answer is 5
- c when I add 12, the answer is 12
- d when I add 5, the answer is 56
- e when I subtract 7, the answer is 1
- f when I subtract 11, the answer is 11
- g when I subtract 4, the answer is 7
- h when I subtract 8, the answer is 0
- i when I multiply by 2, the answer is 12
- j when I multiply by 6, the answer is 30
- k when I multiply by 5, the answer is 30
- l when I multiply by 6, the answer is 12
- m when I divide by 7, the answer is 1
- n when I divide by 3, the answer is 100
- o when I divide by 5, the answer is 2
- p when I divide by 7, the answer is 0
- q when I subtract the number from 15, the answer is 2
- r when I subtract the number from 52, the answer is 8
- s when I divide 21 by the number, the answer is 7
- t when I square the number, the answer is 100.

- 2 WE2** Solve the following equations by inspection.

a  $x + 7 = 18$

b  $y - 8 = 1$

c  $3m = 15$

d  $\frac{m}{10} = 3$

e  $w + 25 = 26$

f  $m - 1 = 273$

g  $4w = 28$

h  $\frac{k}{5} = 0$

i  $b + 15 = 22$

j  $\frac{w}{3} = 6$

k  $5k = 20$

l  $h - 14 = 11$

m  $b - 2.1 = 6.7$

n  $\frac{c}{3} = 1.4$

o  $5x = 14$

- 3 WE3** Use guess, check and improve to solve the equation  $3x + 11 = 5x - 1$ . The process has been started for you.

Guess $x$	Check		Comment
	$3x + 11$	$5x - 1$	
1	14	4	$5x - 1$ is too small
10	41	49	$5x - 1$ is too big
5			

- 4** Use guess, check and improve to solve the following equations.

a  $5x + 15 = x + 27$

b  $2x + 12 = 3x - 2$

c  $x + 20 = 3x$

d  $12x - 18 = 10x$

e  $10(x + 1) = 5x + 25$

f  $x(x + 1) = 21x$

g  $x(x + 7) = 12x$

h  $6(x - 2) = 4x$

i  $3(x + 4) = 5x + 4$

- 5 WE4** Find two numbers whose sum is 21 and whose product is 98. The process has been started for you.

Guess	Check	Comment
Sum (small number first)	Product (P)	
1, 20	$1 \times 20 = 20$	P is too low
10, 11	$10 \times 11 = 110$	P is too high

- 6** Use guess, check and improve to find two numbers whose sum and product are given:

a sum = 26, product = 165

b sum = 27, product = 162

c sum = 54, product = 329

d sum = 45, product = 296

e sum = 178, product = 5712

f sum = 104, product = 2703

g sum = 153, product = 4662

h sum = 242, product = 14 065

i sum = 6.1, product = 8.58

j sum = 8, product = 14.79

k sum = 978, product = 218 957

l sum = 35, product = 274.89.

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Digital docs  
Spreadsheet  
Solving  
equations  
doc-1966

### UNDERSTANDING

- 7** Copy and complete this table by substituting each  $x$ -value into  $x^2 + 4$  and  $4x + 1$ . The first row has been completed for you. Use the table to find a solution to the equation  $x^2 + 4 = 4x + 1$ . (Remember,  $x^2$  means  $x \times x$ .)

$x$	$x^2 + 4$	$4x + 1$
0	4	1
1		
2		9
3	13	
4		

### REASONING

- 8** A football team won 4 more games than it lost. The team played 16 games. How many did it win?  
**9** Lily is half the age of Pedro. Ross is 6 years older than Lily and 6 years younger than Pedro. How old is Pedro?

- 10 A plumber cut a 20-metre pipe into two pieces. One of the pieces is three times as long as the other. What are the lengths of the two pieces of pipe?

- 11 Julie has the same number of sisters as brothers. Her brother Todd has twice as many sisters as brothers. How many children are in the family?

**REFLECTION**

Why is it a good idea to write our guesses down when using guess, check and improve to solve equations?

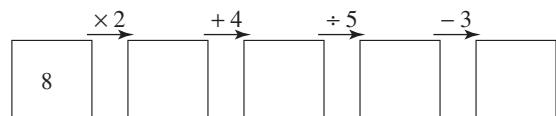


## 13B Using inverse operations

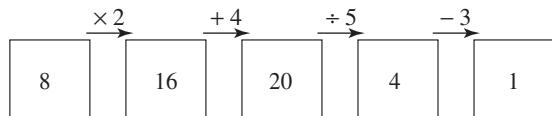
- We can use flowcharts to help us organise harder equations.
- The first number is called the **input number**.
- The last number is called the **output number**.

### WORKED EXAMPLE 5

Complete the flowchart to find the output number.

**THINK**

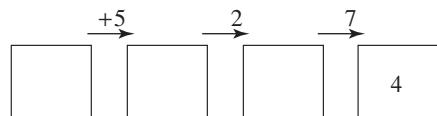
- 1 Follow the steps and fill in the boxes.  
 $8 \times 2 = 16$   
 $16 + 4 = 20$   
 $20 \div 5 = 4$   
 $4 - 3 = 1$

**WRITE**

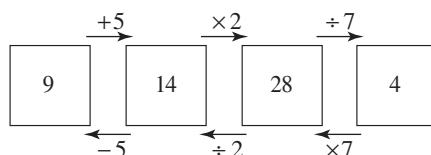
- It is possible to travel in either direction through a flowchart. Working backwards, against the arrows, is called **backtracking**.
- To work backwards, carry out **inverse operations**, or ‘undo’ each step.

### WORKED EXAMPLE 6

Use backtracking and inverse operations to find the input number in this flowchart.

**THINK**

- 1 Fill in the numbers as you backtrack.  
The inverse of  $\div 7$  is  $\times 7$  ( $4 \times 7 = 28$ ).
- 2 The inverse of  $\times 2$  is  $\div 2$  ( $28 \div 2 = 14$ ).
- 3 The inverse of  $+ 5$  is  $- 5$  ( $14 - 5 = 9$ ).

**WRITE**

## REMEMBER

- Flowcharts help us to organise harder equations.
- We can use *backtracking* and *inverse operations* to solve equations.

## EXERCISE

## 13B Using inverse operations

## INDIVIDUAL PATHWAYS

## eBookplus

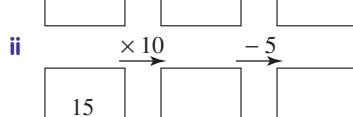
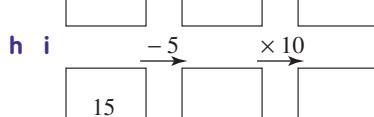
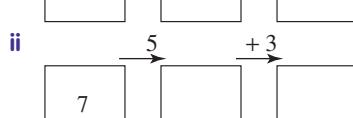
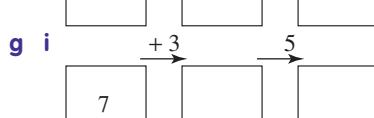
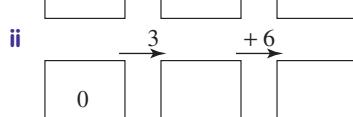
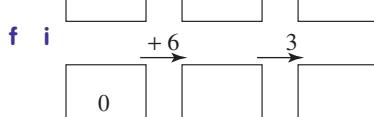
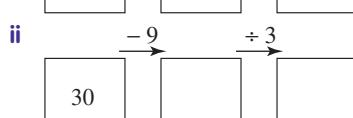
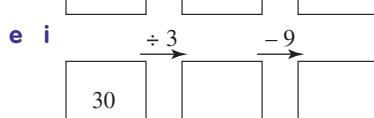
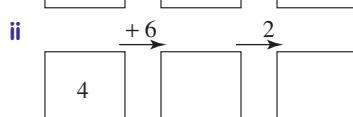
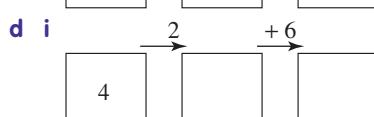
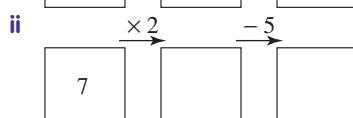
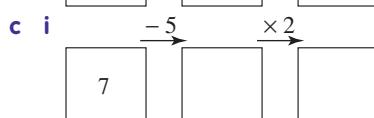
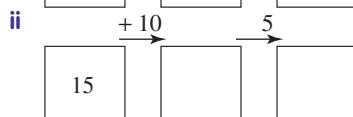
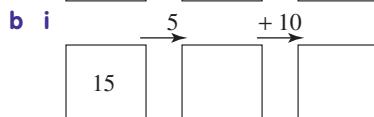
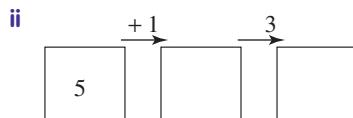
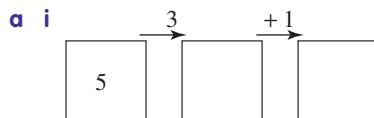
**Activity 13-B-1**  
Inverse operations  
doc-1951

**Activity 13-B-2**  
More inverse  
operations  
doc-1952

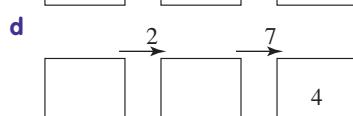
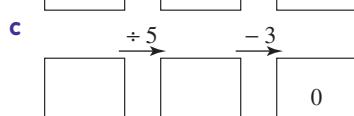
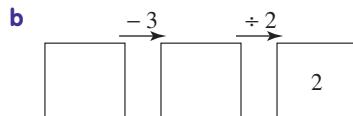
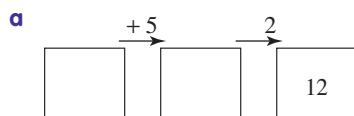
**Activity 13-B-3**  
Advanced operations  
with integers  
doc-1953

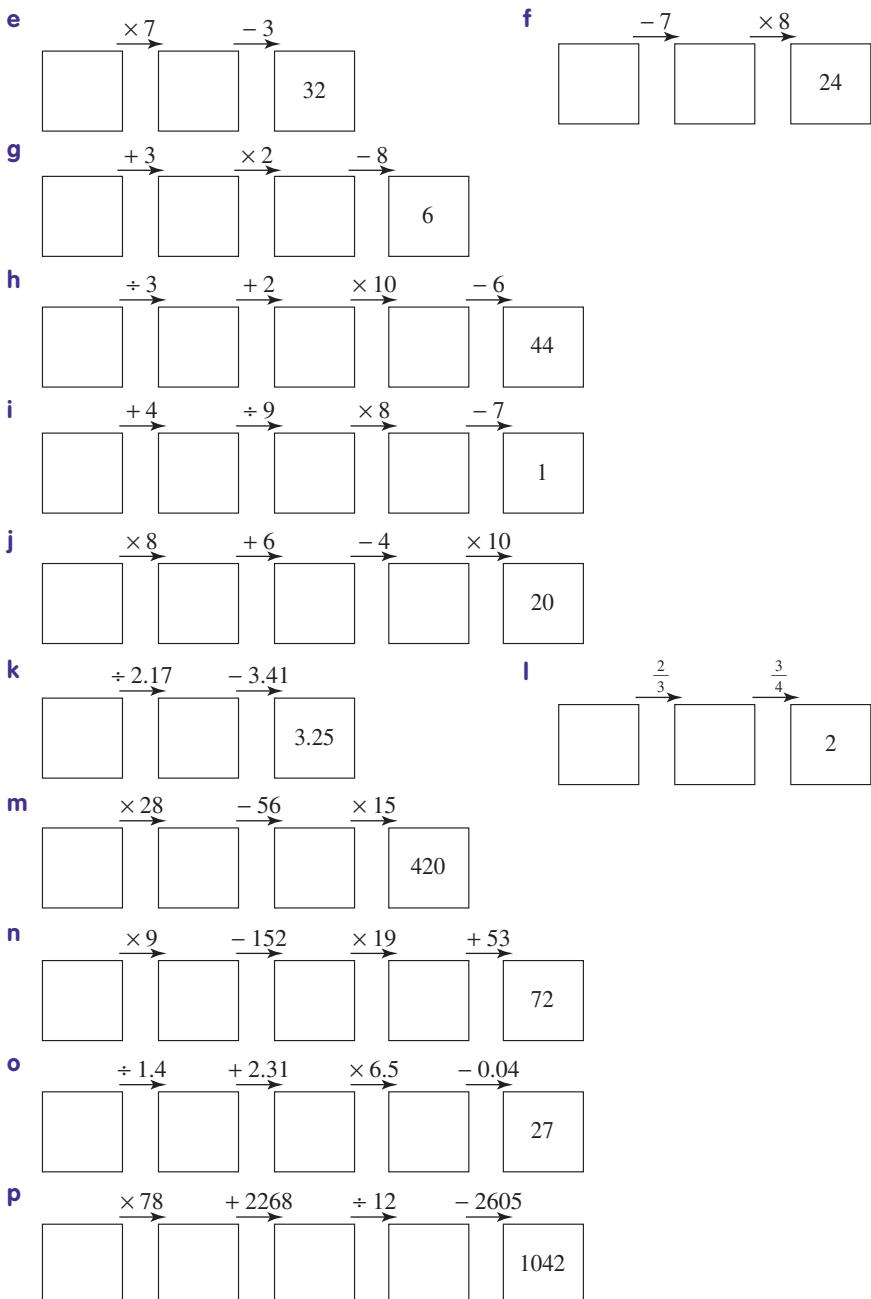
## FLUENCY

- 1 **WE5** Complete the following flowcharts to find the output number.



- 2 **WE6** Use backtracking and inverse operations to find the input number in each of these flowcharts.



**UNDERSTANDING****eBookplus**

**Digital docs**  
Spreadsheet  
Backtracking  
doc-1967

- 3 a** In question 1, what did you notice about each pair of flowcharts?

**b** Does changing the order of operations affect the end result?

- 4** Complete the two statements below:

**a** Adding and \_\_\_\_\_ are inverse operations.

**b** \_\_\_\_\_ and dividing are inverse operations.

**REFLECTION**

When you're backtracking,  
does the order of the  
operations matter?

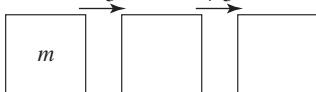
# 13C Building up expressions

■ **Flowcharts** can be used to construct algebraic expressions.

## WORKED EXAMPLE 7

Complete the flowcharts below to find the output number.

a

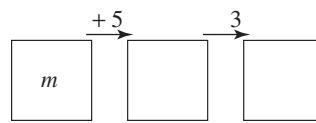


### THINK

a  $m \times 3 = 3m$

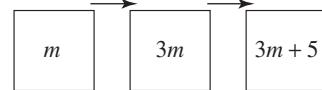
Adding 5 to  $3m$  gives  $3m + 5$ .

b

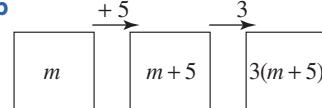


### WRITE

a



b



b Adding 5 to  $m$  results in  $m + 5$ .

To multiply all of  $m + 5$  by 3,  $m + 5$  will need to be inside brackets.

## WORKED EXAMPLE 8

Draw a flowchart whose input number is  $m$  and whose output number is given by the expressions:

a  $2m - 11$

b  $\frac{m+9}{5}$

c  $4\left(\frac{m}{3} + 2\right)$ .

### THINK

a 1 The first step is to obtain  $2m$ ; that is, multiply  $m$  by 2.

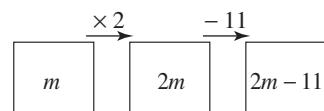
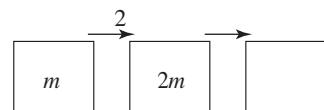
2 This is followed by subtracting 11.

b 1 The expression  $m + 9$  is grouped as in a pair of brackets, so we must obtain this part first. Therefore add 9 to  $m$ .

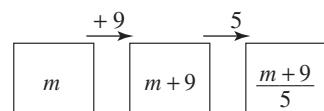
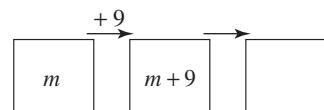
2 Then the whole expression is divided by 5.

### WRITE

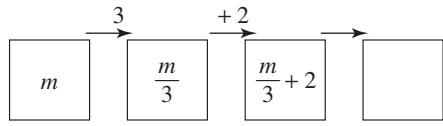
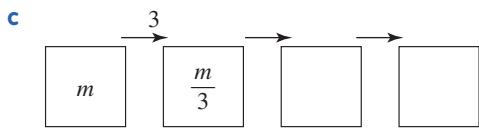
a



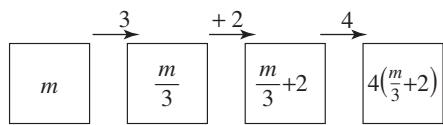
b



- c 1 The pair of brackets indicates we must first work from within the brackets:  
 (a) divide  $m$  by 3  
 (b) then add 2 to this result.



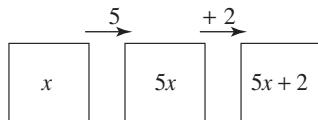
- 2 Multiply the final result obtained in step 1 by 4.



■ Using flowcharts, we can backtrack to our input number using inverse operations.

### WORKED EXAMPLE 9

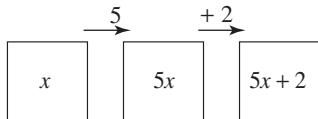
Complete the flowchart at right by writing in the operations which must be carried out in order to backtrack to  $x$ .



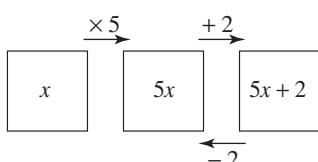
#### THINK

- 1 Copy the flowchart and look at the operations that have been performed.

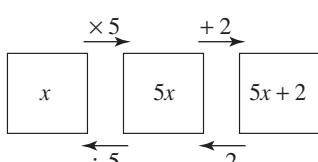
#### WRITE



- 2 The inverse operation of +2 is -2. Show this on the flowchart.



- 3 The inverse operation of  $\times 5$  is  $\div 5$ . Show this on the flowchart.



#### REMEMBER

- 1 We can use flowcharts to construct algebraic expressions.
- 2 As well as constructing an algebraic expression using a flowchart, we can backtrack to our input number using inverse operations.

## EXERCISE

## 13C Building up expressions

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 13-C-1

Building up expressions  
doc-1954

## Activity 13-C-2

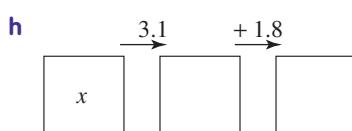
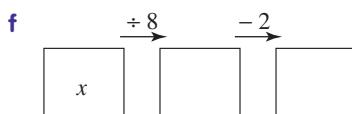
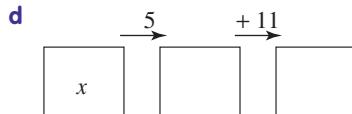
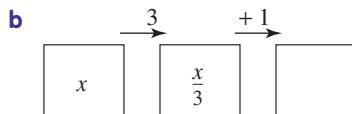
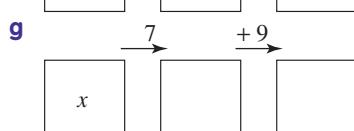
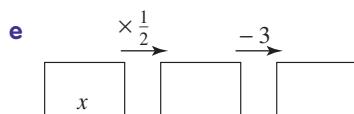
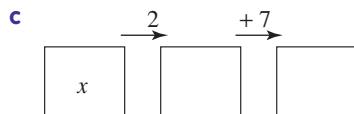
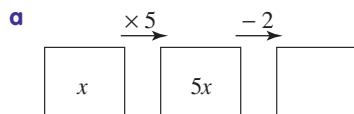
More expressions  
doc-1955

## Activity 13-C-3

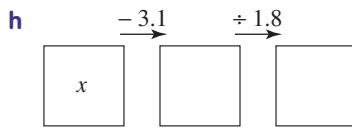
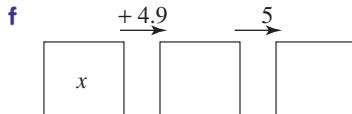
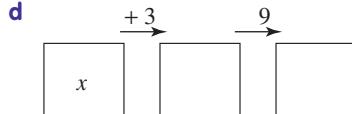
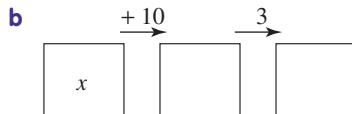
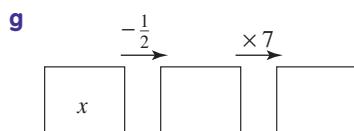
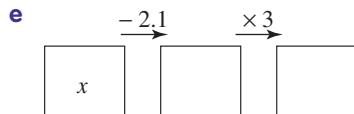
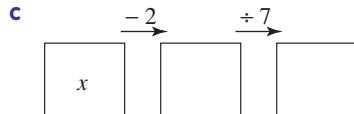
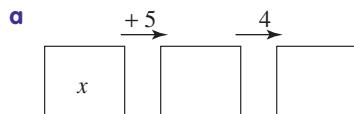
Advanced expressions  
doc-1956

## FLUENCY

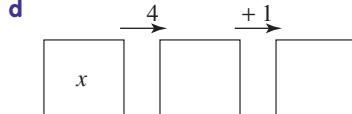
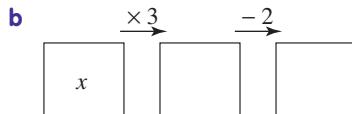
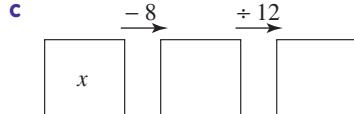
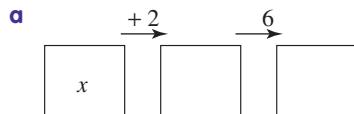
- 1 WE7 Build up an expression by following the instructions on the flowcharts.

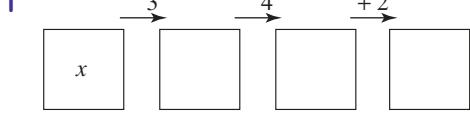
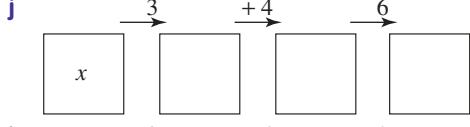
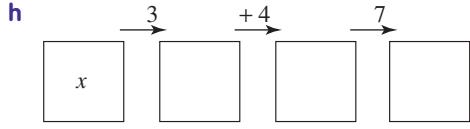
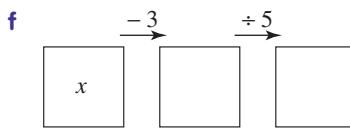
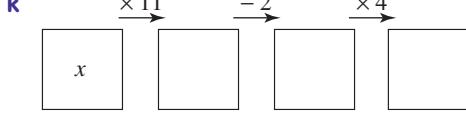
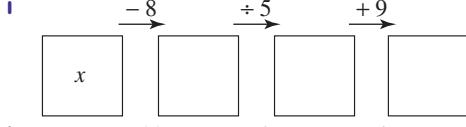
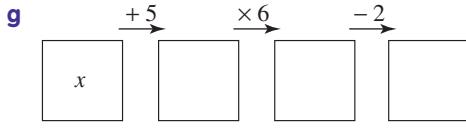
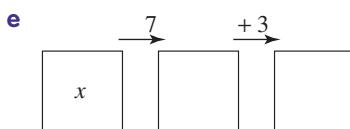


- 2 Build up an expression by following the instructions on the flowcharts. Use a grouping device, such as a pair of brackets or a vinculum; for example,  $2(x + 3)$  or  $\frac{x-5}{4}$ .



- 3 Copy and complete the following flowcharts by filling in the missing expressions.





- 4 **WE8** Draw a flowchart whose input number is  $x$  and whose output is given by the following expressions.

a  $5x + 9$

b  $2(x + 1)$

c  $\frac{x}{6} + 4$

d  $\frac{x-8}{7}$

e  $12(x - 7)$

f  $\frac{x}{5} - 2$

g  $7x - 12$

h  $\frac{x+6}{3}$

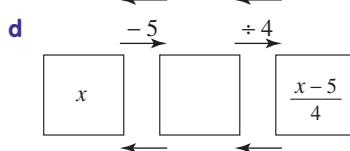
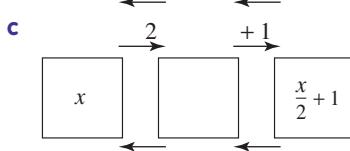
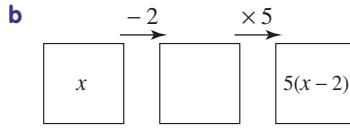
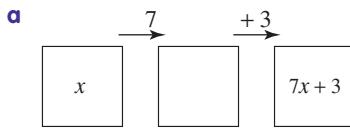
i  $3(x + 7) - 5$

j  $\frac{3x + 7}{2}$

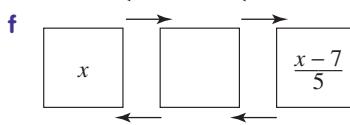
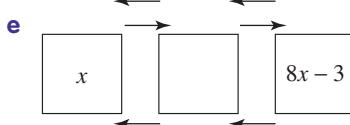
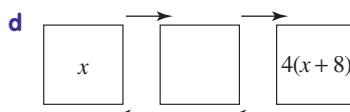
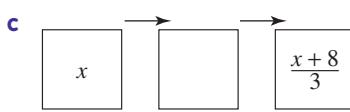
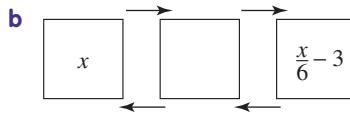
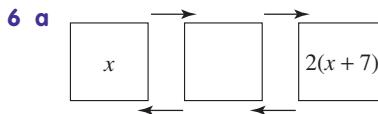
k  $4(3x + 1)$

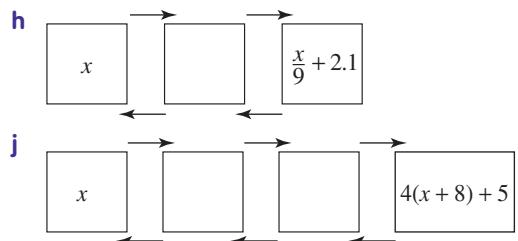
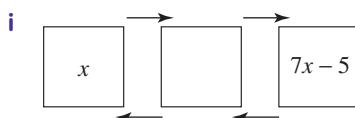
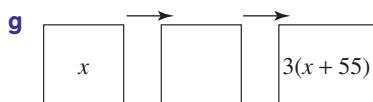
l  $3\left(\frac{x}{5} + 6\right)$

- 5 **WE9** Complete the following flowcharts by writing in the operations which must be carried out in order to backtrack to  $x$ .



### UNDERSTANDING



**REASONING**

- 7** I think of a number, and add 8 to it. I multiply the result by 5 and then divide the result by 4. The answer is 30.  
Build up an expression to write the above in the form of an equation.
- 8** I think of a number, multiply it by 5 and add 15. The result is 3 less than 4 times the original number. If the original number is  $n$ , write down an equation to show the relation.

**REFLECTION**

Do the order of operation laws apply when constructing flowcharts?



eBookplus  
eLesson  
Backtracking  
eles-0010

## 13D Solving equations using backtracking

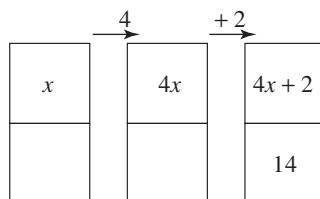
- Backtracking an algebraic expression can be used to find the input number.

**WORKED EXAMPLE 10**

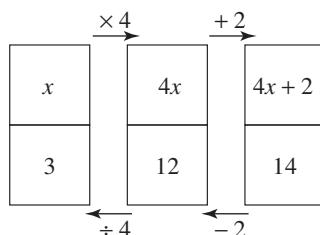
Draw a flowchart to represent the following puzzle and then solve it by backtracking.  
I am thinking of a number. When I multiply it by 4 and then add 2 the answer is 14.

**THINK**

- Build an expression using  $x$  to represent the number.  
Start with  $x$ , multiply by 4 and add 2.  
The output number is 14.

**WRITE**

- Backtrack to find the value of  $x$ .  
The inverse operation of  $+2$  is  $-2$ .  
 $(14 - 2 = 12)$ .  
The inverse operation of  $\times 4$  is  $\div 4$   
 $(12 \div 4 = 3)$ .



- State the answer.

So  $x = 3$ . The number is 3.

## WORKED EXAMPLE 11

Solve the following equations by backtracking.

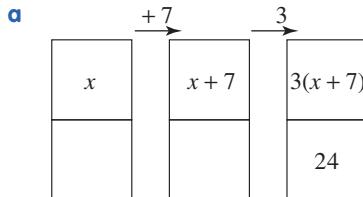
a  $3(x + 7) = 24$

b  $\frac{x}{3} + 5 = 6$

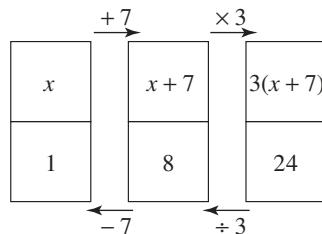
## THINK

- a 1 Build the expression on the left-hand side of the equation.  
Start with  $x$ , add 7 and then multiply by 3.  
The output number is 24.

## WRITE

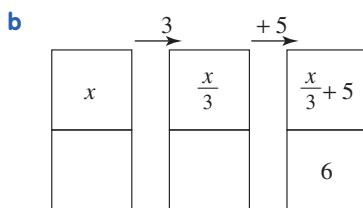


- 2 Backtrack to find  $x$ .  
The inverse operation of  $\times 3$  is  $\div 3$  ( $24 \div 3 = 8$ ).  
The inverse operation of  $+7$  is  $-7$  ( $8 - 7 = 1$ ).

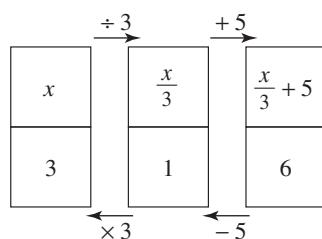


$$x = 1$$

- b 1 Build the expression on the left-hand side of the equation.  
Start with  $x$ , divide by 3 and then add 5.  
The output number is 6.



- 2 Backtrack to solve for  $x$ .  
The inverse operation of  $+5$  is  $-5$  ( $6 - 5 = 1$ ).  
The inverse operation of  $\div 3$  is  $\times 3$  ( $1 \times 3 = 3$ ).



$$x = 3$$

## WORKED EXAMPLE 12

Simplify and then solve the following equation by backtracking.

$$5x + 13 + 2x - 4 = 23$$

## THINK

- 1 Simplify by adding the like terms together on the left-hand side of the equation.  
 $5x + 2x = 7x$ ,  $13 - 4 = 9$

## WRITE

$$5x + 13 + 2x - 4 = 23$$

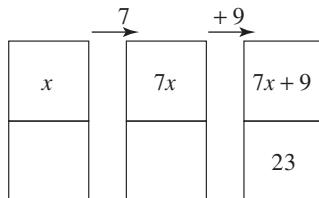
$$7x + 9 = 23$$



- 2** Draw a flowchart and build the expression  $7x + 9$ .

Start with  $x$ , multiply by 7 and add 9.

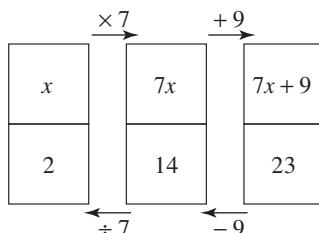
The output number is 23.



- 3** Backtrack to solve for  $x$ .

The inverse operation of  $+9$  is  $-9$  ( $23 - 9 = 14$ ).

The inverse operation of  $\times 7$  is  $\div 7$  ( $14 \div 7 = 2$ ).



- 4** State the answer.

### REMEMBER

We can solve equations by drawing a flowchart and backtracking.

### EXERCISE

## 13D Solving equations using backtracking

### INDIVIDUAL PATHWAYS

#### eBookplus

#### Activity 13-D-1

Backtracking  
doc-1957

#### Activity 13-D-2

More backtracking  
doc-1958

#### Activity 13-D-3

Advanced  
backtracking  
doc-1959

### FLUENCY

- 1 WEIO** Draw a flowchart to represent each of the following puzzles and then solve them by backtracking.

I am thinking of a number.

- When I multiply it by 2 and then add 7 the answer is 11.
- When I add 3 to it and then multiply by 5 the answer is 35.
- When I divide it by 4 and then add 12 the answer is 14.
- When I add 5 to it and then divide by 3 the answer is 6.
- When I subtract 7 from it and then multiply by 6 the answer is 18.
- When I multiply it by 8 and then subtract 11 the answer is 45.
- When I subtract 4 from it and then divide by 9 the answer is 7.
- When I divide it by 11 and then subtract 8 the answer is 0.
- When I add 5 to it and then multiply by 2 the answer is 12.
- When I multiply it by 6 and then add 4 the answer is 34.
- When I multiply it by 5 and then subtract 10 the answer is 30.
- When I subtract 3.1 from it and then multiply by 6 the answer is 13.2.
- When I divide it by 8 and then subtract 0.26 the answer is 0.99.
- When I divide it by 3.7 and then add 1.93 the answer is 7.62.
- When I add  $\frac{2}{5}$  to it and then divide by 6 the answer is  $\frac{4}{5}$ .
- When I subtract  $\frac{3}{4}$  from it and then divide by  $\frac{2}{3}$  the answer is  $\frac{1}{6}$ .

- 2** Draw a flowchart and use backtracking to find the solution to the following equations.

a  $5x + 7 = 22$

d  $4x + 12 = 32$

g  $4w + 5.2 = 28$

b  $9y - 8 = 1$

e  $8w + 2 = 26$

h  $6b - \frac{5}{9} = \frac{2}{3}$

c  $3m - 7 = 11$

f  $11m - 1 = 274$

i  $2a + \frac{1}{3} = \frac{3}{5}$

**3 WE1a** Solve the following equations by backtracking.

a  $3(x + 7) = 24$   
 d  $11(x + 5) = 99$   
 g  $4(w + 5.2) = 26$

b  $2(x - 7) = 22$   
 e  $6(x + 9) = 72$   
 h  $6(b - \frac{5}{9}) = \frac{2}{3}$

c  $5(x - 15) = 40$   
 f  $3(x - 11) = 3$   
 i  $8(x - \frac{1}{4}) = \frac{4}{5}$

**4 WE1b** Solve the following equations by backtracking.

a  $\frac{x}{3} + 5 = 6$   
 d  $\frac{x}{2} - 11 = 6$   
 g  $\frac{x}{5} + 2.3 = 4.9$

b  $\frac{x}{9} - 2 = 3$   
 e  $\frac{x}{7} - 5 = 6$   
 h  $\frac{x}{4} - \frac{3}{11} = \frac{4}{11}$

c  $\frac{x}{4} + 7 = 10$   
 f  $\frac{x}{3} + 1 = 1$   
 i  $\frac{x}{2} + \frac{1}{9} = \frac{4}{9}$

**5** Solve the following equations by backtracking.

a  $\frac{x+4}{3} = 6$   
 d  $\frac{x+11}{2} = 6$   
 g  $\frac{x+2.21}{1.4} = 4.9$

b  $\frac{x-8}{7} = 3$   
 e  $\frac{x-5}{7} = 0$   
 h  $\frac{x-1}{5} = \frac{4}{7}$

c  $\frac{x-8}{7} = 10$   
 f  $\frac{x+100}{17} = 23$   
 i  $\frac{x+1}{4} = \frac{3}{8}$

**6** Use backtracking to find the solution to the following equations.

a  $3x - 7 = 23$

b  $4(x + 7) = 40$

c  $\frac{x+6}{9} = 6$

d  $\frac{x}{5} - 2 = 8$

e  $5(x - 3) = 15$

f  $\frac{x+3}{8} = 3$

g  $6(x - 4) = 18$

h  $\frac{x}{3} + 10 = 12$

i  $\frac{x}{2.1} - 1.7 = 3.6$

j  $4x + \frac{3}{5} = 1$

k  $\frac{x+5}{3} - 3 = 7$

l  $3(2x + 5) = 21$

m  $4(x - 2) + 5 = 21$

n  $3\left(\frac{x}{2} + 1\right) = 15$

o  $2(3x + 4) - 5 = 15$

**7 WE12** Simplify and then solve the following equations by backtracking.

a  $2x + 7 + 3x + 5 = 27$   
 c  $3x + 9 + x - 4 = 17$   
 e  $6x + 6 - x - 4 = 37$   
 g  $5x - 2x + 5 - x = 19$   
 i  $7x - 4x + 8 - x = 10$

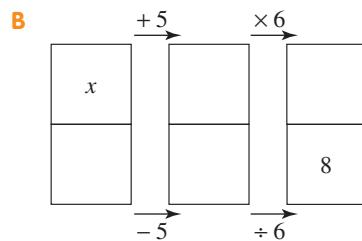
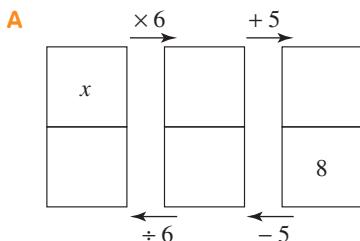
b  $x + x + 1 + x + 2 = 18$   
 d  $3x + 5x + 2x = 40$   
 f  $3x - 11 + 4x = 17$   
 h  $2x + 3x + 4x + 5 = 7$

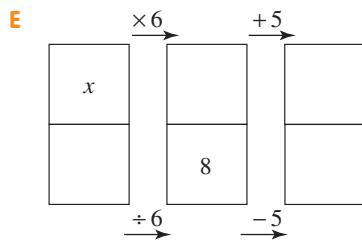
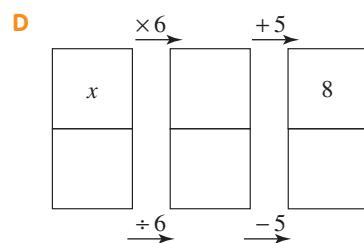
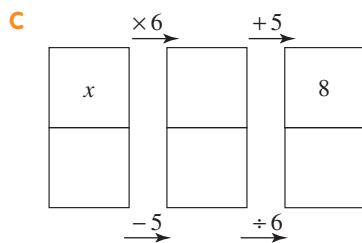
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Digital docs  
WorkSHEET 13.1  
doc-1946

### UNDERSTANDING

**8 MC** The correct flowchart required to solve the equation  $6x + 5 = 8$  is:





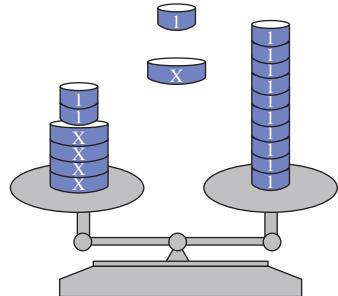
- 9** Given the following balance what operations do you need to do to find the value of  $x$ ? What is the value of  $x$ ?

- 10** Chloë wrote the following explanation to solve the equation  $3(x - 6) + 5 = 8$  by the backtracking method.

*To solve this equation, work backwards and do things in the reverse order. First add 5, then subtract 6, then finally multiply by 3.*

Explain why her instructions are not correct.

$$4x + 2 = 10$$



### REASONING

- 11** Imraan is 5 years older than his brother Gareth and the sum of their ages is 31 years. How old is Gareth? (Let  $x$  represent Gareth's age.)
- 12** In three basketball games Karina has averaged 12 points each game. In the first game she scored 11 points, in the second she scored 17 points, and in the third game she scored  $x$  points.
- From the given information, what is the average of 11, 17 and  $x$ ?
  - Write an equation using the answer to part a.
  - Solve the equation.
  - How many points did Karina score in the third game?
- 13** Melanie and Callie went tenpin bowling together. Melanie scored 15 more pins than Callie, and their total score was 207. What did Callie score?
- 14** The sum of three consecutive whole numbers is 51. Find the numbers.  
*(Hint: Let the smallest number equal  $x$ .)*
- 15** The sum of three consecutive odd numbers is 27. What are the numbers?
- 16** Three consecutive multiples of 5 add up to 90. What are the 3 numbers?
- 17** David is 5 years younger than his twin brothers. If the sum of their ages is 52, then how old is David?
- 18** In the high jump event Chris leapt 12 centimetres higher than Tim, but their two jumps made a total of 3 metres. How high did Chris jump?
- 19** Daniel and Travis are twins, but their sister Phillipa is 3 years older. If the sum of their three ages is 36, how old are the twins?

- 20** Mel loves playing ‘Think of a number’ with friends.

Here’s an example of one of her puzzles.

- Think of a number.
- Double it.
- Add 10.
- Divide by 2.
- Take away the number you first thought of.
- Your answer is . . . 5!

Let’s investigate to see why the answer is always 5, whatever number you first thought of. We can form expressions for each of the steps, using a variable as the starting value.

- |   |                            |
|---|----------------------------|
| • Think of a number .....                         | <i>n</i>                   |
| • Double it .....                                 | $n \times 2 = 2n$          |
| • Add 10 .....                                    | $2n + 10$                  |
| • Divide by 2 .....                               | $(2n + 10) \div 2 = n + 5$ |
| • Take away the number you first thought of ..... | $n + 5 - n = 5$            |
| • Your answer is.....                             | 5                          |

So, your answer will always be 5, for any starting number.

- a Write expressions for each step in the following, showing that you can determine the answer in each case.

#### Puzzle 1

- Take the year in which you were born.
- Subtract 500.
- Multiply by 2.
- Add 1000.
- Divide by 2.
- Your answer is . . . your birth year!

#### Puzzle 2

- Take your age (in years).
- Add 4.
- Multiply by 10.
- Subtract 10.
- Divide by 5.
- Subtract your age.
- Take away 6.
- Your answer will be . . . your age!

#### Puzzle 3

- Think of a number.
- Divide it by 2.
- Add 2.
- Multiply by 4.
- Take away your original number.
- Subtract your original number again.
- Your answer should be . . . 8.

- b Write some ‘Think of a number’ puzzles yourself. Try them out on friends. They will marvel at your mystical powers!



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equations

#### REFLECTION

Can you think of some situations where you might need to be able to solve equations?

## 13E Checking solutions

- It is always a good idea to check if your solution to an equation is correct.

### WORKED EXAMPLE 13

For each of the following, determine whether  $x = 7$  is the solution to the equation.

a  $\frac{x+5}{3} = 4$

b  $2x - 8 = 10$

#### THINK

- a 1 Write the equation.

- 2 Write the left-hand side (LHS) of the equation and substitute  $x = 7$ .

- 3 Perform the calculation.

- 4 Compare this with the right-hand side (RHS) of the equation.

- 5 Comment on the result.

- b 1 Write the equation.

- 2 Write the left-hand side of the equation and substitute  $x = 7$ .

- 3 Perform the calculation.

- 4 Compare this with the right-hand side of the equation.

- 5 Comment on the result.

#### WRITE

a  $\frac{x+5}{3} = 4$

$$\begin{aligned}\text{If } x = 7, \text{ LHS} &= \frac{x+5}{3} \\ &= \frac{7+5}{3} \\ &= \frac{12}{3} \\ &= 4\end{aligned}$$

RHS = 4

The solution is  $x = 7$ , since LHS = RHS.

b  $2x - 8 = 10$

$$\begin{aligned}\text{If } x = 7, \text{ LHS} &= 2x - 8 \\ &= 2(7) - 8 \\ &= 14 - 8 \\ &= 6\end{aligned}$$

RHS = 10

Therefore,  $x = 7$  is not the solution since LHS  $\neq$  RHS.

### WORKED EXAMPLE 14

For each equation below there is a solution given. Is the solution correct?

a  $\frac{x+2}{3} = 2x - 12, x = 10$

b  $3x - 7 = 2x + 3, x = 10$

#### THINK

- a 1 Write the equation.

- 2 Write the left-hand side of the equation and substitute  $x = 10$ .

- 3 Perform the calculation.

#### WRITE

a  $\frac{x+2}{3} = 2x - 12$

$$\begin{aligned}\text{If } x = 10, \text{ LHS} &= \frac{x+2}{3} \\ &= \frac{10+2}{3} \\ &= \frac{12}{3} \\ &= 4\end{aligned}$$

- 4** Write the right-hand side of the equation and substitute  $x = 10$ .
- $$\begin{aligned} \text{RHS} &= 2x - 12 \\ &= 2(10) - 12 \\ &= 20 - 12 \\ &= 8 \end{aligned}$$
- 5** Perform the calculation.
- 6** Comment on the results.
- b** **1** Write the equation.
- 2** Write the left-hand side of the equation and substitute  $x = 10$ .
- 3** Perform the calculation.
- 4** Write the right-hand side of the equation and substitute  $x = 10$ .
- 5** Perform the calculation.
- 6** Comment on the results.
- Therefore  $x = 10$  is not the solution, since  $\text{LHS} \neq \text{RHS}$ .
- b**  $3x - 7 = 2x + 3$
- $$\begin{aligned} \text{If } x = 10, \text{ LHS} &= 3x - 7 \\ &= 3(10) - 7 \\ &= 30 - 7 \\ &= 23 \end{aligned}$$
- $$\begin{aligned} \text{RHS} &= 2x + 3 \\ &= 2(10) + 3 \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

The solution is  $x = 10$ , since  $\text{LHS} = \text{RHS}$ .

### REMEMBER

We can check our solution to an equation by substituting the value for the pronumeral to see if it is true.

### EXERCISE

## 13E Checking solutions

### INDIVIDUAL PATHWAYS

eBookplus

#### Activity 13-E-1

Checking solutions  
doc-1960

#### Activity 13-E-2

More solutions  
doc-1961

#### Activity 13-E-3

Advanced solutions  
doc-1962

### FLUENCY

- 1 WE13** For each of the following determine whether:
- $x = 3$  is the solution to the equation  $x + 2 = 6$
  - $x = 3$  is the solution to the equation  $2x - 1 = 5$
  - $x = 5$  is the solution to the equation  $2x + 3 = 7$
  - $x = 4$  is the solution to the equation  $6x - 6 = 24$
  - $x = 10$  is the solution to the equation  $3x + 5 = 20$
  - $x = 5$  is the solution to the equation  $4(x - 3) = 8$
  - $x = 7$  is the solution to the equation  $3(x - 2) = 25$
  - $x = 8$  is the solution to the equation  $5(x + 1) = 90$
  - $x = 12$  is the solution to the equation  $6(x - 5) = 42$
  - $x = 81$  is the solution to the equation  $3x - 53 = 80$
  - $x = 2.36$  is the solution to the equation  $5x - 7 = 4.8$
  - $x = 4.4$  is the solution to the equation  $7x - 2.15 = 18.64$ .
- 2 WE14** For each equation below there is a solution given. Is the solution correct?
- |                            |          |
|----------------------------|----------|
| <b>a</b> $2x + 1 = 3x - 5$ | $x = 6$  |
| <b>b</b> $5x + 1 = 2x - 7$ | $x = 8$  |
| <b>c</b> $3x - 5 = x + 8$  | $x = 10$ |

- |          |                                 |            |
|----------|---------------------------------|------------|
| <b>d</b> | $5x = 2x + 12$                  | $x = 4$    |
| <b>e</b> | $4x = 3x + 8$                   | $x = 8$    |
| <b>f</b> | $3x = x + 20$                   | $x = 15$   |
| <b>g</b> | $3x - 1.2 = x + 2.9$            | $x = 1.9$  |
| <b>h</b> | $6x + 1.5 = 2x + 41.5$          | $x = 10$   |
| <b>i</b> | $2.4(x + 1) = 9.6$              | $x = 3$    |
| <b>j</b> | $1.2(x + 1.65) = 0.2(x + 9.85)$ | $x = 0.45$ |

**UNDERSTANDING**

- 3** Complete the table below to find the value of  $2x + 3$  when  $x = 0, 1, 2, 3, 4$ .

$x$	0	1	2	3	4
$2x + 3$					

- a** For what value of  $x$  does  $2x + 3 = 11$ ?
- b** What is the solution (that is, the value of  $x$ ) for  $2x + 3 = 11$ ?
- c** What is the solution (that is, the value of  $x$ ) for  $2x + 3 = 5$ ?

- 4** Complete the table below to find the value of  $5(x - 2)$  when  $x = 2, 3, 4, 5, 6$ .

$x$	2	3	4	5	6
$5(x - 2)$					

- a** What is the solution (that is, the value of  $x$ ) to  $5(x - 2) = 10$ ?
- b** What is the solution (that is, the value of  $x$ ) to  $5(x - 2) = 20$ ?
- c** What do you guess is the solution (that is, the value of  $x$ ) to  $5(x - 2) = 30$ ? Check your guess.

- 5 a** Copy and complete the table below.

$x$	$2x + 1$	$3x - 5$
3	7	
4		
5		10
6		
7		

- b** What is the solution to  $2x + 1 = 3x - 5$ ?

- 6 a** Copy and complete the table below.

$x$	$\frac{x+3}{2}$	$2x - 6$
3	3	
5		
7		8
9		
11		

- b** What is the solution to  $\frac{x+3}{2} = 2x - 6$ ?

**eBookplus**

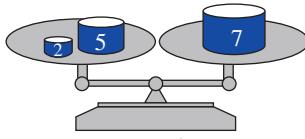
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doc-1947

**REFLECTION**

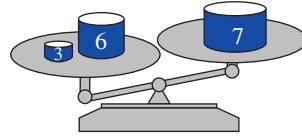
Is it possible for an equation to have a decimal solution?

## 13F Keeping equations balanced

- Equations are mathematical statements that show two equal expressions; that is, the left-hand side and the right-hand side of an equation *are equal*. For example,  $2 + 5 = 7$ .
- A **pan balance scale** can be used to show whether things have equal mass. If the contents of the two pans have the same mass, the pans balance at the same level and the arms of the scale are parallel to the ground. If the contents of the two pans are different in mass, the arms tip so that the side with the greater mass is lower.

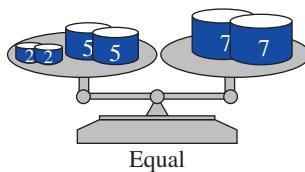


Equal

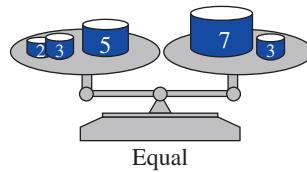


Not equal

- Any change that is made to one side must also be made to the other side if balance is to be maintained.



Equal

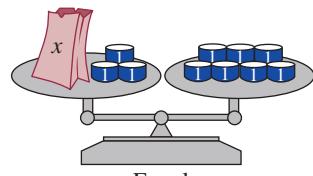


Equal

If we double what is on the left-hand side, we must double the amount on the right-hand side.

If we add 3 to the left-hand side, we must also add 3 to the right-hand side.

- Variables in expressions can be represented on a pan balance scale using bags. Each bag contains a particular number of weights that is equal to value of the variable shown on the front of the bag. For example, the scale below shows the equation  $x + 3 = 7$ .
- Making changes to both sides of the scale allows you to work out how many weights are in the bag. This is the same as working out the value of the variable. On the scale above, if we remove 3 weights from each pan, the bag with  $x$  weights in it weighs the same as 4 weights, so  $x = 4$ .



Equal

### WORKED EXAMPLE 15

For the following pan balance scale:

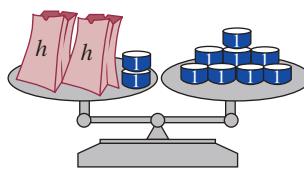
- Write the equation represented by the scale
- Calculate the value of the variable.

#### THINK

- On the LHS, there are 2 bags with  $h$  weights in each, and 2 weights outside the bags. On the RHS, there are 8 weights. Write this as an equation.
- Remove 2 weights from both sides so that the balance will be maintained ( $-2$ ). This leaves the 2 bags on the LHS and 6 weights on the RHS.

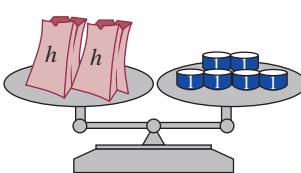
#### WRITE

a  $2h + 2 = 8$



Equal

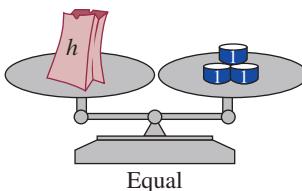
b



$2h = 6$



- 2** Remove half of the contents of each pan ( $\div 2$ ).



Equal

$$h = 3$$

Each bag contains 3 weights.

Therefore,  $h = 3$ .

- Both backtracking and the balance method of solving equations use inverse operations to find the value of a variable. For example, the equation  $2q + 3 = 11$  can be solved as follows.

Backtracking	Balance method
<div style="display: flex; align-items: center; justify-content: space-around;"> <div style="text-align: center;"> <math>q</math>  <span style="border: 1px solid green; padding: 2px;">4</span> </div> <div style="margin: 0 10px;"><math>\xrightarrow{\times 2}</math></div> <div style="text-align: center;"> <math>2q</math>  <span style="border: 1px solid red; padding: 2px;">8</span> </div> <div style="margin: 0 10px;"><math>\xrightarrow{+ 3}</math></div> <div style="text-align: center;"> <math>2q + 3</math>  <span style="border: 1px solid blue; padding: 2px;">11</span> </div> </div> <p>The equivalent equations made when backtracking are:</p> $2q + 3 = 11$ $2q = 8$ $q = 4$	 Equal $2q + 3 = 11$ <p>Remove three counters from both side (<math>-3</math>).</p> Equal $2q = 8$ <p>Halve the contents of each pan (<math>\div 2</math>).</p> Equal $q = 4$

- In both methods:
- the operations and the order in which they are performed to solve the equation are the same
  - the equivalent equations created on the way to the solution are the same
  - the aim is to get the variable by itself on one side of the equation
  - the last operation performed on the variable when building the equation is the first operation undone.

### WORKED EXAMPLE 16

Use inverse operations to solve the following equation.

a  $2y + 3 = 11$

b  $\frac{h}{5} + 1 = 3$

c  $2(k - 4) = 4$

**THINK**

- a** 1 The last operation performed when building  $2y + 3 = 11$  was  $+3$ . Subtract 3 from both sides and then simplify.

- 2 To get  $y$  by itself, divide both sides by 2 and then simplify.

- b** 1 The last operation performed when building  $\frac{h}{5} + 1 = 3$  was  $+1$ . Subtract 1 from both sides and then simplify.

- 2 To get  $h$  by itself, multiply both sides by 5 and then simplify.

- c** 1 The last operation performed when building  $2(k - 4) = 4$  was  $\times 2$ . Divide both sides by 2 and then simplify.

- 2 To get  $k$  by itself, add 4 to both sides and then simplify.

**WRITE**

$$\begin{aligned} \text{a} \quad & 2y + 3 = 11 \\ & 2y + 3 - 3 = 11 - 3 \\ & 2y = 8 \\ & \frac{2^1 y}{2^1} = \frac{8^1}{2^1} \\ & y = 4 \end{aligned}$$

$$\begin{aligned} \text{b} \quad & \frac{h}{5} + 1 = 3 \\ & \frac{h}{5} + 1 - 1 = 3 - 1 \\ & \frac{h}{5} = 2 \\ & \frac{h}{5} \times 5^1 = 2 \times 5 \\ & h = 10 \end{aligned}$$

$$\begin{aligned} \text{c} \quad & 2(k - 4) = 4 \\ & \frac{2^1(k - 4)}{2^1} = \frac{4^1}{2^1} \\ & k - 4 = 2 \\ & k - 4 + 4 = 2 + 4 \\ & k = 6 \end{aligned}$$

## Equations with rational number solutions

- When solving equations, sometimes the solution is a rational number.

### WORKED EXAMPLE 17

Solve the following equations.

**a**  $5d = 4$       **b**  $3c + 1 = 6$

**THINK**

- a** To get  $d$  by itself, divide both sides by 5 and then simplify.

**WRITE**

$$\begin{aligned} \text{a} \quad & 5d = 4 \\ & \frac{5^1 d}{5^1} = \frac{4}{5} \\ & d = \frac{4}{5} \end{aligned}$$

- b** 1 The last operation performed when building  $3c + 1 = 6$  was  $+1$ . Subtract 1 from both sides and simplify.

$$\begin{aligned} \text{b} \quad & 3c + 1 = 6 \\ & 3c + 1 - 1 = 6 - 1 \\ & 3c = 5 \end{aligned}$$

- 2 To get  $c$  by itself, divide both sides by 3 and then simplify.

$$\begin{aligned} & \frac{3^1 c}{3^1} = \frac{5}{3} \\ & c = \frac{5}{3} \\ & = 1\frac{2}{3} \end{aligned}$$

## Negative integers

- When the variable is being subtracted, it is difficult to represent the equation using backtracking or the balance method.
- By adding the variable to both sides, the equation can be solved as normal.

### WORKED EXAMPLE 18

Solve the following equations.

- a  $5 - v = 2$   
b  $17 - 2a = 11$

#### THINK

- a 1 Undo subtracting  $v$  by adding  $v$  and then simplify.

- 2 To get  $v$  by itself, subtract 2 from both sides and simplify.

- b 1 Undo subtracting  $2a$  by adding  $2a$  and then simplify.

- 2 The last operation performed when building  $17 = 11 + 2a$  was  $+11$ . Subtract 11 from both sides and simplify.

- 3 To get  $a$  by itself, divide both sides by 2 and then simplify.

#### WRITE

$$\begin{aligned} a \quad 5 - v &= 2 \\ 5 - v + v &= 2 + v \\ 5 &= 2 + v \end{aligned}$$

$$\begin{aligned} 5 - 2 &= 2 + v - 2 \\ 3 &= v \\ v &= 3 \end{aligned}$$

$$\begin{aligned} b \quad 17 - 2a &= 11 \\ 17 - 2a + 2a &= 11 + 2a \\ 17 &= 11 + 2a \end{aligned}$$

$$\begin{aligned} 17 - 11 &= 11 + 2a - 11 \\ 6 &= 2a \end{aligned}$$

$$\begin{aligned} \frac{6}{2} &= \frac{2a}{2} \\ 3 &= a \end{aligned}$$

#### REMEMBER

- Both backtracking and the balance method of solving equations use inverse operations to find the value of a variable.
- In both methods:
  - the operations and the order in which they are performed to solve the equation are the same
  - the equivalent equations created on the way to the solution are the same
  - the aim is to get the variable by itself on one side of the equation
  - the last operation performed on the variable when building the equation is the first operation undone.
- When solving equations, sometimes the solution is a rational number.
- When the variable is being subtracted, the equation can be solved as usual by adding the variable to both sides.

## EXERCISE

## 13F Keeping equations balanced

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 13-F-1

Balancing equations  
doc-1963

## Activity 13-F-2

More balancing  
equations  
doc-1964

## Activity 13-F-3

Tricky equations  
doc-1965

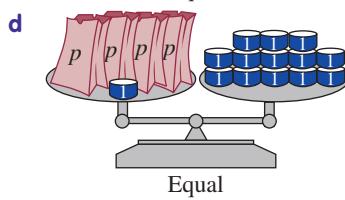
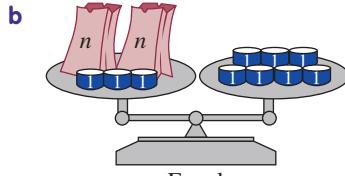
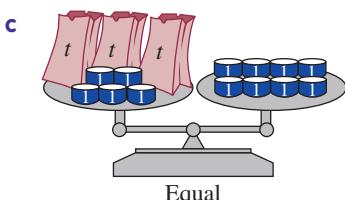
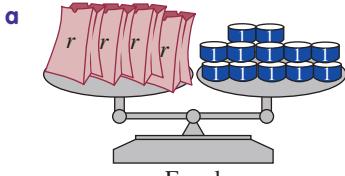
## eBook plus

Interactivity  
Balancing  
equations  
int-0077

## FLUENCY

- 1 WE15 For each of the following pan balance scales:

- i write the equation represented by the scale  
ii calculate the value of the variable.



- 2 WE16 Use inverse operations to solve the following.

a  $3g + 7 = 10$

b  $4m - 6 = 6$

c  $\frac{q}{3} + 8 = 11$

d  $2(n - 5) = 8$

e  $\frac{g-2}{4} = 3$

f  $4(y + 1) = 16$

- 3 WE17 Solve the following equations.

a  $3h = 7$

b  $2k = 5$

c  $2w + 1 = 8$

d  $3t - 4 = 9$

e  $6h - 3 = 10$

f  $3l + 4 = 8$

- 4 WE18 Solve the following equations.

a  $6 - m = 2$

b  $4 - d = 1$

c  $12 - 3v = 6$

d  $13 - 2s = 7$

e  $19 - 3g = 4$

f  $30 - 5k = 20$

## UNDERSTANDING

- 5 Solve the following equations.

a  $2(x + 3) = 11$

b  $3(5 - y) = 12$

c  $p + 3 = 6 \frac{1}{4}$

d  $5(8 - 2h) = 15$

- 6 The formula  $C = \frac{5(F - 32)}{9}$  is used to convert degrees Fahrenheit to degrees Celsius. Use the formula to find:

- a  $45^\circ$  Fahrenheit in degrees Celsius  
b  $45^\circ$  Celsius in degrees Fahrenheit.

## REASONING

- 7 A taxi company charges a \$2.70 flag fall. An additional \$1.87 per km is charged for the journey.

- a Estimate the cost of a 4.6 km journey.  
b If the journey cost \$111.16, how long was the journey.

- 8 While shopping for music online, Olivia found an album she liked. She could purchase the entire album for \$17.95 or songs from the album for \$1.69 each. How many songs would Olivia want to buy for it to be cheaper to buy the entire album?

## REFLECTION

How will you decide the order to undo the operations?

# Summary

## Solving equations using trial and error

- An equation can be used to describe a problem.
- Sometimes equations can be solved by guessing the solution.
- Guess, check and improve is a method where the guess is noted, so that the next guess is closer to the solution.

## Using inverse operations

- *Flowcharts* help us to organise harder equations.
- We can use *backtracking* and *inverse operations* to solve equations.

## Building up expressions

- We can use flowcharts to construct algebraic expressions.
- As well as constructing an algebraic expression using a flowchart, we can backtrack to our input number using inverse operations.

## Solving equations using backtracking

- We can solve equations by drawing a flowchart and backtracking.

## Checking solutions

- We can check our solution to an equation by substituting the value for the variable to see if it is true.

## Keeping equations balanced

- Both backtracking and the balance method of solving equations use inverse operations to find the value of a variable.
- In both methods:
  - the operations and the order in which they are performed to solve the equation are the same
  - the equivalent equations created on the way to the solution are the same
  - the aim is to get the variable by itself on one side of the equation
  - the last operation performed on the variable when building the equation is the first operation undone.
- When solving equations, sometimes the solution is a rational number.
- When the variable is being subtracted, the equation can be solved as usual by adding the variable to both sides.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter.

Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

1 Solve these equations by inspection.

a  $m + 7 = 12$

b  $5h = 30$

c  $s - 12 = 7$

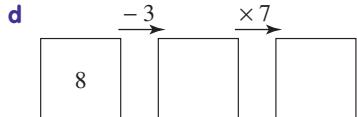
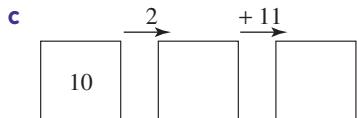
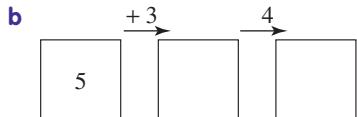
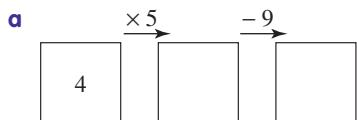
d  $\frac{d}{5} = 4$

2 Use guess, check and improve to find two numbers whose sum and product are given.

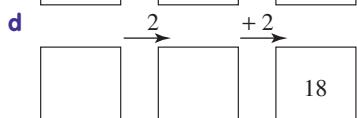
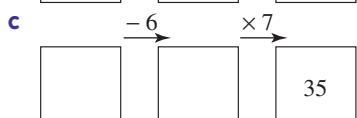
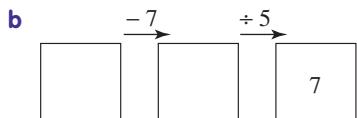
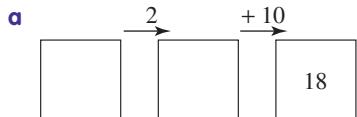
a Sum = 83, product = 1632

b Sum = 86, product = 1593

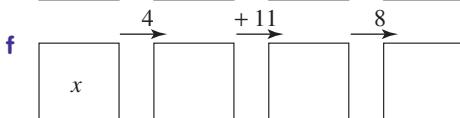
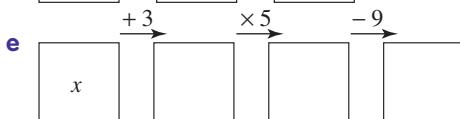
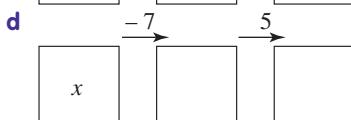
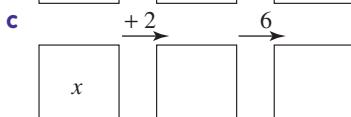
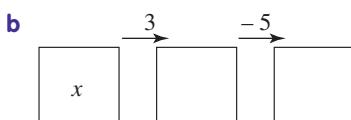
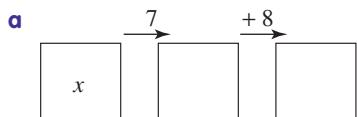
3 Complete these flowcharts to find the output number.



4 Use backtracking and inverse operations to find the input number in each of these flowcharts.



5 Build up an expression by following the instructions on the flowchart.



6 Draw the flowchart whose input is  $x$  and whose output is given by the expression.

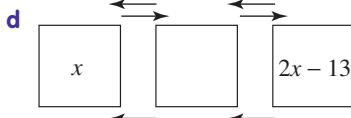
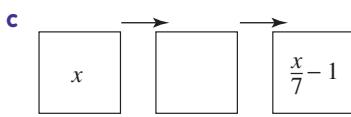
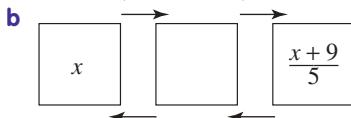
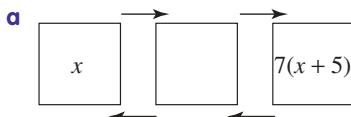
a  $5(x + 7)$

b  $\frac{x}{4} - 3$

c  $6x - 14$

d  $\frac{x + 2}{5}$

7 Complete these flowcharts by writing in the operations which must be carried out in order to backtrack to  $x$ .



8 Draw a flowchart and use backtracking to find the solution to the following equations.

a  $7x + 6 = 20$

b  $9(y - 8) = 18$

c  $\frac{m}{5} - 3 = 9$

d  $\frac{s + 7}{5} = 5$

- 9 Use backtracking to find the solution to these equations.

a  $3(d + 1) = 15$

b  $\frac{t}{4} - 11 = 14$

c  $6d - 3 = 15$

d  $\frac{a + 6}{4} = 3$

- 10 Simplify the expression and then solve the equation for each of the following.

a  $7v + 3 + 3v + 4 = 37$

b  $6c + 15 - 5c - 8 = 19$

- 11 For each equation below there is a solution given. Is the solution correct?

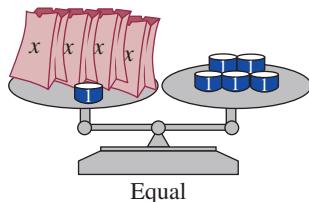
a  $5x - 7 = 2x + 2$        $x = 3$

b  $\frac{x + 9}{2} = 2x - 7$        $x = 5$

- 12 For each of the following pan balance scales:

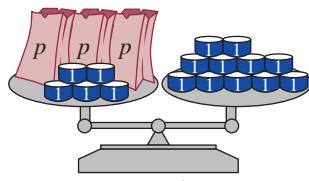
- i write the equation represented by the scale  
ii calculate the value of the variable.

a



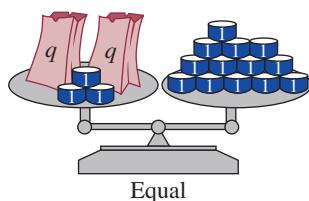
Equal

b



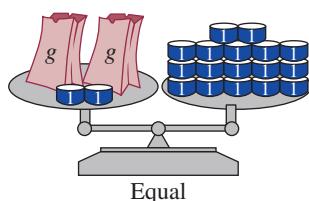
Equal

c



Equal

d



Equal

- 13 Use inverse operations to solve the following.

a  $2v + 1 = 7$

b  $\frac{x}{2} + 3 = 4$

c  $4w - 3 = 9$

d  $2(g + 1) = 10$

- 14 Solve the following equations. (Make sure you check your solutions.)

a  $4k + 3 = 10$

b  $2p - 3 = 8$

c  $3q + 6 = 7$

d  $5t + 1 = 3$

- 15 Solve the following equations. (Make sure you check your solutions.)

a  $3 - x = 1$

b  $7 - 2p = 3$

c  $10 - 4r = 2$

d  $13 - 3z = 7$

### PROBLEM SOLVING

- 1 Sophie and Jackie each have a collection of football cards. Jackie has 5 more cards than Sophie, and together they have 67 cards. By writing and solving an equation, find out how many cards Sophie owns.

- 2 Andreas has completed 2 more pieces of homework than Richard who submitted  $x$  pieces of homework for the semester. If the total number of pieces of homework submitted by the two boys is 12, how many pieces of homework did Andreas submit?



- 3 Keith is 6 years younger than his twin brothers. If the sum of their ages is 48, how old is Keith?

- 4 Margaret bought six 2-hour tickets and four daily tickets for \$28. If the cost of one daily ticket is \$4, write an equation and solve it to find the cost of a 2-hour ticket.

- 5 Jetski hire is \$30 deposit and \$10 per hour.

- a How much will it cost to hire a jetski for:

i 1 hour?

ii 2 hours?

iii 3 hours?

- b Write a rule that could be used to calculate the cost of hiring a jetski for  $h$  hours.

- c Use the rule to calculate the cost of hiring a jetski for 8 hours.

- d** You have \$85 to spend. Write an equation to help you work out how many hours you could hire a jetski for. Solve this equation.
- e** Work out how much money (if any) you would have left over after you pay the hire charge, if you hire the jet ski for 5 hours.
- f** After a great day on the water, you return the jetski with no damage. Would you have enough money for a hamburger on the way home?
- 6** Suppose it cost \$30 for an adult and \$15 for a child to enter the Royal Melbourne Show.
- How much does it cost two adults and three children to enter?
  - How much does it cost one adult and five children to enter?
  - What is the maximum number of people who could get into the Show for \$100?
  - What combination of adults and children could get in for \$300?
- 7** Judy is watching her daughter play in the park. She notices that some children are riding tricycles and some are riding bicycles. Altogether, 19 children are on cycles. She counts that there are 43 wheels on the cycles. How many children are on tricycles and how many are on bicycles?



**8** Two angles in a triangle have the same measure. The sum of the measures of these angles is the measure of the third angle. Find the measure of the angles. Is there something special about the triangle?

**9** A stage screen is 4 times as long as it is wide. If it were 5 metres wider and 4 metres shorter it would be a square. What are the dimensions of the stage screen?

**10** The largest angle in a triangle is 65 degrees more than the smallest angle. The third angle is 10 degrees more than the smallest angle. Find the size of the smallest angle.

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**Weblinks**

Algebra  
puzzle  
Algebra  
games

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**Interactivities**

Test yourself  
Chapter 13  
int-1824  
Word search  
Chapter 13  
int-2605  
Crossword  
Chapter 13  
int-2606

**Chapter Opener****Digital doc** (page 433)

- Hungry brain activity Chapter 13 (doc-6570)

**Are you ready?****Digital docs** (page 434)

- SkillsHEET 13.1 (doc-6571) Completing number sentences
- SkillsHEET 13.2 (doc-6572) Writing number sentences from written information
- SkillsHEET 13.3 (doc-6573) Applying the four operations
- SkillsHEET 13.4 (doc-6574) Inverse operations
- SkillsHEET 13.5 (doc-6575) Combining like terms
- SkillsHEET 13.6 (doc-6576) Checking solutions by substitution

**13A Solving equations using trial and error****Digital docs** (pages 437–8)

- Activity 13-A-1 (doc-1948) Solving equations
- Activity 13-A-2 (doc-1949) More solving equations
- Activity 13-A-3 (doc-1950) Advanced equations
- Spreadsheet Solving equations (doc-1966)

**13B Using inverse operations****Digital docs** (pages 440–1)

- Activity 13-B-1 (doc-1951) Inverse operations
- Activity 13-B-2 (doc-1952) More inverse operations
- Activity 13-B-3 (doc-1953) Advanced operations with integers
- Spreadsheet Backtracking (doc-1967)

**13C Building up expressions****Digital docs** (page 444)

- Activity 13-C-1 (doc-1954) Building up expressions
- Activity 13-C-2 (doc-1955) More expressions
- Activity 13-C-3 (doc-1956) Advanced expressions

**eLesson**

- Backtracking (eles-0010) (page 446)

**13D Solving equations using backtracking****Digital docs** (pages 448–9)

- Activity 13-D-1 (doc-1957) Backtracking
- Activity 13-D-2 (doc-1958) More backtracking
- Activity 13-D-3 (doc-1959) Advanced backtracking
- WorkSHEET 13.1 (doc-1946)

**Weblink**

- Solving equations (page 451)

**13E Checking solutions****Digital docs** (pages 453–4)

- Activity 13-E-1 (doc-1960) Checking solutions
- Activity 13-E-2 (doc-1961) More solutions
- Activity 13-E-3 (doc-1962) Advanced solutions
- WorkSHEET 13.2 (doc-1947)

**13F Keeping equations balanced****Digital docs** (page 459)

- Activity 13-F-1 (doc-1963) Balancing equations
- Activity 13-F-2 (doc-1964) More balancing equations
- Activity 13-F-3 (doc-1965) Tricky equations

**Interactivity**

- Keeping equations balanced (int-0077) (page 459)

**Chapter review****Weblinks** (page 463)

- Algebra puzzle
- Algebra games

**Interactivities** (page 463)

- Test yourself Chapter 13 (int-1824) Take the end-of-chapter test to test your progress.
- Word search Chapter 13 (int-2605)
- Crossword Chapter 13 (int-2606)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# 14

# Representing and interpreting data



- 14A Classifying data
- 14B Displaying data in tables
- 14C Measures of centre and spread
- 14D Representing data graphically
- 14E Comparing data

## WHAT DO YOU KNOW?

- 1 List what you know about data. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of data.

eBook plus

**Digital docs**

Hungry brain activity  
Chapter 14  
doc-6577

## OPENING QUESTION

Estimate the average height of this group of children.

# Are you ready?

Try the questions below. If you have difficulty with any of them, you can find extra help by completing the matching SkillsSHEET located on your eBookPLUS.


**Digital docs**

SkillsHEET 14.1

doc-6578

**Distinguishing qualitative from quantitative data**

- 1 State whether the following pieces of data are numerical or non-numerical.
  - a The monthly temperatures
  - b The brands of soft drinks
  - c The types of dogs
  - d The balance in a bank account


**Digital docs**

SkillsHEET 14.2

doc-6579

**Distinguishing discrete from continuous data**

- 2 State whether the following pieces of data are discrete or continuous.
  - a The daily temperature
  - b Your height
  - c The time taken to swim 100 m
  - d The number of children in your family


**Digital docs**

SkillsHEET 14.3

doc-6580

**Finding the mean of ungrouped data**

- 3 Find the average of the following scores.
  - a 1, 2, 3, 4, 5
  - b 11, 12, 13, 14, 15
  - c 1, 7, 9, 4, 2
  - d 180, 426, 392, 874


**Digital docs**

SkillsHEET 14.4

doc-6581

**Finding the median**

- 4 Arrange the following scores in ascending order, then state the middle score.
  - a 4, 8, 3, 9, 2
  - b 27, 16, 2, 9, 11
  - c 5, 6, 4, 8, 2, 9, 4
  - d 9, 9, 8, 7, 8, 4, 9

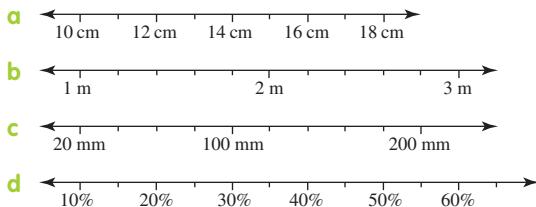

**Digital docs**

SkillsHEET 14.5

doc-6582

**Reading scales**

- 5 How much is each interval worth in the following scales?


**Digital docs**

SkillsHEET 14.6

doc-6583

**Comparing decimals**

- 6 Find the largest number in each of the following.
  - a 1.25, 1.35, 1.32
  - b 0.438, 0.483, 0.484


**Digital docs**

SkillsHEET 14.7

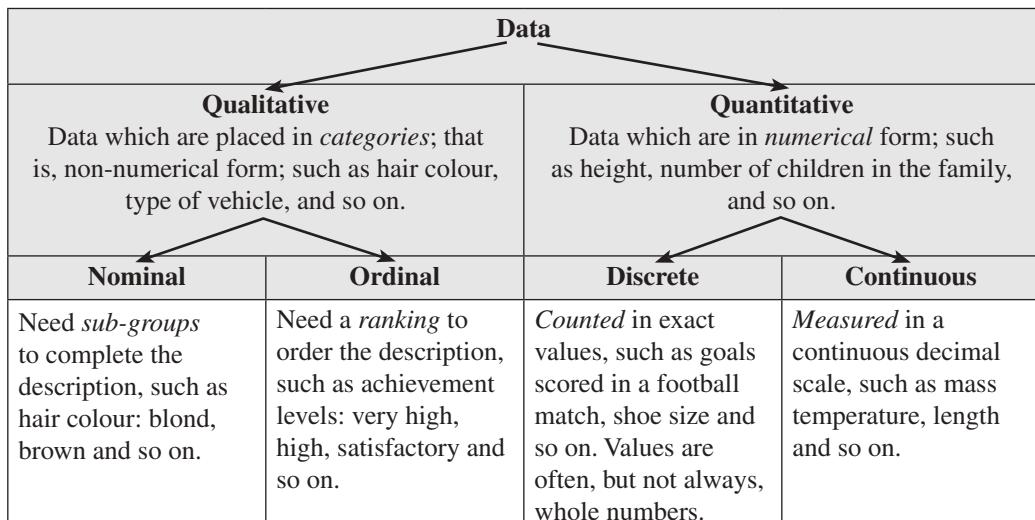
doc-6584

**Converting units to compare lengths or distances**

- 7 Arrange the following in order from smallest to largest.
  - a 25 mm, 2.3 cm, 0.2 m
  - b 1.8 m, 1700 mm, 185 cm

## 14A Classifying data

- Each day, people in all types of professions are presented with various forms of information, called data, which will assist them in answering questions and planning for the future.
  - Statistics is the branch of mathematics that deals with the *collection, organisation, display, analysis* and *interpretation* of data. These data are usually presented in numerical form.
- Data may be classified in the following ways.



### WORKED EXAMPLE 1

Classify each of the following data using two selections from the following descriptive words: qualitative, quantitative, nominal, ordinal, discrete and continuous.

- The number of students absent from school
- The types of vehicle using a certain road
- The various pizza sizes available at a local takeaway
- The room temperature at various times during a particular day

#### THINK

- Determine whether the data are qualitative or quantitative.
  - Determine whether the data are discrete or continuous.
- Determine whether the data are qualitative or quantitative.
  - Determine whether the data are nominal or ordinal.
- Determine whether the data are qualitative or quantitative.
  - Determine whether the data are nominal or ordinal.
- Determine whether the data are qualitative or quantitative.
  - Determine whether the data are discrete or continuous.

#### WRITE

- The data are quantitative as absences are represented by a number.

The data are discrete as the number of absences can be counted and are exact values.
- The data are qualitative as the types of vehicle need to be placed in non-numerical categories.

The data are nominal as there is no ranking or order involved.
- The data are qualitative as the pizza sizes need to be ranked in order ranging from small to family.

The data are ordinal as pizzas are ranked in order of size.
- The data are quantitative as room temperature is represented by a number.

The data are continuous as temperature can assume any value and measurement is involved.

**REMEMBER**

1. Surveys use representative samples of the target population.
2. Samples are chosen at random.
3. Data may be classified under the following headings.
  - (a) *Qualitative*. Data are placed in *categories* (non-numerical form).
    - Nominal: need other *sub-groups*.
    - Ordinal: need a *ranking* system.
  - (b) *Quantitative*. Data are in *numerical* form.
    - Discrete: *counted* in exact values.
    - Continuous: *measured* in a continuous decimal scale.

**EXERCISE****14A****Classifying data****INDIVIDUAL PATHWAYS****eBookplus****Activity 14-A-1**  
Classifying data  
doc-6719**Activity 14-A-2**  
More classification  
of data  
doc-6720**Activity 14-A-3**  
Advanced  
classification of data  
doc-6721**FLUENCY**

- 1 Match each word with its correct meaning:

- |                       |   |
|-----------------------|---|
| <b>a</b> discrete     | <b>i</b> placed in categories or classes                  |
| <b>b</b> qualitative  | <b>ii</b> counted in exact values                         |
| <b>c</b> ordinal      | <b>iii</b> data in the form of numbers                    |
| <b>d</b> continuous   | <b>iv</b> needs further names to complete the description |
| <b>e</b> quantitative | <b>v</b> needs a ranking order                            |
| <b>f</b> nominal      | <b>vi</b> measured in decimal numbers.                    |

**UNDERSTANDING**

- 2 **WE1** Classify each of the following data using two words selected from the following descriptive words: qualitative, quantitative, nominal, ordinal, discrete and continuous.
- a** The population of your town or city
  - b** The types of motorbike in a parking lot
  - c** The heights of people in an identification line-up
  - d** The masses of babies in a group
  - e** The languages spoken at home by students in your class
  - f** The time spent watching TV
  - g** The number of children in the families in your suburb
  - h** The air pressure in your car's tyres
  - i** The number of puppies in a litter
  - j** The types of radio program listened to by teenagers
  - k** The times for swimming 50 metres
  - l** The quantity of fish caught in a net
  - m** The number of CDs you own
  - n** The types of shops in a shopping centre
  - o** The football competition ladder at the end of each round
  - p** The lifetime of torch batteries
  - q** The number of people attending the Big Day Out
  - r** Final Year 12 exam grades



- s The types of magazine sold at a newsagency  
t Hotel accommodation rating
- 3 Write a sentence explaining the difference between discrete and continuous data. Give an example of each.
- 4 List 2 examples of each of the following types of data:
- a quantitative, discrete      b qualitative, ordinal
  - c quantitative, continuous      d qualitative, nominal.
- 5 **MC** Data representing shoe (or rollerblade) sizes can be classified as:
- A quantitative, continuous
  - B qualitative, nominal
  - C qualitative, ordinal
  - D quantitative, discrete
  - E none of the above
- 6 **MC** The data, points scored in a basketball game, can be best described as being:
- A discrete      B continuous      C qualitative      D ordinal      E nominal
- 7 **MC** An example of qualitative, ordinal data would be the:
- A heights of buildings in Melbourne      B number of pets in households
  - C type of pets in households      D birthday month of students in Year 7
  - E number of hours spent playing sport



### REASONING

- 8 A fisheries and wildlife officer released 200 tagged trout into a lake. A week later, the officer took a sample of 50 trout and found that 8 of them were tagged. The officer can use this information to estimate the population of trout in the lake. How many trout are in the lake?

### REFLECTION

Why is it necessary to classify data into different categories?

## 14B Displaying data in tables

- Data can be displayed in a variety of forms.
- Generally, data is first organised into a table; often then, a suitable graph is chosen as a visual representation.

### Frequency distribution tables

- One way of presenting data is by using a frequency distribution table.
- **Frequency** is the number of times a result or piece of data occurs.
- The **frequency distribution table** consists of three columns: score, tally and frequency.

### WORKED EXAMPLE 2

A particular class was surveyed to find out the number of pets per household and the data were recorded. The raw data were: 0, 3, 1, 2, 0, 1, 0, 1, 2, 4, 0, 6, 1, 1, 0, 2, 2, 0, 1, 3, 0, 1, 2, 1, 1, 2.

- a Organise the data into a frequency distribution table.
- b How many households were included in the survey?
- c How many households have fewer than 2 pets?
- d Which is the most common number of pets?
- e How many households have 3 or more pets?
- f What fraction of those surveyed had no pets?





## THINK

- a** **1** Draw a frequency distribution table comprising three columns, headed score (that is, the number of pets), tally and frequency.
- 2** In the first column list the possible number of pets per household (that is, 0 to 6).
- 3** Place a stroke in the tally column each time a particular score is noted.  
*Note:* A score of 5 is denoted as a ‘gate post’ (that is, four vertical strokes and one diagonal stroke |||).
- 4** Write the total tally strokes for each pet in the frequency column.
- 5** Calculate the total of the frequency column.
- b** The total of the frequency column gives the number of households surveyed.
- c** **1** Calculate the number of households which have fewer than 2 pets.  
*Note:* Fewer than 2 means 0 pets or 1 pet.
- 2** Answer the question.
- d** **1** Make a note of the highest value in the frequency column and check which score it corresponds to.
- 2** Answer the question.
- e** **1** Calculate the number of households which have 3 or more pets.  
*Note:* 3 or more means 3, 4, 5 or 6.
- 2** Answer the question.
- f** **1** Write the number of households with no pets.
- 2** Write the total number of households surveyed.
- 3** Define the fraction and substitute the known values into the rule.
- 4** Answer the question.

## WRITE

Score	Tally	Frequency
0		7
1		9
2		6
3		2
4		1
5		0
6		1
	Total	26

**b** Twenty-six households were surveyed.

**c** Fewer than two pets =  $7 + 9 = 16$

Sixteen households have fewer than 2 pets.

**d** The score with the highest frequency (that is, 9) corresponds to one pet.

The most common number of pets is one.

**e** 3 or more pets =  $2 + 1 + 0 + 1 = 4$

Four households have 3 or more pets.

**f** Households with no pets = 7

Total number of households surveyed = 26

$$\frac{\text{Households with no pets}}{\text{Total number of households surveyed}} = \frac{7}{26}$$

Of the households surveyed  $\frac{7}{26}$  have no pets.

Sometimes, the data may contain too many numerical values to list them all individually in the ‘score’ column. In this case, we use a range of values, called a *class interval*, as our category. For example, the range 100–104 may be used to cater for all the values that lie within the range, including 100 and 104.

**WORKED EXAMPLE 3**

The data below show the ages of a number of mobile phone owners:

12, 11, 21, 12, 30, 26, 13, 15, 29, 16, 17, 17, 17, 21, 19, 12, 14, 16, 43, 18, 51, 25, 30, 28, 33, 62, 39, 40, 30, 18, 19, 41, 22, 21, 48, 31, 33, 33, 34, 41, 18, 17, 31, 43, 42, 17, 46, 23, 24, 33, 27, 31, 53, 52, 25

- Draw a frequency table to classify the given data. Use a class interval of 10; that is, ages 11–20, 21–30 and so on, as each category.
- How many people were surveyed?
- Which age group had the largest number of people with mobile phones?
- Which age group had the least number of people with mobile phones?
- How many people in the 21–30 age group own a mobile phone?

**THINK**

- Draw a frequency distribution table.
  - In the first column, list the possible age groups; that is, 11–20, 21–30 etc.
  - Systematically go through the results and place a stroke in the tally column each time a particular age group is noted.
  - Write the total tally of strokes for each age group in the frequency column.
  - Calculate the total of the frequency column.
- The total of the frequency column gives us the number of people surveyed.
- Make note of the highest value in the frequency column and check which age group it corresponds to.
  - Answer the question.
- Make note of the lowest value in the frequency column and check which age group it corresponds to.  
*Note:* There may be more than one answer.
  - Answer the question.
- Check the 21–30 age group in the table to see which frequency value corresponds to this age group.
  - Answer the question.

**WRITE**

a	Age group	Tally	Frequency
	11–20		19
	21–30		15
	31–40		10
	41–50		7
	51–60		3
	over 60		1
	Total		55

- A total of 55 people were surveyed.

- The 11–20 age group has the highest frequency; that is, a value of 19.

The 11–20 age group has the most number of people with mobile phones.

- The over 60 age group has the lowest frequency; that is, a value of 1.

The over 60 age group has the least number of people with mobile phones.

- The 21–30 age group has a corresponding frequency of 15.

Fifteen people in the 21–30 age group own a mobile phone.

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Digital docs  
Investigation  
How many red M&M'S?  
doc-3438

**REMEMBER**

- Frequency is the number of times a result or piece of data occurs.
- A frequency distribution table consists of 3 columns, headed category, tally and frequency.

**EXERCISE****14B Displaying data in tables****INDIVIDUAL PATHWAYS****eBookplus****Activity 14-B-1**Displaying data  
doc-6722**Activity 14-B-2**More data displays  
doc-6723**Activity 14-B-3**Advanced data  
displays  
doc-6724**FLUENCY**

- 1 WE2** The number of children per household in a particular street is surveyed and the data recorded. The raw data are:

0, 8, 6, 4, 0, 0, 0, 2, 1, 3, 3, 3, 1, 2, 3,  
2, 3, 2, 1, 2, 1, 3, 0, 2, 2, 4, 2, 3, 5, 2.

- a** Organise the data into a frequency distribution table.
- b** How many households are included in the survey?
- c** How many households have no children?
- d** How many households have at least 3 children?
- e** Which is the most common number of children?
- f** What fraction of those surveyed have 4 children?

- 2 WE3** Draw a frequency table to classify the following data on house prices. Use a class interval of 10 000; that is prices \$100 000 to \$109 000 and so on for each category. The values are: \$100 000, \$105 000, \$110 000, \$150 000, \$155 000, \$106 000, \$165 000, \$148 000, \$165 000, \$200 000, \$195 000, \$138 000, \$142 000, \$153 000, \$173 000, \$149 000, \$182 000, \$186 000.

**UNDERSTANDING**

- 3** Rosemary decided to survey the participants of her local gym about their preferred sport. She asked each participant to name one preferred sport and recorded her results: hockey, cricket, cricket, tennis, scuba diving, netball, tennis, netball, swimming, netball, tennis, hockey, cricket, lacrosse, lawn bowls, hockey, swimming, netball, tennis, netball, cricket, tennis, hockey, lacrosse, swimming, lawn bowls, swimming, swimming, netball, netball, tennis, golf, hockey, hockey, lacrosse, swimming, golf, hockey, netball, swimming, scuba diving, scuba diving, golf, tennis, cricket, cricket, hockey, lacrosse, netball, golf.

- a** Can you see any problems with the way Rosemary has displayed the data?
- b** Organise Rosemary's results into a frequency table to show the participants' preferred sports.
- c** From the frequency table, find:
  - i** the most preferred sport
  - ii** the least preferred sport.
- d** Did any sport(s) have the same frequency?

- 4** Complete a frequency distribution table for each of the following.

**a** Andrew's scores in mathematics tests this semester are: 6, 9, 7, 9, 10, 7, 6, 5, 8, 9.

**b** The number of children in each household of a particular street are:  
2, 0, 6, 1, 1, 2, 1, 3, 0, 4, 3, 2, 4, 1, 0, 2, 1, 0, 2, 0.

**c** The masses (in kilograms) of students in a certain Year 7 class are:  
46, 60, 48, 52, 49, 51, 60, 45, 54, 54, 52, 58, 53, 51, 54, 50, 50, 56, 53, 57, 55, 48, 56, 53, 58, 53, 59, 57.

**d** The heights of students in a particular Year 7 class are:  
145, 147, 150, 150, 148, 145, 144, 144, 147, 149, 144, 150, 150, 152, 145, 149, 144, 145, 147, 143, 144, 145, 148, 144, 149, 146, 148, 143.

- 5** Use the frequency distribution table to answer the questions.

- a** How many participated in the survey?
- b** What was the most frequent score?
- c** How many scored less than 3?
- d** How many scored 3 or more?
- e** What fraction of those surveyed scored 3?

Score	Tally	Frequency
0		2
1		5
2		3
3		11
4		8
5		4
	Total	

- 6** A random sample of 30 families was surveyed to find the number of high-school-aged children in each family. Below are the raw data collected:  
 2, 1, 1, 0, 2, 0, 1, 0, 2, 0, 3, 1, 1, 0, 0, 0, 1, 4, 1, 0, 0, 1, 2, 1, 2, 0, 3, 2, 0, 1.
- Organise the data into a frequency distribution table.
  - How many families have no children of high school age?
  - How many have 2 or more children of high school age?
  - Which score has the highest frequency?
  - What is the greatest number of high-school-aged children in the 30 families surveyed?
  - What fraction of families had 2 children of high school age?
- 7** Draw a frequency table to classify the following data on students' heights. Use a range of values (such as 140–144) as each category.  
 The values are: 168 cm, 143 cm, 145 cm, 151 cm, 153 cm, 148 cm, 166 cm, 147 cm, 160 cm, 162 cm, 175 cm, 168 cm, 143 cm, 150 cm, 160 cm, 180 cm, 146 cm, 158 cm, 149 cm, 169 cm, 167 cm, 167 cm, 163 cm, 172 cm, 148 cm, 151 cm, 170 cm, 160 cm.
- 8** Complete a frequency table for all vowels in the following paragraphs.  
 Australian Rules Football is a ball game played by two teams of eighteen players with an ellipsoid ball on a large oval field with four upright posts at each end. Each team attempts to score points by kicking the ball through the appropriate posts (goals) and prevent their opponents from scoring. The team scoring the most points in a given time is the winner. Usually this period is divided into four quarters of play.  
 Play begins at the beginning of a quarter or after a goal, with a tap contest between two opposing players (rucks) in the centre of the ground after the umpire either throws the ball up or bounces it down.

Questions **9**, **10** and **11** refer to the following information.

A real estate agent has listed all the properties sold in the area in the last month as shown below. She wants to know what has been the most popular type of property from the following:  
 2 bedroom house, 4 bedroom house, 3 bedroom house, 2 bedroom unit, 4 bedroom house, 1 bedroom unit, 3 bedroom house, 2 bedroom unit, 3 bedroom house, 1 bedroom unit, 2 bedroom unit, 3 bedroom house, 3 bedroom house, 3 bedroom house, 2 bedroom unit, 1 bedroom unit.

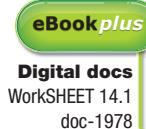
- 9** Complete a frequency table for the list given and work out which type of property was most popular.
- 10 MC** The least popular type of property is the:
- 1 bedroom unit
  - 2 bedroom unit
  - 2 bedroom house
  - 3 bedroom house
  - 4 bedroom house
- 11 MC** The property which is half as popular as a 2 bedroom unit is the:
- 4 bedroom house
  - 3 bedroom house
  - 2 bedroom house
  - 1 bedroom unit
  - none of these
- 12 MC** The frequency column of a frequency table will:
- add up to the total number of categories
  - add up to the total number of results given
  - add up to the total of the category values
  - display the tally
  - none of these

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Digital docs  
Spreadsheet  
Frequency  
tally tables

doc-3437

## REASONING



- 13 This frequency table shows the percentage occurrence of the vowels in a particular piece of text. Two pieces of data are missing — those for O and U.

Vowel	Percentage frequency
A	22.7
E	27.6
I	19.9
O	
U	

The occurrence of O is 2.6 times that of U. What are the two missing values?

## REFLECTION

What do you need to consider when selecting a class interval for a frequency distribution table?

## 14C Measures of centre and spread

### Measures of centre

Three measures of centre are used to show how a set of data is grouped around a central point.

- The **mean** of a set of data is another name for the average. It is calculated by adding all the data values, and dividing by the number of values in the set of data.
- The **median** is the middle value of the data when the values are arranged in numerical order.
- The **mode** of a set of data is the most frequently occurring value.

### Mean

The mean, or average, of a set of values is the sum of all the values divided by the number of values.

#### WORKED EXAMPLE 4

For each of the following sets of data, calculate the mean:

- a 5, 5, 6, 4, 8, 3, 4      b 0, 0, 0, 1, 1, 1, 1, 1, 4, 4, 4, 4, 5, 5, 5, 7, 7.

#### THINK

- a 1 Calculate the total of the given values.  
 2 Count the number of values.  
 3 Define the rule for the mean.  
 4 Substitute the known values into the rule and evaluate.
- b 1 Calculate the total of the given values.  
 Take note of the number of times each value occurs. That is, 0 occurs 3 times ( $3 \times 0$ ), 1 occurs 5 times ( $5 \times 1$ ), 4 occurs 4 times ( $4 \times 4$ ), 5 occurs 3 times ( $3 \times 5$ ), 7 occurs 2 times ( $2 \times 7$ ).  
 2 Count the number of values.  
 Note: Although zero has no numerical value, it is still counted as a piece of data and must be included in the number of values tally.

#### WRITE

$$\text{a} \quad \begin{aligned} \text{Total of values} &= 5 + 5 + 6 + 4 + 8 + 3 + 4 \\ &= 35 \end{aligned}$$

$$\text{Number of values} = 7$$

$$\text{Mean} = \frac{\text{total of values}}{\text{number of values}}$$

$$= \frac{35}{7} \\ = 5$$

$$\text{b} \quad \begin{aligned} \text{Total of values} &= 3 \times 0 + 5 \times 1 + 4 \times 4 \\ &\quad + 3 \times 5 + 2 \times 7 \\ &= 0 + 5 + 16 + 15 + 14 \\ &= 50 \end{aligned}$$

$$\text{Number of values} = 17$$

- 3 Define the rule for the mean.
- 4 Substitute the known values into the rule and evaluate.
- 5 Round the answer to 1 decimal place.  
*Note:* The mean doesn't necessarily have to be a whole number or included in the original data.

$$\begin{aligned}\text{Mean} &= \frac{\text{total of values}}{\text{number of values}} \\ &= \frac{50}{17} \\ &= 2.941\ 176\ 471\end{aligned}$$

= 2.9

## Median

The median is the middle value for an odd number of data and the average of the two middle values for an even number of data.

When determining the median:

1. The values must be arranged in numerical order.
2. There are as many values above the median as there are below it.
3. For an even number of values, the median may not be one of the listed scores.

### WORKED EXAMPLE 5

**Find the median for the following sets of data:**

**a** 5, 4, 2, 6, 3, 4, 5, 7, 4, 8, 5, 5, 6, 7, 5      **b** 8, 2, 5, 4, 9, 9, 7, 3, 2, 9, 3, 7, 6, 8.

#### THINK

- a** 1 Arrange the values in ascending order.

- 2 Select the middle value.

*Note:* There are an odd number of values; that is, 15. Hence, the eighth value is the middle number or median.

- 3 Answer the question.

- b** 1 Arrange the values in ascending order.

- 2 Select the middle values.

*Note:* There are an even number of values; that is, 14. Hence, the sixth and seventh values are the middle numbers.

- 3 Obtain the average of the two middle values.

*Note:* Add the two middle values and divide the result by 2.

- 4 Answer the question.

#### WRITE

**a**

$$2, 3, 4, 4, 4, 5, 5, \underline{\textcircled{5}}, 5, 5, 6, 6, 7, 7, 8$$

The median of the scores is 5.

**b**

$$2, 2, 3, 3, 4, 5, \underline{\textcircled{6}}, \underline{\textcircled{7}}, 7, 8, 8, 9, 9, 9$$

$$\begin{aligned}\text{Median} &= \frac{6+7}{2} \\ &= \frac{13}{2} \text{ (or } 6\frac{1}{2})\end{aligned}$$

The median of the scores is  $6\frac{1}{2}$  or 6.5.

## Mode

The mode is the most common value in a set of data.

Some sets of data have more than one mode, or no mode at all; that is, there is no value which corresponds to the highest frequency, as all values occur once only.

**WORKED EXAMPLE 6**

**Find the mode of the following scores:**

- a 2, 3, 4, 5, 5, 6, 6, 7, 8, 8, 8, 9
- b 12, 18, 5, 17, 3, 5, 2, 10, 12
- c 42, 29, 11, 28, 21.

**THINK**

- a 1 Look at the set of data and highlight any values that have been repeated.
- 2 Choose the value which has been repeated the most.
- 3 Answer the question.
- b 1 Look at the set of data and highlight any values that have been repeated.
- 2 Choose the value(s) which have been repeated the most.
- 3 Answer the question.  
*Note:* Some sets have more than one mode. The data set is called bimodal as two values were most common.
- c 1 Look at the set of data and highlight any values that have been repeated.
- 2 Answer the question.  
*Note:* No mode is not the same as having a mode which equals 0.

**WRITE**

- a 2, 3, 4, 5, 5, 6, 6, 7, 8, 8, 8, 9

The numbers 5 and 6 occur twice. However, the number 8 occurs three times.

The mode for the given set of values is 8.

- b 12, 18, 5, 17, 3, 5, 2, 10, 12

The number 5 occurs twice. The number 12 occurs twice.

The mode for the given set of values is 5 and 12.

- c 42, 29, 11, 28, 21

No values have been repeated. The set of data has no mode since none of the scores correspond to a highest frequency. Each of the numbers occurs once only.

**eBookplus**

**Interactivity**  
Measures of  
centre  
int-2352

**Measures of spread**

These measures indicate how far data values are spread from the centre, or from each other. There are several measures, but the appropriate one to discuss at this stage is the range.

**Range**

The **range** of a set of values is the difference between the highest and lowest values.

**WORKED EXAMPLE 7**

**Find the range of the following data.**

12, 76, 35, 29, 16, 45, 56

**THINK**

- 1 Obtain the highest and lowest values.
- 2 Define the range.
- 3 Substitute the known values into the rule.
- 4 Evaluate.
- 5 Answer the question.

**WRITE**

$$\begin{aligned} \text{Highest value} &= 76 \\ \text{Lowest value} &= 12 \end{aligned}$$

$$\begin{aligned} \text{Range} &= \text{highest value} - \text{lowest value} \\ &= 76 - 12 \\ &= 64 \end{aligned}$$

The set of values has a range of 64.

**REMEMBER**

- To determine the *mean* of data in a list simply obtain the total of all the values and divide by the number of values.
- The *median* is the middle value for an odd number of data and the average of the two middle values for an even number of data which are arranged in numerical order. There are as many values above the median as there are below it.
- The *mode* is the most common value. If two values are most common, they are called bimodal.
- The *range* is the difference between the highest and lowest value in the data.

**EXERCISE****14C****Measures of centre and spread****INDIVIDUAL PATHWAYS****eBookplus****Activity 14-C-1**

Measures of centre  
and spread  
doc-6725

**Activity 14-C-2**

More measures of  
centre and spread  
doc-6726

**Activity 14-C-3**

Advanced measures  
of centre and spread  
doc-6727

**eBookplus****Digital docs**  
Spreadsheets**Mean**

doc-3434

**Median**

doc-3435

**Mode**

doc-3436

**FLUENCY**

- 1 **WE4a** For each of the following sets of data, calculate the mean.

- |                                 |                                 |                                 |
|---------------------------------|---------------------------------|---------------------------------|
| <b>a</b> 3, 4, 5, 5, 6, 7       | <b>b</b> 5, 6, 7, 5, 5, 8       | <b>c</b> 4, 6, 5, 4, 2, 3       |
| <b>d</b> 3, 5, 6, 8, 7, 7       | <b>e</b> 5, 4, 4, 6, 2, 3       | <b>f</b> 2, 2, 2, 4, 3, 5       |
| <b>g</b> 12, 10, 13, 12, 11, 14 | <b>h</b> 11, 12, 15, 17, 18, 11 | <b>i</b> 12, 15, 16, 17, 15, 15 |
| <b>j</b> 10, 14, 12, 12, 16, 14 |                                 |                                 |

- 2 **WE4b** For each of the following sets of data, calculate the mean.

*Hint:* Use the grouping of values to help you.

- |                                    |                                    |                                    |
|------------------------------------|------------------------------------|------------------------------------|
| <b>a</b> 9, 9, 7, 7                | <b>b</b> 2, 2, 2, 4, 4, 4          | <b>c</b> 4, 4, 3, 3, 5, 5          |
| <b>d</b> 1, 1, 2, 3, 3             | <b>e</b> 1, 2, 2, 4, 4, 5          | <b>f</b> 1, 2, 2, 5, 5, 6, 7       |
| <b>g</b> 9, 9, 8, 8, 7, 1, 1, 1, 1 | <b>h</b> 3, 3, 3, 1, 1, 1, 2, 2, 2 | <b>i</b> 4, 4, 5, 5, 8, 8, 1, 1, 9 |
| <b>j</b> 2, 2, 2, 3, 3, 3, 3, 6    |                                    |                                    |

- 3 **WE5a** Find the middle value (median) for the following sets of data, by carefully ordering the values first.

- |                                    |  |
|------------------------------------|--|
| <b>a</b> 3, 3, 4, 5, 5, 6, 7       | <b>b</b> 1, 2, 2, 3, 4, 8, 9             |
| <b>c</b> 1, 2, 5, 6, 8, 8, 9       | <b>d</b> 2, 2, 2, 3, 3, 4, 5             |
| <b>e</b> 5, 5, 6, 6, 7, 7, 8, 9, 9 | <b>f</b> 7, 7, 7, 10, 11, 12, 15, 15, 16 |
| <b>g</b> 4, 3, 5, 3, 4, 4, 3, 5, 4 | <b>h</b> 1, 2, 5, 4, 1, 1, 1, 2, 5       |
| <b>i</b> 1, 2.5, 5, 3.4, 1, 2.4, 5 | <b>j</b> 1.2, 1.5, 1.4, 1.8, 1.9         |

- 4 **WE5b** Find the middle value (median) for the following sets of data, by carefully ordering the values first. Note there is an even number of values.

- |                             |                                 |                           |
|-----------------------------|---------------------------------|---------------------------|
| <b>a</b> 1, 1, 2, 2, 4, 4   | <b>b</b> 1, 2, 2, 2, 4, 5       | <b>c</b> 4, 5, 5, 5, 6, 7 |
| <b>d</b> 4, 5, 7, 7, 8, 9   | <b>e</b> 1, 2, 2, 3, 3, 4       | <b>f</b> 2, 4, 4, 6, 8, 9 |
| <b>g</b> 1, 5, 7, 8         | <b>h</b> 2, 4, 5, 7, 8, 8, 9, 9 | <b>i</b> 1, 4, 7, 8       |
| <b>j</b> 1, 5, 7, 8, 10, 15 |                                 |                           |

- 5 **WE6** Find the mode for each of the following sets of data.

- |   |                                 |                                 |
|---|---------------------------------|---------------------------------|
| <b>a</b> 3, 3, 4, 4, 4, 5, 6                              | <b>b</b> 2, 9, 8, 8, 4, 5       | <b>c</b> 1, 1, 2, 2, 2, 3       |
| <b>d</b> 4, 6, 4, 2, 7, 3                                 | <b>e</b> 2, 4, 3, 6, 2, 4, 2    | <b>f</b> 4, 8, 8, 3, 3, 4, 3, 3 |
| <b>g</b> 6, 2, 12, 10, 14, 14, 10, 12, 10, 12, 10, 12, 10 |                                 |                                 |
| <b>h</b> 7, 9, 4, 6, 26, 71, 3, 3, 3, 2, 4, 6, 4, 25, 4   |                                 |                                 |
| <b>i</b> 2, 2, 3, 4, 4, 9, 9, 9, 6                        | <b>j</b> 3, 7, 4, 3, 4, 3, 6, 3 |                                 |

- 6 **WE7** a Find the range of the following: 15, 26, 6, 38, 10, 17.

- b Find the range of the following: 12.8, 21.5, 1.9, 12.0, 25.4, 2.8, 1.3.

**UNDERSTANDING**

- 7 MC** The mean for the data 5, 5, 6, 7, 2 would be found by:
- adding all the results and multiplying by the number of results
  - adding all the results and dividing by the number of results
  - adding all the results
  - choosing the middle result
  - ordering the results, then choosing the middle result
- 8 MC** When finding the mean of a set of data:
- zeroes do not matter at all
  - zeroes must be counted in the number of results
  - zeroes must be added to the total as they will change it
  - zeroes will make the mean zero
  - none of these is true
- 9 MC** For the following set of data, 2.6, 2.8, 3.1, 3.7, 4.0, 4.2:
- the mean value for the data will be above 4.2
  - the mean value for the data will be below 2.6
  - the mean value for the data will be between 2.6 and 3.0
  - the mean value for the data will be between 3.0 and 4.0
  - the mean value for the data will be between 4.0 and 4.2
- 10 MC** Which of the following is a correct statement?
- The mean, median and mode for any set of data will always be the same value.
  - The mean, median and mode for any set of data will never be the same value.
  - The mean, median and mode for any set of data must always be close in value.
  - The mean, median and mode for any set of data are usually close in value.
  - None of these statements is true.
- 11 MC** The range of the following set of numbers: 16, 33, 24, 48, 11, 30, 15, is:
- A** 48      **B** 59      **C** 37      **D** 20      **E** 11
- 12** Eleanor wanted to know what her mathematics test average was. Her teacher said that she used the mean of her test results to calculate the end-of-year mark. Eleanor's test results (percentages) were: 89, 87, 78, 75, 89, 94, 82, 93, 78.  
What was her mathematics test mean?
- 13** The number of shoes inspected by a factory worker in an hour was counted over a number of days' work. The results are as follows: 105, 102, 105, 106, 103, 105, 105, 102, 108, 110, 102, 103, 106, 107, 108, 102, 105, 106, 105, 104, 102, 99, 98, 105, 102, 101, 97, 100.  
What is the mean number of shoes checked by this worker in one hour? Round your answer to the nearest whole number.
- 14** The number of students in the cafeteria each lunchtime was surveyed for 2 weeks. The results were as follows: 52, 45, 41, 42, 53, 45, 47, 32, 52, 56.  
What was the mean number of students in the cafeteria at lunchtime in that fortnight? Round your answer to the nearest whole number.
- 15** A cricketer had scores of 14, 52, 35, 42 and 47 in her last 5 innings. What is her mean score?
- 16** Tom thinks that the petrol station where he buys his petrol is cheaper than the one where his friend Sarah buys her petrol. They begin to keep a daily watch on the prices for 4 weeks and record the following prices (in cents per litre).
- Tom: 75.2, 72.5, 75.2, 75.3, 75.4, 75.6, 72.8, 73.1, 73.1, 73.2, 73.4, 75.8, 75.6, 73.4, 73.4, 75.6, 75.4, 75.2, 75.3, 75.4, 76.2, 76.2, 76.3, 76.4, 76.4, 76.2, 76.0  
Sarah: 72.6, 77.5, 75.6, 78.2, 67.4, 62.5, 75.0, 75.3, 72.3, 82.3, 75.6, 72.3, 79.1, 70.0, 67.8, 67.5, 70.1, 67.8, 75.9, 80.1, 81.0, 58.5, 68.5, 75.2, 68.3, 75.2, 75.1, 72.0
- a** Calculate the mean petrol prices for Tom and Sarah.  
**b** Which station sells cheaper petrol on average?  
**c** Why might Tom have been misled?

- 17** Peter has calculated his mean score for history to be 89%, based on five tests. If he scores 92% in the sixth test, what will his new mean score be?
- 18** Kim has an average (mean) score of 72 in Scrabble. He has played six games. What must he score in the next game to keep the same average?
- 19** A clothing company wanted to know the size of jeans that should be manufactured in the largest quantities. A number of shoppers were surveyed and asked their jeans size. The results were: 13, 12, 14, 12, 15, 16, 14, 12, 15, 14, 12, 14, 13, 14, 11, 10, 12, 13, 14, 14, 10, 12, 14, 12, 12, 10, 8, 16, 17, 12, 11, 13, 12, 15, 14, 12, 17, 8, 16, 11, 12, 13, 12, 12.
- What is the mode of these data?
  - Why would the company be more interested in the mode than the mean or median values?
- 20** Jennifer wants to ensure that the mean height of her jump in the high jump for 10 jumps is over 1.80 metres.
- If her jumps so far have been (in metres) 1.53, 1.78, 1.89, 1.82, 1.53, 1.81, 1.75, 1.86, 1.82, what is her current mean?
  - What height must she jump on the tenth jump to achieve a mean of 1.80?
  - Is this likely, given her past results?
- 21** The local football team has been doing very well. They want to advertise their average score (to attract new club members). You suggest that they use the mean of their past season's game scores. They ask you to find that out for them. Here are the results.  
Game scores for season (totals):  
110, 112, 141, 114, 112, 114, 95, 75, 58, 115, 116, 115, 75, 114, 78, 96, 78, 115, 112, 115, 102, 75, 79, 154, 117, 62.
- What was their mean score?
  - Would the mode or median have been a 'better' average to use for the advertisement?

**REASONING**

- 22** A group of three children have a mean height of 142 cm. The middle height is the same as the mean. The tallest child leaves the group, and is replaced by a child with the same height as the shortest child. The mean height of this group of three children is now 136 cm.  
What are the heights of the four children?
- 23** Find five numbers that have a mean of 10 and a median of 12.
- 24** The mean of 5 different test scores is 15. What are the largest and smallest possible test scores, given that the median is 12? All test scores are whole numbers.
- 25** The mean of 5 different test scores is 10. What are the largest and smallest possible values for the median? All test scores are whole numbers.
- 26** The mean of 9 different test scores that are whole numbers and range from 0 to 100 is 85. The median is 80. What is the greatest possible range between the highest and lowest possible test scores?

**REFLECTION**

Why do we need to summarise data by calculating measures of centre and spread?

# 14D Representing data graphically

- Graphs are a useful way of displaying data, or numerical information. Newspapers, magazines and TV frequently display data as graphs.
- All graphs should have the following features:
  1. a *title* — to tell us what the graph is about
  2. clear *labels* for the axes — to explain what is being shown
  3. *evenly scaled axes* — if the graph has numerical axes, they must have a scale, which must stay constant for the length of the axes and the units that are being used should be indicated
  4. **legends** — these are not always necessary, but are necessary when any symbols or colours are used to show some element of the graph.

## Column and bar graphs

- Columns and bar graphs use categories to divide the results into groups.
- The frequency for each category determines the length of the bar, or height of the column.
- It is easiest to graph the data from a frequency table.

### Column graphs

**Column graphs** should be presented on graph paper and have:

1. a title
2. labelled axes which are clearly and evenly scaled
3. columns of the same width
4. an even gap between each column
5. the first column beginning half a unit (that is, half the column width) from the vertical axis.

### WORKED EXAMPLE 8

Beth surveyed the students in her class to find out their preferences for the school uniform. Her results are shown in the table at right.

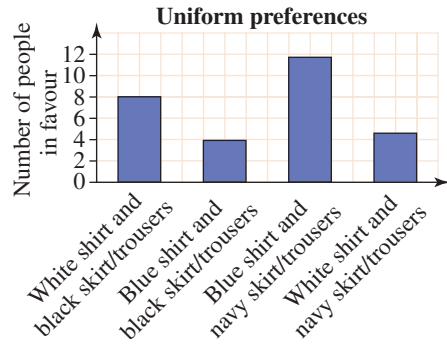
Construct a column graph to display the results.

Type of uniform	Tally	Frequency
White shirt and black skirt/trousers		8
Blue shirt and black skirt/trousers		4
Blue shirt and navy skirt/trousers		12
White shirt and navy skirt/trousers		5
<b>Total</b>		<b>29</b>

### THINK

- 1 Rule a set of axes on graph paper. Provide a title for the graph. Label the horizontal and vertical axes.
- 2 Scale the horizontal and vertical axes. *Note:* Leave a half interval at the beginning and end of the graph; that is, begin the first column half a unit from the vertical axis.
- 3 Draw the first column so that it reaches a vertical height corresponding to 8 people. Label the section of the axis below the column as ‘White shirt and black skirt/trousers’.
- 4 Leave a gap (measuring one column width) between the first column and the second column.
- 5 Repeat steps 3 and 4 for each of the remaining uniform types.

### DRAW



### Bar graphs

**Bar graphs** are drawn in a similar manner to column graphs. However, there is one major difference. To draw a bar graph, numbers are placed on the horizontal axis and categories on the vertical axis. Therefore, instead of having vertical columns we have horizontal bars.

When drawing bar graphs, they should be presented on graph paper and have:

1. a title
2. labelled axes which are clearly and evenly scaled
3. horizontal bars of the same width
4. an even gap between each horizontal bar
5. the first horizontal bar beginning half a unit (that is, half the bar width) above the horizontal axis.

### Dot plots

- **Dot plots** can be likened to picture graphs where each piece of data or score is represented by a single dot.
- Dot plots consist of a horizontal axis that is labelled and evenly scaled, and each data value is represented by a dot.
- Dot plots give a quick overview of a particular distribution. They show clustering, extreme values, and help to determine whether data should be grouped.
- If a score is repeated in a dot plot, a second dot is placed directly above the previous one. Once all values have been recorded, the data points, if neatly drawn and evenly spaced, resemble columns placed over a number line.
- Sometimes extreme values occur in a data set. They appear to be not typical of the rest of the data, and are called **outliers**. Sometimes they occur because measurements of the data have been incorrectly recorded. They serve as a reminder to always check the data collected.

#### WORKED EXAMPLE 9

Over a 2-week period, the number of packets of potato chips sold from a vending machine each day was recorded:

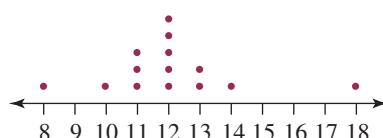
10, 8, 12, 11, 12, 18, 13, 11, 12, 11, 12, 12, 13, 14.

- a Draw a dot plot of the data.
- b Comment on the distribution.

#### THINK

- a
  - 1 Use a scaled number line to include the full range of data recorded.
  - 2 Place a dot above the appropriate scale number for each value recorded.
- b Comment on interesting features of the dot plot, such as the range, clustering, extreme values and any practical conclusions that fit the situation.

#### WRITE



- a For the given dot plot:  
The scores extend from 8 to 18; that is, a range of ten.  
Mostly between 11 to 13 packets were sold.  
Sales of 8 and 18 packets of chips were extremely low.  
A provision of 20 packets of chips each day should cover the most extreme demand.



## Stem-and-leaf plots

- When data are being displayed, a stem-and-leaf plot may be used as an alternative to the frequency distribution table.
- Sometimes ‘stem-and-leaf plot’ is shortened to ‘stem plot’.
- Each piece of data in a stem plot is made up of two components: a stem and a leaf. For example, the value 28 is made up of a tens component (the stem) and the units component (the leaf) and would be written as:

Stem	Leaf
2	8

- It is important to provide a key when drawing up stem plots, as the plots may be used to display a variety of data; that is, values ranging from whole numbers to decimals.

### WORKED EXAMPLE 10

Prepare an ordered stem plot for each of the following sets of data.

- a 129, 148, 137, 125, 148, 163, 152, 158, 172, 139, 168, 121, 134  
 b 1.6, 0.8, 0.7, 1.2, 1.9, 2.3, 2.8, 2.1, 1.6, 3.1, 2.9, 0.1, 4.3, 3.7, 2.6

#### THINK

- a 1 Rule two columns with the headings ‘Stem’ and ‘Leaf’.  
 2 Include a key to the plot that informs the reader of the meaning of each entry.  
 3 Make a note of the smallest and largest values of the data (that is, 121 and 172 respectively). List the stems in ascending order in the first column (that is, 12, 13, 14, 15, 16, 17).  
*Note:* The hundreds and tens components of the number represent the stem.  
 4 Systematically work through the given data and enter the leaf (unit component) of each value in a row beside the appropriate stem.  
*Note:* The first row represents the interval 120–129, the second row represents the interval 130–139 and so on.  
 5 Redraw the stem plot so that the numbers in each row of the leaf column are in ascending order.

#### WRITE

a Key: 12|1 = 121

Stem	Leaf
12	9 5 1
13	7 9 4
14	8 8
15	2 8
16	3 8
17	2

Key: 12|1 = 121

Stem	Leaf
12	1 5 9
13	4 7 9
14	8 8
15	2 8
16	3 8
17	2

- b 1 Rule the stem and leaf columns and include a key.  
 2 Make a note of the smallest and largest values of the data (that is, 0.1 and 4.3 respectively). List the stems in ascending order in the first column (that is, 0, 1, 2, 3, 4).  
*Note:* The units components of the decimal represent the stem.  
 3 Systematically work through the given data and enter the leaf (tenths component) of each decimal in a row beside the appropriate stem.  
*Note:* The first row represents the interval 0.1–0.9, the second row represents the interval 1.0–1.9 and so on.

b Key: 0|1 = 0.1

Stem	Leaf
0	8 7 1
1	6 2 9 6
2	3 8 1 9 6
3	1 7
4	3

- 4** Redraw the stem plot so that the numbers in each row of the leaf column are in ascending order to produce an ordered stem plot.

Key:	0   1 = 0.1
Stem	Leaf
0	1 7 8
1	2 6 6 9
2	1 3 6 8 9
3	1 7
4	3

The advantage of using a stem plot compared with a grouped frequency distribution table is that all the original data are retained. It is therefore possible to identify smallest and largest values, as well as repeated values. Measures of centre (such as mean, median and mode) and spread (range) are able to be calculated. This cannot be done when values are grouped in class intervals. When two sets of data are related, we can present them as back-to-back stem plots.

### WORKED EXAMPLE 11

**The ages of male and female groups using a ten-pin bowling centre are listed.**

**Males:** 65, 15, 50, 15, 54, 16, 57, 16, 16, 21, 17, 28, 17, 27, 17, 22, 35, 18, 19, 22, 30, 34, 22, 31, 43, 23, 48, 23, 46, 25, 30, 21.

**Females:** 16, 60, 16, 52, 17, 38, 38, 43, 20, 17, 45, 18, 45, 36, 21, 34, 19, 32, 29, 21, 23, 32, 23, 22, 23, 31, 25, 28.

**Display the data as a back-to-back stem plot and comment on the distribution.**



#### THINK

- Rule three columns, headed Leaf (female), Stem and Leaf (male).
- Make a note of the smallest and largest values of both sets of data (15 and 65). List the stems in ascending order in the middle column.
- Beginning with the males, work through the given data and enter the leaf (unit component) of each value in a row beside the appropriate stem.
- Repeat step 3 for the females' set of data.
- Include a key to the plot that informs the reader of the meaning of each entry.

#### WRITE

**Key:** 1 | 5 = 15

Leaf (female)	Stem	Leaf (male)
9 8 7 7 6 6	1	5 5 6 6 6 7 7 7 8 9
8 5 3 2 3 3 1 9 1 0	2	1 8 7 2 2 2 3 3 5 1
1 2 2 4 6 8 8	3	5 0 4 1 0
5 5 3	4	3 8 6
2	5	0 4 7
0	6	5



- 6** Redraw the stem plot so that the numbers in each row of the leaf columns are in ascending order.  
*Note:* The smallest values are closest to the stem column and increase as they move away from the stem.

Key:  $1 | 5 = 15$ 

Leaf (female)	Stem	Leaf (male)
9 8 7 6 6	1	5 5 6 6 6 7 7 7 8 9
9 8 5 3 3 3 2 1 1 0	2	1 1 2 2 2 3 3 5 7 8
8 8 6 4 2 2 1	3	0 0 1 4 5
5 5 3	4	3 6 8
2	5	0 4 7
0	6	5

- 7** Comment on any interesting features.

The youngest male attending the ten-pin bowling centre is 15 and the oldest 65; the youngest and oldest females attending the ten-pin bowling centre are 16 and 60 respectively. Ten-pin bowling is most popular for men in their teens and 20s, and for females in their 20s and 30s.

**eBook plus****Digital docs**

Investigations

Walking billboard

doc-3439

Families

doc-3440

**REMEMBER**

- Graphs should be drawn on graph paper for greater accuracy.
- Column graphs* must have:
  - a clear, bold title
  - labelled axes that are clearly and evenly scaled
  - columns of the same width
  - an even gap between each column
  - the first column beginning half a unit from the vertical axis.
- Bar graphs* must have:
  - a clear, bold title
  - labelled axes that are clearly and evenly scaled
  - horizontal bars of the same width
  - an even gap between each horizontal bar
  - the first horizontal bar beginning half a unit above the horizontal axis.
- Dot plots (or line plots) give a quick overview of a distribution: they show clustering, extreme values, and help us to see if data should be grouped.
- A stem plot allows:
  - all the original data to be retained
  - a graphical representation of the data to be seen.
- Back-to-back stem plots present two related sets of data simultaneously.

**EXERCISE****14D****Representing data graphically****INDIVIDUAL PATHWAYS****eBook plus****Activity 14-D-1**Graphing data  
doc-6728**Activity 14-D-2**More graphs of data  
doc-6729**FLUENCY**

- 1 WE8** Beth surveyed the students in her class to find out their method of travelling to school. Her results are shown in the table at right.

Construct a column graph to display the data.

Transport	Tally	Frequency
Car		15
Tram		9
Train		18
Bus		8
Bicycle		3
	Total	53

**INDIVIDUAL PATHWAYS****eBookplus****Activity 14-D-3**

Advanced graphs of data  
doc-6730

**eBookplus**

**Digital docs**  
Spreadsheets  
**Column graphs**  
doc-3441  
**Bar graphs**  
doc-3442  
**Dot plots**  
doc-3443

- 2 Construct a column graph to display the data at right, showing the mean daily maximum temperatures for each month in Cairns, Queensland.
- 3 The table below shows the number of students absent from school each day in a fortnight. Construct a bar graph to display the data.

Day	Number of students absent
Monday	15
Tuesday	17
Wednesday	20
Thursday	10
Friday	14
Monday	16
Tuesday	14
Wednesday	12
Thursday	5
Friday	14

Month	Mean daily maximum temperature (°C)
January	31.8
February	31.5
March	30.7
April	29.3
May	27.6
June	25.9
July	25.6
August	26.4
September	27.9
October	29.7
November	30.8
December	31.8

- 4 **WE9** Over a 2-week period, the number of packets of potato chips sold from a vending machine each day was recorded as follows:  
15, 17, 18, 18, 14, 16, 17, 6, 16, 18, 16, 16, 20, 18.
- a Draw a dot plot of the data.
  - b Comment on the distribution.
- 5 Draw a dot plot for each of the following sets of data:
- a 2, 0, 5, 1, 3, 3, 2, 1, 2, 3
  - b 18, 22, 20, 19, 20, 21, 19, 20, 21
  - c 5.2, 5.5, 5.0, 5.8, 5.3, 5.2, 5.6, 5.3, 6.0, 5.5, 5.6
  - d 49, 52, 60, 55, 57, 60, 52, 66, 49, 53, 61, 57, 66, 62, 64, 48, 51, 60.
- 6 **WE10a** The following data give the number of fruit that have formed on each of 40 trees in an orchard:  
29, 37, 25, 62, 73, 41, 58, 62, 73, 67, 47, 21, 33, 71, 92, 41, 62, 54, 31, 82, 93, 28, 31, 67, 29, 53, 62, 21, 78, 81, 51, 25, 93, 68, 72, 46, 53, 39, 28, 40  
Prepare an ordered stem plot that displays the data.
- 7 The number of errors made each week by 30 machine operators is recorded below:  
12, 2, 0, 10, 8, 16, 27, 12, 6, 1, 40, 16, 25, 3, 12, 31, 19, 22, 15, 7, 17, 21, 18, 32, 33, 12, 28, 31, 32, 14  
Prepare an ordered stem plot that displays the data.
- 8 Prepare an ordered stem plot for each of the following sets of data:
- a 132, 117, 108, 129, 165, 172, 145, 189, 137, 116, 152, 164, 118
  - b 131, 173, 152, 146, 150, 171, 130, 124, 114
  - c 196, 193, 168, 170, 199, 186, 180, 196, 186, 188, 170, 181, 209
  - d 207, 205, 255, 190, 248, 248, 248, 237, 225, 239, 208, 244
  - e 748, 662, 685, 675, 645, 647, 647, 708, 736, 691, 641, 735
- 9 **WE10b** Prepare an ordered stem plot for each of the following sets of data:
- a 1.2, 3.9, 5.8, 4.6, 4.1, 2.2, 2.8, 1.7, 5.4, 2.3, 1.9
  - b 2.8, 2.7, 5.2, 6.2, 6.6, 2.9, 1.8, 5.7, 3.5, 2.5, 4.1



- c 7.7, 6.0, 9.3, 8.3, 6.5, 9.2, 7.4, 6.9, 8.8, 8.4, 7.5, 9.8
- d 14.8, 15.2, 13.8, 13.0, 14.5, 16.2, 15.7, 14.7, 14.3, 15.6, 14.6, 13.9, 14.7, 15.1, 15.9, 13.9, 14.5
- e 0.18, 0.51, 0.15, 0.02, 0.37, 0.44, 0.67, 0.07

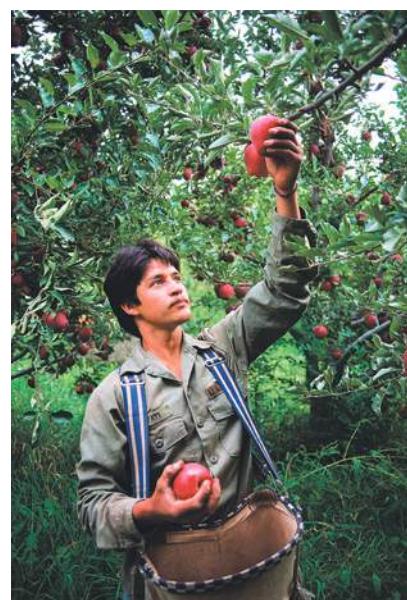
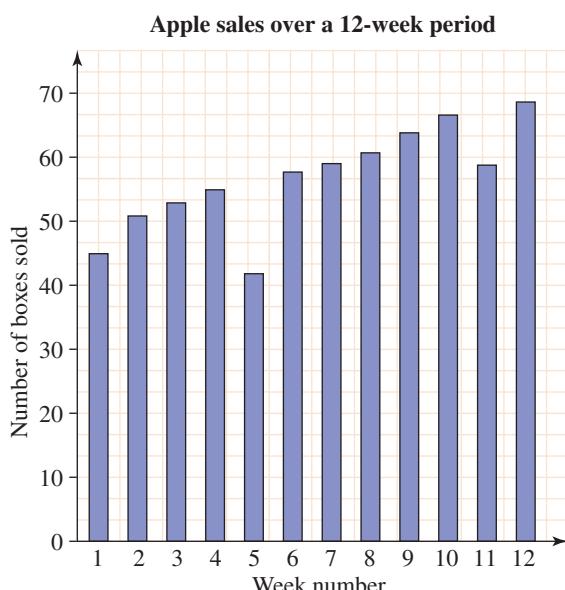
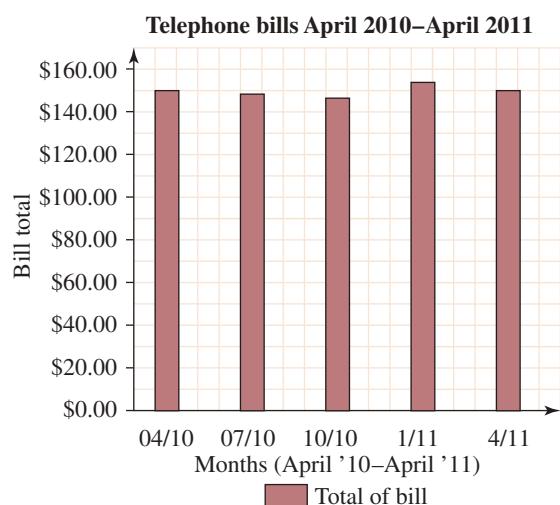
- 10** **WE1** The number of goals scored in football matches by Mitch and Yani were recorded as follows:

<b>Mitch</b>	0	3	1	0	1	2	1	0	0	1
<b>Yani</b>	1	2	0	1	0	1	2	2	1	1

Display the data as a back-to-back stem plot and comment on the distribution.

### UNDERSTANDING

- 11** Telephone bills often include a graph showing how much your previous bill totals. Use the column graph at right to answer the following questions.
- a What is the title of this graph?
  - b What is the horizontal axis label?
  - c What is the vertical axis label?
  - d How often does this person receive a phone bill?
  - e In which month was the bill the highest?
  - f Was each bill for roughly the same amount?
  - g If you answered ‘yes’ to part f, approximately how much was the amount?
  - h Why would it be useful to receive a graph like this with your phone bill?
  - i If the next bill was for \$240.09, would this be normal? Why?
  - j How much (approximately) do phone calls from this phone cost per month?
- 12** An apple producer records his sales for a 12-week period.



- a How many boxes were sold in the first week?
- b How many boxes were sold in the fifth week?
- c How many boxes were sold in the eighth week?
- d The values for some weeks may be unusual. Which ones might be unusual? Explain your answer.
- e What might cause unusual values in a graph like this?
- f Does the graph indicate that apple sales are improving? Explain your answer.

- 13** In their physical education class the girls in a Year 7 class were asked to sprint for 10 seconds. The teacher recorded their results on 2 different days. They are shown in the graph at right.

- a Why are there 2 columns for each girl?
- b Which girl ran the fastest on either day?
- c How far did she run on each day?
- d Which girl improved the most?
- e Were there any students who did not improve? Who were they?
- f Could this graph be misleading in any way? Explain your answer.
- g Why might the graph's vertical axis start at 30 m?

- 14** Compare and comment on the range, clustering and extreme values (if any) for the dot plots in Question 5.

- 15** The following stem plot gives the age of members of a theatrical group.

Key:	2   4 = 24
Stem	Leaf
1	7 8 8 9 9
2	2 4 7 9
3	1 3 3 8
4	0 2 2 2 6 6
5	5 7
6	4

- a How many people are in the theatrical group?
- b What is the age of the youngest member of the group?
- c What is the age of the oldest member of the group?
- d How many people are over 30 years of age?
- e What age is the most common in the group?
- f How many people are over 65 years of age?

- 16** Swim times, in seconds, over 100 metres were recorded for a random sample of 20 swimmers at Mathsville Secondary College:

10.8, 11.0, 12.0, 13.2, 12.4, 13.9, 11.8, 12.8, 14.0, 15.0, 11.2, 12.6, 12.5, 12.8, 13.6, 11.5, 13.6, 10.9, 14.1, 13.9.

- a Show the data as a stem plot.
- b Comment on the range of performance and other interesting points.
- c What conclusions could be drawn about the swimmers' performance?

- 17** Answer the following questions for the back-to-back stem plot in Question 10.

- a How many times did each player score more than 1 goal?
- b Who scored the greatest number of goals in a match?
- c Who scored the greatest number of goals overall?
- d Who is the more-consistent performer?

- 18** Percentages in a mathematics exam for two classes were as follows:

<b>9A</b>	32	65	60	54	85	73	67	65	49	96	57	68
<b>9B</b>	46	74	62	78	55	73	60	75	73	77	68	81

- a Construct a back-to-back stem plot of the data.
- b What percentage of each group scored above 50?
- c Which group had more scores over 80?

- d Compare the clustering for each group.
- e Comment on extreme values.
- f Calculate the *average* percentage for each group.
- g Show a back-to-back dot plot of the data (use colour).
- h Compare class performances by reference to both graphs.

**REASONING**

- 19** Ten randomly chosen students from Class A and Class B each sit for a test in which the highest possible mark is 10. The results of the ten students from the two classes are:

Class A: 1    2    3    4    5    6    7    8    9    10  
 Class B: 1    2    2    3    3    4    4    5    9    10

- a Graphically display the data.
- b Calculate measures of centre and spread.
- c Explain any similarities or differences between the results of the two classes.

- 20** Explain and give an example of the effect that outliers in a set of data have on the:

- a mean
- b median
- c mode
- d range.

**REFLECTION**

Why is it important to use a key with all stem plots? Give an example to illustrate.

## 14E Comparing data

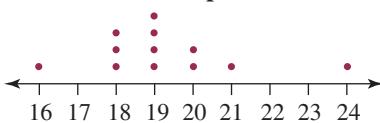
We have considered the calculation of measures of centre and spread from listed data. We also need to know how to calculate these measures from graphs of individual data. We can then make comparisons between data presented in listed form and graphical form.

### Determining measures of centre and spread from graphs

When data are displayed graphically, their spread may be obvious, but we often need to calculate their measures of centre so that we can understand them better. We also need to be able to determine which measure of centre best represents the data.

#### WORKED EXAMPLE 12

Consider this dot plot.



- a Use the dot plot to determine the:

- i mean
- ii median
- iii mode
- iv range.

- b Comment on the most suitable measure of centre for the data.

**THINK**

- a i 1 Find the total of the values.

- 2 Count the number of values.
- 3 Find the mean by dividing the total by the number of values.

**WRITE**

$$\begin{aligned} \text{a Total of values} &= 16 + 3 \times 18 + 4 \times 19 \\ &\quad + 2 \times 20 + 21 + 24 \\ &= 231 \end{aligned}$$

There are 12 values.

$$\begin{aligned} \text{Mean} &= \frac{\text{total of values}}{\text{number of values}} \\ &= \frac{231}{12} \\ &= 19.25 \end{aligned}$$

- ii**
- 1** The values are already in order. The median is the middle value. There are 12 values, so the middle one is the average of the 6th and 7th values. Locate these.
  - 2** Calculate the average of these.
- iii** The mode is the most common value. Look for the one which occurs most frequently.
- iv** The range is the difference between the highest value and the lowest value.
- b** Look at the measures of mean, median and mode to see which one/s best represent the values in terms of their closeness to the centre.

The middle position of the 12 values is between the 6th and 7th values. These are both 19.

The median value is 19.

The mode is 19.

$$\begin{aligned}\text{Range} &= 24 - 16 \\ &= 8\end{aligned}$$

- b** The values of mean (19.25), median (19) and mode (19) are all quite close together, so any of these measures could be used to represent the data. There appear to be two outliers (16 and 24). These two tend to cancel out the effect of each other.

### WORKED EXAMPLE 13

Consider this stem plot.

Key:  $1|8 = 18$

Stem	Leaf
1	8 9
2	2 2 5 7 7 8
3	0 1 4 6 7
4	0 5

#### THINK

- a i** **1** Find the total of the values.

**a** Use the stem plot to determine the:

**i** mean      **ii** median      **iii** mode      **iv** range.

**b** Comment on the most suitable measure of centre for the data.

#### WRITE

$$\begin{aligned}\text{a Total of values} &= 18 + 19 + 22 + 22 \\ &\quad + 25 + 27 + 27 + 28 \\ &\quad + 30 + 31 + 34 + 36 \\ &\quad + 37 + 40 + 45 \\ &= 441\end{aligned}$$

There are 15 values.

$$\begin{aligned}\text{Mean} &= \frac{\text{total of values}}{\text{number of values}} \\ &= \frac{441}{15} \\ &= 29.4\end{aligned}$$

The middle position of the 15 values is the 8th value. This is 28.

The median value is 28.

There are two modes (it is bimodal) — 22 and 27.

$$\begin{aligned}\text{Range} &= 45 - 18 \\ &= 27\end{aligned}$$

- ii** The values are already in order. The median is the middle value. There are 15 values, so the middle one is the 8th value. Locate this.
- iii** The mode is the most common value. Look for the one which occurs most frequently.
- iv** The range is the difference between the highest value and the lowest value.

- b** Look at the measures of mean, median and mode to see which one/s best represent/s the values in terms of their closeness to the centre.
- b** The values of mean (29.4), median (28) and modes (22 and 27) are quite different in this case. There do not appear to be any outliers. The mean or median could be used to represent the centre of this set of data.

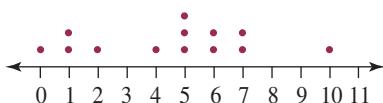
**REMEMBER**

We can determine measures of centre and spread from a dot plot or a stem plot by considering the individual values.

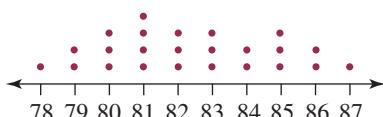
1. The mean can be calculated by dividing the total of the values displayed in the graph by the number of values shown.
2. The median is the middle value for an odd number of data, and the average of the two middle values for an even number of data. The data are already in order, so it is not necessary to order them first.
3. The mode is the most common value displayed. If two values are most common, they are called bimodal.
4. The range is the difference between the highest and lowest values as displayed by the graph.

**EXERCISE****14E Comparing data****INDIVIDUAL PATHWAYS****eBook plus****Activity 14-E-1**  
Comparing data  
doc-6731**Activity 14-E-2**  
More comparisons  
doc-6732**Activity 14-E-3**  
Advanced  
comparisons  
doc-6733**FLUENCY**

- 1 WE12** Consider this dot plot.

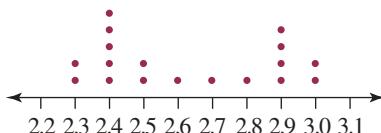


- a** Use the dot plot to determine the:
- i mean
  - ii median
  - iii mode
  - iv range.
- b** Disregard the score of 10, and recalculate each of these values.
- c** Discuss the differences/similarities in your two sets of results.
- 2** Consider this dot plot.



- a** Use the dot plot to determine the:
- i mean
  - ii median
  - iii mode
  - iv range.
- b** Comment on the most suitable measure of centre for the data.

**3** Consider this dot plot.



- a Use the dot plot to determine the:
  - i mean
  - ii median
  - iii mode
  - iv range.
- b Comment on the most suitable measure of centre for the data.

**4** **WE13** Consider this stem plot.

Key:  $1|0 = 10$

Stem	Leaf
1	0 2
2	1 3 3 5
3	
4	4

- a Use the stem plot to determine the:
  - i mean
  - ii median
  - iii mode
  - iv range.
- b Disregard the score of 44, and recalculate each of these values.
- c Discuss the differences/similarities in your two sets of results.

**5** Consider this stem plot.

Key:  $10|0 = 100$

Stem	Leaf
10	0
11	0 2 2 2
12	0 4 6 6
13	3

- a Use the stem plot to determine the:
  - i mean
  - ii median
  - iii mode
  - iv range.
- b Comment on the most suitable measure of centre for the data.

**6** Consider this stem plot.

Key:  $6.1|8 = 6.18$

Stem	Leaf
6.1	8 8 9
6.2	0 5 6 8
6.3	0 1 2 4 4 4

- a Use the graph to determine the:
  - i mean
  - ii median
  - iii mode
  - iv range.
- b Comment on the most suitable measure of centre for the data.

## UNDERSTANDING

- 7** A survey of the number of people in each house in a street produced these data:  
2, 5, 1, 6, 2, 3, 2, 1, 4, 3, 4, 3, 1, 2, 2, 0, 2, 4.
- Prepare a frequency distribution table with an  $f \times x$  column and use it to find the average (mean) number of people per household.
  - Draw a dot plot of the data and use it to find the median number per household.
  - Find the modal number per household.
  - Which of the measures would be most useful to:
    - real estate agents renting out houses?
    - a government population survey?
    - an ice-cream mobile vendor?
- 8** A small business pays these wages (in thousands of dollars) to its employees:  
18, 18, 18, 18, 26, 26, 26, 35, 80 (boss).
- What is the wage earned by most workers?
  - What is the ‘average’ wage?
  - Find the median of the distribution.
  - Which measure might be used in wage negotiations by:
    - the union, representing the employees (other than the boss)?
    - the boss?
- Explain each answer.
- 9** The mean of 12 scores is 6.3. What is the total of the scores?
- 10** Five scores have an average of 8.2. Four of those scores are 10, 9, 8 and 7. What is the fifth score?
- 11** **MC** a The score that shows up most often is the:
- |           |             |        |
|-----------|-------------|--------|
| A median  | B mean      | C mode |
| D average | E frequency |        |
- b The term ‘average’ in everyday use suggests:
- |             |                 |              |
|-------------|-----------------|--------------|
| A the mean  | B the mode      | C the median |
| D the total | E none of these |              |
- c The measure affected by outliers (extreme values) is:
- |              |                 |              |
|--------------|-----------------|--------------|
| A the middle | B the mode      | C the median |
| D the mean   | E none of these |              |
- 12** **MC** The back-to-back stem plot displays the heights of a group of Year 7 students.
- Key: 13|7 = 137 cm
- | Leaf<br>(boys) | Stem | Leaf<br>(girls) |
|----------------|------|-----------------|
| 9 8            | 13   | 7 8             |
| 9 8 8 7 6      | 14   | 3 5 6           |
| 9 8 8          | 15   | 1 2 3 7         |
| 7 6 6 5        | 16   | 3 5 6           |
| 8 7 6          | 17   | 1               |
- The total number of Year 7 students is:
 

A 13	B 17	C 30
D 36	E 27	
  - The tallest male and shortest female heights respectively are:
 

A 186 cm and 137 cm	B 171 cm and 148 cm	C 137 cm and 188 cm
D 178 cm and 137 cm	E none of these	



**REASONING**

- 13** A class of 26 students had a median mark of 54 in Mathematics; however, no-one actually obtained this result.
- Explain how this is possible.
  - Explain how many must have scored below 54.
- 14** A soccer team had averaged 2.6 goals per match after 5 matches. After their sixth match, the average had dropped to 2.5. How many goals did they score in that latest match?
- 15** A tyre manufacturer selects 48 tyres at random from the production line for testing. The total distance travelled during the safe life of each tyre is shown in the table.



<b>Distance in km ('000)</b>	82	78	56	52	50	46
<b>Number of tyres</b>	2	4	10	16	12	4

- Calculate the mean, median and mode.
- Which measure best describes 'average' tyre life? Explain.
- Recalculate the mean with the 6 longest-lasting tyres removed. By how much is it lowered?
- If you selected a tyre at random, what tyre life would it most likely have?
- In a production run of 10 000 tyres, how many could be expected to last for a maximum of 50 000 km?
- As the manufacturer, for what distance would you be prepared to guarantee your tyres? Why?

**REFLECTION**

Why do we need to be able to compare sets of data?

**eBookplus**

**Digital docs**  
WorkSHEET 14.2  
doc-1979

# Summary

## Classifying data

- Surveys use representative samples of the target population.
- Samples are chosen at random.
- Data may be classified under the following headings.
  - (a) *Qualitative*. Data are placed in *categories* (non-numerical form).
    - Nominal: need other *sub-groups*.
    - Ordinal: need a *ranking* system.
  - (b) *Quantitative*. Data are in *numerical* form.
    - Discrete: *counted* in exact values.
    - Continuous: *measured* in a continuous decimal scale.

## Displaying data in tables

- *Frequency* is the number of times a result or piece of data occurs.
- A frequency distribution table consists of 3 columns, headed category, tally and frequency.

## Measures of centre and spread

- To determine the *mean* of data in a list simply obtain the total of all the values and divide by the number of values.
- The *median* is the middle value for an odd number of data and the average of the two middle values for an even number of data which are arranged in numerical order. There are as many values above the median as there are below it.
- The *mode* is the most common value. If two values are most common, they are called bimodal.
- The *range* is the difference between the highest and lowest value in the data.

## Representing data graphically

- Graphs should be drawn on graph paper for greater accuracy.
- *Column graphs* must have:
  - a clear, bold title
  - labelled axes that are clearly and evenly scaled
  - columns of the same width
  - an even gap between each column
  - the first column beginning half a unit from the vertical axis.
- *Bar graphs* must have:
  - a clear, bold title
  - labelled axes that are clearly and evenly scaled
  - horizontal bars of the same width
  - an even gap between each horizontal bar
  - the first horizontal bar beginning half a unit above the horizontal axis.
- Dot plots (or line plots) give a quick overview of a distribution: they show clustering, extreme values, and help us to see if data should be grouped.
- A stem plot allows:
  - all the original data to be retained
  - a graphical representation of the data to be seen.
- Back-to-back stem plots present two related sets of data simultaneously.

**Comparing data**

- We can determine measures of centre and spread from a dot plot or a stem plot by considering the individual values.
  - The mean can be calculated by dividing the total of the values displayed in the graph by the number of values shown.
  - The median is the middle value for an odd number of data, and the average of the two middle values for an even number of data. The data are already in order, so it is not necessary to order them first.
  - The mode is the most common value displayed. If two values are most common, they are called bimodal.
  - The range is the difference between the highest and lowest values as displayed by the graph.

**MAPPING YOUR UNDERSTANDING**

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- 1 MC** The classification of data describing the number of iPods sold during the year is:
- quantitative, discrete
  - qualitative, nominal
  - qualitative, ordinal
  - quantitative, continuous
  - none of these
- 2 MC** The classification of data which describe a person's mass is:
- quantitative, discrete
  - qualitative, nominal
  - qualitative, ordinal
  - quantitative, continuous
  - none of these
- 3 MC** The data which describe sandwich types at a takeaway outlet are:
- qualitative, nominal
  - quantitative, discrete
  - qualitative, ordinal
  - quantitative, continuous
  - none of these

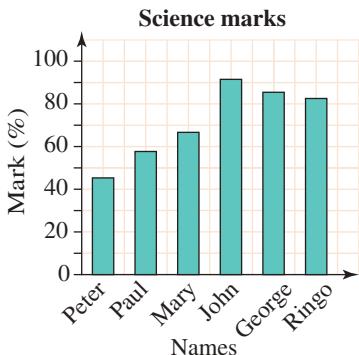


- 4** Eighty students in a school of 800 participated in a survey to find their favourite drinks. The results were: Coke: 20, Pepsi: 14, other soft drink: 10, milk/milk-based: 16, coffee: 9, tea: 4 and fruit juice: 7.
- What was the most popular beverage?
  - What was the least popular beverage?
- 5** Use the following frequency distribution table to answer the following questions.
- How many were in the class?
  - What was the most frequent score?
  - What was the least frequent score?
  - How many scored, at most, 15?
  - How many scored at least 16?

Score	Frequency
13	2
14	9
15	3
16	5
17	6
18	1

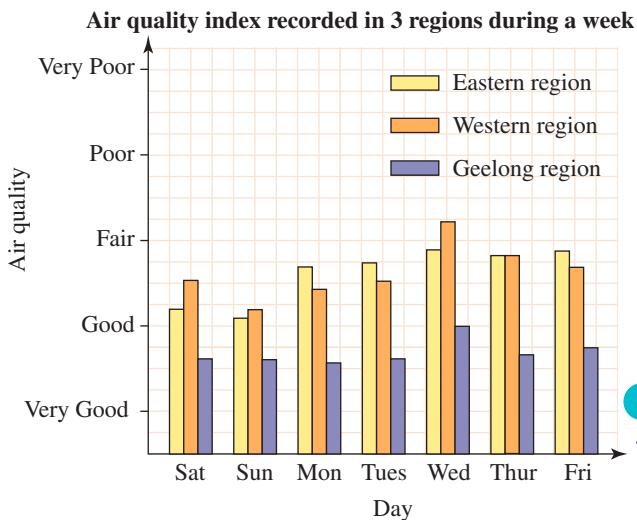
- 6** A random sample of 24 families was surveyed to determine the number of vehicles in each household. Here are the raw data collected:  
2, 0, 3, 2, 1, 0, 2, 3, 4, 2, 2, 1, 0, 1, 3, 2, 1, 0, 0, 0, 2, 2, 3, 3.
- Organise the data into a frequency distribution table.
  - How many families have no vehicles in their household?
  - How many families have 2 or more vehicles in their household?
  - Which score has the highest frequency?
  - Which is the highest score?
  - What fraction of families had 2 vehicles in their household?
- 7** For the following sets of data, find:
- the mean
  - the mode
  - the median.
- 4, 4, 4, 3, 2, 2, 3
  - 2, 4, 2, 4, 3, 7, 4
  - 4, 2, 6, 6, 5, 4, 6
  - 4, 5, 6, 6, 5, 5, 3
  - 1, 2, 1, 4, 1, 5, 1
  - 5, 8, 1, 7, 7, 5, 2, 7, 5, 5
  - 1, 5, 4, 5, 7
  - 2, 4, 2, 5, 7, 6, 3
  - 1, 7, 5, 8, 9, 5, 3
  - 1, 5, 8, 7, 4, 8, 5, 6, 8
- 8** Look at the graph below, showing the test marks of a group of students.
- What is the title for this graph?
  - What is the label on the vertical axis?
  - What is the label on the horizontal axis?
  - What scale is used on the vertical axis?
  - Which student has the highest mark?
  - Which student(s) would have failed if the pass mark was 50%?
  - What was Paul's mark?

**h** What was George's mark?



**9** Look at the graph below, showing air quality.

- What does the horizontal axis represent?
- What does the vertical axis represent?
- Which region had a rating that ranged from very good to good every day?
- Which day did the Western region have a fair air quality rating?
- Which region had a good rating every day?



**10** The following data give the speed of 30 cars recorded by a roadside speed camera along a stretch of road where the speed limit is 80 km/h.  
75, 90, 83, 92, 103, 96, 110, 92, 102, 93, 78, 94, 104, 85, 88, 82, 81, 115, 94, 84, 87, 86, 96, 71, 91, 91, 92, 104, 88, 97

Present the data as an ordered stem plot.

**11** Comment on the data recorded by the roadside speed camera in Question 10.

**12** Complete the specified tasks for the given sets of data.

<b>Set A</b>	64	30	59	1	57	13	45	28
<b>Set B</b>	3	41	38	68	29	32	30	31

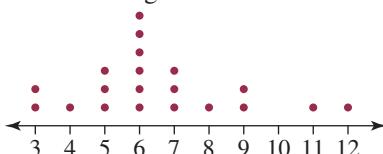
**a** Compile a back-to-back stem plot of the two sets of data.

**b** For each data set, find the:

- lowest score
- highest score
- mean
- median
- range.

**c** Compare the two data sets.

**13** Consider this dot plot, which represents the number of soft drinks sold at lunch time over a 20-day period from a vending machine in a cafeteria.



**a** Use the dot plot to determine the:

- mean
- median
- mode
- range.

**b** Comment on the distribution of the data.

**14** Consider this stem plot, which represents the ages of participants in an aerobics class at a gym.

Key: 1 | 17 = 17

Stem	Leaf
1	4 7 8 8 9
2	0 1 3 5 6 7 9
3	2 4 5 6 7 8
4	2 5
5	3

**a** Use the graph to determine the:

- mean age
- median age
- modal age
- range of ages.

**b** Comment on the distribution of the data.

### PROBLEM SOLVING

**1** This table shows the maximum and minimum daily temperatures in a city over a one-week period.

<b>Day</b>	1	2	3	4	5	6	7
<b>Maximum (°C)</b>	12	13	10	11	9	10	8
<b>Minimum (°C)</b>	3	3	2	1	0	4	2

Use the table to answer the following questions.

- What was the maximum temperature on day 3?
- Which day had the lowest minimum temperature?
- Which day was the coldest?
- Which day had the warmest overnight temperature?
- What was the temperature range (variation) on day 2?
- Which day had the smallest range of temperatures?

- 2** Look at the following part of a teachers' mark book. Use it to answer the questions below.

Name	Test 1	Test 2
John	85	94
Peter	85	63
Mark	95	58
James	82	67
David	76	95
Rachel	62	85
Mary	87	75
Eve	94	63
Esther	68	68

- a What was the class mean for test 1?
- b What was the class mean for test 2?
- c In which test did the class do better?
- d How many students scored above the mean in test 1?
- e How many students scored above the mean in test 2?
- f What was the mode score for test 1?
- g What was the mode score for test 2?
- h What was the median score for test 1?
- i What was the median score for test 2?
- j Calculate the mean score for the two tests for each student and list them.
- k What is the range of values for these means?

- 3** Look at the following results of height measurements (in cm) for the students in class 9A: 145, 152, 148, 152, 163, 148, 165, 158, 159, 162, 145, 153, 156, 158, 157, 159, 169, 156, 156, 156, 152, 154, 128, 141, 154, 153, 156, 156, 165, 168.
- a What is the range of heights?
  - b What is the mean height of the class?
  - c What is the mode for the heights?
  - d What is the median height of the class?
  - e Are these three values (mean, mode, median) similar?
  - f Why might this be so?



- 4** Use the following table of values to answer the questions below.

Month	Number of flies (thousands)
January	24
February	28
March	26
April	20
May	18
June	15
July	12
August	10
September	11
October	12
November	15
December	19

- a In drawing a column graph using these data, what would be on the horizontal axis?
- b In drawing a column graph, what would be on the vertical axis?
- c What is the minimum value required on the vertical axis?
- d What is the maximum value required on the vertical axis?
- e What scale could be used for the vertical axis?
- f Draw a column graph to display these data.



- 5** Use the following table of values of students enrolled in elective classes to answer the questions below.

Subject	Number enrolled
Modern history	15
China in focus	12
Beginner's photography	16
First aid and fitness	18
Business enterprises	15
Consumer studies	12
Outdoor education	16

- a** What is the minimum value which would be required on the horizontal axis if a bar graph was to be drawn using these data?
- b** What is the maximum value which would be required on the horizontal axis if a bar graph was to be drawn?
- c** Which class has the largest number of students?
- d** If the maximum number of students for each of these electives is 15 students, which ones can take more students?
- e** Which electives have too many students?
- f** Draw a bar graph of the data.



- 6** The number of pets cared for by each of 20 families was surveyed, giving these data:  
1, 2, 3, 2, 0, 1, 0, 2, 5, 3, 2, 1, 2, 0, 2, 0, 1, 3, 2, 1.
  - a** Organise the data into a frequency distribution table.
  - b** Draw a dot plot of the data and comment on the distribution.
  - c** Find the mode, median and mean.

- 7 a** For the given set of scores, 9, 3, 8, 5, 6, 9, 4, 5, 5, 25, find the:
- i** mode
  - ii** median
  - iii** mean
  - iv** outliers, if any
  - v** mean if outliers are omitted
  - vi** mean and median if 10 is added to each of the given scores.

- b** Comment on any discoveries from part **a**.
- 8** This back-to-back stem plot shows the heights of a group of boys and girls.

Key: 13 | 7 = 137 cm

<i>Leaf (boys)</i>	<i>Stem</i>	<i>Leaf (girls)</i>
9 8	13	7 8
9 8 8 7 6	14	3 5 6
9 8 8	15	1 2 3 7
7 6 6 5	16	3 5 6
8 7 6	17	1

Give a full description of the heights of the boys and the girls.

#### eBook plus

##### Interactivities

- Test yourself
- Chapter 14
- int-1825
- Word search
- Chapter 14
- int-2607
- Crossword
- Chapter 14
- int-2608

#### eBook plus

- Weblink**  
**Data picking**

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 14 (doc-6577) (*page 465*)

**Are you ready?****Digital docs** (*page 466*)

- SkillsHEET 14.1 (doc-6578) Distinguishing qualitative from quantitative data
- SkillsHEET 14.2 (doc-6579) Distinguishing discrete from continuous data
- SkillsHEET 14.3 (doc-6580) Finding the mean of ungrouped data
- SkillsHEET 14.4 (doc-6581) Finding the median
- SkillsHEET 14.5 (doc-6582) Reading scales
- SkillsHEET 14.6 (doc-6583) Comparing decimals
- SkillsHEET 14.7 (doc-6584) Converting units to compare lengths or distances

**14A Classifying data****Digital docs** (*page 468*)

- Activity 14-A-1 (doc-6719) Classifying data
- Activity 14-A-2 (doc-6720) More classification of data
- Activity 14-A-3 (doc-6721) Advanced classification of data

**14B Displaying data in tables****Digital docs**

- Activity 14-B-1 (doc-6722) Displaying data (*page 472*)
- Activity 14-B-2 (doc-6723) More data displays (*page 472*)
- Activity 14-B-3 (doc-6724) Advanced data displays (*page 472*)
- Investigation How many red M&M'S? (doc-3438) (*page 471*)
- Spreadsheet Frequency tally tables (doc-3437) (*page 473*)
- WorkSHEET 14.1 (doc-1978) (*page 474*)

**14C Measures of centre and spread****Digital docs** (*page 477*)

- Activity 14-C-1 (doc-6725) Measures of centre and spread
- Activity 14-C-2 (doc-6726) More measures of centre and spread

- Activity 14-C-3 (doc-6727) Advanced measures of centre and spread
- Spreadsheet Mean (doc-3434)
- Spreadsheet Median (doc-3435)
- Spreadsheet Mode (doc-3436)

**Interactivity**

- Measures of centre (int-2352) (*page 476*)

**14D Representing data graphically****Digital docs**

- Activity 14-D-1 (doc-6728) Graphing data (*page 484*)
- Activity 14-D-2 (doc-6729) More graphs of data (*page 484*)
- Activity 14-D-3 (doc-6730) Advanced graphs of data (*page 485*)
- Investigation Walking billboard (doc-3439) (*page 484*)
- Investigation Families (doc-3440) (*page 484*)
- Spreadsheet Column graphs (doc-3441) (*page 485*)
- Spreadsheet Bar graphs (doc-3442) (*page 485*)
- Spreadsheet Dot plots (doc-3443) (*page 485*)

**14E Comparing data****Digital docs**

- Activity 14-E-1 (doc-6731) Comparing data (*page 490*)
- Activity 14-E-2 (doc-6732) More comparisons (*page 490*)
- Activity 14-E-3 (doc-6733) Advanced comparisons (*page 490*)
- WorkSHEET 14.2 (doc-1979) (*page 493*)

**Chapter review****Interactivities** (*page 499*)

- Test yourself Chapter 14 (int-1825) Take the end-of-chapter test to test your progress.
- Word search Chapter 14 (int-2607)
- Crossword Chapter 14 (int-2608)

**Weblink** (*page 499*)

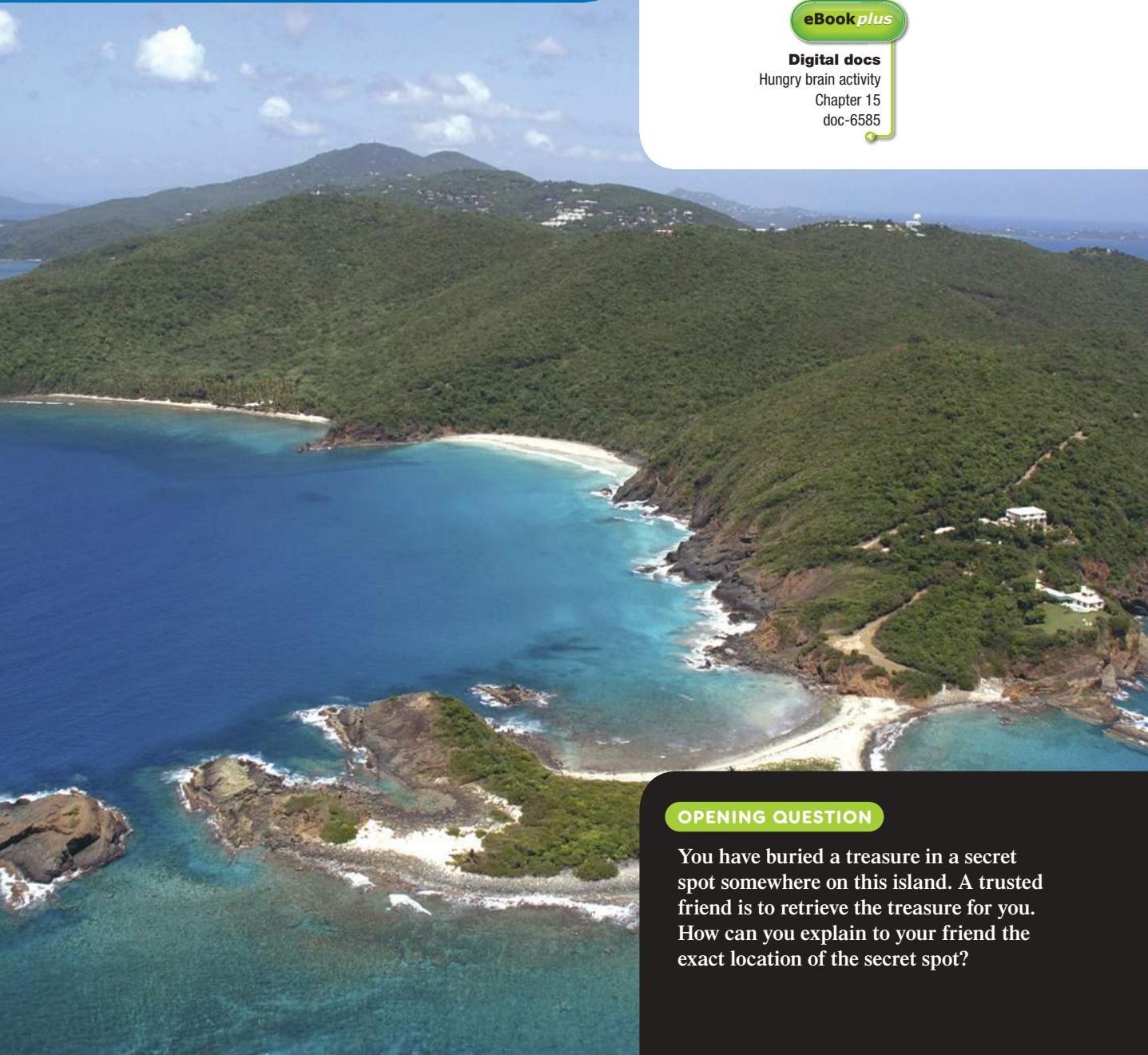
- Data picking

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# 15

# Coordinates and the Cartesian plane



- 15A** Alphanumeric grid references on a map
- 15B** The Cartesian plane
- 15C** Plotting simple linear relationships
- 15D** Interpreting graphs

## WHAT DO YOU KNOW?

- 1 List what you know about plotting coordinates. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of plotting coordinates

eBook plus

Digital docs

Hungry brain activity

Chapter 15

doc-6585

## OPENING QUESTION

You have buried a treasure in a secret spot somewhere on this island. A trusted friend is to retrieve the treasure for you. How can you explain to your friend the exact location of the secret spot?

# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.

## Finding locations

eBookplus  
Digital docs  
SkillsHEET 15.1  
doc-6586

- 1 Study the grid on the right. Which letters can be found by starting at point O and moving:
- 2 squares to the left and 3 squares up,
  - 2 squares to the right and 1 down,
  - 1 square up and 3 squares to the right,
  - three squares to the right and three up?

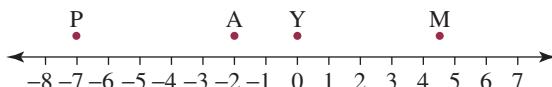
L	E	N	A	I	A
M	O	L	S	K	I
R	P	H	T	A	N
E	A	O	H	A	L
S	K	C	Y	E	N
O	J	A	H	A	M

## Reading scales

eBookplus  
Digital docs  
SkillsHEET 15.2  
doc-6587

- 2 Consider the diagram below. What value is indicated by each of the following letters?

a M      b A      c Y      d P

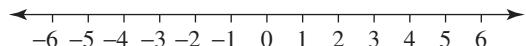


## Marking points on the line

eBookplus  
Digital docs  
SkillsHEET 15.3  
doc-6588

- 3 Copy the number line shown below into your workbook, and use it to mark the points whose values are as follows:

a R = 3.5      b A = -5  
c C =  $-1\frac{3}{4}$       d H =  $-\frac{1}{2}$



## Drawing scales

eBookplus  
Digital docs  
SkillsHEET 15.4  
doc-6589

- 4 For each of the following, draw a 10-cm-long line, and scale it evenly to show:
- all integers from 1 to 5
  - all integers from -7 to 2
  - all integers from -15 to -10
  - all integers from -4 to 4.

## Substituting values into the rule

eBookplus  
Digital docs  
SkillsHEET 15.5  
doc-6590

- 5 Substitute the given values of  $b$  into the rule  $a = 2b - 4$ , and find the corresponding value of  $a$ .

a  $b = 4$       b  $b = 0$       c  $b = -1$       d  $b = -3$

## Finding patterns

eBookplus  
Digital docs  
SkillsHEET 15.6  
doc-6591

- 6 For each of the following sets of numbers, find the pattern and hence write the next two numbers.

a 15.5, 19, 22.5, 26, ...      b -11, -7, -3, 1, ...  
c -4, -8, -16, -32, ...      d 0.75, 1.5, 3, 6, ...

## Establishing patterns connecting two numbers

eBookplus  
Digital docs  
SkillsHEET 15.7  
doc-6592

- 7 In each of the following tables, the values of  $a$  (that is, the numbers in the left column) are connected to the values of  $b$  (that is, the numbers in the right column) in the same way. Find the pattern for each table and use it to find the missing values.

a	b
4	8
2	4
5	10
0	
	14

a	b
7	5
10	8
-4	-6
3	
	0

a	b
2	7
-2	3
0	5
8	
	4

a	b
3	18
0	0
2	12
1	
	-6

## 15A Alphanumeric grid references on a map

- Areas of land can be displayed by using a grid reference map, where the map is divided into squares.
  - Because grid references refer to a square region rather than a point on a map, they are often called **area references**.
  - The reference for each square on a map is given by two coordinates — a letter and a number (hence the name *alphanumeric*).
  - Maps with alphanumeric grid references are usually found in a street directory or an atlas.
  - An example of a map with an alphanumeric grid is shown below.



© Gregory's Road Atlas of Australia, Universal Publishers Pty Ltd

**WORKED EXAMPLE 1**

Joe is holidaying in Brisbane and has planned to visit various sites. Using the map of the Gold Coast on page 503, give the grid reference for each of the following sites Joe plans to visit.

**a** Southport**b** Movie World**c** Bond University**THINK**

- a** 1 Locate Southport.
- 2 Read the horizontal and vertical coordinates.
- 3 Write the answer.
- b** 1 Locate Movie World.
- 2 Read the horizontal and vertical coordinates.
- 3 Write the answer.
- c** 1 Locate Bond University.
- 2 Read the horizontal and vertical coordinates.
- 3 Write the answer.

**WRITE**

- a** C5
- b** B4
- c** C6

**WORKED EXAMPLE 2**

Name the major feature located at each of the following grid references in the Gold Coast map (see page 503).

**a** D3**b** E8**c** B3**THINK**

- a** 1 Look for the major feature in column D and row 3.
- 2 Write the answer.
- b** 1 Look for the major feature in column E and row 8.
- 2 Write the answer.
- c** 1 Look for the major feature in column B and row 3.
- 2 Write the answer.

**WRITE**

- a** South Stradbroke Island
- b** Tweed Heads
- c** Dreamworld

**REMEMBER**

1. Areas of land can be displayed by using a grid reference map, where the map is divided into squares.
2. The reference for each square on a map is given by two coordinates: a letter and a number (hence the name *alphanumeric*).
3. Maps with alphanumeric grid references are usually found in street directories and atlases.

**EXERCISE****15A****Alphanumeric grid references on a map****INDIVIDUAL PATHWAYS****eBook plus****Activity 15-A-1**Grid references  
doc-2006**FLUENCY**

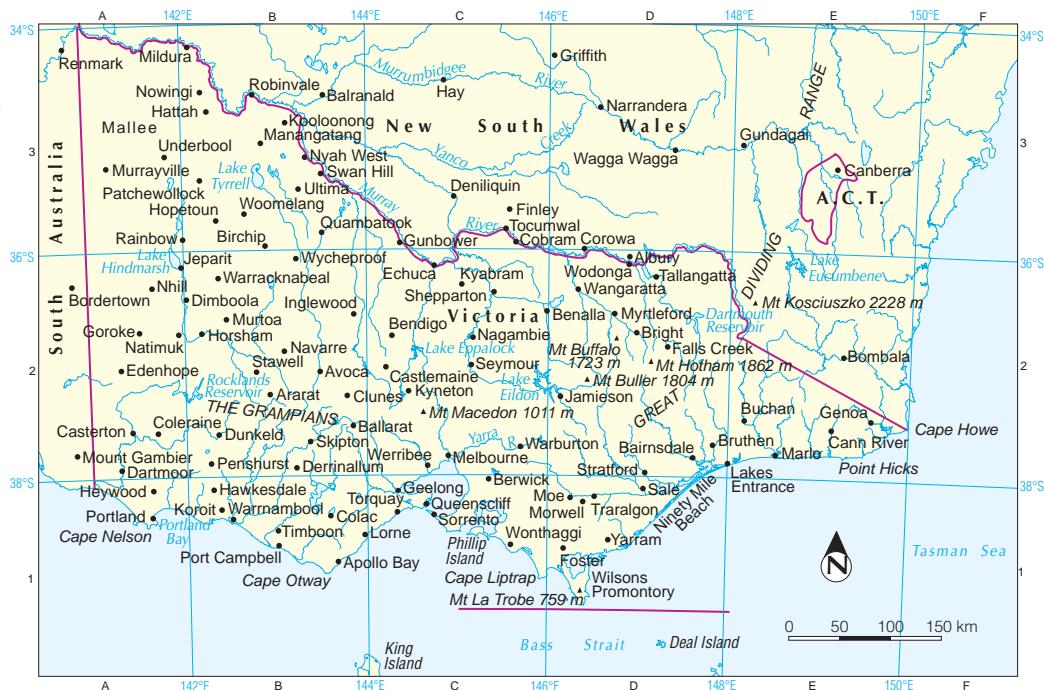
Use the grid map of Victoria and its surrounding states (opposite) to answer Questions 1 and 2.

- 1** **(WEI)** Give the grid reference for each of the following, using the map of Victoria and its surrounding states above.

- |                             |                        |                            |
|-----------------------------|------------------------|----------------------------|
| <b>a</b> Mt Kosciuszko      | <b>b</b> Port Campbell | <b>c</b> Lake Tyrrell      |
| <b>d</b> Wilsons Promontory | <b>e</b> The Grampians | <b>f</b> Phillip Island    |
| <b>g</b> Mt Hotham          | <b>h</b> Portland Bay  | <b>i</b> Ninety Mile Beach |

## INDIVIDUAL PATHWAYS

eBookplus

**Activity 15-A-2**More grid references  
doc-2007**Activity 15-A-3**Advanced grid  
references  
doc-2008

- 2 WE2** Name the major city/town(s) located at each of the following grid references in the map of Victoria and its surrounding states.

- a C1   b B2   c C2   d B1   e A1   f B3   g D2   h A2   i D3

Use this Melway map 206 to answer Questions 3 and 4.



- 3** In which square would you find each of the following:
- Hoppers Crossing station?
  - Glen Orden Primary School?
  - Kookaburra Avenue?
- 4** On Melway map 206, (page 505) what is the name of the longest street passing through:
- F2?
  - C2?
  - B5?

### UNDERSTANDING

- 5** The group of American tourists will start from the tourist office next to Parliament House and visit the Rialto Towers Observation Deck, Shrine of Remembrance, Crown Entertainment Complex, Melbourne Aquarium, Melbourne Cricket Ground, Queen Victoria Market and the Victorian Arts Centre.
- Use the map below to list the map references for each attraction.
  - Since they have limited time, the tour group want to visit each one but not pass any of the attractions more than once. They will start and finish at the tourist office (Parliament House).

Suggest possible travel routes they could take.



- 6** Consider the following chess board and answer the questions that follow.
- One of the black rooks is on a1. Give the location of the white rooks.
  - Which chess piece is on c6?
  - One of the white bishops is on c1. Where is the second white bishop at the moment?
  - A knight makes an L-shaped move (that is, one square to the side and two vertically, or two squares to the side and one vertically).
    - If it is White's turn, which pawn can the knight capture in the next move? (Specify the move.)
    - If it is the Black's turn, which pawn can the knight capture in the next move? (Specify the move.)

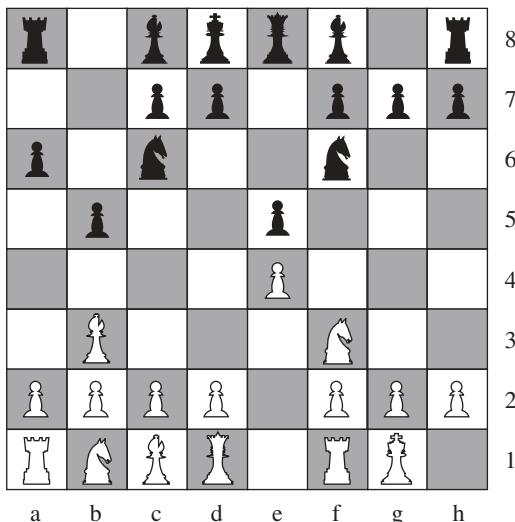
**eBookplus**

**Weblink**  
**Battleship Game**

- e A bishop can move diagonally any number of (unoccupied) squares at a time. Complete the following sentence: ‘With its next move, the bishop at b3 could capture the ..... (name the chess piece) at ..... (name the position).’
- 7 The game of *Battleship* uses alphanumeric grids. Navigate to the *Battleship* weblink in your eBookPLUS. Play the game to test your skills with alphanumeric references.

**REFLECTION**

When we specify an alphanumeric grid reference, does order matter? (That is, does it matter whether the letter is stated first and the number second, or the other way around?)



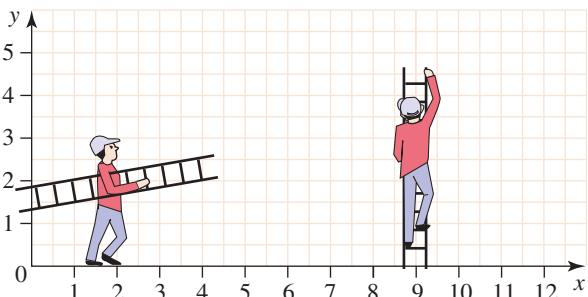
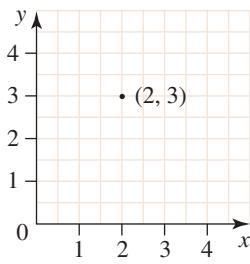
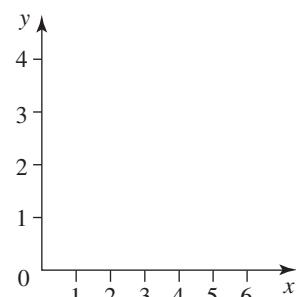
## 15B The Cartesian plane

**eBookplus**

**eLesson**  
Coordinates and the  
Cartesian plane  
eles-0008

- The **Cartesian plane** is named after its inventor, mathematician René Descartes. It is a visual means of describing locations on a plane by using two numbers as coordinates (rather than a letter and a number).
- The Cartesian plane is formed by two perpendicular lines. The horizontal line is called the *x*-axis, while the vertical line is referred to as the *y*-axis. The point where the two axes intersect is called the origin.
- Both axes must be marked (with marks being evenly spaced) and numbered. The distance between each mark is one unit.
- To locate any point on the Cartesian plane we use a pair of numbers called *Cartesian coordinates*. The numbers are written in a set of brackets and are separated by a comma. The first number in brackets is called the *x*-coordinate of the point; this shows how far across from the origin the point is located. The second number is called the *y*-coordinate; this shows how far up (or down) from the origin the point is.
- If the Cartesian coordinates of the point are known, it can be easily located by moving across and up (or down) from the origin the specified number of units. For example, to find the point with coordinates (2, 3), start from the origin and move 2 units to the right and 3 units up.

*Hint:* To help remember the order in which Cartesian coordinates are measured, think about using a ladder. Remember we must always walk across with our ladder and then climb up it.

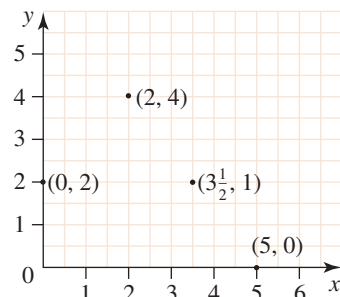


**WORKED EXAMPLE 3**

Draw a Cartesian plane with axes extending from 0 to 6 units. Mark the following points with a dot, and label them.

a  $(2, 4)$ b  $(5, 0)$ c  $(0, 2)$ d  $(3\frac{1}{2}, 1)$ **THINK**

- 1 First rule up and label the axes.
- 2 Mark each point.
  - a  $(2, 4)$  means starting at the origin, go across 2 units, and then up 4 units.
  - b  $(5, 0)$  means go across 5 units and up 0 units. It lies on the  $x$ -axis.
  - c  $(0, 2)$  means go across 0 units and up 2 units. It lies on the  $y$ -axis.
  - d  $(3\frac{1}{2}, 1)$  means go across  $3\frac{1}{2}$  units and up 1 unit. Label each point.

**DRAW****WORKED EXAMPLE 4**

Find the Cartesian coordinates for each of the points A, B, C and D.

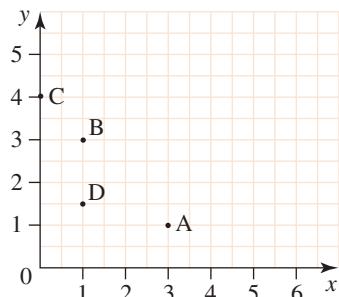
A, B, C and D.

**THINK**

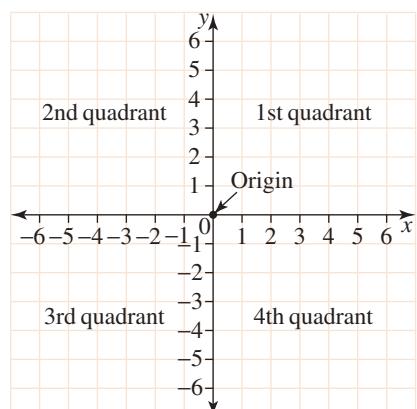
- Point A is 3 units across and 1 unit up  
 Point B is 1 unit across and 3 units up.  
 Point C is 0 units across and 4 units up.  
 Point D is 1 unit across and  $1\frac{1}{2}$  units up.

**WRITE**

- A is at  $(3, 1)$   
 B is at  $(1, 3)$   
 C is at  $(0, 4)$   
 D is at  $(1, 1\frac{1}{2})$

**Extending the axes**

- The Cartesian **axes** can extend infinitely in both directions.
- On the  $x$ -axis, the values to the left of the origin are negative and decreasing. Likewise, on the  $y$ -axis the values below the origin are negative and decreasing.
- The axes divide the Cartesian plane into four sections called **quadrants**. The quadrants are numbered in an anti-clockwise direction, starting with the top right corner.
- If both  $x$ - and  $y$ -coordinates of the point are positive, it will be located in the first quadrant; if the  $x$ -coordinate is negative but the  $y$ -coordinate is positive, the point will be in the second quadrant. If the point is in the third quadrant, both the  $x$ - and  $y$ -coordinates of the point will be negative. Finally, if the point is in the fourth quadrant, its  $x$ -coordinate is positive, while its  $y$ -coordinate is negative.



- If the point is located on the  $x$ -axis, its  $y$ -coordinate is always 0. Likewise, if the point is on the  $y$ -axis, its  $x$ -coordinate is always 0.

**WORKED EXAMPLE 5**

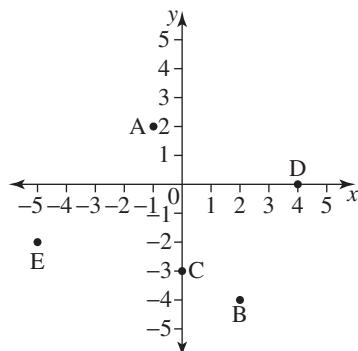
**Plot the following points on the Cartesian plane.**

A( $-1, 2$ ), B( $2, -4$ ), C( $0, -3$ ), D( $4, 0$ ), E( $-5, -2$ )

**State the location of each point on the plane (that is, the quadrant, or the axis on which it sits).**

**THINK**

- Draw a set of axes, ensuring that they are long enough to fit all the values.
- Plot the points. The first point is one unit to the left and two units up from the origin. The second point is two units to the right and four units down from the origin (and so on).

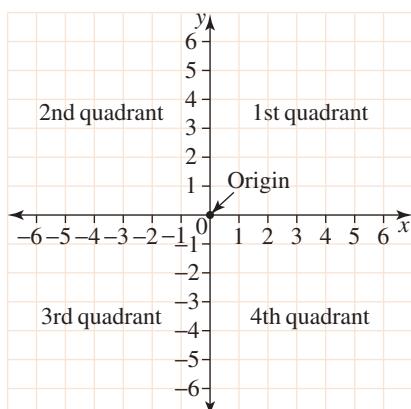
**WRITE**

- Look at the plane and state the location of each point. Remember that the quadrants are numbered in an anti-clockwise direction, starting at the top right. If the point is on the axis, specify which axis it is.

Point A is in the second quadrant.  
 Point B is in the fourth quadrant.  
 Point C is on the  $y$ -axis.  
 Point D is on the  $x$ -axis.  
 Point E is in the third quadrant.

**REMEMBER**

- Cartesian coordinates* can be used to locate any point on a plane.
- The Cartesian plane is formed by two perpendicular lines called *axes*. The horizontal axis is called the  $x$ -axis and the vertical axis is called the  $y$ -axis. The axes intersect at the point called the *origin*.
- Both axes must be marked (with marks being evenly spaced) and numbered. The distance between each mark is one unit. The axes can extend infinitely in both directions.
- The location of any point on the Cartesian plane is given by its Cartesian coordinates. The Cartesian coordinates are a pair of numbers that are separated by a comma and are shown within brackets. The first number is called the  $x$ -coordinate of the point; this shows how far across (that is, to the left or to the right) from the origin the point is located. The second number is called the  $y$ -coordinate; this shows how far up (or down) from the origin the point is. For example, the point  $(2, 3)$  is located 2 units to the right and 3 units up from the origin.



## EXERCISE

## 15B The Cartesian plane

## INDIVIDUAL PATHWAYS

## eBookplus

## Activity 15-B-1

Introducing the  
Cartesian plane  
doc-2009

## Activity 15-B-2

More of the  
Cartesian plane  
doc-2010

## Activity 15-B-3

3D planes  
doc-2011

## eBookplus

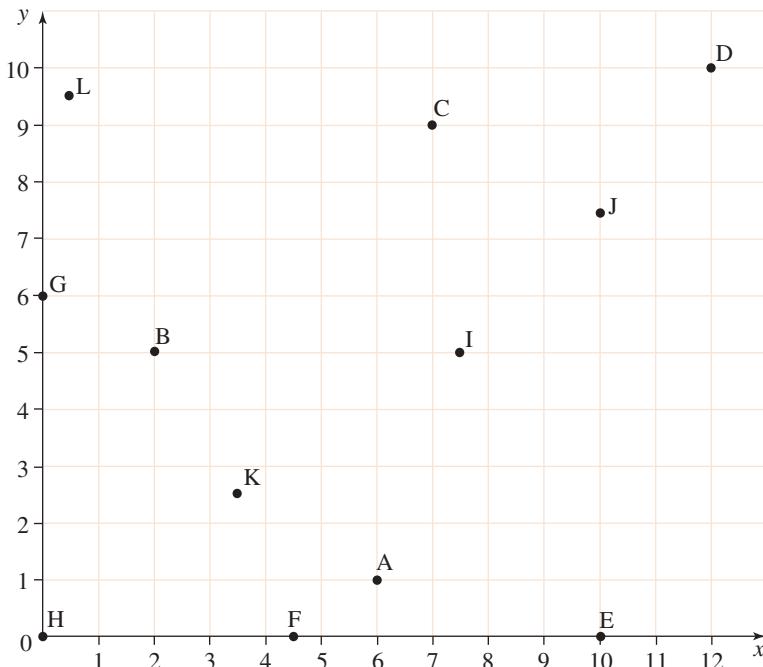
Weblink  
The coordinate  
plane

## FLUENCY

- 1 **WE3** Draw a Cartesian plane with axes extending from 0 to 5 units. Mark the following points with a dot, and label them.

a (4, 3)      b (1, 4)      c (3, 3)      d (2, 0)      e (0, 4)      f (0, 0)

- 2 **WE4** Find the Cartesian coordinates for each of the points A–L.



- 3 **WE5** Plot the following points on the Cartesian plane.

A(-1, -3), B(-2, 5), C(3, -3), D(0, -4), E(-2, -2), F(-5, 0), G(3, 1), H(3, 0), I(-4, -2), J(4, -5)

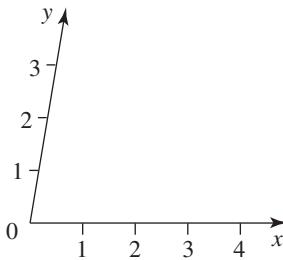
State the location of each point on the plane (that is, the quadrant, or the axis it sits on).

## UNDERSTANDING

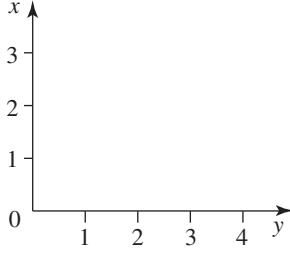
- 4 Each of these sets of Cartesian axes (except one) has something wrong with it. From the list below, match the mistake in each diagram with one of the sentences.

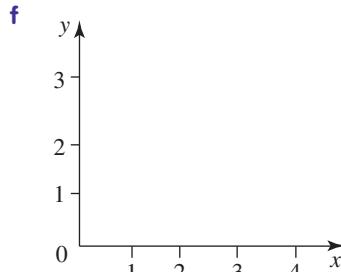
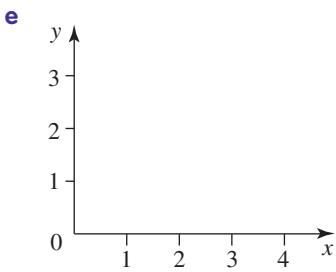
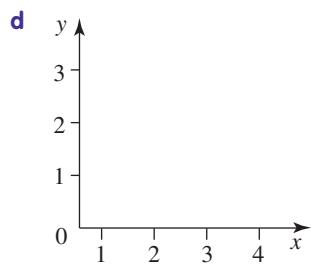
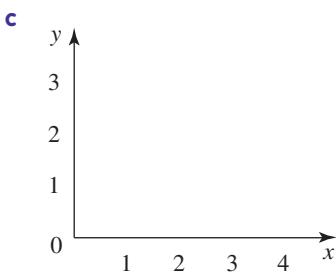
- A** The units are not marked evenly.
- B** The  $y$ -axis is not vertical.
- C** The axes are labelled incorrectly.
- D** The units are not marked on the axes.
- E** There is nothing wrong.

**a**



**b**





- 5** From the diagram at right, write down the coordinates of 2 points which:

- a** have the same  $x$ -coordinate
- b** have the same  $y$ -coordinate.

- 6** Messages can be sent in code using a grid like the one drawn below, where the letter B is represented by the coordinates  $(2, 1)$ .

Use the diagram to decode the answer to the following riddle.

- Q** Where did they put the man who was run over by a steamroller?

- A**  $(4, 2)(4, 3)(3, 2)(5, 3)(4, 4)(1, 4)(4, 2)(5, 4)(1, 1)$   
 $(2, 3)(4, 2)(4, 3)(3, 5)(1, 1)(3, 4)(4, 1)(4, 4)(4, 4)(4, 2)$   
 $(4, 5)(4, 4)(5, 1)(2, 5)(5, 1)(4, 3)(5, 1)(4, 2)(2, 2)(3, 2)$   
 $(5, 4)(1, 1)(4, 3)(4, 1)(4, 3)(4, 2)(4, 3)(5, 1)$

- 7** Rule up a Cartesian plane with both axes extending from 0 to 10 units. Plot the following points and join them in the order given to make a geometric figure. Name each shape.

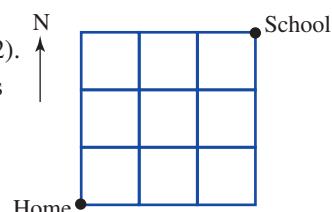
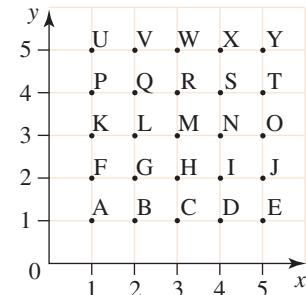
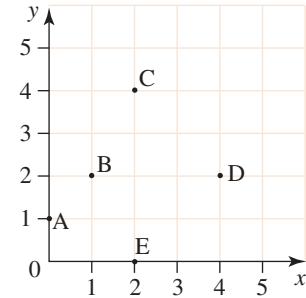
- a**  $(2, 2)-(5, 2)-(2, 6)-(2, 2)$
- b**  $(4, 4)-(8, 4)-(6, 8)-(4, 4)$
- c**  $(1, 1)-(10, 1)-(8, 9)-(2, 9)-(1, 1)$
- d**  $(0, 0)-(8, 0)-(10, 10)-(2, 10)-(0, 0)$

- 8** Here is an exercise which may require care and concentration. On graph paper or in your exercise book rule up a pair of Cartesian axes. The  $x$ -axis must go from 0 to 26 and the  $y$ -axis from 0 to 24. Plot the following points and join them in the order given.

$(0, 15)-(4, 17)-(9, 22)-(10, 21)-(12, 24)-(16, 22)-(15, 21)-(18, 19)-(20, 24)-$   
 $(22, 18)-(26, 12)-(26, 10)-(23, 4)-(20, 3)-(18, 4)-(14, 7)-(11, 7)-(4, 6)-(2, 7)-$   
 $(2\frac{1}{2}, 8)-(0, 15)$

Complete the picture by joining  $(19, 2)-(21, 2)-(20, 0)-(19, 2)$ .

- 9** What is the area of a rectangle formed by connecting the points  $(2, 1), (7, 1), (7, 4)$  and  $(2, 4)$  on a Cartesian plane?

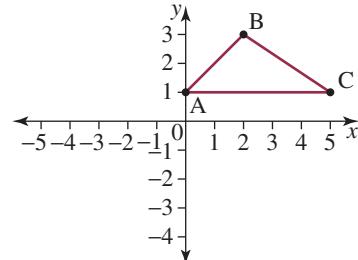


eBook plus

Digital docs  
Spreadsheet  
Plotting points  
doc-0002

- 10** Consider the following set of points: A(2, 5) B(−4, −12) C(3, −7) D(0, −2) E(−10, 0) F(0, 0) G(−8, 15) H(−9, −24) I(18, −18) J(24, 0).
- Which of the following statements is true?
- Points A and J are in the first quadrant.
  - Only point I is in the fourth quadrant.
  - Point F is at the origin.
  - Point D is two units to the left of point F.

- 11** Consider the triangle ABC at right.
- State the coordinates of the vertices of the triangle ABC.
  - Find the area of the triangle.
  - Reflect the triangle in the  $x$ -axis. (You need to copy it into your workbook first.) What are the new coordinates of the vertices?
  - Now reflect the triangle you have obtained in part c into the  $y$ -axis, and state the new coordinates of the vertices.

**eBook plus**

**Digital docs**  
WorkSHEET 15.1  
doc-2004

**REFLECTION**

Why must the  $x$ -coordinate always be written first and the  $y$ -coordinate second?

## 15C Plotting simple linear relationships

- When a set of points is plotted on the Cartesian plane, a pattern may be formed. If a pattern forms a straight line, we call it a *linear pattern*.
- The coordinates of the points that form a pattern can be presented as a set, or in a table. If shown in a table (similar to the one shown below), the coordinates of each point should be read ‘in columns’; that is, the top number in each column gives the  $x$ -coordinate and the bottom number gives the corresponding  $y$ -coordinate of the point. Consider, for example, the table of values and the set of points below. Both show the same information.

<b><math>x</math></b>	0	1	2	3
<b><math>y</math></b>	8	6	4	2

(0, 8) (1, 6) (2, 4) (3, 2)

- The Cartesian coordinates of the points are ordered pairs. That is, the first number always represents the  $x$ -coordinate, and the second always represents the  $y$ -coordinate of a point. A set of ordered pairs forms a relation between  $x$  and  $y$ .
- If the points form a linear pattern when plotted, we say that the relation between  $x$  and  $y$  is linear.

**WORKED EXAMPLE 6**

Plot the following set of points on the Cartesian plane, and comment on any pattern formed.

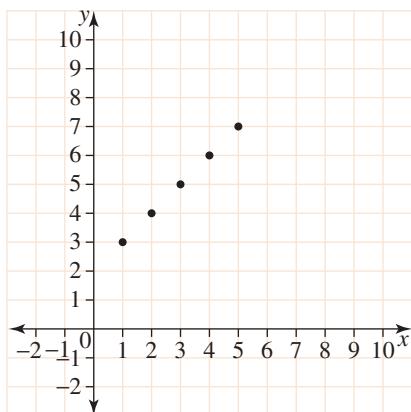
(1, 3) (2, 4) (3, 5) (4, 6) (5, 7)

**THINK**

- Look at the coordinates of the points in the set: the  $x$ -values range between 1 and 5, while the  $y$ -values range between 3 and 7. Draw a set of axes, ensuring they are long enough to fit all the values.

**WRITE**

- 2 Plot the points.



- 3 Comment on the pattern that the points form.

The points form a linear pattern.

### WORKED EXAMPLE 7

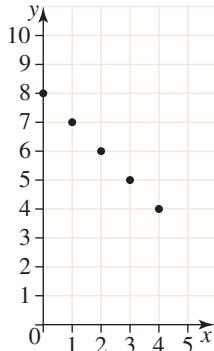
Plot the following points on a Cartesian plane and decide whether or not the relationship is linear.

$x$	0	1	2	3	4
$y$	8	7	6	5	4

#### THINK

- The  $x$ -values range from 0 to 4; the  $y$ -values range from 4 to 8. Draw a set of axes that will include all these values. (The axes need not be the same length. In this case you can extend the  $y$ -axis a bit more than the  $x$ -axis, as the  $y$ -values are higher.)
- The entries in the table can be rewritten as a set of points. The top number in each column gives the  $x$ -coordinate and the bottom number gives the corresponding  $y$ -coordinate of the point. Thus the first column gives the point  $(0, 8)$ , the second  $(1, 7)$ , and so on. Plot all points.

#### WRITE



- Consider the pattern formed by the points. It is a straight line. Draw a suitable conclusion about the relationship between  $x$  and  $y$ .

The relationship between  $x$  and  $y$  is linear.

### REMEMBER

- When a set of points is plotted on the Cartesian plane, a pattern may be formed. If a pattern forms a straight line, we call it a *linear pattern*.
- If coordinates of the points are shown in a table, they should be read ‘in columns’; that is, the top number in each column gives the  $x$ -coordinate and the bottom number gives the corresponding  $y$ -coordinate of the point.

-  3. The Cartesian coordinates of the points are ordered pairs. A set of ordered pairs forms a relation between  $x$  and  $y$ .
4. If the points form a linear pattern when plotted, we say that the relation between  $x$  and  $y$  is *linear*.

**EXERCISE****15C****Plotting simple linear relationships****INDIVIDUAL PATHWAYS****eBook plus****Activity 15-C-1**  
Plotting relationships  
doc-2012**Activity 15-C-2**  
More relationships  
doc-2013**Activity 15-C-3**  
Advanced relationships  
doc-2014**FLUENCY**

- 1 WE6** Plot the following sets of points on the Cartesian plane, and comment on any pattern formed.

**a** (0,1) (1,2) (2,3) (3,4) (4,5)      **b** (0,3) (1,4) (2,5) (3,6) (4,7)

**c** (2,2) (3,3) (4,4) (5,5) (6,6)      **d** (0,2) (1,3) (2,1) (3,0) (4,5)

**e** (1,10) (2,8) (3,6) (4,4) (5,2)      **f** (2,3) (3,3) (4,3) (5,3) (6,3)

- 2** Plot the following sets of points on the Cartesian plane. (Note that some coordinates are negative, so you will need to use all four quadrants.) Comment on any pattern formed.

**a** (-2,0) (-1,2) (0,4) (1,6) (2,8)      **b** (-3,-3) (-2,0) (-1,3) (0,6) (1,9)

**c** (-2,7) (-1,6) (0,4) (1,3) (2,1)      **d** (-6,8) (-5,6) (-4,4) (-3,2) (-2,0)

- 3 WE7** Plot the points in each table on a Cartesian plane, and decide whether or not the relationship is linear.

<b>a</b>	<table border="1"> <thead> <tr> <th><math>x</math></th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th></tr> </thead> <tbody> <tr> <td><math>y</math></td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> </tbody> </table>	$x$	0	1	2	3	4	$y$	3	4	5	6	7
$x$	0	1	2	3	4								
$y$	3	4	5	6	7								

<b>b</b>	<table border="1"> <thead> <tr> <th><math>x</math></th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th></tr> </thead> <tbody> <tr> <td><math>y</math></td><td>0</td><td>1</td><td>4</td><td>9</td><td>16</td></tr> </tbody> </table>	$x$	0	1	2	3	4	$y$	0	1	4	9	16
$x$	0	1	2	3	4								
$y$	0	1	4	9	16								

<b>c</b>	<table border="1"> <thead> <tr> <th><math>x</math></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr> </thead> <tbody> <tr> <td><math>y</math></td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td></tr> </tbody> </table>	$x$	1	2	3	4	5	$y$	7	6	5	4	3
$x$	1	2	3	4	5								
$y$	7	6	5	4	3								

<b>d</b>	<table border="1"> <thead> <tr> <th><math>x</math></th><th>1</th><th>2</th><th>3</th><th>4</th><th>6</th></tr> </thead> <tbody> <tr> <td><math>y</math></td><td>12</td><td>6</td><td>4</td><td>3</td><td>2</td></tr> </tbody> </table>	$x$	1	2	3	4	6	$y$	12	6	4	3	2
$x$	1	2	3	4	6								
$y$	12	6	4	3	2								

<b>e</b>	<table border="1"> <thead> <tr> <th><math>x</math></th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th></tr> </thead> <tbody> <tr> <td><math>y</math></td><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td></tr> </tbody> </table>	$x$	0	1	2	3	4	$y$	0	3	6	9	12
$x$	0	1	2	3	4								
$y$	0	3	6	9	12								

<b>f</b>	<table border="1"> <thead> <tr> <th><math>x</math></th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr> </thead> <tbody> <tr> <td><math>y</math></td><td>0</td><td>-2</td><td>-4</td><td>-6</td><td>-8</td></tr> </tbody> </table>	$x$	2	3	4	5	6	$y$	0	-2	-4	-6	-8
$x$	2	3	4	5	6								
$y$	0	-2	-4	-6	-8								

**UNDERSTANDING**

- 4 a** Plot the following points on the set of axes: (0, 10) (1, 8) (2, 6) (3, 4).

**b** Describe the pattern formed by the points.

**c** Extend the pattern by plotting the next two points.

**d** What are the coordinates of the two points you have plotted?

- 5** The points in a table below form a linear pattern. The  $y$ -coordinate of the middle point is missing.

$x$	1	2	3	4	5
$y$	0	3		9	12

**a** Plot the four points (whose  $x$ - and  $y$ -coordinates are both known) on the Cartesian plane.

**b** Use your graph to predict the missing  $y$ -coordinate of the middle point.

**c** Add the middle point to your graph and check whether it fits the pattern.

- 6 Consider the linear pattern shown.
- If the pattern is continued to the right, what will be the coordinates of the next point?
  - If the pattern is extended to the left, what will be the coordinates of the previous point?

- 7 Which of the following sets of points, when plotted, will form a linear pattern?

- (0, 3) (1, 5) (2, 7) (3, 9) (4, 11)
- (−1, 8) (−2, 7) (−3, 6) (−4, 5) (−5, 4)
- (1, 6) (2, 5) (3, 7) (4, 8) (5, 9)
- (2, 3) (3, 5) (4, 7) (5, 9) (6, 11)
- (0, 2) (1, 4) (2, 4) (3, 5) (4, 6)
- (10, 7) (9, 4) (8, 3) (7, 1) (6, 0)

- 8 Rachel planted a cherry tomato in her vegetable patch. She measured the height of her plant every week for four consecutive weeks, obtaining the following data: 20 mm, 24 mm, 28 mm, 32 mm. Rachel forgot to measure her plant in week 5, but the following week she found the height of the cherry tomato to be 40 mm.

- Fill in the table of values below.

Week						
Height (mm)						

- Plot the heights of the cherry tomato plant on a set of axes.
- Is the relationship between the number of the week and the height of the plant linear? Explain your answer.
- Assuming the pattern was the same for the first 6 weeks, what was the height of the tomato plant in week 5?
- If the pattern continues, predict the height of the cherry tomato plant in two weeks time.

### REASONING

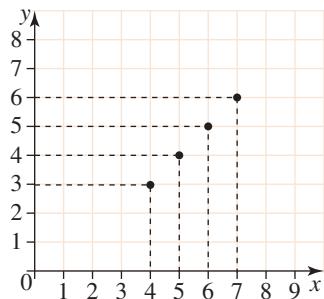
- 9 The relation between the two variables  $x$  and  $y$  is described by the rule  $y = 2x - 3$ .
- Complete the following statement: ‘The rule describes a set of points such that the  $y$ -coordinate of each point is three less than .....’
  - Construct a table of values for  $-2 \leq x \leq 2$  using the given rule.
  - Plot the points from the table on a set of axes.
  - Is the relation linear? Justify your answer.
  - The  $x$ -values of the relation form the *domain* of that relation, while the  $y$ -values form its *range*. The domain of the given relation is  $-2 \leq x \leq 2$ . What is its range?

### REFLECTION

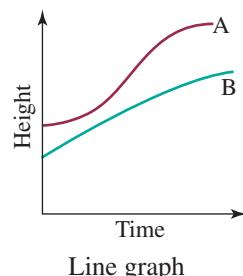
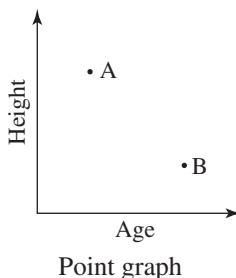
Is there a way to predict whether the points will form a linear pattern without plotting them?

## 15D Interpreting graphs

- Graphs are found in many areas of everyday life; therefore, being able to interpret graphs is a useful skill.
- When interpreting graphs, consider the following points:
  - Look at the overall picture. Where are the points situated? Does the line go up from left to right or go down from left to right?
  - Look at what horizontal and vertical axes represent, the values included (if any) and their units of measurement.
  - The values are increasing as you move up the vertical axis and to the right along the horizontal axis.



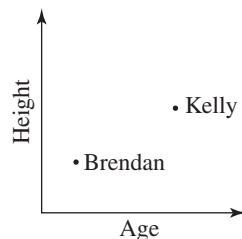
- A point graph shows pieces of information only. A line graph can show how the information changes. Time is often used on the horizontal axis.



### WORKED EXAMPLE 8

Use the graph shown to answer the following questions about Brendan and Kelly.

- Who is taller, Brendan or Kelly?
- Who is younger?



#### THINK

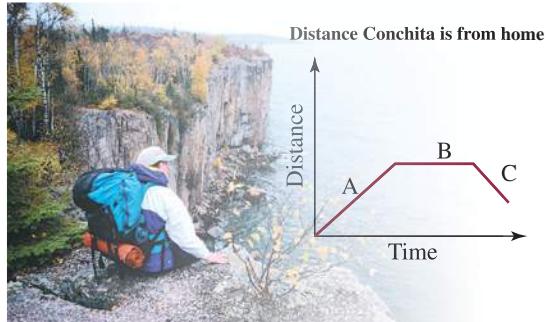
- Height is on the vertical axis. Kelly is above Brendan, so Kelly is taller.
- The horizontal axis shows age. Brendan is to the left of Kelly, so Brendan is younger.

#### WRITE

- Kelly is taller.**
- Brendan is younger.**

### WORKED EXAMPLE 9

This graph shows the distance Conchita is from home when she is out hiking. Describe what each straight line section of the graph is showing.



#### THINK

- As you move along section A, the distance is increasing and time is increasing, so Conchita is moving further away from home.
- As you move along section B, the distance is staying the same but time is still increasing.
- As you move along section C, the distance from home is decreasing and time is increasing.

#### WRITE

- Section A:** Conchita is walking away from home.
- Section B:** Conchita could be resting.
- Section C:** Conchita is walking towards home.

**REMEMBER**

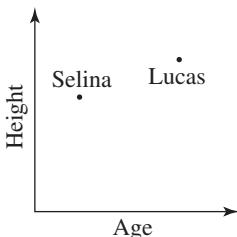
When interpreting graphs consider the following points.

1. Look at the overall picture. Where are the points situated? Does the line increase or decrease?
2. Look at what the horizontal and vertical axes represent, the values included (if any) and their units of measurement.
3. The values are increasing as you move up the vertical axis and to the right along the horizontal axis.

**EXERCISE****15D****Interpreting graphs****INDIVIDUAL PATHWAYS****eBookplus****Activity 15-D-1**Interpreting graphs  
doc-6592**Activity 15-D-2**More graphs  
doc-6593**Activity 15-D-3**Advanced graphs  
doc-6594**FLUENCY**

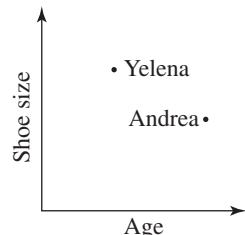
- 1 WE8** Use the graph shown to answer the following questions about Lucas and Selina.

- a Who is taller?
- b Who is younger?



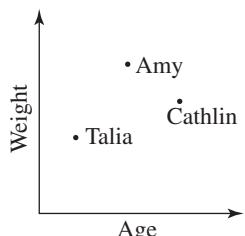
- 2** Use the graph at right to answer the following questions.

- a Does Yelena have a larger shoe size?
- b Is Yelena older than Andrea?



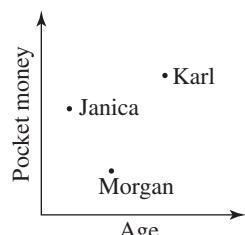
- 3** Use the graph at right to answer the following questions.

- a Who is the youngest?
- b Who weighs the most?
- c Is the oldest person the heaviest?

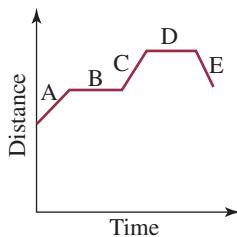


- 4** Use the given graph to answer the following questions.

- a Who is given the most pocket money?
- b Is the youngest person given the least amount of pocket money?
- c Is the oldest person given the most pocket money?

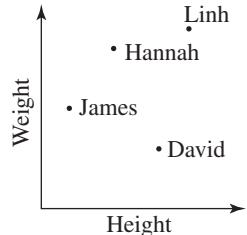


- 5 WE9** The graph at right shows the distance Claire is from home when she is out rockclimbing. Describe what each straight line section of the graph is showing.

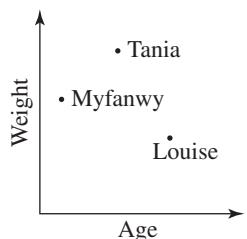


### UNDERSTANDING

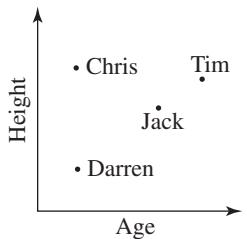
- 6 a** Use the graph at right to answer the following questions.
- Who weighs the most?
  - Who is the tallest?
  - Does the shortest person weigh the least?
- b** Copy this graph and plot the height and weight of Georgia, who is taller than Linh and weighs less than Hannah but more than James.



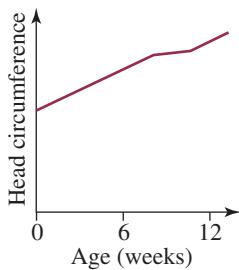
- 7 MC** Which of the following is shown by this graph?
- Myfanwy is the lightest and youngest.
  - Tania is the oldest.
  - Myfanwy is older than Louise and younger than Tania.
  - Tania and Louise are the same age.
  - Louise weighs the least.



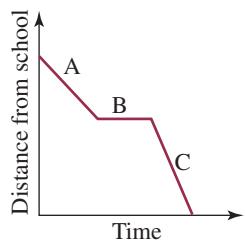
- 8 MC** Which of the following is shown by this graph?
- Jack is the tallest.
  - Chris is the oldest.
  - Darren and Chris are the same age.
  - Tim is the tallest.
  - Darren is the same height as Chris.



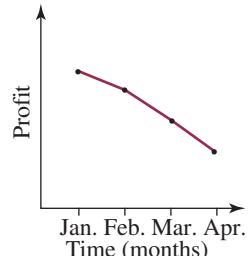
- 9** The graph below shows Thomas's head circumference for the first 12 weeks after birth.
- Why doesn't the graph start at zero?
  - What has happened to his head circumference during the first 12 weeks?
  - What would you expect the graph to look like beyond the first 12 weeks?



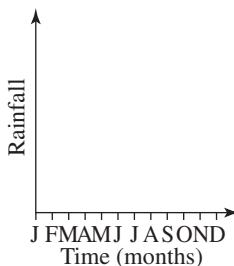
- 10** The graph at right shows Monique's journey as she walks to school.
- In which section of the graph is Monique not moving?
  - The line in section C is steeper than the line in section A. What is this telling us about section C?
  - Where is Monique at the end of section C?



- 11** Yupa owns an ice-cream shop. The graph at right shows his profit for the first 4 months of the year.
- In which month did Yupa make the most profit?
  - In which month did Yupa make the least profit?
  - Describe what has happened to Yupa's profit over the 4 months.



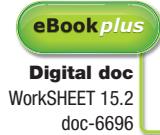
- 12** Copy the axis shown below and draw a line graph that you think would show rainfall over one year.



- 13** Cathy has been training hard for the 100 m. She has kept a record of the fastest time she has run each week. Draw a graph that shows the following section:
- Section A: Cathy increased her speed for the 100 m each week for the first 4 weeks.
- Section B: After 4 weeks Cathy's speed stayed the same for the next 2 weeks.
- Section C: Cathy had a cold and her speed decreased for 2 weeks.
- Section D: Having recovered from her cold, Cathy trained hard and increased her speed again.

#### REFLECTION

Think of some everyday situations where being able to interpret graphs would come in handy.



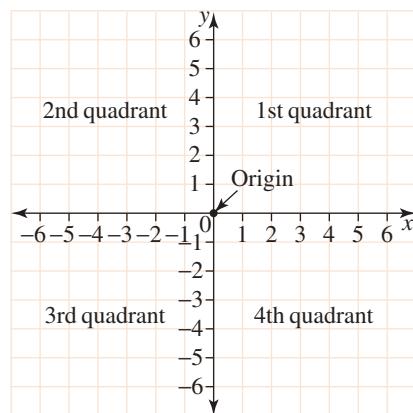
# Summary

## Alphanumeric grid references on a map

- Areas of land can be displayed by using a grid reference map, where the map is divided into squares.
- The reference for each square on a map is given by two coordinates: a letter and a number (hence the name *alphanumeric*).
- Maps with alphanumeric grid references are usually found in street directories and atlases.

## The Cartesian plane

- *Cartesian coordinates* can be used to locate any point on a plane.
- The Cartesian plane is formed by two perpendicular lines called *axes*. The horizontal axis is called the *x-axis* and the vertical axis is called the *y-axis*. The axes intersect at the point called the *origin*.
- Both axes must be marked (with marks being evenly spaced) and numbered. The distance between each mark is one unit. The axis can extend infinitely in both directions.
- The location of any point on the Cartesian plane is given by its Cartesian coordinates. The Cartesian coordinates are a pair of numbers that are separated by a comma and are shown within brackets. The first number is called the *x*-coordinate of the point; this shows how far across (that is, to the left or to the right) from the origin the point is located. The second number is called the *y*-coordinate; this shows how far up (or down) from the origin the point is. For example, the point  $(2, 3)$  is located 2 units to the right and 3 units up from the origin.



## Plotting simple linear relationships

- When a set of points is plotted on the Cartesian plane, a pattern may be formed. If a pattern forms a straight line, we call it a *linear pattern*.
- The Cartesian coordinates of the points are ordered pairs. A set of ordered pairs forms a relation between *x* and *y*.
- If the points form a linear pattern when plotted, we say that the relation between *x* and *y* is *linear*.

## Interpreting graphs

- When interpreting graphs consider the following points.
  - Look at the overall picture. Where are the points situated? Does the line increase or decrease?
  - Look at what the horizontal and vertical axes represent, the values included (if any) and their units of measurement.
  - The values are increasing as you move up the vertical axis and to the right along the horizontal axis.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

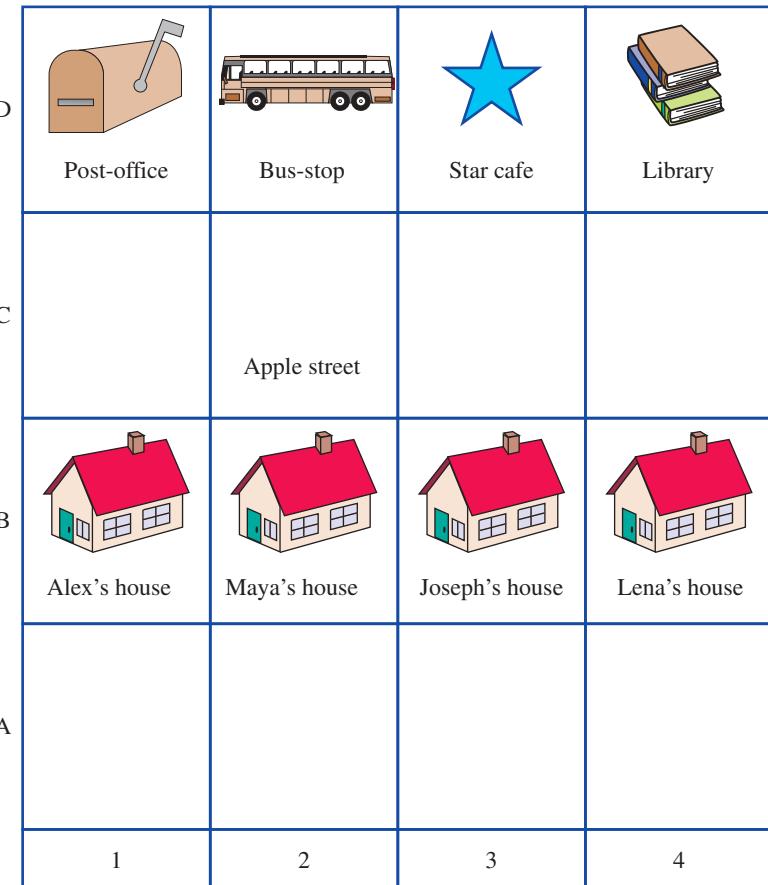
## FLUENCY

- 1 The grid at right shows a simplified map that uses alphanumeric grid references.

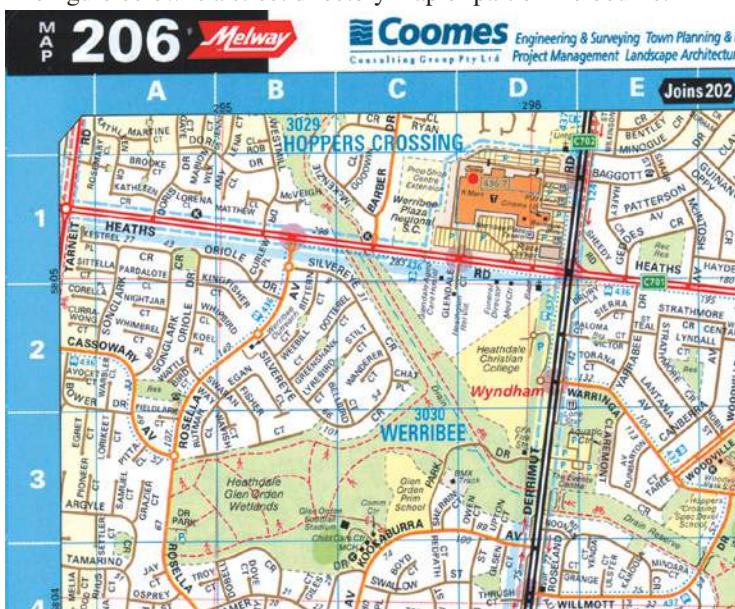
a Fill in the blanks below.

Location	Grid reference
	B3
Alex's house	
	D1
Lena's house	C

- b A playground is to be built at A3. Mark its location on the grid with a X.



- 2 The figure below is a street directory map of part of Melbourne.



- a What is the name of the major road passing through A1?
- b What is the map reference of the Heathdale Glen Orden Wetlands?

**3** Give the grid reference for each of these places using the map of Australia below.



**a** Rockhampton

**b** Canberra

**c** Broome

**d** Hobart

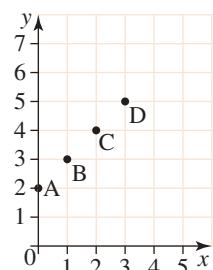
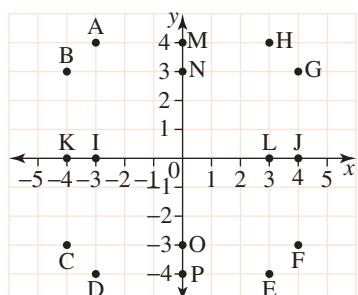
- 4 Use the tourist map of central London to answer the questions below.



- a Give the location of the following places of interest:
- Cleopatra's needle
  - Westminster Abbey
  - British Museum
- b Which rail stations are located in F9?
- c What can be found in A4?
- d Where is Buckingham Palace?
- 5 Study the diagram at right to answer the following questions.
- a Which letter in the diagram represents the point  $(-3, 4)$ ?
- b Which points are on the  $x$ -axis?
- c Name all points whose  $x$ -coordinate is zero.
- d Which points are in the second quadrant? Give their coordinates.
- e What are the coordinates of C and F?
- 6 Use the graph below to answer the following questions.
- a Fill in the blanks.

Coordinates	Point
$(0, 2)$	
	C
	D

- b Mark E  $(4, 6)$  on the graph. Join all the points with a straight line.

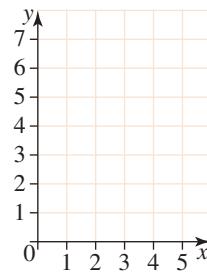


- 7** The table below gives the  $x$  and  $y$  coordinates of some points.

$x$	0	1	2	3
$y$	0	2	4	6

- a** Using the axes at right, plot the points given in the table and join them.

- b** Does the relation between  $x$  and  $y$  appear linear? Explain your answer.



### PROBLEM SOLVING

- 1** A game consists of a grid, and a dart which is thrown to land on a square of the grid.

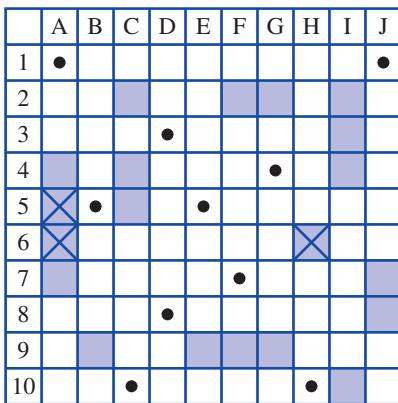
The dart is thrown and lands on a letter which:

- has horizontal symmetry
- also has vertical symmetry
- and is not a vowel.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

What are the possibilities for the landing position of the dart? Give instructions for locating any possibilities on the grid.

- 2** Rachel and Nathan are playing a battleship game. Rachel's field (in progress, up to this moment) is shown below.



- a** For each of the following attempts, state whether Nathan will miss (M), sink (S) or hit (H) one of the Rachel's ships.

i A7

ii F9

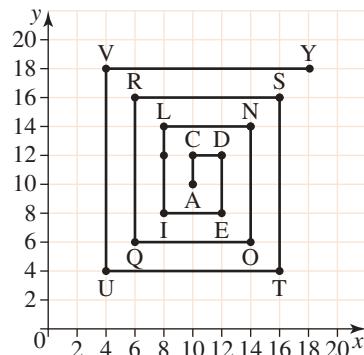
iii I 10

iv C3

- b** Specify the location of all of Rachel's one-berth ships.

- c** State the set of grid references that Nathan needs if he is to sink Rachel's largest ship.

- 3** Answer the following questions, then translate each letter of your answer into an ordered pair by using the grid below.

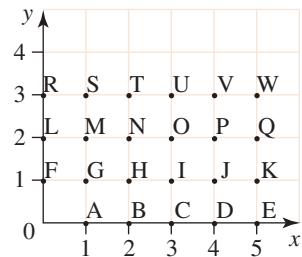


- a** In which Australian state is the Great Barrier Reef?

- b** Which Australian state is known as the Garden State?

- c** The Opera House is in which city?

- 4** The diagram below gives the key to a code by matching letters to coordinates. Use the key to write in code this famous statement by Descartes: 'I think therefore I am.'



- 5** On a carefully ruled pair of Cartesian axes join the following points in the order given. It reveals a symbol used by an ancient mathematical secret society.

(1, 0)–(6, 4)–(0, 4)–(5, 0)–(3, 6)–(1, 0)

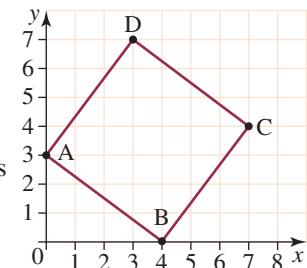
- 6** ABCD is a square drawn in the first quadrant as shown.

If A(0, 3) and

B(4, 0) and

C(7, 4) find

the coordinates of D.



- 7 a** Plot the points A(3, 0), B(6, 7), C(10, 7), D(7, 0). What type of quadrilateral is ABCD?

Find its area.

- b** Draw a set of axes and on them plot points so that the encompassing triangle has an area of 20 square units. List the points and show why the area is 20 square units.

- 8** Four points — A, B, C and D — are drawn on a Cartesian plane, and are joined in order to form a square. A is the point  $(-4, 0)$ , B is  $(1, 5)$  and C is  $(6, 0)$ . What are the coordinates of point D?

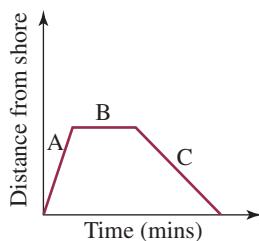
- 9** Does this table represent a relationship between  $x$  and  $y$  which is linear?

$x$	1	2	3	4
$y$	1	4	9	16

Explain why or why not.

- 10** Consider the following statement: ‘A linear relation, when plotted on a Cartesian plane, produces a straight line which cuts the  $x$ -axis at one point, and the  $y$ -axis at one point.’ Discuss whether or not this statement is true.

- 11** The following graph shows Anton’s distance from the shore during a surf life-saving drill.



- a** Describe what each section of the graph is showing.  
**b** The line in section A is steeper than the line in section C. What does this tell you about Anton’s swimming speed in section A compared to his speed in section C?

### eBook plus

#### Interactivities

Test yourself

Chapter 15

int-1826

Word search

Chapter 15

int-2609

Crossword

Chapter 15

int-2610

**Chapter Opener****Digital docs (page 501)**

- Hungry brain activity Chapter 15 (doc-6585)

**Are you ready?****Digital docs (page 502)**

- SkillsSHEET 15.1 (doc-6586) Finding locations
- SkillsSHEET 15.2 (doc-6587) Reading scales
- SkillsSHEET 15.3 (doc-6588) Marking points on the line
- SkillsSHEET 15.4 (doc-6589) Drawing scales
- SkillsSHEET 15.5 (doc-6590) Substituting values into the rule
- SkillsSHEET 15.6 (doc-6591) Finding patterns
- SkillsSHEET 15.7 (doc-6592) Establishing patterns connecting two numbers

**15A Alphanumeric grid references on a map****Digital docs (pages 504–5)**

- Activity 15-A-1 (doc-2006) Grid references
- Activity 15-A-2 (doc-2007) More grid references
- Activity 15-A-3 (doc-2008) Advanced grid references

**Weblink**

- Battleship Game (page 507)

**15B The Cartesian plane****Digital docs**

- Activity 15-B-1 (doc-2009) Introducing the Cartesian plane (page 510)
- Activity 15-B-2 (doc-2010) More of the Cartesian plane (page 510)
- Activity 15-B-3 (doc-2011) 3D planes (page 510)

- Spreadsheet Plotting points (doc-0002) (*page 511*)
- WorkSHEET 15.1 (doc-2004) (*page 512*)

**eLesson**

- Coordinates and the Cartesian plane (eles-0008) (*page 507*)

**Weblink**

- The coordinate plane (*page 510*)

**15C Plotting simple linear relationships****Digital docs (page 514)**

- Activity 15-C-1 (doc-2012) Plotting relationships
- Activity 15-C-2 (doc-2013) More relationships
- Activity 15-C-3 (doc-2014) Advanced relationships

**15D Interpreting graphs****Digital docs**

- Activity 15-D-1 (doc-6592) Interpreting graphs (*page 517*)
- Activity 15-D-2 (doc-6593) More graphs (*page 517*)
- Activity 15-D-3 (doc-6594) Advanced graphs (*page 517*)
- WorkSHEET 15.2 (doc-6696) (*page 519*)

**Chapter review****Interactivities (page 525)**

- Test yourself Chapter 15 (int-1826) Take the end-of-chapter test to test your progress.
- Word search Chapter 15 (int-2609)
- Crossword Chapter 15 (int-2610)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# 16

# Transformations



- 16A Axes of symmetry
- 16B Translations
- 16C Reflections
- 16D Rotations and combined transformations

## WHAT DO YOU KNOW?

- 1 List what you know about transforming shapes by translation, reflection and rotation. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of transformations.

eBook plus

### Digital docs

Hungry brain activity

Chapter 16

doc-6595

## OPENING QUESTION

If this photo were to be rotated  $180^\circ$ , how could you tell the original from the new photo?

# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.

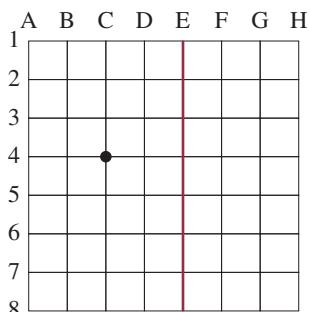
Use the grid shown at right for questions 1, 2 and 4.



**Digital docs**  
SkillsHEET 16.1  
doc-6596

### Translation of a point

- In the grid shown, the point at position C4 is moved 3 units right and 2 units down. What is the final position of the point?



**Digital docs**  
SkillsHEET 16.2  
doc-6597

### Reflection of a point

- In the grid shown, the point at position C4 is reflected in the red line. What is the final position of the point?



**Digital docs**  
SkillsHEET 16.3  
doc-6598

### Reflection of an object

- What would the following letters look like in a mirror?

F R P Z A



**Digital docs**  
SkillsHEET 16.4  
doc-6599

### Rotation of a point

- If the point at position C4 in the above grid is rotated a half turn about a point at position E4, what is the final position of the point?



**Digital docs**  
SkillsHEET 16.5  
doc-6600

### Rotation of an object through 180°

- Which of the following letters are the same when you look at them upside down?

O S H N B

## 16A Axes of symmetry

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eLesson  
Transformations  
eles-0012

- If it is possible to draw a line which would divide a shape into two parts, so that when folded along that line the parts would coincide exactly, the shape is said to be symmetrical along that line.
- The line itself is called the **line** (or the **axis**) **of symmetry**.
- If a mirror is placed along the axis of symmetry, the half of the shape that faces the mirror, together with its reflection, will form the whole shape (that is, the mirror reflection will look exactly like the other half of the shape).
- We can think of the axis of symmetry as the line that acts like a mirror.
- Many things in nature are symmetrical. Some objects have more than one axis of symmetry, while others have none.

### WORKED EXAMPLE 1

Decide whether the front of the house shown at right is symmetrical and, if so, show where the line of symmetry is.



#### THINK

A line of symmetry divides a shape into two parts so that if the shape is folded along it, the two parts will coincide. Imagine folding the picture along a line drawn through the top point of the roof and the midpoint of the base of the house. The two halves will coincide exactly. Show this line of symmetry using a dotted line.

#### WRITE

The shape of the house is symmetrical. The dotted line shows the axis of symmetry.

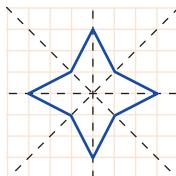
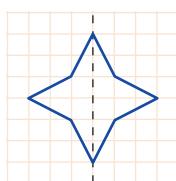
### WORKED EXAMPLE 2

How many axes of symmetry does this shape have?

#### THINK

- An axis of symmetry is a line that acts as a mirror. A vertical line through the top and bottom vertices of the given shape would divide it into halves that are mirror images of each other. So it is the axis of symmetry.
- Look for other axes of symmetry. Indicate all the axes of symmetry with dotted lines. State the number.

#### WRITE/DRAW



There are 4 axes of symmetry.

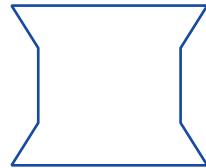
- A figure has rotational symmetry if it can be turned less than  $360^\circ$  to match the original figure.
- The number of times a match occurs within the  $360^\circ$  is known as the *order* (or *degree*) of rotational symmetry.

**WORKED EXAMPLE 3**

Determine if the figure at right has rotational symmetry and, if it does, state the order.

**THINK**

- 1 The shape is rotated less than  $360^\circ$  to match the original and so it has rotational symmetry.
- 2 Work out how many turns are needed to return to the initial position. Two turns are needed.

**WRITE**

The figure has rotational symmetry of order 2.

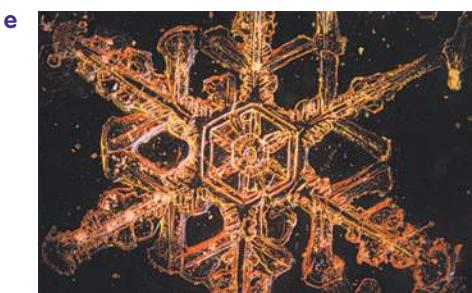
**REMEMBER**

1. A *line* (or *axis*) of *symmetry* is a line that can be drawn on a shape so that it acts like a mirror. One side of the object appears reflected in the line of symmetry.
2. If the shape is folded along the line of symmetry, the two sides will coincide exactly (that is, the shape will fold onto itself).
3. Some shapes have more than one axis of symmetry, while others have none.
4. If a figure can be turned less than  $360^\circ$  so that it matches the original figure, it has rotational symmetry. The number of turns that provide a match and return the figure to its initial position is the order of the rotational symmetry.

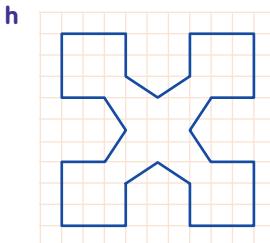
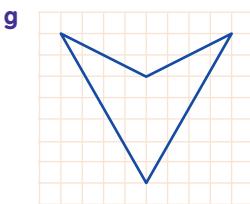
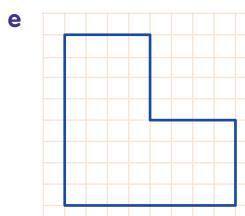
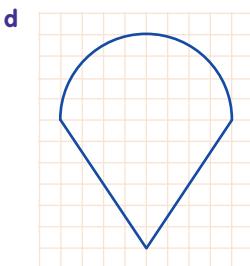
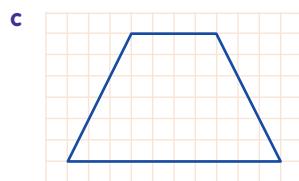
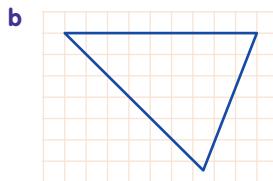
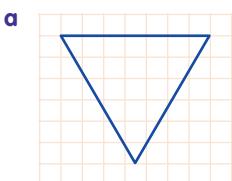
**EXERCISE****16A Axes of symmetry****INDIVIDUAL PATHWAYS****eBook plus****Activity 16-A-1**  
Symmetry  
doc-2035**Activity 16-A-2**  
More symmetry  
doc-2036**Activity 16-A-3**  
Advanced symmetry  
doc-2037**FLUENCY**

- 1 **WEI** Decide whether each of the following is symmetrical and, if so, show the line of symmetry.

**a****b****c****d**



- 2 Copy each of the following shapes on to grid or squared paper. Carefully cut out each shape. Fold it to find the axes of symmetry. State the number of axes of symmetry for each shape. (Some shapes will have more than one axis of symmetry.)



- 3 **WE2** How many axes of symmetry do each of the following shapes have?



- 4 **WE3** Determine which of the figures in Question 2 have rotational symmetry and, for those that do, state the order.

- 5 For each figure in Question 3, state the order of rotational symmetry.

## UNDERSTANDING

**6** A B C D E F G H I J K L M  
N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m  
n o p q r s t u v w x y z

Copy the letters and draw in the axes of symmetry for each if appropriate.

- a** Which upper case letters of the alphabet have a line of symmetry?
- b** Which lower case letters of the alphabet have a line of symmetry?
- c** Do any of the letters have more than one line of symmetry? If so, which ones?

**7 MC** The number of axes of symmetry in the following shape is:

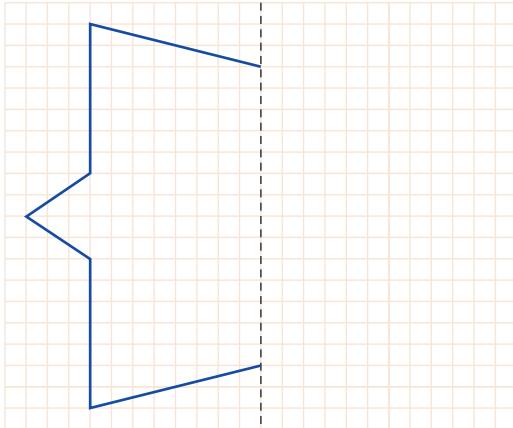
- A 0      B 1      C 2      D 4      E 3

**8 MC** The number of axes of symmetry in the following shape is:

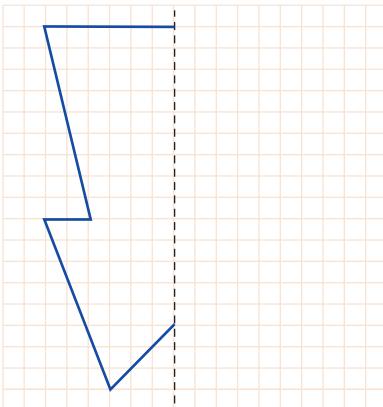
- A 0      B 1      C 2      D 3      E 4

**9** For each of the following diagrams, use the dotted line as an axis of symmetry to complete the picture.

**a**



**b**



**eBook plus**

**Weblink**  
All about symmetry

- 10 For each of the following, use the dotted line as an axis of symmetry to complete the picture.

**a****b****REFLECTION**

What strategy would you use to determine the number of axes in a shape?

## 16B Translations

### Transformations

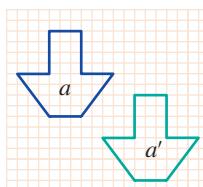
- Shapes can be shifted by an ordered set of moves from one place to another. Such ordered sets of moves are called **transformations**.
- Prior to transformation, the shape is usually called the *original object*; after the transformation, the shape is called the *image*.
- When dealing with various transformations, it is customary to denote the image with the same letters as the original shape, but with an added apostrophe. For example, if the original object is a rectangle, named ABCD, then its image after the transformation is usually named A'B'C'D'.
- If, after the transformation, the image has exactly the same size and shape as the original object, such a transformation is called **isometric**.
- Isometric transformations discussed in this chapter include **translations**, **reflections** and **rotations**. They result in **congruent figures**.

### Translations

- A translation is the movement of an object up, down, left or right (U, D, L or R) without flipping, turning or changing size.
- After a translation the object has been carried across a plane in any direction, parallel to its original position. Translations are also known as ‘slides’.

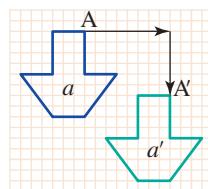
#### WORKED EXAMPLE 4

State the translation of shape  $a$  to  $a'$ .



**THINK**

- Select any point on the boundaries of the original object,  $a$  (any vertex is good to choose) and name it A. Locate the corresponding point on the image and name it  $A'$ . From point A draw a horizontal line, until it is directly above  $A'$ ; then draw a vertical line, so that it meets  $A'$ .
- Count the number of units that the object has moved across (to the right) and down and, hence, record the translation that took place.

**WRITE/DRAW**

Translation: 8R 6D

- If an object has been translated several times in both horizontal and vertical directions, its final position (relative to the original one) can be described by the total horizontal and the total vertical translations.
- When finding the total horizontal translation, we treat translations (that is, the number of units) *to the right* as *positive numbers* and *translations to the left* as *negative*. The number of units the object has been moved to the right and to the left (with appropriate signs) can be added together to give the total horizontal translation.
- To find the total vertical translation, we add together the number of units the object has been moved upward and downward, treating *upward moves* as *positive numbers* and *downward moves* as *negative*.

**WORKED EXAMPLE 5**

**State the final position of an object (relative to its original position) after the translation: 4U 2L, 2D 5L, 6U 2R and 3D 2L.**

**THINK**

- Find the total vertical translation.  
*Note:* Think of upward movements as positive and downward movements as negative.
- Find the total horizontal translation.  
*Note:* Treat movements to the right as positive and movements to the left as negative.
- State the final position of the object, relative to its position prior to translation.

**WRITE**

$$\begin{aligned}4U + 2D + 6U + 3D \\= 4 - 2 + 6 - 3 \\= 5\end{aligned}$$

So, the vertical translation is 5U.

$$\begin{aligned}2L + 5L + 2R + 2L \\= -2 - 5 + 2 - 2 \\= -7\end{aligned}$$

So, the horizontal translation is 7L.

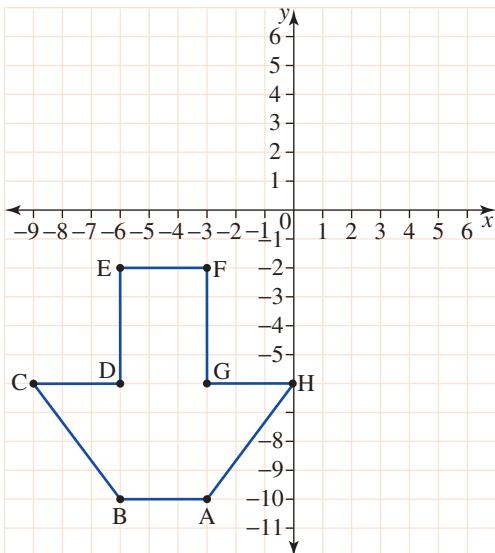
The position after translation, relative to the original position, is 5U 7L.

When you are drawing the image of an object after a series of translations, the following steps might be of assistance.

- Find the total vertical and the total horizontal translations.
- Select a point on the boundaries of the original object. (The vertices are usually the most convenient points to choose.)
- Translate the selected point as required.
- If the shape is complicated, translate a few more points.
- Complete the image of the object, using the points translated previously as a guide.

## WORKED EXAMPLE 6

- Give the coordinates of the points, A to H, on the object shown at right.
- Translate the object 5 units up, 2 units left, 2 units up and 8 units right.
- Give the coordinates of the new points, A' to H'.

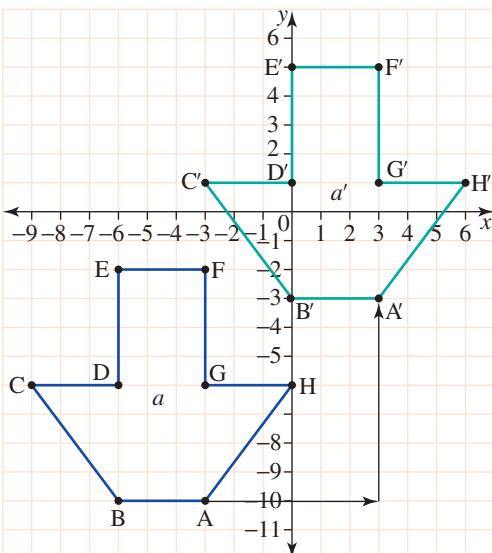


## THINK

- Write the coordinates of the points A to F.
- Find the total vertical move.
  - Find the total horizontal move.
  - State the total translation (that is, the simplified version of the required translation).
  - Select any point on the object say point A. Translate point A 7 units up and 6 units to the right. Call the translated (image) point A'. Complete the image of the object, using point A' as a guide. Label the original object  $a$  and its image  $a'$ . (You may wish to highlight the image, using a different colour.)
- Write the coordinates of the points A' to F'.

## WRITE/DRAW

- The coordinates of the vertices are:  
 $A(-3, -10)$ ,  $B(-6, -10)$ ,  $C(-9, -6)$ ,  $D(-6, -6)$ ,  
 $E(-3, -2)$ ,  $F(-3, -2)$ ,  $G(-3, -6)$ ,  $H(0, -6)$ .
  - Vertical move: 5 up and 2 up =  $5 + 2 = 7$  (up)  
Horizontal move:  
2 to the left and 8 to the right =  $-2 + 8 = 6$  (to the right)
- Translation: 7U 6R



- The coordinates of the vertices are:  
 $A'(3, -3)$ ,  $B'(0, -3)$ ,  $C'(-3, 1)$ ,  $D'(0, 1)$ ,  $E'(0, 5)$ ,  
 $F'(3, 5)$ ,  $G'(3, 1)$ ,  $H'(6, 1)$ .

## REMEMBER

- A translation is a movement up, down, left or right without changing size, flipping or turning. We can specify the translation that has occurred by stating how many units the object has moved vertically (up or down) and horizontally (left or right).
- When finding the total translations, movements (the number of units) to the right and up are considered as positive, while movements to the left and down are treated as negative numbers.

## EXERCISE

## 16B Translations

## INDIVIDUAL PATHWAYS

## eBook plus

## Activity 16-B-1

Translations  
doc-2026

## Activity 16-B-2

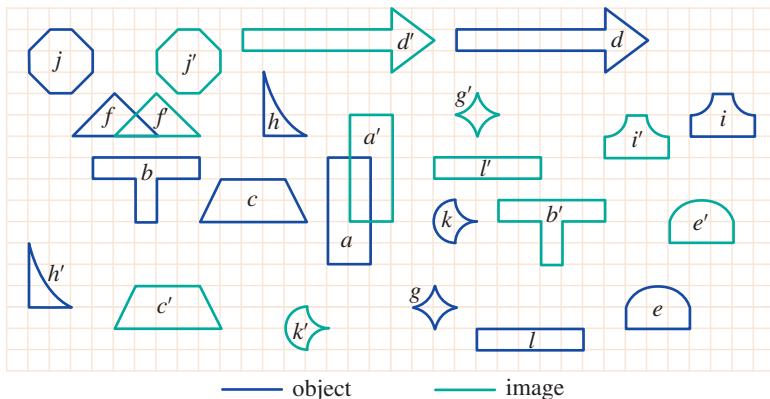
More translations  
doc-2027

## Activity 16-B-3

Advanced  
translations  
doc-2028

## FLUENCY

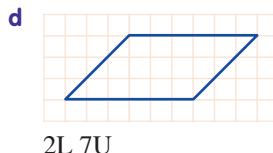
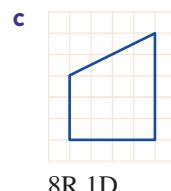
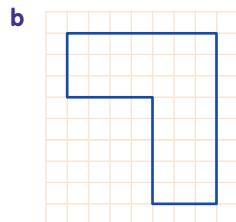
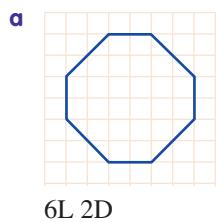
- 1 WE4** State the translation that has occurred to each of the shapes labelled *a* to *l* in the figure below.



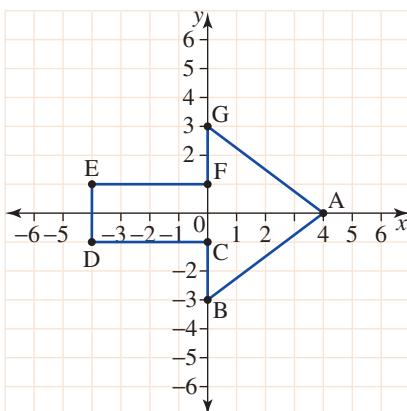
- 2 WES** State the final position of an object (relative to its original position) after each of the following translations:

- 4L 2U, 3R 4U
- 6R 3D, 4L 5D
- 8L 2U, 4R 3D, 6R 4U
- 10R 3D, 4L 5D, 6R 8U
- 12L 4D, 2L 3D, 8R 4D, 6L 2U
- 3U, 4D 3R, 4D 6L, 2R, 7L 2U, 3L
- 8L 2D, 4R 9U, 6L 2D, 4R 3U, 2L 9D
- 6R 2U, 3L 5U, 3R 4D, 6L 3D, 2R 4U, 6L 3U

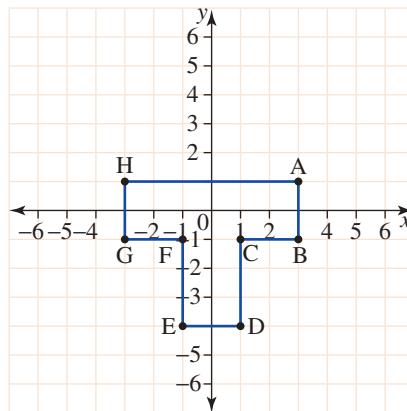
- 3** Draw the following translations to the shapes shown.



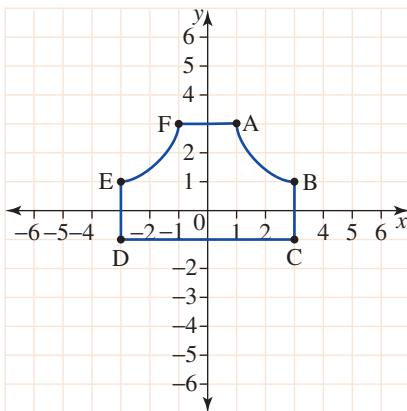
**4 i WE6** Translate the following objects as specified under the figures.

**a**

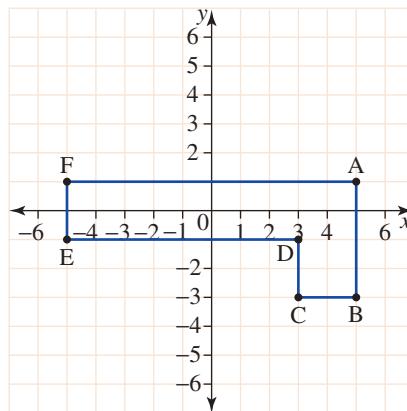
4L 2D, 3R 4U

**b**

6L 2D, 8R 3D

**c**

2R 4D, 6R 3D, 2R 2D

**d**

7R 5U, 2R 3U, 8L 2D, 7L 1D

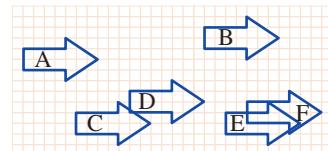
**ii** Give the coordinates of the vertices of each object in its final position.

### UNDERSTANDING

**5** Specify each of the following translations of the labelled arrows at right.

- a** A to C
- c** F to E
- e** D to B

- b** A to F
- d** E to F
- f** E to A



**6** Using the diagram in Question 5, state which arrow has moved where (that is, which letter represents the object and which one represents the image), if the following translations have occurred.

- a** 2U 5R
- d** 2L 8U

- b** 2D 5L
- e** 4U 10L

- c** 2U 9L
- f** 14L

**7 MC** If an object is translated 3U 2L, 5R 4D, 6L, its final position (related to the original one) is:

- A** 7U 3R
- D** 1D 3L

- B** 1U 3L
- E** 1D 3R

- C** 7U 3L

**8 MC** If, relative to the original position, the final position of an object is 3L 4U, then the series of translations that could have occurred is:

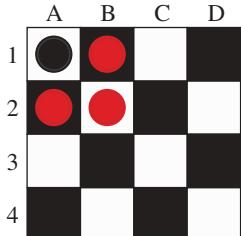
- A** 4R 2U, 3L 2U
- D** 4R 2U, 3L 3U

- B** 4R 1U, 7L 3U
- E** 4R 1U, 4L 3U

**REASONING**

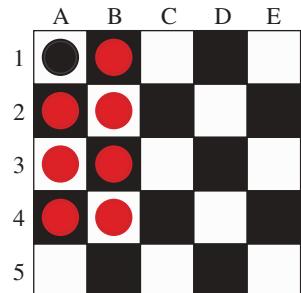
- 9** In a game of draughts, a counter is removed from the board when another counter jumps over it. Four counters are arranged on the corner of a draughts board as shown. Use three horizontal or vertical jumps to remove all three red counters, leaving the single black counter. Copy and complete the table to show your working.

Jump	From	To
1		
2		
3		



- 10** In another game, any counter can jump any other counter. Use seven horizontal or vertical jumps to remove all seven red counters, leaving the single black counter. Copy and complete the table to show your working.

Jump	From	To
1		
2		
3		
4		
5		
6		
7		

**REFLECTION**

What strategy would you use to translate an object from one position to another?

## 16C Reflections

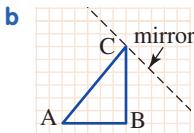
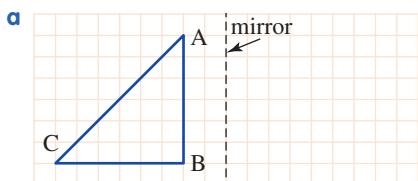
- A reflection is the image of an object, as seen in the mirror.
- Reflections are often called ‘mirror images’ and the lines in which the objects are reflected — ‘mirror’ lines.
- Mirror images always have reverse orientation; that is, left appears to be right and vice versa.
- The points of the image and the corresponding points of the object lie on the same line, which is perpendicular to the mirror line.
- Each point of the image is the same distance from the mirror line as is the object on the other side of the mirror.
- Reflections are also known as ‘flips’.

When reflecting shapes in the given line, the following steps can be of assistance:

- Step 1 Select some key points on the original object (the vertices are usually convenient to choose).
- Step 2 From each point draw a line, perpendicular (that is, at right angles) to the mirror line. Extend each line beyond the mirror line.
- Step 3 For each selected point, measure its distance along the line from the mirror. Then measure out exactly the same distance along the line on the other side of the mirror to find the position of the corresponding point of the image.
- Step 4 Complete the image, using the previously reflected points as a guide.

## WORKED EXAMPLE 7

For each of the following shapes, find the reflected image in the line given.

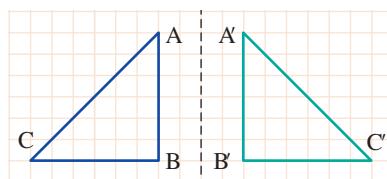
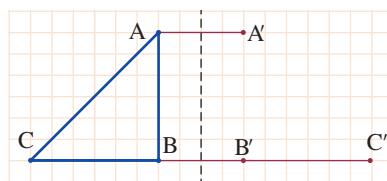


## THINK

- a 1 From each vertex of the given triangle, draw the lines perpendicular to and extending beyond the mirror line.
- 2 Points A and B are both 2 units from the mirror to the left of it. Since the image is reversed, the vertices  $A'$  and  $B'$  are 2 units from the mirror on the right side of it. Furthermore, point C is 8 units to the left of the mirror. The corresponding point of the image  $C'$  is 8 units to the right of the mirror.
- 3 Join the vertices  $A'$ ,  $B'$  and  $C'$  to complete the image.

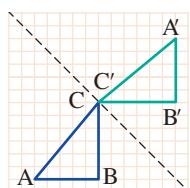
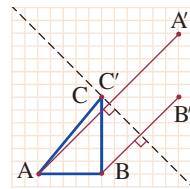
## DRAW

a



- b 1 Draw the lines from points A and B so that they extend beyond the mirror line and are perpendicular to it. (Point C of the given triangle is already on the mirror line, so nothing needs to be done to it.)
- 2 Any point of the object that is on the mirror line will reflect onto itself. So  $C'$  will coincide with point C, since point C is on the mirror line. Points  $A'$  and  $B'$  are the same distance from the mirror as points A and B respectively, but on the other side of it. (The distances must be measured along the perpendicular lines.)
- 3 Join the vertices  $A'$ ,  $B'$  and  $C'$  to complete the image.

b

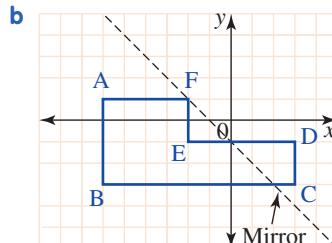
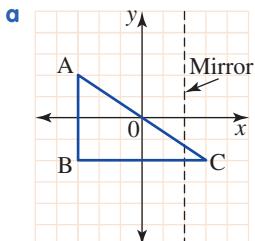


Sometimes the line in which the object is to be reflected is placed over the object so that the object is divided in two parts. Since the image is reversed, the part of the object that is to the left of the mirror line when reflected will appear to the right of it. The part of the object to the right of the mirror line will appear to the left of it after the reflection.

## WORKED EXAMPLE 8

For each of the following objects:

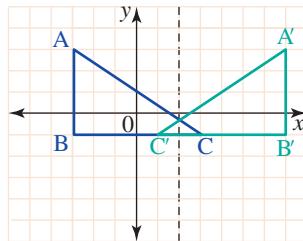
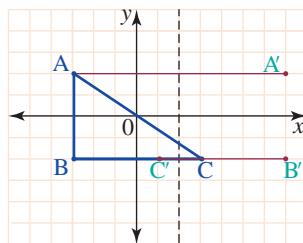
- Give the coordinates of the vertices of the object.
- Draw the image reflected in the mirror.
- Give the coordinates of the vertices of the image.

**THINK**

- a** **i** Write the coordinates of each of the vertices of the object.
- ii** **1** Extend line BC (it is perpendicular to the mirror line). From point A, draw the line extending beyond the mirror and perpendicular to it.  
**2** The mirror line is placed over the object, so one part of the image will appear to the left of the mirror and the other part to the right of it. Points A and B are both 5 units to the left of the mirror, so  $A'$  and  $B'$  will be 5 units to the right of it. Point C is 1 unit to the right of the mirror, so its image,  $C'$ , will be 1 unit to the left of the mirror line.  
**3** Complete the image by joining the reflected points appropriately.

**DRAW**

- a** The coordinates of the vertices of the object are:  
 $A(-3, 2)$ ,  $B(-3, -2)$ ,  $C(3, -2)$



- iii** Write the coordinates of each of the vertices of the image.
- b** **i** Write the coordinates of each of the vertices of the object.

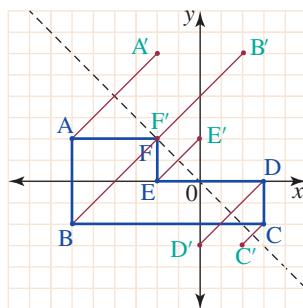
- ii** **1** From every vertex of the object, draw the lines extending beyond and perpendicular to the mirror line.

- The coordinates of the vertices of the image are:  
 $A'(7, 2)$ ,  $B'(7, -2)$ ,  $C'(1, -2)$

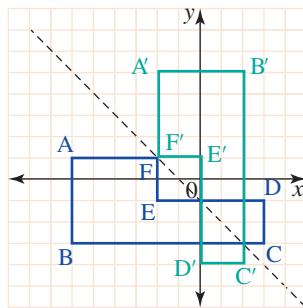
- b** The coordinates of the vertices of the object are:  
 $A(-6, 1)$ ,  $B(-6, -3)$ ,  $C(3, -3)$ ,  $D(3, -1)$ ,  $E(-2, -1)$ ,  $F(-2, 1)$

- 2 The mirror is placed over the object, so parts of the image will appear on either sides of the mirror line. Points A, B and E are to the left of the mirror, so their corresponding images (that is,  $A'$ ,  $B'$  and  $E'$ ) will be the same distances from the mirror line, but to the right of it. Points D and C are to the right of the mirror, so their images ( $D'$  and  $C'$ ) will appear to the left of the mirror line. Finally,  $F'$  will coincide with point F, as F is exactly on the mirror line.

- 3 Complete the image, using the reflected vertices as a guide.



- iii Write the coordinates of each of the vertices of the image.



The coordinates of the vertices of the image are:

$A(-2, 5)$ ,  $B'(2, 5)$ ,  $C'(2, -4)$ ,  $D'(0, -4)$ ,  $E'(0, 1)$ ,  $F'(-2, 1)$

### REMEMBER

1. A reflection is the image of the object as seen in the mirror.
2. A reflection always has reverse orientation: left appears right and vice versa.
3. The lines joining each point of the image with the corresponding points of the object are perpendicular to the mirror line. The image points are the same distance as the original points from the mirror along the perpendicular lines but on the opposite side of it.
4. Any point of the object that is on the mirror line will reflect onto itself.
5. If the line in which the object is to be reflected is placed over the object, part of the image will appear to the left of the mirror line and part to the right of it.

### EXERCISE

## 16C Reflections

### INDIVIDUAL PATHWAYS

eBookplus

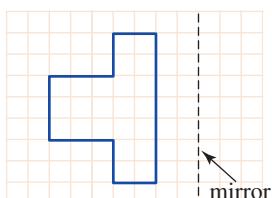
**Activity 16-C-1**  
Reflections  
doc-2029

**Activity 16-C-2**  
More reflections  
doc-2030

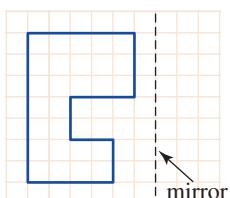
### FLUENCY

- 1 **WE7** For each of the following shapes, find the reflected image in the line given.

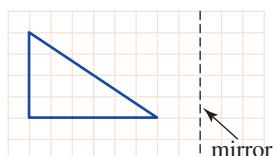
a



b



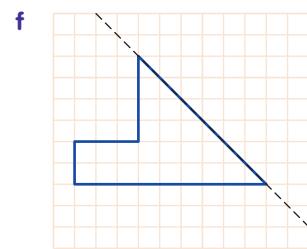
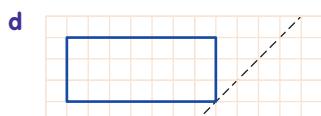
c



## INDIVIDUAL PATHWAYS

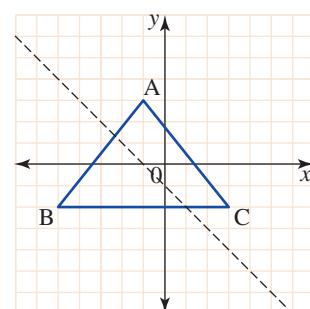
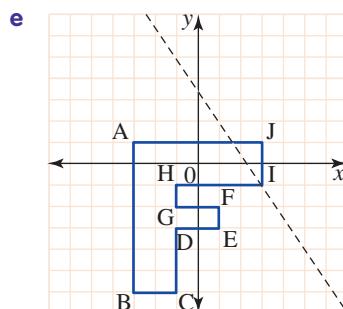
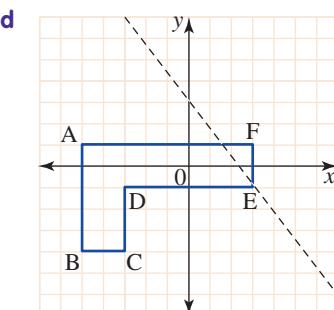
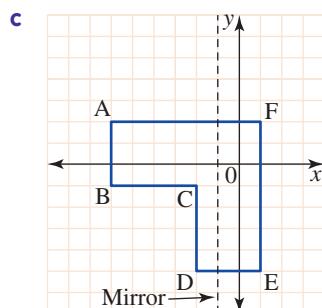
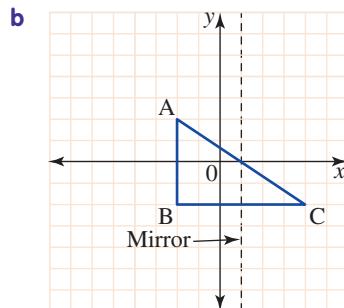
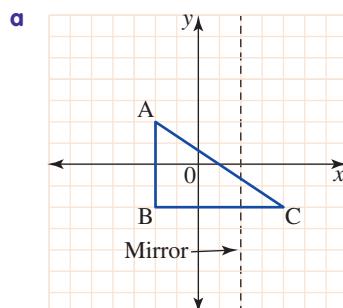
eBook plus

**Activity 16-C-3**  
Advanced reflections  
doc-2031

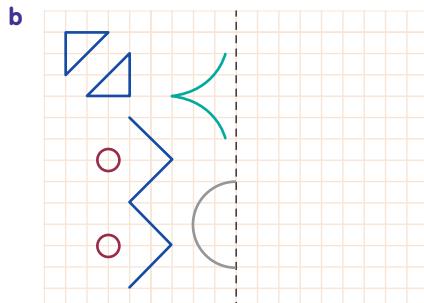
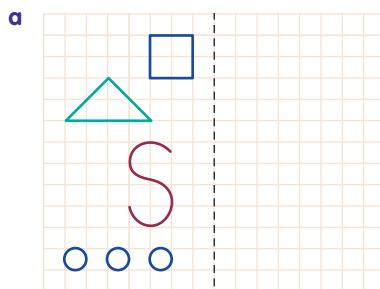


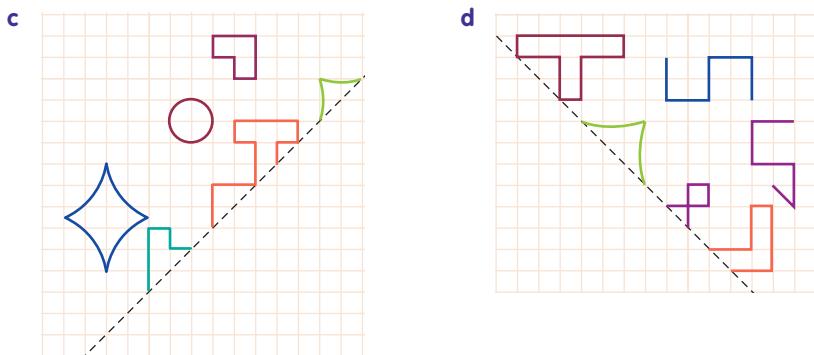
**2 WEB** For each of the following shapes:

- draw the image reflected in the mirror
- give the coordinates of the vertices of the image.



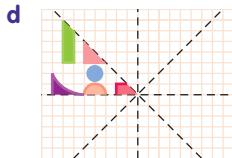
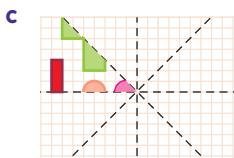
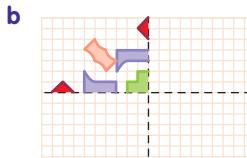
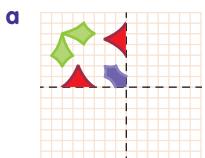
**3** Reflections in mirrors transform objects. Complete each of the patterns by finding images of the objects reflected in a single mirror, as shown in the following.



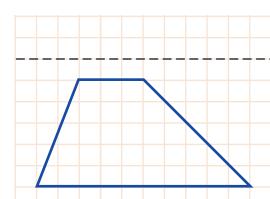
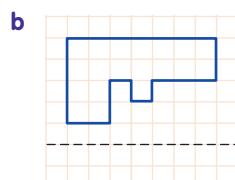
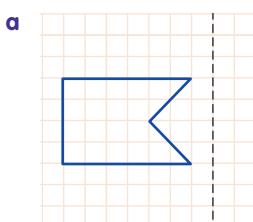


## UNDERSTANDING

- 4** A kaleidoscope uses more than one mirror to create wonderful patterns. To help design a kaleidoscope, reflect each object in each of the mirrors to complete the pattern in each of these figures.



- 5** Find the image for each of the following objects, using the transformations specified under the figures.



Reflected in the mirror  
and then translated 2 units  
down and 3 units left

Translated 3U 4R  
and then reflected in  
the mirror

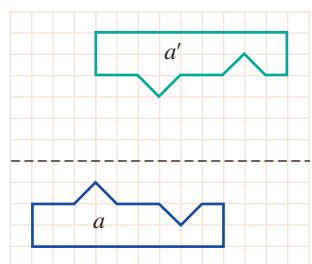
Translated 1U 4L,  
reflected in the mirror and  
then translated 6U 2R

- 6 MC** The diagram shows an object and its image after a certain set of transformations. The object was:

- A reflected in the mirror; then translated 3R 2D
- B reflected in the mirror; then translated 4R 2D
- C translated 3R 2D; then reflected in the mirror
- D translated 4R 2D;  
then reflected in  
the mirror
- E translated 3R 2U;  
then reflected in  
the mirror

## REFLECTION

What strategy would you use to draw the reflection of a shape in a mirror?



# 16D Rotations and combined transformations

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**Interactivity**  
int-2745  
Multiple  
transformations

- A figure can be rotated about a point called the **centre of rotation**.
- To specify the rotation, we need to give the angle through which the object is to be turned and the direction of the rotation (clockwise or anticlockwise).
- Rotations are also known as ‘turns’.

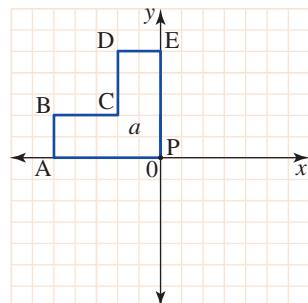
To perform a rotation, the following steps could be of assistance:

- Step 1 Select any point on the object (vertices are most convenient to choose).
- Step 2 Join the selected point and the centre of rotation with a straight line.
- Step 3 Using a protractor and a ruler (or a pair of compasses), find the image of the selected point after the required rotation. The image point is as far from the centre of rotation as is its corresponding point on the object.
- Step 4 Rotate some other points if necessary.
- Step 5 Complete the image, using previously rotated points as a guide. Label the image appropriately.

## WORKED EXAMPLE 9

Consider the shape shown at right.

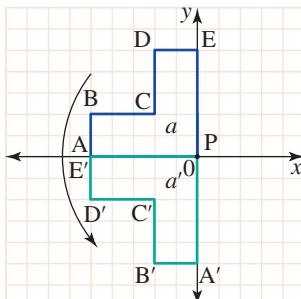
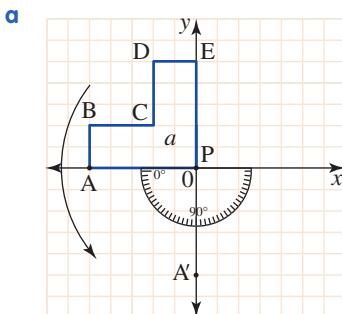
- Show the image of the shape after a rotation of  $90^\circ$  (quarter turn) anticlockwise about point P.
- Compare the coordinates of the vertices of the object with those of its image.



### THINK

- Select any point on the object say A. Join point A with the centre of rotation, point P. (In this case the points are already joined by the side of the shape.) Place the protractor so that its centre is on P, its zero is on the line AP and its scale increases in an anticlockwise direction. Measure the  $90^\circ$  angle. The image point A' is as far from the centre of rotation, P, as is point A (5 units in this case).
- Repeat step 1 for some other points. Use these points as a guide to complete the image. Label the image a'. (You may wish to highlight it, using a different colour.)

### DRAW



- b** 1 Write the coordinates of the object and the image.

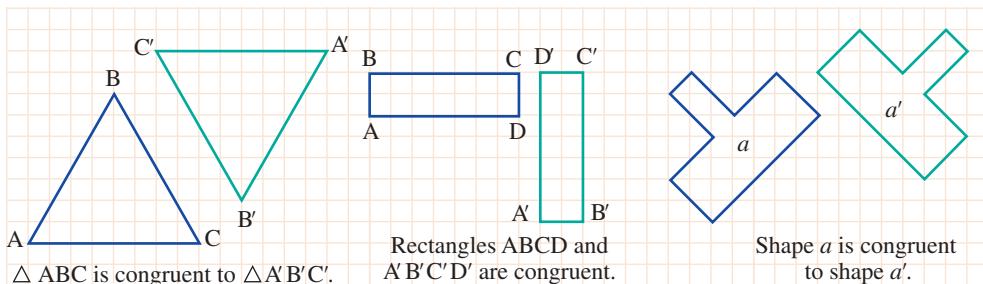
- 2 Compare the values of the coordinates.

- b** The coordinates of each point of the object and its corresponding position on the image are:  
 $A(-5, 0) \rightarrow A'(0, -5)$   
 $B(-5, 2) \rightarrow B'(-2, -5)$   
 $C(-2, 2) \rightarrow C'(-2, -2)$   
 $D(-2, 5) \rightarrow D'(-5, -2)$   
 $E(0, 5) \rightarrow E'(-5, 0)$

The coordinates of P do not change as it is the point of rotation.

The x-coordinate of the object is the same as the y-coordinate of its image, while the x-coordinate of the image is the negative value of the y-coordinate of the object.

*Note:* This relationship does not apply to all rotations. The relationship depends on the type of rotation, and the position of the object on the Cartesian plane.



During a rotation the figures do not change their shape or size and, therefore, the images are said to be congruent to their object.

Instead of the lengthy word *congruent*, the sign  $\cong$  is used. For example, in the above diagram,  $\triangle ABC \cong \triangle A'B'C'$ .

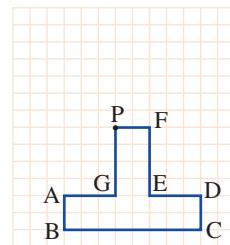
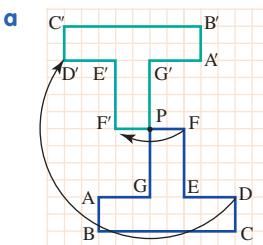
### WORKED EXAMPLE 10

- a** Show the image of the shape at right after a rotation about point P by  $180^\circ$  (half turn) in a clockwise direction. Label the image appropriately.  
**b** State whether the image is congruent to the object.

#### THINK

- a** Rotate the vertices of the object  $180^\circ$  in a clockwise direction. Label the image of point A, using A', the image of point B with B' and so on. Join the vertices appropriately to complete the image.

#### WRITE/DRAW

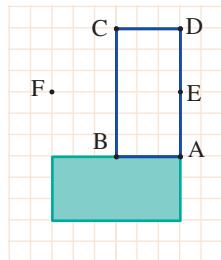


- b** Determine whether the two figures are exactly the same size and shape, and answer the question.

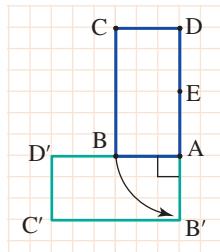
- b** Yes, the object and its image are congruent; therefore,  $ABCDEF \cong A'B'C'D'E'F'G'$ .

**WORKED EXAMPLE 11**

The rectangle at right has been rotated  $90^\circ$  about one of the points A, B, C, D, E or F in an anticlockwise direction. State the centre of rotation that has been used to form the shaded image.

**THINK**

- As the image has not moved from point A, test rotation about this point. That is, copy the rectangle ABCD and perform the rotation of  $90^\circ$  in an anticlockwise direction about point A.
- Answer the question.  
*Note:* The image obtained by rotating the object about point A is exactly the same as the given one. Therefore, A is the centre of rotation that has been used.

**WRITE/DRAW**

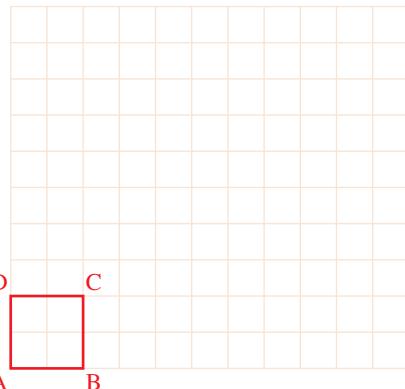
The rectangle has been rotated about point A.

**Combined transformations**

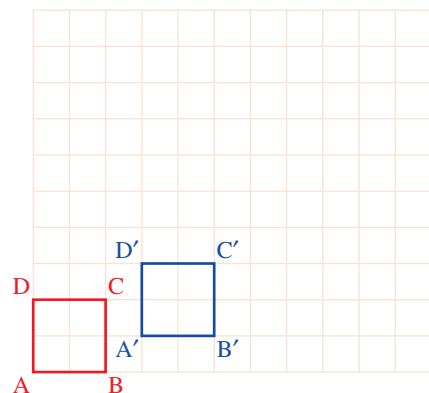
- Transformations can be combined by performing two or more transformations one after the other.

**WORKED EXAMPLE 12**

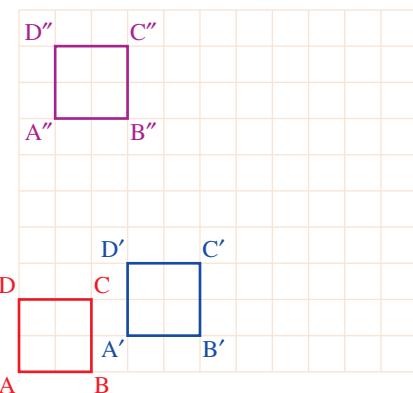
- Translate the square ABCD 3 units right and 1 unit up, then translate the image 2 units left and 6 units up.
- Determine a single transformation that would have produced the same result.

**THINK**

- Take each corner point of the square and translate it 3 units to the right and 1 unit up. Mark each new corner with the appropriate label to represent the image. So, A is translated to A' and so on.

**DRAW**

- 2** Take each image point from the previous step and translate it 2 units left and 6 units up.  
Mark each new corner with the appropriate label to represent the image.  
So,  $A'$  is translated to  $A''$  and so on.



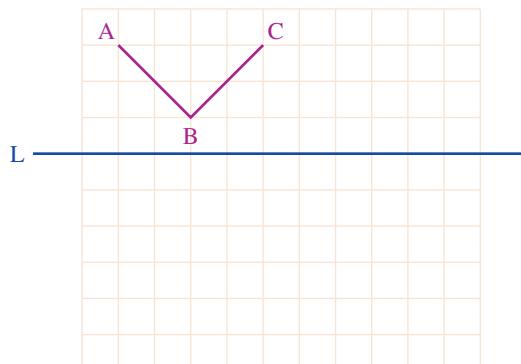
- b**
- 1** Starting at the original square ABCD, count the number of units up and to the right from A to  $A''$ . Check that each this translation works for each corner.
  - 2** State the combined transformation.

**b** 1 unit right and 7 units up.

The combined transformation is the same as a translation of 1 unit right and 7 units up.

### WORKED EXAMPLE 13

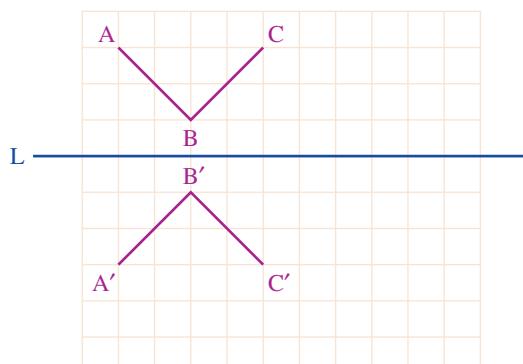
Reflect the V in the line L, then translate the image 6 units right and 3 units down, then rotate the image  $90^\circ$  anti-clockwise.



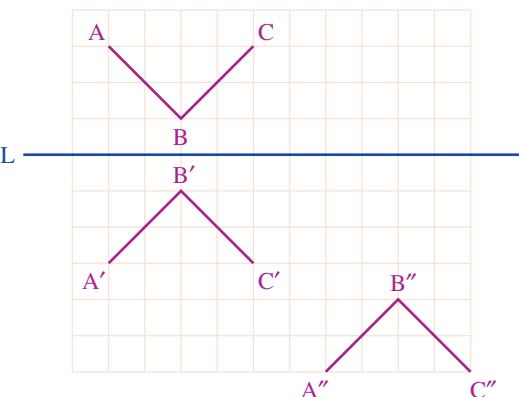
#### THINK

- 1** Reflect the V in the line L and label the vertices  $A'$ ,  $B'$ ,  $C'$ .

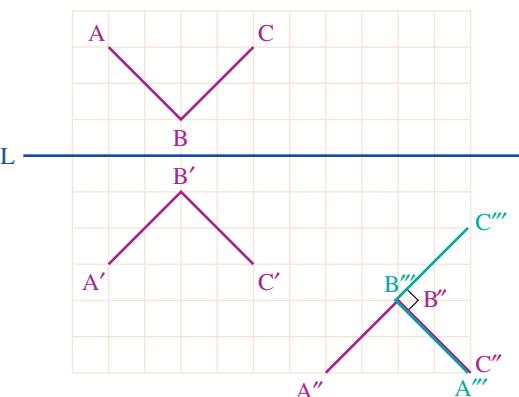
#### DRAW



- 2 Translate the image 6 units to the right and 3 units down.  
Label this new image  $A'', B'', C''$ .



- 3 Rotate this image  $90^\circ$  anti-clockwise.  
Label this new image  $A''', B''', C'''$ .



#### REMEMBER

1. A rotation is the process of turning an object around a fixed point without changing its shape or size.
2. The fixed point about which the shape is turned is called the centre of rotation.
3. To specify the rotation about the given point, we need to state the direction (clockwise or anticlockwise) of rotation and the angle through which the object is to be turned.
4. Figures that have the same shape and size are said to be congruent.
5. The image formed by rotation is congruent ( $\cong$ ) to the original object.

#### EXERCISE

## 16D

## Rotations and combined transformations

#### INDIVIDUAL PATHWAYS

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##### Activity 16-D-1

Rotations  
doc-2032

##### Activity 16-D-2

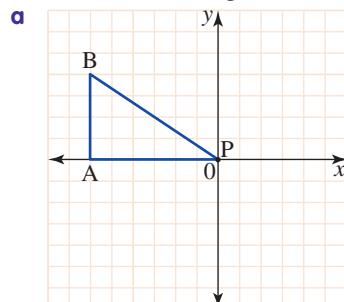
More rotations  
doc-2033

##### Activity 16-D-3

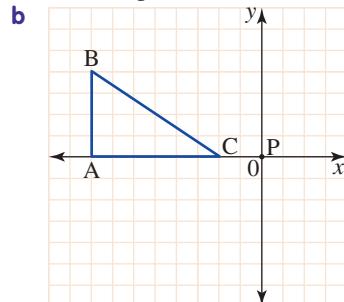
Advanced rotations  
doc-2034

#### FLUENCY

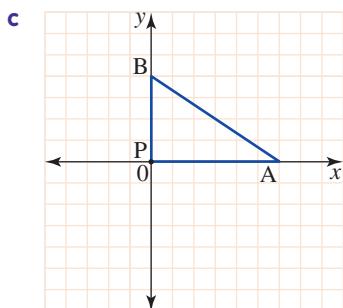
- 1 i **WE9** Show the image of the following shapes, rotated about point P.



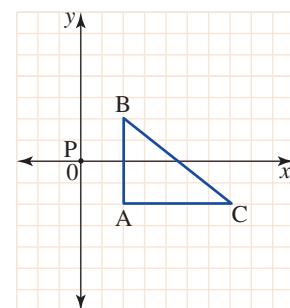
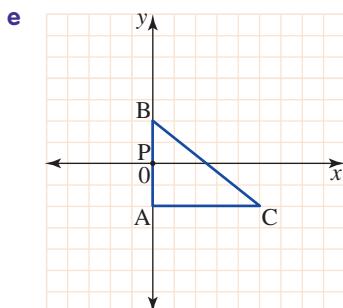
Rotated  $90^\circ$  clockwise



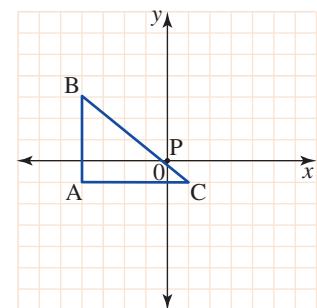
Rotated  $90^\circ$  anticlockwise



Rotated 180° anticlockwise

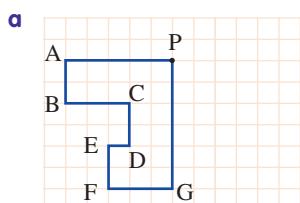
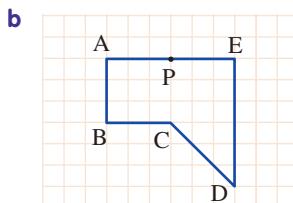
Rotated 270° ( $\frac{3}{4}$  turn) clockwise

Rotated 270° anticlockwise

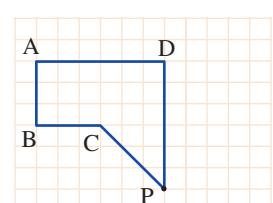
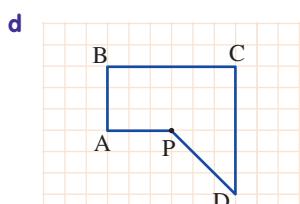
Rotated 450° ( $1\frac{1}{4}$  turns) clockwise

ii Compare the coordinates of the vertices of the object with those of its image.

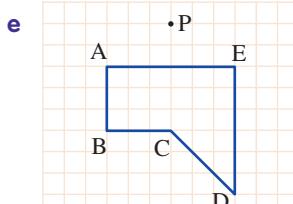
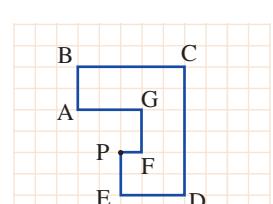
- 2 WE10** Show the image of each of the following shapes after rotation about point P. Label the images appropriately. In each case state whether the image is congruent to the object.

Rotated 90°  
anticlockwise

Rotated 270° clockwise

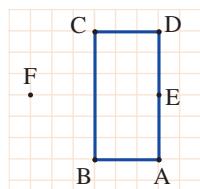
Rotated 180°  
anticlockwise

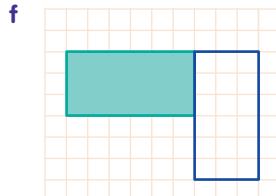
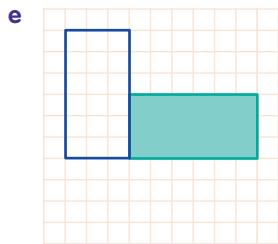
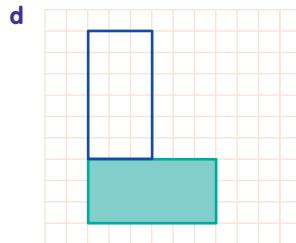
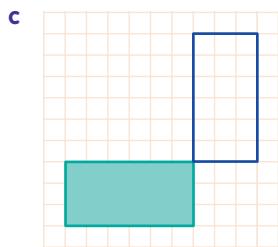
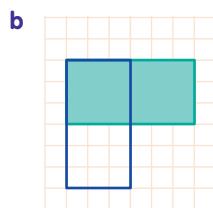
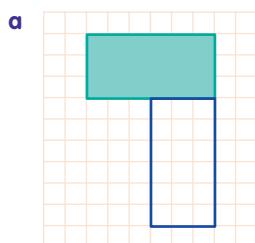
Rotated 90° clockwise

Rotated 270°  
anticlockwise

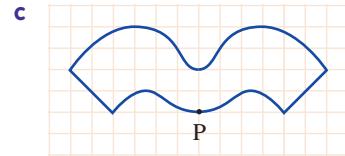
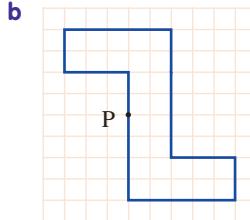
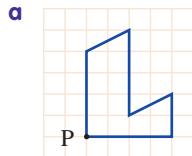
Rotated 450° clockwise

- 3 WE11** The rectangle at right has been rotated 90° about A, B, C, D, E or F in a clockwise direction. State the centre of rotation that has been used to form each of the images (shaded) at the top of the next page.

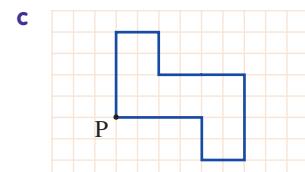
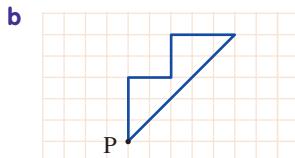
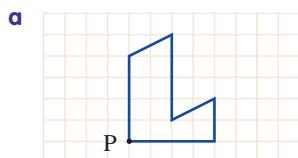


**UNDERSTANDING**

- 4** Patterns can be made by rotating images. For each of these shapes, show the pattern after a  $180^\circ$  turn in a clockwise direction about point P.

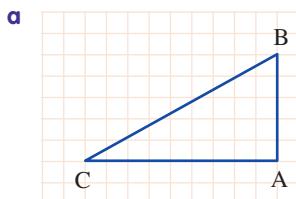


- 5** Some patterns can be made by rotating images several times. For each of the following, rotate the object *three times* about point P,  $90^\circ$  in a clockwise direction. That is, rotate the object first, then rotate the image of the object and, finally, the image of the image, so that you have 4 shapes altogether.

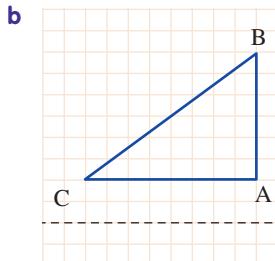


- 6** Create your own designs by drawing an interesting shape and rotating it several times.

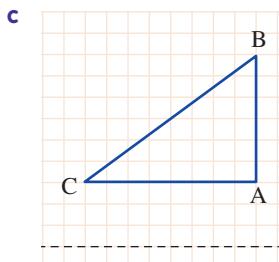
**7** Show the final image after the following sets of transformations.



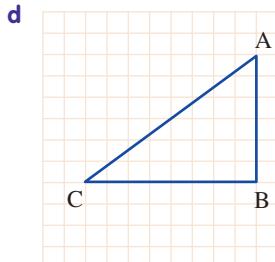
Translated 2U 2R, rotated 90° about the image of C in a clockwise direction



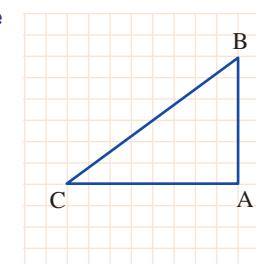
Reflected in the mirror line and then rotated 180° clockwise about the image of C



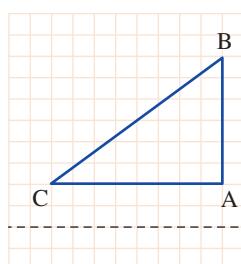
Rotated 270° anticlockwise about C and then reflected in the mirror line



Rotated 90° anticlockwise about C, then translated 2D 5L

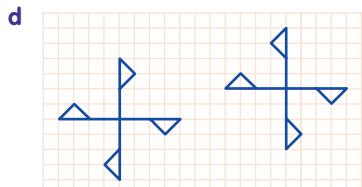
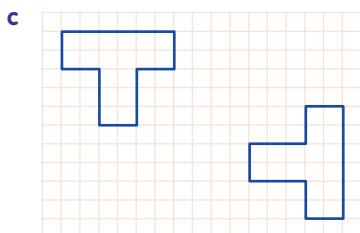
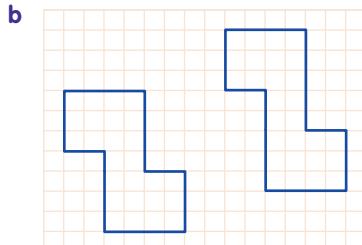
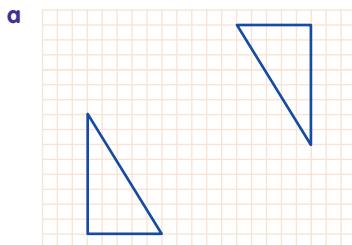


Translated 3D 4L; then rotated 180° clockwise about the image of C



Rotated 180° about C, translated 5R 3U and then reflected in the mirror line

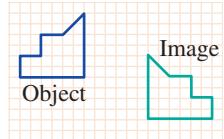
**8** Which of the following pairs of shapes are congruent?



- 9 State whether the following statements are true or false.
- A shape that has been translated is congruent to the original one.
  - A shape that has been rotated is congruent to the original one.
  - A shape that has been reflected is congruent to the original one.
  - A shape that has been translated and then rotated is not congruent to the original one.
  - A shape that has been enlarged is congruent to the original one.
  - A shape that has been reduced is not congruent to the original one.

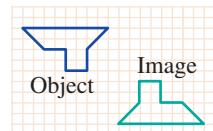
- 10 **MC** The diagram at right shows the object and its image after a certain transformation(s). The object was:

- A** reflected                           **B** translated  
**C** rotated                               **D** reflected and then translated  
**E** reflected and then rotated

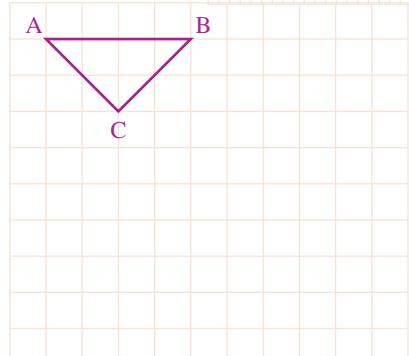


- 11 **MC** To obtain the image, shown in the diagram at right, the object was:

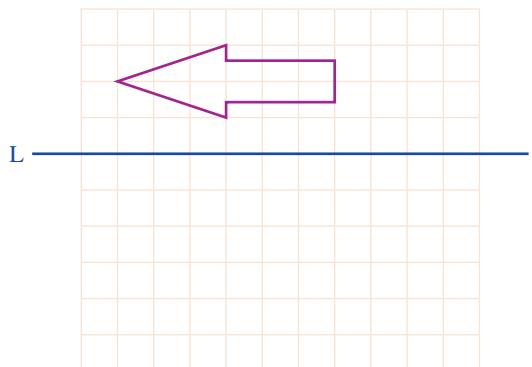
- A** reflected                           **B** translated  
**C** rotated                               **D** rotated and then reflected  
**E** rotated and then translated



- 12 **WE12** a Translate the triangle ABC 6 units right and 7 units down and then translate the image 4 units left and 2 units up.  
b Determine a single transformation that would have produced the same result.



- 13 **WE13** Reflect the arrow in the line L, then translate the image 4 units right and 2 units down and then rotate this image  $90^\circ$  anti-clockwise.



### REASONING

#### eBookplus

**Digital docs**  
WorkSHEET 16.2  
doc-2025  
Investigation  
Designing a  
patchwork  
quilt  
doc-3444

- 14 In the photograph shown, a spoke of a wheel undergoes rotation about the centre of the wheel. What angle has spoke A rotated through to move to the position of spoke B?

#### REFLECTION

What strategy would you use when drawing the rotated image of an object?



# Summary

## Axes of symmetry

- A *line (or axis) of symmetry* is a line that can be drawn on a shape so that it acts like a mirror. One side of the object appears reflected in the line of symmetry.
- If the shape is folded along the line of symmetry, the two sides will coincide exactly (that is, the shape will fold onto itself).
- Some shapes have more than one axis of symmetry, while others have none.
- If a figure can be turned less than  $360^\circ$  so that it matches the original figure, it has rotational symmetry. The number of turns that provide a match and return the figure to its initial position is the order of the rotational symmetry.

## Translations

- A translation is a movement up, down, left or right without changing size, flipping or turning. We can specify the translation that has occurred by stating how many units the object has moved vertically (up or down) and horizontally (left or right).
- When finding the total translations, movements (the number of units) to the right and up are considered as positive, while movements to the left and down are treated as negative numbers.

## Reflections

- A reflection is the image of the object as seen in the mirror.
- A reflection always has reverse orientation: left appears right and vice versa.
- The lines joining each point of the image with the corresponding points of the object are perpendicular to the mirror line. The image points are the same distance as the original points from the mirror along the perpendicular lines but on the opposite side of it.
- Any point of the object that is on the mirror line will reflect onto itself.
- If the line in which the object is to be reflected is placed over the object, part of the image will appear to the left of the mirror line and part to the right of it.

## Rotations and combined transformations

- A rotation is the process of turning an object around a fixed point without changing its shape or size.
- The fixed point about which the shape is turned is called the centre of rotation.
- To specify the rotation about the given point, we need to state the direction (clockwise or anticlockwise) of rotation and the angle through which the object is to be turned.
- Figures that have the same shape and size are said to be congruent.
- The image formed by rotation is congruent ( $\cong$ ) to the original object.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary above, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter.

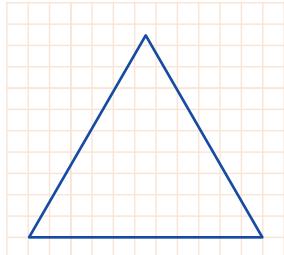
Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

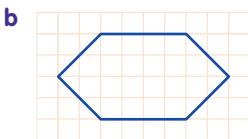
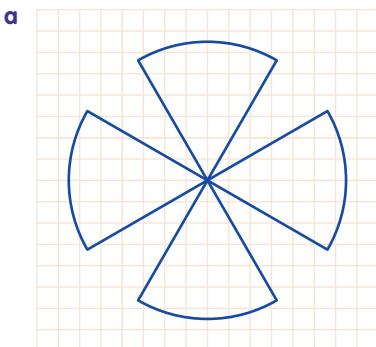
## FLUENCY

- 1 MC How many axes of symmetry does this shape have?



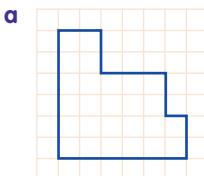
- A 0
- B 1
- C 2
- D 3
- E 4

- 2 Copy each of the shapes below and draw in its axis of symmetry.



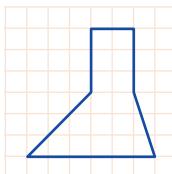
- 3 For the figures in Question 2, state the degree of rotational symmetry.

- 4 Show the following translations by copying the shapes and producing an image on a grid.



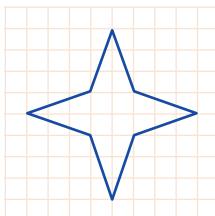
7R 2U

b



2L 3D

c



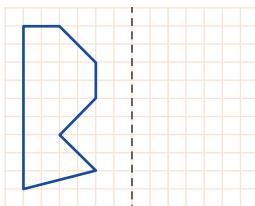
5R 2D

- 5 State the final position after each set of translations given below.

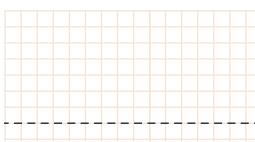
- a 3U 2R, 4D 6R, 2U 3L
- b 2D 3L, 4U 5R, 2D 6L, 3D 2R
- c 3L 2U, 9R 5D, 2L 3D, 4R 7U
- d 12L 3U, 4R 2D, 6L 5U, 2R 2D

- 6 For each of the following shapes, find the reflected image in the mirror line shown.

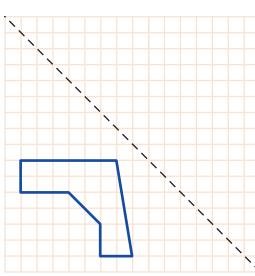
a



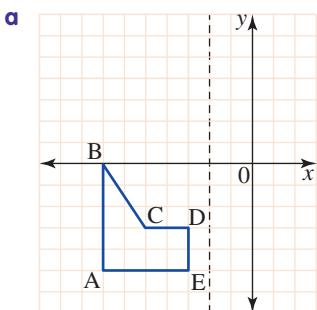
b



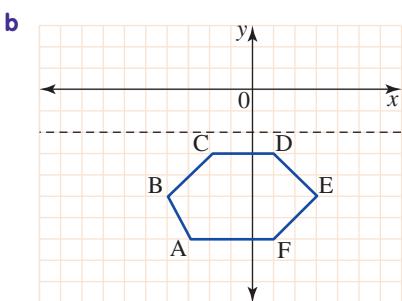
c



- 7 i** Find the image after the sets of transformations shown in these objects.

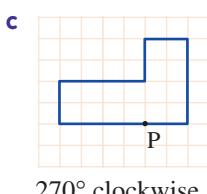
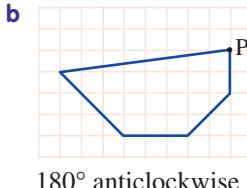
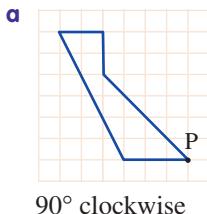


Translated 3L 4U; then reflected in the mirror

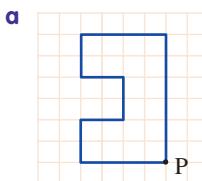


Reflected in the mirror; then translated 3U 5L

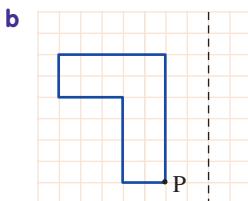
- ii** Give the coordinates of the vertices of the object and its image.  
**iii** Discuss the relationship between these coordinates in light of the transformation that took place.
- 8** For each of the following shapes, show the image after the rotation about point P as specified below the figures.



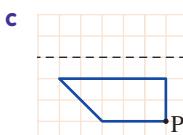
- 9** Show the final image after each of the following shapes have gone through these sets of transformations.



Translated 3U 6L; then rotated 90° anticlockwise around the image of P.



Reflected in the mirror; then rotated 90° clockwise around the image of P.



Translated 2D 4R, rotated 180° clockwise around the image of P and then reflected.

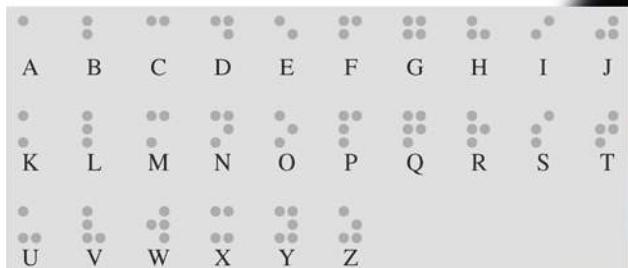
### PROBLEM SOLVING

- 1** This array of 9 nines contains 8 straight lines, each totalling 27 (3 across, 3 down, 2 diagonal). Show how to move 4 nines to new positions to make 10 straight lines totalling 27.

$$\begin{array}{ccc} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{array}$$

- 2** Braille is a code of raised dots that can be read by touch. It was developed by a 15-year-old blind French student named Louis Braille. The Braille alphabet is based on a cell three dots high and two dots wide.

- a** Compare the first 10 letters of the Braille alphabet with the second 10 letters. What patterns do you see?  
**b** Which Braille letters are reflections of each other?  
**c** Which Braille letters are rotations of each other?

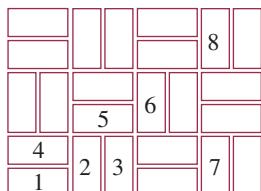


- d** There are no Braille letters that are translations of each other. Can you explain why?
- e** A word like MUM has reflection symmetry. Find two words that have reflection symmetry when written in Braille.
- f** A word like SWIMS has rotational symmetry. Find two words that have rotational symmetry when written in Braille.
- 3** Many paths and entertaining areas are paved with bricks of the same size and shape laid in an interesting design.

MUM  
SWIMS



One such pattern is shown in the picture (above) and in the diagram (below).



To design this pattern, the basic shape of the brick undergoes one or more transformations to different positions.

- a** What transformation has occurred to brick 1 to obtain the position of brick 2?
  - b** What transformation has occurred to brick 2 to obtain the position of brick 3?
  - c** Describe how the position of brick 3 can be obtained in relation to brick 1.
  - d** For each of the other numbered bricks in the pattern, describe what transformations to brick 1 are needed to obtain the position of that brick.
- 4** Design an image on grid paper. Perform three successive transformations on this image.

eBook plus

 Weblink  
Brain box

eBook plus

<b>Interactivities</b>
Transformations in the plane
int-2353
Test yourself
Chapter 16
int-1827
Word search
Chapter 16
int-2611
Crossword
Chapter 16
int-2612

**Chapter Opener****Digital docs**

- Hungry brain activity Chapter 16 (doc-6595) (*page 527*)

**Are you ready?****Digital docs** (*page 528*)

- SkillsHEET 16.1 (doc-6596) Translation of a point
- SkillsHEET 16.2 (doc-6597) Reflection of a point
- SkillsHEET 16.3 (doc-6598) Reflection of an object
- SkillsHEET 16.4 (doc-6599) Rotation of a point
- SkillsHEET 16.5 (doc-6600) Rotation of an object through  $180^\circ$

**16A Axes of symmetry****Digital docs** (*page 530*)

- Activity 16-A-1 (doc-2035) Symmetry
- Activity 16-A-2 (doc-2036) More symmetry
- Activity 16-A-3 (doc-2037) Advanced symmetry

**eLesson**

- Transformations (eles-0012) (*page 529*)

**Weblink**

- All about symmetry (*page 533*)

**16B Translations****Digital docs**

- Activity 16-B-1 (doc-2026) Translations (*page 536*)
- Activity 16-B-2 (doc-2027) More translations (*page 536*)
- Activity 16-B-3 (doc-2028) Advanced translations (*page 536*)
- WorkSHEET 16.1 (doc-2024) (*page 538*)

**16C Reflections****Digital docs**

- Activity 16-C-1 (doc-2029) Reflections (*page 541*)
- Activity 16-C-2 (doc-2030) More reflections (*page 541*)
- Activity 16-C-3 (doc-2031) Advanced reflections (*page 542*)

**16D Rotations and combined transformations****Digital docs**

- Activity 16-D-1 (doc-2032) Rotations (*page 548*)
- Activity 16-D-2 (doc-2033) More rotations (*page 548*)
- Activity 16-D-3 (doc-2034) Advanced rotations (*page 548*)
- WorkSHEET 16.2 (doc-2025) (*page 552*)
- Investigation Designing a patchwork quilt (doc-3444) (*page 552*)

**Interactivity**

- Multiple transformations (int-2745) (*page 544*)

**Chapter review****Weblink**

- Brain box (*page 557*)

**Interactivities** (*page 557*)

- Transformations in the plane (int-2353)
- Test yourself Chapter 16 (int-1827) Take the end-of-chapter test to test your progress.
- Word search Chapter 16 (int-2611)
- Crossword chapter 16 (int-2612)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

17

# Views of 3-D shapes

- ## **17A** Scales and house plans **17B** Plans and views **17C** Isometric drawing

## WHAT DO YOU KNOW?

- 1 List what you know about reading scales. Create a concept map to show your list.
  - 2 Share what you know with a partner and then with a small group.
  - 3 As a class, create a large concept map that shows your class's knowledge of scales.

eBook *plus*

Digital doc

Hungry brain activity  
Chapter 17  
doc-6545



### OPENING QUESTION

How do builders know the exact size to build a house simply by looking at a drawing of the plans?

# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.

## Metric units of length

- 1 a** For each metric unit of length, match the name with the correct abbreviation.

- |               |    |
|---------------|----|
| i centimetre  | m  |
| ii kilometre  | mm |
| iii metre     | cm |
| iv millimetre | km |

- b** Give an example to describe the length of each unit in part **a**.

## Reading scales (how much is each interval worth?)

- 2** For the scales shown, how much is each interval worth?



## Measuring the length of a line

- 3 a** Measure the length of this line to the nearest mm. \_\_\_\_\_

- b** Measure the length of this line in cm. \_\_\_\_\_

## Relationship between unit size and the number of units used

- 4** The distance from your home to school is measured in both metres and kilometres. Which unit would you have more of for this measurement?

## Converting units (metres and kilometres)

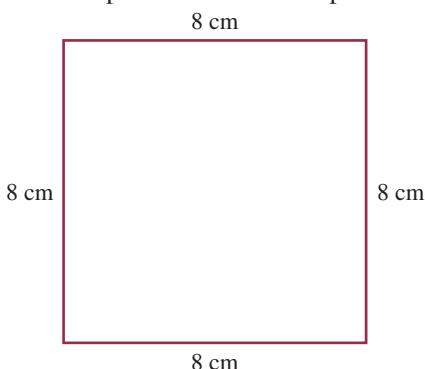
- 5 a** Convert 18 km to m. \_\_\_\_\_

- b** Convert 6000 m to km. \_\_\_\_\_

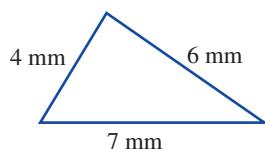
## Perimeter

- 6** Calculate the perimeter of each shape shown below. (Note: The figures are not drawn to scale.)

**a**



**b**



# 17A Scales and house plans

## Understanding scales

- A scale on a map or plan describes the ratio between the distance on the map or plan and the actual distance on the Earth's surface.
- A scale may be written several ways.
  - A ratio such as  $1 : 100\,000$  means that 1 cm on a map or plan represents a distance of 100 000 cm on the Earth, or 1 mm on the map or plan represents 100 000 mm on the Earth, etc.
  - A statement such as  $1 \text{ cm} \Leftrightarrow 500 \text{ km}$  means that 1 cm on the map or plan represents a distance of 500 km on the Earth.
  - A graphical bar scale, such as the one shown in the diagram, can be used. In this case, we need to measure the length of the bar to determine the scale ratio.

### WORKED EXAMPLE 1

- The scale on a house plan is  $1 : 2000$ . How many metres does a length of 1 cm on the map represent?
- Write the scale  $1 \text{ cm} \Leftrightarrow 5000 \text{ m}$  as a ratio scale.
- A map shows a scale of  $1 \text{ cm} \Leftrightarrow 1500 \text{ m}$ . What distance is represented by 18 mm on the map?
- Use the graphical scale shown below to determine the distance that 1 cm on the map represents.



#### THINK

- 1 Use the ratio to add the same units to both sides. Since the answer asks for cm, use this unit.
  - 2 Bring the length to an appropriate unit. In this case, the answer is required in metres.
  - 3 Write the answer.
- 1 Convert the ground units to the same as those of the map.
  - 2 Write the scale as a ratio.
- 1 Write a statement explaining this scale.
  - 2 Change the question distance to the same units as those for the map.
  - 3 Multiply by this scale factor.

#### WRITE

- $1 : 2000$  is the same as:  
 $1 \text{ cm} \Leftrightarrow 2000 \text{ cm}$   
 $2000 \text{ cm} = (2000 \div 100) \text{ m}$   
 $= 20 \text{ m}$   
So,  $1 \text{ cm} \Leftrightarrow 20 \text{ m}$ .  
1 cm on the map represents a length of 20 m in reality.
- $5000 \text{ m} = (5000 \times 100) \text{ cm}$   
 $= 500\,000 \text{ cm}$   
So,  $1 \text{ cm} \Leftrightarrow 5000 \text{ m}$  is the same as  
 $1 \text{ cm} \Leftrightarrow 500\,000 \text{ cm}$ .  
Expressed as a ratio, this is  $1 : 500\,000$ .
- $1 \text{ cm} \Leftrightarrow 1500 \text{ m}$  means that 1 cm on the map represents a distance of 1500 m.  
 $18 \text{ mm} = (18 \div 10) \text{ cm}$   
 $= 1.8 \text{ cm}$   
1.8 cm represents  $(1.8 \times 1500)$  m, which is 2700 m or 2.7 km.

**d**

- 1 Measure the length of the bars with a ruler.
- 2 Relate this measurement to the length shown in the scale.
- 3 Write as a scale.
- 4 Calculate a 1-cm map distance and write the answer.

**d** The bars measure 3.5 cm.  
This represents a length of 15 km.  
 $3.5 \text{ cm} \Leftrightarrow 15 \text{ km}$   
So,  $1 \text{ cm} \Leftrightarrow \frac{15}{35} \text{ km}$   
1 cm on the map represents a distance of 4.3 km.

- Sometimes it is necessary to do the reverse, and convert from a real length to a plan measurement.

### WORKED EXAMPLE 2

The length of a wall for a particular house is 12 m. If the scale of a house plan is 1 : 200, what length, on the plan, would represent the actual length of the wall.

#### THINK

- 1 Write the scale as a ratio using units of cm
- 2 Convert the actual length from cm to m units.
- 3 Divide the actual length by 2 to find how many lots of 2 m there are in 12 m.
- 4 Multiply both sides of the scale by this scale factor then simplify
- 5 Answer the question.

#### WRITE

1 : 200 means that 1 cm on the plan represents an actual length of 200 cm.  
 $1 \text{ cm} \Leftrightarrow 200 \text{ cm}$

$$\begin{aligned} 200 \text{ cm} &= (200 \div 100) \text{ m} \\ &= 2 \text{ m} \end{aligned}$$

So 1 cm on the plan represents an actual length of 2 m.

$$1 \text{ cm} \Leftrightarrow 2 \text{ m}$$

$$\frac{12 \text{ m}}{2 \text{ m}} = 6$$

$$\begin{aligned} 1 \text{ cm} \times 6 &\Leftrightarrow 2 \text{ m} \times 6 \\ 6 \text{ cm} &\Leftrightarrow 12 \text{ m} \end{aligned}$$

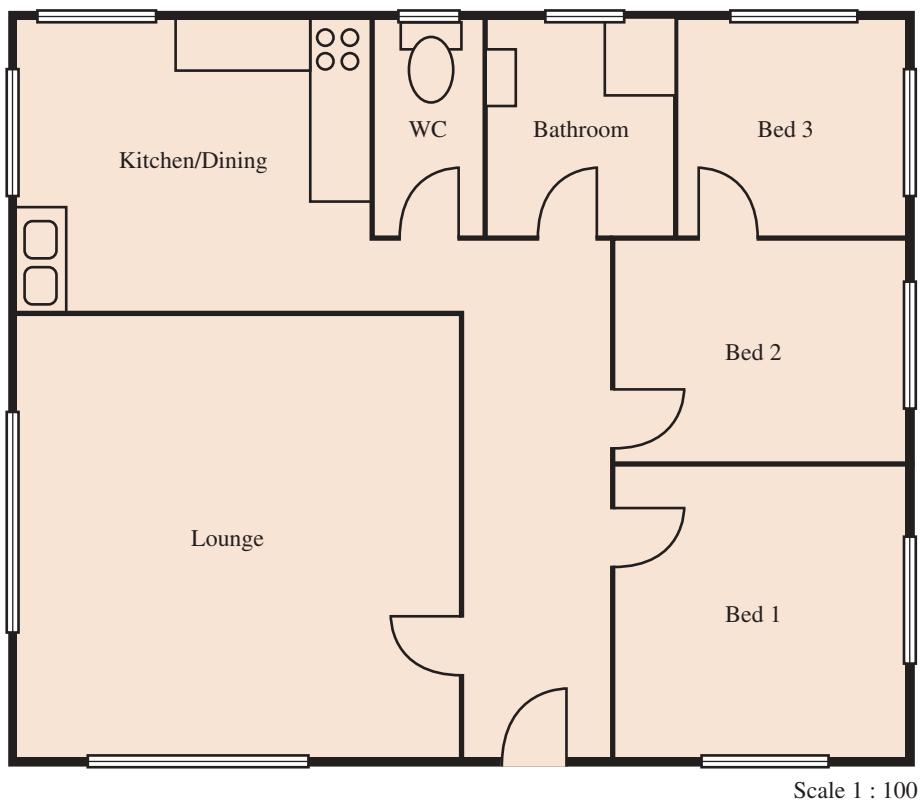
The length of the wall on the house plan would be 6 cm.

## House plans

- Whenever a house is being constructed, floor plans are supplied to the builder so that the footings, walls, roof, etc. are all placed in the correct positions and are the correct length. The scale of the plan is generally given as a ratio. This enables us to calculate all dimensions within the house.

## WORKED EXAMPLE 3

Below is a plan for a house.



- a** Calculate the dimensions of the house.
- b** Calculate the area of the lounge room.

## THINK

- a**
  - 1** Measure the length and width of the house on the plan.
  - 2** Multiply each of these measurements by 100.
  - 3** Write your answer.

## WRITE

- a**

Length of house on plan = 12 cm  
Width of house on plan = 10 cm

Actual length of house =  $12 \text{ cm} \times 100$   
= 1200 cm  
= 12 m

Actual width =  $10 \text{ cm} \times 100$   
= 1000 cm  
= 10 m

The dimensions of the house are 12 m by 10 m.

- b**
  - 1** Measure the length and width of the lounge room on the plan.
  - 2** Multiply each of these measurements by 100.

- b**

Length of lounge room on plan = 6 cm  
Width of lounge room on plan = 6 cm

Actual length of lounge room =  $6 \text{ cm} \times 100$   
= 600 cm  
= 6 m

Actual width of lounge room is also 6 m.



- 3 Calculate the area of the lounge room.

$$\begin{aligned}A &= 6^2 \\&= 36 \text{ m}^2\end{aligned}$$

- 4 Write your answer.

The area of the lounge room is 36 m<sup>2</sup>.

- In addition to a floor plan, a builder needs to know what the house will look like if it is viewed from the front, back and sides. These plans are called **elevations**. These are also drawn to scale.

#### WORKED EXAMPLE 4

The diagram at right shows the front elevation of a house.

- a Calculate the height of the eaves on the lower side of the house.  
b Measure the angle of the pitch of the roof.



Scale 1 : 100

#### THINK

- a 1 Measure the height on the plan for the lower side of the house.  
2 Multiply the plan measurement by 100.  
3 Write your answer.  
b 1 Measure the angle that the slope of the roof makes with the horizontal.  
2 Write your answer.

#### WRITE

- a Height on the plan = 3.5 cm  
Actual height = 3.5 cm × 100  
= 350 cm  
= 3.5 m  
The height of the eaves is 3.5 m.  
b Angle to horizontal = 45°  
The angle of the pitch of the roof is 45°.

- With an understanding of house plans, you should now be able to draw plans of objects familiar to you, such as your home, bedroom or classroom. It is important to include a scale with each plan. Sometimes a legend is also appropriate.

#### REMEMBER

- A scale may be written several ways:
  - as a ratio
  - as a statement
  - as a graphical bar scale.
- House plans are drawn with a ratio as the scale factor.
- Using the scale factor, we can calculate the dimensions for the house from the plans.
- An elevation is the view of a house from one side.
- From an elevation, we can measure all lengths and angles and use the scale to calculate the actual measurements.

## EXERCISE

## 17A

## Scales and house plans

## INDIVIDUAL PATHWAYS

eBook plus

## Activity 17-A-1

House plans  
doc-6552

## Activity 17-A-2

More house plans  
doc-6553

## Activity 17-A-3

Advanced house  
plans  
doc-6554

## FLUENCY

- 1 **WE1** Rewrite the following ratio scales in the form  $1\text{ cm} \leftrightarrow \underline{\hspace{1cm}}$  using the most appropriate units.

- a  $1 : 10$
- b  $1 : 100$
- c  $1 : 1000$
- d  $1 : 10000$
- e  $1 : 100000$
- f  $1 : 5000$
- g  $1 : 60000$
- h  $1 : 400$
- i  $1 : 750000$
- j  $1 : 2200000$

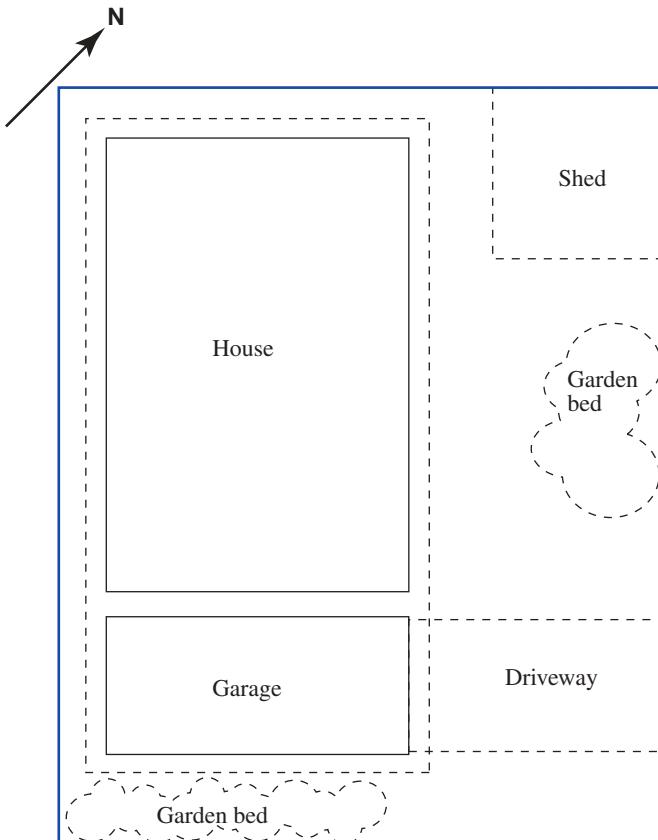
- 2 Write the following as ratio scales.

- a  $1\text{ cm} \leftrightarrow 20\text{ cm}$
- b  $1\text{ cm} \leftrightarrow 50000\text{ cm}$
- c  $1\text{ cm} \leftrightarrow 10\text{ m}$
- d  $1\text{ cm} \leftrightarrow 200\text{ m}$
- e  $1\text{ cm} \leftrightarrow 5\text{ km}$
- f  $1\text{ cm} \leftrightarrow 50\text{ km}$
- g  $1\text{ cm} \leftrightarrow 5.5\text{ km}$
- h  $2\text{ cm} \leftrightarrow 2\text{ km}$
- i  $2\text{ cm} \leftrightarrow 2.5\text{ km}$
- j  $3\text{ cm} \leftrightarrow 6\text{ km}$

- 3 **WE2** The scale of a town plan is given as  $1:100000$ . Use this to find the distances on the plan between:

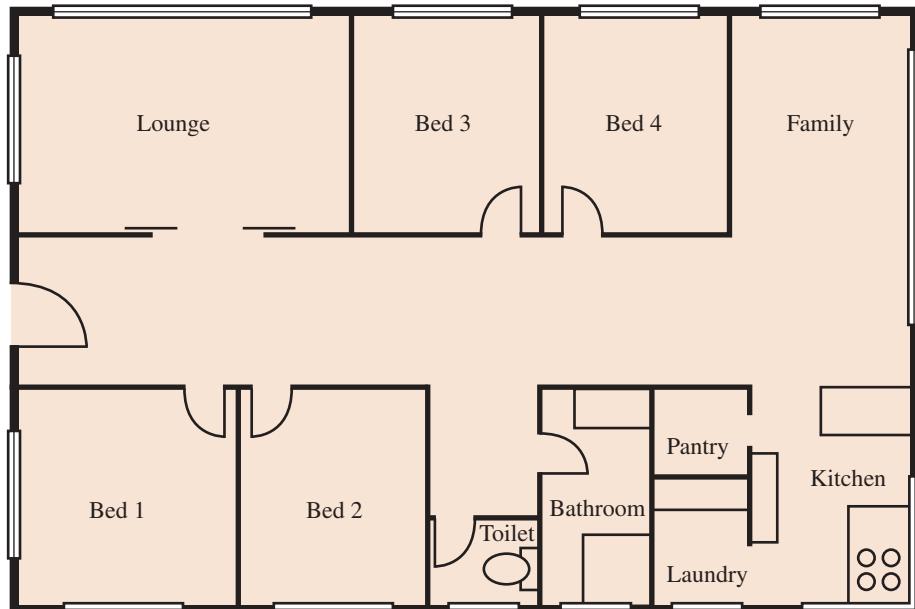
- a two intersections that are  $1\text{ km}$  apart
- b two department stores that are  $200\text{ m}$  apart
- c an airport and a motel that are  $15\text{ km}$  apart
- d a service station and a bank that are  $2.5\text{ km}$  apart
- e the northern and southern extremities, which are  $22\text{ km}$  apart
- f two shops that are  $500\text{ m}$  apart.

- 4 **WE3** Below is the site plan for a block of land.

Scale  $1 : 250$ 

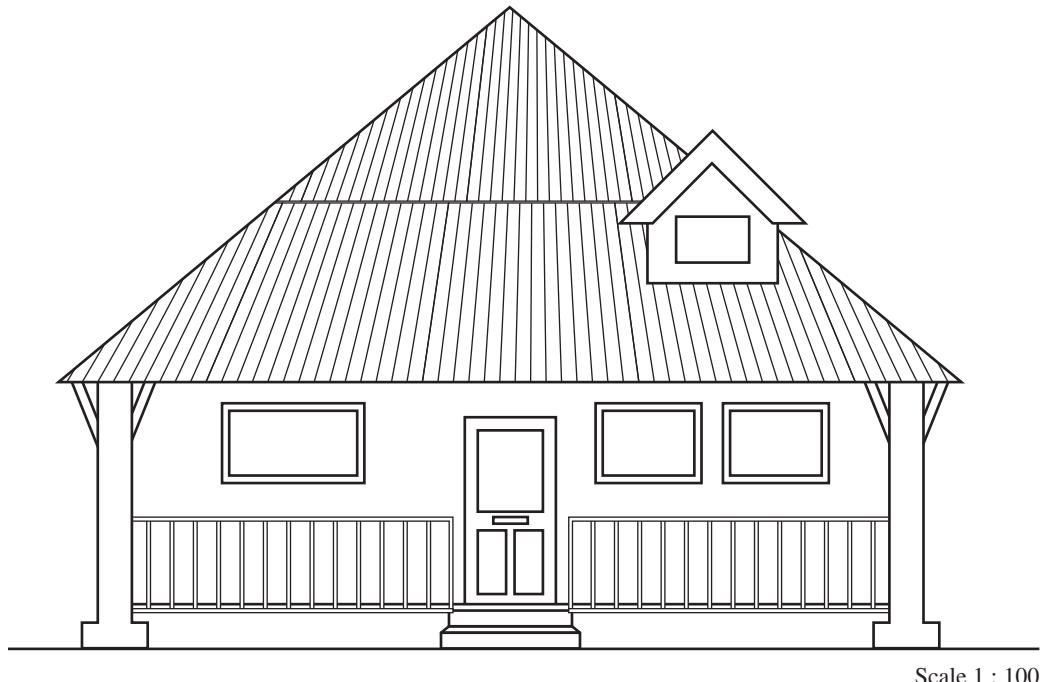
(continued)

- a What are the dimensions of the block of land?  
 b What are the dimensions of the house?  
 5 A house plan is shown below.



Scale 1 : 150

- a Calculate the dimensions of the house.  
 b What are the dimensions of the lounge room?  
 c Which bedroom is the largest? What are its dimensions?  
 6 **WE4** Below is the front elevation of a house, drawn to scale.

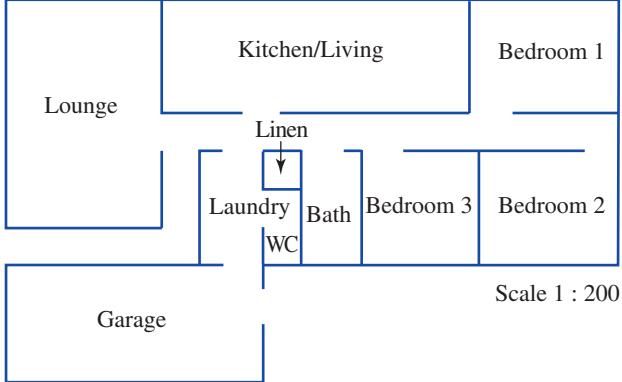


Scale 1 : 100

- a Calculate the height of the peak of the roof.
- b Calculate the height of the eaves.
- c Measure the angle of the pitch of the roof.

### UNDERSTANDING

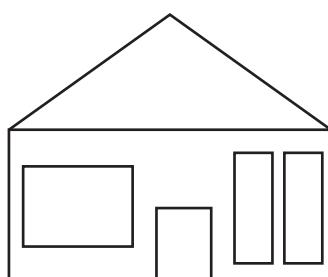
- 7 If a scale of 1 : 50 is used to make each of the following drawings:
- a what would be the width of the drawing of a house which is 25 m wide?
  - b what would be the height of the drawing of a man who is 2 m tall?
  - c what would be the dimensions of the drawing of a rectangular pool 50 m by 20 m?
  - d what would be the dimensions of the drawing of a table 4 m by 1.5 m?
  - e what is the actual height of a tree whose height in the drawing is 10 cm?
  - f what is the actual length of a truck whose length in the drawing is 25 cm?
  - g what are the dimensions of a television set whose dimensions on the drawing are 10 mm by 8 mm?
  - h what are the dimensions of a bed whose dimensions on the drawing are 4.5 cm long by 3 cm wide by 1.2 cm high?
- 8 Referring to the plan of the house shown below, find:



- a the dimensions of the lounge
- b the width of the hallway
- c the length of the garage
- d the floor area of the kitchen/living room
- e the total floor area of the largest bedroom.

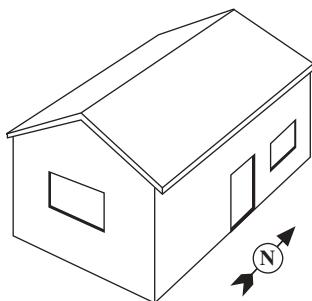
(Hint: Use a ruler to find the required lengths on the plan first.)

- 9 Trace the front elevation of the house at right into your book.  
On your diagram write all lengths and angles necessary for the construction of the house.

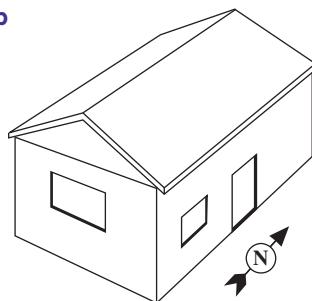


Scale 1 : 200

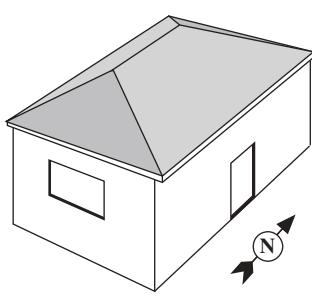
- 10** The following diagrams are representations of houses with a variety of roof types.  
Draw a plan of the south and east elevations of these houses. The direction of north is given.

**a**

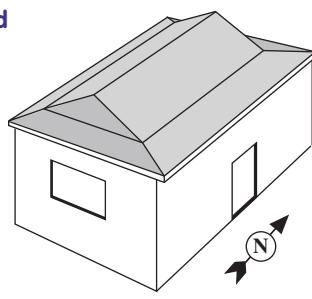
Gable roof

**b**

Boxed gable roof

**c**

Hip roof

**d**

Gambrel

(a roof combining the addition of small gables to a hip roof)

**eBookplus****Weblinks**  
**Maps and scales****Engineering drawings**

- 11** Choose an area which is familiar to you, such as your home, bedroom, classroom or school grounds. Draw a plan of your area. Include a scale and a legend. Don't forget to show the northerly direction.

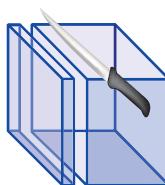
**REFLECTION**

Why is it important to show the position of north on a plan?

## 17B Plans and views

### Prisms

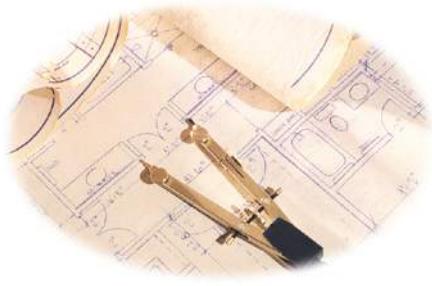
- A prism is a 3-dimensional figure that can be cut into parallel slices or cross-sections that are identical in size and shape.



- Objects or buildings are often constructed by combining prisms.

## Plans and views

- An object can be viewed from different angles.
- Architects and draftspersons often draw plans of building sites and various objects when viewed from the front, the side or the top.
- The **front view**, or **front elevation**, is what you see if you are standing directly in front of an object.
- The **side view**, or **side elevation**, is what you see if you are standing directly to one side of the object. You can draw the left view or the right view of an object.
- The **top view**, or **bird's eye view**, is what you see if you are hovering directly over the top of the object looking down on it.

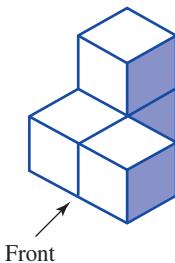


### WORKED EXAMPLE 5

The following object is made from 4 cubes.

Draw plans of it showing:

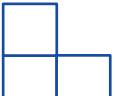
- the front view
- the right view
- the top view.



#### THINK

- Make this shape using cubes. Place the shape at a considerable distance and look at it from the front (this way you can see only the front face of each cube). Draw what you see. (Or simply imagine looking at the shape from the front and draw what you see.)
- Look at your model from the right, or imagine that you can see only the right face of each cube and draw what you see.
- Look at your model from the top, or imagine that you can see only the top face of each cube. Draw what you see.

#### DRAW

-  Front view
  -  Right view
  -  Top view
- ↑  
Front

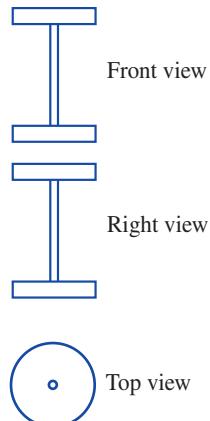
- Figures such as the one in Worked example 13 can be drawn using isometric dot paper. This will help to give the 3-dimensional perspective of the object.

**WORKED EXAMPLE 6****Draw:**

- the front view**
- the right view**
- the top view of this solid.**

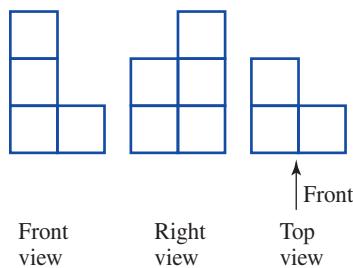
**THINK**

- Find an object of similar shape, or visualise the object in your head.
- Whether viewed from the front, or from the right of the object, the cylindrical shaft will appear as a long thin rectangle. The circular discs will also be seen as a pair of identical rectangles. So the front view and the right view are the same.
- When the object is viewed from above, all we can see is the flat surface of the top disc; that is, a large and a small circle with the same centre. (Note that such circles are called concentric.)

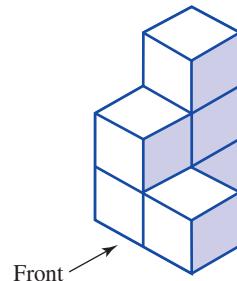
**DRAW****WORKED EXAMPLE 7**

The front, right and top views of a solid are shown.

Use cubes to construct the solid.

**THINK**

- Use cubes to construct the solid.
- Check carefully that your solid matches each of the 3 views you are given. Make adjustments if necessary.

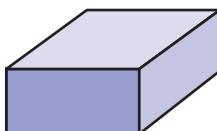
**CONSTRUCT**

## Perspective drawing

- A **2-dimensional representation** of a **3-dimensional object** can be made using a single-point perspective. This shows the object as it appears to the eye in reality. When we look at a picture of a scene, the objects seem to converge or move towards a point in the distance. This point is called a vanishing point.

### WORKED EXAMPLE 8

Construct a perspective drawing of the rectangular prism shown.



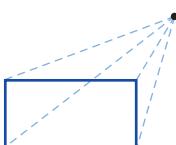
#### THINK

- Draw the front face of the prism as a rectangle.

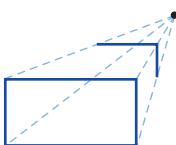
#### WRITE



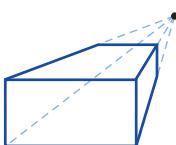
- Take a vanishing point in the distance behind the object and slightly to the right. Draw dotted lines from this point to the corners of the front face of the prism.



- On these dotted lines, make the following markings. Draw the top edge of the back face parallel to the top edge of the front face. Also draw the right edge of the back face parallel to the right edge of the front face.



- Join the top- and right-side vertices of the front face to the corresponding vertices of the back face.



#### REMEMBER

- The *front view*, or *front elevation*, is what you see if you are standing directly in front of an object.
- The *side view*, or *side elevation*, is what you see if you are standing directly to one side of the object. You can draw the left view or the right view of an object.
- The *top view*, or *bird's eye view*, is what you see if you are hovering directly over the top of the object looking down on it.
- A 2-dimensional representation of a 3-dimensional object can be made using a single-point perspective.

## EXERCISE

## 17B

## Plans and views

## INDIVIDUAL PATHWAYS

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## Activity 17-B-1

Plans and views

doc-6555

## Activity 17-B-2

More plans and

views

doc-6556

## Activity 17-B-3

Advanced plans and

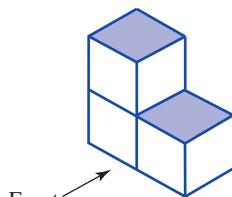
views

doc-6557

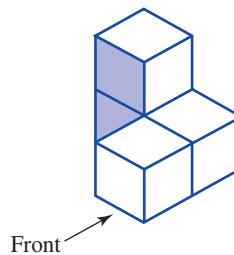
## FLUENCY

- 1 WES** The following objects are made from cubes. For every object draw the plans, showing the front view, the right view and the top view. (You may wish to use a set of cubes or building blocks to help you.)

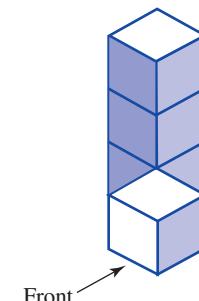
a



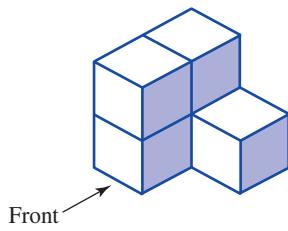
b



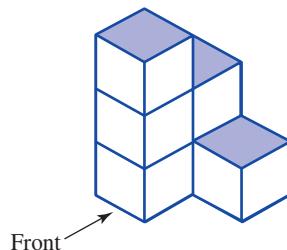
c



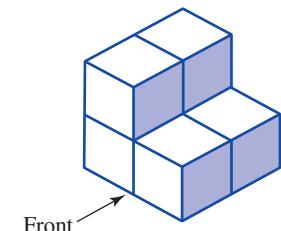
d



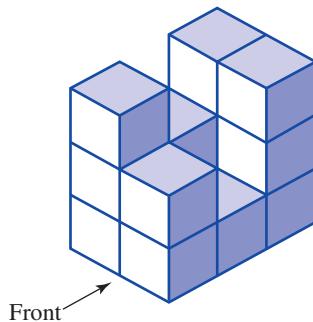
e



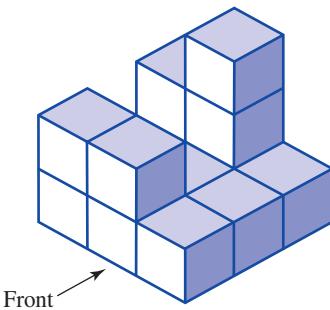
f



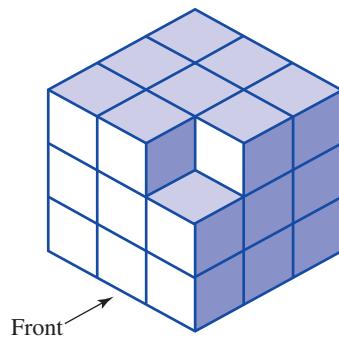
g



h



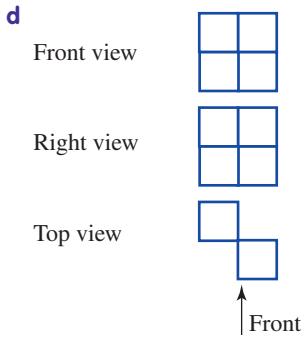
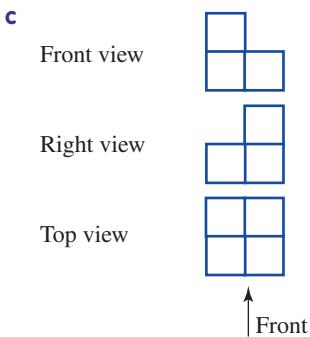
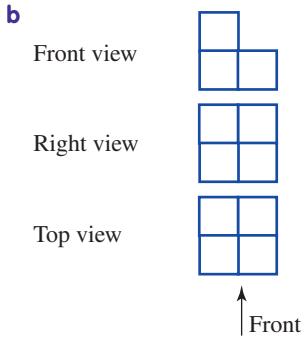
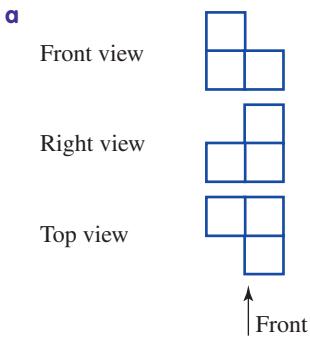
i



- 2 WE6** Draw the front, right and top views of each solid shown.



- 3 WE7** The front, right and top views of a solid are shown. In each case, use cubes to construct the solid.



- 4 WE8** Construct a perspective drawing of a cube.

## UNDERSTANDING

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**Interactivity**  
Plans,  
elevations and  
cross-sections  
int-0009

- 5 MC The front, right and top views of a solid are shown. Which of the given drawings could represent the solid?

Front view



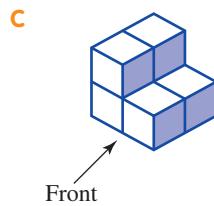
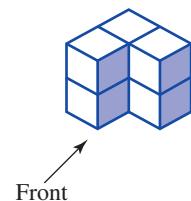
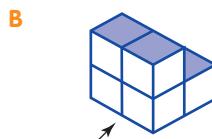
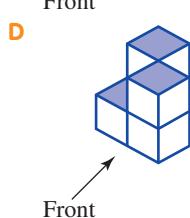
Right view



Top view



Front



- 6 a What shape is the top view of a telephone pole?  
 b What shape is the top view of the Melbourne Cricket Ground?  
 c What shape is the side view of a bucket?  
 d What shape is the top view of a car?  
 7 a Draw the side view of this pool table.  
 b Draw the front view of your house (seen from the street).  
 c Draw the side view of an electric kettle.  
 d Draw the top view of your television set.



- 8 A shape is made using only 4 cubes. Its front view, right view and top view are shown.

Front view



Right view



Top view



Front

- a Is it possible to construct this solid?  
 b Describe or draw what this solid would look like.

## REFLECTION

Why is the technique of perspective drawing frequently used in artistic paintings?

## 17C Isometric drawing

- When working with **3-dimensional** models and designs, it is often useful to have the design or model drawn on paper (that is, in 2 dimensions).
- An isometric drawing is a 2-dimensional drawing of a 3-dimensional object.
- This picture shows an architect's drawing of a beach hut and environs in isometric view superimposed on the actual hut. Architects and draftspersons often use **isometric drawings** to give their clients a clear picture of the proposed design.



### WORKED EXAMPLE 9

**First copy the incomplete figure at far right onto isometric dot paper. Complete the isometric drawing of the object shown at near right.**



#### THINK

Study the object and identify the lines that have already been drawn. Fill in the missing lines on your isometric drawing to match the object.

#### DRAW



### WORKED EXAMPLE 10

**Draw the following object on isometric dot paper. (You could construct it first from a set of cubes.)**



#### THINK

- Use cubes to make the object shown (optional). Draw the front face of the object. The vertical edges of the 3-dimensional object are shown with vertical lines on the isometric drawing; the horizontal edges are shown with the lines at an angle (by following the dots on the grid paper).

#### DRAW





- 2 Draw the left face of the object.



- 3 Add the top face to complete the isometric drawing of the object.



#### REMEMBER

1. An isometric drawing is a 2-dimensional drawing of a 3-dimensional object.
2. If possible, construct the solid from the set of cubes prior to drawing its isometric view.
3. Draw the front face first.
4. In isometric drawings, vertical edges of a 3-dimensional object are shown with vertical lines, while horizontal edges are shown with the lines drawn at an angle.

#### EXERCISE

## 17C Isometric drawing

### INDIVIDUAL PATHWAYS

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**Activity 17-C-1**  
Isometric drawing  
doc-6558

**Activity 17-C-2**  
More isometric drawing  
doc-6559

**Activity 17-C-3**  
Advanced isometric drawing  
doc-6560

### FLUENCY

- 1 **WE9** Copy the following figures onto isometric dot paper and complete the isometric drawing of the objects shown.

a



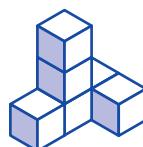
b



c



d



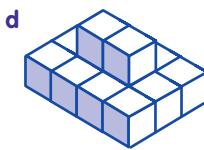
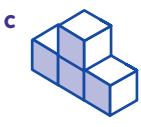
- 2 **WE10** Draw each of the following objects on isometric dot paper. (You might wish to make them first from a set of cubes.)

a



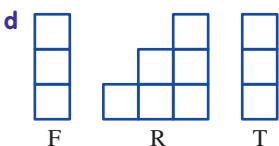
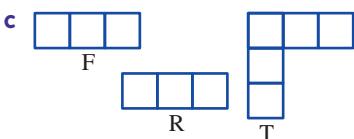
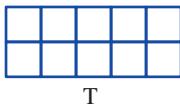
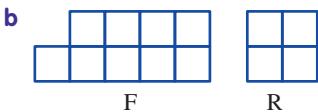
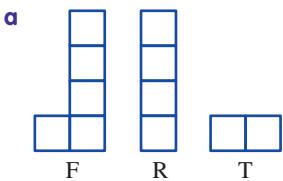
b



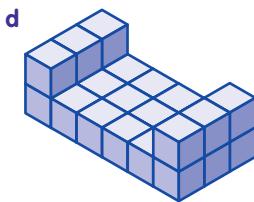
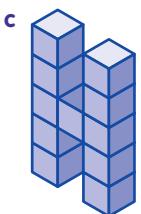
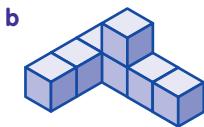
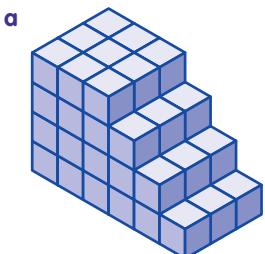


### UNDERSTANDING

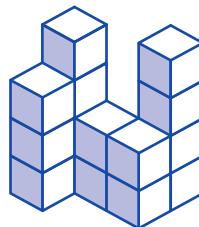
- 3 Construct the following letters using cubes, and then draw the solids on isometric dot paper:
- the letter T with 5 cubes
  - the letter L with 7 cubes
  - the letter E with 10 cubes
  - the letter H with 7 cubes.
- 4 Draw these objects, whose front (F), right (R) and top (T) views are given, on isometric dot paper.



- 5 Draw the front, right and top views of these objects.



- 6** Draw the following figure on isometric dot paper.



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**Digital doc**  
WorkSHEET 17.1  
doc-6561

- 7** Draw a selection of buildings from this photograph of the Melbourne skyline on isometric dot paper.



**REFLECTION**

How does isometric dot paper turn two-dimensional lines into a three-dimensional image?

# Summary

## Scales and house plans

- A scale may be written several ways:
  - as a ratio
  - as a statement
  - as a graphical bar scale.
- House plans are drawn with a ratio as the scale factor.
- Using the scale factor, we can calculate the dimensions for the house from the plans.
- An elevation is the view of a house from one side.
- From an elevation, we can measure all lengths and angles and use the scale to calculate the actual measurements.

## Plans and views

- The *front view*, or *front elevation*, is what you see if you are standing directly in front of an object.
- The *side view*, or *side elevation*, is what you see if you are standing directly to one side of the object. You can draw the left view or the right view of an object.
- The *top view*, or *bird's eye view*, is what you see if you are hovering directly over the top of the object looking down on it.
- A 2-dimensional representation of a 3-dimensional object can be made using a single-point perspective.

## Isometric drawing

- An isometric drawing is a 2-dimensional drawing of a 3-dimensional object.
- If possible, construct the solid from the set of cubes prior to drawing its isometric view.
- Draw the front face first.
- In isometric drawings, vertical edges of a 3-dimensional object are shown with vertical lines, while horizontal edges are shown with the lines drawn at an angle.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare your concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



Homework  
Book

# Chapter review

## FLUENCY

- The scale on a map is given as 1:100000. Find the actual distances between:
  - two schools that are 5 cm apart on the map
  - two parks that are 2.5 cm apart on the map
  - two farms that are 4 cm apart on the map.
- A scale of 1 : 200 is used to make models of a tree and a car.
  - What would be the length of the model car if the car is actually 2.5 metres long?
  - What would be the height of the model tree if the tree is actually 6.5 metres tall?

## PROBLEM SOLVING

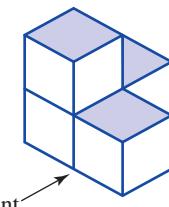
- On a town plan, the scale is given as 1:200000.
  - Find the actual distances between two schools that are 4.5 cm apart on the plan.
  - Find the distance on the plan between two parks that are 2.5 km apart.
- The scale diagram below is a site plan for a block of land on which a house is to be built. Use this diagram to answer the questions on the next page.
 

Scale 1 cm = 4 m

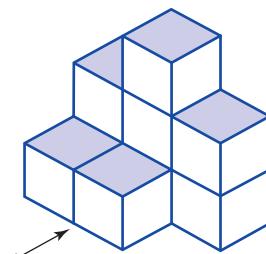
- Calculate the dimensions of the block of land.
- Calculate the dimensions of the house.

- Calculate the area of the house in square metres.
- In which direction would you be travelling as you drive into the garage?
- Draw the front, side and top views of each of these solids.

a

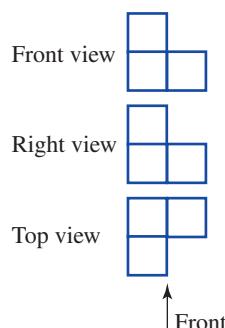


b

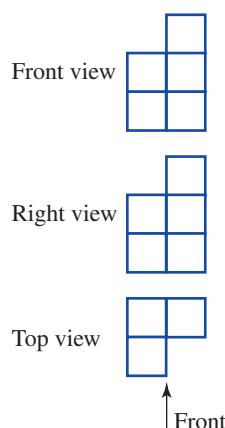


- The front, side and top view of a solid are shown. Construct this solid, using blocks.

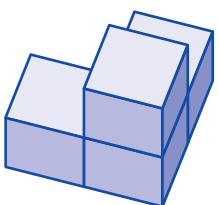
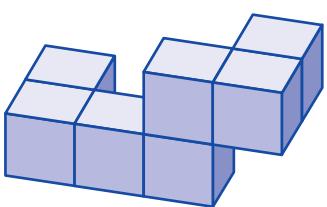
a



b



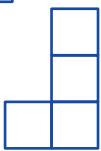
- 5 Draw the front, right and top views of these objects.

**a****b**

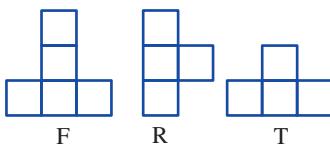
- 6 Draw isometric views of the objects, whose front, right and top views are given below.

**a**

R



F

**b**

- 7 A rectangular prism, constructed from the set of cubes, is 3 cubes long, 2 cubes wide and 4 cubes high. Draw an isometric view of the prism.

**eBook plus****Interactivities**

Test yourself

Chapter 17

int-2741

Word search

Chapter 17

int-2742

Crossword

Chapter 17

int-2743

**Chapter opener****Digital docs (page 559)**

- Hungry brain activity Chapter 17 (doc-6545)

**Are you ready?****Digital docs (page 560)**

- SkillsHEET 17.1 (doc-6546) Metric units of length
- SkillsHEET 17.2 (doc-6547) Reading scales (how much is each interval worth?)
- SkillsHEET 17.3 (doc-6548) Measuring the length of a line
- SkillsHEET 17.4 (doc-6549) Relationship between unit size and the number of units used
- SkillsHEET 17.5 (doc-6550) Converting units (metres and kilometres)
- SkillsHEET 17.6 (doc-6551) Perimeter

**17A Scales and house plans****Digital docs (page 565)**

- Activity 17-A-1 (doc-6552) House plans
- Activity 17-A-2 (doc-6553) More house plans
- Activity 17-A-3 (doc-6554) Advanced house plans

**Weblinks (page 568)**

- Maps and scales
- Engineering drawings

**17B Plans and views****Digital docs (page 572)**

- Activity 17-B-1 (doc-6555) Plans and views
- Activity 17-B-2 (doc-6556) More plans and views
- Activity 17-B-3 (doc-6557) Advanced plans and views

**Interactivity (page 574)**

- Plans, elevations and cross-sections (int-0009)

**17C Isometric drawing****Digital docs (page 576)**

- Activity 17-C-1 (doc-6558) Isometric drawing
- Activity 17-C-2 (doc-6559) More isometric drawing
- Activity 17-C-3 (doc-6560) Advanced isometric drawing
- WorkSHEET 17.1 (doc-6561) (page 578)

**Chapter review****Interactivities (page 581)**

- Test yourself Chapter 17 (int-2741) Take the end-of-chapter test to test your progress.
- Word search Chapter 17 (int-2742)
- Crossword Chapter 17 (int-2743)

To access eBookPLUS activities, log on to

[www.jacplus.com.au](http://www.jacplus.com.au)

# 18

# Money



- 18A Money
- 18B Money and percentages
- 18C Unitary method

## WHAT DO YOU KNOW?

- 1 List what you know about money. Create a concept map to show your list.
- 2 Share what you know with a partner and then with a small group.
- 3 As a class, create a large concept map that shows your class's knowledge of money.

eBook plus

Digital doc

Hungry brain activity  
Chapter 18  
doc-6601

## OPENING QUESTION

Money makes the world go round.  
Discuss.

# Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET located on your eBookPLUS.



## Rounding money to the nearest 5 cents

- 1 Round the following amounts to the nearest 5 cents.

a \$23.48      b \$207.91

**Digital doc**

SkillsHEET 18.1

doc-6602



## Converting a percentage to a decimal fraction

- 2 Convert each of the following percentages to a decimal fraction.

a 34%      b 79%      c 4%  
d 67.2%      e 8.25%      f 17.5%

**Digital doc**

SkillsHEET 18.2

doc-6603



## Decreasing a quantity by a percentage

- 3 Decrease 100% by the following percentages.

a 15%      b 12.5%      c 90%      d 5.5%

**Digital doc**

SkillsHEET 18.3

doc-6604



## Finding a percentage of a quantity (money)

- 4 Find each of the following.

a 10% of \$350      b 25% of \$1424      c 18% of \$9000      d 12.5% of \$4570

**Digital doc**

SkillsHEET 18.4

doc-6605



## Expressing one quantity as a percentage of another

- 5 For each of the following pairs, express the first quantity as a percentage of the second quantity.

a \$56, \$400      b \$13, \$20      c \$125, \$625

**Digital doc**

SkillsHEET 18.5

doc-6606



## Increasing a quantity by a percentage

- 6 Increase 100% by the following percentages.

a 25%      b 5%      c 100%      d 12.5%

**Digital doc**

SkillsHEET 18.6

doc-6607

## 18A Money

- In most modern societies, money is used as a medium of exchange for goods and services.

### Operations with money

- Australian currency is a decimal system using dollars and cents. There are 100 cents in a dollar.
- To change dollars to cents, you multiply by 100. Multiply, because there will be more cents than dollars.
- To change cents to dollars, you divide by 100. Divide, because there will be fewer dollars than there are cents.
- The Australian government withdrew one-cent and two-cent coins in 1990. Prices are still expressed in one-cent increments. When payment is made by cheque, credit card or electronic means, the exact amount is paid in dollars and cents.
- When cash is being used, amounts are rounded to the nearest 5 cents. For example, 61 cents and 62 cents would be rounded down to 60 cents while 63 cents and 64 cents would be rounded up to 65 cents. Similarly, 66 cents and 67 cents would be rounded down to 65 cents while 68 cents and 69 cents would be rounded up to 70 cents.

#### WORKED EXAMPLE 1

- a Change \$2.45 into cents.  
b Write 20c in dollars.



#### THINK

- a 1 Write the given amount and change dollars to cents by multiplying by 100.  
2 Write the answer.  
b 1 Write the given amount and change to dollars by dividing by 100.  
2 Write the answer.

#### WRITE

$$\begin{aligned} \text{a } \$2.45 &= (2.45 \times 100) \text{ cents} \\ &= 245 \text{ cents} \\ \text{b } 20\text{c} &= (\$20 \div 100) \\ &= \$0.20 \end{aligned}$$

- To do our shopping, we must be able to use money, to estimate our total shopping bill and calculate the change that we can expect. Most importantly, we need to be able to budget. This means thinking about our available funds before we make a purchase.

#### WORKED EXAMPLE 2

A \$100 note is used to pay for the following:

- 2 packs of socks @ \$16.95 per packet  
3 packs of golf balls @ \$19.95 per packet.

Estimate the total bill and the change, then find the exact amount for each.



**THINK**

- 1 Round the cost of the socks to the nearest dollar.
- 2 Round the cost of the golf balls to the nearest dollar.
- 3 Find the total estimated cost of the socks and golf balls.
- 4 Find the change by subtracting the total cost of the purchases from the amount tendered.
- 5 Answer the question in a sentence.
- 6 Find the exact cost of the socks and the golf balls.
- 7 Find the change given.
- 8 Answer the question in a sentence.

**WRITE**

Socks: about \$17 per pack

Golf balls: about \$20 per pack

$$\begin{aligned}\text{Total estimated cost in dollars} \\ &= 2 \times 17 + 3 \times 20 \\ &= 94\end{aligned}$$

$$\begin{aligned}\text{Estimated change in dollars} \\ &= 100 - 94 \\ &= 6\end{aligned}$$

The estimated total cost is \$94 and the change from \$100 is \$6.

$$\begin{aligned}\text{Total cost} &= 2 \times 16.95 + 3 \times 19.95 \\ &= 33.90 + 59.85 \\ &= 93.75\end{aligned}$$

$$\begin{aligned}\text{Change} &= \$100 - \$93.75 \\ &= \$6.25\end{aligned}$$

The total cost of the socks and the golf balls is \$93.75 and the change from \$100 is \$6.25.

- Each pair of socks and each pack of golf balls costs 5 cents less than the estimated cost, so the total cost of the goods is  $(5 \times 5)$  cents or 25 cents less than the estimated cost.

**REMEMBER**

- 1 To change dollars to cents, multiply by 100.
- 2 To change cents to dollars, divide by 100.
- 3 To add, subtract, multiply or divide using money, use the same methods as are used with decimals.
- 4 Money is rounded to the nearest 5 cents when cash is used because there are no longer 1 or 2 cent coins.
- 5 Exact amounts are used for cheques, credit cards and electronic transactions.
- 6 When solving real-life applications using money, clearly state what is being done at each step of the solution.

**EXERCISE****18A Money****FLUENCY**

- 1 **WE1a** Change each of the following into cents.
 

<b>a</b> \$7.55	<b>b</b> \$3.05
<b>c</b> \$2.40	<b>d</b> \$9
- 2 **WE1b** Change each of the following to dollars.
 

<b>a</b> 685 cents	<b>b</b> 450 cents
<b>c</b> 5 cents	<b>d</b> 805 cents
- 3 Round the following to the nearest 5 cents.
 

<b>a</b> \$4.76	<b>b</b> \$12.61
<b>c</b> \$159.28	<b>d</b> \$83.44

## INDIVIDUAL PATHWAYS

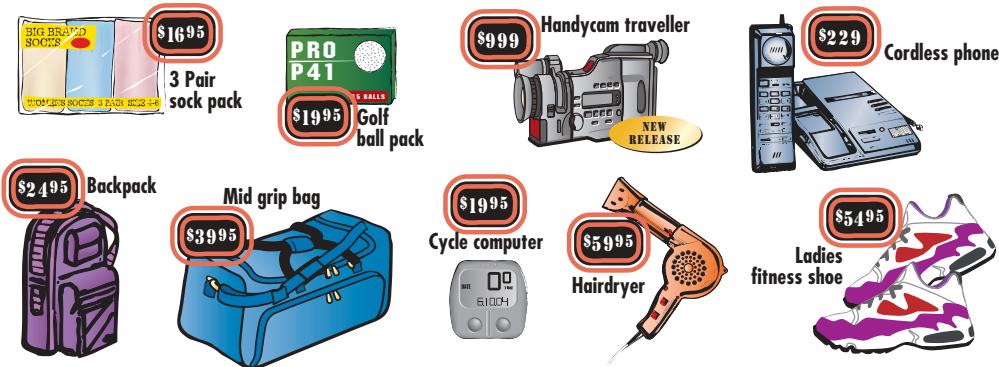
## eBookplus

**Activity 18-A-1**Money  
doc-6608**Activity 18-A-2**More money  
doc-6609**Activity 18-A-3**Advanced  
calculations  
with money  
doc-6610

## UNDERSTANDING

- 4** The students of Year 9 have raised \$245.40 for charity and want to share the money equally among the Children's Hospital, the Guide Dog Appeal and the Red Cross Shield Appeal. How much money will be given to each charity?
- 5** The cost of 5 packets of chocolate biscuits and a 2-litre bottle of milk was \$15.60. If the bottle of milk cost \$4.35, how much did a packet of chocolate biscuits cost?

Questions **6–12** refer to these advertised products.



- 6** How much change would you receive from \$50 if you purchased:
- the socks?
  - the cycle computer?
  - the backpack?
- 7** Rani can save \$15 per week. How many weeks will it take for her to save enough to buy the fitness shoes?
- 8** John has just selected a new set of golf clubs worth \$489.95. He then buys two golf ball packs. What is his total outlay?
- 9** Janine has just started full-time work. She pays a \$100 deposit on the Handycam and the balance in 10 equal monthly instalments. How much is each instalment, assuming she pays no interest charges?
- 10** Nicole needs a new hairdryer. Her dad lends her \$60 to buy the one shown. She agrees to work for him in order to pay him back. They agree on \$8 per hour. For how long will she have to work to clear the debt?
- 11** Veda buys the Panasonic cordless phone with a \$50 deposit and 6 payments of \$35 each. How much more than the advertised price does she finally pay? Why would this be?
- 12** For her birthday, Cristi was given the advertised fitness shoes, the backpack, the socks and the hairdryer by her family. If four people split the expense evenly, how much did each person pay?

For questions **13** and **14**, estimate, then find, the exact value for:

- the total bill
- the change.

- 13** **WE2** A \$20 note is used to pay the supermarket bill after purchasing these goods:

- 3 packets of soup @ \$1.89 per packet
- 2 packs of pasta @ \$1.24 per packet
- 2 boxes of cereal @ \$2.95 per box.

- 14** \$30 is used to pay the fruit shop for:

- 3 kg bananas @ \$1.99 per kg
- 4 kg apples @ \$2.95 per kg
- 2 kg oranges @ \$3.25 per kg
- $\frac{1}{2}$  kg mushrooms @ \$3.90 per kg.

- 15** A Petrol Rewards card gives a discount of 4 cents per litre. If I purchase 37.62 litres of petrol at a price of \$1.379 per litre then:
- How much is the cost of the fuel before the discount?
  - Using the Petrol Rewards card, what is the discount?
  - How much change will I receive if I used \$60.00 cash to make the purchase?

**REASONING**

- eBookplus**
- Digital docs**  
Investigation  
**How much is one million dollars?**  
doc-2226  
Investigation  
**Movie**  
**Munchies dollars**  
doc-2227
- 16** Sam and Georgia were paid a total of \$167.00 for painting a fence. Sam worked for 12 hours and Georgia worked for 8 hours. How much should each person receive?
- 17** Olivia has 78 fifty-cent coins and 43 two-dollar coins. What percentage of her money is made up of two-dollar coins?
- 18** Fifty people, half of whom were females, were asked if they liked yoghurt. Twenty-seven of the people, including two-fifths of all the males, said they did like yoghurt. How many of the females did not like yoghurt?
- 19** Ingrid offered to pay her brothers \$2 for doing her share of the housework each day, but fined them \$5 if they forgot to do it. After 4 weeks, Ingrid discovered that she owed her brothers nothing. For how many days did Ingrid's brothers do her share of the housework?

**REFLECTION**

In what situations would estimation be useful?

## 18B Money and percentages



Signs like this are common when stores are having a sale. To find the percentage of an amount, for example 20% of \$50, follow the steps given below.

- Write the percentage as an amount out of 100.  
 $\frac{20}{100}$  of 50
  - Change the 'of' to a multiplication sign and put the second amount over 1.  
 $\frac{20}{100} \times \frac{50}{1}$
  - Simplify the fractions if possible.  
 $\frac{2}{1} \times \frac{5}{1}$
  - Multiply the numerators and the denominators.  
 $\frac{10}{1}$
  - If left with a fraction, divide the denominator into the numerator.  
\$10
- (A calculator may be helpful here.)

**WORKED EXAMPLE 3****Find 22% of \$40.****THINK**

- 1** Write the question.
- 2** Write the percentage as an amount out of 100, change ‘of’ to ‘×’ and write the second amount over 1.
- 3** Simplify the fractions.
- 4** Multiply numerators and denominators and write the answer as a mixed number or decimal if appropriate (2 decimal places for money).
- 5** Write the answer.

**WRITE**

22% of 40

$$= \frac{22}{100} \times \frac{40}{1}$$

$$= \frac{11}{5} \times \frac{4}{1}$$

$$= \frac{44}{5}$$

$$= 8\frac{4}{5} \text{ or } 8.8$$

22% of \$40 = \$8.80

**REMEMBER**

To find a percentage of an amount:

1. Write the percentage as a fraction out of 100.
2. Multiply this fraction by the amount expressed as a fraction.

**EXERCISE****18B Money and percentages****INDIVIDUAL PATHWAYS****eBookplus****Activity 18-B-1**Basic money and percentages  
doc-6611**Activity 18-B-2**Money and percentages  
doc-6612**Activity 18-B-3**Advanced money and percentages  
doc-6613**FLUENCY**

- 1** **WE3** Find the following percentages, rounding your answer to 2 decimal places, if appropriate.
- |                      |                         |                       |
|----------------------|-------------------------|-----------------------|
| <b>a</b> 10% of \$80 | <b>b</b> 20% of \$50    | <b>c</b> 50% of \$200 |
| <b>d</b> 8% of \$30  | <b>e</b> 12.5% of \$200 | <b>f</b> 25% of \$48  |
| <b>g</b> 6% of \$75  | <b>h</b> 48% of \$20    | <b>i</b> 35% of \$15  |
| <b>j</b> 45% of \$18 | <b>k</b> 5% of \$142    | <b>l</b> 55% of \$44  |
| <b>m</b> 7% of \$350 | <b>n</b> 95% of \$28    | <b>o</b> 12% of \$35  |
- 2** Find 10% of the following amounts. (Money answers must be rounded to 2 decimal places.)
- |                   |                  |                   |
|-------------------|------------------|-------------------|
| <b>a</b> \$32.00  | <b>b</b> \$12.50 | <b>c</b> \$75.00  |
| <b>d</b> \$167.00 | <b>e</b> \$1.45  | <b>f</b> \$15.98  |
| <b>g</b> \$21.09  | <b>h</b> \$77.77 | <b>i</b> \$164.20 |
- 3** Look at your answer to each of the problems in question **2**. Copy and complete the following sentence.
- To find 10% of an amount, move the position of the decimal point \_\_\_\_\_ place to the \_\_\_\_\_.
- 4** Calculate 10% of the following amounts by moving the position of the decimal point.
- |                   |                   |                    |
|-------------------|-------------------|--------------------|
| <b>a</b> \$23.00  | <b>b</b> \$57.00  | <b>c</b> \$130.00  |
| <b>d</b> \$64.00  | <b>e</b> \$81.50  | <b>f</b> \$14.80   |
| <b>g</b> \$149.60 | <b>h</b> \$345.90 | <b>i</b> \$1354.50 |
- 5** Calculate 5% of the following by finding 10% and halving your answer.
- |                  |                  |                   |
|------------------|------------------|-------------------|
| <b>a</b> \$12.00 | <b>b</b> \$36.00 | <b>c</b> \$68.00  |
| <b>d</b> \$24.00 | <b>e</b> \$90.00 | <b>f</b> \$120.00 |

**g** \$45.00  
**j** \$33.60

**h** \$27.40  
**k** \$508.20

**i** \$12.80  
**l** \$235.50

- 6** Calculate 1% of the following by finding 10% and dividing by 10.
- |                   |                    |                   |
|-------------------|--------------------|-------------------|
| <b>a</b> \$26.00  | <b>b</b> \$42.00   | <b>c</b> \$166.00 |
| <b>d</b> \$406.00 | <b>e</b> \$1620.00 | <b>f</b> \$14.25  |
| <b>g</b> \$7.00   | <b>h</b> \$5.50    | <b>i</b> \$16.80  |

- 7** Copy and complete the following sentence.

To find 1% of an amount, move the position of the decimal point \_\_\_\_\_ places to the \_\_\_\_\_.

- 8** Find 15% of the following by finding 10% and 5% then adding them together.
- |                   |                   |                   |
|-------------------|-------------------|-------------------|
| <b>a</b> \$100.00 | <b>b</b> \$220.00 | <b>c</b> \$40.00  |
| <b>d</b> \$8.00   | <b>e</b> \$6.20   | <b>f</b> \$15.00  |
| <b>g</b> \$19.80  | <b>h</b> \$568.20 | <b>i</b> \$150.00 |

- 9 a MC** 10% of \$180.00 is:

**A** \$1.80                           **B** \$9.00  
**C** \$18.00                           **D** \$90.00

- b** 15% of \$340.00 is:

**A** \$34.00                           **B** \$17.00  
**C** \$68.00                           **D** \$51.00

- c** 12% of \$12.00 is:

**A** \$1.00                           **B** \$1.20  
**C** \$1.44                           **D** \$2.00

- d** 64% of \$75 is:

**A** \$48.00                           **B** \$4.80  
**C** \$0.48                           **D** \$6.40

### UNDERSTANDING

- 10** Merryn's yearly salary is \$46 500. How much does she pay towards her superannuation each year if 5% of her salary goes into superannuation?
- 11** K Mart has a 15% discount storewide on all items. How much would I save on a pair of shoes which normally costs \$45?
- 12** Coles has cut the price of Cadbury's chocolate bars by 20%. Their normal price is \$3.45. A shareholder who has a Coles-Myer discount card is also entitled to a further discount of 7.5%. What would a shareholder save on the normal price?

### REASONING

- 13** Sam operates a fruit shop. He marks all his fruit up by 40% of their cost. If the fruit does not sell readily, he tags it with a sign 'Reduced 40%'. He has such a sign on punnets of strawberries which cost him 75 cents. How much would a customer pay for these strawberries? Justify your answer.

### REFLECTION

Why is it important to have knowledge of percentages when dealing with money?



## 18C Unitary method

- Many consumers assume that if they buy the larger quantity of a product, they will be getting better value for money. This is not always the case.
- The **unitary method** can be used in everyday life situations to work out which item is the best buy by comparing the unit price. Supermarkets in Australia are now obliged to show the unit cost on most products.
- The **unit price** is the price per unit. This may mean the price per 100 mL, the price per 100 g or the price of a single item.
- Remember to represent the same units when you are making comparisons.

### WORKED EXAMPLE 4

Three shampoos are sold in the following quantities.

**Brand A:** 200 mL for \$5.38

**Brand B:** 300 mL for \$5.98

**Brand C:** 400 mL for \$8.04

Which shampoo is the best buy?

#### THINK

- Determine the number of 100-mL units for each shampoo.

#### WRITE

$$\text{Brand A} = \frac{200}{2}$$

$$= 2 \text{ units}$$

$$\text{Brand B} = \frac{300}{3}$$

$$= 3 \text{ units}$$

$$\text{Brand C} = \frac{400}{4}$$

$$= 4 \text{ units}$$

- Determine the price per unit for each shampoo.

$$\text{Price per unit} = \frac{\text{price}}{100 - \text{mL units}}$$

$$\text{Brand A} = \frac{538}{2}$$

$$= \$2.69 \text{ per 100 mL}$$

$$\text{Brand B} = \frac{598}{3}$$

$$= \$1.99 \text{ per 100 mL}$$

$$\text{Brand C} = \frac{804}{4}$$

$$= \$2.01 \text{ per 100 mL}$$

- Answer the question for the best buy.

Brand B: 300 mL of shampoo for \$5.98

- The smaller the unit cost, the better value of the item.

### Unitary method in percentages

- The first step is to find 1% of the amount.
- The next step is to multiply the value of the goods by 100 to get the whole amount (100%).

**WORKED EXAMPLE 5**

Find the whole amount if 5% is represented by \$25.

**THINK**

- 1 Five per cent of a number equals 25.  
Find 1% by dividing the number by 5.
- 2 Find 100% by multiplying the amount by 100.

**WRITE**

$$\begin{aligned} 5\% &= \$25 \\ 1\% &= 25 \div 5 \\ &= 5 \\ 100\% &= 5 \times 100 \\ &= \$500 \end{aligned}$$

**WORKED EXAMPLE 6**

Ronan operates a sports store at a fixed profit margin of 65%. Ronan sells a tracksuit for \$99.95. What would Ronan have paid for the tracksuit?

**THINK**

- 1 Write the selling price as 165% of the cost price.
- 2 Find 1% of the cost price by dividing both sides by 165.
- 3 Find 100% of the cost price by multiplying both sides by 100. Give your answer to the nearest cent.
- 4 Answer in a sentence.

**WRITE**

$$\begin{aligned} 165\% \text{ of cost price} &= \$99.95 \\ 1\% \text{ of cost price} &= \$0.605\,757\,6 \\ 100\% \text{ of cost price} &= \$60.58 \end{aligned}$$

Ronan would have paid \$60.58 for the tracksuit.

**REMEMBER**

1. Using the unitary method means to find the value for one unit.
2. It allows for an easy comparison of the cost of the same quantity of items that come in different sizes.
3. When using the unitary method for percentages, determine 1% of an amount first.

**EXERCISE****18C Unitary method****INDIVIDUAL PATHWAYS****eBookplus****Activity 18-C-1**

Unitary method  
doc-6614

**Activity 18-C-2**

More applications of  
the unitary method  
doc-6615

**FLUENCY**

- 1 How much will you pay for 4.5 kg of apples, given that 2 kg cost \$3.80?
- 2 Compare the cost of 400 g of biscuits for \$2.48 and 500 g for \$3.10. Which is the better buy?
- 3 **WE4** Which of the following is the best buy?

a

Chocolate weight	Cost
150 g	\$3.25
250 g	\$4.75
325 g	\$5.50

**INDIVIDUAL PATHWAYS****eBookplus****Activity 18-C-3**

Advanced applications of the  
unitary method  
doc-6616

**eBookplus**

**Interactivity**  
**Unitary**  
**method**  
int-2365

No. of pages	Cost
80	\$1.98
160	\$3.38
200	\$3.98

- 4 **WE5** Calculate the original amount if:

- a 10% is \$18
- c 25% is \$60
- e 34% is \$60
- g 200% is \$800
- i 9% is \$54.
- b 20% is \$6
- d 40% is \$900
- f 90% is \$380
- h 120% is \$420

- 5 Calculate:

- a 60%, if 40% is \$120
- c 50%, if 25% is \$60
- e 12%, if 75% is \$250
- b 38%, if 20% is \$6
- d 150%, if 50% is \$900
- f 86%, if 14% is \$4200.

**UNDERSTANDING**

- 6 **WE6** The profit on a certain mp3 player is \$240. If this is 60% of the cost price, what is:
- a the cost price?
  - b the selling price?
- 7 You spend 40% of your monthly allowance on your mobile phone. What is your monthly allowance if you spend \$20 on your mobile phone?
- 8 Hanh saves 32% of his wages each week. If he saves \$220, what is his weekly wage?
- 9 The retail price of rollerblades is \$320. This represents 200% of the cost price. What is the cost price?
- 10 Alex receives a 15% pay rise. He now receives \$97 290 a year. How much was his annual income before the pay rise?
- 11 A company made a profit of \$238 000. This represents a 10% increase in profit on the previous year. What was the profit that year?
- 12 A camping goods shop operates on a profit margin of 85%. How much would the shop have paid for a sleeping bag that sells for \$89.95?
- 13 Barry's electricity bill for 3 months is \$870 inclusive of the GST (10% tax).
- a What is the initial cost of electricity?
  - b How much is the GST?
- 14 After a discount of 15%, an mp3 player was worth \$183. What was its value before the discount?
- 15 During a sale, a retailer allows a discount of 15% off the marked price. His sale price of \$60 still gives him a profit of 10%.
- a What did the article cost him?
  - b What was the marked price?
- 16 A discount of 15% reduced the price of a CD by \$3.20.
- a What was the original price of the CD?
  - b What was its selling price?
- 17 A discount of 22% reduced the price of an outfit by \$48.
- a What was its original price?
  - b At what reduced price was it selling?

- 18** You buy ten i-phone earplugs for \$15. At what price should you sell a dozen earplugs if you wish to make a profit of 25%?
- 19** A water bill, including GST (10%), is \$216.30. What is the cost excluding GST?

**REASONING**

- 20** An art dealer sold two paintings at an auction. The first painting sold for \$7600, making a 22% loss on its cost. The second painting was sold for \$5500, making a profit of 44%. Did the art dealer make an overall profit or loss?
- 21** Café Noir charges a 1% levy on the bill for trading on Sundays. If the final bill is \$55.55, what was the original price, taking into account that the 10% GST has been added and then the levy?

**eBookplus****Digital doc**  
WorkSHEET 18.1  
doc-6617**REFLECTION**

What are the different situations in which you would use the unitary method?

# Summary

## Money

- To change dollars to cents, multiply by 100.
- To change cents to dollars, divide by 100.
- To add, subtract, multiply or divide using money, use the same methods as are used with decimals.
- Money is rounded to the nearest 5 cents when cash is used because there are no longer 1 or 2 cent coins.
- Exact amounts are used for cheques, credit cards and electronic transactions.
- When solving real-life applications using money, clearly state what is being done at each step of the solution.

## Money and percentages

To find a percentage of an amount:

- Write the percentage as a fraction out of 100.
- Multiply this fraction by the amount expressed as a fraction.

## Unitary method

- Using the unitary method means that you find the value for one unit.
- The unitary method allows for an easy comparison of the cost of the same quantity of items that come in different sizes.
- When using the unitary method for percentages, determine 1% of an amount first.

## MAPPING YOUR UNDERSTANDING

Using terms from the summary, and other terms if you wish, construct a concept map that illustrates your understanding of the key concepts covered in this chapter. Compare this concept map with the one that you created in *What do you know?* on this chapter's opening page. Have you completed the two *Homework sheets*, the *Rich task* and two *Code puzzles* in your *Maths Quest 7 Homework book*?



# Chapter review

## FLUENCY

- Calculate these amounts.
  - $\$2.45 + \$13.20 + \$6.05$
  - $\$304.60 - \$126.25$
  - $\$9.65 \times 7$
- What is  $\$65.50 + \$11$ ? (Round your answer to the nearest 5 cents.)
- Find 10% of each of the following by moving the position of the decimal point. Round your answer to the nearest 5 cents.
 

a \$63.00	b \$42.00
c \$105.00	d \$216
e \$3.45	f \$42.68
g \$118.55	h \$2125.85
- Find 5% of the following by finding 10% and halving your answer. Round your answer to the nearest 5 cents.
 

a \$8.00	b \$21.00
c \$64.00	d \$104.00
e \$35.00	f \$52.00
g \$205.50	h \$77.30
- Calculate the following. Round your answer to the nearest 5 cents.
 

a 1% of \$16.00
b 1% of \$28.00
c 12% of \$42.00
d 30% of \$90.00
e 22% of \$220.00
f 43% of \$27
g 15% of \$19.50
h 8% of \$37
- Mentally calculate 12% of \$15.

- Heo buys a new television set marked \$495. He pays a \$100 deposit and 12 payments of \$40 each. How much more than the marked price does he finally pay?



- Estimate the total bill and change, then find the exact amounts when \$150 is used to buy:
  - 10 disks at \$0.95 each
  - 2 games at \$59.95 each
  - 1 file box at \$8.95.
- William owns a hairdressing salon and raises the price of haircuts from \$26.50 to \$29.95. By what percentage did he increase the price of haircuts?
- The price of milk increased by 8%. If the original price was \$1.84, what is the new price?
- Find:
 

a 20% of \$130
b 10% of \$25.80
c 15% of \$68.60
d 42% of \$20
e 1% of \$16.70
f 5% of \$186.40
g 27% of \$250
h 12% of \$36.
- If you deposit your money in a bank, the bank will give you a small amount of money called interest. The amount of interest they give you depends on how much you have saved and for how long. It will be a certain percentage of the amount deposited. How much interest would you receive if you have \$1200 in the bank and the bank calculates interest of 5% on your account balance?
- Last year Brisbane Roar won 70% of its home and away matches. If there are 20 matches in a season, how many matches did Brisbane Strikers win?

## PROBLEM SOLVING

- Natalie went shopping and bought a pair of bathers for \$38.95, a football for \$75.50, four pot plants at \$8.75 each and a photograph album for \$14.90. How much money did Natalie spend in total?
- Sally bought a motorbike costing \$2785. She paid a deposit of \$160 then paid the remainder in 15 equal instalments. How much was each instalment?
- Jacques' furniture shop had a sale with  $\frac{1}{3}$  off the usual price of lounge suites. If the original price of a suite was \$5689, what will the sale price be?

- 11** Sets of 90-minute CDs are sold as in the following packs. Which is the best buy?

Cost (\$)	Number of CDs
\$3.25	6
\$4.99	10
\$7.50	15

- 12** Which is a better buy — 400 g of biscuits costing \$2.98, or a pack of biscuits with 400 g + 25% extra, costing \$3.28?

- 13** Antwert buys a pair of jeans for \$59.95. The original price tag was covered by a 30% sticker but the sign on top of the rack said an ‘Additional 15% off already reduced prices.’

- a** How could Antwert work out how much he had saved?
- b** What percentage of the original cost did he end up saving?


**eBook plus**
**Interactivities**

Test yourself

Chapter 18

int-2746

Word search

Chapter 18

int-2747

Crossword

Chapter 18

int-2748



**Chapter opener****Digital doc (page 583)**

- Hungry brain activity Chapter 18 (doc-6601)

**Are you ready?****Digital docs (page 584)**

- SkillsHEET 18.1 (doc-6602) Rounding money to the nearest 5 cents
- SkillsHEET 18.2 (doc-6603) Converting a percentage to a decimal fraction
- SkillsHEET 18.3 (doc-6604) Decreasing a quantity by a percentage
- SkillsHEET 18.4 (doc-6605) Finding a percentage of a quantity (money)
- SkillsHEET 18.5 (doc-6606) Expressing one quantity as a percentage of another
- SkillsHEET 18.6 (doc-6607) Increasing a quantity by a percentage

**18A Money****Digital docs (page 587)**

- Activity 18-A-1 (doc-6608) Money
- Activity 18-A-2 (doc-6609) More money
- Activity 18-A-3 (doc-6610) Advanced calculations with money
- Investigation (doc-2226) How much is one million dollars? (page 588)
- Investigation (doc-2227) Movie Munchies dollars (page 588)

**18B Money and percentages****Digital docs (page 589)**

- Activity 18-B-1 (doc-6611) Basic money and percentages
- Activity 18-B-2 (doc-6612) money and percentages
- Activity 18-B-3 (doc-6613) Advanced money and percentages

**18C Unitary method****Digital docs (page 592–3)**

- Activity 18-C-1 (doc-6614) Unitary method
- Activity 18-C-2 (doc-6615) More applications of the unitary method
- Activity 18-C-3 (doc-6616) Advanced applications of the unitary method
- WorkSHEET 18.1 (doc-6617) (page 594)

**Interactivity (page 593)**

- Unitary method (int-2365)

**Chapter review****Interactivities (page 597)**

- Test yourself Chapter 18 (int-2746) Take the end-of-chapter test to test your progress.
- Word search Chapter 18 (int-2747)
- Crossword Chapter 18 (int-2748)

To access eBookPLUS activities, log on to

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# 19

## Problem solving II

- 19A Problem solving A
- 19B Problem solving B
- 19C Problem solving C
- 19D Problem solving D
- 19E Problem solving E
- 19F Problem solving F
- 19G Problem solving G
- 19H Problem solving H
- 19I Problem solving I
- 19J Problem solving J

### OPENING QUESTION

During a storm, 13 mm of rain fell on a particular roof. How much rainwater could a 52-m<sup>2</sup> roof collect?

For some strategies that may help to solve the following problems, see Chapter 11.

## 19A

## Problem solving A

- 1** In a survey, 24 people were asked: ‘What is the maximum distance you would walk to a train station?’. A person who says ‘800 m’ will walk up to 800 m, but no more. What is the longest distance (as a multiple of 100 m) that at least  $\frac{3}{4}$  of the people would walk to the train station?

The survey responses are:

100 m	200 m	250 m	1 km	$\frac{1}{2}$ km	$\frac{3}{4}$ km
600 m	500 m	450 m	100 m	1.2 km	800 m
1.5 km	1.4 km	200 m	300 m	1.2 km	350 m
900 m	750 m	300 m	650 m	320 m	100 m.

- 2** Michelle studies elective music. Her assignment this term is to compose a piece of music using as many instruments as she chooses, but only those that she can play. Michelle plays the acoustic guitar, the piano, the double bass and the electric bass. How many choices does Michelle have?



- 3** In a class of 30 students, each chooses to study French or German or both languages. If 24 study French and 18 study German, find the number of students who study both languages.
- 4** Christine and Con made a certain number of telephone calls over a 3-month period. There is a charge of 22 cents for each call, and the monthly rental of the phone line is \$15.20. The bill for the 3 months comes to \$138. How many calls did they make in the 3-month period?
- 5** What is the angle sum (sum of all the angles) of a decagon?
- 6** Geoff wants to establish a rectangular vegetable garden bed on his farm. His 20-m long work shed will act as one of the boundaries. (*Note:* The garden will not necessarily be this long.) He has 24 metres of fencing which he plans to use to fence the other three sides. How could Geoff use this fencing to enclose the largest possible area?
- 7** A group of people arrive for a meeting. Each person shakes hands (only once) with every other person as they arrive. If there are 15 handshakes altogether, how many people arrived for the meeting?
- 8** A rectangular lawn measures 40 m by 20 m. Your lawn mower can cut a path 30 cm wide. If you start along the outer edge of the lawn and mow around the outer portion of the uncut lawn twenty times, what fractional part of the lawn have you mowed?

- 9** A gift box has dimensions 30 cm by 22 cm by 15 cm. If a ribbon is wrapped around this box as shown using 25 cm for a bow, what is the total length of ribbon needed?
- 10** Cameron wants to finish the 11-match cricket season with an average of at least 27. With 5 matches already played, his average is 22. How many runs does he still need to make if he is to achieve his goal?



**EXERCISE**

## 19B Problem solving B

- 1** Alex has one brother, one sister, a mother and a father. The family owns a 5-seater car. When the family goes out, the parents always sit in the front two seats. There are three seats behind that. How many different seating arrangements are there?
- 2** Vernon works in a restaurant where people choose to work Monday to Friday, or weekends, or Monday to Sunday. There are 15 employees altogether. If 13 work Monday to Friday and 14 work weekends, find the number of employees who do not work Monday to Sunday.
- 3** Stan bought 2 bottles of juice and 6 packets of chips. The total bill was \$14. If the juice cost \$2.80 per bottle, what did he pay for a packet of chips?
- 4** The club coach at a local cycling track was overheard saying that he felt at least half the cyclists were cycling at a speed of 30 km/h or more. The speeds (in km/h) of the club cyclists were recorded as follows.  
31, 22, 40, 12, 26, 39, 49, 23, 24, 38, 27, 16, 25, 37, 19, 25, 45, 23, 17, 20, 34, 19, 24, 15, 40, 39, 11, 29, 33, 44, 29, 50, 18, 22, 51, 24, 19, 20, 30, 49, 29, 17, 25, 37, 25, 18, 34, 21, 20, 18  
Is the coach correct in making this statement? First round each of these speeds to the nearest 5 km/h.
- 5** How many different three-way telephone calls are possible among a group of five friends?
- 6** The land-line telephone numbers in Victoria start with 03 (the area code) followed by 8 digits:  
03 \_\_\_\_\_.  
 a How many telephone numbers are possible using this system?  
 b How many telephone numbers are possible if the first digit following the area code can only be an 8 or a 9?



- 7** Form an equation for the following statement.  
One third of a certain number is 12.  
Solve the equation to find the number.
- 8** *Squeezy fruit drink* is 36% pure orange juice and 64% water.  
If 1 litre of *Squeezy fruit drink* is mixed with 600 mL of water, what is the concentration of juice in the new mixture?
- 9** A piece of paper is 22 cm long, including a 2.2-cm sticky strip at each end. Five strips of paper are stuck together so that the sticky parts overlap exactly to make a loop of paper. What is the circumference of the loop?

**EXERCISE**

**19C**

## Problem solving C

- 1** Sam has a set of wooden cube blocks he uses to build different shapes. His sister Chris challenged him to see how many different ways he could stack 4 cubes. He can only stack them on top of each other or side by side — not one behind the other. The arrangements must be different and not simply a mirror image or rotation of another shape. Draw the different arrangements possible. Ignore the numbers on the blocks in your answer.
- 2** There are 20 chocolate bars on the table. If 12 contain nuts and 14 contain fruit, find the numbers of chocolate bars that contain both fruit and nut.
- 3** Alvin bought 3 bottles of juice and 8 packets of chips. The total bill was \$18.50. If the chips cost \$1.75 per packet, what did he pay for a bottle of juice?
- 4** Imagine a simplified form of car numberplate where the plate consists of a letter followed by a number; that is, A0 up to Z9. How many numberplates could be issued under this system? Remember that all numberplates issued have to be different.
- 5** A clothing store records the dress sizes sold during a day in order to cater for the popular sizes. The results for a particular day are:  
12, 14, 10, 12, 8, 12, 16, 10, 8, 12,  
10, 12, 18, 10, 12, 14, 16, 10, 12, 12,  
12, 14, 18, 10, 14, 12, 12, 14, 14, 10.

Rebecca is in charge of marketing and sales. She uses these figures in ordering future stock. From these figures she decides on the following ordering strategy.

Order:

- the same number of size 8, 16 and 18 dresses
- three times this number of size 10 and size 14 dresses
- five times as many size 12 dresses as size 8, 16 and 18.

Comment on Rebecca's strategy.

- 6** A new rectangular patio has been built on the end of the house. It measures 3.8 m by 1.9 m. You want to plant a garden that is 1.5 m wide around the patio on three sides. The garden beds are to be 0.5 metres deep. If you order 7.3 cubic metres, explain with a diagram whether enough soil has been ordered.

- 7** Form an equation to represent the following statement.

When 12 is subtracted from 7 times a number, the result is 9.

Solve your equation to find the number.

- 8** A mother leaves out a bowl of cherries for her 3 daughters with a note saying 'Share'.

Talisia comes home first and realises that if she eats a cherry, the remaining cherries can be divided evenly. She eats the cherry, takes her share, and leaves.

Rinasha comes home next. She also realises that if she eats a cherry, the remaining cherries can be divided evenly. She eats the cherry, takes what she thinks is her share, and leaves.



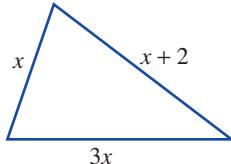
Preeya comes home last. She also finds that if she eats a cherry, the remaining cherries can be divided evenly. She eats the cherry, takes what she thinks is her share and leaves 6 cherries in the bowl.

How many cherries were left out by the girls' mother?

Note: Each sister thinks that she is the first one home, and is sharing between 3 people.

- 9 A number,  $x$ , becomes the fifth member of the set of numbers  $\{3, 7, 9, 13\}$  to make the mean of the five numbers equal to their median. Find the possible values for  $x$ ?

- 10 Given that the perimeter of the following triangle is 22 cm, find  $x$ .



#### EXERCISE

## 19D Problem solving D

- 1 A new vending machine stocking water and sports drinks was installed at the local gymnasium. The number of bottles sold each day over the first 5 weeks after its installation is shown.  
4, 39, 31, 31, 50, 43, 60, 45, 57, 61, 18, 26, 3, 52, 51, 59, 33, 51, 27, 62, 30, 40, 3, 30, 37, 9, 33, 44, 53, 16, 22, 6, 42, 33, 19

The gymnasium realises that the number of daily sales will vary, depending particularly on the weather. The machine is stacked with 60 bottles at the start of each day. The supplier advises that, because of electricity costs and other running expenses, the machine is not worthwhile having unless at least three-quarters of the bottles are sold on average 2 days per week. Round these sales to the nearest 5 and analyse the figures to determine whether the gymnasium should keep the vending machine.

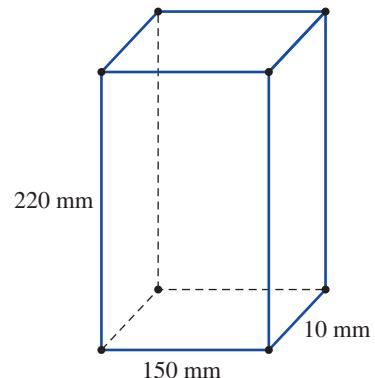


- 2 Kelly has a piece of string 32 cm long. He uses it to form rectangular shapes each with the length and width a whole number of centimetres. How many different rectangular shapes can he form?
- 3 The students in a Year 7A were voting for a class captain. There were only two candidates. When the votes were counted, Rachel received 84% of the votes and Susi received 28% of the votes. It was discovered that some students had voted twice. If the class consisted of 25 students, how many of them voted twice?
- 4 A jug is  $\frac{1}{5}$  full of water. Jack added more water to the jug until it was  $\frac{3}{4}$  full. At that stage there was 1.5 L of water in the jug. How much water was originally in the container?

- 5** Ten people were sitting around a round table. How many handshakes were required so that each person had shaken hands with everyone else exactly once?
- 6** A standard Domino set consists of a set of rectangular tiles, each with a line dividing its face into two square ends. Each end is marked with a number of black dots (similar to those on dice), or is blank. A standard domino set has ends ranging from zero dots to six dots. The back side of a domino tile is plain. How many tiles make up a full set of dominoes?



- 7** The perimeter of a rectangle is 32 cm. Using integers to represent the length and width, predict what the sides of the rectangle will be to give the largest possible area. Check by drawing different rectangles to see if your prediction is correct.
- 8** Form an equation from the following statements.  
I thought of a number and added 5. Three times the result was equal to 27.  
Use your equation to find the number that I first thought of?
- 9** An ant crawls along the edges of a rectangular cereal box that measures 220 mm  $\times$  150 mm  $\times$  10 mm.  
What is the greatest distance that the ant can travel without retracing its steps?
- 10** You leave home riding your bike at 8 kilometres per hour. An hour later your brother leaves home trying to catch up to you. He rides at 10 kilometres per hour. Using both a table of values and a graph, find out how long it will take for him to catch up to you.



#### EXERCISE

## 19E Problem solving E

- 1** Glenn remembered that his mother's car registration plate had 2 letters followed by 3 numbers. He could remember that the letters were R and B and that the numbers were 5, 1 and 4, but he couldn't remember the order. What combination of letters and numbers could his mother's car registration plate have? Make a list of the possibilities.

- 2 Given that OD bisects COE, that  $\angle COD = 4x + 16$ , and that  $\angle DOE = 6x + 4$ , explain if there is enough information for you to determine the angles for COD and DOE? If so what are the measures for each?

- 3 A big cargo box measures 12 m by 6 m by 5 m. How many small boxes of 40 cm by 25 cm by 10 cm can be put into the cargo box?

- 4 Laurie connected a water tank to his  $5 \text{ m} \times 3 \text{ m}$  roof in order to collect rain water. In four hours, 2 cm of rain fell. The water tank held 280 litres. Did the tank overflow?  
*Hint:*  $1000 \text{ L} = 1 \text{ m}^3$ .

- 5 Farmer Jones wants to build a rectangular paddock in which to keep her sheep. Farmer Jones has 500 metres of fencing to build the paddock.

- a If the length of the paddock is to be  $x$  and the breadth of the paddock is to be  $y$ , write an equation connecting  $x$  and  $y$ .
- b On a number plane draw a graph of your equation.
- c Choose three points on your graph. Taking these values of  $x$  and  $y$ , find the area of the rectangular paddock.

- 6 Without using a calculator, find the average of the numbers 0.1, 0.11 and 0.111?

- 7 A sheet of paper is 0.012 cm thick. Would you notice the difference in the height between a stack of 500 sheets and a stack of 550 sheets?

- 8 Find the average of  $\frac{1}{4}, \frac{1}{5}, \frac{1}{10}$  and  $\frac{1}{20}$ .

- 9 A statement has been made: ‘No matter what size or thickness, no piece of paper can be folded in half more than 7 times.’

Investigate what happens to the thickness of the paper as the folding progresses and develop a formula which allows you to calculate the thickness of the paper for any particular number of folds.

- 10 A *cryptarithm* is a puzzle where each digit of a number has been replaced by a pronumeral or letter. Solve the following cryptarithm by determining the digit that corresponds to each letter to make the calculation true.

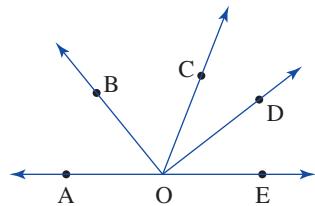
This cryptarithm has several solutions. Find at least 4 solutions.

$$\begin{array}{r} \text{T W O} \\ + \text{T W O} \\ \hline \text{F O U R} \end{array}$$

### EXERCISE

## 19F Problem solving F

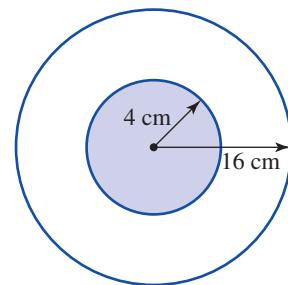
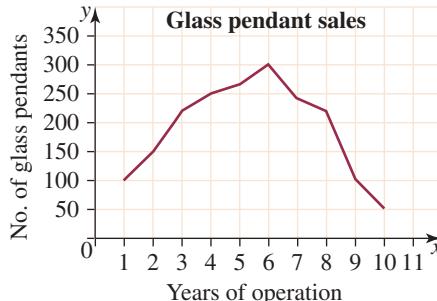
- 1 Mrs Makeit sells stuffed teddy bears to a stall holder at the Queen Victoria Market for \$10 more than they cost her to make. The stall holder sells the teddy bears to customers for twice what he pays for them.
- a Complete this rule: Price = (Mrs Makeit's costs +  $\underline{\hspace{2cm}}$ )  $\times \underline{\hspace{2cm}}$ .
  - b Write the rule using  $c$  to stand for the price that customers pay in dollars, and  $n$  to stand for Mrs Makeit's cost in dollars
  - c If a teddy bear costs Mrs Makeit \$20 to make, how much will a customer at the market stall pay for it?



- 2** You have a piece of paper that measures 24 cm by 36 cm. You want to cut out a net that can be folded into a box. Using whole numbers, is there more than one way to create a box with the same maximum volume? Explain.

- 3** At a fun fair, a prize is given if the dart lands in the inner circle of a dart board. Given that the inner circle has a radius of 4 cm, and that the dart board has a radius of 16 cm, what is the probability of winning a prize?

- 4** Lisa created a line graph to illustrate the number of glass pendants she has sold for each of the last ten years. She intends to use the graph for a presentation at her bank, in order to obtain a loan.



- a** In what year of operation did she sell the most pendants? How many did she sell?  
**b** In what year of operation did she sell the least number of pendants? How many did she sell?  
**c** Is there a trend in the sales of the pendants?  
**d** Will this trend help Lisa obtain the loan?

- 5** A book has pages numbered 1 to 140. This means that the numbers on each page have either one, two or three of the digits 0 to 9. How many digits would be used in total to number the pages?

- 6** A die is rolled 30 times, and gives the following results.

4	3	5	4	3	5	2	1	1	5
3	2	2	4	1	3	1	6	1	3
2	1	6	6	3	5	1	3	5	3



- a** Display these results in a frequency table.  
**b** What is the probability of obtaining a 6 when you roll a die?  
**c** How many times would you expect to obtain a 6 in 30 rolls of a die?  
**d** Explain the difference between your expected results and the actual results shown above.

- 7** You love to make bracelets. To make them, you need to buy some beads. Brand A costs \$7.50 for a box that contains 100 beads; Brand B costs \$17.50 for a box that contains 2500 beads. Which brand is the better buy? Explain how you decide.

- 8** In an amusement park, 0.3 of the children were girls. If there were 80 more boys than girls, how many children were there in the park?

- 9** The same rule applies to each of these three sets of numbers.

Set A: 1, 2, 5

Set B: 2, 5, 26

Set C: 3, 10,   .

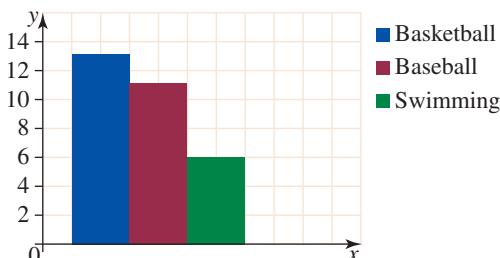
Explain how you can determine the missing number.

- 10** \$125 is shared between James and Alison, but Alison is to receive \$19 more than James. Set up an equation to describe this situation. How much money do James and Alison each receive?

- 1 A tap drips water at a rate of one drop every three seconds. If it takes 750 drops of water to fill a 100-ml container, how many litres of water will be lost in one year?

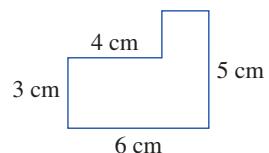
- 2 In a survey, a group of boys was asked to name their favourite sport. Part of the data collected is shown in the bar chart below.

- a On the same chart, draw a bar to show that 10 boys named soccer as their favourite sport.



- b A boy is chosen at random. What is the probability that his favorite sport is baseball?

- 3 a Calculate the area of the figure. Show all work.  
 b Calculate the area of the figure using an alternative method. Show all work.  
 c Explain the differences between the two methods.



- 4 This is a backward clock. It has the numbers in reverse order, and the hands sweep anticlockwise.

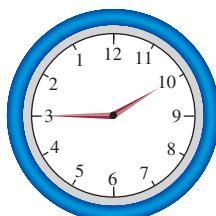
What time does the clock show?

- 5 It's an interesting fact that the sum of the digits of a cube of an integer, when reduced to a single digit, can only be 1, 8 or 9.

For example,  $12^3 = 1728$

$$\begin{aligned}1 + 7 + 2 + 8 &= 18 \\1 + 8 &= 9\end{aligned}$$

There is a pattern in the sequence of these numbers. Try this rule with the cubes of the first 10 integers, then explain how you could determine the single-digit sum of  $35^3$  without performing the actual calculation.



- 6 The cost of hiring a taxi is \$3.80 plus \$1.20 per kilometre travelled. Write an equation, and use it to find the maximum distance that can be travelled in a taxi for \$20.

- 7 Apples cost twice as much as pears. If an apple and a pear together cost \$1.20, find the cost of each.

- 8 The first two numbers in a series are  $x$  and  $y$ . The next number is found by dividing the term before it by the term before that.

Write down the first ten terms of the series.

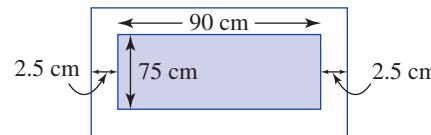
- 9 The Body Mass Index (BMI) is a number that is used in the medical field to determine whether an individual is underweight, has a healthy weight or is overweight. Your index is calculated by dividing your weight (in kg) by your height (in metres) squared.

$$\text{BMI} = \frac{\text{weight (kg)}}{\text{height}^2 (\text{m}^2)}$$

What is the height of a person whose BMI is 20 and whose weight is 45 kg?

- 10 Equations are solved by making sure that the left-hand side exactly balances the right-hand side. You have three coins in your hand, two of which weigh the same, the third being slightly heavier. Explain how, with just one weighing on a pair of scales, you can determine which of the coins is the heavier one.

## 19H Problem solving H

- 1** Ellen wants to frame her cross-stitch work. Her cross-stitch work is 90 cm long and 75 cm wide. The frame's border measures 2.5 cm wide (see the figure at right). One company charges \$120 for the frame. What is the cost per square centimetre? Round off your answer to 2 decimal places.
- 
- 2** Cory records the fact that it has rained on 65 out of 365 days in a year.
- Write the number of days that it has rained as a simple fraction.
  - Karen says that since any day can be wet or dry, the probability of rain on any day is  $\frac{1}{2}$ . Is Karen correct?
  - What is the experimental probability of rain on any given day, expressed as a decimal correct to two decimal places?
- 3** An internet company charges a connection fee of \$12 and then \$1.50 for every hour of usage. Write an equation that represents the total cost,  $C$ , in dollars for using the internet for  $t$  hours. Find the cost for 7 hours.
- 4** Draw up a quadrilateral, PQRS. Label angle P as  $100^\circ$  and angle R as a right angle. Angle Q is two thirds of angle S. Determine the measure of angle S.
- 5** Marcus eats one of the strawberries in a basket, then he and his two friends split them evenly. If his friends get 15 strawberries each, how many were in the basket to begin with?
- 6** When half a number is increased by 15 the result is 50. Find the original number.
- 7** Six consecutive numbers add to 393. Find the smallest of these numbers.
- 8** Some players from a soccer club went to a café to celebrate their win. Each had a burger and a drink. The bill came to \$80. When it came time to pay the bill, 3 people had left without paying, so the remaining members had to pay an extra \$2 each to cover the bill. If there were originally  $n$  people in the group, and the burger and drink deal cost  $b$ , write an equation using these pronumerals to show how the bill was settled. Do not attempt to solve your equation.
- 9** Jodie bought two packets of chips and a drink. From \$10 she received \$1.40 in change. Given that the drink cost \$3.40 find the cost of a packet of chips.
- 10** A *cryptarithm* is a puzzle where each digit of a number has been replaced by a pronumeral or letter. Solve the following cryptarithm by determining the digit that corresponds to each letter to make the calculation true.

$$\begin{array}{r}
 \text{S E N D} \\
 + \text{M O R E} \\
 \hline
 \text{M O N E Y}
 \end{array}$$

- 1** Gavin records the amount of rainfall each day in millimetres each day over a two-week period. Gavin's results are:  
 11, 24, 0, 6, 15, 0, 0, 0, 0, 12, 0, 0, 127, 15, 0.
- What is the mean rainfall for the two-week period?
  - What is the median rainfall?
  - What is the mode of the rainfall?
  - Which of the mean, median and mode is the best measure of the typical rainfall? Explain your choice.
- 2** If the sum of three consecutive integers is 39, what are the three consecutive integers?
- 3** Athena the artist paints portraits. For her present subject, Seth, she uses the following measurement relationships.
- Seth's arm span is the same length as his height.
  - The length of Seth's hand is one-tenth of his height.
  - The distance across Seth's shoulders is one-quarter his height.
- What is the relationship between Seth's arm span, the length of his hand and the distance across his shoulders?



- 4** Andrew thinks of a number, adds 9 and multiplies the result by 3.  
 Andrea thinks of a number, multiplies it by 3 and adds 9 to the result.  
 Both Andrew and Andrea get an answer of 60.  
 Did Andrew and Andrea begin by selecting the same number? Show working out to support your answer.
- 5** While solving a puzzle, Tiana worked out that  $a > m$ ,  $z > a$ ,  $a < g$  and  $z < g$ . Tiana had to place them in numeric order. What is the numeric order of  $a$ ,  $g$ ,  $m$  and  $z$ ?
- 6** While shopping in a sale, Carly spots a sign that says 'Buy 2 items, get the third half price (the least expensive item will be counted as the third item)'. If she finds a shirt for \$25.99, a skirt for \$87.99 and a belt for \$15.97 and pays cash for the items, how much will Carly end up paying.
- 7** A pin, 14 mm long, is made of wire. How many pins could be made from 1 km of wire?

- 8** Tienna needs to convert her Grandma's favourite cake recipe to metric units. She notes that 1 oz = 30 g and 1 pint = 568 mL. Help her convert the following quantities to metric units.

Water  $\frac{3}{4}$  pint

Butter 6 oz

Sugar 5 oz

Flour 10 oz

Eggs 2

- 9** Use the digits 1, 2, 3 and 4 and the operators  $+$ ,  $-$ ,  $\times$ , and  $\div$  to construct equations that result in the numbers 1 to 5 (the numbers 2 and 4 are already done for you). You must use each digit in each expression, and you may not use the digits more than once. You may combine the digits to form larger numbers (like 21). You may also use brackets to make sure the operations are done in the correct order.

$$1 =$$

$$2 = 4 - 3 + 2 - 1$$

$$3 =$$

$$4 = 4 - 2 + 3 - 1$$

$$5 =$$

- 10** A *cryptarithm* is a puzzle in which each digit of a number is replaced by a letter. Using the digits 1, 2, 3, 4, 5 and 6 solve the following product.  
 $A \times BC = DEF$

#### EXERCISE

## 19J Problem solving J

- 1** A *cryptarithm* is a puzzle where each digit of a number has been replaced by a prounomial or letter. Solve the following cryptarithm by determining the digit that corresponds to each letter to make the calculation true.

$$\begin{array}{r} \text{O N E} \\ + \quad \text{T W O} \\ + \quad \text{F O U R} \\ \hline \text{S E V E N} \end{array}$$

- 2** Glen and Cathy were married on a Saturday in December 1992. In what year will their wedding anniversary fall on a Saturday for the first time?
- 3** Use the digits 1, 2, 3 and 4 and the operators  $+$ ,  $-$ ,  $\times$ , and  $\div$  to construct equations that result in the numbers 5 to 10 (the numbers 7 and 10 have already been done for you). You must use each digit in each expression and you may not use the digits more than once. You may combine the digits to form larger numbers (like 21). You may also use brackets to make sure the operations are done in the correct order.

$$5 =$$

$$6 =$$

$$7 = 24 \div 3 - 1$$

$$8 =$$

$$9 =$$

$$10 = 1 + 2 + 3 + 4$$

- 4** Find two consecutive numbers with a product of:

a 650

b 5256

c 14 520

- 5** The number 123451234512345... has 103 digits. What is the sum of the 103 digits?

- 6** Hollie could buy 2 packets of chips and 3 chocolates for \$5.10. She could buy 3 packets of chips and 2 chocolates for \$5.90. Determine the cost of each item.

- 7** Candice, Sara, Jordyn and Georgia are friends. They are all different ages: 11, 12, 13 and 14 years. Use the clues to work out the ages of the friends.  
Sara's age is an even number.  
Jordyn and Georgia's ages added together are double Sara's age.  
Jordyn's age is half of Candice and Sara's ages added together.  
Jordyn and Georgia's ages are both odd numbers.  
Candice is the eldest.



**eBook plus**

**Weblinks**

**Problem**

**solving 1**

**Problem**

**solving 2**

**Problem**

**solving 3**

**Problem**

**solving 4**

**Problem**

**solving 5**

- 8** If you had any number of ordinary dice, how many different ways could you roll the dice and obtain a total of 6?
- 9** Find two prime numbers with a product of:  
**a** 21  
**b** 26  
**c** 323
- 10** Exactly 87 digits have been used to number the pages in a book. How many pages are there?

# Answers

## CHAPTER 1

### Numeracy

#### Set 1A

##### 1A Calculator allowed

- |                   |                           |               |
|-------------------|---------------------------|---------------|
| <b>1</b> A        | <b>2</b> B                | <b>3</b> C    |
| <b>4</b> 17 000   | <b>5</b> D                | <b>6</b> B    |
| <b>7</b> B        | <b>8</b> C                | <b>9</b> B    |
| <b>10</b> 54 days | <b>11</b> D               | <b>12</b> C   |
| <b>13</b> D       | <b>14</b> \$240           | <b>15</b> 276 |
| <b>16</b> 62.5 km | <b>17</b> \$140           | <b>18</b> D   |
| <b>19</b> B       | <b>20</b> C               | <b>21</b> B   |
| <b>22</b> A       | <b>23</b> US\$460<br>€580 | <b>24</b> B   |
| <b>25</b> A       | <b>26</b> 27              | <b>27</b> B   |
| <b>28</b> 6.22 kg | <b>29</b> D               | <b>30</b> 24  |

#### Set 1B

##### 1B Non-calculator

- |                   |                   |                   |
|-------------------|-------------------|-------------------|
| <b>1</b> B        | <b>2</b> 10.50 am | <b>3</b> 100 mL   |
| <b>4</b> C        | <b>5</b> D        | <b>6</b> D        |
| <b>7</b> C        | <b>8</b> C        | <b>9</b> D        |
| <b>10</b> A       | <b>11</b> B       | <b>12</b> B       |
| <b>13</b> C       | <b>14</b> A       | <b>15</b> C       |
| <b>16</b> B       | <b>17</b> B       | <b>18</b> D       |
| <b>19</b> 8280 L  | <b>20</b> C       | <b>21</b> 8.48 am |
| <b>22</b> B       | <b>23</b> 5 hours | <b>24</b> 76.6 m  |
| <b>25</b> 6.45 am | <b>26</b> \$440   | <b>27</b> A       |
| <b>28</b> B       | <b>29</b> B       | <b>30</b> C       |

#### Set 1C

##### 1C Calculator allowed

- |                               |                                   |                              |
|-------------------------------|-----------------------------------|------------------------------|
| <b>1</b> D                    | <b>2</b> B                        | <b>3</b> C                   |
| <b>4</b> \$108.70<br>\$258.62 | <b>5</b> \$66.37                  | <b>6</b> B                   |
| <b>7</b> East                 | <b>8</b> B                        | <b>9</b> D                   |
| <b>10</b> D                   | <b>11</b> B                       | <b>12</b> 250 g              |
| <b>13</b> D                   | <b>14</b> 52                      | <b>15</b> C                  |
| <b>16</b> A                   | <b>17</b> Thursday<br>80 students | <b>18</b> \$3                |
| <b>19</b> D                   | <b>20</b> D                       | <b>21</b> 84 pages           |
| <b>22</b> $\frac{3}{8}$       | <b>23</b> 5 metres                | <b>24</b> C                  |
| <b>25</b> 54.5                | <b>26</b> D                       | <b>27</b> C                  |
| <b>28</b> 60 and 30           | <b>29</b> D                       | <b>30</b> 25 cm <sup>2</sup> |

#### Set 1D

##### 1D Non-calculator

- |                              |                             |                                   |
|------------------------------|-----------------------------|-----------------------------------|
| <b>1</b> C                   | <b>2</b> C                  | <b>3</b> D                        |
| <b>4</b> $\frac{3}{5}$       | <b>5</b> 160                | <b>6</b> D                        |
| <b>7</b> 44                  | <b>8</b> B                  | <b>9</b> B                        |
| <b>10</b> $x + 5 = 2x + 4$   | <b>11</b> D                 | <b>12</b> -3 and -0.5             |
| <b>13</b> 2.35               | <b>14</b> C                 | <b>15</b> A                       |
| <b>16</b> 14                 | <b>17</b> 20 km             | <b>18</b> A                       |
| <b>19</b> D                  | <b>20</b> C                 | <b>21</b> $\frac{5}{12}$ teaspoon |
| <b>22</b> C                  | <b>23</b> 46 °C             | <b>24</b> C                       |
| <b>25</b> D                  | <b>26</b> $5\frac{1}{2}$ km | <b>27</b> C                       |
| <b>28</b> $14\frac{1}{2}$ km | <b>29</b> C                 | <b>30</b> A                       |

## Set 1E

### 1E Calculator allowed

- |                         |                   |                |
|-------------------------|-------------------|----------------|
| <b>1</b> 10%            | <b>2</b> C        | <b>3</b> \$46  |
| <b>4</b> 8              | <b>5</b> A        | <b>6</b> 26 cm |
| <b>7</b> C              | <b>8</b> 1680     | <b>9</b> A     |
| <b>10</b> C             | <b>11</b> B       | <b>12</b> B    |
| <b>13</b> C             | <b>14</b> C       | <b>15</b> C    |
| <b>16</b> C             | <b>17</b> A       | <b>18</b> D    |
| <b>19</b> D             | <b>20</b> 7909 km | <b>21</b> A    |
| <b>22</b> B             | <b>23</b> B       | <b>24</b> B    |
| <b>25</b> 41, 29 and 25 | <b>26</b> 1920 L  | <b>27</b> B    |
| <b>28</b> C             | <b>29</b> C       | <b>30</b> B    |

## Set 1F

### 1F Non-calculator

- |                    |             |             |
|--------------------|-------------|-------------|
| <b>1</b> B         | <b>2</b> D  | <b>3</b> C  |
| <b>4</b> C         | <b>5</b> C  | <b>6</b> A  |
| <b>7</b> 69°       | <b>8</b> D  | <b>9</b> B  |
| <b>10</b> C        | <b>11</b> B | <b>12</b> A |
| <b>13</b> C        | <b>14</b> D | <b>15</b> A |
| <b>16</b> A        | <b>17</b> C | <b>18</b> C |
| <b>19</b> B        | <b>20</b> D | <b>21</b> A |
| <b>22</b> \$2244   | <b>23</b> B | <b>24</b> D |
| <b>25</b> D        | <b>26</b> A | <b>27</b> C |
| <b>28</b> 12 times | <b>29</b> D | <b>30</b> B |

## CHAPTER 2

### Positive integers

#### Are you ready?

- |  |                                  |               |
|--|----------------------------------|---------------|
| <b>1</b> a Sixty (60)                    | <b>b</b> Fifty thousand (50 000) |               |
| <b>c</b> Three hundred (300)             | <b>d</b> Eight thousand (8000)   |               |
| <b>2</b> a 19, 91, 99, 199, 919, 991     |                                  |               |
| <b>b</b> 201, 121, 120, 112, 102, 21, 12 |                                  |               |
| <b>3</b> a 15                            | <b>b</b> 27                      | <b>c</b> 26   |
| <b>d</b> 4                               | <b>e</b> 7                       | <b>f</b> 9    |
| <b>4</b> a 32                            | <b>b</b> 30                      | <b>c</b> 63   |
| <b>d</b> 36                              | <b>e</b> 16                      | <b>f</b> 66   |
| <b>5</b> a 315                           | <b>b</b> 322                     | <b>c</b> 5652 |
| <b>6</b> a 14                            | <b>b</b> 163 remainder 1         | <b>c</b> 93   |
| <b>7</b> a 19                            | <b>b</b> 26                      | <b>c</b> 22   |
| <b>8</b> a 4, 8, 12, 16, 20              |                                  |               |
| <b>b</b> 6, 12, 18, 24, 30               |                                  |               |
| <b>9</b> a 1                             | <b>b</b> 8                       | <b>c</b> 4    |
| <b>10</b> a 500                          | <b>b</b> 2000                    | <b>c</b> 70   |

#### 2A Place value

- |   |                                   |                   |                  |
|---|-----------------------------------|-------------------|------------------|
| <b>1</b> a 900 + 20 + 5                                   | <b>b</b> 1000 + 60 + 2            |                   |                  |
| <b>c</b> 20 000 + 8000 + 400 + 60 + 9                     |                                   |                   |                  |
| <b>d</b> 40 + 3   | <b>e</b> 500 000 + 2000 + 30 + 9  |                   |                  |
| <b>f</b> 800 000 + 2                                      | <b>g</b> 1 000 000 + 80 000 + 100 |                   |                  |
| <b>h</b> 20 000 + 2000 + 200 + 20 + 2                     |                                   |                   |                  |
| <b>2</b> a Seven hundred and sixty-five                   |                                   |                   |                  |
| <b>b</b> Nine thousand, one hundred and five              |                                   |                   |                  |
| <b>c</b> Ninety thousand, four hundred and fifty          |                                   |                   |                  |
| <b>d</b> One hundred thousand, two hundred and thirty-six |                                   |                   |                  |
| <b>3</b> a 495  | <b>b</b> 2670                     | <b>c</b> 24 000   | <b>d</b> 109 605 |
| <b>4</b> a i Two hundred                                  | <b>ii</b> Forty                   | <b>iii</b> Eight  |                  |
| <b>b</b> i Two  | <b>ii</b> One hundred             | <b>iii</b> Thirty |                  |
| <b>c</b> i Zero   | <b>ii</b> Two hundred             |                   |                  |

- d** i Eighty      ii Seven hundred  
iii One thousand
- 5 a** E
- b** All 5 numbers have 4 in the thousands place; A, C, D and E have 8 in the hundreds place; A and E have 8 in the tens place and only E has 8 in the units place. E has the largest number.
- 6 B**
- 7 a** 48 987, 8569, 742, 647, 28  
**b** 72 167, 58 625, 47 890, 32 320, 12 947  
**c** 7647, 7476, 6747, 6477, 4776  
**d** 8880, 8808, 8800, 8088, 8080, 8008
- 8 a** 9, 58, 743, 68 247, 1 258 647  
**b** 34 108, 58 610, 60 000, 78 645, 84 364  
**c** 1290, 1902, 2019, 2910, 9021, 9201  
**d** 111, 211, 212, 221, 222, 1112
- 9 a** i \$6000      ii \$340 000      iii \$58 000  
**b** i \$430K      ii \$7K      iii \$800K  
**c** Check with your teacher.
- 2B Adding and subtracting positive integers**
- |  |  |                    |
|--|--|--------------------|
| <b>1 a</b> 15  | <b>b</b> 24  | <b>c</b> 37        |
| <b>d</b> 121   | <b>e</b> 210   | <b>f</b> 240       |
| <b>g</b> 472   | <b>h</b> 1730  | <b>i</b> 7458      |
| <b>j</b> 18 220                                      | <b>k</b> 175 000                                       | <b>l</b> 20        |
| <b>m</b> 40  | <b>n</b> 50  |                    |
| <b>2 a</b> 99  | <b>b</b> 999   | <b>c</b> 4183      |
| <b>d</b> 72 648                                      | <b>e</b> 3 485 551                                     | <b>f</b> 13 629    |
| <b>3 a</b> 978                                       | <b>b</b> 1155  | <b>c</b> 766       |
| <b>d</b> 257   | <b>e</b> 1335  | <b>f</b> 11 747    |
| <b>g</b> 14 462                                      | <b>h</b> 332 941                                       | <b>i</b> 1 701 553 |
| <b>j</b> 1 053 944                                   |  |                    |
| <b>4 a</b> 180                                       | <b>b</b> 200   | <b>c</b> 280       |
| <b>d</b> 460   | <b>e</b> 790   | <b>f</b> 827       |
| <b>5 a</b> 6   | <b>b</b> 8   | <b>c</b> 23        |
| <b>d</b> 5   | <b>e</b> 60  | <b>f</b> 125       |
| <b>g</b> 814   | <b>h</b> 900   | <b>i</b> 700       |
| <b>j</b> 10 500                                      | <b>k</b> 70  | <b>l</b> 35        |
| <b>m</b> 500   | <b>n</b> 63  | <b>o</b> 5         |
| <b>p</b> 26  | <b>q</b> 39  |                    |
| <b>6 a</b> 4   | <b>b</b> 20  | <b>c</b> 26        |
| <b>d</b> 55  | <b>e</b> 40  | <b>f</b> 0         |
| <b>g</b> 73  | <b>h</b> 1150  |                    |
| <b>7 a</b> 44  | <b>b</b> 35  | <b>c</b> 35 123    |
| <b>d</b> 86  | <b>e</b> 584 599                                       | <b>f</b> 1831      |
| <b>g</b> 267   | <b>h</b> 4413  | <b>i</b> 65 439    |
| <b>8 a</b> 40 672                                    | <b>b</b> 2634  | <b>c</b> 69 397    |
| <b>d</b> 56 176                                      | <b>e</b> 1553  | <b>f</b> 517 526   |
| <b>g</b> 349 818                                     | <b>h</b> 19 088 744                                    |                    |
| <b>9</b> \$247                                       |  |                    |
| <b>10</b> 3032 pages                                 |  |                    |
| <b>11</b> \$71                                       |  |                    |
| <b>12 a</b> 8 days                                   | <b>b</b> 2003 km                                       |                    |
| <b>c</b> 3492 km                                     | <b>d</b> 2754 km                                       |                    |
| <b>13</b> 6670 km                                    |  |                    |
| <b>14 a</b> 7, 6, 14                                 | <b>b</b> 15, 10, 15                                    | <b>c</b> 8, 20, 36 |
| <b>15</b> 15   | <b>16</b> 41 551                                       | <b>17</b> 31       |
| <b>18 a</b> 215 million                              | <b>b</b> 220 million                                   |                    |
| <b>19 a</b> 26 m                                     | <b>b</b> 26 m  | <b>c</b> 52 m      |
| <b>d</b> Check with your teacher.                    |  |                    |
| <b>20 a</b> 172 km                                   | <b>b</b> 353 km  |                    |
| <b>21 a</b> $638\ 321 - 488\ 417 = 149\ 904$         | <b>b</b> $3492 - 1685 = 1807$ (Other answers possible) |                    |
| <b>22</b> 5 849 318                                  | <b>23</b> 7 days                                       |                    |
| <b>24</b> $735 + 1689 = 2424$ or $641 + 8752 = 9393$ |  |                    |

## 2C Multiplying and dividing positive integers

- |  |                              |                    |               |
|--|------------------------------|--------------------|---------------|
| <b>1 a</b> 12  | <b>b</b> 45                  | <b>c</b> 22        |               |
| <b>d</b> 56  | <b>e</b> 96                  | <b>f</b> 110       |               |
| <b>g</b> 54  | <b>h</b> 132                 | <b>i</b> 72        |               |
| <b>2 a</b> 26  | <b>b</b> 45                  | <b>c</b> 50        |               |
| <b>d</b> 39  | <b>e</b> 100                 | <b>f</b> 90        |               |
| <b>g</b> 32  | <b>h</b> 70                  | <b>i</b> 42        |               |
| <b>j</b> 63  | <b>k</b> 108                 | <b>l</b> 75        |               |
| <b>m</b> 72  | <b>n</b> 90                  | <b>o</b> 27        |               |
| <b>p</b> 90  | <b>q</b> 100                 | <b>r</b> 80        |               |
| <b>3 a</b> 128   | <b>b</b> 116                 | <b>c</b> 1233      |               |
| <b>d</b> 2571  | <b>e</b> 24 600              | <b>f</b> 111 888   |               |
| <b>g</b> 63 104  | <b>h</b> 1888                | <b>i</b> 16 120    |               |
| <b>j</b> 63 582  | <b>k</b> 374 055             | <b>l</b> 492 720   |               |
| <b>4 a</b> 2288  | <b>b</b> 3007                | <b>c</b> 1652      |               |
| <b>d</b> 912   | <b>e</b> 7093                | <b>f</b> 64 600    |               |
| <b>g</b> 21 571  | <b>h</b> 97 478              | <b>i</b> 2 161 485 |               |
| <b>j</b> 4 870 460                                     | <b>k</b> 27 529 460          | <b>l</b> 1 725 805 |               |
| <b>5 a</b> 80  | <b>b</b> 240                 | <b>c</b> 1900      |               |
| <b>d</b> 4500  | <b>e</b> 900                 | <b>f</b> 67 000    |               |
| <b>6 a</b> 900   | <b>b</b> 1830                | <b>c</b> 3100      |               |
| <b>d</b> 16 800  | <b>e</b> 9500                | <b>f</b> 172 000   |               |
| <b>7 a</b> 8000  | <b>b</b> 21 000              | <b>c</b> 480 000   |               |
| <b>d</b> 7200  | <b>e</b> 48 000              | <b>f</b> 5 500 000 |               |
| <b>g</b> 6 300 000 000                                 | <b>h</b> 144 000 000         |                    |               |
| <b>i</b> 5 600 000                                     | <b>j</b> 54 000 000          |                    |               |
| <b>k</b> 440 000                                       | <b>l</b> 13 200 000          |                    |               |
| <b>8 a</b> 306   | <b>b</b> 747                 | <b>c</b> 5652      |               |
| <b>d</b> 7425  | <b>e</b> 456                 | <b>f</b> 208       |               |
| <b>9 a</b> 5600  | <b>b</b> 560                 | <b>c</b> 5040      |               |
| <b>10 a</b> 4320                                       | <b>b</b> 6660                | <b>c</b> 11 250    |               |
| <b>d</b> 28 800  | <b>e</b> 3600                | <b>f</b> 57 600    |               |
| <b>11 a</b> 150  | <b>b</b> 300                 | <b>c</b> 300       |               |
| <b>d</b> Multiply first by 6 then double the answer.   |                              |                    |               |
| <b>12 a</b> 180  | <b>b</b> 840                 | <b>c</b> 216       | <b>d</b> 1260 |
| <b>e</b> 350   | <b>f</b> 640                 | <b>g</b> 198       | <b>h</b> 680  |
| <b>13 a</b> 90   | <b>b</b> 27                  | <b>c</b> 117       |               |
| <b>d</b> Multiply by 10 and by 3 then add the answers. |                              |                    |               |
| <b>14 a</b> 325  | <b>b</b> 390                 | <b>c</b> 312       | <b>d</b> 1326 |
| <b>15 a</b> 4  | <b>b</b> 3                   | <b>c</b> 4         |               |
| <b>d</b> 9   | <b>e</b> 7                   | <b>f</b> 8         |               |
| <b>g</b> 12  | <b>h</b> 8                   | <b>i</b> 4         |               |
| <b>j</b> 4   | <b>k</b> 20                  | <b>l</b> 13        |               |
| <b>m</b> 3   | <b>n</b> 2                   | <b>o</b> 3         |               |
| <b>p</b> 2   | <b>q</b> 2                   | <b>r</b> 4         |               |
| <b>s</b> 2   | <b>t</b> 5                   | <b>u</b> 15        |               |
| <b>16 a</b> 10   | <b>b</b> 6                   | <b>c</b> 56        |               |
| <b>d</b> 35  | <b>e</b> 84                  | <b>f</b> 60        |               |
| <b>g</b> 3   | <b>h</b> 44                  | <b>i</b> 54        |               |
| <b>17 a</b> 485  | <b>b</b> 6942                | <b>c</b> 6208      |               |
| <b>d</b> 57 319  | <b>e</b> 2761                | <b>f</b> 80 045    |               |
| <b>g</b> 85 064 remainder 2                            | <b>h</b> 86 047 remainder 3  |                    |               |
| <b>i</b> 8643 remainder 1                              | <b>j</b> 904 075 remainder 5 |                    |               |
| <b>k</b> 93 155 remainder 2                            | <b>l</b> 60 074 remainder 2  |                    |               |
| <b>18 a</b> 700  | <b>b</b> 10                  | <b>c</b> 7         |               |
| <b>d</b> 900   | <b>e</b> 9                   | <b>f</b> 500       |               |
| <b>g</b> 5000  | <b>h</b> 7                   | <b>i</b> 800       |               |
| <b>19 a</b> \$72                                       | <b>b</b> \$1728              |                    |               |
| <b>20</b> \$132 000                                    |                              |                    |               |
| <b>21 a</b> \$675                                      | <b>b</b> \$1200              | <b>c</b> \$2700    |               |
| <b>22</b> 490 km                                       |                              |                    |               |
| <b>23 a</b> 4420 litres                                | <b>b</b> 22 100 litres       |                    |               |
| <b>24</b> 524 seconds                                  |                              |                    |               |
| <b>25 a</b> 34 hours                                   | <b>b</b> \$1105              | <b>c</b> \$1885    |               |

- 26** a 10 km      b \$175      c 120 km  
 d \$9100      e 400 papers      f 20 800 papers  
**27** a 58      b 29      c 29  
 d Divide by 10 and halve the answer.  
**28** a 214      b 92      c  $39\frac{1}{2}$       d 32

- 29** a 28 km      b 14 km  
**30** a \$208      b \$48

**31** 186 cows

**32** Number of tables required: 19; total number of courses to be served: 608; number of waiters required: 8; total charge for catering: \$8360

**33** a 950 blocks

b \$794 750

**34** \$16 per night

**35** 3 goals, 9 behinds; 4 goals, 8 behinds; 7 goals, 7 behinds

- 36** a 1089, 2178, 3267, 4356, 5445, 6534, 7623, 8712, 9801  
 b 10 989, 21 978, 32 967, 43 956, 54 945, 65 934,  
 76 923, 87 912, 98 901  
 c 109 989, 219 978, 329 967, 439 956, 549 945,  
 659 934, 769 923, 879 912, 989 901; 1 099 989,  
 2 199 978, 3 299 967, 4 399 956, 5 499 945,  
 6 599 934, 7 699 923, 8 799 912, 9 899 901

## 2D Long division

- 1** a 15      b 14  
 c 32      d 19  
 e 421      f 658  
 g 325      h 362  
 i 62      j 5502  
**2** a  $65 \text{ r } 2$  or  $65\frac{2}{13}$       b  $63 \text{ r } 6$  or  $63\frac{2}{5}$   
 c  $71 \text{ r } 3$  or  $71\frac{3}{17}$       d  $84 \text{ r } 4$  or  $84\frac{4}{19}$   
 e  $123 \text{ r } 7$  or  $123\frac{7}{41}$       f  $251 \text{ r } 6$  or  $251\frac{2}{11}$   
 g  $325 \text{ r } 6$  or  $325\frac{2}{7}$       h  $249 \text{ r } 2$  or  $249\frac{2}{31}$   
 i  $652 \text{ r } 1$  or  $652\frac{1}{23}$       j  $741 \text{ r } 4$  or  $741\frac{4}{27}$

**3** a  $19 \overline{)448} \underline{)4}$

b  $17 \overline{)82} \underline{)79}$

c  $21 \overline{)512} \underline{)0}$

## 2E Order of operations

- 1** a 5      b 9      c 3      d 30  
 e 59      f 25      g 7      h 96  
 i 15      j 24      k 32      l 18  
 m 76      n 27      o 26      p 7  
 q 12      r 96      s 27      t 0  
 u 18      v 3

- 2** a  $(12 - 8) \div 4 = 1$   
 b  $(4 + 8) \times 5 - 4 \times 5 = 40$   
 c  $3 + 4 \times (9 - 3) = 27$   
 d  $3 \times (10 - 2) \div 4 + 4 = 10$   
 e  $12 \times (4 + 2) - 12 = 60$   
 f  $17 - 8 \times 2 + 6 \times (11 - 5) = 37$   
 g  $10 \div (5 + 5) \times 9 \times 9 = 81$   
 h  $(18 - 3) \times 3 \div 5 = 9$

- 3** C      **4** B  
**5** a  $6 + 2 \times (4 - 3) \times 2 = 10$   
 b  $(6 + 2) \times 4 - 3 \times 2 = 26$   
 c  $6 + (2 \times 4 - 3) \times 2 = 16$   
 d  $6 + 2 \times 4 - 3 \times 2 = 8$ , no brackets required  
**6** James  
**7** 43 pebbles  
**8** Same as the digit you started with. This is because you are dividing by 111 111 each time.

## 2F Factors and multiples

- 1** a 3, 6, 9, 12, 15      b 6, 12, 18, 24, 30  
 c 100, 200, 300, 400, 500      d 11, 22, 33, 44, 55  
 e 15, 30, 45, 60, 75      f 4, 8, 12, 16, 20  
 g 21, 42, 63, 84, 105      h 25, 50, 75, 100, 125  
 i 14, 28, 42, 56, 70      j 12, 24, 36, 48, 60  
 k 50, 100, 150, 200, 250      l 30, 60, 90, 120, 150  
 m 33, 66, 99, 132, 165      n 120, 240, 360, 480, 600  
 o 45, 90, 135, 180, 225      p 72, 144, 216, 288, 360  
**2** 10, 20, 100, 70      **3** 21, 7, 70, 35  
**4** 16, 64, 160, 32, 80      **5** 70, 35, 140, 105, 700  
**6** Three of: 8, 24, 32, 48, 56, 72, 80, 88, 96  
**7** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99  
**8** 102, 108, 114, 120, 126, 132, 138, 144, 150, 156  
**9** a C      b A  
 c  $15 \times 1 = 15$ ,  $15 \times 2 = 30$ ,  $15 \times 3 = 45$   
**10** a 1, 2, 3, 4, 6, 12  
 b 1, 2, 4, 8  
 c 1, 2, 4, 5, 8, 10, 20, 40  
 d 1, 5, 7, 35  
 e 1, 2, 4, 7, 14, 28  
 f 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60  
 g 1, 2, 4, 5, 10, 20, 25, 50, 100  
 h 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72  
 i 1, 3, 13, 39  
 j 1, 5, 17, 85  
 k 1, 2, 4, 19, 38, 76  
 l 1, 3, 23, 69  
 m 1, 3, 9, 11, 33, 99  
 n 1, 2, 5, 10, 25, 50, 125, 250  
 o 1, 3, 17, 51  
 p 1, 3, 5, 7, 15, 21, 35, 105  
**11** a 20: 1, 20; 2, 10; 4, 5  
 b 18: 1, 18; 2, 9; 3, 6  
 c 36: 1, 36; 2, 18; 3, 12; 4, 9; 6, 6  
 d 132: 1, 132; 2, 66; 3, 44; 4, 33; 6, 22; 11, 12

**12** 15

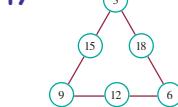
**13** a B      b A

**14** 3, 4

**15** a 6, 12, 18 (or any other multiple of 6)  
 b 6      c 30

**16** 4 rows of 5 students or 5 rows of 4 students;  
 2 rows of 10 students or 10 rows of 2 students;  
 1 row of 20 students or 20 rows of 1 student

**17**



**18** a D

b  $4 \times 8 \neq 24$ ,  $2 \times 12 = 24$ ,  $4 \times 6 = 24$ ,  $6 \times 5 \neq 24$ ,  $3 \times 8 = 24$

**19** 84

**20** 60

**21** 12

**22** 60

**23** 12 or 30

**24** a 25 cm      b 22      c 13      d 286

## 2G Lowest common multiple and highest common factor

- 1** **a** 4, 8, 12, 16, 20, 24, 28, 32, 36, 40  
**b** 6, 12, 18, 24, 30, 36, 42, 48, 54, 60  
**c** 12, 24, 36  
**d** 12
- 2** **a** 3, 6, 9, 12, 15, 18  
**b** 9, 18, 27, 36, 45, 54  
**c** 9, 18  
**d** 9
- 3** **a** 3  
**b** 30  
**c** 70  
**d** 48
- 4** **a** F    **b** T  
**c**  $HCF = 7$
- 5** **a** 1, 3, 7, 21  
**b** 1, 2, 4, 7, 8, 14, 28, 56  
**c**  $HCF = 7$
- 6** **a** 1, 3, 9, 27  
**b** 1, 3, 5, 15  
**c**  $HCF = 3$
- 7** **a** 1, 7  
**b** 1, 2, 4, 7, 14, 28  
**c**  $HCF = 7$
- 8** **a** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48  
**b** 1, 2, 3, 5, 6, 10, 15, 30  
**c**  $HCF = 6$
- 9** **a** 9    **b** 10    **c** 7  
**d** 11    **e** 4    **f** 9  
**g** 12    **h** 176
- 10** **a** 300    **b** 420    **c** 660  
**d** 154    **e** 165    **f** 300  
**g** 360    **h** 630
- 11** **a** 900    **b** 1100    **c** 600  
**d** 3100    **e** 3300    **f** 2800  
**g** 1800    **h** 6000
- 12** **a** 13 000  
**b** 6 days  
**c** 6 is the smallest multiple of both 2 and 3.
- 13** **a** 35 minutes  
**b** 35 is the smallest multiple of both 5 and 7.
- 14** **a** 12 hours  
**b** 12 is the smallest multiple of both 4 and 6.
- 15** 20 seconds
- 16** **a** 12    **b** 6    **c** 7
- 17** **a** 3 metres  
**b** Daniella
- 18** **a** Two semicircles  
**b** The starting line for a 100-m race has been positioned in the extended section so the athletes can all start from the same starting line.  
**c** The finish line for a 100-m race has been positioned so that all athletes can run in a direct line without having to have staggered starting positions.  
**d** There is a need to stagger the starting blocks in the 200-m, 400-m and 800-m track events so that the outside runners will not be running further than the inside runners.  
**e** and **f**

Lane number	Distance travelled	Difference
1	400 m	0
2	408 m	8
3	416 m	16
4	424 m	24
5	432 m	32
6	440 m	40
7	448 m	48
8	456 m	56

- g** The runner in Lane 8 is running 56 metres further than the runner in Lane 1.

## 2H Estimation

- 1** 28 000 000  
**2** Estimation table:

	Estimate	Estimated answer	Actual answer
<b>a</b>	$500 + 1000$	1500	1449
<b>b</b>	$30\ 000 + 80\ 000$	110 000	115 670
<b>c</b>	$200\ 000 + 700\ 000$	900 000	907 448
<b>d</b>	$1000 + 6000$	7000	7429
<b>e</b>	$30 + 80$	110	112
<b>f</b>	$60\ 000 + 50\ 000$	110 000	104 266
<b>g</b>	$90\ 000 - 40\ 000$	50 000	55 880
<b>h</b>	$300 - 200$	100	127
<b>i</b>	$500\ 000 - 200\ 000$	300 000	296 049
<b>j</b>	$900\ 000 - 400\ 000$	500 000	553 120
<b>k</b>	$60\ 000 - 20\ 000$	40 000	44 682
<b>l</b>	$90\ 000 - 40\ 000$	50 000	50 888
<b>m</b>	$40 \times 200$	8000	7128
<b>n</b>	$9000 \times 9$	81 000	77 679
<b>o</b>	$90 \times 400$	36 000	37 584
<b>p</b>	$600 \times 10\ 000$	6 000 000	7 894 033
<b>q</b>	$30\ 000 \times 40$	1 200 000	1 149 954
<b>r</b>	$200 \times 60$	12 000	13 320
<b>s</b>	$30\ 000 \div 1000$	30	32.907 58
<b>t</b>	$60\ 000 \div 3000$	20	24.325 483
<b>u</b>	$900\ 000 \div 3000$	300	263.003 03
<b>v</b>	$8000 \div 4000$	2	2.295 287 4
<b>w</b>	$70\ 000 \div 1000$	70	53.385 505
<b>x</b>	$40\ 000 \div 200$	200	205.605 56

- 3** **a** D    **b** B    **c** B  
**d** 8000    **e** 200    **f** 100  
**g** 400 000    **h** 700 000    **i** 11 000  
**j** 6000    **k** 40 000 000    **l** 8 000 000  
**m** 100    **n** 500    **o** D  
**5** **a** B    **b** D    **c** A  
**d** 1500 tickets    **e** \$11 250  
**6** **a** Bread rolls    **b** \$75  
Hamburgers    **c** \$50  
Tomato sauce    **d** \$4  
Margarine    **e** \$4  
Onions    **f** \$4  
Tomatoes    **g** \$8  
Lettuce    **h** \$5  
**7** **a** \$150    **b** \$600    **c** \$450  
**d** Answers will vary.

## Chapter review

### Fluency

- 1** a 7      b 40 000  
c 1 000 000      d 90

**2** a  $300 + 90 + 2$

$$b \quad 4000 + 100 + 9$$

$$c \quad 40\,000 + 2000 + 1$$

$$d \quad 100\,000 + 20\,000$$

**3** 349, 394, 934, 943, 3499, 3994

**4** 1101, 1100, 1011, 110, 101

**5** a 127      b 3187

$$d \quad 4\,217\,656$$

**6** a 2      b 35

$$d \quad 14$$

**7** a 415      b 217

$$d \quad 37\,758$$

**8** a 150      b 8400

$$d \quad 16\,800$$

**9** a 596      b 8420

$$d \quad 8$$

**10** a 8      b 3

$$d \quad 45$$

**11** a 700      b 7

$$d \quad 632$$

**12** a 362      b 327

$$c \quad 632 \qquad d \quad 198$$

**13** a  $335 \text{ r } 7$  or  $335\frac{7}{17}$

$$b \quad 301 \text{ r } 9 \text{ or } 301\frac{9}{23}$$

$$c \quad 911 \text{ r } 9 \text{ or } 911\frac{9}{13}$$

$$d \quad 454 \text{ r } 21 \text{ or } 454\frac{21}{29}$$

**14** Brackets

Indices

Multiplication and division

Addition and subtraction

**15** a 5      b 38

$$d \quad 5 \qquad e \quad 70$$

$$g \quad 54 \qquad h \quad 29$$

**16** a 14 000      b 5000

$$d \quad 420\,000 \qquad e \quad 120\,000$$

$$g \quad 3 \qquad h \quad 210\,000$$

**17** a 11, 22, 33, 44, 55

$$c \quad 5, 10, 15, 20, 25$$

$$e \quad 13, 26, 39, 52, 65$$

**18** a 12      b 30

$$d \quad 40$$

**19** a 1, 2, 4, 8, 16

$$b \quad 1, 3, 9, 27$$

$$c \quad 1, 2, 5, 10, 25, 50$$

$$d \quad 1, 2, 3, 6, 7, 14, 21, 42$$

$$e \quad 1, 2, 3, 4, 6, 9, 12, 18, 36$$

$$f \quad 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$$

**20** a 1, 24; 2, 12; 3, 8; 4, 6

$$b \quad 1, 40; 2, 20; 4, 10; 5, 8$$

$$c \quad 1, 48; 2, 24; 3, 16; 4, 12; 6, 8$$

$$d \quad 1, 21; 3, 7$$

$$e \quad 1, 99; 3, 33; 9, 11$$

$$f \quad 1, 100; 2, 50; 4, 25; 5, 20; 10, 10$$

### Problem solving

**1** a 306 km      b 322 km      c 482 km

**2** a 448      b 3136      c 98

**3** a 200 s

b Hung 8 laps

Frank 5 laps

**4** As long as one of the factors in a multiplication is even, the result will be even. Otherwise, the answer will be odd.

**5** a 12, 24, 36      b 12      c 60

**6** Six possible answers.

Example

(6) (2) (5)

(4) (3)

(1)

**7** The sum of its factors (except for 496) add to give 496.

**8** 
$$\begin{array}{r} 8 \ 2 \ 4 \ 0 \\ -3 \ 6 \ 8 \ 1 \\ \hline 4 \ 5 \ 5 \ 9 \end{array}$$

**9** 95

**10** 20 458

**11** 120 seconds or 2 minutes

**12** The numbers bounce up and down like hailstones.

**13** a  $8 \times 3 + 12 \times 2$

$$b \quad \$48$$

**14** 20 499

## CHAPTER 3

### Indices and primes

#### Are you ready?

- |                |               |                         |                     |
|----------------|---------------|-------------------------|---------------------|
| <b>1</b> a 16  | <b>b</b> 144  | <b>c</b> 49             | <b>d</b> 8          |
| <b>2</b> a 6   | <b>b</b> 9    | <b>c</b> 5              | <b>d</b> 27         |
| <b>3</b> a 125 | <b>b</b> 1331 | <b>c</b> 2              | <b>d</b> 4          |
| <b>4</b> a 6   | <b>b</b> 9    | <b>c</b> 26             | <b>d</b> 2          |
| <b>5</b> a 107 | <b>b</b> 85   | <b>c</b> 14, 16, 18, 20 | <b>d</b> 37, 39, 41 |

#### 3A Index notation

- |   |   |   |                                     |
|---|---|---|-------------------------------------|
| <b>1</b> $7^4$  | <b>b</b> $8^6$  | <b>c</b> $10^5$   | <b>d</b> $3^9$                      |
| <b>2</b> a $2^4$  | <b>e</b> $13^3$   | <b>g</b> $12^6$   | <b>h</b> $9^8$                      |
| <b>3</b> $4^5 \times 6^3$   | <b>4</b> a $2^2 \times 3$   | <b>b</b> $2^2 \times 3^4$   | <b>c</b> $2^4 \times 5^2$           |
| <b>5</b> $6 \times 6 \times 6 \times 6 \times 6$  | <b>d</b> $2^3 \times 7$   | <b>e</b> $3^3 \times 5 \times 11^2$                                 | <b>f</b> $5^3 \times 7^2 \times 13$ |
| <b>6</b> a $11 \times 11 \times 11$   | <b>g</b> $2^3 \times 3^2 \times 5$  | <b>h</b> $2^2 \times 3^2 \times 5^3$                                |                                     |
| <b>7</b> a $300 = 3 \times 10^2$  | <b>b</b> $4500 = 4 \times 10^3 + 5 \times 10^2$   | <b>c</b> $6785 = 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 5$ | <b>d</b> $10\,000 = 1 \times 10^4$  |
| <b>e</b> $31\,702 = 3 \times 10^4 + 1 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 2$                  | <b>f</b> $200\,301 = 2 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 1$ |   |                                     |
| <b>8</b> a $5^{11}$   | <b>b</b> $7^8$  | <b>c</b> $4^{13}$   | <b>f</b> $9^{15}$                   |
| <b>d</b> $2^{21}$   | <b>e</b> $11^{24}$  | <b>g</b> $5^5 \times 7^5$   |                                     |
| <b>g</b> $3^6 \times 13^6$  | <b>h</b> $2^4 \times 5^4$   | <b>i</b> $5^5 \times 7^5$   |                                     |
| <b>9</b> A  | <b>10</b> D   | <b>11</b> 72  |                                     |
| <b>12</b> a 5184  | <b>b</b> 972  | <b>c</b> 16   | <b>f</b> 112                        |
| <b>d</b> 567  | <b>e</b> 2717   |   |                                     |
| <b>13</b> $41^2 \neq 691$ . Notice that $31^2 = 961$ , therefore $41^2$ will need to be larger than 961 |   |   |                                     |
| <b>14</b> $75^2 = 5625$ , $95^2 = 9025$   |   |   |                                     |

### 3B Prime numbers and composite numbers

**1** 23, 29, 31, 37

**2 a** Composite

**b** Prime

**c** Composite

**d** Composite

**e** Prime

**f** Composite

**g** Composite

**h** Composite

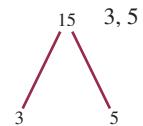
**i** Composite

**j** Prime

**k** Composite

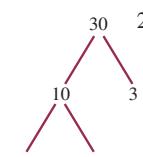
**l** Prime

**3 a i**



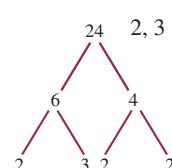
**ii**  $15 = 3 \times 5$

**b i**



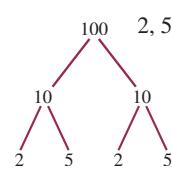
**ii**  $30 = 2 \times 3 \times 5$

**c i**



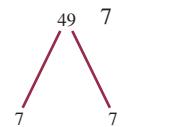
**ii**  $24 = 2 \times 2 \times 2 \times 3$

**d i**



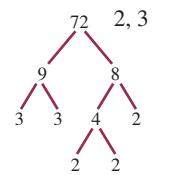
**ii**  $100 = 2 \times 2 \times 5 \times 5$

**e i**



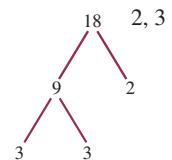
**ii**  $49 = 7 \times 7$

**f i**



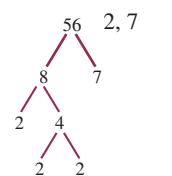
**ii**  $72 = 2 \times 2 \times 2 \times 3 \times 3$

**g i**



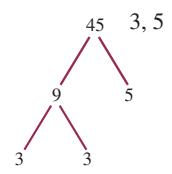
**ii**  $18 = 2 \times 3 \times 3$

**h i**



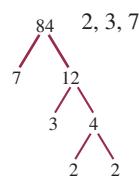
**ii**  $56 = 2 \times 2 \times 2 \times 7$

**i i**



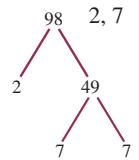
**ii**  $45 = 3 \times 3 \times 5$

**j i**



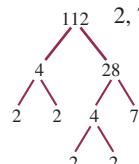
**ii**  $84 = 2 \times 2 \times 3 \times 7$

**k i**



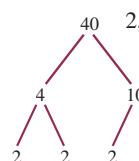
**ii**  $98 = 2 \times 7 \times 7$

**l i**



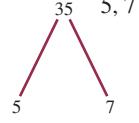
**ii**  $112 = 2 \times 2 \times 2 \times 2 \times 7$

**4 a i**



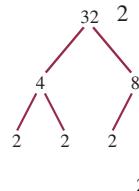
**ii**  $40 = 2 \times 2 \times 2 \times 5$

**b i**



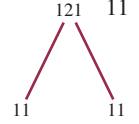
**ii**  $35 = 7 \times 5$

**c i**



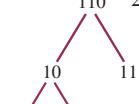
**ii**  $32 = 2 \times 2 \times 2 \times 2 \times 2$

**d i**



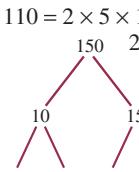
**ii**  $121 = 11 \times 11$

**e i**



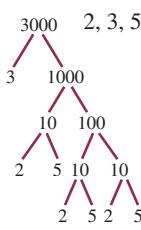
**ii**  $110 = 2 \times 5 \times 11$

**f i**



**ii**  $150 = 2 \times 3 \times 5 \times 5$

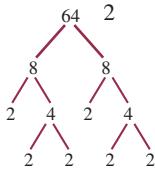
**g** i



2, 3, 5

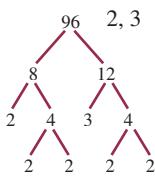
ii  $3000 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

**h** i



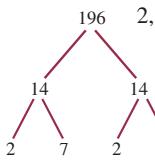
ii  $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

i i



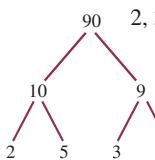
ii  $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

j i



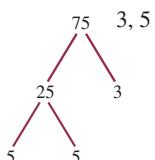
ii  $196 = 2 \times 2 \times 7 \times 7$

k i



ii  $90 = 2 \times 3 \times 3 \times 5$

l i



ii  $75 = 3 \times 5 \times 5$

5 a 2, 3

b 2, 5

c 2, 3, 7

d 3

e 2, 3

f 3, 13

g 3

h 2, 3, 5

i 2, 3, 5

j 2, 3, 5

k 2, 5

l 2, 5

6 a  $2^2 \times 3 \times 5$

b  $2 \times 5^2$

c  $3 \times 5^2$

d  $2^2 \times 5 \times 11$

e  $2^6 \times 3$

f  $2^3 \times 3^2$

g  $2^2 \times 31$

h  $2^3 \times 5^2$

7 a LCM is 252; HCF is 12

b LCM is 240; HCF is 12

c LCM is 1200; HCF is 40

d LCM is 8800; HCF is 20

8 No. The number 2 would always be a factor if the number is even.

9 a F

b F

c F

d T

e T

f F

g F

h F

10 a A

b C

c C

d C

11 11 and 13; 17 and 19

12 a 3, 5, 7. Prime numbers greater than 2 are always odd numbers and the difference between 2 odd numbers is an even number. So 2, 4 and 6 can be differences between consecutive primes.

b 2: 17 and 19; 4: 19 and 23; 6: 23 and 29; other answers possible

13 a  $365 = 5 \times 73$

b  $187 = 11 \times 17$

14 a-f

1	2	3	4	5	6	7	8	9	10
(1)	(2)	(3)	4	(5)	6	(7)	8	(9)	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	47	48	49	50
51	52	(53)	54	55	56	57	58	(59)	60
(61)	62	63	64	65	66	(67)	68	69	70
(71)	72	(73)	74	75	76	77	78	(79)	80
81	82	(83)	84	85	86	87	88	(89)	90
(91)	92	93	94	95	96	(97)	98	99	100

g It is not necessary to check primes larger than 7. When you check multiples of 7, you know that  $7 \times 2$  to  $7 \times 6$  have already been crossed out. The first number to be crossed out will be 49. The next prime is 11. All composite numbers from  $11 \times 2$  to  $11 \times 10$  will have already been crossed out. The first composite number not crossed out will be  $11 \times 11 = 121 > 100$ .

15 a A

b D

c C

d B

16 773

17 29 or 47 or 83 or 137. Others may be possible.

18 61

### 3C Squares and square roots

1 a 64      b 121      c 225      d 625

2 a 64, 81      b 169, 196

3 a 36 and 49      b 49 and 84

c 81 and 100      d 144 and 169

4 a 5      b 9      c 12      d 20

5 a 70      b 120      c 600      d 400

6 7 and 8

7 a 3 and 4      b 9 and 10

c 14 and 15      d 1 and 2

8 a 16, 36, 64      b 81

9 B

10 A

11 a 9      b 75      c 700      d 25

e 14      f 45

12 An exact value of  $\sqrt{10}$  cannot be found because 10 is not a square number.

13 169, 196, 225      14 7, 9

### 3D Cubes and cube roots

1 64

2 a 8      b 27      c 216      d 1000

3 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

4 2

5 a 4      b 6      c 7      d 9

6 For cubic numbers, it was necessary to find the third difference. If we look at power 4, it will be necessary to find the fourth difference.

### Chapter review

#### Fluency

1 a 36      b 196      c 361      d 6400

2 a 9 and 16      b 25 and 36

c 100 and 121      d 225 and 256

3 a 7      b 16      c 30      d 37

4 a 64      b 343      c 3375      d 27000

5 a 3      b 5      c 10      d 20

- 6** a  $2 \times 3^3$     b  $5^2 \times 6^4$     c  $2 \times 5^3 \times 9^3$   
**7** a 243    b 343    c 4096    d 161 051  
**8** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29  
**9** 4  
**10** 53  
**11** a 3, 11    b 3, 7    c 5    d 2, 31  
**12**  $280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$   
**13**  $144 = 2^4 \times 3^2$   
**14** a  $1344 = 1 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 4$   
b  $30\,601 = 3 \times 10^4 + 0 \times 10^3 + 6 \times 10^2 + 0 \times 10^1 + 1$   
**15** a HCF is 24; LCM is 1920  
b HCF is 8; LCM is 2016

### Problem solving

- 1** 432  
**2** 10 is not a perfect square, so we can only find an approximate answer. It will be between 3 and 4.  
**3**  $2, \sqrt[3]{27}, 5, \sqrt{64}$   
**4** 30, 225  
**5** a 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; 71, 73  
b 5, 11; 7, 13; 11, 17; 13, 19; 17, 23; 23, 29; 31, 37; 37, 43;  
41, 47; 47, 53; 53, 59; 61, 67; 67, 73; 73, 79; 83, 89  
c For 2, the number 7 higher is 9, which is not prime.  
For odd primes, the number 7 higher is even and therefore not prime.

- 6** 1, 64  
**7** 6  
**8** 2, 3, 5, 7, 11, 13, 17, 19, 23  
**9** 5; 125  
**10** 45 000  
**11** a 4 regions  
b 

n (number of folds)	0	1	2	3
R (number of regions made)	1	2	4	8

c Each time the page is folded the number of regions is multiplied by 2 (doubled).

d  $R = 2^n$

$2^5 = 32$	$3^5 = 243$
$2^4 = 16$	$3^4 = 81$
$2^3 = 8$	$3^3 = 27$
$2^2 = 4$	$3^2 = 9$
$2^1 = 2$	$3^1 = 3$
$2^0 = 1$	$3^0 = 1$
$2^{-1} = \frac{1}{2}$	$3^{-1} = \frac{1}{3}$

## CHAPTER 4

### Rational numbers

#### Are you ready?

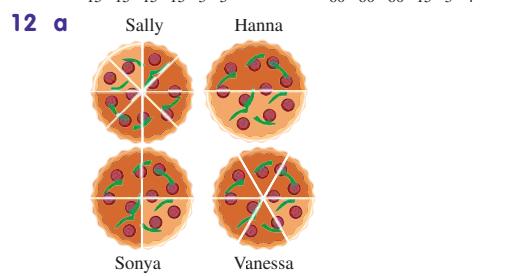
- 1** a Proper fraction    b Improper fraction  
c Mixed number    d Improper fraction  
e Mixed number    f Proper fraction
- 2** a  $\frac{10}{15}$     b  $\frac{6}{7}$
- 3**  $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$
- 4** a 3, 6, 9, 12, 15, 18, 21, 24  
b 4, 8, 12, 16, 20, 24, 28, 32  
c 12, 24  
d 12
- 5** a  $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}, 1, \frac{7}{5}$     b  $0, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}, \frac{13}{12}$

- 6** a 1, 2, 3, 6, 9, 18  
c 1, 2, 3, 6  
**7**  $2\frac{1}{2}$   
**8**  $\frac{4}{3}$   
**9** a  $\frac{8}{6} = \frac{4}{3}$  or  $1\frac{1}{3}$     b  $\frac{2}{12} = \frac{1}{6}$   
c  $\frac{2}{3}$     d  $3\frac{3}{4}$

### 4A What are rational numbers?

- 1** a  $\frac{3}{4}$     b  $\frac{15}{16}$     c  $\frac{7}{8}$     d  $\frac{5}{9}$
- 2** a  $\frac{1}{3}$     b  $\frac{3}{4}$
- c** The flags belong to Italy and Taiwan.
- 3** a i  $\frac{3}{11}$     ii  $\frac{2}{11}$     b  $\frac{3}{11}$
- 4** a 3    b 3    c 4    d 5  
e 6    f 9    g 11    h 12  
i 7
- 5** a  $\frac{20}{30}, \frac{14}{21}, \frac{40}{60}$     b  $\frac{12}{15}, \frac{36}{45}, \frac{16}{20}, \frac{28}{35}, \frac{80}{100}$   
c  $\frac{56}{64}, \frac{21}{24}$     d  $\frac{35}{50}, \frac{21}{30}, \frac{14}{20}, \frac{140}{200}$
- 6** a  $\frac{9}{15}, \frac{12}{20}, \frac{21}{35}$     b  $\frac{30}{40}, \frac{18}{24}, \frac{27}{36}$   
c  $\frac{7}{28}, \frac{6}{24}, \frac{3}{12}$     d  $\frac{15}{18}, \frac{35}{42}, \frac{40}{48}$
- 7** a  $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24}$     b  $\frac{5}{8} = \frac{10}{16} = \frac{15}{24} = \frac{20}{32}$   
c  $\frac{3}{10} = \frac{6}{20} = \frac{9}{30} = \frac{12}{40}$     d  $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$   
e  $\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32}$     f  $\frac{7}{8} = \frac{14}{16} = \frac{21}{24} = \frac{28}{32}$
- 8** a  $\frac{3}{5}$     b  $\frac{7}{8}$     c  $\frac{1}{5}$     d  $\frac{1}{8}$     e  $\frac{1}{2}$   
f  $\frac{3}{4}$     g  $\frac{5}{8}$     h  $\frac{5}{8}$     i  $\frac{4}{5}$
- 9** a >    b <    c >    d >  
e <    f <
- 10** a D    b B

- 11** a  $\frac{3}{10}, \frac{5}{10}, \frac{2}{10}, \frac{1}{5}, \frac{3}{10}, \frac{1}{2}$     b  $\frac{9}{24}, \frac{12}{24}, \frac{8}{24}, \frac{1}{3}, \frac{3}{8}, \frac{1}{2}$   
c  $\frac{10}{15}, \frac{12}{15}, \frac{6}{15}, \frac{6}{15}, \frac{2}{5}, \frac{4}{5}$     d  $\frac{45}{60}, \frac{40}{60}, \frac{28}{60}, \frac{7}{15}, \frac{2}{3}, \frac{3}{4}$



- b** Vanessa Vegetarian  
c They ate the same amount.

### 4B Simplifying rational numbers

- 1** a  $\frac{1}{2}$     b  $\frac{2}{3}$     c  $\frac{7}{8}$     d  $\frac{4}{5}$     e  $\frac{4}{5}$
- f**  $\frac{3}{4}$     g  $\frac{9}{10}$     h  $\frac{7}{8}$     i  $\frac{5}{6}$     j  $\frac{24}{25}$
- k**  $\frac{7}{8}$     l  $\frac{7}{10}$     m  $\frac{11}{12}$     n  $\frac{11}{25}$     o  $\frac{3}{8}$
- 2** a  $4\frac{3}{5}$     b  $7\frac{5}{6}$     c  $10\frac{1}{2}$     d  $5\frac{1}{6}$     e  $3\frac{7}{8}$
- f**  $1\frac{11}{12}$     g  $6\frac{3}{4}$     h  $5\frac{1}{3}$     i  $3\frac{1}{5}$     j  $2\frac{1}{4}$

3 E  
4  $\frac{21}{28} = \frac{3}{4}$

5 a  $\frac{5}{9}$  b  $\frac{4}{9}$

6 7A:  $\frac{1}{5}$ , 7B:  $\frac{3}{20}$ , 7C:  $\frac{8}{25}$ , 7D:  $\frac{2}{25}$ , 7E:  $\frac{1}{4}$

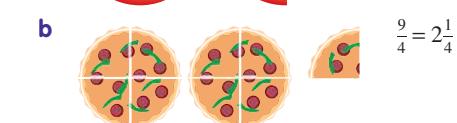
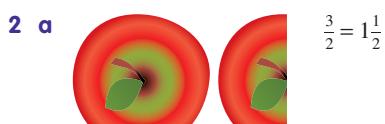
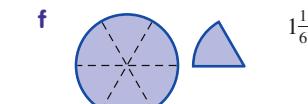
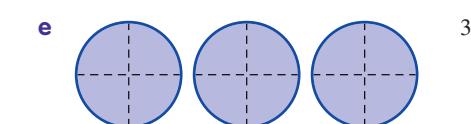
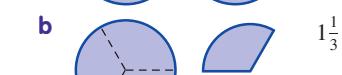
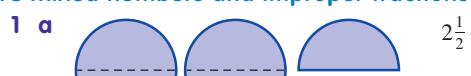
7 a 40

b Mark:  $\frac{5}{40} = \frac{1}{8}$ , David:  $\frac{10}{40} = \frac{1}{4}$ , Samantha:  $\frac{8}{40} = \frac{1}{5}$ , Jules:  $\frac{6}{40} = \frac{3}{20}$ , Ahmed:  $\frac{4}{40} = \frac{1}{10}$ , Darren:  $\frac{7}{40}$

8 a  $\frac{7293}{14586}, \frac{6792}{13584}$

9 Geelong to Melbourne is more reliable.

#### 4C Mixed numbers and improper fractions



3 a  $1\frac{2}{5}$  b  $2\frac{3}{4}$  c  $10\frac{1}{2}$  d  $3\frac{9}{10}$  e  $4\frac{1}{4}$   
f  $8\frac{4}{11}$  g  $2\frac{3}{13}$  h  $2\frac{7}{8}$  i  $33\frac{1}{3}$  j  $12\frac{1}{2}$

k  $6\frac{2}{3}$

4 a  $1\frac{3}{5}$  b  $3\frac{3}{8}$  c  $5\frac{1}{2}$  d  $10\frac{7}{9}$  e  $8\frac{2}{7}$   
f  $11\frac{7}{10}$  g  $8\frac{3}{8}$  h  $8\frac{2}{9}$  i  $2\frac{7}{12}$  j  $14\frac{5}{6}$

5 a C b B c D



7 a  $\frac{8}{7}$  b  $\frac{9}{5}$  c  $\frac{8}{3}$  d  $\frac{10}{3}$  e  $\frac{25}{4}$

f  $\frac{37}{9}$  g  $\frac{23}{2}$  h  $\frac{31}{8}$  i  $\frac{39}{5}$  j  $\frac{47}{5}$

8 a  $\frac{7}{2}$  b  $\frac{21}{5}$  c  $\frac{29}{5}$  d  $\frac{47}{7}$  e  $\frac{29}{10}$

f  $\frac{43}{12}$  g  $\frac{28}{5}$  h  $\frac{67}{7}$  i  $\frac{21}{11}$  j  $\frac{53}{6}$

9 a D b D c A

10 Kim:  $2\frac{1}{4}$ , Carly:  $2\frac{3}{4}$

11  $6\frac{3}{8}$

12  $4\frac{3}{4}$

13 a > b > c < d > e <

f < g < h < i <

14 a 25 b 77

15  $10\frac{1}{2}$

#### 4D Adding and subtracting rational numbers

1 a  $\frac{4}{5}$  b  $\frac{6}{11}$  c  $\frac{5}{6}$  d  $\frac{31}{50}$  e  $\frac{9}{25}$

f  $\frac{11}{12}$  g  $\frac{7}{8}$  h  $\frac{6}{7}$  i  $\frac{29}{100}$

2 a  $\frac{1}{2}$  b  $\frac{1}{4}$  c  $\frac{1}{3}$  d  $\frac{1}{2}$  e  $\frac{1}{2}$

f 1 g  $\frac{3}{4}$  h  $\frac{3}{4}$  i  $\frac{8}{9}$

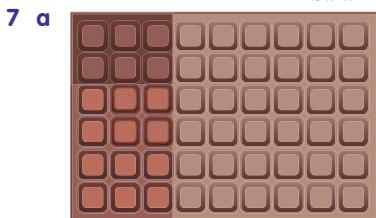
3 a $\frac{3}{4}$	b $\frac{1}{4}$	c $\frac{3}{7}$	d $\frac{3}{4}$	e $\frac{2}{3}$
f $\frac{37}{40}$	g $\frac{13}{27}$	h $\frac{16}{35}$	i $\frac{2}{5}$	
4 a 6	b 15	c 10	d 12	e 30
f 18	g 8	h 24	i 35	
5 a $\frac{1}{6}$	b $\frac{4}{15}$	c $\frac{9}{10}$	d $\frac{7}{12}$	e $\frac{19}{30}$
f $\frac{17}{18}$	g $\frac{5}{8}$	h $\frac{11}{24}$	i $\frac{11}{35}$	
6 a $\frac{11}{12}$	b $\frac{19}{56}$	c $\frac{1}{20}$	d $\frac{5}{6}$	e $\frac{5}{24}$
f $\frac{5}{18}$	g $\frac{23}{36}$	h $\frac{39}{44}$	i $\frac{34}{55}$	j $1\frac{8}{15}$
k $\frac{1}{4}$	l $\frac{11}{40}$	m $1\frac{7}{15}$	n $1\frac{13}{28}$	o $\frac{17}{63}$
p $\frac{23}{39}$	q $\frac{13}{60}$	r $1\frac{17}{42}$	s $\frac{2}{15}$	t $\frac{7}{33}$
7 a D	b C	c	d	e
8 a $\frac{7}{12}$	b $\frac{2}{5}$	c $\frac{19}{24}$	d $\frac{7}{30}$	e $\frac{7}{16}$
f $\frac{1}{3}$	g $1\frac{11}{12}$	h $1\frac{11}{12}$		
9 a $\frac{4}{15}$	b $\frac{11}{15}$			
10 $1\frac{1}{4}$ hours				
11 a $\frac{3}{4}$	b $\frac{7}{12}$	c 1		
12 a $\frac{2}{3}$	b $\frac{23}{30}$			

#### 4E Multiplying rational numbers

1 a $\frac{1}{8}$	b $\frac{2}{9}$	c $\frac{6}{25}$	d $\frac{20}{27}$	
e $\frac{11}{26}$	f $\frac{54}{77}$	g $\frac{121}{144}$	h $\frac{40}{81}$	
i $\frac{5}{24}$	j $\frac{4}{9}$	k $\frac{55}{72}$	l $\frac{25}{132}$	
m $\frac{49}{64}$	n $\frac{8}{39}$	o $\frac{9}{50}$	p $\frac{8}{21}$	
2 a $\frac{1}{6}$	b $\frac{1}{5}$	c $\frac{5}{14}$	d $\frac{1}{10}$	e $\frac{4}{5}$
f 1	g $\frac{1}{2}$	h $\frac{12}{35}$	i $\frac{4}{9}$	j $\frac{1}{7}$
k $\frac{1}{3}$	l $\frac{2}{5}$	m $\frac{4}{21}$	n $\frac{1}{20}$	o $\frac{1}{4}$
3 a 2	b $2\frac{1}{2}$	c $7\frac{1}{2}$	d $1\frac{1}{4}$	
4 a $\frac{1}{8}$	b $\frac{1}{2}$	c $\frac{25}{36}$	d $\frac{9}{14}$	e $\frac{2}{9}$
f $\frac{1}{5}$	g 16	h 28	i 45	j 18
k 15	l 72	m $4\frac{1}{2}$	n $13\frac{1}{2}$	o $\frac{5}{6}$
p $5\frac{1}{7}$				

5 Emily drank 2 litres. Tracy drank  $\frac{1}{2}$  litre.  
Jonathan drank  $\frac{1}{3}$  litre.

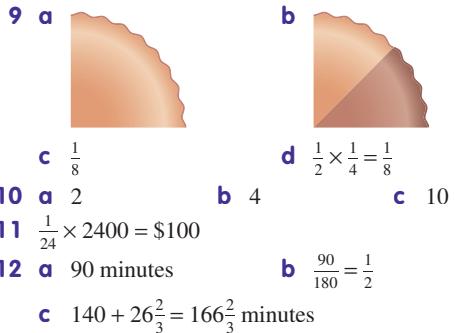
6 a Zoe 32 minutes b Sarah 35 minutes



Answers may vary.

b  $\frac{1}{9}$

8 a  $\frac{3}{4}$  b 36 litres



#### 4F Dividing rational numbers

1 a $\frac{4}{3}$	b $\frac{7}{2}$	c $\frac{3}{5}$	d $\frac{12}{5}$	e $\frac{5}{6}$
f $\frac{2}{9}$	g $\frac{3}{5}$	h $\frac{3}{10}$	i 5	j 12
k 5	l 11	m $\frac{1}{5}$	n $\frac{1}{20}$	o $\frac{1}{13}$
p 1				
2 a $\frac{3}{5}$	b $\frac{7}{31}$	c $\frac{2}{15}$	d $\frac{5}{22}$	e $\frac{7}{65}$
f $\frac{7}{27}$	g $\frac{8}{27}$	h $\frac{3}{16}$	i $\frac{9}{92}$	j $\frac{10}{69}$
3 a 1	b 1	c 1	d 1	e
d 1	e 1	f 1		
4 a $\frac{3}{8}$	b $\frac{16}{21}$	c $1\frac{4}{5}$	d $\frac{1}{8}$	e $\frac{24}{55}$
f $\frac{77}{108}$	g $4\frac{2}{3}$	h $\frac{3}{10}$	i $\frac{1}{2}$	j 1
k $\frac{2}{35}$	l $\frac{1}{32}$	m $\frac{1}{9}$	n $\frac{3}{11}$	o $\frac{7}{270}$
5 $\frac{5}{12}$	6 34	7 $\frac{4}{5}$		

#### 4G Fractions as ratios

1 a 4:5	b 5:4	c 5:9	d 9:4	e 1:2
2 a 5:3	b 3:5	c 1:5	d 5:1	e 1:3
f 5:4	g 1:8	h 5:9	i 1:9	j 3:9
3 a 5:7	b 7:5	c 5:12		
4 a 4:3	b 3:4	c 6:1	d 4:1	e 2:5
5 a 3:5	b 6:19	c 4:11	d 7:9	
e 1:5	f 9:4	g 3:4	h 3:10	
i 17:60	j 53:100	k 11:100	l 1:1000	
m 1:2000	n 7:24	o 5:12	p 1000:27	
q 7:12	r 13:24	s 3:5	t 1:22	
6 a C	b D	c D	d B	e B
7 a 1:2	b 1:3	c 1:2	d 1:3	e 3:4
f 5:6	g 3:2	h 3:2	i 5:3	j 1:2
k 3:7	l 3:4	m 4:5	n 3:2	o 5:6
p 10:3	q 7:8	r 3:4	s 7:12	t 2:3
8 a 1:3	b 2:1	c 2:3	d 64:32	e 48:64
2:6	4:2	4:6	32:16	24:32
3:9	8:4	6:9	16:8	12:16
4:12	16:8	8:12	8:4	6:8
5:15	20:10	16:24	2:1	3:4
9 a 2:3	b 1:3	c 5:3	d 2:5	e 1:4
f 8:3	g 3:40	h 1:5	i 9:4	j 8:13
k 5:4	l 5:4	m 1:5	n 11:2	o 2:5
p 16:9	q 3:10	r 3:2	s 7:3	t 1:6
10 a 4:5	b 2:3	c 3:2	d 4:1	e 4:3
f 5:4	g 1:8	h 10:3	i 8:1	j 9:2
11 a 1:2	b 5:6	c 1:2	d 4:5	e 16:9
f 10:9	g 3:10	h 5:1	i 5:6	j 4:3

- 12** a \$400, \$600      b \$750, \$250  
 c \$200, \$800      d \$500, \$500  
 e \$375, \$625      f \$625, \$375  
 g \$300, \$700      h \$900, \$100  
 i \$350, \$650      j \$450, \$550
- 13** a \$10 000, \$40 000      b \$15 000, \$35 000  
 c \$20 000, \$30 000      d \$25 000, \$25 000  
 e \$12 500, \$37 500
- 14** a  $2 \text{ m}^3$       b  $0.5 \text{ m}^3$       c  $1.6 \text{ m}^3$   
 d  $1.6 \text{ m}^3$       e  $1.2 \text{ m}^3$
- 15** a \$90 000, \$210 000, \$300 000  
 b \$180 000, \$180 000, \$240 000  
 c \$30 000, \$240 000, \$330 000  
 d \$150 000, \$180 000, \$270 000  
 e \$150 000, \$225 000, \$225 000
- 16**  $30^\circ, 60^\circ, 90^\circ$   
**17** a 90      b 75  
**18** a 120      b 45      c \$6      d \$19.50  
**19** \$12.50, \$10, \$7.50  
**20**  $96^\circ$

#### 4H Working with mixed numbers

- |  |                      |   |                      |
|--|----------------------|---|----------------------|
| <b>1</b> a $4\frac{7}{10}$                     | b $1\frac{1}{6}$     | c $3\frac{5}{12}$                       | d $13\frac{3}{8}$    |
| e $1\frac{1}{10}$                              | f $6\frac{1}{4}$     |   |                      |
| <b>2</b> a $1\frac{1}{3}$                      | b $1\frac{1}{2}$     | c $1\frac{5}{8}$                        | d $1\frac{7}{10}$    |
| e $3\frac{2}{3}$                               | f $1\frac{11}{12}$   |   |                      |
| <b>3</b> a $2\frac{17}{35}$                    | b $3\frac{7}{8}$     | c $3\frac{13}{18}$                      | d $3\frac{1}{24}$    |
| <b>4</b> a $7\frac{7}{12}; >7$                 | b $3\frac{2}{3}; >3$ | c $4\frac{1}{2}; >4$                    | d $4\frac{1}{4}; >4$ |
| e $3\frac{3}{10}; >3$                          | f $4\frac{1}{2}; >4$ |   |                      |
| <b>5</b> a $\frac{5}{6}$                       | b $1\frac{7}{32}$    | c $2\frac{1}{4}$                        | d $4\frac{19}{32}$   |
| e $7\frac{7}{10}$                              | f $7\frac{4}{15}$    | g $5\frac{5}{8}$                        | h $1\frac{17}{35}$   |
| i $3\frac{3}{10}$                              | j $2\frac{3}{5}$     | k 18                                    | l $16\frac{1}{2}$    |
| m $17\frac{1}{2}$                              | n 26                 | o $9\frac{33}{50}$                      | p $37\frac{5}{21}$   |
| q $34\frac{2}{3}$                              | r 10                 | s $2\frac{11}{12}$                      | t $26\frac{18}{35}$  |
| <b>6</b> a $1\frac{2}{3}$                      | b 2                  | c $\frac{9}{14}$                        | d 1                  |
| e $1\frac{9}{11}$                              | f $22\frac{1}{3}$    | g $\frac{1}{20}$                        | h $\frac{18}{35}$    |
| i $2\frac{10}{77}$                             | j 3                  | k $2\frac{29}{40}$                      | l $1\frac{1}{6}$     |
| m $1\frac{8}{15}$                              | n 4                  | o $2\frac{1}{14}$                       |                      |
| <b>7</b> $1\frac{5}{6} \text{ L}$              |                      | <b>8</b> $1\frac{5}{16} \text{ inches}$ |                      |
| <b>9</b> $1\frac{1}{12} \text{ months}$        |                      | <b>10</b> $6\frac{1}{4} \text{ days}$   |                      |
| <b>11</b> $\frac{3}{8} \text{ cup sugar}$      |                      |   |                      |
| $\frac{3}{8} \text{ cup margarine}$            |                      |   |                      |
| $\frac{3}{4} \text{ cup peanut butter}$        |                      |   |                      |
| 3 eggs   |                      |   |                      |
| $2\frac{1}{4} \text{ cups milk}$               |                      |   |                      |
| $3\frac{3}{4} \text{ cups self-raising flour}$ |                      |   |                      |
| $\frac{3}{8} \text{ teaspoon baking soda}$     |                      |   |                      |

- 12**  $\frac{3}{16} \text{ kg}$   
**13** a 22  
 b 96  
**14**  $3\frac{1}{10} \text{ kg}$   
**15** 7 containers

- 16** \$52.50  
**17** a  $3\frac{15}{16} \text{ cm}$   
 b  $2\frac{15}{16} \text{ cm}$   
**18** a  $6\frac{2}{3} \text{ laps}$   
 b 12 minutes  
**19**  $\frac{5}{12}, 10$

#### Chapter review

##### Fluency

- 1**  $\frac{3}{20}$   
**2** a  $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$   
 b  $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}$   
 c  $\frac{10}{22}, \frac{15}{33}, \frac{20}{44}$   
 d  $\frac{12}{14}, \frac{18}{21}, \frac{24}{28}$
- 3** a  $\frac{2}{4} = \frac{3}{6}$   
 b  $\frac{3}{9} = \frac{5}{15}$   
 c  $\frac{9}{12} = \frac{21}{28}$   
 d  $\frac{4}{10} = \frac{16}{40}$   
 e  $\frac{5}{20} = \frac{25}{100}$   
 f  $\frac{30}{100} = \frac{300}{1000}$
- 4** a Leo  
 b Ricky
- 5** a  $\frac{6}{7}$   
 b  $\frac{3}{4}$   
 c  $\frac{9}{10}$   
 d  $\frac{3}{4}$   
 e  $\frac{1}{2}$   
 f  $\frac{1}{3}$   
 g  $\frac{3}{5}$   
 h  $\frac{1}{40}$
- 6** a  $6\frac{4}{5}$   
 b  $3\frac{2}{3}$   
 c  $4\frac{3}{4}$   
 d  $1\frac{1}{5}$
- 7**  $\frac{3}{12} = \frac{1}{4}$   
**8**  $7.1 = \frac{1}{8}$       7.2 =  $\frac{1}{4}$       7.3 =  $\frac{5}{32}$       7.4 =  $\frac{5}{32}$       7.5 =  $\frac{7}{40}$       7.6 =  $\frac{11}{80}$   
**9** a  $5\frac{1}{3}$   
 b  $4\frac{1}{5}$   
 c  $1\frac{5}{7}$   
 d  $5\frac{1}{2}$   
 e  $1\frac{1}{4}$   
 f  $11\frac{3}{7}$   
 g  $6\frac{1}{9}$   
 h  $7\frac{1}{5}$
- 10** a  $\frac{11}{4}$   
 b  $\frac{79}{8}$   
 c  $\frac{26}{7}$   
 d  $\frac{35}{6}$   
 e  $\frac{11}{3}$   
 f  $\frac{99}{10}$   
 g  $\frac{47}{12}$   
 h  $\frac{17}{2}$
- 11** 4 trays
- 12** a  $\frac{23}{24}$   
 b  $\frac{17}{45}$   
 c  $1\frac{1}{15}$
- 13** a  $\frac{20}{63}$   
 b  $\frac{6}{25}$   
 c  $\frac{5}{8}$   
 d  $\frac{7}{36}$   
 e 44  
 f 22  
 g  $\frac{1}{6}$   
 h  $50\frac{50}{3} \text{ or } 16\frac{2}{3}$
- 14** a  $\frac{2}{5}$   
 b  $\frac{7}{2}$   
 c 9  
 d  $\frac{1}{12}$   
 e  $\frac{1}{124}$   
 f 4  
 g  $\frac{4}{23}$   
 h  $\frac{2}{17}$   
 i  $\frac{5}{37}$   
 j  $\frac{9}{22}$   
 k  $\frac{8}{29}$   
 l  $\frac{5}{21}$
- 15** a  $\frac{3}{2} \text{ or } 1\frac{1}{2}$   
 b  $\frac{4}{3} \text{ or } 1\frac{1}{3}$   
 c  $\frac{7}{36}$   
 d  $\frac{3}{70}$
- 16** a  $4\frac{17}{24}$   
 b  $1\frac{13}{14}$   
 c  $3\frac{7}{9}$   
 d  $2\frac{2}{3}$   
 e  $6\frac{13}{40}$   
 f  $3\frac{43}{72}$   
 g  $1\frac{47}{70}$   
 h  $\frac{76}{99}$
- 17** a  $3\frac{2}{3}$   
 b  $6\frac{5}{18}$
- 18** a 14  
 b 64
- 19** a  $\frac{3}{5}$   
 b  $13\frac{2}{3}$   
 c  $1\frac{1}{2}$
- 20** a  $\frac{5}{16}$   
 b  $1\frac{3}{5}$   
 c  $1\frac{53}{115}$   
 d  $\frac{217}{267}$   
 e  $1\frac{73}{147}$

**21** \$10, \$15

**22** \$880

### Problem solving

**1** \$150

**2** 2 kg

**3**  $\frac{1}{4}^\circ\text{C}$

**4** Yes

**5** **a**  $\frac{1}{6}$

**b**  $\frac{1}{4}$

**c**  $\frac{5}{9}$

**d**  $\frac{48}{25}$

**6** Alice \$80, Will \$64, James \$72, Jessica and Alex \$52

**7**  $\frac{12}{21}$

**8** Tomas

**9**  $\frac{1}{2} + \frac{1}{4} + \frac{1}{20}$

**10** 2 h 11 min

**11** Approx. 25 years

**12** 2961 m

**13** Mr Thompson \$3000, Mr Goody \$2500

**14**  $4\frac{1}{2}$  minutes

**15**  $\frac{3}{5}$  of the standard spots were filled.

**16** **a** 4 complete surfboards

**b** 2.25 kg of resin, 0 catalyst

## CHAPTER 5

### Geometry

#### Are you ready?

**1**  $27^\circ$

**2** **a** C

**b** B

**3** **a**  $45^\circ$  and  $55^\circ$

**b**  $30^\circ$  and  $40^\circ$

**c**  $90^\circ$  and  $100^\circ$

**4** **A**  $52^\circ$

**B**  $38^\circ$

**C**  $97^\circ$

**5** Check with your teacher.

**6** **a** B

**b** C

**c** F

**d** E

**e** A

**f** D

**7** **a** PQR or RQP

**b** KDL or LDK

**8** **a** Isosceles

**b** Equilateral

**c** Scalene

**9** **i** Triangle *a* is obtuse-angled.

**ii** Triangle *b* is acute-angled.

**iii** Triangle *c* is right-angled.

#### 5A Measuring angles

**1** **a**  $40^\circ$

**b**  $81^\circ$

**c**  $75^\circ$

**d**  $13^\circ$

**e**  $142^\circ$

**f**  $107^\circ$

**g**  $162^\circ$

**h**  $103^\circ$

**i**  $240^\circ$

**j**  $201^\circ$

**k**  $316^\circ$

**l**  $356^\circ$

**2** Discuss with your teacher as answers will vary.

**3** **a** B

**b** B

**4** **a**  $45^\circ$

**b**  $155^\circ$

**c**  $35^\circ$

**d**  $180^\circ$

**e**  $77^\circ$

**f**  $63^\circ$

**5** **a** Difficult, as angle is approximately  $7^\circ$

**b** Same as bottom section

**6** **a, b, c, e, f, h** — various answers

**d, g**

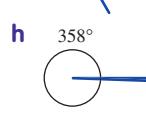
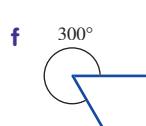
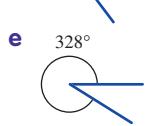
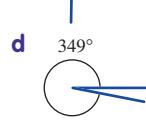
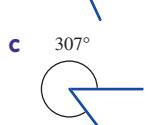
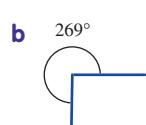
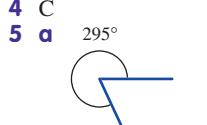
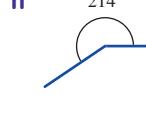
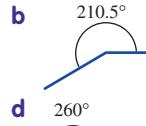
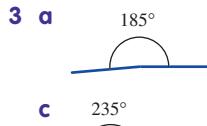
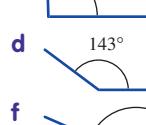
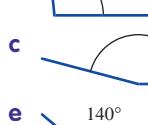
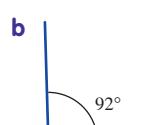
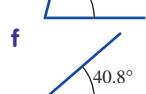
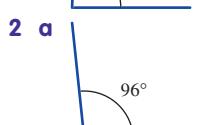
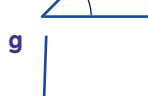
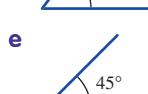
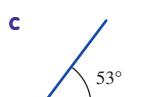
#### Diagram Measured angle

1	$190^\circ$
2	$130^\circ$
3	$110^\circ$
4	$40^\circ$
5	$300^\circ$
6	$200^\circ$
7	$140^\circ$
8	$290^\circ$

#### 5B Constructing angles with a protractor

**1** **a**

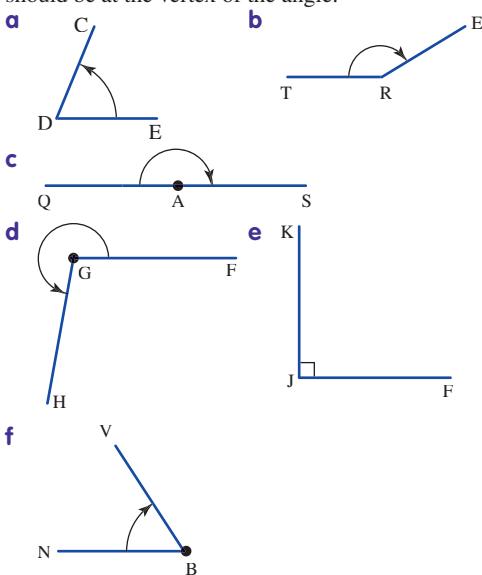
**b**



## 5C Types of angles and naming angles

- 1** a Acute      b Reflex      c Obtuse  
 d Obtuse      e Straight      f Acute  
 g Reflex      h Right      i Acute  
 j Revolution      k Obtuse      l Acute  
 m Acute      n Reflex      o Acute  
 p Reflex
- 2** a  $4^\circ, 3^\circ, 45^\circ, 65^\circ, 69^\circ$       b  $2; 123^\circ, 165^\circ$   
 c Yes,  $90^\circ$       d Yes,  $180^\circ$   
 e  $4; 234^\circ, 300^\circ, 270^\circ, 210^\circ$       f Yes,  $360^\circ$   
 g  $25^\circ$       h  $57^\circ$
- 3** a Right      b Straight line      c Obtuse  
 d Acute      e Reflex      f Revolution
- 4** a  $\angle XUV$  or  $\angle VUX$       b  $\angle PQR$  or  $\angle RQP$   
 c  $\angle SDX$  or  $\angle XDS$       d  $\angle GPZ$  or  $\angle ZPG$   
 e  $\angle PFH$  or  $\angle HFP$       f  $\angle ZRO$  or  $\angle ORZ$

**5** Answers will vary; however, in each case, the middle letter should be at the vertex of the angle.



- 6** a Acute:  $\angle NOM, \angle MOY$ ;  
 obtuse:  $\angle MOX$   
 b Acute:  $\angle PQS, \angle RQO$ ;  
 obtuse:  $\angle SQR, \angle SQO, \angle RQP$   
 c Acute:  $\angle MLN, \angle NLO, \angle OLP, \angle PLK, \angle NLP, \angle OLK$ ;  
 obtuse:  $\angle MLO, \angle MLP, \angle NLK$   
 d Acute:  $\angle POR, \angle ROT, \angle TOU, \angle UOV$ ;  
 obtuse:  $\angle POU, \angle ROU, \angle ROV$   
 e Acute:  $\angle YOX, \angle WOQ, \angle POQ$ ;  
 obtuse:  $\angle XOW, \angle YOP, \angle YOW, \angle XOP, \angle YOQ, \angle POW$   
 f Acute:  $\angle VOT, \angle TOR, \angle ROP, \angle POZ, \angle ZOW, \angle WOW$ ;  
 obtuse:  $\angle VOR, \angle TOP, \angle ROZ, \angle POW, \angle ZOV, \angle WOT$   
 g Acute:  $\angle FOG, \angle GOK$ ;  
 obtuse:  $\angle DOG$   
 h Acute:  $\angle JOH, \angle KOH, \angle POK$ ;  
 obtuse:  $\angle JOK, \angle HOP$
- 7** a Right      b Acute      c Revolution  
 d Obtuse
- 8** a Acute      b Right      c Obtuse  
 d Straight      e Reflex      f Revolution
- 9** a i Acute      ii Obtuse      iii Straight      iv Right  
 v Obtuse      vi Obtuse  
 b Various answers

- 10** a Angle 1: obtuse; angle 2: acute; angle 3: straight

- b Angle 1:  $140^\circ$ ; angle 2:  $48^\circ$ ; angle 3:  $180^\circ$   
 c Various answers

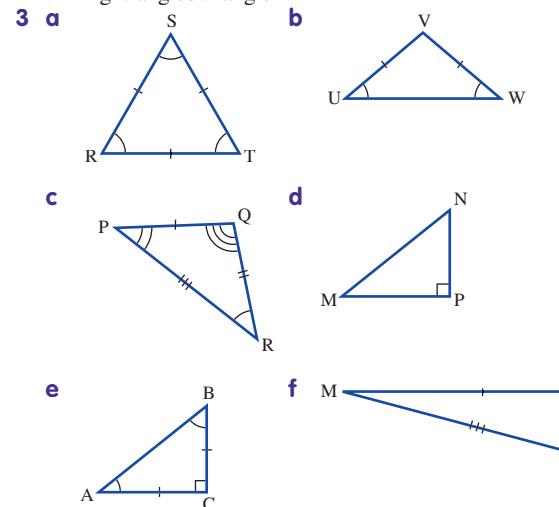
**11** In each case the middle letter should be at the vertex of the angle.

- a i Acute:  $\angle HUT$   
 ii Right:  $\angle ABC, \angle GHI$   
 iii Obtuse:  $\angle DEF$   
 iv Reflex:  $\angle MNO$
- b i  $\angle DEF$   
 ii  $\angle ABC, \angle XWY$   
 iii  $\angle PQR$   
 iv Various answers possible.

## 5D Triangles

- 1** a  $\triangle ABC$  Scalene      b  $\triangle LMN$  Equilateral  
 c  $\triangle XYZ$  Isosceles      d  $\triangle PQR$  Isosceles  
 e  $\triangle RST$  Scalene      f  $\triangle EFG$  Isosceles

- 2** a Right-angled triangle  
 b Acute-angled triangle  
 c Acute-angled triangle  
 d Acute-angled triangle  
 e Obtuse-angled triangle  
 f Right-angled triangle



- 4** a A      b B  
 5 a D      b A  
 6 a  $x = 57^\circ$       b  $g = 125^\circ$       c  $t = 44^\circ$   
 d  $k = 60^\circ$       e  $f = 93^\circ$       f  $z = 90^\circ$   
 7 a  $d = 45^\circ$       b  $b = 65^\circ$       c  $a = 50^\circ$   
 8 a  $c = 70^\circ$       b  $e = 76^\circ$       c  $n = 52^\circ$   
 9 a  $u = 66^\circ$       b  $k = 45^\circ$       c  $d = 76^\circ$   
 d  $t = 74^\circ$       e  $f = 57^\circ$       f  $p = 70^\circ$

- 10** a  $p = 50^\circ; n = 70^\circ$       b  $a = 22^\circ; b = 28^\circ$   
 c  $x = 80^\circ; y = 50^\circ$       d  $t = 35^\circ; s = 55^\circ$   
 e  $b = 46^\circ$       f  $n = 21^\circ$   
 g  $m = 60^\circ$       h  $t = 28^\circ$

**11** Isosceles/acute-angled triangle

**12** Check with your teacher.

**13** Check with your teacher.

- 14** a 5      b 6  
 c 2

- 15** c 10 cm (a, b, d; check with your teacher)

- 16** a  $b = 20^\circ$       b  $p = 60^\circ$       c  $k = 30^\circ$       d  $p = 56^\circ$   
 17 a  $70^\circ$       b  $156^\circ$       c  $50^\circ$       d  $64^\circ$

**18** Check with your teacher.

- 19** **a** Yes, angles add to  $180^\circ$ .  
**b** No, angles add to  $175^\circ$ .  
**c** No, angles add to  $185^\circ$ .  
**d** Yes, angles add to  $180^\circ$ .

**20** The sum of the angles will be greater than  $180^\circ$ .

- 21** **a** 42  
**b** 12 scalene triangles, 6 equilateral triangles and 24 isosceles triangles

**c** Yes

**d** The largest angle is always opposite the longest side (likewise, the smallest angle is opposite the shortest side and the middle size angle is opposite the middle length side). Also, angles that are opposite to sides of equal length are equal in size.

**e** The sum of any two side lengths must be greater than the length of the third side.

### 5E Quadrilaterals and their properties

- |                          |                                  |
|--------------------------|----------------------------------|
| <b>1</b> <b>a</b> Square | <b>b</b> Trapezium               |
| <b>c</b> Kite            | <b>d</b> Rhombus                 |
| <b>e</b> Rectangle       | <b>f</b> Irregular quadrilateral |

- |                     |            |            |
|---------------------|------------|------------|
| <b>2</b> <b>a</b> E | <b>b</b> A | <b>c</b> E |
|---------------------|------------|------------|

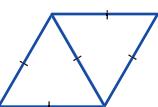
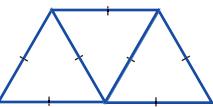
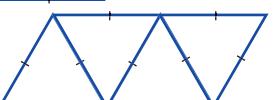
- |                                   |                         |                         |
|-----------------------------------|-------------------------|-------------------------|
| <b>3</b> <b>a</b> $t = 138^\circ$ | <b>b</b> $b = 85^\circ$ | <b>c</b> $t = 90^\circ$ |
| <b>d</b> $t = 202^\circ$          | <b>e</b> $p = 92^\circ$ | <b>f</b> $m = 90^\circ$ |
| <b>g</b> $s = 53^\circ$           | <b>h</b> $c = 66^\circ$ | <b>i</b> $k = 67^\circ$ |

- 4** **a**  $m = 78^\circ$  (opposite angles of parallelogram equal)  
**b**  $u = 75^\circ$  (opposite angles of parallelogram equal)  
**c**  $t = 132^\circ$  (opposite angles of a kite equal)  
**d**  $f = 108^\circ$  (opposite angles of a kite equal)  
**e**  $p = 63^\circ$  (opposite angles of parallelogram equal)  
**f**  $z = 73^\circ$  (opposite angles of parallelogram equal)

- 5** **a**  $m = 113^\circ$
- |  |
|--|
| <b>b</b> $c = 98^\circ$ , $d = 82^\circ$ |
|--|
- c**  $t = 116^\circ$
- |   |
|---|
| <b>d</b> $p = 52^\circ$ , $m = 106^\circ$ |
|---|
- e**  $t = 103^\circ$ ;  $x = 91^\circ$
- |  |
|--|
| <b>f</b> $m = 65^\circ$ ; $p = 115^\circ$ ; $n = 65^\circ$ |
|--|

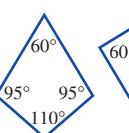
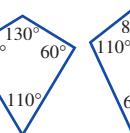
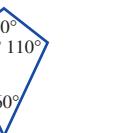
- 6** D      **7** E      **8** **a** True      **b** True      **c** False  
**d** True      **e** True

**9** D

- 10** **a**   
**b**   
**c** 

- 11**   
**12** Check with your teacher.

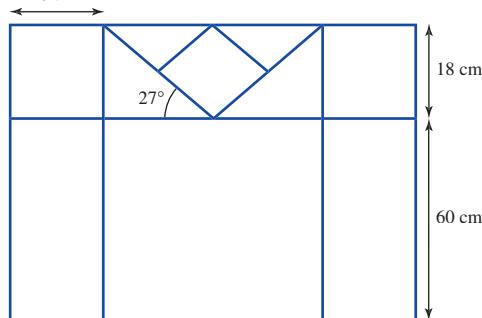
- 13**  $p = 61^\circ$       **14**  $115^\circ$       **15**  $45^\circ$  and  $135^\circ$

- 16**   
  


- 17** **a** Yes, angles add up to  $360^\circ$ .  
**b** No, angles add up to  $270^\circ$ .  
**18** **a**  $130^\circ$       **b** Infinite number  
**c** Various answers  
**19** **a** Two smaller obtuse-angled isosceles triangles; one larger size obtuse-angled isosceles triangle and two right-angled scalene triangles.

- b** Two types: rectangles (of different sizes) and a rhombus.

**c**

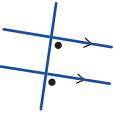
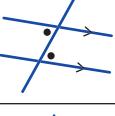
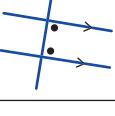


- d** In both right-angled triangles angles are:  $27^\circ$ ,  $63^\circ$  and  $90^\circ$ ; in all isosceles triangles angles are:  $126^\circ$ ,  $27^\circ$ ,  $27^\circ$ ; in a rhombus angles are:  $126^\circ$ ,  $54^\circ$ ,  $126^\circ$ ,  $54^\circ$ ; in all rectangles all angles are  $90^\circ$  each.

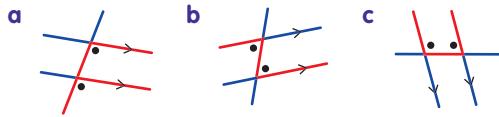
- e** Various answers

### 5F Parallel and perpendicular lines

**1**

Picture	Type of angle	Associated shape	Rule
	Corresponding	F	Are equal
	Alternate	Z	Are equal
	Co-interior	C	Add to 180°

**2**



**3**

- a** C      **b** D      **c** A

- 4** **a**  $a$  and  $d$ ,  $c$  and  $b$ ,  $e$  and  $h$ ,  $f$  and  $g$   
**b**  $a$  and  $e$ ,  $c$  and  $g$ ,  $b$  and  $f$ ,  $d$  and  $h$   
**c**  $c$  and  $e$ ,  $d$  and  $f$   
**d**  $c$  and  $f$ ,  $d$  and  $e$

- 5** **a** C      **b** D      **c** B  
**d** A      **e** D

- 6** **a** i Corresponding angles

$$\text{ii} \quad p = 60^\circ$$

- b** i Alternate angles

$$\text{ii} \quad q = 45^\circ$$

- c** i Vertically opposite angles

$$\text{ii} \quad s = 65^\circ$$

- d** i Corresponding angles

$$\text{ii} \quad t = 72^\circ$$

- e** i Co-interior angles

$$\text{ii} \quad m = 110^\circ$$

- f** i Co-interior angles

$$\text{ii} \quad n = 48^\circ$$

- 7** **a**  $y = 116^\circ$ , corresponding

- b**  $z = 38^\circ$ , alternate

- c**  $b = 136^\circ$ , supplementary

- d**  $g = 68^\circ$ , vertically opposite

- e**  $h = 90^\circ$ , corresponding

- f**  $k = 120^\circ$ , alternate

- 8** a i  $b = 123^\circ$  ii  $x = 43^\circ$  iii  $y = 28^\circ$   
 iv  $z = 50^\circ$  v  $p = 70^\circ$  vi  $q = 45^\circ$

b In iii, AH  $\perp$  GC  
 In v, PW  $\perp$  VR  
 In vi, NR  $\perp$  QV.

- 9  $x = 45^\circ$  10  $y = 55^\circ$  11  $60^\circ$   
 12 a  $130^\circ$  b  $50^\circ$   
 13 No, as  $48^\circ + 133^\circ \neq 180^\circ$ . For parallel lines, co-interior angles need to add to  $180^\circ$  (or alternate angles must be equal).

14 Check with teacher.

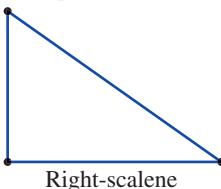
### Chapter review

#### Fluency

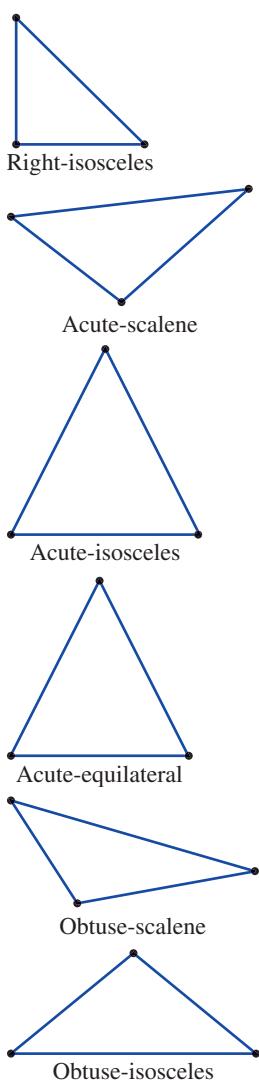
- |   |                  |                  |
|---|------------------|------------------|
| 1 a $42^\circ$  | b $105^\circ$    | c $298^\circ$    |
| 2 Constructions   |                  |                  |
| 3 a Revolution  | b Right angle    | c Straight line  |
| d Obtuse  | e Reflex         | f Acute          |
| 4 C   | 5 A              |                  |
| 6 B   | 7 D              |                  |
| 8 $\angle EDF$ or $\angle FDE$  |                  |                  |
| 9 a Equilateral triangle  |                  |                  |
| b Scalene triangle  |                  |                  |
| c Isosceles triangle  |                  |                  |
| 10 a Obtuse-angled triangle   |                  |                  |
| b Acute-angled triangle   |                  |                  |
| c Right-angled triangle   |                  |                  |
| 11 a $t = 67^\circ$   | b $b = 28^\circ$ | c $x = 75^\circ$ |
| 12 a $p = 56^\circ$   | b $m = 64^\circ$ | c $n = 60^\circ$ |
| 13 a Rectangle  |                  |                  |
| b Parallelogram   |                  |                  |
| c Trapezium   |                  |                  |
| d Kite  | e Square         | f Rhombus        |
| 14 a $75^\circ$   | b $55^\circ$     | c $42^\circ$     |
| 15 a $66^\circ$   | b $112^\circ$    | c $126^\circ$    |
| 16 a $x = 15^\circ$ , vertically opposite angles are equal  |                  |                  |
| b $x = 50^\circ$ , corresponding angles are equal, angle sum of a straight line is $180^\circ$ ; $y = 25^\circ$ , corresponding angles are equal  |                  |                  |
| c $y = 100^\circ$ , co-interior angles add to $180^\circ$ ; $x = 16^\circ$ , angles opposite the equal sides of an isosceles triangle are equal, angle sum of a triangle is $180^\circ$ |                  |                  |
| 17 a A, B   | b A              |                  |
| c B, D  | d D              |                  |
| 18 a i Alternate angles   |                  |                  |
| ii $x = 60^\circ$   |                  |                  |
| b i Vertically opposite angles  |                  |                  |
| ii $y = 135^\circ$  |                  |                  |
| c i Vertically opposite angles then co-interior angles, then vertically opposite angles   |                  |                  |
| ii $t = 50^\circ$   |                  |                  |

#### Problem solving

- 1  $67^\circ$  2  $115^\circ$   
 3 As an equilateral triangle has 3 congruent angles, right-equilateral and obtuse-equilateral are not possible.  
 Examples of the other triangles:



Right-scalene

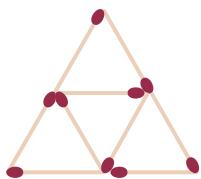


- 4 As the base and height of the triangle do not change as X moves along the fence line, the area of the triangle won't change. The volume of soil needed won't change.  
 5 When he first arranges the timber, the opposite sides are congruent. He has made a parallelogram. A rectangle can be thought of as a parallelogram with equal diagonals. Hence, it is necessary to measure the diagonals to verify that a rectangle has been created.

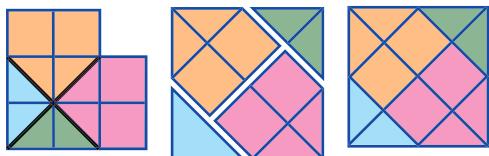
6 4, 5 and 6

7 24

8



9



- 10**  $165^\circ$   
**11** 4  
**12** BOE, BOD, BOC, COE, COD, DOE.  
**13**  $75^\circ$   
**14** **a**  $15^\circ$   
**b**  $AD \perp CD$ ,  $CD \perp BC$ ,  $AB \perp AD$ ,  $AB \perp BC$   
**15**  $35^\circ$

## CHAPTER 6

### Decimals

#### Are you ready?

- |                          |                |                    |
|--------------------------|----------------|--------------------|
| <b>1</b> <b>a</b> 0.3    | <b>b</b> 0.05  | <b>c</b> 0.007     |
| <b>2</b> <b>a</b> 1.35   | <b>b</b> 0.484 |                    |
| <b>3</b> <b>a</b> 5      | <b>b</b> 18    | <b>c</b> 57        |
| <b>4</b> <b>a</b> 6.7    | <b>b</b> 1.37  | <b>c</b> 14.02     |
| <b>5</b> <b>a</b> 3.96   | <b>b</b> 5.576 | <b>c</b> 26.77     |
| <b>6</b> <b>a</b> 5.3    | <b>b</b> 0.32  | <b>c</b> 5.17      |
| <b>7</b> <b>a</b> 5.55   | <b>b</b> 0.45  | <b>c</b> 7.496     |
| <b>8</b> <b>a</b> 5.0    | <b>b</b> 11.04 | <b>c</b> 63.55     |
| <b>9</b> <b>a</b> 14.56  | <b>b</b> 35.1  | <b>c</b> 16.039    |
| <b>10</b> <b>a</b> 2.3   | <b>b</b> 4.16  | <b>c</b> 3.147     |
| <b>11</b> <b>a</b> 1.435 | <b>b</b> 0.271 | <b>c</b> 0.038 664 |

#### 6A Place value and comparing decimals

- |   |   |                            |                             |
|---|---|----------------------------|-----------------------------|
| <b>1</b> <b>a</b> $\frac{2}{10}$  | <b>b</b> $\frac{2}{100}$                                  | <b>c</b> $\frac{2}{100}$   | <b>d</b> $\frac{2}{10}$     |
| <b>e</b> $\frac{2}{1000}$   | <b>f</b> $\frac{2}{1000}$                                 | <b>g</b> $\frac{2}{10000}$ | <b>h</b> $\frac{2}{100000}$ |
| <b>i</b> 20   | <b>j</b> 2  | <b>k</b> $\frac{2}{10000}$ | <b>l</b> $\frac{2}{100000}$ |
| <b>2</b> <b>a</b> $\frac{9}{10}$  | <b>b</b> $\frac{9}{10}$                                   | <b>c</b> $\frac{9}{100}$   | <b>d</b> $\frac{9}{100}$    |
| <b>e</b> $\frac{9}{100}$  | <b>f</b> $\frac{9}{1000}$                                 | <b>g</b> $\frac{9}{10000}$ | <b>h</b> $\frac{9}{100000}$ |
| <b>i</b> 9  | <b>j</b> 9  | <b>k</b> 900               | <b>l</b> 90                 |
| <b>3</b> <b>a</b> Four units, 4; one tenth, $\frac{1}{10}$ ; $4 + \frac{1}{10}$   |   |                            |                             |
| <b>b</b> One unit, 1; eight tenths, $\frac{8}{10}$ ; five hundredths, $\frac{5}{100}$ ; $1 + \frac{8}{10} + \frac{5}{100}$  |   |                            |                             |
| <b>c</b> Two tenths, $\frac{2}{10}$ ; seven hundredths, $\frac{7}{100}$ ; one thousandth, $\frac{1}{1000}$ ; $\frac{2}{10} + \frac{7}{100} + \frac{1}{1000}$                                      |   |                            |                             |
| <b>d</b> Nine units, 9; two hundredths, $\frac{2}{100}$ ; $9 + \frac{2}{100}$   |   |                            |                             |
| <b>e</b> One ten, 10; six units, 6; one thousandth, $\frac{1}{1000}$ ; $10 + \frac{6}{1000}$  |   |                            |                             |
| <b>f</b> Three units, 3; four tenths, $\frac{4}{10}$ ; two thousandths, $\frac{2}{1000}$ ; seven hundred thousandths, $\frac{7}{100000}$ ; $3 + \frac{4}{10} + \frac{2}{1000} + \frac{7}{100000}$ |   |                            |                             |
| <b>4</b> <b>a</b> $\frac{4}{10}$  | <b>b</b> $2 + \frac{7}{10}$                               |                            |                             |
| <b>c</b> $6 + \frac{8}{10}$   | <b>d</b> $5 + \frac{2}{10} + \frac{3}{100}$               |                            |                             |
| <b>e</b> $\frac{7}{10} + \frac{6}{100} + \frac{3}{1000}$  | <b>f</b> $2 + \frac{1}{10} + \frac{8}{1000}$              |                            |                             |
| <b>g</b> $10 + 9 + \frac{9}{10} + \frac{1}{100}$  | <b>h</b> $\frac{1}{10} + \frac{1}{100} + \frac{1}{10000}$ |                            |                             |
| <b>i</b> $7 + \frac{2}{10} + \frac{9}{100} + \frac{6}{1000} + \frac{4}{10000}$  |   |                            |                             |
| <b>j</b> $\frac{3}{10} + \frac{3}{100} + \frac{2}{10000} + \frac{4}{100000}$  |   |                            |                             |
| <b>k</b> $300 + \frac{3}{100}$  |   |                            |                             |
| <b>l</b> $10 + 2 + \frac{2}{10} + \frac{7}{100} + \frac{6}{1000} + \frac{4}{10000} + \frac{3}{1000000}$   |   |                            |                             |

- |  |  |  |            |            |           |
|--|--|--|------------|------------|-----------|
| <b>5</b> <b>a</b> $(2 \times 1) + (4 \times \frac{1}{10}) + (7 \times \frac{1}{100})$                    | <b>b</b> $(3 \times 1) + (6 \times \frac{1}{10}) + (9 \times \frac{1}{100})$                             | <b>c</b> $(1 \times 1) + (2 \times \frac{1}{10}) + (5 \times \frac{1}{100})$   |            |            |           |
| <b>d</b> $(5 \times 10) + (6 \times 1) + (1 \times \frac{1}{100})$                                       | <b>e</b> $(3 \times 10) + (9 \times 1) + (1 \times \frac{1}{100})$                                       | <b>f</b> $(1 \times 10) + (6 \times 1) + (7 \times \frac{1}{100})$   |            |            |           |
| <b>g</b> $(7 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (3 \times \frac{1}{1000})$ | <b>h</b> $(5 \times 1) + (9 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$ | <b>i</b> $(1 \times 10) + (3 \times 1) + (4 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$ |            |            |           |
| <b>j</b> $(3 \times \frac{1}{10}) + (6 \times \frac{1}{1000}) + (2 \times \frac{1}{10000})$              | <b>k</b> $(1 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (5 \times \frac{1}{10000})$               | <b>l</b> $(5 \times \frac{1}{10}) + (2 \times \frac{1}{10000})$  |            |            |           |
| <b>6</b> <b>a</b> C  | <b>b</b> D   |  |            |            |           |
| <b>7</b>   |  |  |            |            |           |
|  | Tens   | Units  | Tenths     | Hundredths | Thousands |
| <b>a</b> 0.205   | 0  | 0  | .          | 2          | 0         |
| <b>b</b> 1.06  | 0  | 1  | .          | 0          | 6         |
| <b>c</b> 74.108  | 7  | 4  | .          | 1          | 0         |
| <b>d</b> 0.108   | 0  | 0  | .          | 1          | 0         |
| <b>e</b> 50.080  | 5  | 0  | .          | 0          | 8         |
| <b>8</b> <b>a</b> 0.32   | <b>b</b> 0.91  | <b>c</b> 0.59  |            |            |           |
| <b>d</b> 0.99  | <b>e</b> 0.901   | <b>f</b> 0.666   |            |            |           |
| <b>g</b> 0.1440  | <b>h</b> 0.039 90  | <b>i</b> 2.559   |            |            |           |
| <b>j</b> 10.0233   | <b>k</b> 0.110 49  | <b>l</b> 0.102 36  |            |            |           |
| <b>9</b> <b>a</b> >  | <b>b</b> <   | <b>c</b> <   | <b>d</b> < |            |           |
| <b>e</b> <   | <b>f</b> <   | <b>g</b> <   | <b>h</b> > |            |           |
| <b>i</b> >   | <b>j</b> <   | <b>k</b> <   | <b>l</b> > |            |           |
| <b>10</b> <b>a</b> 0.17, 0.21, 0.33, 0.39, 0.45  |  |  |            |            |           |
| <b>b</b> 0.12, 0.16, 0.19, 0.22, 0.27  |  |  |            |            |           |
| <b>c</b> 0.314, 0.391, 0.413, 0.420, 0.502   |  |  |            |            |           |
| <b>d</b> 0.613, 0.624, 0.677, 0.690, 0.710   |  |  |            |            |           |
| <b>e</b> 0.803, 0.807, 0.811, 0.821, 0.902   |  |  |            |            |           |
| <b>f</b> 0.1002, 0.1033, 0.1164, 0.1196, 0.1245  |  |  |            |            |           |
| <b>g</b> 0.9087, 0.9189, 0.9812, 0.9864, 0.9943  |  |  |            |            |           |
| <b>h</b> 0.4004, 0.4076, 0.4100, 0.4139, 0.4826  |  |  |            |            |           |
| <b>i</b> 4.0292, 4.5097, 4.6031, 4.6249, 4.802   |  |  |            |            |           |
| <b>j</b> 13.0229, 13.0291, 13.0294, 13.0299, 13.0929   |  |  |            |            |           |
| <b>k</b> 0.003, 0.004 65, 0.005 02, 0.0056, 0.009  |  |  |            |            |           |
| <b>l</b> 0.507, 0.5079, 0.595, 0.61, 0.617   |  |  |            |            |           |
| <b>11</b> <b>a</b> 0.48, 0.39, 0.36, 0.31, 0.19  |  |  |            |            |           |
| <b>b</b> 0.38, 0.35, 0.27, 0.16, 0.02  |  |  |            |            |           |
| <b>c</b> 0.99, 0.97, 0.95, 0.91, 0.90  |  |  |            |            |           |
| <b>d</b> 0.29, 0.13, 0.09, 0.07, 0.02  |  |  |            |            |           |
| <b>e</b> 1.291, 1.288, 1.279, 1.273, 1.264   |  |  |            |            |           |
| <b>f</b> 0.491, 0.442, 0.437, 0.433, 0.406   |  |  |            |            |           |
| <b>g</b> 0.390, 0.372, 0.318, 0.317, 0.309   |  |  |            |            |           |
| <b>h</b> 0.602, 0.591, 0.573, 0.556, 0.502   |  |  |            |            |           |
| <b>i</b> 0.8889, 0.8823, 0.8448, 0.8217, 0.8207  |  |  |            |            |           |
| <b>j</b> 0.7695, 0.7657, 0.6024, 0.1079, 0.0307  |  |  |            |            |           |
| <b>k</b> 1.843 21, 1.702 96, 1.486 59, 1.486 13, 1.349 54  |  |  |            |            |           |
| <b>l</b> 12.9092, 12.392, 12.289 50, 12.208 64, 12.002 36  |  |  |            |            |           |
| <b>12</b> E  | <b>13</b> D  | <b>14</b> E  |            |            |           |

- 15** a i Tenths      ii Yes  
 b i Units      ii Yes  
 c i Thousandths      ii No  
 d i Hundredths      ii No  
 e i Tenths      ii No  
 f i Tenths      ii Yes  
 g i Units      ii No  
 h i Hundredths      ii Yes  
 i i Units      ii Yes  
 j i Tens      ii Yes
- 16** a F      b T      c F  
 d T      e F      f T
- 17** a Shelley, 0.21 seconds  
 b Carolyn, 0.06 seconds  
 c Mara, 0.11 seconds  
 d Shelley, Carolyn, Robyn  
 e Carolyn, Shelley, Kyah  
 f Mara, Jenika, Shelley  
 g Yes, Shelley

## 6B Converting decimals to fractions and fractions to decimals

- |                            |                       |                         |                        |
|----------------------------|-----------------------|-------------------------|------------------------|
| <b>1</b> a $\frac{3}{10}$  | b $\frac{1}{2}$       | c $\frac{9}{10}$        | d $\frac{21}{100}$     |
| e $\frac{2}{5}$            | f $\frac{4}{5}$       | g $\frac{6}{25}$        | h $\frac{11}{25}$      |
| i $\frac{49}{100}$         | j $\frac{63}{100}$    | k $\frac{251}{500}$     | l $\frac{617}{1000}$   |
| m $\frac{3}{25}$           | n $\frac{3}{10}$      | o $\frac{16}{25}$       | p $\frac{7}{25}$       |
| q $\frac{441}{500}$        | r $\frac{591}{625}$   | s $\frac{9209}{10000}$  | t $\frac{4621}{10000}$ |
| u $\frac{3}{4}$            | v $\frac{3}{25}$      | w $\frac{143}{500}$     | x $\frac{117}{250}$    |
| <b>2</b> a $1\frac{3}{10}$ | b $1\frac{3}{5}$      | c $2\frac{7}{10}$       | d $9\frac{2}{5}$       |
| e $1\frac{1}{5}$           | f $2\frac{4}{5}$      | g $4\frac{1}{5}$        | h $8\frac{1}{2}$       |
| i $2\frac{13}{100}$        | j $6\frac{12}{25}$    | k $5\frac{27}{100}$     | l $19\frac{91}{500}$   |
| m $12\frac{21}{50}$        | n $3\frac{3}{20}$     | o $6\frac{1}{4}$        | p $9\frac{7}{50}$      |
| q $12\frac{843}{1000}$     | r $16\frac{341}{500}$ | s $2\frac{4917}{10000}$ | t $4\frac{1693}{5000}$ |
| u $37\frac{41}{200}$       | v $18\frac{129}{200}$ | w $24\frac{69}{200}$    | x $100\frac{3}{625}$   |
| 3 a B                      | b A                   | c C                     |                        |
| d D                        | e E                   |                         |                        |
| 4 a 0.75                   | b 0.5                 | c 0.8                   |                        |
| d 0.05                     | e 0.2                 | f 0.25                  |                        |
| g 0.375                    | h 0.02                | i 0.32                  |                        |
| 5 a 0.375                  | b 0.875               | c 0.0625                |                        |
- 6** If the denominator only contains factors of 2 or 5, the decimal will terminate.

## 6C Rounding and repeating decimals

- |                 |          |         |          |
|-----------------|----------|---------|----------|
| <b>1</b> a 0.32 | b 0.86   | c 1.25  | d 13.05  |
| e 7.13          | f 100.81 | g 71.26 | h 0.01   |
| i 0.19          | j 19.70  | k 0.40  | l 1.00   |
| <b>2</b> a 0.4  | b 0.9    | c 9.3   | d 25.3   |
| e 300.1         | f 12.8   | g 99.9  | h 8.9    |
| i 17.6          | j 0.9    | k 94.0  | l 1.0    |
| <b>3</b> a 2.39 | b 14.0   | c 0.03  | d 0.8765 |
| e 64.2952       | f 0.382  | g 96.3  | h 3.04   |
| i 8.90          | j 47.880 | k 0.10  | l 0.49   |
| <b>4</b> a C    | b D      | c D     | d A      |
| <b>5</b> a 11   | b 8      | c 4     | d 93     |
| e 112           | f 22     | g 42    | h 2138   |
| i 0             | j 1      | k 1     | l 41     |

- |                         |                 |                        |                   |
|-------------------------|-----------------|------------------------|-------------------|
| <b>6</b> a 2. $\dot{5}$ | b 0. $\dot{6}$  | c 12. $\dot{8}$        | d 49.1            |
| e 0.2 $\dot{6}$         | f 0.4 $\dot{1}$ | g 0.9 $\dot{1}\dot{3}$ | h 8.641 $\dot{8}$ |
| i 0.04012               | j 133.9462      | k 1.83                 | l 0.127           |
| <b>7</b> a 0.17         | b 0.3           | c 0.1                  | d 0.13            |
| e 0.18                  | f 0.4           | g 0.417                |                   |
| h 0.142857              | i 0.47          |                        |                   |
| <b>8</b> a B            | b A             | c D                    | d A               |
| 9 a 0.5                 | b 0. $\dot{3}$  | c 0.25                 |                   |
| d 0.2                   | e 0. $\dot{1}$  |                        |                   |
| <b>10</b> a 3.5         | b 4.6           | c 0.75                 |                   |
| d 3.8                   | e 0. $\dot{7}$  |                        |                   |
| <b>11</b> a 10          | b 80            | c 50                   | d 140             |
| e 260                   | f 180           | g 310                  | h 1480            |
| i 10 020                | j 209 720       | k 20                   | l 10              |
| <b>12</b> a 300         | b 200           | c 900                  | d 100             |
| e 100                   | f 6800          | g 900                  | h 9800            |
| i 1700                  | j 1462 800      | k 100                  | l 0               |
| <b>13</b> a 3000        | b 6000          | c 12 000               | d 19 000          |
| e 28 000                | f 9000          | g 1000                 | h 497 000         |
| i 13 000                | j 1000          | k 130 000              | l 50 000          |
| <b>14</b> \$27.70       |                 |                        |                   |

**15** A more reasonable measurement would be 3.568 m.  
 With the instruments we have available, it would not be possible to cut the timber more accurately.

**16** 0.295, 0. $\dot{2}\dot{9}\dot{5}$ , 0.29 $\dot{5}$ , 0.29 $\dot{5}$

**17** Answers will vary. An example is 0.543216.

**18** Check with your teacher

**19** 25°C

## 6D Adding and subtracting decimals

- |                            |                          |
|----------------------------|--------------------------|
| <b>1</b> a 3.5             | b 2.69                   |
| c 13.239                   | d 102.1858               |
| <b>2</b> a 6.77            | b 21.906                 |
| c 2012.333                 | d 261.44906              |
| <b>3</b> a 4.6             | b 6.9                    |
| d 3.471                    | e 17.280                 |
| g 17.3271                  | h 20.4329                |
| <b>4</b> a 11.69           | b 105.22                 |
| d 18.951                   | e 105.958                |
| g 20.9916                  | h 2618.9564              |
| <b>5</b> a B               | b A                      |
| c A                        | d D                      |
| <b>6</b> a 0.60            | b 4.205                  |
| e 5.02                     | f 89.19                  |
| <b>7</b> a 3.34            | b 5.93                   |
| e 4.146                    | f 27.83                  |
| i 1.2509                   | j 14.5326                |
| m 4.1136                   | n 0.1951                 |
| <b>8</b> a C               | b B                      |
| c B                        | d B                      |
| <b>9</b> \$167.00          | <b>10</b> \$4.70         |
| <b>11</b> 20.05 kilometres | <b>12</b> 8.1 kilometres |
| <b>13</b> \$6.60           | <b>14</b> \$10.35        |
| <b>15</b> 50.48 kilograms  | <b>16</b> 11.73 metres   |
| <b>17</b> 48.38 seconds    | <b>18</b> 31.28 metres   |

<b>19</b> a	Team	1	2	3	4	5	6
Time (seconds)	153	159.3	148.9	166.6	155.9	155.8	
<b>b</b> Team 3				<b>c</b> 4.1 seconds			

## 6E Multiplying decimals (including by multiples of 10)

- |                 |            |               |
|-----------------|------------|---------------|
| <b>1</b> a 14.0 | b 125.6    | c 146.7       |
| d 61.2          | e 111.70   | f 428.67      |
| g 190.26        | h 518.96   | i 26.068      |
| j 6.384         | k 0.794 43 | l 113.541 464 |

- 2** a 0.84      b 2.94      c 0.32  
 d 7.76      e 0.140      f 0.192  
 g 0.385      h 0.441      i 0.326  
 j 4.304      k 0.2082      l 0.3486  
 m 2.8992      n 4.905 64      o 10.016 395  
**3** a 0.0001      b 0.000 012      c 0.063  
 d 0.000 222      e 0.000 002 44      f 0.000 36  
 g 0.392      h 0.000 6062      i 0.000 0042  
**4** a 0.3      b 0.851      c 2.538  
 d 6.557      e 16.848      f 60.300  
 g 55.872      h 28.652      i 12.3095  
 j 18.9644      k 79.7720      l 35.6896  
 m 4.2036      n 1.699 24      o 14.892  
**5** a E      b B      c A      d B  
**6** a 0.0004      b 1.69      c 4.2025  
 d 0.3      e 0.4      f 0.07  
**7** a 64.8      b 1389.6  
 c 58 906.43      d 270.8  
 e 217 148.96      f 842 619  
 g 82 049.6783      h 3 268 904.3267  
 i 984 326.641      j 59 027 683.017  
 k 278.498 32      l 460  
 m 5 290 000      n 39 486 000  
**8** a 1092      b 4548      c 13 450      d 38 340  
 e 1016      f 5973      g 147 200      h 437 400  
**9** a 3500      b 12 700      c 1100      d 2535  
 e 5820      f 11 015  
**10** \$750      **11** 108 L  
**12** 260.7569 m<sup>2</sup>      **13** \$5.55  
**14** a 3205c  
 b \$32.05  
**15** 0.375 L      **16** 13
- 6F Dividing decimals (including by multiples of 10)**
- 1** a 0.6      b 3.1      c 2.9      d 0.54  
 e 1.61      f 4.71      g 2.42      h 0.788  
 i 1.983      j 1.147      k 4.0701      l 8.3729  
 m 6.8594      n 31.6176      o 3.13      p 3.12  
 q 2.231      r 1.222      s 6.421      t 11.0311  
**2** a 1.85      b 4.75      c 1.46      d 2.45  
 e 1.25      f 6.95  
**3** a 1.407      b 9.6813  
 c 9.8506      d 0.6208  
 e 3.592 87      f 0.023 4978  
 g 0.053 320 667      h 0.000 090 769  
 i 0.103 454 97      j 0.802 4056  
 k 0.000 1527      l 0.000 583  
 m 0.000 072 05      n 0.000 000 0032  
 o 0.000 000 048 7  
**4** a 0.265      b 0.79      c 0.235  
 d 0.595      e 0.001 465      f 0.001 74  
**5** a 5      b 7      c 8      d 9  
 e 4      f 11.1      g 0.31      h 5.1  
 i 2.2      j 66      k 72.1      l 12.31  
 m 0.852      n 0.336 77      o 1.367  
**6** a 1564.08      b 4.254      c 20.01  
 d 12.13      e 143.134      f 1786.1  
 g 0.4782      h 15 771.987      i 1630.95  
**7** a D      b D      c B      d D  
**8** a \$3.65      b \$1.70      c \$56.85  
 d \$0.75      e \$0.90      f \$63.50  
**9** 10.10      **10** \$0.60  
**11** \$4.50      **12** 0.675 m  
**13** 20      **14** 71  
**15** 26      **16** 201

**Chapter review****Fluency**

- 1** a 7 tenths      b 7 hundredths  
 c 7 tenths      d 7 thousandths  
 e 7 ten thousandths      f 7 hundred thousandths  
 g 7 units      h 7 thousandths
- 2** a  $(2 \times 1) + (6 \times \frac{1}{10}) + (4 \times \frac{1}{100})$   
 b  $(3 \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$   
 c  $(1 \times 10) + (8 \times 1) + (4 \times \frac{1}{10}) + (6 \times \frac{1}{1000})$   
 d  $(9 \times 10) + (6 \times 1) + (3 \times \frac{1}{10})$   
 $+ (4 \times \frac{1}{100}) + (2 \times \frac{1}{1000}) + (8 \times \frac{1}{10000})$
- 3** a 6.4      b 0.943      c 12.26      d 4.11  
**4** a 0.459      b 12.806      c 1.603      d 2.793  

**5** a >      b <      c <      d <  
 e >      f <      g <      h >

**6** a 0.13, 0.22, 0.34, 0.71, 0.86  
 b 0.124, 0.247, 0.258, 0.274, 0.285  
 c 0.826, 0.834, 0.859, 0.888, 0.891  
 d 0.217, 0.356, 0.358, 0.365, 0.385

**7** a  $\frac{4}{5}$       b  $\frac{17}{100}$       c  $\frac{9}{25}$       d  $\frac{187}{1000}$   
 e  $\frac{1}{8}$       f  $\frac{71}{125}$       g  $\frac{41}{200}$       h  $\frac{19}{20}$

**8** a  $1\frac{1}{2}$       b  $4\frac{3}{5}$       c  $3\frac{12}{25}$       d  $5\frac{1}{4}$   
 e  $2\frac{3}{4}$       f  $2\frac{5}{8}$       g  $1\frac{14}{25}$       h  $8\frac{8}{25}$

**9** a 1.3      b 2.05      c 13.86  
 d 0.048      e 1.9260      f 17.90

**10** a 14      b 29      c 87      d 100

**11** a 9.1      b 10.09      c 6.993  
 d 15.647      e 215.72      f 41.737 45

**12** a 5.3      b 9.07      c 2.04  
 d 90.659      e 0.949      f 1.575

**13** a 18.6      b 42.03      c 66.0180      d 5.3494  
**14** a 2.53      b 667.48      c 5122.32      d 0.926 08  

**15** a 2.6      b 13.45      c 6.45      d 182.94  
 e 14 662.81      f 48.0643      g 839.204      h 368

**16** a 1.312      b 0.516      c 0.783  
 d 0.000 18      e 0.87      f 2.7168  
 g 14.9084      h 1.023 36

**17** a 6.25      b 0.0009      c 0.8      d 0.05  
**18** a 0.3      b 0.41      c 2.08  
 d 73.62      e 0.16      f 33.11

**19** a 1.4623      b 10.236      c 9.612 347  
 d 0.200 32      e 264.983 0026      f 34.6294

**20** a 0.05      b 0.625  
 c 0.3125      d 0.125

**21** a 4.5      b 0.82  
 c 19.2781      d 83.0162

**22** a 8      b 51      c 11  
 d 34.3      e 894.73      f 6200.762  
 g 2.413      h 21 426.78

**Problem solving**

**1** \$53.30      **2** 1.125 L  
**3** \$3.70 (Rounded to nearest 5c) or \$3.68

**4** a  $\underline{0.076\ 923}$       b  $\underline{0.384\ 615}$   
**5** 5c      **6** 1.25  
**7** 3.7, 6.8, 12.5      **8** 13.84  
**9** 20      **10** 7 min 12 s

## CHAPTER 7

### Percentages

#### Are you ready?

- 1 a  $\frac{1}{4}$   
 2 a  $\frac{27}{10}$   
 3 a  $\frac{80}{100}$   
 4 a 75  
 5 a 137.5  
 6 a  $17\frac{1}{2}$   
 7 a 2  
 8 a \$23.50
- b  $\frac{8}{25}$   
 b  $\frac{167}{20}$   
 b  $\frac{48}{100}$   
 b  $87\frac{1}{2}$   
 b 2  
 b 183  
 b 6  
 b \$207.90

### 7A Percentages as fractions

- 1 a  $\frac{17}{100}$   
 b  $\frac{29}{100}$   
 c  $\frac{81}{100}$   
 d  $\frac{79}{100}$   
 e  $\frac{99}{100}$   
 f  $\frac{43}{100}$   
 g  $\frac{3}{100}$   
 h  $\frac{19}{100}$   
 i  $\frac{67}{100}$   
 j  $\frac{33}{100}$   
 k  $\frac{9}{100}$   
 l  $\frac{189}{100}$   
 m  $\frac{243}{100}$   
 n  $\frac{127}{100}$   
 o  $\frac{353}{100}$
- 2 a  $\frac{1}{2}$   
 b  $\frac{4}{5}$   
 c  $\frac{1}{4}$   
 d  $\frac{7}{20}$   
 e  $\frac{3}{5}$   
 f  $\frac{17}{20}$   
 g  $\frac{1}{10}$   
 h  $\frac{9}{20}$   
 i  $\frac{49}{50}$   
 j  $\frac{3}{25}$   
 k  $\frac{1}{20}$   
 l  $\frac{14}{25}$   
 m  $\frac{37}{50}$   
 n  $\frac{1}{50}$   
 o  $1\frac{1}{10}$   
 p  $1\frac{1}{2}$   
 q  $\frac{9}{10}$   
 r  $1\frac{4}{5}$   
 s  $\frac{2}{1} = 2$   
 t  $\frac{5}{1} = 5$   
 u  $1\frac{3}{25}$
- 3 a  $\frac{1}{200}$   
 b  $\frac{1}{500}$   
 c  $\frac{3}{400}$   
 d  $\frac{1}{150}$   
 e  $\frac{1}{1000}$   
 f  $\frac{1}{160}$   
 g  $\frac{3}{550}$   
 h  $\frac{33}{400}$   
 i  $\frac{3}{80}$   
 j  $\frac{31}{150}$   
 k  $\frac{29}{300}$   
 l  $\frac{57}{400}$   
 m  $\frac{241}{400}$   
 n  $\frac{31}{200}$   
 o  $\frac{9}{40}$   
 p  $\frac{14}{125}$   
 q  $\frac{83}{800}$   
 r  $\frac{7}{60}$   
 s  $1\frac{101}{200}$   
 t  $1\frac{41}{200}$   
 u  $\frac{1}{3}$
- 4 a  $\frac{7}{200}$   
 b  $\frac{9}{125}$   
 c  $\frac{59}{500}$   
 d  $\frac{197}{1000}$   
 e  $\frac{81}{250}$   
 f  $\frac{143}{200}$   
 g  $\frac{629}{1000}$   
 h  $\frac{31}{200}$   
 i  $\frac{87}{1000}$   
 j  $\frac{81}{500}$   
 k  $\frac{283}{1000}$   
 l  $\frac{13}{80}$   
 m  $\frac{2069}{5000}$   
 n  $\frac{5799}{10000}$   
 o  $\frac{1079}{1250}$   
 p  $\frac{907}{5000}$   
 q  $\frac{243}{2000}$   
 r  $\frac{1}{2000}$

- 5 a B  
 b E  
 c C  
 d D
- 6  $\frac{20}{100} = \frac{1}{5}$   
 7  $\frac{35}{100} = \frac{7}{20}$   
 8  $\frac{45}{100} = \frac{9}{20}$   
 9 32% are girls,  $\frac{8}{25}$
- 10 a  $\frac{2}{25}$   
 b  $\frac{23}{25}$   
 c 8 people

- 11 a  $\frac{17}{100}$   
 b  $\frac{83}{100}$   
 c 17  
 12 Less than  $\frac{1}{2}$   
 13 a  $\frac{7}{20}$   
 b  $\frac{81}{100}$   
 c  $\frac{17}{20}$   
 d  $\frac{13}{200}$

### 7B Percentages as decimals

- 1 a 0.36  
 b 0.14  
 c 0.19  
 d 0.28  
 e 0.73  
 f 0.92  
 g 0.66  
 h 0.59  
 i 0.11  
 j 0.99  
 k 0.09  
 l 0.07  
 m 0.04  
 n 0.01  
 o 0.25  
 p 2.00  
 q 1.5  
 r 3.6  
 2 a 0.123  
 b 0.316  
 c 0.592  
 d 0.849  
 e 0.376  
 f 0.421  
 g 0.219  
 h 0.169  
 i 0.107  
 j 0.111  
 k 0.031  
 l 0.046  
 m 0.092  
 n 0.059  
 o 0.068  
 p 0.088  
 q 0.1425  
 r 0.3175  
 s 0.2355  
 t 0.4575  
 v 0.0102  
 w 0.0401  
 x 0.0002
- 3 a B  
 b C  
 c A  
 d D
- 4 a  $\frac{17}{100}$   
 b 0.17
- 5 a  $\frac{1}{2000}$   
 b 0.0005
- 6 0.0285  
 7 2.18  
 8 0.138  
 9 0.0835

### 7C Fractions and decimals to percentages

- 1 a 21%  
 b 48%  
 c 9%  
 d 93%  
 e 28%  
 f 72%  
 g 16%  
 h 80%  
 i 12%  
 j 60%  
 k 40%  
 m 10%  
 n 35%  
 o 5%  
 p 20%  
 q 80%  
 r 20%  
 s 40%  
 t 80%  
 v 65%  
 w 30%  
 x 60%  
 2 a 50%  
 b 25%  
 c 60%  
 d 25%  
 e 40%  
 f 20%  
 g 50%  
 h 50%  
 i 75%  
 j 80%  
 k 10%  
 l 20%  
 3 a  $33\frac{1}{3}\%$   
 b  $16\frac{2}{3}\%$   
 c  $22\frac{2}{9}\%$   
 d  $44\frac{4}{9}\%$   
 e  $27\frac{3}{11}\%$   
 f  $46\frac{2}{3}\%$   
 g  $55\frac{5}{9}\%$   
 h  $56\frac{1}{4}\%$   
 i  $91\frac{2}{3}\%$   
 j  $87\frac{1}{2}\%$   
 k  $7\frac{9}{13}\%$   
 l  $58\frac{14}{17}\%$   
 m  $14\frac{2}{7}\%$   
 n  $83\frac{1}{3}\%$   
 o  $66\frac{2}{3}\%$
- 4 a D  
 b B  
 c A  
 d E
- 5 a 45%  
 b 32%  
 c 56%  
 d 68%  
 e 90%  
 f 84%  
 g 12%  
 h 8%  
 i 2%  
 j 10%  
 k 99%  
 l 5%  
 m 29%  
 n 9%  
 o 19%  
 p 10.5%  
 q 0.1%  
 r 6.7%
- 6 a 30%  
 b 80%  
 c 90%  
 d 10%  
 e 0.2%  
 f 0.7%  
 g 0.5%  
 h 0.9%  
 i 132%  
 j 150%  
 k 865%  
 l 205%  
 m 450%  
 n 0.015%  
 o 200%  
 p 1000%  
 q 500%  
 r 10 000%
- 7 a D  
 b A  
 c E  
 d C
- 8  $\frac{3}{4}\%$ , 12.5%, 15%, 25%, 50%, 85%, 94%  
 ↓ ↓ ↓ ↓ ↓ ↓ ↓  
 $\frac{3}{4}\%$ ,  $\frac{125}{1000}$ , 15%,  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{85}{100}$ ,  $\frac{94}{100}$

- 9** 20%      **10** 32%  
**11** 7.5%      **12** 58%  
**13** 30%      **14** 40%  
**15**  $36\frac{4}{11}\%$       **16**  $52\frac{3}{11}\%$   
**17** 70%      **18** 40%  
**19** 43%      **20** 3.1%

Student name	Directed numbers test	
	Mark	Percentage
Mandy Adams	0.86	86
Sandra Bazumik	0.72	72
Malcolm Boncev	0.64	64
James Callan	0.91	91
Kate D'Arpa	0.79	79
Louise Edmonds	0.92	92
Chris Edwards	0.95	95
Thomas Evancik	0.10	10
Jessie Farmer	0.88	88
Mia Yazzett	0.46	46

**7D Finding percentages of an amount**

- 1** **a** 180      **b** 4      **c** 60      **d** 18      **e** 45  
**f** 12      **g**  $1\frac{1}{2}$       **h** 44      **i**  $10\frac{1}{2}$       **j** 78
- 2** **a** 10      **b** 16      **c** 3  
**d** 3      **e** 18      **f** 93  
**g** 6      **h** 6      **i** 28  
**j** 77      **k** 39      **l** 63  
**m** 4000      **n** 66      **o** 17  
**p** 42      **q** 95      **r** 190  
**s** 55      **t** 12      **u** 25
- 3** **a**  $3\frac{3}{5}$       **b**  $4\frac{4}{5}$       **c**  $7\frac{7}{10}$       **d**  $9\frac{3}{5}$       **e**  $18\frac{1}{2}$   
**f**  $13\frac{1}{5}$       **g**  $1\frac{9}{10}$       **h**  $24\frac{3}{4}$       **i**  $11\frac{1}{10}$       **j**  $70\frac{2}{5}$   
**k**  $58\frac{9}{10}$       **l**  $20\frac{4}{5}$       **m**  $14\frac{2}{5}$       **n**  $98\frac{2}{5}$       **o**  $16\frac{1}{5}$
- 4** **a** A      **b** D      **c** E      **d** E
- 5** 210 students
- 6** **a** 4      **b** 18      **c** 4  
**d** 5      **e** 7      **f** 41  
**g** 37      **h** 5      **i** 144  
**j** 40.5      **k** 50.4      **l** 372  
**m** 4.5      **n** 254.4      **o** 22  
**p** 3.5      **q** 236      **r** 12.48
- 7** **a** 7.2      **b** 1.74      **c** 23.529  
**d** 2.196      **e** 12.127      **f** 11.088  
**g** 11.723      **h** 17.766      **i** 1.072  
**j** 66.528      **k** 32.708      **l** 324.159  
**m** 268.763      **n** 110.1375      **o** 1.243 75
- 8** **a** 0.464      **b** 2.816      **c** 5.264  
**d** 22.407      **e** 2.720      **f** 17.424  
**g** 3.8775      **h** 30.6037      **i** 11.8335  
**j** 286      **k** 4423.5      **l** 932.4  
**m** 38.2655      **n** 13.665 75      **o** 88.2
- 9** **a** B      **b** E      **c** D      **d** C
- 10** 73 days
- 11** 1 person
- 12** **a** 80%
- 13** **a** \$17.10
- 14** \$1 600 000
- 15** 1600 people
- 16** \$587.10
- 17** \$12 000

- 16** 47 species      **17** 600 000 people  
**18** **a** Peter calculated 35% of 80 mL rather than 800 mL.  
**b** 280 mL
- 19** **a** 18 adults      **b** 1 adult  
**20** 8 houses      **21** 38 workers  
**22** **a** 12 houses      **b** 11 houses  
**23** 2 women

**7E One amount as a percentage of another**

- | Topic      | Prob-ability    | Measure-ment    | Algebra         | Equa-tions      | Geom-etry        |
|------------|-----------------|-----------------|-----------------|-----------------|------------------|
| Score      | $\frac{15}{20}$ | $\frac{13}{15}$ | $\frac{27}{30}$ | $\frac{70}{80}$ | $\frac{95}{100}$ |
| Percentage | 75              | 87              | 90              | 88              | 95               |
- b** Geometry  
**c** 87%
- 7F Common percentages and shortcuts**
- 1** **a** \$1.00      **b** \$1.80      **c** \$4.50  
**d** \$8.10      **e** \$15.00      **f** \$11.20  
**g** \$9.30      **h** \$7.90      **i** \$4.70  
**j** \$2.20      **k** \$1.65      **l** \$1.70

<b>m</b>	\$1.25	<b>n</b>	\$0.15	<b>o</b>	\$3.30	<b>4</b>	<b>a</b>	71%	<b>b</b>	84%	<b>c</b>	3%	<b>d</b>	20%
<b>p</b>	\$4.80	<b>q</b>	\$8.15	<b>r</b>	\$19.25	<b>e</b>	50%	<b>f</b>	0.8%	<b>g</b>	164%	<b>h</b>	380%	
<b>s</b>	\$50.70	<b>t</b>	\$462.00	<b>u</b>	\$192.60	<b>5</b>	<b>a</b>	6	<b>b</b>	12	<b>c</b>	39	<b>d</b>	39
<b>v</b>	\$304.15	<b>w</b>	\$721.95	<b>x</b>	\$200.00	<b>e</b>	$6\frac{3}{10}$	<b>f</b>	$37\frac{1}{10}$	<b>g</b>	$4\frac{4}{5}$	<b>h</b>	$13\frac{3}{5}$	
<b>2 a</b>	\$1.50	<b>b</b>	\$5.10	<b>c</b>	\$1.70	<b>6 a</b>	60%	<b>b</b>	30%	<b>c</b>	40%	<b>d</b>	40%	
<b>d</b>	\$0.90	<b>e</b>	\$13.70	<b>f</b>	\$17.20	<b>e</b>	$33\frac{1}{3}\%$	<b>f</b>	$12\frac{1}{2}\%$	<b>g</b>	$46\frac{2}{3}\%$	<b>h</b>	$87\frac{1}{2}\%$	
<b>g</b>	\$0.45	<b>h</b>	\$0.65	<b>i</b>	\$0.80	<b>i</b>	$7\frac{1}{7}\%$	<b>j</b>	$11\frac{1}{9}\%$					
<b>j</b>	\$3.90	<b>k</b>	\$7.50	<b>l</b>	\$1.40	<b>7 a</b>	5	<b>b</b>	3.72	<b>c</b>	2.25	<b>d</b>	6.9	
<b>m</b>	\$10.30	<b>n</b>	\$6.80	<b>o</b>	\$4.30	<b>e</b>	19.26	<b>f</b>	38.76	<b>g</b>	6.12	<b>h</b>	40.6926	
<b>p</b>	\$51.80	<b>q</b>	\$30.50	<b>r</b>	\$62.85	<b>8 a</b>	\$6.30	<b>b</b>	\$4.20	<b>c</b>	\$10.50			
<b>s</b>	\$10.05	<b>t</b>	\$20.70			<b>d</b>	\$21.60	<b>e</b>	\$0.35	<b>f</b>	\$4.25			
<b>3 a</b>	\$0.40	<b>b</b>	\$0.30	<b>c</b>	\$0.10	<b>g</b>	\$11.85	<b>h</b>	\$212.60					
<b>d</b>	\$0.10	<b>e</b>	\$7.00	<b>f</b>	\$4.05	<b>9 a</b>	\$0.40	<b>b</b>	\$1.05	<b>c</b>	\$3.20	<b>d</b>	\$5.20	
<b>g</b>	\$2.10	<b>h</b>	\$0.55	<b>i</b>	\$12.15	<b>e</b>	\$1.75	<b>f</b>	\$2.60	<b>g</b>	\$10.30	<b>h</b>	\$3.85	
<b>j</b>	\$15.25	<b>k</b>	\$50	<b>l</b>	\$32.15	<b>10 a</b>	\$0.15	<b>b</b>	\$0.30	<b>c</b>	\$5.05	<b>d</b>	\$27.00	
<b>m</b>	\$5.15	<b>n</b>	\$1.60	<b>o</b>	\$0.30	<b>g</b>	\$48.40	<b>f</b>	\$11.60	<b>g</b>	\$2.95	<b>h</b>	\$2.95	
<b>4 a</b>	\$4.30	<b>b</b>	\$8.45	<b>c</b>	\$1.65									
<b>d</b>	\$0.65	<b>e</b>	\$14.80	<b>f</b>	\$0.20									
<b>g</b>	\$0.15	<b>h</b>	\$3.30	<b>i</b>	\$27.15									
<b>j</b>	\$52.35	<b>k</b>	\$247.40	<b>l</b>	\$1013.80									
<b>5 a</b>	\$1.80	<b>b</b>	\$1.20	<b>c</b>	\$3.00									
<b>d</b>	\$9.00	<b>e</b>	\$7.50	<b>f</b>	\$11.25									
<b>g</b>	\$22.50	<b>h</b>	\$55.00	<b>i</b>	\$4.50									
<b>j</b>	\$7.50	<b>k</b>	\$14.25	<b>l</b>	\$30.65									
<b>6 a</b>	\$2.70	<b>b</b>	\$7.15	<b>c</b>	\$5.75									
<b>d</b>	\$6.05	<b>e</b>	\$0.05	<b>f</b>	\$0.10									
<b>g</b>	\$0.15	<b>h</b>	\$0.15	<b>i</b>	\$0.20									
<b>j</b>	\$0.80	<b>k</b>	\$0.20	<b>l</b>	\$4.30									
<b>m</b>	\$0.05	<b>n</b>	\$0.05	<b>o</b>	\$0.10									
<b>p</b>	\$0.00	<b>q</b>	\$0.00	<b>r</b>	\$12.65									
<b>7 a</b>	\$1.30	<b>b</b>	\$10.50	<b>c</b>	\$3.30									
<b>d</b>	\$0.65	<b>e</b>	\$2.40	<b>f</b>	\$2.20									
<b>g</b>	\$1.80	<b>h</b>	\$73.50	<b>i</b>	\$18.00									
<b>j</b>	\$1.55	<b>k</b>	\$1.05	<b>l</b>	\$2.05									
<b>m</b>	\$32.20	<b>n</b>	\$4.80	<b>o</b>	\$1.60									
<b>p</b>	\$0.45	<b>q</b>	\$14.40	<b>r</b>	\$492									
<b>8 a</b>	D		B	<b>c</b>	E			<b>d</b>	C					
<b>9</b>	\$855													
<b>10</b>	54 000 residents													
<b>11 a</b>	\$322.50													
<b>b</b>	\$152.30													
<b>12</b>	110 students													
<b>13</b>	27.9 seconds													
<b>14 a</b>	2 people			<b>b</b>	38 people									
<b>15 a</b>	13 608 people			<b>b</b>	17 820 people									
<b>16</b>	20 years old													
<b>17</b>	\$0.80													
<b>18</b>	8.2 kg													
<b>19</b>	50 years old													
<b>20</b>	9 years old, 90 years old													

## Chapter review

### Fluency

<b>1 a</b>	$\frac{13}{100}$	<b>b</b>	$\frac{7}{10}$	<b>c</b>	$\frac{13}{50}$
<b>d</b>	$\frac{33}{25} = 1\frac{8}{25}$	<b>e</b>	$\frac{1}{400}$	<b>f</b>	$\frac{3}{125}$
<b>g</b>	$\frac{97}{1000}$	<b>h</b>	$\frac{7317}{10\,000}$		
<b>2 a</b>	0.42	<b>b</b>	0.05	<b>c</b>	0.94
<b>e</b>	0.067	<b>f</b>	0.197	<b>g</b>	0.5803
<b>3 a</b>	15%	<b>b</b>	55%	<b>c</b>	60%
<b>e</b>	$37\frac{1}{2}\%$	<b>f</b>	$53\frac{1}{3}\%$	<b>g</b>	$83\frac{1}{3}\%$
				<b>h</b>	$36\frac{4}{11}\%$

<b>4 a</b>	71%	<b>b</b>	84%	<b>c</b>	3%	<b>d</b>	20%
<b>e</b>	50%	<b>f</b>	0.8%	<b>g</b>	164%	<b>h</b>	380%
<b>5 a</b>	6	<b>b</b>	12	<b>c</b>	39	<b>d</b>	39
<b>e</b>	$6\frac{3}{10}$	<b>f</b>	$37\frac{1}{10}$	<b>g</b>	$4\frac{4}{5}$	<b>h</b>	$13\frac{3}{5}$
<b>6 a</b>	60%	<b>b</b>	30%	<b>c</b>	40%	<b>d</b>	40%
<b>e</b>	$33\frac{1}{3}\%$	<b>f</b>	$12\frac{1}{2}\%$	<b>g</b>	$46\frac{2}{3}\%$	<b>h</b>	$87\frac{1}{2}\%$
<b>i</b>	$7\frac{1}{7}\%$	<b>j</b>	$11\frac{1}{9}\%$				
<b>7 a</b>	5	<b>b</b>	3.72	<b>c</b>	2.25	<b>d</b>	6.9
<b>e</b>	19.26	<b>f</b>	38.76	<b>g</b>	6.12	<b>h</b>	40.6926
<b>8 a</b>	\$6.30	<b>b</b>	\$4.20	<b>c</b>	\$10.50		
<b>d</b>	\$21.60	<b>e</b>	\$0.35	<b>f</b>	\$4.25		
<b>g</b>	\$11.85	<b>h</b>	\$212.60				
<b>9 a</b>	\$0.40	<b>b</b>	\$1.05	<b>c</b>	\$3.20	<b>d</b>	\$5.20
<b>e</b>	\$1.75	<b>f</b>	\$2.60	<b>g</b>	\$10.30	<b>h</b>	\$3.85
<b>10 a</b>	\$0.15	<b>b</b>	\$0.30	<b>c</b>	\$5.05	<b>d</b>	\$27.00
<b>e</b>	\$48.40	<b>f</b>	\$11.60	<b>g</b>	\$2.95	<b>h</b>	\$2.95

### Problem solving

- 1**  $\frac{19}{20}$
- 2 a**  $\frac{6}{25}$       **b**  $\frac{19}{25}$       **c** 96
- 3 a** 62%      **b** 75%
- 4**  $62\frac{1}{2}\%$       **5** 0.41%
- 6** 24 students      **7** 270 children
- 8** 1 425 000 km<sup>2</sup>
- 9** 3%      **10** 8%
- 11** \$50.85      **12** \$11 600
- 13** 400 g      **14** 200 g      **15 b, d**
- 16** 13.5 litres of milk would be needed to make the fat of 500 g of butter.
- 17 a** 1194 people      **b** 1.4%
- 18** \$850
- 19** No. (Check with your teacher for explanation.)
- 20** Check with your teacher.
- 21** 45%      **22** 720
- 23** \$16      **24** 20%
- 25 a** Yes. If he paid \$10 GST then the total cost = \$110 – \$10 = \$100 and 10% of \$100 = \$10, so the calculation is valid.
- b** The total price (110% of original price) is 11 times that of the GST (10% of original price), therefore if you divide the total price by 11, you will get the GST amount.
- c** No  $\frac{112\%}{12\%} \neq 11$
- 26 a** \$2250      **b** \$2025      **c** 19%
- d** The second 10% discount is not 10% of the original price but 10% of an amount which is only 90% of the original.
- 27** \$80
- 28** No. 10% off each item gives a total saving of \$8.50, while 20% of the combined purchase would give a saving of \$19.
- 29** No. Overall, after two months your salary will decrease by 1% using either option. However, option 1 gives an extra income (10% of the original salary) in the first month.
- 30 a** \$1575
- b** \$1575
- c** It doesn't matter which discount is applied first; the final price will be the same.
- 31 a** Nathan is 30 years old now.
- b** \$0.63

## CHAPTER 8

### Algebra

#### Are you ready?

- 1 a** Add 2 to obtain the next number in the sequence. The next three numbers are 15, 17, 19.
- b** Subtract 4 to obtain the next number in the sequence. The next three numbers are 12, 8, 4.
- c** Multiply by 2 to obtain the next number in the sequence. The next three numbers are 48, 96, 192.
- d** Divide by 10 to obtain the next number in the sequence. The next three numbers are 100, 10, 1.
- 2 a** 16                   **b** 23  
**c** 2                   **d** 2

- 3 a** Each first number is one more than the previous first number and each second number is one more than the previous second number. Each second number is always 6 more than the matching first number.
- b** Each first number is two more than the previous first number and each second number is six more than the previous second number. Each second number is three times the matching first number.

- 4 a**  $3 + 5$                    **b**  $7 \times 8$   
**c**  $6 - 2$                    **d**  $5 + 9$

<b>5 a</b>	Number of squares	1	2	3	4	5	6
	Number of sides	4	8	12	16	20	24

**b** 40

#### 8A Using rules

- 1 a** Add 2 to each term to get the next term. 10, 12, 14
- b** Add 5 to each term to get the next term. 23, 28, 33
- c** Subtract 3 from each term to get the next term. 15, 12, 9
- d** Multiply each term by 3 to get the next term. 81, 243, 729
- e** Divide each term by 2 to get the next term. 8, 4, 2
- f** The term number is squared. 25, 36, 49

<b>2 a i</b>	Number of squares	1	2	3	4
	Number of matchsticks	4	7	10	13

**ii** The number matchsticks equals 3 times the number of squares plus 1.

**iii** 61

<b>b i</b>	Number of triangles	1	2	3	4
	Number of matchsticks	3	5	7	9

**ii** The number matchsticks equals 2 times the number of triangles plus 1

**iii** 41

<b>c i</b>	Number of houses	1	2	3	4
	Number of matchsticks	6	11	16	21

**ii** The number of matchsticks equals 5 times the number of squares plus 1.

**iii** 101

<b>d i</b>	Number of panels	1	2	3	4
	Number of matchsticks	4	7	10	13

**ii** The number of matchsticks equals 3 times the number of squares plus 1.

**iii** 61

<b>3 a</b>	Input	4	5	6	10	14
	Output	0	1	2	6	10

<b>b</b>	Input	1	2	6	13	107
	Output	13	14	18	25	119

<b>c</b>	Input	1	10	51	60	144
	Output	2	20	102	120	288

<b>d</b>	Input	3	12	21	66	141
	Output	1	4	7	22	47

<b>4 a</b>	Input	2	3	4	10	17
	Output	9	11	13	25	39

<b>b</b>	Input	2	4	5	20	100
	Output	12	28	36	156	796

<b>c</b>	Input	2	3	5	12	43
	Output	25	30	40	75	230

<b>d</b>	Input	3	4	5	8	25
	Output	0	11	22	55	242

<b>e</b>	Input	0	1	5	8	10
	Output	0	1	25	64	100

<b>f</b>	Input	0	1	4	7	12
	Output	4	5	20	53	148

<b>5 a</b>	Input	1	3	27	2	53	191
	Output	3	5	29	4	55	193

<b>b</b>	Input	3	25	56	0	74	1753
	Output	23	45	76	20	94	1773

<b>c</b>	Input	7	15	96	9	17	109
	Output	2	10	91	4	12	104

<b>d</b>	Input	1	6	321	2	7	103
	Output	4	24	1284	8	28	412

<b>6 a</b>	Input	3	13	21	1	10	50
	Output	8	28	44	4	22	102

<b>b</b>	Input	2	5	16	3	5	20
	Output	3	18	73	8	18	93

<b>c</b>	Input	3	15	273	6	63	162
	Output	1	5	91	2	54	21

<b>d</b>	Input	1	5	17	2	7	12
	Output	1	25	289	4	49	144

<b>e</b>	<b>Input</b>	2	7	15	3	9	11
	<b>Output</b>	34	134	294	54	174	214

<b>f</b>	<b>Input</b>	7	19	31	23	51	63
	<b>Output</b>	1	4	7	5	12	15

### 8B Finding formula

- 1** a  $b = 4h$       b  $m = 4f$   
 c  $r = 5a$       d  $m = 4t$   
 e  $x = 5(k + 4)$       f  $k = 6w - 2$   
 g  $t = 4(20 - g)$       h  $b = 10a - 5$   
 i  $d = 6f + 7$       j  $h = 9(x + 5)$   
 k  $y = 8(3b + 6)$       l  $y = 8p - 6$   
 m  $g = 2(3r + 17)$       n  $j = 18(5h - 4)$
- 2** a  $g = f + 9$   
 b  $b = a - 3$   
 c  $t = 2k + 1$   
 d  $y = 7x - 5$   
 e  $g = 4a + 2$   
 f  $t = 9m + 11$   
 g  $w = 3 \times p + 2$  ( $w = 3p + 2$ )  
 h  $x = 5 \times t - 8$  ( $x = 5t - 8$ )  
 i  $p = 3m - 1$   
 j  $b = s \times 10 - 7$  ( $b = 10s - 7$ )
- 3** a  $b = a + 3$       b  $w = t + 5$   
 c  $a = m - 4$       d  $t = 2s + 3$   
 e  $g = 2f - 1$       f  $c = 3s + 1$   
 g  $a = 4d - 3$       h  $g = 5s - 4$   
 i  $e = 11f - 20$       j  $q = 20p + 4$   
 k  $t = 4b$       l  $p = 3u - 1$
- 4** C      **5** C      **6** D  
**7** E      **8** C      **9** D

**10** A      **11** D      **12** C

**13** Total biscuits =  $3n + 9$

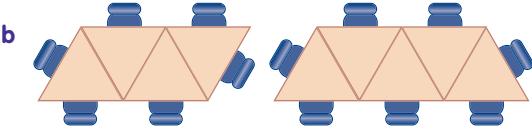
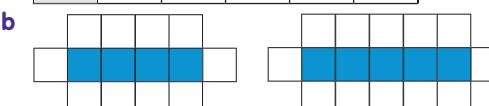
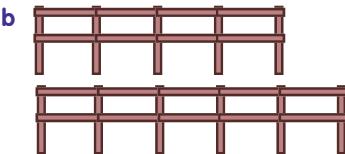
**14** a  $t = a^2 - 1$       b  $x = \frac{b+1}{2}$   
 c  $p = 2w^2$

### 8C Substitution

- |          |          |            |              |               |              |
|----------|----------|------------|--------------|---------------|--------------|
| <b>1</b> | <b>a</b> | i $m = 2$  | ii $m = 3$   | iii $m = 0$   | iv $m = 100$ |
|          | b        | i $m = 11$ | ii $m = 1$   | iii $m = 197$ | iv $m = 13$  |
|          | c        | i $m = 19$ | ii $m = 7$   | iii $m = 67$  | iv $m = 247$ |
|          | d        | i $m = 21$ | ii $m = 13$  | iii $m = 1$   | iv $m = 25$  |
| <b>2</b> | <b>a</b> | i $m = 4$  | ii $m = 2$   | iii $m = 26$  | iv $m = 152$ |
|          | b        | i $m = 20$ | ii $m = 40$  | iii $m = 45$  | iv $m = 0$   |
|          | c        | i $m = 40$ | ii $m = 36$  | iii $m = 0$   | iv $m = 4$   |
|          | d        | i $m = 5$  | ii $m = 125$ | iii $m = 35$  | iv $m = 75$  |
|          | e        | i $m = 7$  | ii $m = 1$   | iii $m = 15$  | iv $m = 63$  |
|          | f        | i $m = 17$ | ii $m = 23$  | iii $m = 32$  | iv $m = 77$  |
|          | g        | i $m = 9$  | ii $m = 3$   | iii $m = 303$ | iv $m = 216$ |
|          | h        | i $m = 16$ | ii $m = 28$  | iii $m = 13$  | iv $m = 49$  |
|          | i        | i $m = 26$ | ii $m = 8$   | iii $m = 44$  | iv $m = 2$   |
- 3** a Fat 0.4 g, Protein 4.5 g, Carbohydrate 13.1 g  
 b 74 calories  
 c Multiply the number of fat grams by 9 and to this, add the sum of the number of protein and carbohydrate grams multiplied by 4.  
 d i  $f$       ii  $p$       iii  $c$       iv  $e$   
 e  $e = 9f + 4(p + c)$   
 f (see below)  
 g 100 grams of chocolate cake  
 h In general, fruits and vegetables  
 i Number of calories:  $e$ , number of kilojoules  $k$ ,  $k = 4.2e$   
 j 311 kJ  
 k (see below)

### f and k

Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrate	Number of calories (f)	Number of Kilojoules (k)
1 hot cross bun	3	3	19	115	483
100 grams of chocolate cake	16	4	56	384	1613
100 grams of roast chicken	14	26	0	230	966
70 grams of bacon	8	21	0	156	655
2 grilled sausages	17	13	15	265	1113
1 piece of fish (flake), no batter	1	21	0	93	391
10 grams of sultanas	0	0	6	24	101
25 grams of dried apricots	0	0	17	68	286
1 banana	0	1	20	84	353
1 apple	0	0	17	68	286
1 serve of carrots	0	0	5	20	84
1 serve of potatoes	0	0	17	68	286
250 millilitres of milk	10	8	12	170	714
210 grams of tinned tomato noodle soup	0.8	2.6	14.9	77.2	324

- 4 a**
- |          |   |   |   |   |   |
|----------|---|---|---|---|---|
| <i>T</i> | 1 | 2 | 3 | 4 | 5 |
| <i>D</i> | 3 | 4 | 5 | 6 | 7 |
- b** 
- c**  $D = T + 2$
- d**  $D = 26$
- 5 a**
- |          |   |   |   |    |    |
|----------|---|---|---|----|----|
| <i>B</i> | 1 | 2 | 3 | 4  | 5  |
| <i>W</i> | 4 | 6 | 8 | 10 | 12 |
- b** 
- c**  $W = 2B + 2$
- d**  $W = 22$
- 6 a**
- |          |   |   |    |    |    |
|----------|---|---|----|----|----|
| <i>S</i> | 1 | 2 | 3  | 4  | 5  |
| <i>P</i> | 4 | 7 | 10 | 13 | 16 |
- b** 
- c** Formula is  $P = 3S + 1$
- d**  $P = 661$  pieces of timber
- e** \$2544.85

- 7 a, b**
- |          |   |   |    |    |    |
|----------|---|---|----|----|----|
| <i>A</i> | 1 | 2 | 3  | 4  | 5  |
| <i>L</i> | 4 | 7 | 10 | 13 | 16 |
- c** 
- d** Formula is  $L = 3A + 1$
- e**  $L = 226$  light panels

- 8 a** 1225  
**b** 2485

#### 8D Terms, expressions and equations

- 1 a**  $c + 3$       **b**  $2c$       **c**  $3c + 2$   
**d**  $2c + 5$       **e**  $4c + 3$   
**2 a**  $4c + 34$       **b** 242      **c** \$48.40  
**3 a**  $2m$       **b**  $2m + n$       **c**  $4m + 3n$   
**d**  $m + 2n + 5$       **e**  $3m + 2n + 8$   
**4 a** 34      **b** 46      **c** 104  
**d** 46      **e** 83  
**5 a**  $B + 2$       **b**  $T - 3$       **c**  $D + 6$   
**d**  $K - 5$       **e**  $G + N + W$       **f**  $D + H$   
**g**  $2N$       **h**  $C + H$       **i**  $12 - G$   
**j**  $4D$       **k**  $H + 6$       **l**  $Z - G$   
**m**  $BF$       **n**  $3M + Y$   
**6 a**  $A + B + C$       **b**  $A - C$       **c**  $AB$   
**d**  $ABC$       **e**  $\frac{B}{C}$       **f**  $\frac{A+C}{B}$   
**g**  $A + 3$   
**7 a**  $T - X$       **b** \$4B      **c**  $45G$   
**d**  $\frac{\$T}{5}$       **e**  $R + M$       **f**  $\frac{Y}{6}$

- g** \$11*M*      **h**  $A + H$   
 $B + G$   
 $\frac{2}{2}$       **i**  $52 - J$   
**j**  $12B$       **k**  $\frac{4R}{2}$       **l**  $A + H + K$   
**m**  $4R$
- 8 a**  $x + 8 = 11$       **b**  $x - 11 = 63$   
**c**  $8x = 128$       **d**  $\frac{x}{7} = 5$   
**e**  $A + H = 6.49$       **f**  $52 - Q = 19$   
**g**  $99M = 792$       **h**  $4R + 4D = 164$
- 9 a** True      **b** True      **c** False      **d** True  
**e** True      **f** True      **g** True      **h** False  
**i** False      **j** True      **k** True      **l** False
- 10 a** B      **b** C      **c** D      **d** D  
**e** B      **f** A      **g** A      **h** D
- 11 a**  $x + 5$       **b**  $x - 8$       **c**  $2x$       **d**  $\frac{x}{4}$   
**e**  $x + 1$       **f**  $100x$       **g**  $\frac{x}{100}$       **h**  $7x$   
**i**  $\frac{x+20}{2}$       **j** 10*x* dollars      **k**  $\frac{k+23}{3}$
- 12 a** D      **b** C      **c** C  
**d** E      **e** B      **f** C
- 13 a**  $(2x + 7)$       **b**  $3x$   
**c**  $4x + 3$       **d**  $3x + 3$   
**e**  $3x + 6$       **f**  $3x + 6$
- 14 a** Cost of a hamburger + cost of a hamburger + cost of a drink = total cost or cost of the drink + twice the cost of a hamburger = total cost  
**b**  $\$3.75 + 2h = \$7.25$
- 15 a** **i** \$62  
**ii** \$74  
**iii** \$86  
**b** Cost =  $50 + 12n$   
**c** \$146  
**d**  $100 = 50 + 12n$
- 8E Simplifying and the distributive law**
- 1 a**  $5a$       **b**  $14y$       **c**  $15c$   
**d**  $2u$       **e**  $20e$       **f**  $5t$   
**g**  $14ab$       **h**  $6fg$       **i**  $3e$   
**j**  $7t$       **k**  $5f$       **l** 0  
**m**  $18x$       **n**  $12m$       **o**  $8a$
- 2 a**  $25ab - 7$       **b**  $3y$   
**c** Cannot be simplified      **d**  $30i - 2$   
**e**  $14t - 3$       **f**  $13r$   
**g** Cannot be simplified      **h** Cannot be simplified  
**i**  $16f + 5$       **j**  $4 + 2g$   
**k**  $t$       **l** 0  
**m**  $2mno$       **n**  $14pq$   
**o**  $11pr + 2$       **p** Cannot be simplified  
**q**  $11t - 5$       **r**  $t$   
**s**  $8aw$       **t** Cannot be simplified  
**u**  $11t - 8$       **v**  $4t + 3$   
**w**  $10g - 2$       **x**  $22r + 6$
- 3 a** 0      **b** 0      **c** 0  
**d** 8      **e** 7      **f** 2  
**g** 0      **h** 7      **i** 6y
- 4 a**  $5a, 35$       **b**  $9a, 63$       **c**  $4a, 28$   
**d**  $10a, 70$       **e**  $10a, 70$       **f**  $10a, 70$   
**g**  $17 + 8a, 73$       **h**  $5a + 2, 37$       **i**  $2a, 14$   
**j**  $a, 7$       **k**  $0, 0$       **l**  $17a - 16, 103$
- 5 a**  $3a + 6b$       **b**  $5x - 20z$       **c**  $20g + 30h$   
**d**  $2bc - 3ab$       **e**  $16cd - 2cq$       **f**  $9b^2 - 6bg$   
**g**  $9ax + 7bx$       **h**  $7kt + 21km$   
**6 a**  $2a + 10b$       **b**  $5n - 8c$       **c**  $18c$

- d**  $ax - bx + x$     **e**  $5y + 2yz$     **f**  $3dx - 5x$   
**g**  $12fu + 14u$     **h**  $20n + 21hn$
- 7 a** True    **b** True    **c** False  
**d** False    **e** True    **f** False  
**g** True    **h** False    **i** True  
**j** True    **k** True    **l** False  
**m** True    **n** True    **o** True  
**p** True    **q** False    **r** False
- 8 a** E    **b** B    **c** C    **d** D  
**e** C    **f** A    **g** D
- 9 a** True    **b** False    **c** False  
**d** True    **e** False    **f** True  
**g** False    **h** True    **i** True  
**j** True
- 10 a**  $13x + 2y$     **b**  $12a - 10y + 6ay$   
**c**  $9cg + 18cd + dg$     **d**  $12dy + 3ry$
- 11 a**  $2(2x + 1)$     **b**  $a(b + 3)$   
**c**  $3a(4y + 3b)$     **d**  $3b(d - 2c + 3a)$

### 8F The associative law

- 1 a**  $w + (r + 6y) = ([w] + [r]) + 6y$   
**b**  $6t + (4r + [3s]) = (6t + 4r) + [3s]$   
**c**  $(9y + [2r]) + 3w = [9y] + (2r + 3w)$   
**d**  $(z + 2p) + [6t] = z + ([2p] + 6t)$
- 2 a**  $6t + (3w + [7v]) = [3w] + (6t + 7v)$   
**b**  $s + (9r + [2c]) = 2c + ([s] + [9r])$   
**c**  $(3c + w) + [d] = d + ([3c] + [w])$   
**d**  $(g + [3b]) - 2y = 2y - (3b - g)$
- 3 a** T    **b** F    **c** T  
**d** F    **e** F    **f** F  
**g** T    **h** F
- 4 a** i 33    ii 33    They are the same.  
**b** i 21    ii 1    They are different.  
**c** i 48    ii 48    They are the same.  
**d** i 2    ii  $\frac{1}{8}$     They are different.
- 5** \$48    **6** Check with your teacher.

### Chapter review

#### Fluency

- 1 a**
- | Input  | 1 | 3  | 4  | 7  | 13 |
|--------|---|----|----|----|----|
| Output | 4 | 12 | 16 | 28 | 52 |
- b**
- | Input  | 1 | 4  | 6  | 7  | 15 |
|--------|---|----|----|----|----|
| Output | 8 | 11 | 13 | 14 | 22 |
- c**
- | Input  | 3 | 4 | 7  | 10 | 13 |
|--------|---|---|----|----|----|
| Output | 3 | 5 | 11 | 17 | 23 |
- d**
- | Input  | 0 | 4 | 8 | 16 | 20 |
|--------|---|---|---|----|----|
| Output | 0 | 1 | 2 | 4  | 5  |
- 2 a**
- | Input  | 15 | 16 | 33 | 10 | 14 |
|--------|----|----|----|----|----|
| Output | 8  | 9  | 26 | 7  | 3  |
- b**
- | Input  | 1  | 4  | 15 | 5  | 33  |
|--------|----|----|----|----|-----|
| Output | 10 | 19 | 52 | 22 | 106 |

<b>c</b>	<b>Input</b>	1	3	6	4	43
	<b>Output</b>	12	16	22	18	96

<b>d</b>	<b>Input</b>	8	11	15	13	38
	<b>Output</b>	11	17	25	21	7

- 3 a** C    **b** B  
**4 a**  $y = 9x - 4$     **b**  $q = 3p + 6$   
**5 a**  $y = x + 6$     **b**  $d = c - 5$   
**c**  $h = 5g + 2$     **d**  $n = 4m - 1$

<b>6 a</b>	<b>x</b>	1	3	4	7	105
	<b>y</b>	0	2	3	6	104

<b>b</b>	<b>x</b>	1	2	6	7	12
	<b>d</b>	3	6	18	21	36

<b>c</b>	<b>x</b>	0	2	4	5	6
	<b>h</b>	6	4	2	1	0

<b>d</b>	<b>x</b>	0	1	3	4	7
	<b>n</b>	3	14	36	47	80

- 7 a**  $35x + 18$     **b** 1068  
**8 a**  $M - C$     **b** \$3B  
**c**  $XY$     **d**  $G + 15$   
**e**  $D + 1$     **f** 12H cents  
**g**  $5T$
- 9 a**  $7g$     **b**  $6y$     **c**  $9h$   
**d**  $5ag$     **e**  $3gy$     **f** 0  
**g**  $5y$     **h**  $7t + 6$     **i**  $18gh$   
**j**  $11t - 2m$     **k**  $4m$     **l**  $15g + 12$   
**m**  $4h + 4t$     **n**  $10b + 7c$     **o**  $8axy$   
**10 a**  $10x, 50$     **b**  $5x - 4, 21$     **c**  $23x, 115$   
**d**  $3x, 15$     **e**  $3x, 15$     **f**  $x + 16, 21$   
**g**  $8x + 7, 47$     **h**  $11 + 7x, 46$     **i**  $6x, 30$
- 11 a**  $7m - 21k$     **b**  $gw + 9aw$   
**c**  $16hy - 14y$     **d**  $70mr - 20mp$   
**e**  $6ag + 6cg$     **f**  $24jx - 12jy$
- 12 a**  $2t - 10s$     **b**  $4a + 10g$   
**c**  $3df + 7d$     **d**  $15z - 4az$   
**e**  $3fp - 6pq + 7p$     **f**  $10ab + 65b$

**13** In this question, the order of the terms is important.

- a**  $a + (3b + [5c]) = ([a] + [3b]) + 5c$   
**b**  $\frac{1}{a} + (2b + [7g]) = \left(\frac{1}{a} + 2b\right) + [7g]$   
**c**  $(d - 3y) - z = [d] - ([3y] - [z])$   
**d**  $\left(\frac{a}{b} \times c\right) \times [f] = \frac{a}{b} \times ([c] \times f)$
- 14 a**  $g + ([3m] + 2k) = (2k + [g]) + 3m$   
**b**  $(w + [5g]) + 4r = [4r] + (5g + w)$   
**c**  $\frac{2}{3}([2d] - [r]) = (r - 2d) \cdot \frac{2}{3}$   
**d**  $(3z - 5b) \cdot [6m] = [3z] \cdot (6m - 5b)$

## Problem solving

<b>1 a</b>	<b>B</b>	1	2	3	4	5
	<b>S</b>	6	11	16	21	26

**b**



**c** Formula is  $S = 5B + 1$

**d**  $S = 61$  sleepers

<b>2 a</b>	<b>T</b>	1	2	3	4
	<b>S</b>	3	5	7	9

**b**

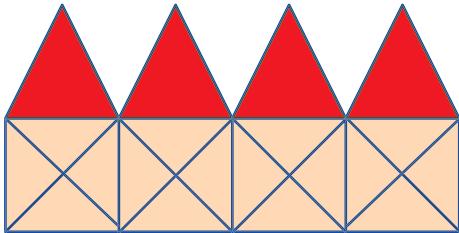


**c**  $S = 2T + 1$

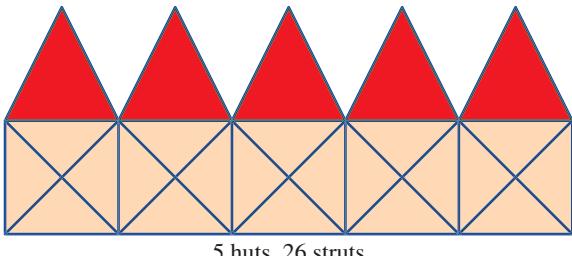
**d**  $S = 51$

<b>3 a</b>	<b>H</b>	1	2	3
	<b>S</b>	6	11	16

**b**



4 huts, 21 struts

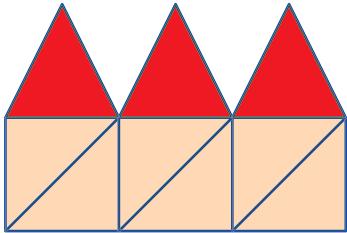


5 huts, 26 struts

**c**  $S = 5H + 1$

**d** 171

**e**



3 huts, 13 struts

<b>f</b>	<b>H</b>	1	2	3
	<b>S</b>	5	9	13

$S = 4H + 1$

**g** 137

- 4 a** 8.0 cm      **b** 9.4 cm  
**c** Let  $d$  be the number of days,  $h$  be the height,  
 $h = 1.4d + 2.4$

**d** 12.2 cm

**e** 27.6 cm

**f** 37.4 cm

**g** Approximately 21 days later

**h** The plant will eventually stop growing. According to the formula, it will grow indefinitely.

**5 a** **i**  $m - 5$

**ii**  $3m$

**b** Kate 2, Bianca 7, Paul 15, Aunt Theresa 21

**c** Kate 8, Bianca 13, Paul 21, Aunt Theresa 39

**6**  $x \times \frac{1}{x} = 1$

**7** 9 tables

**8**  $5y + 20$

**9** 346

**10**  $\frac{3x+8y}{20}$

**11**  $\frac{5x-36}{8}$

**12** 84 coins

**13** 3 cm

**14** 9 years

**15** 586

## CHAPTER 9

### Measurement

#### Are you ready?

**1 a** **i** cm      **ii** km      **iii** m      **iv** mm

**b** Check with your teacher.

**2 a** 1 unit      **b** 0.1 of a unit

**3 a** 43 mm      **b** 3.7 cm

**4** Metres

**5 a** 18 000 m      **b** 6 km

**6 a** 2.3 cm, 25 mm, 0.2 m

**b** 1700 mm, 1.8 m, 185 cm

**7 a** 32 cm      **b** 17 mm

**8 a** **i**  $m^2$       **ii**  $cm^2$       **iii**  $km^2$       **iv** ha

**b** Check with your teacher.

**9 a** 4  $cm^2$       **b** 12  $cm^2$       **c** 13  $cm^2$

**10 a** The area of a rectangle is found by multiplying the length by the width.

**b** 15  $cm^2$

**11 a** The area of a triangle is half the base multiplied by the height.

**b** 20  $cm^2$

**12 a** **i**  $cm^3$       **ii** L      **iii** mL      **iv**  $m^3$

**b** Check with your teacher.

**13 a** 6  $cm^3$       **b** 18  $cm^3$

#### 9A Units of measurement and converting units of measurement

**1 a** **m, cm**      **b** **m, km**      **c** **mm, cm**      **d** **m**

**e** **m**      **f** **m, cm**      **g** **cm**      **h** **m, cm**

**i** **cm, m**      **j** **km**

**2 a** **m**      **b** **cm**      **c** **cm**      **d** **mm**

**e** **cm, mm**      **f** **km**      **g** **m, cm**      **h** **cm, mm**

**i** **cm, mm**      **j** **mm**

**k** **km, m**      **l** **cm, mm**

**3 A**

**4 C**

**5** Various answers

**6** Various answers

**7 a** 2000 m

**b** 7000 m

**c** 65 000 m

**d** 5300 m

**e** 660 m

**f** 900 cm

**g** 25 cm

**h** 280 mm

**i** 2000 mm

**j** 70 000 cm

**8 a** 8 km

**b** 6.5 km

**c** 0.7 km

**d** 0.05 km

**e** 0.0105 km

**f** 60 m

**g** 3 m

**h** 0.57 m

**i** 0.09 m

**j** 4.5 cm

**k** 83.5 cm

**l** 2560 cm

- 9** **a** 800 000 cm      **b** 5.4 m  
**c** 101 000 mm      **d** 0.078 km  
**e** 60 250 000 mm      **f** 1.128 m  
**g** 72 330 mm      **h** 4.05 cm  
**i** 5050 m      **j** 0.030 35 mm

- 10** D      **11** A

- 12** **a** 67 700 m      **b** 0.0452 km      **c** 0.56 m  
**d** 4.5 km      **e** 87.5 mm      **f** 60 cm  
**g** 7210 mm      **h** 309 000 cm      **i** 4.8 cm  
**j** 11 655 mm

- 13** 868 cm, 10.8 cm      About 80 times longer

- 14** Everest 8.863 km, K2 8.607 km

- 15** 1630 cm      **16** 448 m

- 17** **a** 150 cm, 12.5 m, 0.02 km  
**b** 0.445 m, 3000 mm, 350 cm  
**c** 50 000 mm, 500 m, 50 km  
**d** 1.7 m, 1700 cm, 0.17 km  
**e** 0.0052 mm, 0.000 052 m, 0.052 cm  
**f** 0.909 m, 9000 mm, 990 cm

- 18** **a** 3.75 m (375 cm)      **b** 10.2 km (10 200 m)  
**c** 432 cm (4320 mm)      **d** 2.242 m (224.2 cm)  
**e** 3.91 m (391 cm)      **f** 25.6 m (25 600 mm)  
**g** 47.8 km (47 800 m)      **h** 0.001 553 km (155.3 cm)

- 19** **a** 66.4 km (66 400 m)      **b** 410.4 m (410 400 mm)  
**c** 0.104 m (10.4 cm)      **d** 1.7 km (170 000 cm)

- 20** 2.76 cm      **21** 44      **22** 216 m

- 23** 1.593 m      **24** Yes, 36 cm

- 25** 215 cm, 105 cm, 155 cm, 170 cm      **26** 2.4 m

- 27** 198 cm      **28** 40 cm      **29** 38

## 9B Reading scales and measuring length

- |                          |                          |
|--------------------------|--------------------------|
| <b>1</b> <b>a</b> i 4 cm | ii N/A                   |
| <b>b</b> i 7 cm          | ii N/A                   |
| <b>c</b> i 2 cm          | ii N/A                   |
| <b>d</b> i 9.5 cm        | ii $9\frac{1}{2}$ cm     |
| <b>e</b> i 6.4 cm        | ii $6\frac{2}{5}$ cm     |
| <b>f</b> i 10.1 cm       | ii $10\frac{1}{10}$ cm   |
| <b>g</b> i 5.8 cm        | ii $5\frac{4}{5}$ cm     |
| <b>h</b> i 11.6 cm       | ii $11\frac{3}{5}$ cm    |
| <b>i</b> i 2.3 units     | ii $2\frac{3}{10}$ units |
| <b>j</b> i 1.6 units     | ii $1\frac{3}{5}$ units  |

- 2** **a** i  $30^\circ$       ii  $165^\circ$       iii  $288^\circ$   
**b** i  $45^\circ\text{F}$       ii  $34^\circ\text{C}$       iii  $10^\circ\text{C}$       iv  $26^\circ\text{C}$   
**c** i  $45 \text{ km/h}$       ii  $80 \text{ km/h}$       iii  $100 \text{ km/h}$   
**d** i 6.5 cm      ii 19.8 cm      iii 27.5 cm  
**e** i 1250 mL (5 cups)      ii 1800 mL (7 cups)  
 iii 500 mL (2 cups)      iv 1000 mL (4 cups)

- f** 11

- g** i  $130^\circ\text{C}$       ii  $220^\circ\text{C}$       iii  $360^\circ\text{F}$

- 3** B      **4** C

- |   |                                  |                                  |                                  |
|---|----------------------------------|----------------------------------|----------------------------------|
| <b>5</b> <b>a</b> 5 cm                    | <b>b</b> 7 cm                    | <b>c</b> 3 cm                    | <b>d</b> 8 cm                    |
| <b>e</b> 9 cm                             | <b>f</b> 2.5 cm                  | <b>g</b> 4.5 cm                  | <b>h</b> 6.5 cm                  |
| <b>6</b> <b>a</b> 45 mm                   | <b>b</b> 86 mm                   | <b>c</b> 90 mm                   | <b>d</b> 35 mm                   |
| <b>e</b> 25 mm                            | <b>f</b> 80 mm                   | <b>g</b> 42 mm                   | <b>h</b> 65 mm                   |
| <b>7</b> <b>a</b> $4\frac{1}{2}\text{cm}$ | <b>b</b> $8\frac{3}{5}\text{cm}$ | <b>c</b> 9 cm                    | <b>d</b> $3\frac{1}{2}\text{cm}$ |
| <b>e</b> $2\frac{1}{2}\text{cm}$          | <b>f</b> 8 cm                    | <b>g</b> $4\frac{1}{5}\text{cm}$ | <b>h</b> $6\frac{1}{2}\text{cm}$ |
| <b>8</b> <b>a</b> 150 million km          | <b>b</b> 60 m                    | <b>c</b> 4.6 m                   | <b>d</b> 1.74 m                  |
| <b>e</b> i 187.5 km                       | ii 412.5 km                      | iii 540 km                       |                                  |
| <b>f</b> 136 m                            |                                  | <b>g</b> 13.32 m                 |                                  |

- 9** Measure the thickness of the book using a ruler, then divide the thickness of the book by the number of pages (two-sided) in the book to find the thickness of each page.

**10** Answers will vary.

## 9C Perimeter

- |                           |                   |                   |                   |
|---------------------------|-------------------|-------------------|-------------------|
| <b>1</b> <b>a</b> 8 units | <b>b</b> 16 units | <b>c</b> 18 units | <b>d</b> 10 units |
| <b>e</b> 18 units         | <b>f</b> 22 units | <b>g</b> 16 units | <b>h</b> 12 units |
| <b>i</b> 14 units         | <b>j</b> 14 units |                   |                   |
| <b>2</b> <b>a</b> 40 cm   | <b>b</b> 92 mm    | <b>c</b> 108 mm   | <b>d</b> 30 m     |
| <b>e</b> 44 cm            | <b>f</b> 29.5 m   | <b>g</b> 74 cm    | <b>h</b> 173 mm   |
| <b>i</b> 68 cm            | <b>j</b> 28 cm    |                   |                   |
| <b>3</b> <b>a</b> 16 cm   | <b>b</b> 16 cm    | <b>c</b> 18 cm    | <b>d</b> 8 cm     |
| <b>e</b> 22 cm            | <b>f</b> 22 cm    | <b>g</b> 15 cm    | <b>h</b> 25 cm    |
| <b>i</b> 22 cm            | <b>j</b> 18 cm    |                   |                   |
| <b>4</b> <b>a</b> 108 mm  | <b>b</b> 173 mm   | <b>c</b> 18 units |                   |
| <b>d</b> 2995 cm          | <b>e</b> 408 mm   | <b>f</b> 180 mm   |                   |
| <b>g</b> 14 433 m         | <b>h</b> 17 579 m |                   |                   |

- 5** B

- 6** D

- 7** A

- 8** **a** 156 cm      **b** 664.2 cm

- c**  $12\frac{9}{10}$  m or 12.9 m

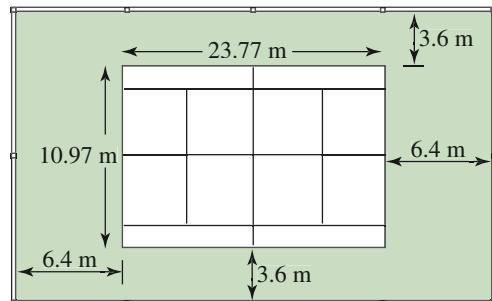
- 9** **a** 14.8 cm      **b** 8.56 m

- c**  $32\frac{4}{5}$  mm or 32.8 mm

- 10** 48.5 m      **11** 252 cm

- 12** **a** 193 m      **b** \$3667

- 13** **a**



- b** 109.48 m

- c** \$4050.76

- 14** 325.8 m

- 15** 139.8 m

- |                          |                    |                |
|--------------------------|--------------------|----------------|
| <b>16</b> <b>a</b> 120 m | <b>b</b> \$228     | <b>c</b> \$810 |
| <b>d</b> 25 posts        | <b>e</b> \$341.25  | <b>f</b> 1600  |
| <b>g</b> \$1680          | <b>h</b> \$4059.25 |                |

## 9D Area

- |                                     |                                      |                            |                            |
|-------------------------------------|--------------------------------------|----------------------------|----------------------------|
| <b>1</b> <b>a</b> $\text{cm}^2$     | <b>b</b> $\text{cm}^2$               | <b>c</b> $\text{m}^2$      | <b>d</b> $\text{mm}^2$     |
| <b>e</b> $\text{km}^2$              | <b>f</b> $\text{cm}^2$               | <b>g</b> $\text{m}^2$      | <b>h</b> $\text{m}^2$      |
| <b>i</b> $\text{km}^2$              | <b>j</b> $\text{m}^2$                | <b>k</b> ha                | <b>l</b> $\text{mm}^2$     |
| <b>2</b> <b>a</b> $14 \text{ cm}^2$ | <b>b</b> $12 \text{ cm}^2$           | <b>c</b> $10 \text{ cm}^2$ | <b>d</b> $10 \text{ cm}^2$ |
| <b>e</b> $10 \text{ cm}^2$          | <b>f</b> $8 \text{ cm}^2$            |                            |                            |
| <b>3</b> <b>a</b> $16 \text{ cm}^2$ | <b>b</b> $8 \text{ cm}^2$            | <b>c</b> $7 \text{ cm}^2$  | <b>d</b> $9 \text{ cm}^2$  |
| <b>e</b> $6 \text{ cm}^2$           | <b>f</b> $9\frac{1}{2} \text{ cm}^2$ | <b>g</b> $12 \text{ cm}^2$ | <b>h</b> $10 \text{ cm}^2$ |

- 4** Approximate answers — accept answers within 2  $\text{cm}^2$  of those listed here.

- a**  $19 \text{ cm}^2$       **b**  $24 \text{ cm}^2$       **c**  $19 \text{ cm}^2$

- 5** **a** i Length 4 cm, width 2 cm

- ii  $8 \text{ cm}^2$       iii  $8 \text{ cm}^2$       iv same

- b** i Length 6 cm, width 1 cm

- ii  $6 \text{ cm}^2$       iii  $6 \text{ cm}^2$       iv same

- c** i Length 6 cm, width 4 cm  
ii  $24 \text{ cm}^2$       iii  $24 \text{ cm}^2$   
**d** i Length 8 cm, width 5 cm  
ii  $40 \text{ cm}^2$       iii  $40 \text{ cm}^2$   
**6** **a**  $15 \text{ cm}^2$       **b**  $4 \text{ cm}^2$       **c**  $15 \text{ m}^2$   
**d**  $14 \text{ km}^2$       **e**  $24 \text{ cm}^2$       **f**  $10 \text{ m}^2$   
**7** **a**  $l = 1.8 \text{ cm}$       **w**  $= 1.2 \text{ cm}$       **A**  $= 2.16 \text{ cm}^2$   
**b**  $l = 3.6 \text{ cm}$       **w**  $= 0.7 \text{ cm}$       **A**  $= 2.52 \text{ cm}^2$   
**c**  $l = 31 \text{ mm}$       **w**  $= 12 \text{ mm}$       **A**  $= 372 \text{ mm}^2$   
**d**  $l = 25 \text{ mm}$       **w**  $= 0.4 \text{ mm}$       **A**  $= 10 \text{ mm}^2$   
**e**  $l = 2.5 \text{ cm}$       **w**  $= 1.3 \text{ cm}$       **A**  $= 3.25 \text{ cm}^2$   
**f**  $l = 25 \text{ mm}$       **w**  $= 25 \text{ mm}$       **A**  $= 625 \text{ mm}^2$   
**8** **a**  $20 \text{ cm}^2$       **b**  $48 \text{ m}^2$       **c**  $261 \text{ mm}^2$   
**d**  $56 \text{ m}^2$       **e**  $400 \text{ mm}^2$       **f**  $72 \text{ cm}^2$   
**g**  $76 \text{ m}^2$       **h**  $18 \text{ cm}^2$       **i**  $135 \text{ mm}^2$   
**j**  $686 \text{ mm}^2$   
**9** **a**  $4 \text{ cm}^2$       **b**  $3 \text{ cm}^2$       **c**  $40.5 \text{ cm}^2$   
**d**  $8 \text{ cm}^2$       **e**  $50 \text{ cm}^2$       **f**  $1 \text{ m}^2$   
**10** **a** C      **b** A      **c** D  
**d** A  
**11** **a**  $416 \text{ cm}^2$       **b**  $4 \text{ m}^2$       **c**  $3.125 \text{ m}^2$   
**d**  $2.4 \text{ m}^2$       **e**  $493 \text{ mm}^2$       **f**  $583 \text{ cm}^2$   
**g**  $600 \text{ km}^2$       **h**  $337.5 \text{ cm}^2$       **i**  $152 \text{ cm}^2$   
**12**  $28.14 \text{ mm}^2$

Base	Height	Area
<b>a</b> 5 mm	3 mm	$15 \text{ mm}^2$
<b>b</b> 8 cm	7 cm	$56 \text{ cm}^2$
<b>c</b> 10 m	5.2 m	$52 \text{ m}^2$
<b>d</b> 6 km	2 km	$12 \text{ km}^2$
<b>e</b> 4 cm	30 mm	$1200 \text{ mm}^2$
<b>f</b> 4.9 m	8.7 m	$42.63 \text{ m}^2$

- 14** **a**  $23 \text{ m}^2$       **b**  $27 \text{ cm}^2$   
**c**  $7952\,400 \text{ km}^2$       **d**  $22 \text{ cm}^2$   
**15**  $200\,000 \text{ m}^2$   
**16** **a**  $16\,000\,000\,000 \text{ m}^2$       **b** 1 600 000 hectares  
**17**  $17.5 \text{ m}^2$   
**18**  $800 \text{ cm}^2$   
**19**  $9.66 \text{ m}^2$   
**20** **a**  $7.5 \text{ m}^2$   
**b** 15 boxes  
**c** \$528  
**21**  $104 \text{ m}^2$ , \$1300  
**22** \$141.64  
**24** 3 sheets  
**26**  $864 \text{ cm}^2$   
**28**  $900 \text{ cm}^2$   
**30**  $12.6 \text{ m}^2$   
**32** **a**  $212\,500 \text{ km}^2$   
**b** Check with your teacher.  
**33** \$2560  
**34** Length 9 cm, width 4 cm  
**35** 48 mm

### 9E Area of composite shapes, using addition and subtraction

- 1** **a**  $81 \text{ cm}^2$       **b**  $5 \text{ m}^2$       **c**  $16 \text{ cm}^2$   
**d**  $56 \text{ cm}^2$       **e**  $588 \text{ cm}^2$       **f**  $13.5 \text{ m}^2$   
**2** **a**  $48 \text{ m}^2$   
**3** **a**  $23.625 \text{ m}^2$   
**4** **a**  $27 \text{ m}^2$   
**5**  $13.05 \text{ m}^2$   
**b**  $\$283.50$   
**b** \$675

- 6**  $39.28 \text{ m}^2$   
**7**  $315 \text{ cm}^2$   
**8**  $14.25 \text{ m}^2$   
**9** \$2157.60 (rounded to the nearest 5 cents)  
**10**  $75 \text{ m}^2$   
**11**  $39 \text{ m}^2$

### 9F Volume

- |                                     |                                |                             |
|-------------------------------------|--------------------------------|-----------------------------|
| <b>1</b> <b>a</b> $8 \text{ cm}^3$  | <b>b</b> $8 \text{ cm}^3$      | <b>c</b> $6 \text{ cm}^3$   |
| <b>d</b> $12 \text{ cm}^3$          | <b>e</b> $12 \text{ cm}^3$     | <b>f</b> $48 \text{ cm}^3$  |
| <b>2</b> B                          |                                |                             |
| <b>3</b> <b>a</b> $4 \text{ cm}^3$  | <b>b</b> $24 \text{ cm}^3$     | <b>c</b> $15 \text{ cm}^3$  |
| <b>d</b> $8 \text{ cm}^3$           | <b>e</b> $13 \text{ cm}^3$     | <b>f</b> $7 \text{ cm}^3$   |
| <b>g</b> $35 \text{ cm}^3$          | <b>h</b> $14 \text{ cm}^3$     | <b>i</b> $17 \text{ cm}^3$  |
| <b>4</b> A                          |                                |                             |
| <b>5</b> <b>a</b> $12 \text{ cm}^3$ | <b>b</b> $12 \text{ m}^3$      | <b>c</b> $243 \text{ cm}^3$ |
| <b>d</b> $6144 \text{ cm}^3$        | <b>e</b> $4 \text{ cm}^3$      | <b>f</b> $192 \text{ m}^3$  |
| <b>g</b> $270 \text{ cm}^3$         | <b>h</b> $101.25 \text{ cm}^3$ |                             |

**6**

Prism	Area of base ( $\text{cm}^2$ )	Height (cm)	Volume ( $\text{cm}^3$ )
<b>a</b>	6	2	12
<b>b</b>	5	2	10
<b>c</b>	3	3	9
<b>d</b>	5	2	10
<b>e</b>	5	3	15
<b>f</b>	4	4	16
<b>g</b>	3	2	6
<b>h</b>	7	3	21
<b>i</b>	8	3	24
<b>j</b>	6	2	12

- 7** D  
**9** **a**  $126.9 \text{ cm}^3$   
**c**  $720 \text{ mm}^3$   
**e**  $81.9 \text{ cm}^3$   
**10** **a**  $186 \text{ cm}^3$   
**c**  $189 \text{ cm}^3$   
**11**  $9690 \text{ cm}^3$   
**13** 20 cm  
**15** 12 cm  
**16** **a**  $2700 \text{ cm}^3$   
**17**  $4.875 \text{ m}^3$   
**18** **a**  $28.875 \text{ cm}^3$   
**c**  $176 \text{ mm}^3$   
**19**  $768 \text{ m}^3$   
**21** Each guest receives  $267 \text{ cm}^3$  of cake.  
**22**  $2277 \text{ cm}^3$   
**23**  $2500 \text{ m}^3$   
**24** **a**  $1920 \text{ cm}^3$       **b** 12

### 9G Capacity

- |  |  |                              |
|--|--|------------------------------|
| <b>1</b> <b>a</b> 2000 mL                      | <b>b</b> 3 L                                 | <b>c</b> $13\,000 \text{ L}$ |
| <b>d</b> 7 L                                   | <b>e</b> 5.5 L                               | <b>f</b> $0.26 \text{ kL}$   |
| <b>g</b> 2500 mL                               | <b>h</b> 32 L                                | <b>i</b> $0.055 \text{ L}$   |
| <b>j</b> 35 mL                                 | <b>k</b> $420\,000 \text{ mL}$               | <b>l</b> 990 L               |
| <b>m</b> 1870 mL                               | <b>n</b> 22.5 L                              | <b>o</b> $0.35 \text{ L}$    |
| <b>p</b> 100 mL                                | <b>q</b> 0.025 kL                            | <b>r</b> 7 mL                |
| <b>2</b> <b>a</b> 750 mL                       | <b>b</b> $2500 \text{ kL}$                   |                              |
| <b>c</b> $800 \text{ mL} = 0.8 \text{ L}$      | <b>d</b> $40\,000 \text{ mL} = 40 \text{ L}$ |                              |
| <b>e</b> $6000 \text{ mL} = 6000 \text{ cm}^3$ | <b>f</b> $5.2 \text{ kL} = 5.2 \text{ m}^3$  |                              |
| <b>g</b> $2450 \text{ cm}^3$                   | <b>h</b> 78 L                                |                              |

- 3** D  
**4** D  
**5** **a** 0.25 L, 2.45 L, 2.5 L, 25 000 mL  
**b** 7.65 mL, 760 mL, 0.765 L, 7.60 L  
**c** 0.011 L, 0.1 L, 110 mL, 1.1 L  
**6** 16 bottles  
**7** 1750 mL (1.75 L)      **8** 1620 mL (1.62 L)  
**9** 6 L      **10** 50 doses  
**11** 225 bottles  
**12** The 185 mL container is the better buy.  
**13** 9.9 L      **14** 4.8 L  
**15** 616 mL      **16** No  
**17** Yes      **18** 50.4 L  
**19** Approximately 27 L (27.09 L)  
**20** Answers will vary.  $16.4 \text{ kL} = 16.4 \text{ m}^3$ . An example could be length 2 m, width 2 m, height 4.1 m.  
**21** 300 mL

### Chapter review

#### Fluency

- 1** **a** Kilometres or metres  
**b** Centimetres  
**c** Millimetres  
**2** **a** 0.56 m      **b** 0.023 km      **c** 1700 cm  
**d** 750 000 mm      **e** 2090 mm      **f** 0.068 m  
**g** 2.25 cm      **h** 6.3 m      **i** 82 000 km  
**j** 0.59 cm  
**3** 0.004 45 km, 455 cm, 44.5 m, 455 000 mm  
**4** **a** 51 200 m (51.2 km)  
**b** 3700 cm (37 m)  
**5** **a** 114 kg      **b** 42 kg  
**c** 80 kg      **d** 95 kg  
**6** **a** **i** 4.5 cm      **b** **i**  $4\frac{1}{2}$  cm  
**b** **i** 3.2 cm      **b** **ii**  $3\frac{1}{5}$  cm  
**c** **i** 2.9 cm      **b** **ii**  $2\frac{9}{10}$  cm  
**7** 5.525 m  
**8** **a**  $\text{m}^2$       **b**  $\text{km}^2$       **c**  $\text{mm}^2$   
**d** ha      **e**  $\text{cm}^2$       **f**  $\text{cm}^2$  or  $\text{mm}^2$   
**g**  $\text{m}^2$   
**9** **a** 12  $\text{cm}^2$       **b** 8  $\text{cm}^2$       **c** 13  $\text{cm}^2$   
**d** 2  $\text{cm}^2$       **e** 5  $\text{cm}^2$       **f** 8  $\text{cm}^2$   
**g**  $6\frac{1}{2} \text{cm}^2$   
**h** 15  $\text{cm}^2$   
**10** **a** 15  $\text{cm}^2$       **b** 23  $\text{cm}^2$   
**11** **a** 150  $\text{m}^2$       **b** 6  $\text{cm}^2$       **c** 2914  $\text{mm}^2$   
**12** **a** 5.04  $\text{cm}^2$       **b** 1.98  $\text{cm}^2$       **c** 4.8  $\text{cm}^2$   
**13** **a** 9  $\text{cm}^2$       **b** 9  $\text{cm}^2$       **c** 19  $\text{cm}^2$   
**d** 52  $\text{cm}^2$       **e** 380  $\text{cm}^2$       **f** 11  $\text{cm}^2$   
**14** **a** 627  $\text{cm}^2$       **b** 96  $\text{m}^2$       **c** 1344  $\text{cm}^2$   
**15** **a** 16  $\text{m}^2$       **b** 432  $\text{mm}^2$   
**c** 87.5  $\text{cm}^2$       **d** 41  $\text{cm}^2$   
**16** **a** 18  $\text{cm}^3$       **b** 12  $\text{cm}^3$       **c** 30  $\text{cm}^3$   
**d** 16  $\text{cm}^3$       **e** 20  $\text{cm}^3$       **f** 28  $\text{cm}^3$   
**17** **a** 36  $\text{cm}^3$       **b** 24  $\text{cm}^3$       **c** 15  $\text{cm}^3$   
**d** 24  $\text{cm}^3$       **e** 24  $\text{cm}^3$       **f** 8  $\text{cm}^3$   
**18** **a** 112  $\text{cm}^3$       **b** 384  $\text{cm}^3$   
**c** 40  $\text{cm}^3$       **d** 72  $\text{cm}^3$   
**19** **a** 8000 mL      **b** 420 mL      **c** 3.3 L  
**d** 1012 mL      **e** 4300 L      **f** 3400 mL  
**g** 4.755 kL      **h** 0.432 L

#### Problem solving

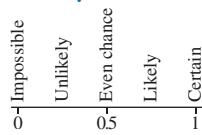
- 1** 25 cm  
**2** 8.7 m (870 cm)

- 3** **a** 46 cm      **b** 20 cm  
**c** 191 mm (19.1 cm)      **d** 21 cm (210 mm)  
**4** 243 m  
**6** **a** 41 m      **b** \$346.45  
**7** 870 cm      **8** 540 m  
**9** Answers will vary.  
**11** \$582.40      **12** 3.6  $\text{m}^2$   
**13** 0.72  $\text{m}^2$       **14** 60  $\text{m}^3$   
**15** 144  $\text{m}^3$       **16** 28 cm  
**17** 8.5 cm      **18** 15 960  $\text{cm}^3$   
**19** 4.9 L  
**20** **a** 1725 mL (1.725 L)      **b** 8 glasses  
**21** Approximately 3 L  
**23** The perimeter is 48  
**25** 188 mm      **26** 39.6 cubic metres.  
**27** 0.704  $\text{m}^3$ .  
**28** 10 cm by 30 cm.  
**29** Area of paths is 78  $\text{m}^2$ ; 4680 bricks are needed.  
**30** Possible sides dimensions: 9 × 1; 8 × 2; 7 × 3; 6 × 4; 5 × 5. Largest area is 25 square units.  
**31** Water level will rise approximately by 0.1 m. The largest cube will have a side length of 1.016 m  
**32** \$5 is 130 mm long; \$10 is 0.137 m long; \$20 is 14.4 cm long; \$50 is 15.1 cm long; \$100 is 0.158 m long.  
**33** 12 000 000 pools  
**34** 8 cm

### CHAPTER 10

#### Probability

##### Are you ready?



**b** Check with your teacher.

- 2** **ii, i, iii**  
**3** **a** More than a half  
**b** Less than a half  
**c** More than a half  
**4** **a** {Head, Tail}  
**b** {1, 2, 3, 4, 5, 6}  
**c** {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}  
**5** **a** 52  
**b** Hearts, diamonds, clubs, spades  
**c** 4  
**6** **a**  $\frac{3}{5}$       **b**  $\frac{1}{6}$

##### 10A The language of chance

- 1** **a** Certain      **b** Likely      **c** Impossible  
**d** Impossible      **e** Certain      **f** Likely  
**g** Unlikely      **h** Unlikely      **i** Impossible  
**j** Even chance      **k** Certain      **l** Certain  
**m** Impossible      **n** Even chance      **o** Likely  
**2** D, C, A, B  
**3** **a**  $\frac{1}{2}$  (or 0.5)      **b** 0      **c**  $\frac{1}{5}$  (or 0.2)  
**d**  $\frac{1}{2}$  (or 0.5)      **e**  $\frac{1}{2}$  (or 0.5)      **f**  $\frac{1}{4}$  (or 0.25)  
**g**  $\frac{4}{5}$  (or 0.8)      **h** 1      **i** 1  
**j** 0.3–0.7. Any value within this range is reasonable.  
 Discuss.

**4** Class discussion or check with your teacher.

**5** A      **6** A

- 7** **a** 1      **b** 0.75      **c** 0.25      **d** 0.75  
**e** 0.25      **f** 1      **g** 0      **h** 0.25  
**i** 1      **j** 0.25      **k** 0      **l** 0.75  
**m** 0.5      **n** 0.5

### 10B The sample space

- 1**  $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$   
**2** **a**  $S = \{\text{Heads, Tails}\}$   
**b**  $S = \{\text{A, B, C, D, E}\}$   
**c**  $S = \{\text{win, loss, draw}\}$   
**d**  $S = \{\text{ace of clubs, ace of spades, ace of hearts, ace of diamonds}\}$   
**e**  $S = \{\text{A, B, C, D, E, F}\}$   
**3** **a**  $S = \{\text{clubs, spades, diamonds, hearts}\}$   
**b**  $S = \{\text{ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king}\}$   
**c**  $S = \{\text{red, black}\}$   
**4**  $S = \{\text{red, green, orange}\}$   
**5** 2  
**6** **a** 52      **b** 45      **c** 16  
**d** 365 (or 366 in a leap year)  
**e** 26  
**f** 180 000      **g** 4      **h** 3

**7** E

**8** 12 different ways:

- 20, 20, 10; 20, 20, 5, 5; 20, 10, 10, 10;  
 20, 10, 10, 5, 5; 20, 10, 5, 5, 5; 20, 5, 5, 5, 5, 5;  
 10, 10, 10, 10, 10; 10, 10, 10, 10, 5, 5;  
 10, 10, 10, 5, 5, 5; 10, 10, 5, 5, 5, 5, 5;  
 10, 5, 5, 5, 5, 5; 5, 5, 5, 5, 5, 5, 5, 5, 5

- 9** SC 378      CS 378  
 SC 387      CS 387  
 SC 738      CS 738  
 SC 783      CS 783  
 SC 837      CS 837  
 SC 873      CS 873

### 10C Simple probability

- 1** **a** 12: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12  
**b** 5: 1, 2, 3, 4, 5  
**c** 2: C, L  
**d** 3: red, blue, black  
**e** 5: black, blue, green, red, yellow  
**f** 3: 2, 4, 6  
**g** 2: 4, 6  
**h** 10: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19  
**2** **a** 1, 2, 3, 4, 5, 6. All equally likely.  
**b** Heads, Tails. Each is equally likely.  
**c** White, black. Each is equally likely.  
**d** White, blue, red. It is more likely you will get white rather than red or blue, because it takes up a bigger area.  
**e** 1, 2, 3, 4, 5. It is more likely that a five will be rolled because it appears twice on the die.  
**f** Blue, green, red. Each is equally likely.  
**g** a, e, i. It is more likely that a consonant would be obtained as there are 7 consonants and 4 vowels.

- 3** **a** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10  
**b** **i**  $\frac{1}{10}$       **ii**  $\frac{3}{5}$       **iii**  $\frac{2}{5}$       **iv**  $\frac{7}{10}$   
**v**  $\frac{3}{10}$       **vi** 0      **vii**  $\frac{3}{10}$       **viii**  $\frac{1}{5}$   
**4** **a** 20  
**b** **i**  $\frac{3}{10}$       **ii**  $\frac{1}{5}$       **iii**  $\frac{7}{20}$       **iv**  $\frac{3}{20}$   
**c** 10      **d**  $\frac{1}{2}$       **e**  $\frac{1}{2}$       **f**  $\frac{13}{20}$   
**g** 1

**h** When the probability of a particular result is certain (in this case, that the marble must be either blue, red, green or black), it equals 1.

**5**  $\frac{2}{5}$

**6** E

**8** **a**  $\frac{1}{2}$

**b**  $\frac{1}{13}$

**c**  $\frac{1}{4}$

**d**  $\frac{3}{13}$

**e**  $\frac{1}{52}$

**f**  $\frac{3}{4}$

**g**  $\frac{12}{13}$

**h** 1

**i**  $\frac{3}{4}$

**j**  $\frac{1}{26}$

**9** **a** 36 square units

**b**

A 20 square units

B 12 square units

C 4 square units

**c** **i**  $\frac{5}{9}$

**ii**  $\frac{1}{3}$

**iii**  $\frac{1}{9}$

**d** **i**  $\frac{37}{351}$ , which is approx 0.11. This is very close to the theoretical probability.

**ii**  $\frac{631}{1872}$ , which is approx 0.34. This is very close to the theoretical probability.

**iii**  $\frac{3131}{5616}$ , which is approx 0.56. This is very close to the theoretical probability.

**10** **a**  $\frac{1}{100\ 000}$

**b**  $\frac{1}{5000}$

**c**  $\frac{1}{2000}$

**d** 1

**e** It would not be a very wise purchase because the total cost would be \$500 000 unless of course the car was worth more than \$500 000.

### 10D Using a table to show sample spaces

- 1** **a** {1, 2, 3, 4, 5, 6}  
**b** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}  
**c** {fruit salad, cheesecake, mudcake, cheese platter}  
**d** {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}  
**e** {Victoria, New South Wales, Queensland, Northern Territory, South Australia, Tasmania, Western Australia, Australian Capital Territory}  
**f** {true, false}  
**g** {king of hearts, king of diamonds, king of spades, king of clubs}  
**h** {guitar, drum, saxophone, piano, trumpet}

**2** **a**

Spinner

	A	B	C
1	A1	B1	C1
2	A2	B2	C2
3	A3	B3	C3
4	A4	B4	C4
5	A5	B5	C5
6	A6	B6	C6

{(A, 1), (A, 2), (A, 3), (A, 4), (A, 5), (A, 6), (B, 1), (B, 2), (B, 3), (B, 4), (B, 5), (B, 6), (C, 1), (C, 2), (C, 3), (C, 4), (C, 5), (C, 6)}

**b** 18

**c** **i**  $\frac{1}{3}$

**ii**  $\frac{1}{6}$

**iii**  $\frac{2}{3}$

**iv**  $\frac{1}{3}$

**v**  $\frac{1}{2}$

**vi**  $\frac{1}{6}$

**vii** 1

**viii**  $\frac{5}{6}$

**ix**  $\frac{1}{9}$

**x** 0

3 a

		Coin 1	
		Head	Tail
Coin 2	Head	H H	T H
	Tail	H T	T T

b  $\frac{4}{4}$

c  $\frac{1}{4}$

d  $\frac{1}{4}$

e  $\frac{1}{2}$

f  $\frac{1}{4}$

g  $\frac{1}{4}$

h  $\frac{1}{2}$

i  $\frac{1}{2}$

j  $\frac{1}{4}$

4 a

		5c Coin	
		Head	Tail
10c Coin	Head	H H	T H
	Tail	H T	T T

b  $\frac{4}{4}$

c  $\frac{1}{4}$

d  $\frac{1}{4}$

e  $\frac{1}{4}$

f  $\frac{1}{4}$

5 a

		Coin	
		Head	Tail
Die	1	H 1	T 1
	2	H 2	T 2
	3	H 3	T 3
	4	H 4	T 4
	5	H 5	T 5
	6	H 6	T 6
	7	H 7	T 7
	8	H 8	T 8
	9	H 9	T 9
	10	H 10	T 10

b  $\frac{20}{20}$

c  $\frac{1}{20}$

d  $\frac{1}{4}$

e  $\frac{1}{2}$

f  $\frac{1}{20}$

g  $\frac{1}{5}$

h  $\frac{1}{2}$

i  $\frac{1}{4}$

j  $\frac{1}{1}$

6 a

		Red die					
		1	2	3	4	5	6
Blue die	1	1 1	2 1	3 1	4 1	5 1	6 1
	2	1 2	2 2	3 2	4 2	5 2	6 2
	3	1 3	2 3	3 3	4 3	5 3	6 3
	4	1 4	2 4	3 4	4 4	5 4	6 4
	5	1 5	2 5	3 5	4 5	5 5	6 5
	6	1 6	2 6	3 6	4 6	5 6	6 6

Note: Your table may list the blue die along the top and therefore it would be listed first.

b  $\frac{36}{36}$

c  $\frac{1}{36}$

d  $\frac{1}{36}$

e  $\frac{11}{36}$

f  $\frac{1}{4}$

g  $\frac{1}{4}$

h  $\frac{1}{4}$

i  $\frac{5}{36}$

j  $\frac{0}{36}$

k  $\frac{0}{36}$

7 a

		Marble colour		
		Red	Blue	Green
Coin	Head	R H	B H	G H
	Tail	R T	B T	G T

b  $\frac{6}{6}$

c  $\frac{1}{6}$

d  $\frac{1}{6}$

e  $\frac{1}{6}$

f  $\frac{1}{3}$

g  $\frac{1}{3}$

h  $\frac{1}{2}$

i  $\frac{1}{2}$

j  $\frac{1}{3}$

k  $\frac{1}{3}$

8 C

9 Note: You may have chosen the opposite number for rows and columns. This is also correct.

a i 31 rows, 2 columns ii 62

b i 2 rows, 3 columns ii 6

c i 72 rows, 6 columns ii 432

d i 10 rows, 6 columns ii 60

e i 30 rows, 6 columns ii 180

f i 100 rows, 2 columns ii 200

10 16 576 000

11  $\frac{1}{18}$

## 10E Experimenting with chance

1  $\frac{1}{5}$ 

2 a

Exp. No.	Heads		Tails	
	Tally	Count	Tally	Count
1		4		6
2		1		9
3		6		4
4		8		2
5		4		6
6		7		3
	Total	30	Total	30

Note: This is only one possible solution. Answers will differ each time.

b  $\frac{2}{5}$  (40%)

c  $\frac{3}{5}$  (60%)

d The theoretical value for both results is  $\frac{1}{2}$  (50%). They differ by 10%.

e Refer to table.

f The combined result equals the theoretical value.

3 4

4 Divide the spinner into 10 sections; 3 of these sections will be shaded in 1 colour and 7 sections will be shaded in another colour.

5  $\frac{3}{8}$ 

6 See Worked example 9.

Note: These values will differ each time the experiment is performed.

Spreadsheets could also be used.

7 Values will differ for each group. As you complete more trials, you will probably notice the values you get experimentally are closer to those you would expect from theoretical probability ( $\frac{1}{2}$  or 50%). If this is not occurring, though, you will probably need more trials.

**8 a**  $\frac{1}{5}$

Colour	1	2	3	4	5
Number of times it occurs					

c  $P(1) = 0$     $P(2) = \frac{1}{5}$     $P(3) = \frac{1}{5}$

$P(4) = \frac{2}{5}$     $P(5) = \frac{1}{5}$

Note: This is the only possible solution. Answers will differ each time.

d Your spinner may not be evenly balanced and this may lead to an increased likelihood of getting one result or another.

e-f Values will differ for each group. As you complete more trials, you will probably notice the values you get experimentally are closer to those you would expect from theoretical probability ( $\frac{1}{5}$  or 20%). If this is not occurring, though, you will probably need more trials.

**9** 12 triangles, 24 squares

**10 a** Each die would need to be thrown about 120 times to get some impression of whether it is biased or not. The same person will need to roll the die each time in the same manner and onto the same surface.

b

Number on die	Black	White
1	20	19
2	19	21
3	20	20
4	21	20
5	19	20
6	21	20
Total	120	120

Note: These values will differ each time the experiment is conducted.

c

Number on die	Probability of black	Probability of white
1	$\frac{20}{120} = \frac{1}{6}$	$\frac{19}{120}$
2	$\frac{19}{120}$	$\frac{21}{120} = \frac{7}{40}$
3	$\frac{20}{120} = \frac{1}{6}$	$\frac{20}{120} = \frac{1}{6}$
4	$\frac{21}{120} = \frac{7}{40}$	$\frac{20}{120} = \frac{1}{6}$
5	$\frac{19}{120}$	$\frac{20}{120} = \frac{1}{6}$
6	$\frac{21}{120} = \frac{7}{40}$	$\frac{20}{120} = \frac{1}{6}$
Total	1	1

The long-term trend suggests that the probability of obtaining each value on either dice will be  $\frac{1}{6}$ .

d The 2 dice appear to be fair as each value occurred approximately 20 times, which is what we would expect in 120 throws. More trials however could be conducted. It is very important when conducting an experiment such as this that the devices used are

even in shape and size and that one doesn't have an advantage over the other.

**11 a** Answers will vary for each packet.

b Check with your teacher.

c Smarties must be replaced each time they are removed, otherwise the chance of taking out 1 of a certain colour would be changed.

d You could use a spinner with sections coloured to give the same probability for each colour as you would get for removing the Smarties from the box. (This is only one possible answer.)

### Chapter review

#### Fluency

**1 a** Certain

b Unlikely

c Even chance

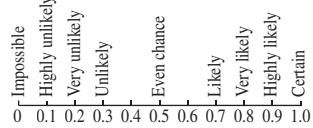
d Impossible

e Likely

f Impossible

**2** Discuss in class.

**3**



**4 a** {Head, Tail}

b {red marble, green marble, blue marble}

c {1, 2, 3, 4, 5, 6}

d {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

e {4 of clubs, 4 of diamonds, 4 of hearts, 4 of spades}

f {true, false} or {correct, incorrect}

g {1, 2, 3, 4, 5, 6, 7, 8, 9}

h {Year 7, Year 8, Year 9, Year 10, Year 11, Year 12}

**5 a** 5000

b 26

c 5

d 31

**6 a**  $\frac{1}{6}$

b  $\frac{1}{3}$

c  $\frac{1}{3}$

d  $\frac{2}{3}$

e  $\frac{2}{3}$

f  $\frac{1}{3}$

g 0

h 0

**7 a**  $\frac{1}{4}$

b  $\frac{1}{4}$

c  $\frac{3}{13}$

d  $\frac{3}{13}$

Note: Do not count aces as a number.

**8 a**  $\frac{1}{10}$

b  $\frac{1}{2}$

c  $\frac{3}{10}$

d  $\frac{1}{10}$

e  $\frac{7}{10}$

f  $\frac{7}{10}$

g  $\frac{1}{5}$

h 0

**9**

Coin

	Head	Tail
1	H 1	T 1
2	H 2	T 2
3	H 3	T 3
4	H 4	T 4
5	H 5	T 5
6	H 6	T 6

**a**  $\frac{1}{6}$

**b**  $\frac{1}{4}$

**c**  $\frac{1}{4}$

**d**  $\frac{1}{4}$

10

Spinner

Die	1	2	3	4	5	6	7	8	9	10
1	11	21	31	41	51	61	71	81	91	101
2	12	22	32	42	52	62	72	82	92	102
3	13	23	33	43	53	63	73	83	93	103
4	14	24	34	44	54	64	74	84	94	104
5	15	25	35	45	55	65	75	85	95	105
6	16	26	36	46	56	66	76	86	96	106

**a**

$$\frac{1}{2}$$

**b**

$$\frac{1}{4}$$

**c**

$$\frac{4}{15}$$

**d**

$$\frac{1}{5}$$

**e**

$$\frac{3}{20}$$

**f**

$$\frac{23}{60}$$

**11**

$$\frac{9}{10}$$

**12** **a**

$$\frac{1}{2}$$

**b** Since both opponents have not played chess before and are given the same instructions on the rules of the game, it is safe to say they will be evenly matched. Neither has an advantage over the other.

**c**

$$\frac{1}{4}$$

**d** No, the probabilities are not the same; they have changed because Jane has practised and developed skills while her father has not.

### Problem solving

**1** **a** 6 socks. Worst case scenario is first picking 4 blue socks, then 2 black socks.

**b** 7 socks. Worst case scenario is first picking 5 black socks then 2 blue socks.

**c** 7 socks. Minimum needed to guarantee a pair of black socks is 6 (from part **a**). Picking one more guarantees a pair of blue socks will be included (from part **b**).

**2** Danielle will need to obtain a fair die with 6 sides to represent each card. She will need to roll the die in the same manner and on the same surface each time. Danielle must continue to roll the die until each number 1, 2, 3, 4, 5, 6 has appeared. This will represent one experiment. She will need to repeat this process at least 20 times until she can obtain the average number of cereal packets she must purchase in order to obtain each of the cards. Danielle may also use a circular spinner with 6 equal sectors or 6 tennis balls labelled 1 to 6 in a box and follow the same procedure outlined above. She may also generate random numbers using the calculator or Excel spreadsheet.

**3** **a** 12**b** 4, 6, 8, 10, 12**c**  $2 + 6, 6 + 2, 4 + 4$ **d** 0**e**

$$\frac{2}{9}$$

**f**

	2	4	6
2	4	6	8
4	6	8	10
6	8	10	12

4	6	8	10	12
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

**4** Yes

$$\frac{1}{3}$$

$$\frac{2}{3}$$

**5** If one container has one gold marble, if that container is selected, Charles is certain to receive \$100. This leaves 9 gold marbles and 10 green marbles in the second container. The probability that the gold is selected from this container is  $\frac{9}{20}$ . The overall probability that Charles

will receive \$100:  $\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{9}{20} = \frac{29}{40}$

$$\frac{3}{8}$$

**6** Red, green, orange, yellow, blue, purple

$$\frac{3}{5}$$

$$\frac{91}{216}$$

$$\frac{1}{20}$$

### CHAPTER 11

#### Problem solving I

##### 11A Introduction to problem solving — create a table or diagram

**1** Half a minute

**2** Balance at end of month \$10. During the second week, her balance was negative.

**3** 2nd floor. It travelled 22 floors in 10 minutes.

**4** **a**

<i>t</i>	1	2	3	4	5	6	7	8
<i>C</i>	21	22	23	24	25	26	27	28

$$b \quad C = t + 20$$

$$c \quad \$1.40$$

**d** 3 minutes

$$5 \quad 64$$

**6** Angus will be 27 and his father will be 81.

$$7 \quad A = 4, B = 8, C = 16, D = 13$$

**8** 6 and 10

**9** 32.04 g or 65.53 g or 43.43 g

**10**

Base	Height	Number of hits	Number of squares
6	4	5	24
3	4	7	12
3	5	8	15
3	6	3	6
4	8	3	8
<i>b</i>	<i>h</i>	$\frac{(b+h)}{\text{H.C.F.}(b,h)}$	$\frac{b \times h}{\text{H.C.F.}(b,h)}$

##### 11B Draw a diagram

**1** Nadia can communicate in 6 different ways.

**2** 15 pins

**3** Trixie

$$4 \quad \frac{19}{60}, \frac{1}{3}$$

$$5 \quad \frac{1}{3}$$

**6** 2 hours

**7** \$6.30

**8** 2 cm × 4 cm, 2 cm × 8 cm, 2 cm × 12 cm, 2 cm × 16 cm

**9** 70 cm × 70 cm concrete pavers

**10** 4520 km

### 11C Look for a pattern — using technology

- 1 Three students attend both swimming practice and lifesaving.
- 2 55
- 3 2.6 m, 6th bounce
- 4 8 years old
- 5 3 kg
- 6 72 questions
- 7 0.06
- 8 a i 2.25      ii 6.25      iii 12.25

b Multiply the units digit by the next higher units digit, then add 0.25 to that number.

- c 72.25
- 9 a 121                          b 10 201
  - c 1 002 001                          d 1 000 002 000 001
  - e 484                                  f 4 008 004

10  $\frac{3}{7} = 0.\dot{4}2857\dot{1}$ ,  $\frac{4}{7} = 0.\dot{5}7142\dot{8}$ ,  $\frac{5}{7} = 0.\dot{7}1428\dot{5}$ ,  $\frac{6}{7} = 0.\dot{8}5714\dot{2}$

### 11D Work backwards from the answer

- 1 49 calls
- 2 135
- 3 64
- 4 Boy 6, grandfather 66
- 5 52 GB
- 6 500 women
- 7 36
- 8 307
- 9 4
- 10 
$$\begin{array}{r} + 92 \\ \hline 121 \end{array}$$

### 11E Elimination

- 1 French L2, German K1, Visual Arts L1
- 2 Two 2-litre containers to house number 5; three 1-litre containers to house number 7; skim milk to house number 18.
- 3 Phone, Eugenie; SMS, Florence; email, Anthea; mail, Kim.
- 4 Alan has a \$2 coin, Bob a 50c piece and Cam a \$1 coin.
- 5 Penny's brother is Matt, Teresa's brother is Fred and Raelene's brother is Steve.
- 6 a-e 416
- 7 a 8, 9                                  b 60 060
- 8 49    9 8.264

10	The Alexander	Master Duncan Sinclair	195 male convicts
	The Borrowdale	Master Readthorn Hobson	No convicts
	The Charlotte	Master Thomas Gilbert	88 male and 20 female convicts
	The Fishburn	Master Robert Brown	No convicts
	The Friendship	Master Francis Walton	76 male and 21 female convicts
	The Golden Grove	Master Sharp	No convicts; Reverend Richard Johnson and his wife
	The Lady Penrhyn	Master William Sever	101 female convicts
	The Prince of Wales	Master John Mason	1 male and 49 female convicts
	The Scarborough	Master John Marshall	208 male convicts
	HMS Sirius	Captain John Hunter	
	HMS Supply	Captain Henry Bull	50 people

### 11F Simplify the problem

- 1 35
- 2 12 345 678 987 654 321
- 3 55
- 4 \$229.20
- 5 325
- 6 25
- 7 180
- 8 120
- 9  $\frac{1}{10}$
- 10 256

### 11G Guess and check

- 1 22 years old
- 2  $10\frac{3}{4}$  hours
- 3 \$1215
- 4 The two numbers are 19 and 13.
- 5 A hamburger costs \$3.75 and a drink costs \$2.45.

6	8	1	6
	3	5	7
	4	9	2

- 7 7, 3, 4, 6
- 8 a 64 footballers would receive a call.  
b Each person would call 6 people and Karin would have 2 extra.
- 9 11, 13, 15
- 10 5, 13, 17, 10

### 11H Mixed problems 1

- 1 Two possible solutions: 2 packets with 3 ice-creams and 5 packets with 5 ice-creams, or 7 packets with 3 ice-creams and 2 packets with 5 ice-creams.  
With further investigation of the number pattern obtained by summing 3s and 5s, it appears that there are combinations of 3 packs and 5 packs that will amount to any quantity of ice-creams equal to or larger than 8.
- 2 Bronwyn's 1st birthday will be on a Thursday unless the next year is a leap year, in which case her 1st birthday will be on a Friday.

There is more chance that the answer will be a Thursday because leap years come around only every 4 years. Since she was born in a non-leap year, then there is a 1-in-3 chance of the following year being a leap year.

- 3 8 students play volleyball only.
- 4 The two numbers are 34 and 35. Their sum is 69.
- 5  $\frac{5}{12}$
- 6  $9 + 8 + 7 + 65 + 4 + 3 + 2 + 1 = 99$ . Other solutions may be possible.
- 7 a \$546.70  
b \$5.90  
c \$2
- 8 2961 m
- 9 0.2 m
- 10 0.2 km

### 11I Mixed problems 2

- 1 9 trips
- 2 
$$\begin{array}{r} 5 & 6 \\ \times 5 & \text{or } \times 6 \\ \hline 25 & 36 \end{array}$$
- 3 8.9494 grains

- 4**  $\frac{2}{5}$   
**5**  $6 \div (1 - 5 \div 7) = 21$   
**6**  $P = 6a - 2b$   
**7** No, second jump was best.  
**8** **a** 12      **b**  $12x$   
**c**  $x^y$   
**9** The older brother carries a 2-kg backpack, 4 kg of books, 6 kg of clothes, 2 kg of snacks, 2 kg of water; a total of 16 kg. The younger brother carries a 2-kg backpack, 3 kg of toiletries, 3 kg of shoes; a total of 8 kg.  
**10** 506 aluminium cans.

### 11J Mixed problems 3

- 1** 5 years  
**2** 16 boys  
**3** 98 641  
**4** Increase 51c  
**5** 38.5 km  
**6**  $145^\circ$   
**7** 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73  
**8** 3, 5 and 7  
**9**  $2^3 \times 1 \div 4 = 2$   
**10** 112 Hip Hop songs.

## CHAPTER 12

### Positive and negative integers

#### Are you ready?

- 1** **a**  $5 > 1$       **b**  $17 < 71$       **c**  $1 < 10$   
**2** **a** 0, 3, 5, 15, 120      **b** 42, 25, 24, 19, 4  
**3** **b**      **a**      **c**  
  
**4** **a** 2 and 4      **b** 6 and 10  
**5** **a** 9      **b** 8      **c** 3  
**6** **a** 19      **b** 12      **c** 1  
**7** **a**  $\frac{5}{6}$       **b**  $\frac{11}{35}$       **c**  $\frac{4.5}{12}$   
**d**  $\frac{5}{14}$       **e**  $1\frac{1}{2}$       **f**  $1\frac{9}{25}$

### 12A Integers on the number line

- 1** **a** -7      **b** 10 or +10  
**c** -422      **d** 2228 or +2228  
**e** -3      **f** -30  
**2**   
**3** **a** -6, -2, 3, 5, 7      **b** -6, -4, -1, 0, 3  
**c** -5, -3, -2, 1, 7      **d** -4, -3, 1, 2, 5  
**4** **a** 6      **b** -7  
**c** -1      **d** 8  
**5** **a** >      **b** <      **c** >  
**d** >      **e** <      **f** <  
**6** **a** i -5      **b** ii 7  
**b** i -4      **b** ii 5  
**c** i -10      **b** ii 7  
**d** i -4      **b** ii 3  
**7** -2 °C  
**8** 14  
**9** **a** 2000 years after the birth of Christ.  
**b** i 10      **b** ii -106  
**c** i -18      **b** ii +15, +36, +41, +47, +48  
**d** Check with your teacher.  
**10** Answers will vary.

### 12B Integers on the number plane

- 1** **a** (3, 4)      **b** (8, 4)      **c** (4, 8)  
**d** (0, 7)      **e** (5, 0)  
**2** **a** G      **b** F  
**c** H      **d** J  
**3** **a** (3, 1); 1st quadrant      **b** (-2, 4); 2nd quadrant  
**c** (-6, -5); 3rd quadrant      **d** (0, -3); y-axis  
**e** (6, -6); 4th quadrant  
**4** **a** C; 2nd quadrant      **b** L; y-axis  
**c** D; 4th quadrant      **d** E; 1st quadrant  
**e** G; x-axis  
**5** **a** 3      **b** 3      **c** 7      **d** 0  
**6** **a** 2      **b** -6      **c** 0      **d** -3  
**7** H      **b** 0      **9** 0      **10** 4  
**11** **a** T      **b** T      **c** T  
**d** F      **e** F      **f** T  
**12** **a** D      **b** B      **c** C  
**13**

- a** Parallelogram      **b** Isosceles triangle  
**c** Kite      **d** Square

- 14** **a** (2, 1)      **b** (1, -2)      **c** Draw line DO  
**d** (-3, 1), (-2, 1), (-1, 1) etc      **e** (-5, -2)

### 12C Addition and subtraction of integers

- |                                 |                         |              |              |
|---------------------------------|-------------------------|--------------|--------------|
| <b>1</b> <b>a</b> $-4 + +4 = 0$ | <b>b</b> $-1 + +3 = +2$ |              |              |
| <b>c</b> $-2 + -4 = -6$         | <b>d</b> $+2 + -4 = -2$ |              |              |
| <b>e</b> $+6 + -4 = +2$         | <b>f</b> $-1 + -3 = -4$ |              |              |
| <b>2</b> <b>a</b> -1            | <b>b</b> -1             | <b>c</b> 0   |              |
| <b>d</b> +1                     | <b>e</b> -2             | <b>f</b> -2  |              |
| <b>3</b> <b>a</b> $5 + -2 = 3$  | <b>b</b> $-3 + -4 = -7$ |              |              |
| <b>c</b> $-2 + 2 = 0$           | <b>d</b> $6 + -5 = 1$   |              |              |
| <b>e</b> $-5 + 5 = 0$           | <b>f</b> $4 + -6 = -2$  |              |              |
| <b>g</b> $-5 + 7 = 2$           | <b>h</b> $6 + -9 = -3$  |              |              |
| <b>i</b> $-4 + 6 = 2$           | <b>j</b> $3 + -3 = 0$   |              |              |
| <b>k</b> $-8 + -2 = -10$        | <b>l</b> $0 + -6 = -6$  |              |              |
| <b>4</b> <b>a</b> -7            | <b>b</b> -2             | <b>c</b> 0   | <b>d</b> -4  |
| <b>e</b> 4                      | <b>f</b> -10            | <b>g</b> 4   | <b>h</b> -3  |
| <b>i</b> -100                   | <b>j</b> 10             | <b>k</b> 2   | <b>l</b> -22 |
| <b>5</b>                        |                         |              |              |
| <b>6</b> <b>a</b> 2             | <b>b</b> 10             | <b>c</b> -10 | <b>d</b> 2   |
| <b>e</b> -9                     | <b>f</b> -10            | <b>g</b> 0   | <b>h</b> -4  |
| <b>i</b> 3                      | <b>j</b> 10             | <b>k</b> -14 | <b>l</b> 0   |
| <b>7</b> <b>a</b> -10           | <b>b</b> 13             | <b>c</b> 3   | <b>d</b> 12  |
| <b>e</b> -16                    | <b>f</b> -17            | <b>g</b> 23  | <b>h</b> -12 |
| <b>8</b> <b>a</b> -8            | <b>b</b> -6             | <b>c</b> 10  | <b>d</b> -7  |
| <b>e</b> 3                      | <b>f</b> 1              | <b>g</b> 9   | <b>h</b> -25 |
| <b>9</b> <b>a</b> 3             | <b>b</b> -10            | <b>c</b> 7   | <b>d</b> 6   |
| <b>e</b> -5                     | <b>f</b> 15             | <b>g</b> 15  | <b>h</b> -11 |
| <b>i</b> -1                     |                         |              |              |

10 a	+ -2 -1 0 1 2
	-2 -4 -3 -2 -1 0
	-1 -3 -2 -1 0 1
	0 -2 -1 0 1 2
	1 -1 0 1 2 3
	2 0 1 2 3 4

- b Even numbers;  
doubles  
c Zeroes (addition of  
opposites)  
d Yes  
e Discuss

- 11 a 4      b -3      c  $7^{\circ}\text{C}$   
 d  $-1^{\circ}\text{C}$       e -1      f -7  
 12 a  $-2 + -3 = -5$       b  $-3 + 5 = 2$   
 c  $5 + -6 = -1$       d  $-50 + 63 = 13$   
 e  $3 + -5 = -2$       f  $1 + -6 = -5$   
 g  $-50 + 26 = -24$       h  $200 + -350 = -150$

13 B

14 a to e Discuss

- 15 a T      b T      c T      d T  
 16 a 1      b -11      c -7      d -9  
 e 6      f 12  
 17 a D      b B  
 18  $-2^{\circ}\text{C}$       19 12 m

### 12D Extension: Multiplication and division of integers

- 1 a -10      b -18      c -63      d -30  
 e 6      f 20      g 25      h 0  
 i 8      j -24      k -10      l -30  
 m -54      n 200      o 36      p  
 2 a 4      b -4      c -4      d -4  
 e -3      f 2      g 9      h -8  
 i -6      j 6      k 0      l -21  
 m -92      n 25      o 67      p -43  
 q -71      r -56  
 3 a -3      b 3      c -1  
 d 1      e -2      f  $1\frac{1}{2}$   
 4 a -24      b 36      c 144  
 5 a 18      b -3      c -6  
 d 1      e -1      f 0  
 g -1  
 6 a 3      b 32      c -72  
 d -11      e 0      f -16  
 g 7      h -48      i 6  
 7 a -7      b 10      c -3      d 10  
 e 30      f -11      g 1  
 8 a -8, -8      b -6, -6  
 c -6, -6      d -20, -20  
 9 a B      b A  
 10 a  $6p$       b  $-12t$       c  $10b$   
 d  $8a$       e  $-15c$       f  $14d$   
 g  $-12ab$       h  $10xg$   
 11 a 2      b -3      c 1  
 d 1      e 9      f -3  
 g 4      h 1      i 4  
 12 E      13 -\$6      14 8 flies

### 12E Extension: Combined operations

- 1 a -6      b 22      c -6  
 d 6      e -2      f -27  
 g 0      h -1      i 3  
 j -18      k -40      l 35  
 2 a -19      b -1      c -2  
 d -9      e 0      f 21  
 g 2      h -5      i 9  
 j -3      k -7      l 32

- 3 a -20      b -1  
 d -36      e -68  
 4 D  
 5 a  $-100 + 60 + -25 = -65$   
 b  $274 + 2 \times -68 + 3 \times 50 = 288$

- c 600  
 d 8 kg  
 6 a Melbourne 9 h ahead  
 b Singapore 6 h ahead  
 7 a Merlin:  $-3 \times 10 = -30$ , 30 km west of bridge  
 Morgan:  $-3 \times -8 = 24$ , 24 km east of bridge  
 b Merlin:  $2 \times 10 = 20$ , 20 km east of bridge  
 Morgan:  $2 \times -8 = -16$ , 16 km west of bridge  
 c 36 km      d 72 km

### Chapter review

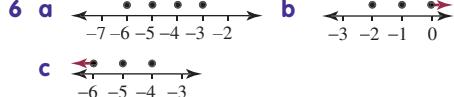
#### Fluency

- 1 a No      b No  
 c Yes      d Yes  
 2 a <      b <      c >      d <

3  $-20, -19, -18, -17, -16$

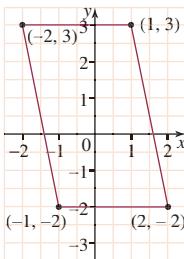
4  $2, 0, -3, -15$

- 5 a  $< 1$  or  $\leq 0$       b Between  $-6$  and  $-3$   
 c  $> -12$  or  $\geq -11$



- 6 a Both axes      b y-axis  
 c x-axis      d First quadrant

Parallelogram



- 9 a Second      b Fourth      c Third  
 d y-axis      e x-axis      f First  
 10 a -5      b -17      c 8      d 6  
 11 a -2      b 7      c 1      d  
 e 0

12  $3 + -2 + 4 + -1 = 4$

The snail is 4 cm from the bottom of the bucket.

- 13 a -8      b 26      c 3  
 d -2      e -8      f 2  
 14 a -42      b -32      c 10  
 d 64      e -64      f -128  
 15 a -12      b 3      c -5  
 d 9      e -16      f -4  
 16 a -2      b -3      c 27  
 d -8      e 5      f -20  
 g 12      h 15      i -125

- 17 a  $-5 < -3$       b  $-22 \times -2 = 44$   
 c  $4 > 2$       d  $-5 \times -3 > -15$   
 e  $0 > -7$       f  $-2(-4 + 7) < 6$   
 g  $-2 \times 5 < -9$       h  $5 \times (-2 - 18) \div 4 < 25$   
 i  $12 \div -4 < 3$       j  $5 \times (-2 - 18) \div 4 = -25$   
 k  $-10 \times -5 = 50$       l  $5 \times -2 - 18 \div 4 < 2.5$   
 18  $-52 + 3 \times 15 = -7$       19 601 years  
 20  $148^{\circ}\text{C}$       21 -\$94

### Problem solving

**1 a**

+	6	10	3
5	11	15	8
12	18	22	15
9	15	19	12

**b**

×	4	-8	-6
-3	-12	24	18
5	20	-40	-30
2	8	-16	-12

**2 a**

Negative

**b** Positive

**c** Negative

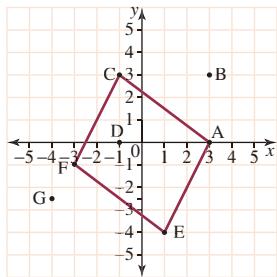
Two negative numbers added together will always be negative. When a positive and a negative number are added together if the larger number is positive the result will be positive, if the larger number is negative the result will be negative.

**3 a** D(-1, 0)

**b i** CBA, BAD others are possible; check with your teacher

**ii** ADE or BCE

**c** F(-3, -1)



**d** (1, 1.5)

**e** (2, -2)

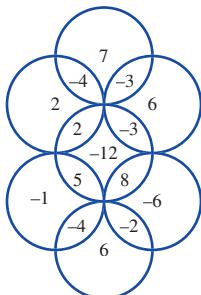
**f** G(-4, -2.5)

**4 a** Arctic Circle 17 °C, London 34 °C, Mexico 22 °C

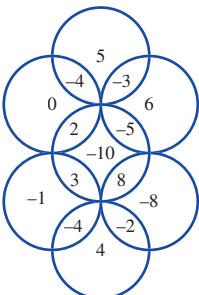
**b** 24 °C

**c** -21 °C

**5 a**



**b**



**6 a** Start  $-4 + 6 + -1 + 3 + 2 + -2 - 4 = 0$  There are more possibilities — check with your teacher

**b** There are several possibilities — check with your teacher

**c** There are several possibilities — check with your teacher

**7** 21 Questions for 3 marks

**8** X is positive and Y is negative

**9**

8	-	4	×	6	=	-16
-		×		-		
3	-	5	+	9	=	7
×		÷		÷		
7	-	2	-	1	=	4
=		=		=		
-13		10		-3		

**10** Column 2

**11**

2009 and 2008

**12 a**

Integer ( $x$ )	$x^2$	$x^3$	$x^4$	$x^5$
2	4	8	16	32
-2	4	-8	16	-32
3	9	27	81	243
-3	9	-27	81	-243
4	16	64	256	1024
-4	16	-64	256	-1024

**b** Positive

**c** If  $x$  was negative then  $x^3$  is negative.

**d** For  $x^4$ , all the results are positive, for  $x^5$ , if  $x$  is negative then  $x^5$  is negative.

**e** The sign in the  $x^2$  is the same as the sign in the  $x^4$  column. The sign in the  $x^3$  column is the same as the matching term in the  $x^5$  column.

**f** When a positive number is raised to any power, the sign of the answer is positive.

When a negative number is raised to an even power, the sign of the answer is positive.

When a negative number is raised to an odd power the sign of the answer is negative.

**g** The square root of 100 is 10 or -10.

**h** It will give an error message.

**i** -5

**j**

Number ( $x$ )	$\sqrt{x}$	$\sqrt[3]{x}$	$\sqrt[4]{x}$	$\sqrt[5]{x}$
16	$\pm 4$		$\pm 2$	
-16	Not possible		Not possible	
27		3		
-27		-3		
32				2
-32				-2
81	$\pm 9$		$\pm 3$	
-81	Not possible		Not possible	
64	$\pm 8$	4		
-64	Not possible	-4		

- k** When taking an even root of a positive number the answer can be positive or negative.  
 It is not possible to take the even root of a negative number.  
 When taking the odd root of a positive number the answer is always positive.  
 When taking the odd root of a negative number the answer is always negative.

## CHAPTER 13

### Linear equations

#### Are you ready?

- |                                |                    |            |
|--------------------------------|--------------------|------------|
| <b>1</b> a 3                   | <b>b</b> 11        |            |
| c 15                           | d 27               |            |
| <b>2</b> a $\square + 12 = 30$ |                    |            |
| b $\square - 9 = 6$            |                    |            |
| c $\square \times 4 = 48$      |                    |            |
| d $\square \div 8 = 7$         |                    |            |
| <b>3</b> a 22                  | <b>b</b> 9         | <b>c</b> 1 |
| d 7                            | e 5                | f 12       |
| <b>4</b> a -2                  | <b>b</b> +5        |            |
| c $\div 7$                     | d $\times 3$       |            |
| <b>5</b> a $5x + 6y$           | <b>b</b> $7x + 9y$ |            |
| c $10x + 4$                    | d $4x + 6$         |            |
| <b>6</b> a True                | <b>b</b> False     |            |
| c False                        | d True             |            |
| e False                        | f True             |            |

#### 13A Solving equations using trial and error

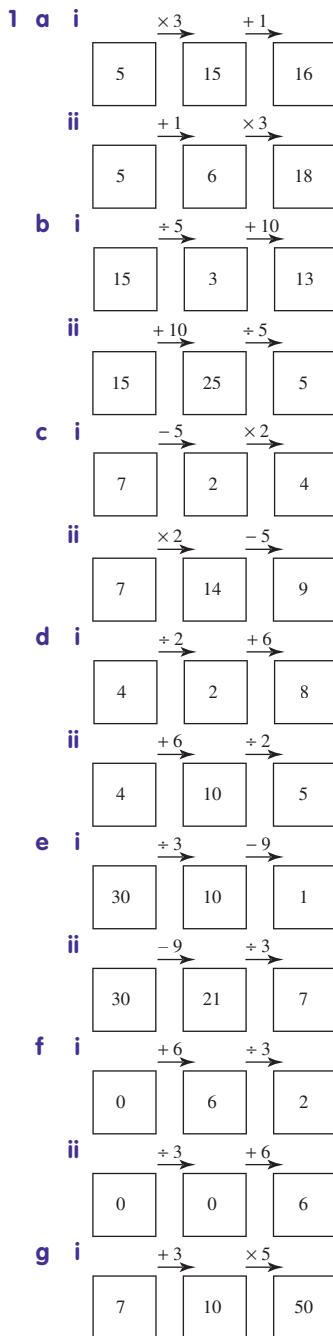
- |                         |                       |
|-------------------------|-----------------------|
| <b>1</b> a $x + 7 = 11$ | <b>b</b> $x + 3 = 5$  |
| c $x + 12 = 12$         | d $x + 5 = 56$        |
| e $x - 7 = 1$           | f $x - 11 = 11$       |
| g $x - 4 = 7$           | h $x - 8 = 0$         |
| i $2x = 12$             | j $6x = 30$           |
| k $5x = 30$             | l $6x = 12$           |
| m $\frac{x}{7} = 1$     | n $\frac{x}{3} = 100$ |
| o $\frac{x}{5} = 2$     | p $\frac{x}{7} = 0$   |
| q $15 - x = 2$          | r $52 - x = 8$        |
| s $\frac{21}{x} = 7$    | t $x^2 = 100$         |
| <b>2</b> a $x = 11$     | <b>b</b> $y = 9$      |
| c $m = 5$               | d $m = 30$            |
| e $w = 1$               | f $m = 274$           |
| g $w = 7$               | h $k = 0$             |
| i $b = 7$               | j $w = 18$            |
| k $k = 4$               | l $h = 25$            |
| m $b = 8.8$             | n $c = 4.2$           |
| o $x = 2.8$             |                       |
| <b>3</b> $x = 6$        |                       |
| <b>4</b> a $x = 3$      | <b>b</b> $x = 14$     |
| c $x = 10$              | d $x = 9$             |
| e $x = 3$               | f $x = 20$ or $x = 0$ |
| g $x = 5$ or $x = 0$    | h $x = 6$             |
| i $x = 4$               |                       |
| <b>5</b> 7 and 14       |                       |
| <b>6</b> a 11 and 15    | <b>b</b> 9 and 18     |
| c 7 and 47              | d 8 and 37            |
| e 42 and 136            | f 51 and 53           |
| g 42 and 111            | h 97 and 145          |
| i 2.2 and 3.9           | j 2.9 and 5.1         |
| k 347 and 631           | l 11.9 and 23.1       |

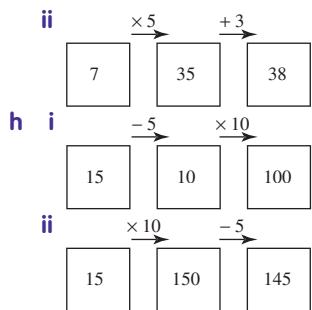
<b>7</b> $x$	$x^2 + 4$	$4x + 1$
0	4	1
1	5	5
2	8	9
3	13	13
4	20	17

$x = 1$  and  $x = 3$  are both solutions.

- 8** 10 **9** 24 years old  
**10** 5 metres, 15 metres **11** 7

#### 13B Using inverse operations

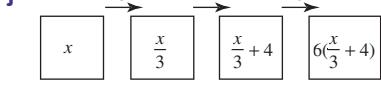
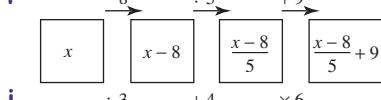
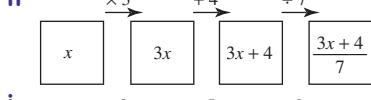
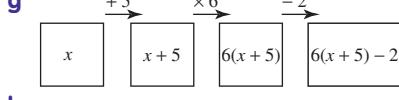
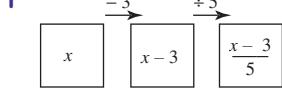
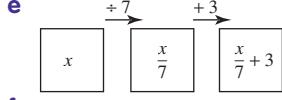
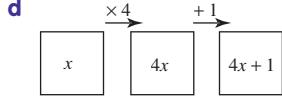
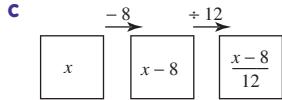
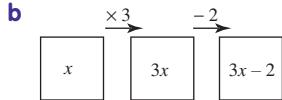
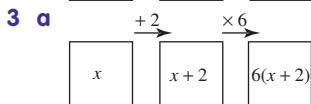
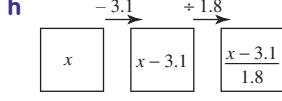
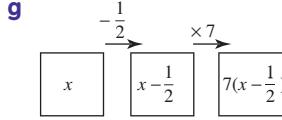
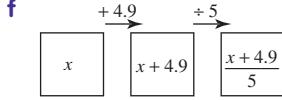
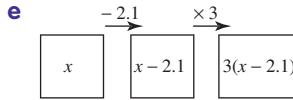
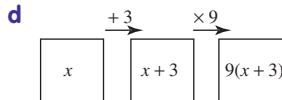
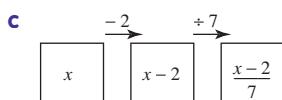
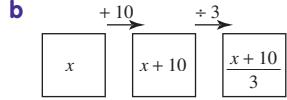
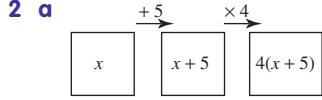
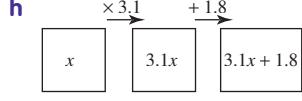
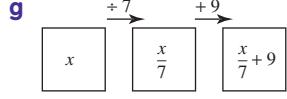
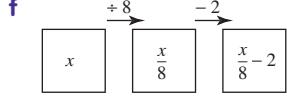
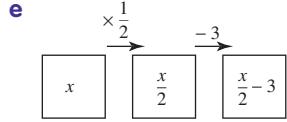
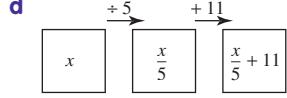
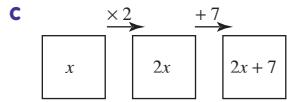
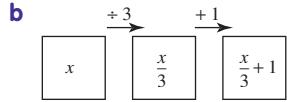
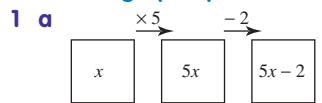




- 2** **a** 1      **b** 7      **c** 15      **d** 14  
**e** 5      **f** 10      **g** 4      **h** 9  
**i** 5      **j** 0      **k** 14.4522      **l** 4  
**m** 3      **n** 17      **o** 2.59      **p** 532

- 3** **a** Each pair of flowcharts has the same operations, but in the reverse order.  
**b** The results are quite different.  
**4** **a** Adding and subtracting are inverse operations.  
**b** Multiplying and dividing are inverse operations.

### 13C Building up expressions



**k**

$$\begin{array}{cccc} & \xrightarrow{\times 11} & \xrightarrow{-2} & \xrightarrow{\times 4} \\ x & 11x & 11x - 2 & 4(11x - 2) \end{array}$$

**l**

$$\begin{array}{cccc} & \xrightarrow{\times 3} & \xrightarrow{\div 4} & \xrightarrow{+2} \\ x & 3x & \frac{3x}{4} & \frac{3x}{4} + 2 \end{array}$$

**4 a**

$$\begin{array}{ccc} & \xrightarrow{\times 5} & \xrightarrow{+9} \\ x & 5x & 5x + 9 \end{array}$$

**b**

$$\begin{array}{ccc} & \xrightarrow{+1} & \xrightarrow{\times 2} \\ x & x + 1 & 2(x + 1) \end{array}$$

**c**

$$\begin{array}{ccc} & \xrightarrow{\div 6} & \xrightarrow{+4} \\ x & \frac{x}{6} & \frac{x}{6} + 4 \end{array}$$

**d**

$$\begin{array}{ccc} & \xrightarrow{-8} & \xrightarrow{\div 7} \\ x & x - 8 & \frac{x - 8}{7} \end{array}$$

**e**

$$\begin{array}{ccc} & \xrightarrow{-7} & \xrightarrow{\times 12} \\ x & x - 7 & 12(x - 7) \end{array}$$

**f**

$$\begin{array}{ccc} & \xrightarrow{\div 5} & \xrightarrow{-2} \\ x & \frac{x}{5} & \frac{x}{5} - 2 \end{array}$$

**g**

$$\begin{array}{ccc} & \xrightarrow{\times 7} & \xrightarrow{-12} \\ x & 7x & 7x - 12 \end{array}$$

**h**

$$\begin{array}{ccc} & \xrightarrow{+6} & \xrightarrow{\times 3} \\ x & x + 6 & \frac{x + 6}{3} \end{array}$$

**i**

$$\begin{array}{ccc} & \xrightarrow{+7} & \xrightarrow{\times 3} \quad \xrightarrow{-5} \\ x & x + 7 & 3(x + 7) \quad 3(x + 7) - 5 \end{array}$$

**j**

$$\begin{array}{cccc} & \xrightarrow{\times 3} & \xrightarrow{+7} & \xrightarrow{\div 2} \\ x & 3x & 3x + 7 & \frac{3x + 7}{2} \end{array}$$

**k**

$$\begin{array}{cccc} & \xrightarrow{\times 3} & \xrightarrow{+1} & \xrightarrow{\times 4} \\ x & 3x & 3x + 1 & 4(3x + 1) \end{array}$$

**l**

$$\begin{array}{cccc} & \xrightarrow{\div 5} & \xrightarrow{+6} & \xrightarrow{\times 3} \\ x & \frac{x}{5} & \frac{x}{5} + 6 & 3(\frac{x}{5} + 6) \end{array}$$

**5 a**

$$\begin{array}{ccc} & \xrightarrow{\times 7} & \xrightarrow{+3} \\ x & 7x & 7x + 3 \end{array}$$

**b**

$$\begin{array}{ccc} & \xrightarrow{-2} & \xrightarrow{\times 5} \\ x & x - 2 & 5(x - 2) \end{array}$$

**c**

$$\begin{array}{ccc} & \xrightarrow{+2} & \xrightarrow{+1} \\ x & \frac{x}{2} & \frac{x}{2} + 1 \end{array}$$

**d**

$$\begin{array}{ccc} & \xrightarrow{-5} & \xrightarrow{\div 4} \\ x & x - 5 & \frac{x - 5}{4} \end{array}$$

**6 a**

$$\begin{array}{ccc} & \xrightarrow{+7} & \xrightarrow{\times 2} \\ x & x + 7 & 2(x + 7) \end{array}$$

**b**

$$\begin{array}{ccc} & \xrightarrow{\div 6} & \xrightarrow{-3} \\ x & \frac{x}{6} & \frac{x}{6} - 3 \end{array}$$

**c**

$$\begin{array}{ccc} & \xrightarrow{\times 6} & \xrightarrow{+3} \quad \xrightarrow{\div 3} \\ x & x + 8 & \frac{x + 8}{3} \end{array}$$

**d**

$$\begin{array}{ccc} & \xrightarrow{+8} & \xrightarrow{\times 4} \\ x & x + 8 & 4(x + 8) \end{array}$$

**e**

$$\begin{array}{ccc} & \xrightarrow{\times 8} & \xrightarrow{-3} \\ x & 8x & 8x - 3 \end{array}$$

**f**

$$\begin{array}{ccc} & \xrightarrow{\div 7} & \xrightarrow{\div 5} \\ x & x - 7 & \frac{x - 7}{5} \end{array}$$

**g**

$$\begin{array}{ccc} & \xrightarrow{+55} & \xrightarrow{\times 3} \\ x & x + 55 & 3(x + 55) \end{array}$$

**h**

$$\begin{array}{ccc} & \xrightarrow{\div 9} & \xrightarrow{+2.1} \\ x & \frac{x}{9} & \frac{x}{9} + 2.1 \end{array}$$

**i**

$$\begin{array}{ccc} & \xrightarrow{\times 7} & \xrightarrow{-5} \\ x & 7x & 7x - 5 \end{array}$$

**j**

$$\begin{array}{cccc} & \xrightarrow{+8} & \xrightarrow{\times 4} & \xrightarrow{+5} \\ x & x + 8 & 4(x + 8) & 4(x + 8) + 5 \end{array}$$

**7**  $\frac{5(x + 8)}{4} = 30$

**8**  $5n + 15 = 4n - 3$

### 13D Solving equations using backtracking

- |                            |                              |                             |                            |
|----------------------------|------------------------------|-----------------------------|----------------------------|
| <b>1</b> <b>a</b> 2        | <b>b</b> 4                   | <b>c</b> 8                  |                            |
| <b>d</b> 13                | <b>e</b> 10                  | <b>f</b> 7                  |                            |
| <b>g</b> 67                | <b>h</b> 88                  | <b>i</b> 1                  |                            |
| <b>j</b> 5                 | <b>k</b> 8                   | <b>l</b> 5.3                |                            |
| <b>m</b> 10                | <b>n</b> 21.053              | <b>o</b> $4\frac{2}{5}$     |                            |
| <b>p</b> $\frac{31}{36}$   |                              |                             |                            |
| <b>2</b> <b>a</b> $x = 3$  | <b>b</b> $y = 1$             | <b>c</b> $m = 6$            |                            |
| <b>d</b> $x = 5$           | <b>e</b> $w = 3$             | <b>f</b> $m = 25$           |                            |
| <b>g</b> $w = 5.7$         | <b>h</b> $b = \frac{11}{54}$ | <b>i</b> $a = \frac{2}{15}$ |                            |
| <b>3</b> <b>a</b> $x = 1$  | <b>b</b> $x = 18$            | <b>c</b> $x = 23$           |                            |
| <b>d</b> $x = 4$           | <b>e</b> $x = 3$             | <b>f</b> $x = 12$           |                            |
| <b>g</b> $w = 1.3$         | <b>h</b> $b = \frac{2}{3}$   | <b>i</b> $x = \frac{7}{20}$ |                            |
| <b>4</b> <b>a</b> $x = 3$  | <b>b</b> $x = 45$            | <b>c</b> $x = 12$           |                            |
| <b>d</b> $x = 34$          | <b>e</b> $x = 77$            | <b>f</b> $x = 0$            |                            |
| <b>g</b> $x = 13$          | <b>h</b> $x = \frac{28}{11}$ | <b>i</b> $x = \frac{2}{3}$  |                            |
| <b>5</b> <b>a</b> $x = 14$ | <b>b</b> $x = 29$            | <b>c</b> $x = 78$           |                            |
| <b>d</b> $x = 1$           | <b>e</b> $x = 5$             | <b>f</b> $x = 291$          |                            |
| <b>g</b> $x = 4.65$        | <b>h</b> $x = \frac{27}{7}$  | <b>i</b> $x = \frac{1}{2}$  |                            |
| <b>6</b> <b>a</b> $x = 10$ | <b>b</b> $x = 3$             | <b>c</b> $x = 48$           | <b>d</b> $x = 50$          |
| <b>e</b> $x = 6$           | <b>f</b> $x = 21$            | <b>g</b> $x = 7$            | <b>h</b> $x = 6$           |
| <b>i</b> $x = 11.13$       | <b>j</b> $x = \frac{1}{10}$  | <b>k</b> $x = 25$           | <b>l</b> $x = 1$           |
| <b>m</b> $x = 6$           | <b>n</b> $x = 8$             | <b>o</b> $x = 2$            |                            |
| <b>7</b> <b>a</b> $x = 3$  | <b>b</b> $x = 5$             | <b>c</b> $x = 3$            | <b>d</b> $x = 4$           |
| <b>e</b> $x = 7$           | <b>f</b> $x = 4$             | <b>g</b> $x = 7$            | <b>h</b> $x = \frac{2}{9}$ |
| <b>i</b> $x = 1$           |                              |                             |                            |
- 8** A
- 9** First subtract 2 from both sides, then divide both side by 4;  $x = 2$ .
- 10** Inverse operations must be performed in the reverse order. The forward order is:  
First subtract 6, then multiply by 3 and finally add 5.  
The reverse order is:  
Subtract 5, divide by 3, then add 6.
- 11** Gareth is 13 years old.

**12** **a** 12      **b**  $\frac{x+28}{3} = 12$       **c**  $x = 8$

**d** Karina scored 8 points in the third game.

**13** Callie scored 96 pins.

**14** 16, 17 and 18

**15** 7, 9 and 11

**16** 25, 30 and 35

**17** David is 14 years old.

**18** Chris jumped 156 centimetres.

**19** The twins are 11 years old.

- 20** **a** Puzzle 1  $[(n - 500) \times 2 + 1000] \div 2 = n$   
Puzzle 2  $[(n + 4) \times 10 - 10] \div 5 - n - 6 = n$   
Puzzle 3  $(n \div 2 + 2) \times 4 - n - n = 8$

**b** Ask your teacher.

### 13E Checking solutions

- |                       |              |              |              |
|-----------------------|--------------|--------------|--------------|
| <b>1</b> <b>a</b> No  | <b>b</b> Yes | <b>c</b> No  | <b>d</b> No  |
| <b>e</b> No           | <b>f</b> Yes | <b>g</b> No  | <b>h</b> No  |
| <b>i</b> Yes          | <b>j</b> No  | <b>k</b> Yes | <b>l</b> No  |
| <b>2</b> <b>a</b> Yes | <b>b</b> No  | <b>c</b> No  | <b>d</b> Yes |
| <b>e</b> Yes          | <b>f</b> No  | <b>g</b> No  | <b>h</b> Yes |
| <b>i</b> Yes          | <b>j</b> No  |              |              |

<b>3</b>	<b>x</b>	0	1	2	3	4
	<b>2x + 3</b>	3	5	7	9	11

**a**  $x = 4$

**b**  $x = 4$

**c**  $x = 1$

<b>4</b>	<b>x</b>	2	3	4	5	6
	<b>5(x - 2)</b>	0	5	10	15	20

**a**  $x = 4$

**b**  $x = 6$

**c**  $x = 8$

<b>5</b> <b>a</b>	<b>x</b>	<b>2x + 1</b>	<b>3x - 5</b>
	3	7	4
	4	9	7
	5	11	10
	6	13	13
	7	15	16

**b**  $x = 6$

<b>6</b> <b>a</b>	<b>x</b>	<b><math>\frac{x+3}{2}</math></b>	<b><math>2x - 6</math></b>
	3	3	0
	5	4	4
	7	5	8
	9	6	12
	11	7	16

**b**  $x = 5$

### 13F Keeping equations balanced

- |                                      |                   |                  |
|--------------------------------------|-------------------|------------------|
| <b>1</b> <b>a</b> <b>i</b> $4r = 12$ | <b>ii</b> $r = 3$ |                  |
| <b>b</b> <b>i</b> $2n + 3 = 7$       | <b>ii</b> $n = 2$ |                  |
| <b>c</b> <b>i</b> $3t + 5 = 8$       | <b>ii</b> $t = 1$ |                  |
| <b>d</b> <b>i</b> $4p + 1 = 13$      | <b>ii</b> $p = 3$ |                  |
| <b>2</b> <b>a</b> $g = 1$            | <b>b</b> $m = 3$  | <b>c</b> $q = 9$ |
| <b>d</b> $n = 9$                     | <b>e</b> $g = 14$ | <b>f</b> $y = 3$ |

**3** **a**  $h = 2\frac{1}{3}$

**b**  $k = 2\frac{1}{2}$

**c**  $w = 3\frac{1}{2}$

**d**  $t = 4\frac{1}{3}$

**e**  $h = 2\frac{1}{6}$

**f**  $l = 1\frac{1}{3}$

**4** **a**  $m = 4$

**b**  $d = 3$

**c**  $v = 2$

**d**  $s = 3$

**e**  $g = 5$

**f**  $k = 2$

**5** **a**  $x = 2\frac{1}{2}$

**b**  $y = 1$

**c**  $p = 3\frac{1}{4}$

**d**  $h = 2\frac{1}{2}$

**6** **a**  $7\frac{2}{9}^{\circ}\text{C}$

**b**  $113^{\circ}\text{F}$

**7** **a** \$11.30

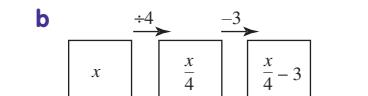
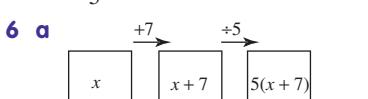
**b** 58 km

**8** 11 songs

### Chapter review

#### Fluency

- |                            |                            |                            |                   |
|----------------------------|----------------------------|----------------------------|-------------------|
| <b>1</b> <b>a</b> $m = 5$  | <b>b</b> $h = 6$           | <b>c</b> $s = 19$          | <b>d</b> $d = 20$ |
| <b>2</b> <b>a</b> 32, 51   | <b>b</b> 27, 59            |                            |                   |
| <b>3</b> <b>a</b> 11       | <b>b</b> 2                 | <b>c</b> 16                | <b>d</b> 35       |
| <b>4</b> <b>a</b> 16       | <b>b</b> 42                | <b>c</b> 11                | <b>d</b> 8        |
| <b>5</b> <b>a</b> $7x + 8$ | <b>b</b> $\frac{x}{3} - 5$ | <b>c</b> $6(x + 2)$        |                   |
| <b>d</b> $\frac{x-7}{5}$   | <b>e</b> $5(x+3)-9$        | <b>f</b> $\frac{4x+11}{8}$ |                   |



c

d

7 a

b

c

d

8 a  $x = 2$  b  $y = 10$

c  $m = 60$  d  $s = 18$

9 a  $d = 4$  b  $t = 100$

c  $d = 3$  d  $a = 6$

10 a  $v = 3$  b  $c = 12$

11 a Yes b No

12 a i  $4x + 1 = 5$  ii  $x = 1$

b i  $3p + 5 = 11$  ii  $p = 2$

c i  $2q + 3 = 13$  ii  $q = 5$

d i  $2g + 2 = 17$  ii  $g = 7\frac{1}{2}$

13 a  $v = 3$  b  $x = 2$

c  $w = 3$  d  $g = 4$

14 a  $k = 1\frac{3}{4}$  b  $p = 5\frac{1}{2}$

c  $q = \frac{1}{3}$  d  $t = \frac{2}{5}$

15 a  $x = 2$  b  $p = 2$

c  $r = 2$  d  $z = 2$

### Problem solving

1 Sophie owns 31 cards

2 7 pieces of homework

3 12 years old

4 \$2

5 a i \$40 ii \$50 iii \$60

b Cost =  $30 + 10h$

c \$110

d  $85 = 30 + 10h; h = 5.5$  hours

e \$5

f Your deposit is returned (\$30). You could buy a hamburger on the way home.

6 a \$105

b \$105

c 6 children

d 10 adults; 9 adults and 2 children; 8 adults and 4 children; 7 adults and 6 children; 6 adults and 8 children; 5 adults and 10 children; 4 adults and

12 children; 3 adults and 14 children; 2 adults and 16 children; 1 adult and 18 children; 20 children;

7 14 bicycles and 5 tricycles

8  $45^\circ, 45^\circ, 90^\circ$ . This is a right-angled isosceles triangle.

9 Length 12 m, width 3 m

10  $35^\circ$

## CHAPTER 14

### Representing and interpreting data

#### Are you ready?

- |                          |                          |         |       |
|--------------------------|--------------------------|---------|-------|
| 1 a Numerical            | b Non-numerical          |         |       |
| c Non-numerical          | d Numerical              |         |       |
| 2 a Continuous           | b Continuous             |         |       |
| c Continuous             | d Discrete               |         |       |
| 3 a 3                    | b 13                     | c 4.6   | d 468 |
| 4 a 4                    | b 11                     | c 5     | d 8   |
| 5 a 1 cm                 | b 0.2 m                  | c 20 mm | d 5%  |
| 6 a 1.35                 | b 0.484                  |         |       |
| 7 a 2.3 cm, 25 mm, 0.2 m | b 1700 mm, 1.8 m, 185 cm |         |       |

#### 14A Classifying data

- |                            |                            |
|----------------------------|----------------------------|
| 1 a ii                     | b Qualitative, nominal     |
| b i                        | c Quantitative, continuous |
| c v                        | d Quantitative, continuous |
| d vi                       | e Qualitative, nominal     |
| e iii                      | f Quantitative, continuous |
| f iv                       | g Quantitative, discrete   |
| 2 a Quantitative, discrete | h Quantitative, continuous |
| b Quantitative, continuous | i Quantitative, discrete   |
| c Qualitative, nominal     | j Qualitative, nominal     |
| d Quantitative, continuous | k Quantitative, continuous |
| e Quantitative, discrete   | l Quantitative, discrete   |
| f Qualitative, nominal     | m Quantitative, discrete   |
| g Quantitative, discrete   | n Qualitative, nominal     |
| h Quantitative, continuous | o Qualitative, ordinal     |
| i Quantitative, discrete   | p Quantitative, continuous |
| j Qualitative, nominal     | q Quantitative, discrete   |
| k Qualitative, continuous  | r Qualitative, ordinal     |
| l Quantitative, discrete   | s Qualitative, nominal     |
| m Quantitative, discrete   | t Qualitative, ordinal     |

3 Discrete data deal with values which are exact and must be counted, such as the number of people attending a football match.

Continuous data deal with values which are measured and may assume decimal form such as the length of each football quarter.

4 Discuss with your teacher.

5 D 6 A

7 D 8 1250

#### 14B Displaying data in tables

1 a	Score	Tally	Frequency
0			5
1			4
2			9
3			7
4			2
5			1
6			1
7			0
8			1
	Total		30

**b** 30  
**d** 12  
**f**  $\frac{2}{30} = \frac{1}{15}$

**c** 5  
**e** 2

**2**

Value (thousand dollars)	Tally	Frequency
100–109		3
110–119		1
120–129		0
130–139		1
140–149		3
150–159		3
160–169		2
170–179		1
180–189		2
190–199		1
200–209		1
	Total	18

- 3 a** The list is messy; it is difficult to see how many different sports there are and to gauge how many people prefer a particular sport.

**b**

Sport	Tally	Frequency
Hockey		8
Cricket		6
Tennis		7
Netball		9
Swimming		7
Golf		4
Scuba diving		3
Lacrosse		4
Lawn bowls		2
	Total	50

- c i** Netball  
**ii** Lawn bowls  
**d** Yes, tennis and swimming had a frequency of 7 and golf and lacrosse had a frequency of 4.

**4 a**

Score	Tally	Frequency
5		1
6		2
7		2
8		1
9		3
10		1
	Total	10

**b**

Score	Tally	Frequency
0		5
1		5
2		5
3		2
4		2
5		0
6		1
	Total	20

**c**

Score	Tally	Frequency
45		1
46		1
47		0
48		2
49		1
50		2
51		2
52		2
53		4
54		3
55		1
56		2
57		2
58		2
59		1
60		2
	Total	28

**d**

Score	Tally	Frequency
143		2
144		6
145		5
146		1
147		3
148		3
149		3
150		4
151		0
152		1
	Total	28

- 5 a** 33    **b** 3    **c** 10    **d** 23    **e**  $\frac{1}{3}$

**6 a**

Score	Tally	Frequency
0		11
1		10
2		6
3		2
4		1
	Total	30

- b** 11  
**c** 9  
**d** 0  
**e** 4  
**f**  $\frac{1}{5}$

**7**

Height	Tally	Frequency
140–144		2
145–149		6
150–154		4
155–159		1
160–164		5
165–169		6
170–174		2
175–179		1
180–184		1
	Total	28

Vowel	Tally	Frequency
A		42
E		55
I		38
O		40
U		15
	Total	190

Type	Tally	Frequency
1 bedroom unit		3
2 bedroom unit		4
2 bedroom house		1
3 bedroom house		6
4 bedroom house		2
	Total	181

A 3 bedroom house was the most popular.

10 C

11 A

12 B

13 O 21.5%, U 8.3%

#### 14C Measures of centre and spread

1 a	5	b	6	c	4	d	6	E	4
f	3	g	12	h	14	i	15	j	13
2 a	8	b	3	c	4	d	2	e	3
f	4	g	5	h	2	i	5	j	3
3 a	5	b	3	c	6	d	3	e	7
f	11	g	4	h	2	i	2.5	j	1.5
4 a	2	b	2	c	5	d	7	e	2.5
f	5	g	6	h	7.5	i	5.5	j	7.5
5 a	4	b	8	c	2	d		e	4
e	2	f	3	g	10	h		i	4
i	9	j	3						
6 a	32	b	24.1						
7 B		8 B		9 D					
10 D		11 C		12 85%					
13 104 shoes		14 47 students		15 38					

16 a Tom's 75.0 Sarah's 72.8

b Sarah's sells cheaper petrol.

c Tom may have seen the price on a very expensive day.

There's a greater range of values at Sarah's petrol station.

17 89.5%                    18 72

19 a Mode = 12

b The mode shows which size to order more of. The mean and median would not show the more common sizes and would give only an indication of the middle of the range.

20 a 1.75 m                    b 2.21 m

c Based on her past performances and her past range of values, she cannot jump this high.

21 a 101.9

b Both the mode (115) and median (112) give a better impression of how the team has performed, even though they could give a 'misleading' impression of the team's performance.

22 133 cm, 133 cm, 142 cm, 151 cm

23 One possible answer is 5, 6, 12, 13, 14. If the five numbers are in ascending order, the third number must be 12 and the other 4 numbers must total 38.

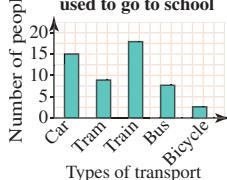
24 Highest score is 49 and lowest score is 0. The scores would be 0, 1, 12, 13, 49.

25 Largest value for median is 15. Scores would be 0, 2, 15, 16, 17. Smallest value for median is 2. Scores would be 0, 1, 2, a, b where  $a + b = 47$ .

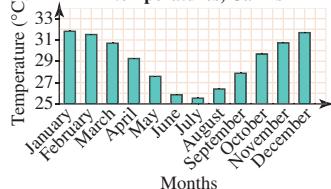
26 43; highest score: 100; lowest score: 57. The scores would be 57, 77, 78, 79, 80, 97, 98, 99, 100.

#### 14D Representing data graphically

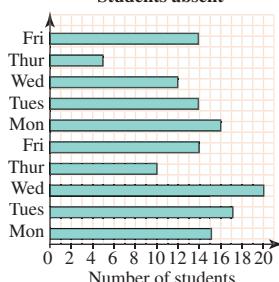
- 1 Method of transport used to go to school



- 2 Mean daily maximum temperatures, Cairns



- 3 Students absent



- 4 a

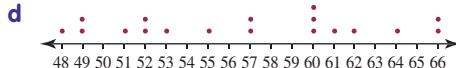
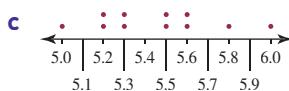


b The scores lie between and include 6 to 20; that is, a range of fourteen.

Mostly between 16 to 18 packets were sold. Sales of 6 and 20 packets of chips were extremely low.

A provision of 20 packets of chips each day should cover the most extreme demands.

- 5 a



**6** Key: 2 | 7 = 27

Stem	Leaf
2	1 1 5 5 8 8 9 9
3	1 1 3 7 9
4	0 1 1 6 7
5	1 3 3 4 8
6	2 2 2 2 7 7 8
7	1 2 3 3 8
8	1 2
9	2 3 3

**7** Key: 3 | 6 = 36

Stem	Leaf
0	0 1 2 3 6 7 8
1	0 2 2 2 2 4 5 6 6 7 8 9
2	1 2 5 7 8
3	1 1 2 2 3
4	0

**8 a** Key: 12 | 7 = 127

Stem	Leaf
10	8
11	6 7 8
12	9
13	2 7
14	5
15	2
16	4 5
17	2
18	9

**b** Key: 13 | 2 = 132

Stem	Leaf
11	4
12	4
13	0 1
14	6
15	0 2
16	
17	1 3

**c** Key: 18 | 5 = 186

Stem	Leaf
16	8
17	0 0
18	0 1 6 6 8
19	3 6 6 9
20	9

**e** Key: 65 | 2 = 652

Stem	Leaf
64	1 5 7 7
65	
66	2
67	5
68	5
69	1
70	8
71	
72	
73	5 6
74	8

**9 a** Key: 1 | 7 = 1.7

Stem	Leaf
1	2 7 9
2	2 3 8
3	9
4	1 6
5	4 8

**b** Key: 2 | 6 = 2.6

Stem	Leaf
1	8
2	5 7 8 9
3	5
4	1
5	2 7
6	2 6

**c** Key: 6 | 7 = 6.7

Stem	Leaf
6	0 5 9
7	4 5 7
8	3 4 8
9	2 3 8

**d** Key: 13 | 7 = 13.7

Stem	Leaf
13	0 8 9 9
14	3 5 5 6 7 7 8
15	1 2 6 7 9
16	2

**e** Key: 1 | 5 = 0.15

Stem	Leaf
0	2 7
1	5 8
2	
3	7
4	4
5	1
6	7

**10** Key: 0 | 0 = 0

Leaf (Mitch)	Stem	Leaf (Yani)
3 2 1 1 1 1 0 0 0 0	0	0 0 1 1 1 1 1 2 2 2

Mitch scored between 0 and 3 goals inclusive.

Yani scored between 0 and 2 goals inclusive.

**11 a** Telephone bills April 2010 to April 2011

**b** Months

**c** Bill total

**d** Every 3 months (quarterly)

**e** January 2011

**f** Yes

**g** \$150

**h** A graph like this is useful to monitor the spending pattern over time and to decide whether there have been any unusual increases.

**i** It would not be normal, but there might be reasons to explain why it was so much higher, such as overseas phone calls.

**j** This phone costs about \$50 per month.

**12 a** 45

**b** 42

**c** 61





- Set B    **i** 3  
**ii** 68  
**iii** 34  
**iv** 31.5  
**v** 65

**c** The two sets have similar minimum and maximum values, giving them a similar range. The mean of Set A is slightly higher than that of Set B. The scores of Set A are spread out more evenly than those in Set B, which tend to be clustered towards the middle.

- 13 a** **i** 6.55  
**ii** 6  
**iii** 6  
**iv** 9

**b** There appear to be two outliers (11 and 12). The mean, median and mode values are quite close, indicating that it is common for 6 soft drinks to be sold quite frequently. The minimum number sold over the period is 3, while the maximum is 12.

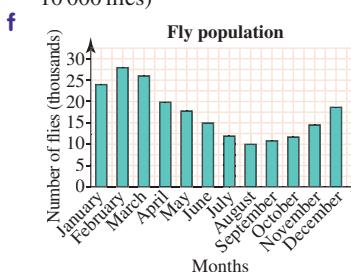
- 14 a** **i** 29 years  
**ii** 27 years  
**iii** 18 years  
**iv** 39 years

**b** The ages of the group range from 14 years to 53 years, with most of the group being under 40. The mean or median values would be a good representation of the age of the group.

### Problem solving

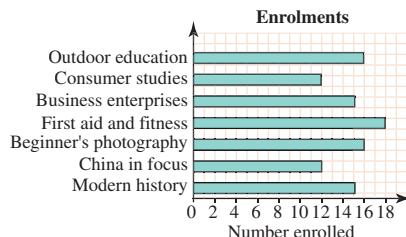
- |            |  |          |              |
|------------|--|----------|--------------|
| <b>1 a</b> | 10 °C  | <b>b</b> | Day 5        |
| <b>c</b>   | Day 7  | <b>d</b> | Day 6        |
| <b>e</b>   | 10 °C  | <b>f</b> | Days 6 and 7 |
| <b>2 a</b> | 81.56  | <b>b</b> | 74.22        |
| <b>c</b>   | Test 1   | <b>d</b> | 6            |
| <b>e</b>   | 4  | <b>f</b> | 85           |
| <b>g</b>   | 63   | <b>h</b> | 85           |
| <b>i</b>   | 68   |          |              |
| <b>j</b>   | John 89.5, Peter 74, Mark 76.5, James 74.5, David 85.5, Rachel 73.5, Mary 81, Eve 78.5, Esther 68.     |          |              |
| <b>k</b>   | 21.5   |          |              |
| <b>3 a</b> | 41 cm  |          |              |
| <b>b</b>   | 154.8 cm   |          |              |
| <b>c</b>   | 156 cm   |          |              |
| <b>d</b>   | 156 cm   |          |              |
| <b>e</b>   | Yes  |          |              |
| <b>f</b>   | There are quite a lot of values which are very similar, so the mean, median and mode are fairly close. |          |              |

- 4 a** Months  
**b** Number of flies  
**c** 10 000  
**d** 28 000  
**e** 1 cm = 5000 flies (if beginning from 0 flies) or 1 cm = 2000 flies (if beginning from 10 000 flies)



- 5 a** 12  
**b** 18  
**c** First aid and fitness  
**d** China in focus, Consumer studies  
**e** Beginners' photography, First aid and fitness, Outdoor education

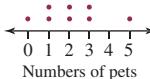
**f**



- 6 a**

<b>x</b>	<b>f</b>
0	4
1	5
2	7
3	3
4	0
5	1
Total	20

- b**



- c** 2, 2, 1.65

- 7 a**

- i** 5  
**ii** 5.5  
**iii** 7.9  
**iv** 25  
**v** 6  
**vi** 17.9, 15.5

- b** Discuss in class

- 8** The boys have a mean height of 158 cm, with a median of 158 cm and a mode of 148 cm. They range in height from 138 cm to 178 cm.  
The girls have a mean height of 153 cm, with a median of 152 cm and a mode of 137 cm. They range in height from 137 cm to 171 cm.  
The shortest person is a girl, while the tallest person is a boy. On the whole, the boys are taller than the girls.

## CHAPTER 15

### Coordinates and the Cartesian plane

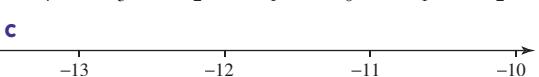
#### Are you ready?

- |                |             |            |             |
|----------------|-------------|------------|-------------|
| <b>1 a</b> L   | <b>b</b> E  | <b>c</b> N | <b>d</b> A  |
| <b>2 a</b> 4.5 | <b>b</b> -2 | <b>c</b> 0 | <b>d</b> -7 |
| <b>3</b>       | A           | C H        | R           |
- 

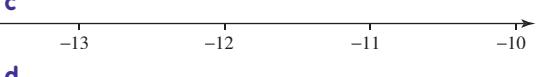
- 4 a**



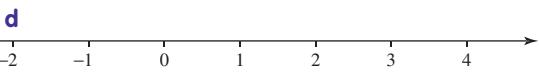
- b**



- c**



- d**

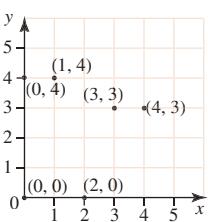


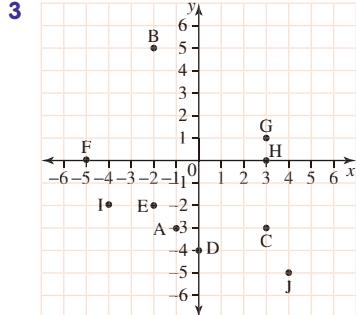
- 5** **a** 4      **b** -4  
**c** -6      **d** -10  
**6** **a** 29.5, 33      **b** 5, 9  
**c** -64, -128      **d** 12, 24  
**7** **a**  $b = 0$ ;  $a = 7$       **b**  $b = 1$ ;  $a = 2$       **c**  $b = 13$ ;  $a = -1$       **d**  $b = 6$ ;  $a = -1$

### 15A Alphanumeric grid references on a map

- 1** **a** E2      **b** B1      **c** B3  
**d** D1      **e** B2      **f** C1  
**g** D2      **h** A1      **i** D1  
**2** **a** Geelong      **b** Ballarat  
**c** Melbourne      **d** Wamambool/Colac  
**e** Portland      **f** Mildura  
**g** Albury/Wodonga/Wangaratta/Bairnsdale  
**h** Mount Gambier      **i** Griffith/Wagga Wagga  
**3** **a** C1      **b** C3  
**c** C5, C4, C3, D3, D4, B5  
**4** **a** Heaths Rd  
**b** Silvereye Crescent  
**c** Rosella Avenue  
**5** **a** Parliament House: O5; Rialto Towers Observation Deck: G11; Shrine of Remembrance: O17; Crown Entertainment Complex: F16; Melbourne Aquarium: G14; Melbourne Cricket Ground: U12; Queen Victoria Market: F2; Victorian Arts Centre: L14  
**b** Answers will vary.  
**6** **a** A1, F1  
**b** Black knight  
**c** B3  
**d** **i** F3 to E5      **ii** F6 to E4  
**e** Pawn at F7

### 15B The Cartesian plane

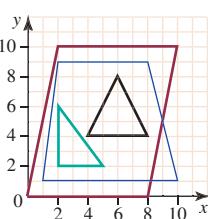
- 1**   
(1, 4), (0, 4), (3, 3), (4, 3), (0, 0), (2, 0)
- 2** A(6, 1) B(2, 5) C(7, 9) D(12, 10) E(10, 0) F( $4\frac{1}{2}$ , 0) G(0, 6)  
H(0, 0) I( $7\frac{1}{2}$ , 5) J(10,  $7\frac{1}{2}$ ) K( $3\frac{1}{2}$ ,  $2\frac{1}{2}$ ) L( $\frac{1}{2}$ ,  $9\frac{1}{2}$ )

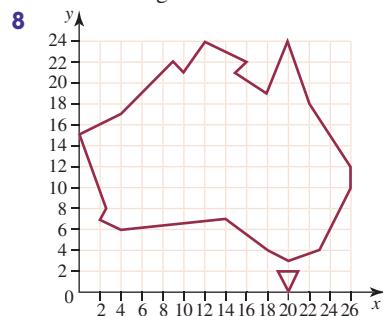


Quadrant I: point G; Quadrant II: point B; Quadrant III: points A, E, I; Quadrant IV: points C, J; x-axis: points F, H; y-axis: point D.

- 4** **a** B  
**b** C  
**c** D

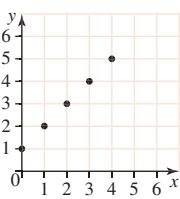
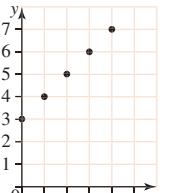
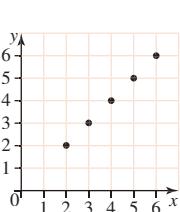
- d** A  
**e** E  
**f** A  
**5** **a** E, C  
**b** B, D  
**6** In hospital; in wards six, seven, eight and nine

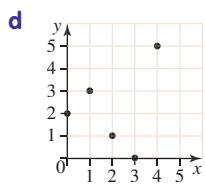
- 7**   
**a** Right-angled triangle  
**b** Isosceles triangle  
**c** Trapezium  
**d** Parallelogram



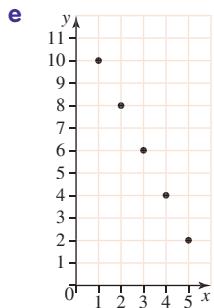
- 9** 15 square units  
**10** **b, d, e**  
**11** **a** A(0, 1)      B(2, 3)      C(5, 1)  
**b** 5 square units  
**c** A(0, -1) B(2, -3) C(5, -1)  
**d** A(0, -1) B(-2, -3) C(-5, -1)

### 15C Plotting simple linear relationships

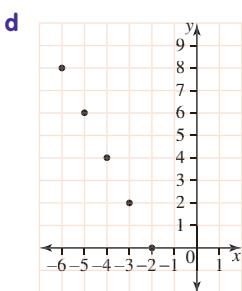
- 1** **a**  Linear pattern  
**b**  Linear pattern  
**c**  Linear pattern



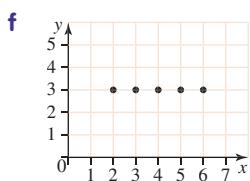
Non-linear pattern



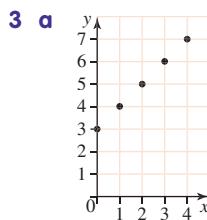
Linear pattern



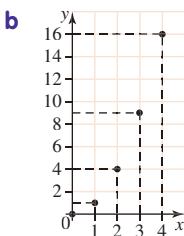
Linear pattern



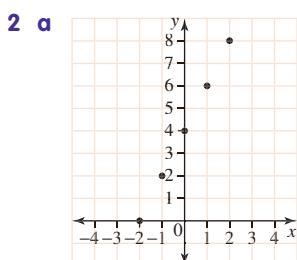
Linear pattern



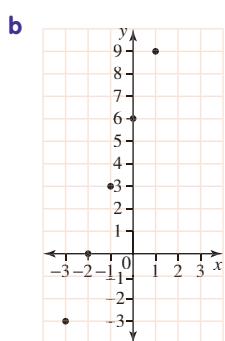
Linear relationship



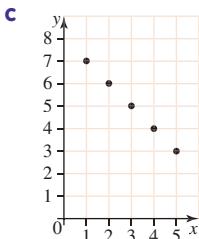
Non-linear relationship



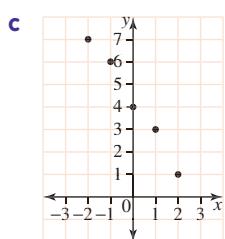
Linear pattern



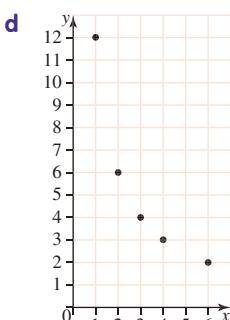
Linear pattern



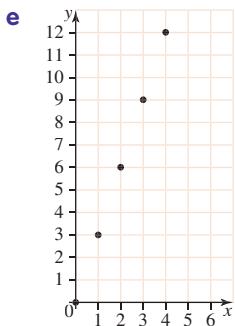
Linear relationship



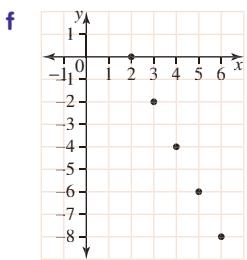
Non-linear pattern



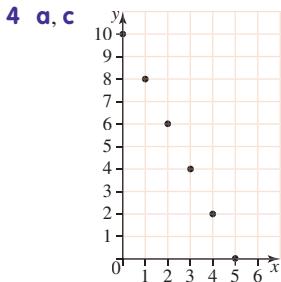
Non-linear relationship



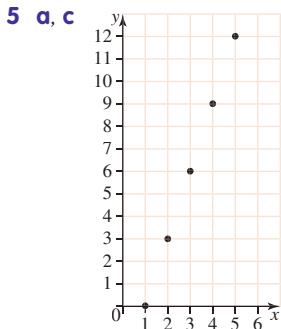
Linear relationship



Linear relationship



- b** Linear pattern  
**d**  $(4, 2), (5, 0)$

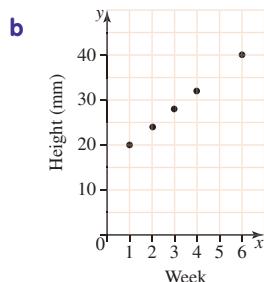


- b**  $y = 6$   
**d**  $(8, 7)$   
**e**  $(3, 2)$

- 7 a, b, d**

**8 a**

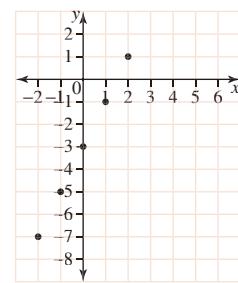
Week	1	2	3	4	5	6
Height (mm)	20	24	28	32		40



- c** Yes, the points form a straight line.  
**d** 36 mm  
**e** 48 mm  
**9 a** The rule describes a set of points such that the  $y$ -coordinate of each point is three less than twice the  $x$ -coordinate.

**b**

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-7	-5	-3	-1	1



- d** Yes; the points form a straight line.  
**e**  $-7 \leq y \leq 1$

### 15D Interpreting graphs

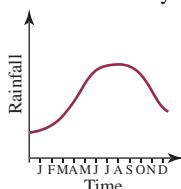
- 1 a** Lucas  
**b** Selina  
**2 a** Yes  
**b** No  
**3 a** Talia  
**b** Amy  
**c** No  
**4 a** Karl  
**b** No  
**c** Yes  
**5** A: Claire is moving away from home.  
B: Claire could be resting as she has stopped.  
C: Claire is moving further away from home.  
D: Claire could be resting again as she has stopped.  
E: Claire is travelling home.

- 6 a** i Linh  
ii Linh  
iii No  
**b**
- 
- | Name    | Weight | Height |
|---------|--------|--------|
| Linh    | 75     | 180    |
| Hannah  | 65     | 160    |
| Georgia | 60     | 160    |
| James   | 55     | 150    |
| David   | 50     | 150    |

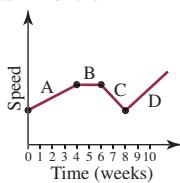
- 7** E  
**8** C  
**9 a** A baby is not born with a zero head circumference.  
**b** The head circumference has increased steadily over the 12 weeks.  
**c** You would expect the graph to continue to show some increase in head circumference, but this would eventually slow down.  
**10 a** B  
**b** Monique is walking faster in section C; therefore, the distance from school is decreasing more quickly.  
**c** At school

- 11** **a** January      **b** April  
**c** Yupa's profit has gone down each month.

**12** Answers will vary.



- 13**



## Chapter review

### Fluency

- 1** **a** Joseph's House; B1; Post office; B4

**b**

Post-office	Bus-stop	Star cafe	Library
C	Apple street		
B	Alex's house	Maya's house	Joseph's house
A			Lena's house
1	2	3	4

- 2** **a** Heaths Rd      **b** B3

- 3** **a** F3      **b** E2

- c** C4

- d** i G5      ii F2      iii F7

- e** St Pancras and King's Cross

- f** Hyde Park

- g** C3 – C4

- 5** **a** A

- b** K, I, L, J

- c** M, N, O, P

- d** A(-3, 4) B(-4, 3)

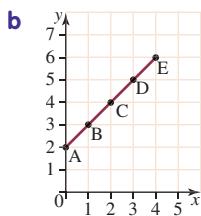
- e** C(-4, -3) F(4, -3)

- 6** **a** A

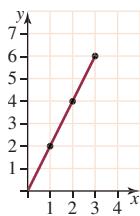
- (2, 4)

- (3, 5)

- b**



- 7** **a**



- b** Yes, the points form a straight line.

### Problem solving

- 1** H or X

H lies in column 3 and row 2.

X lies in column 4 and row 5.

- 2** **a** i H

- ii H

- iii S

- iv M

**b** B9, C2, H6, I10

**c** A5 and A6 are already hit, so he needs A4 and A7 to sink it.

- 3** **a** QUEENSLAND

(6, 6), (4, 4), (12, 8), (12, 8), (14, 14), (16, 16), (8, 14), (10, 10), (14, 14), (12, 12)

- b** VICTORIA

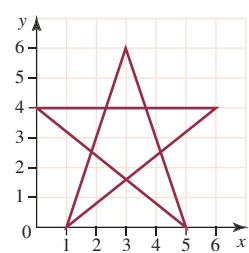
(4, 18), (8, 8), (10, 12), (16, 4), (14, 6), (6, 16), (8, 8), (10, 10)

- c** SYDNEY

(16, 16), (18, 18), (12, 12), (14, 14), (12, 8), (18, 18)

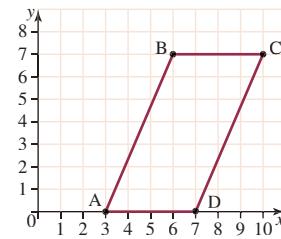
- 4** (3, 1) (2, 3) (2, 1) (3, 1) (2, 2) (5, 1) (2, 3) (2, 1) (5, 0) (0, 3) (5, 0) (0, 1) (3, 2) (0, 3) (5, 0) (3, 1) (1, 0) (1, 2)

- 5**

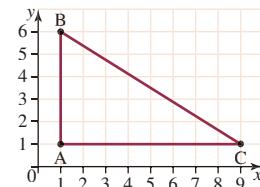


- 6** D is (3, 7)

- 7** **a** Parallelogram area = 28 square units



- b**



One example could be Triangle ABC where A is (1, 1), B is (1, 6) and C is (9, 1).

The area of this triangle is  $\frac{1}{2} \times \text{base} \times \text{height}$   
 $= \frac{1}{2} \times 8 \times 5 = 20$  square units as required.

- 8** D(1, -5)

- 9** No; the relation is  $y = x^2$ , which is not in the form  $y = mx + c$

- 10** Not true, as a straight line could:

- be parallel to the y-axis and only cut the x-axis once
- be parallel to the x-axis and only cut the y-axis once
- pass through the origin, cutting the axes at only that point.

- 11** **a** A: Anton is swimming out from the shore.

- B: Anton is staying the same distance from the shore.

- C: Anton is swimming back to the shore.

- b** Anton is swimming faster in section A than in section C.

## CHAPTER 16

### Transformations

**Are you ready?**

1 F6

2 G4

3 **F R P S A**

4 G4

5 **O S H N**

### 16A Axes of symmetry

1 Check with your teacher.

- |       |     |     |     |
|-------|-----|-----|-----|
| 2 a 3 | b 0 | c 1 | d 1 |
| e 1   | f 0 | g 1 | h 4 |

- |       |     |     |     |
|-------|-----|-----|-----|
| 3 a 2 | b 2 | c 6 | d 8 |
| e 5   | f 0 |     |     |

- 4 a Order 3      f Order 2      h Order 4

- 5 a Order 2      b Order 2      c Order 6

- d Order 8      e Order 5      f Order 3

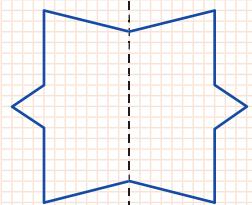
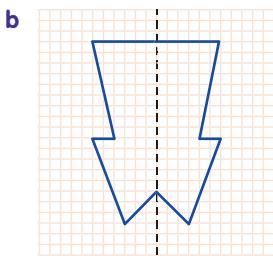
- 6 a A B C D E H I K M O T U V W X Y

- b c i l o v w x      c H I O X l o x

7 C

8 A

9 a

10 Check with your teacher.

### 16B Translations

Key — object — image

- 1 a 1R 2U      b 19R 2D

- d 10L      e 2R 4U

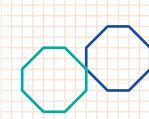
- g 2R 9U      h 11L 8D

- j 6R      k 7L 5D

- 2 a 1L 6U      b 2R 8D

- d 12R      e 12L 9D

- g 8L 1D      h 4L 7U

- 3 a
- 

- c 4L 5D

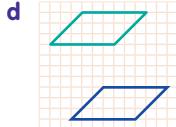
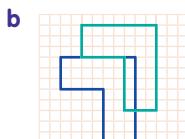
- f 2R

- i 4L 1D

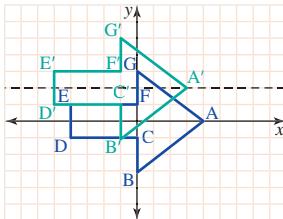
- l 2L 8U

- c 2R 3U

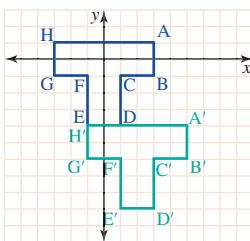
- f 11L 3D



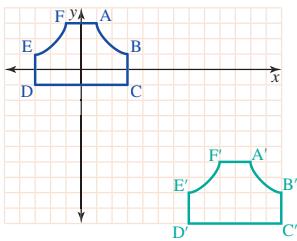
4 i a



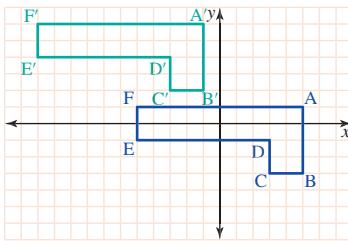
b



c



d



- ii a  $A'(3, 2)$ ,  $B'(-1, -1)$ ,  $C'(-1, 1)$ ,  $D'(-5, 1)$ ,  
 $E'(-5, 3)$ ,  $F'(-1, 3)$ ,  $G'(-1, 5)$

- b  $A'(5, -4)$ ,  $B'(5, -6)$ ,  $C'(3, -6)$ ,  $D'(3, -9)$ ,  
 $E'(1, -9)$ ,  $F'(1, -6)$ ,  $G'(-1, -6)$ ,  $H'(-1, -4)$

- c  $A'(11, -6)$ ,  $B'(13, -8)$ ,  $C'(13, -10)$ ,  $D'(7, -10)$ ,  
 $E'(7, -8)$ ,  $F'(9, -6)$

- d  $A'(-1, 6)$ ,  $B'(-1, 2)$ ,  $C'(-3, 2)$ ,  $D'(-3, 4)$ ,  
 $E'(-11, 4)$ ,  $F'(-11, 6)$

- 5 a 5R 6D      b 21R 5D

- c 2L 1D      d 2R 1U

- e 7R 6U      f 19L 6U

- 6 a C to D      b D to C

- c E to D      d E to B

- e D to A      f E to C

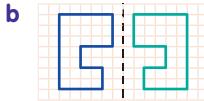
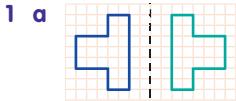
- 7 D      8 B

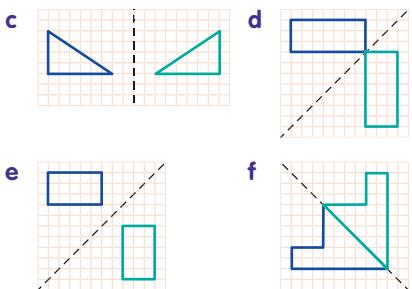
9 Check with your teacher.

10 Check with your teacher.

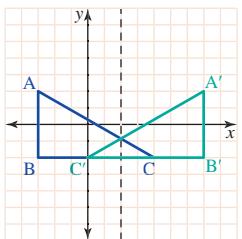
### 16C Reflections

Key — object — image

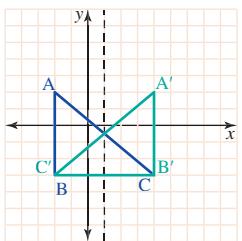




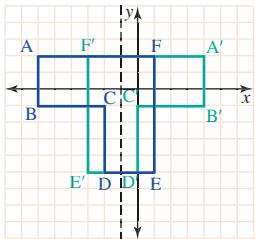
2 i a



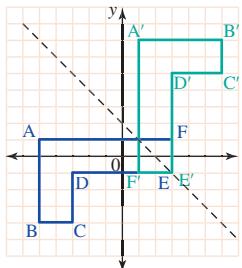
b



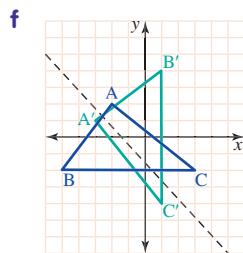
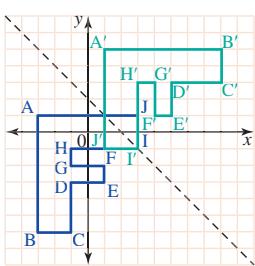
c



d



e


 ii a  $A'(6, 2)$ ,  $B'(6, -2)$ ,  $C'(0, -2)$ 

 b  $A'(4, 2)$ ,  $B'(4, -2)$ ,  $C'(-2, -2)$ 

 c  $A'(4, 2)$ ,  $B'(4, -1)$ ,  $C'(0, -1)$ ,  $D'(0, -5)$ ,  
 $E'(-3, -5)$ ,  $F'(-3, 2)$ 

 d  $A'(1, 7)$ ,  $B'(6, 7)$ ,  $C'(6, 5)$ ,  $D'(3, 5)$ ,  $E'(3, -1)$ ,  
 $F'(1, -1)$ 

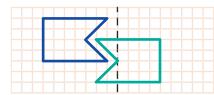
 e  $A'(1, 5)$ ,  $B'(8, 5)$ ,  $C'(8, 3)$ ,  $D'(5, 3)$ ,  $E'(5, 1)$ ,  
 $F'(4, 1)$ ,  $G'(4, 3)$ ,  $H'(3, 3)$ ,  $I'(3, -1)$ ,  $J'(1, -1)$ 

 f  $A'(-4, 1)$ ,  $B'(1, 4)$ ,  $C'(-1, -4)$ 

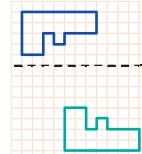
3 Check with your teacher.

4 Check with your teacher.

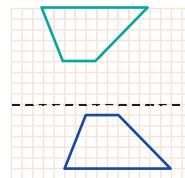
5 a



b



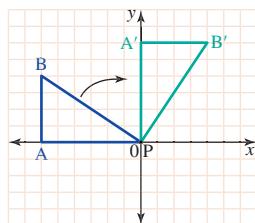
c



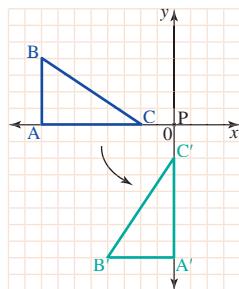
6 C

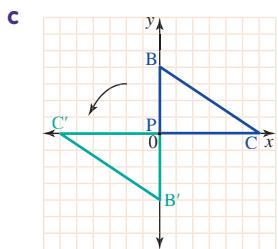
### 16D Rotations and combined transformations

1 i a



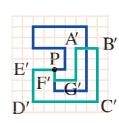
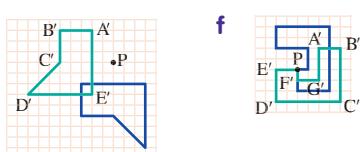
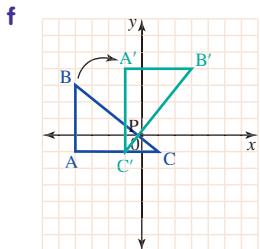
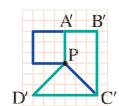
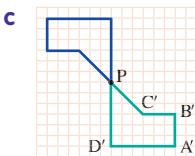
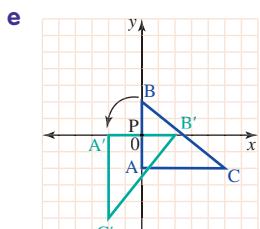
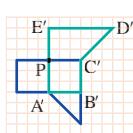
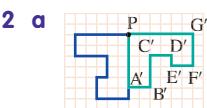
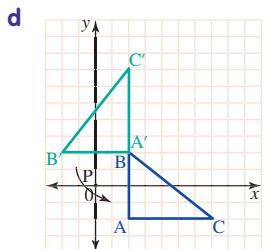
b





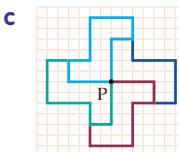
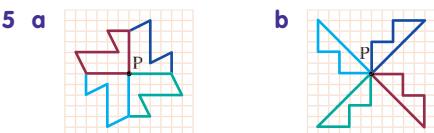
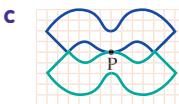
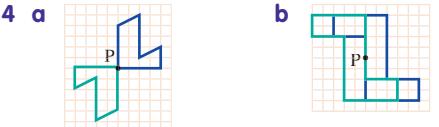
- f  $A(-4, -1) \rightarrow A'(-1, 4)$ ,  $B(-4, 3) \rightarrow B'(3, 4)$ ,  
 $C(1, -1) \rightarrow C'(-1, -1)$

The  $x$ - and  $y$ -values interchange, with the  $x$ -value of the object changing sign as it becomes the  $y$ -value of the image.

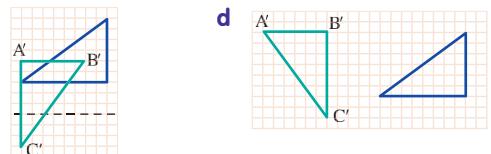
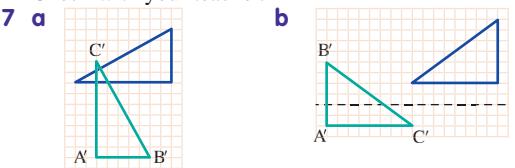


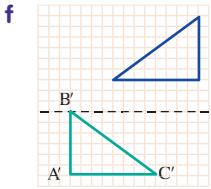
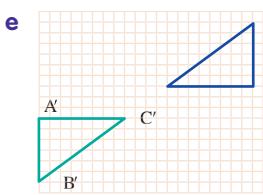
All images are congruent to the original shapes.

- 3 a D  
b E  
c F  
d B  
e A  
f C

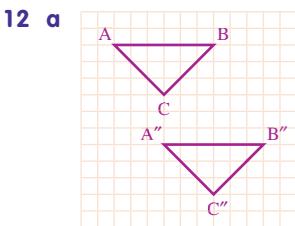


6 Check with your teacher.

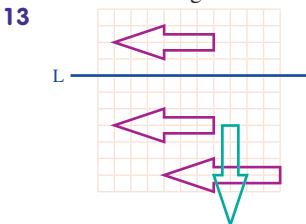




- 8 a Congruent  
c Congruent  
9 a True  
c True  
e False  
10 D  
11 E



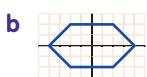
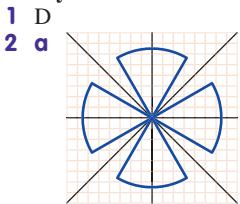
- b Two units right and 4 units down.



14 Angle  $\approx 79^\circ$

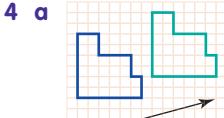
### Chapter review

#### Fluency

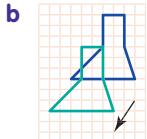


3 a 4

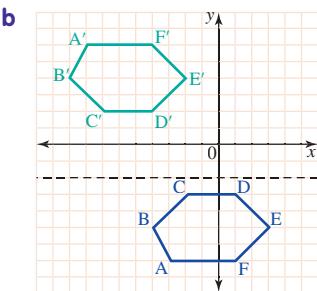
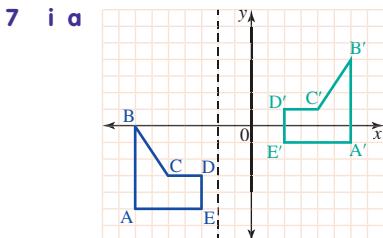
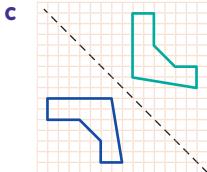
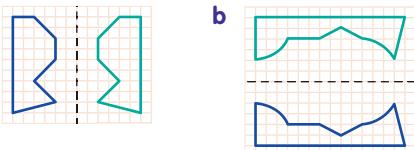
Key — object — image



b 2



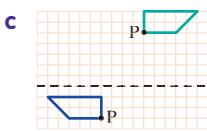
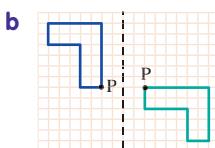
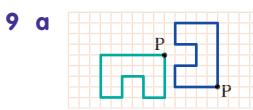
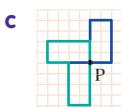
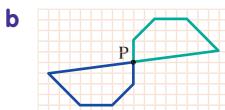
- 5 a 1U 5R  
6 a 3D 2L  
b 8R 1U  
d 12L 4U



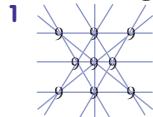
- ii a  $A(-7, -5) \rightarrow A'(6, -1)$ ,  $B(-7, 0) \rightarrow B'(6, 4)$ ,  
 $C(-5, -3) \rightarrow C'(4, 1)$ ,  $D(-3, -3) \rightarrow D'(2, 1)$ ,  
 $E(-3, -5) \rightarrow E'(2, -1)$

- b  $A(-3, -7) \rightarrow A'(-8, 6)$ ,  $B(-4, -5) \rightarrow B'(-9, 4)$ ,  
 $C(-2, -3) \rightarrow C'(-7, 2)$ ,  $D(1, -3) \rightarrow D'(-4, 2)$ ,  
 $E(3, -5) \rightarrow E'(-2, 4)$ ,  $F(1, -7) \rightarrow F'(-4, 6)$

iii Discuss as a class.



## Problem solving



- 1**
- 2** **a** Each of the second group of 10 letters has the same pattern as the corresponding letter but with an extra dot underneath.
- b** D and F, E and I, H and J, M and U, N and Z, P and V, R and W
- c** B, C, E and I
- d** Braille is read by touch and translations would not be determined by touch.
- e** Answers will vary.
- f** Answers will vary.
- 3** **a** Rotation of  $90^\circ$ .
- b** A translation of one brick-width to the right.
- c** A rotation of  $90^\circ$  followed by a translation of one brick-width to the right.
- d** Answers may include:
- Brick 4: translation of 12 brick-widths up
  - Brick 5: translation of two brick-widths up and one brick-length to the right
  - Brick 6: translation of two brick-widths up, one brick-length to the right and a rotation of  $90^\circ$  to the right
  - Brick 7: translation of two brick-lengths to the right and a rotation of  $90^\circ$  to the right
  - Brick 8: translation of three brick-lengths to the right, 4 brick-widths up and a rotation of  $90^\circ$  to the left
- 4** Answers will vary.

## CHAPTER 17

### Views of 3-D shapes

#### Are you ready?

- 1** **a** i cm      ii km      iii m      iv mm  
**b** Check with your teacher.
- 2** **a** 1 unit      b 0.1 of a unit  
**3** **a** 43 mm      b 3.7 cm  
**4** Metres  
**5** **a** 18 000 m      b 6 km  
**6** **a** 32 cm      b 17 mm

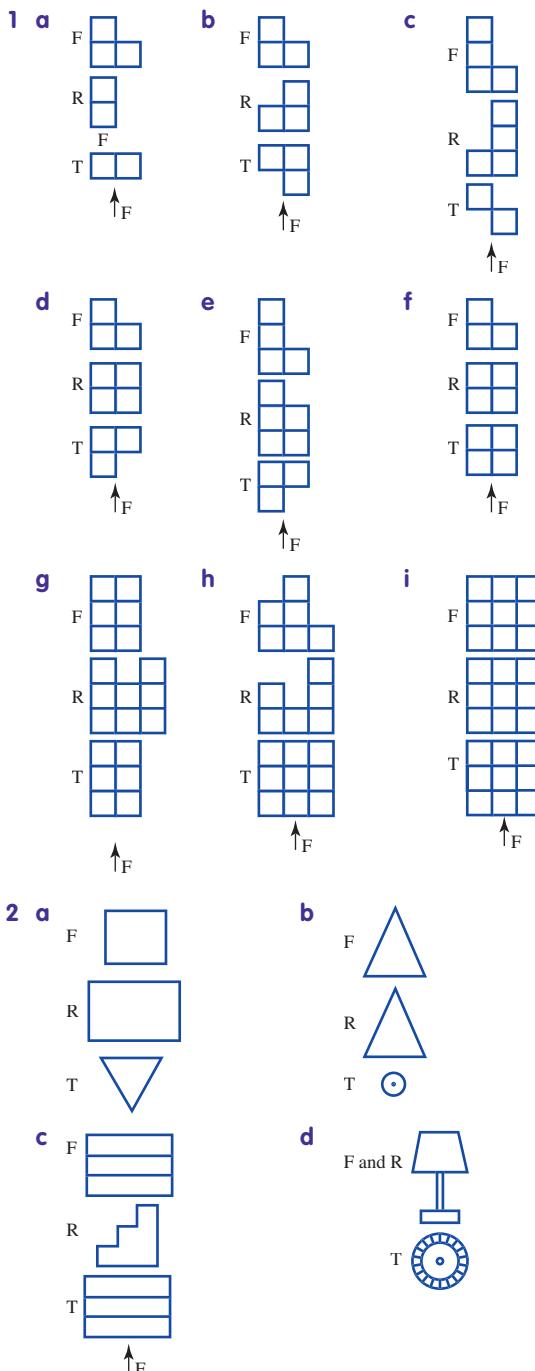
### 17A Scales and house plans

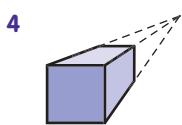
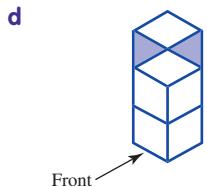
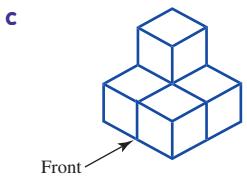
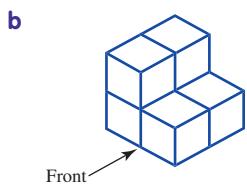
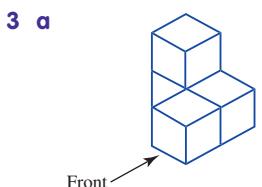
- 1** **a**  $1 \text{ cm} \leftrightarrow 10 \text{ cm}$       b  $1 \text{ cm} \leftrightarrow 1 \text{ m}$   
**c**  $1 \text{ cm} \leftrightarrow 10 \text{ m}$       d  $1 \text{ cm} \leftrightarrow 100 \text{ m}$   
**e**  $1 \text{ cm} \leftrightarrow 1 \text{ km}$       f  $1 \text{ cm} \leftrightarrow 50 \text{ m}$   
**g**  $1 \text{ cm} \leftrightarrow 600 \text{ m}$       h  $1 \text{ cm} \leftrightarrow 4 \text{ m}$   
**i**  $1 \text{ cm} \leftrightarrow 7.5 \text{ km}$       j  $1 \text{ cm} \leftrightarrow 22 \text{ km}$
- 2** **a** 1 : 20      b 1 : 50 000  
**c** 1 : 1000      d 1 : 20 000  
**e** 1 : 500 000      f 1 : 5 000 000  
**g** 1 : 550 000      h 1 : 100 000  
**i** 1 : 125 000      j 1 : 200 000
- 3** **a** 1 cm      b 2 mm  
**c** 15 cm      d 2.5 cm  
**e** 22 cm      f 5 mm  
**4** **a**  $20 \text{ m} \times 25 \text{ m}$       b  $10 \text{ m} \times 15 \text{ m}$   
**5** **a**  $18 \text{ m} \times 12 \text{ m}$       b  $6.75 \text{ m} \times 4.5 \text{ m}$   
**c** Bed 1 —  $4.5 \text{ m} \times 4.5 \text{ m}$
- 6** **a** 8.5 m      b 3.5 m  
**c**  $40^\circ$

- 7** **a** 50 cm      b 4 cm  
**c**  $100 \text{ cm} \times 40 \text{ cm}$       d  $8 \text{ cm} \times 3 \text{ cm}$   
**e** 5 m      f 12.5 m  
**g**  $50 \text{ cm} \times 40 \text{ cm}$       h  $2.25 \text{ m} \times 1.5 \text{ m} \times 0.6 \text{ m}$
- 8** **a**  $6 \text{ m} \times 4 \text{ m}$       b 1 m  
**c** 6.8 m      d  $24 \text{ m}^2$   
**e**  $12 \text{ m}^2$

- 9** Check with your teacher.  
**10** Check with your teacher.  
**11** Check with your teacher.

### 17B Plans and views





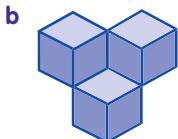
5 D

- 6 a Circle  
b Oval  
c Trapezium  
d Rectangle



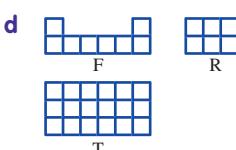
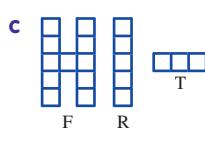
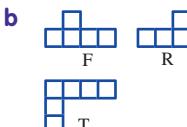
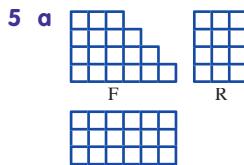
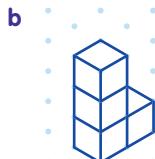
b-d Various answers

8 a It would be difficult to construct this solid.



The solid would look like the one shown. The 4th cube is behind the front bottom cube, resting on the surface. The two top cubes have no cubes supporting them.

### 17C Isometric drawing



6



7 Check with your teacher.

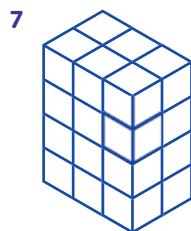
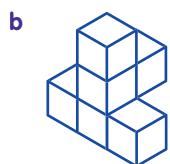
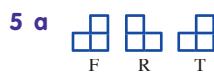
**Chapter review****Fluency**

- 1 a** 5 km      **b** 2.5 km      **c** 4 km  
**2 a** 1.25 cm      **b** 3.25 cm

**Problem solving**

- 1 a** 9 km      **b** 1.25 cm  
**2 a**  $30 \text{ m} \times 34 \text{ m}$       **b**  $10.8 \text{ m} \times 14 \text{ m}$   
**c**  $151.2 \text{ m}^2$       **d** SW

- 3 a** F   
**b** F   
**c** S   
**d** T

**CHAPTER 18****Money****Are you ready?**

- 1 a** \$23.50      **b** \$207.90  
**2 a** 0.34      **b** 0.79      **c** 0.04  
**d** 0.672      **e** 0.0825      **f** 0.175

**3 a** 85%**b** 87.5%**c** 10%**d** 94.5%**4 a** \$35**b** \$356**c** \$1620**d** \$571.25**5 a** 14%**b** 65%**6 a** 125%**b** 105%**c** 200%**d** 112.5%**e** 20%**18A Money**

- 1 a** 755 cents      **b** 305 cents  
**c** 240 cents      **d** 900 cents  
**2 a** \$6.85      **b** \$4.50  
**c** \$0.05      **d** \$8.05  
**3 a** \$4.75      **b** \$12.60  
**c** \$159.30      **d** \$83.45

**4** \$81.80**5** \$2.25**6 a** \$33.05**b** \$30.05**c** \$25.05**7**  $3.7 = 4$  wk**8** \$529.85**9** \$89.90**10**  $7\frac{1}{2}$  hours**11** \$31; interest charges**12** \$39.20**13 a** \$14, \$14.05**b** \$6, \$5.95**14 a** \$26, \$26.22**b** \$4, \$3.80 after rounding to nearest 5 cents**15 a** \$51.88      **b** \$1.50      **c** \$9.63**16** Sam receives \$100.20, Georgia receives \$66.80.**17** 68.8%**18** 8**19** 20 days**18B Money and percentages**

- |                    |                  |                   |
|--------------------|------------------|-------------------|
| <b>1 a</b> \$8     | <b>b</b> \$10    | <b>c</b> \$100    |
| <b>d</b> \$2.40    | <b>e</b> \$25    | <b>f</b> \$12     |
| <b>g</b> \$4.50    | <b>h</b> \$9.60  | <b>i</b> \$5.25   |
| <b>j</b> \$8.10    | <b>k</b> \$7.10  | <b>l</b> \$24.20  |
| <b>m</b> \$24.50   | <b>n</b> \$26.60 | <b>o</b> \$4.20   |
| <b>2 a</b> \$3.20  | <b>b</b> \$1.25  | <b>c</b> \$7.50   |
| <b>d</b> \$16.70   | <b>e</b> \$0.15  | <b>f</b> \$1.60   |
| <b>g</b> \$2.11    | <b>h</b> \$7.78  | <b>i</b> \$16.42  |
| <b>3</b> One, left |                  |                   |
| <b>4 a</b> \$2.30  | <b>b</b> \$5.70  | <b>c</b> \$13.00  |
| <b>d</b> \$6.40    | <b>e</b> \$8.15  | <b>f</b> \$1.48   |
| <b>g</b> \$14.96   | <b>h</b> \$34.59 | <b>i</b> \$135.45 |
| <b>5 a</b> \$0.60  | <b>b</b> \$1.80  | <b>c</b> \$3.40   |
| <b>d</b> \$1.20    | <b>e</b> \$4.50  | <b>f</b> \$6.00   |
| <b>g</b> \$2.25    | <b>h</b> \$1.37  | <b>i</b> \$0.64   |
| <b>j</b> \$1.68    | <b>k</b> \$25.41 | <b>l</b> \$11.78  |
| <b>6 a</b> \$0.26  | <b>b</b> \$0.42  | <b>c</b> \$1.66   |
| <b>d</b> \$4.06    | <b>e</b> \$16.20 | <b>f</b> \$0.14   |
| <b>g</b> \$0.07    | <b>h</b> \$0.06  | <b>i</b> \$0.17   |
| <b>7</b> Two, left |                  |                   |
| <b>8 a</b> \$15    | <b>b</b> \$33    | <b>c</b> \$6      |
| <b>d</b> \$1.20    | <b>e</b> \$0.93  | <b>f</b> \$2.25   |
| <b>g</b> \$2.97    | <b>h</b> \$85.23 | <b>i</b> \$22.5   |
| <b>9 a</b> C       | <b>b</b> D       | <b>c</b> C        |
| <b>d</b> A         |                  |                   |
| <b>10</b> \$2325   |                  |                   |
| <b>11</b> \$6.75   |                  |                   |
| <b>12</b> 90c      |                  |                   |
| <b>13</b> 63c      |                  |                   |

**18C Unitary method**

- 1** \$8.55  
**2** Neither is better (they are both 62 cents per 100 g).  
**3 a** The 325-g option is the best buy.  
**b** The 200-page option is the best buy.  
**4 a** \$180      **b** \$30      **c** \$240  
**d** \$2250      **e** \$176.47      **f** \$422.22  
**g** \$400      **h** \$350      **i** \$600  
**5 a** \$180      **b** \$11.40      **c** \$120  
**d** \$2700      **e** \$40      **f** \$25 800  
**6 a** \$400      **b** \$640  
**7** \$50  
**8** \$687.50  
**9** \$160  
**10** \$84 600  
**11** \$216 364  
**12** \$48.62  
**13 a** \$790.91      **b** \$79.09  
**14** \$215.29  
**15 a** \$54.55      **b** \$70.59  
**16 a** \$21.33      **b** \$18.13  
**17 a** \$218.18      **b** \$170.18  
**18** \$22.20  
**19** \$196.64  
**20** Loss of \$463  
**21** \$50

**Chapter review****Fluency**

- 1 a** \$21.70      **b** \$178.35      **c** \$67.55  
**2** \$5.95  
**3 a** \$6.30      **b** \$4.20      **c** \$10.50  
**d** \$21.60      **e** \$0.35      **f** \$4.25  
**g** \$11.85      **h** \$212.60  
**4 a** \$0.40      **b** \$1.05      **c** \$3.20  
**d** \$5.20      **e** \$1.75      **f** \$2.60  
**g** \$10.30      **h** \$3.85  
**5 a** \$0.15      **b** \$0.30      **c** \$5.05  
**d** \$27.00      **e** \$48.40      **f** \$11.60  
**g** \$2.95      **h** \$2.95  
**6** \$1.80

**Problem solving**

- 1** \$164.35      **2** \$175  
**3** \$3792.67      **4** \$85  
**5** Estimates are total, \$140; change, \$10; exact amounts are \$138.35, \$11.65.  
**6** 13%      **7** \$1.99  
**8 a** \$26      **b** \$2.58      **c** \$10.29  
**d** \$8.40      **e** \$0.17      **f** \$9.32  
**g** \$67.50      **h** \$4.32  
**9** \$60      **10** 14  
**11** Options are 50c per CD, 49.9c per CD, 54.17c per CD. Therefore the \$4.99 pack is best buy.  
**12** First pack costs 76c per 100 g; second pack costs 66c per 100 g; so the second pack is the best buy.  
**13 a** Saved \$40.81  
**b** 40.5% saved

**CHAPTER 19****Problem solving II****19A Problem solving A**

- 1** 800 m  
**2** 14 choices

- 3** 12 students study both French and German.

- 4** 420 calls  
**5** 1440°  
**6** Two sides of the fencing are 6 m long and the other side is 12 m, providing an area of 72 m<sup>2</sup>.  
**7** 6 people  
**8**  $\frac{18}{25}$   
**9** 293 cm  
**10** 187 runs

**19B Problem solving B**

- 1** 12  
**2** 3 people do not work Monday to Sunday  
**3** \$1.40  
**4** The coach's statement was not correct.  
**5** 10  
**6 a** 100 000 000  
**b** 20 000 000  
**7** 36  
**8** 22.5%  
**9** 99 cm

**19C Problem solving C**

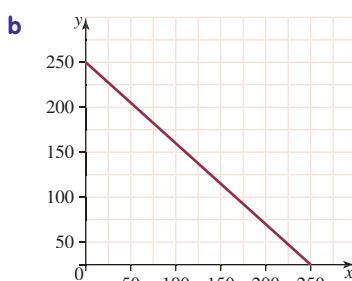
- 1** 4  
**2** 6  
**3** \$1.50  
**4** 260  
**5** Rebecca's strategy seems reasonable.  
**6** 0.65 m<sup>3</sup> more soil is needed (or 2.075 m<sup>3</sup>)  
**7** 3  
**8** 25 cherries  
**9** 3, 8, 13  
**10** 4 cm

**19D Problem solving D**

- 1** The gymnasium should keep the drink machine.  
**2** 8 different rectangles  
**3** 3 students voted twice.  
**4** 0.4 L  
**5** 45 handshakes  
**6** 28 dominoes  
**7** Square sides 8 cm  
**8** 4  
**9** 1490 mm  
**10** 4 hours

**19E Problem solving E**

- 1** RB145, RB154, RB415, RB451, RB514, RB541, BR145, BR154, BR415, BR451, BR514, BR541  
**2**  $\angle COD = \angle DOE = 40^\circ$   
**3** 36 000  
**4** The tank overflowed.  
**5 a**  $x + y = 250$



- c** Check with your teacher

- 6** 0.107  
**7** The 50 extra sheets are 6 mm thick. You probably wouldn't notice the difference.

**8**  $\frac{3}{20}$

**9** Thickness =  $2^n$  (Number of folds)

**10** 
$$\begin{array}{r} 734 & 765 & 836 & 867 & 928 & 938 \\ + 734 & + 765 & + 836 & + 867 & + 928 & + 938 \\ \hline 1468 & 1530 & 1672 & 1734 & 1856 & 1876 \end{array}$$

### 19F Problem solving F

- 1** **a** (Mrs Makeit's costs + 10)  $\times 2$   
**b**  $c = 2(n + 10)$   
**c** \$60  
**2** Yes, 24 cm  $\times$  6 cm  $\times$  6 cm, or 12 cm  $\times$  12 cm  $\times$  6 cm  
**3**  $\frac{1}{16}$   
**4** **a** In year 6 she sold 300.  
**b** In year 10 she sold 50.  
**c** There are two trends. The sales of the glass pendants gradually increase from year 1 to year 6. However, after year 6 the sales decrease quickly.  
**d** No, the second trend is down, and will not help Lisa obtain the loan because the lowest sales are her most recent ones.

**5** 312 digits

**6** **a**

Score	Frequency
1	7
2	4
3	8
4	3
5	5
6	3

**b**  $P(6) = \frac{1}{6}$

**c** 5

**d** The result differs by pure chance. The more often we roll the die the closer we can expect the results to match the expected number of sixes.

**7** Brand B

**8** 200

**9** The missing number is 101.

**10** James \$53 and Alison \$72

### 19G Problem solving G

- 1** 1401.6 L
- 2** **a**
- 
- | Sport      | Participants |
|------------|--------------|
| Basketball | 12           |
| Baseball   | 11           |
| Swimming   | 6            |
| Soccer     | 10           |
- b**  $\frac{11}{40}$
- 3**  $22 \text{ cm}^2$
- 4** 10:15
- 5** 8
- 6** 13.5 km
- 7** Pear costs \$0.40, apple costs \$0.80.

**8**  $x, y, \frac{y}{x}, \frac{1}{x}, \frac{1}{y}, x, y, \frac{y}{x}, \frac{1}{x}$

**9** 1.5 m

- 10** Select any two coins and place one coin on each side of the pair of scales. If the scales balance, then the third coin is the heavier one. Otherwise, the scales will be unbalanced and you can see which one is the heavier.

### 19H Problem solving H

**1**  $\$0.14/\text{cm}^2$

**2** **a**  $\frac{13}{73}$

**b** Karen is not correct, as a wet day (65 out of 365) and a dry day (300 out of 365) are not equally likely.

**c** 0.18

**3** \$22.50

**4**  $102^\circ$

**5** 46

**6** 70

**7** 63

**8**  $(n - 3)(b + 2) = 80$

**9** \$2.60

$$\begin{array}{r} 9567 \\ + 1085 \\ \hline 10652 \end{array}$$

### 19I Problem solving I

**1** **a** 15

**b** 3

**c** 0

**d** The median is the best measure as the mode is the lowest of all scores and the mean is inflated by one much larger figure.

**2** 12, 13, 14

**3** Seth's arm span is 10 times the length of his hand and 4 times the distance across his shoulders.

**4** They did not start with the same number

**5**  $m, a, z, g$

**6** \$121.97

**7** 71 428 pins

**8** Water 426 mL, butter 180 g, sugar 150 g, flour 300 g, eggs 2

**9**  $1 = (4 - 3) \div (2 - 1); 3 = 21 \div (3 + 4); 5 = 3 + 4 - (2 \times 1)$

**10** A = 3, B = 5, C = 4, D = 1, E = 6, F = 2

### 19J Problem solving J

**1** 
$$\begin{array}{r} 940 \\ 739 \\ + 8925 \\ \hline 10604 \end{array}$$

**2** 1998

**3**  $5 = 4 + 3 - (2 \times 1), 6 = 3 \times 4 \div (2 \times 1), 8 = 4 \times (3 - 2 + 1), 9 = 4 + 3 + 2 \div 1$

**4** **a** 25, 26

**b** 72, 73

**c** 120, 121

**5** 306

**6** Chips \$1.50, chocolate \$0.70

**7** Candice 14, Sara 12, Jordyn 13, Georgia 11

**8** 11

**9** **a** 3, 7

**b** 2, 13

**c** 17, 19

**10** 48

# Glossary

- 2-dimensional representation (net):** plan or net that can be folded to form a polyhedron
- 3-dimensional object:** shape that has depth, as well as length and width; also called a polyhedron
- Acute angle:** angle greater than  $0^\circ$  but less than  $90^\circ$
- Acute-angled triangle:** triangle where each angle is acute; that is, less than  $90^\circ$
- Adjacent angles:** angles lying next to another angle
- Algebra:** the branch of mathematics that deals with solving mathematical problems involving an unknown value
- Allied angles (or co-interior angles):** angles on the same side of the transversal positioned between the parallel lines. These angles are supplementary.
- Alternate angles:** angles on opposite sides of the transversal positioned between the parallel lines. These angles are equal in size.
- Angle:** rotation between a pair of lines that meet at a point
- Anticlockwise:** movement in the opposite direction to that of the hands of a clock
- Area:** amount of surface enclosed by the shape; measured in square units, such as square metres,  $m^2$ , and square kilometres,  $km^2$
- Area references:** grid references on a map that refer to a whole square and not a single point
- Arithmagon:** triangular figure in which the two numbers at the ends of each line add to the number along the line
- Arms:** lines that meet at a point to form an angle
- Ascending:** increasing from smallest to largest
- Associative law for addition:** a number law that refers to the order in which three numbers may be added or subtracted
- Associative law for multiplication:** a number law that refers to the order in which three numbers may be multiplied or divided
- Axes:** horizontal and vertical lines with scales that form the Cartesian plane
- Axis of symmetry:** line that divides a shape into two identical parts
- Backtracking:** process of working backwards through a flow chart. Each step is the inverse operation of the corresponding step in the flow chart. It can be used to solve equations.
- Bar graph:** graph drawn in a similar way to column graphs with horizontal bars instead of vertical columns. Categories are graphed on the vertical axis and the frequencies (numbers) on the horizontal axis.
- Base:** the number being repeatedly multiplied in a power term. The power is the number of times the base is written.
- Bearings:** directions from one place to another
- BIDMAS:** the order in which calculations are performed. The order is: brackets; index (or power); division, multiplication from left to right; addition and subtraction from left to right.
- Bird's eye view:** view observed from the top of the object looking down on it
- Bisect (angles):** to divide an angle into two equal, smaller angles
- Capacity:** maximum amount of fluid that can be contained in an object. The units may be the same as for volume or litres and millilitres may be used.
- Cartesian plane:** area formed by a horizontal line with a scale ( $x$ -axis) joined to a vertical line with a scale ( $y$ -axis). The point of intersection of the lines is called the origin.
- Categorical (data):** data that involve grouping or classifying things, not numbers; for example, grouping hair colour
- Centre of rotation:** a specific point around which an object is rotated
- Certain:** in probability, an outcome that will happen
- Classify:** to group objects based on similarities
- Clockwise:** movement in the same direction to that of the hands of a clock
- Coefficient:** the number written in front of a term
- Co-interior angles (or allied angles):** angles on the same side of the transversal positioned between the parallel lines. These angles are supplementary.
- Column graph:** graph in which equal width columns are used to represent the frequencies (numbers) of different categories
- Common factor:** a factor common to more than one number
- Commutative law for addition:** a number law that refers to the order in which two numbers may be added or subtracted
- Commutative law for multiplication:** a number law that refers to the order in which two numbers may be multiplied or divided
- Compass:** instrument with magnetic needle pointing north — used to measure direction
- Complementary angles:** sum (add) to  $90^\circ$
- Composite numbers:** numbers with more than two factors
- Composite shape:** figure made up of more than one basic shape
- Congruent figures:** figures that have exactly the same size and shape
- Constant:** a value that is not a variable
- Continuous (data):** data that take on any value and are able to be measured; for example, the heights of people in a class
- Contour lines:** lines that join points of equal elevation above sea level
- Conventional compass bearing:** directions measured in degrees from the north-south line in either a clockwise or anticlockwise direction. To write the bearing we need to state whether the angle is measured from the north or south, how many degrees there are in the angle and whether the angle is measured in the direction of east or west.
- Conversion factors:** in measurement, they are used to convert one unit of measurement to another
- Convert:** to change from one unit to another
- Coordinates ( $x, y$ ):** two numbers that give the position of a point on the Cartesian plane. The first number is the  $x$ -coordinate and the second number is the  $y$ -coordinate.

**Corresponding angles:** angles on the same side of the transversal, and which are both either above or below the parallel lines. The angles are equal in size.

**Cross-sectional area (shape):** area of the face created by slicing a 3-dimensional solid

**C (Roman numeral):** represents 100

**Cube:** polyhedron with 6 faces; all faces are squares of the same size

**Cube root ( $\sqrt[3]{\cdot}$ ):** number that multiplies by itself three times to equal the original number. Finding a cube root is the opposite of cubing a number.

**Cubic numbers:** numbers with three equal factors; for example,  $8 = 2 \times 2 \times 2$  and  $27 = 3 \times 3 \times 3$

**Data:** numerical information

**Decagon:** 10-sided polygon. It also has 10 angles.

**Decomposition method:** method of subtraction. The larger number is decomposed by taking 10 from the tens column and adding it to the units column. It may also involve taking 100 from the hundreds column and adding it to the tens column.

**Degrees ( $^\circ$ ):** units used to measure the size of an angle

**Degrees Celsius ( $^{\circ}\text{C}$ ):** units used to measure temperature

**Denominator:** bottom term of a fraction. It shows the total number of parts into which the whole has been divided.

**Dependent value:** the value on the vertical axis of a graph

**Descartes:** René Descartes was a mathematician in the 17th century (1596–1650) who first proposed the Cartesian plane and the use of Cartesian coordinates to locate points on a plane.

**Descending:** decreasing from largest to smallest

**Digits:** first nine counting numbers and zero

**Directed number:** a number that has both size and direction, for example, +3 and -7

**Discrete (data):** data that take on exact values that may be counted; for example, the number of televisions sold each week

**Distributive law:** a rule which states that each term inside a pair of brackets is to be multiplied by the term outside the brackets

**Dividend:** the quantity in a division which is to be divided

**Divisor:** the quantity in a division by which the dividend is divided

**Dodecagon:** 12-sided polygon. It also has 12 angles.

**Dodecahedron:** polyhedron with 12 faces

**Dot plot:** consists of a horizontal axis labelled and evenly scaled, with each data value represented by a dot

**Edges (networks):** lines in networks that join the vertices to one another

**D (Roman numeral):** represents 500

**Edges (shapes):** lines where two faces of a polyhedron meet

**Equation:** mathematical statement containing a left-hand side, a right-hand side and an equals sign between them

**Equilateral triangle:** triangle with all sides equal in length. All angles will be equal too.

**Equivalent fractions:** fractions that look different, but represent the same amount. Equivalent fractions can be formed by multiplying (or dividing) both the numerator and denominator by the same (non-zero) value.

**Estimation:** approximate answer to a question based on information given

**Euler's Rule:** rule that links the number of faces, F, the number of vertices, V, and the number of edges, E, of a polyhedron: Euler's Rule is  $F + V - E = 2$ .

**Evaluate:** to obtain an answer

**Even chance:** in probability, an outcome that has a 50% chance of happening

**Even numbers:** numbers divisible by 2; numbers that end in 0, 2, 4, 6 or 8

**Event (probability):** result that may be obtained

**Experiment:** the process in probability of performing repeated trials of an activity for the purpose of obtaining data in order to be able to predict the chances of certain things happening

**Experimental probability:** probability determined by conducting an experiment and gathering data

**Expression:** mathematical statement made up of pronumerals (letters) and numbers. Expressions do not contain an equals sign.

**Exterior angle:** angle formed when any side of a closed shape is extended outwards. The exterior angle and the interior angle adjacent to it are supplementary (add up to  $180^\circ$ ).

**Faces (shapes):** flat surfaces that make a polyhedron

**Factor pairs:** pairs of numbers which multiply together to equal another number; they are factors of the same number.

**Factor tree:** diagram that displays the prime factors of a composite number. Each branch shows a factor of all numbers above it. The last numbers at the bottom of the tree are the prime factors of the original number.

**Factors:** whole numbers that divide exactly into another whole number, without any remainder

**Fair game (probability):** game that allows each player an equal chance of winning

**Favourable outcome (probability):** outcome that we are looking for

**Finite decimal:** a decimal with a fixed number of decimal places

**Flowcharts:** diagrams that illustrate the process by which an input number becomes an output number in an equation or formula

**Formula:** rule that is written using pronumerals

**Fraction:** used to describe parts of a whole number

**Frequency:** number of times a result or piece of data occurs

**Frequency distribution table:** table used to organise data by recording the number of times each data value occurs.

**Front elevation:** view observed from directly in front of an object

**Front view:** see front elevation

**Greater than ( $>$ ):** symbol used to represent one number being larger than another number

**Hectare (ha):** area equal to  $10\,000 \text{ m}^2$ , area of a square of side length of 100 m, unit commonly used to measure land area

**Heptagon:** 7-sided polygon. It also has 7 angles.

**Hexagon:** 6-sided polygon. It also has 6 angles.

- Highest Common Factor (HCF):** largest factor which is common to all the numbers given
- Horizontal:** lying flat or level (with the horizon). A horizontal line is drawn straight across the page.
- Horizontal axis:** the  $x$ -axis
- I (Roman numeral):** represents 1
- Icosagon:** 20-sided polygon. It also has 20 angles.
- Icosahedron:** polyhedron with 20 faces
- Illustration:** example or picture which makes something clearer
- Impossible:** in probability, an outcome that can never happen
- Improper fraction:** fraction whose numerator is larger than its denominator, for example,  $\frac{5}{4}$
- Independent value:** the value on the horizontal axis of a graph
- Index (or power):** indicates the number of times a base is to be multiplied when written in index form
- Index notation (index form):** form of writing the product of prime factors. It has a base and an index or power. The index indicates how many times the base is being multiplied by itself.
- Infinite recurring decimal:** a decimal with a repeating pattern of decimal places that never ends
- Input number:** the first number in a flowchart
- Integers:** positive whole numbers, negative whole numbers and zero
- Interior angle:** angle inside a shape
- Interpret:** to understand or give meaning to; to interpret a graph means to understand what the graph is telling us
- Inverse operation:** operation that ‘undoes’ a previous operation. Addition and subtraction are inverse operations; and multiplication and division are inverse operations of each other.
- Irregular quadrilateral:** quadrilateral that does not have any special properties
- Isobars:** lines on synoptic maps that connect places of equal air pressure
- Isometric:** a transformation that does not alter the size or shape of the original object
- Isosceles triangle:** triangle that has two sides of equal length. The two angles opposite the equal sides will also be equal.
- Kite:** quadrilateral in which two pairs of adjacent sides are equal in length and one pair of opposite angles (those between the sides of unequal length) are equal
- L (Roman numeral):** represents 50
- Legends:** definitions of different symbols or colours used when drawing graphs
- Less than ( $<$ ):** symbol used to represent one number being smaller than another number; for example,  $\frac{1}{5} < \frac{3}{5}$
- Like terms:** terms that contain exactly the same pronominal (letter) part; for example,  $5a$  and  $33a$  are like terms but  $4ab$  is not.
- Line graph:** graph containing points joined with line segments
- Line of symmetry:** a line which divides a shape into two parts that are mirror images of each other
- Line plot:** see dot plot
- Long run (probability):** see long-term trend
- Long-term trend (probability):** trend observed for results from a very large number of trials. This tends to match the results of theoretical probability.
- Lowest Common Multiple (LCM):** lowest multiple that two or more numbers have in common
- M (Roman numeral):** represents 1000
- Mass:** describes how much matter makes up an object. The base unit of mass is the gram, g.
- Mean (summary statistics):** sum of all the values (scores) divided by the number of values — also called the average
- Median (summary statistics):** middle value if the number of data is odd; or the average of the two middle values if the number of data is even. Data must first be in numerical order.
- Meridian:** imaginary line on the Earth’s surface passing through the North and South Poles and running perpendicular to the equator
- Meridians of longitude:** imaginary lines running on the Earth’s surface from the North Pole to the South Pole: for example, the Greenwich Meridian has a longitude of  $0^\circ$  and the International Date Line has  $180^\circ\text{E}$  or  $180^\circ\text{W}$  longitude
- Metric:** system of measurement based on the number 10. The base unit of length in this system is the metre, m.
- Mixed number:** number that is made up of a whole number and a fraction, for example  $2\frac{3}{4}$
- Mode (summary statistics):** most common value in a set of data
- Multiples:** answers obtained when a number is multiplied by other whole numbers; for example, 4, 8, 12, 16 are multiples of 4
- Negative integer:** whole numbers less than zero
- Nets:** 2-dimensional plans of polyhedra
- Network:** series of dots (vertices) joined together in some way by lines (edges). The lines can be straight lines, curves or loops and the dots are labelled. Networks can be used as maps that show how features are joined together.
- Nonagon:** 9-sided polygon. It also has 9 angles.
- Numerator:** top term of a fraction. It shows how many parts there are.
- Obtuse angle:** angle greater than  $90^\circ$  but less than  $180^\circ$
- Obtuse-angled triangle:** triangle that has one angle greater than  $90^\circ$
- Octagon:** 8-sided polygon. It also has 8 angles.
- Octahedron:** polyhedron with 8 faces
- Odd numbers:** numbers not divisible by 2; numbers that end in 1, 3, 5, 7 or 9
- Opposite integer:** numbers equidistant from zero, but one is positive and one is negative
- Ordered pair (probability):** pair of items written in brackets separated by a comma; for example, when tossing a coin and rolling a die, an outcome might be a ‘Head’ and a ‘6’. This would be written as  $(\text{H}, 6)$  in ordered pair notation.

- Ordered pair ( $x, y$ ):** see coordinates
- Ordinal data:** data that involve a rating or ranking; for example, the ratings or grades on a report card where the scale may range from A<sup>+</sup> to UG
- Origin:** point where the  $x$ -axis and the  $y$ -axis intersect on the Cartesian plane; that is, the point with coordinates (0, 0)
- Outcomes (probability):** particular result of an experiment
- Outlier:** extreme value in a set of data
- Output number:** the last number in a flowchart
- Pair of compasses:** instrument that can be used to draw angles, circles and distances
- Palindromes:** numbers or words that read the same forwards and backwards; for example, 898 or mum
- Pan balance scale:** a scale with two pans that can be used to show whether things have equal mass. It can be used in Mathematics to show whether two expressions are equal in value.
- Parallel (lines):** lines running in exactly the same direction and therefore never meeting. They are usually drawn with arrowheads.
- Parallelogram:** quadrilateral with both pairs of opposite sides parallel to each other. Rectangles, squares and rhombuses are parallelograms.
- Parallels of latitude:** imaginary lines running around the Earth parallel to the equator: for example, the equator is of latitude 0°, the North Pole is of latitude 90°N and the South Pole is of latitude 90°S
- Pentagon:** 5-sided polygon. It also has 5 angles.
- Percentage:** amount out of 100 or per hundred; for example, 50% means 50 out of 100 or  $\frac{50}{100}$ . They are used to represent part of a quantity.
- Perigon (revolution):** angle that equals 360° (a full circle)
- Perimeter:** distance around the outside (border) of a shape
- Perpendicular lines:** lines that intersect at right angles (90°)
- Pie graph (pie charts or sector graphs):** data are graphed on a circle with each category being represented by a sector of the circle
- Place value:** the position of a digit in a number
- Polygon:** 2-dimensional, closed shape with 3 or more sides, each of which is a straight line
- Polyhedron:** 3-dimensional shape in which each flat surface (face) is a polygon. The plural term is polyhedra.
- Positive integers:** whole numbers greater than zero
- Power (or index):** see index
- Prime factors:** factors that are only prime numbers
- Prime number:** counting number that has exactly two factors, itself and 1
- Prism:** solid shapes with identical opposite ends joined by straight edges. Rectangular prisms have rectangles as identical ends and triangular prisms have triangles.
- Probability:** likelihood or chance of a particular event (result) occurring
- Probability:**  $= \frac{\text{number of favourable outcome}}{\text{number of possible outcome}}$ . The probability of an event occurring ranges from 0 (impossible — will not occur) to 1 (certainty — will definitely occur).
- Product:** result of a multiplication
- Pronumeral:** letter used in place of a number
- Proper fraction:** fraction whose numerator is smaller than its denominator, for example  $\frac{3}{4}$
- Protractor:** instrument used to measure angles in degrees
- Quadrant:** one of four regions of the Cartesian plane produced by the intersection of the  $x$ - and  $y$ -axes
- Quadrilateral:** 2-dimensional, closed shape formed by four straight lines
- Quotient:** result of a division
- Random numbers:** a selection of numbers that follow no particular order or pattern
- Range (summary statistics):** difference between the highest and lowest values (scores)
- Rates:** measurements used to describe what can occur in a certain amount of time or how quickly a quantity changes:  

$$\text{rate} = \frac{\text{quantity}}{\text{time}}$$
- Rational numbers:** numbers that can be written as fractions, where the denominator is not zero
- Reciprocal:** the reciprocal of a number is obtained by first expressing it as a fraction then tipping the fraction upside down; for example,  $\frac{3}{4}$  is the reciprocal of  $\frac{4}{3}$ . A mixed number must be converted to an improper fraction first and then tipped.
- Rectangle:** parallelogram in which the opposite sides are equal in length and all angles are equal to 90°
- Recurring decimals:** decimals that have one or more digits repeated continuously; for example, 0.999... They can be expressed exactly by placing a dot or horizontal line over the repeating digits as in this example.  $8.\overline{343\,434} = 8.34$  or 8.34
- Reflection:** exact image of an object, as seen in a mirror; as far behind the mirror as the object is in front of it. Reflections always have reversed orientations; right appears left and left appears right.
- Reflex angle:** angle greater than 180° but less than 360°
- Regular polygon:** polygon with all sides equal in length and all angles equal in size
- Revolution:** see perigon
- Rhombus (plural: Rhombi):** parallelogram in which all sides are equal and opposite angles are equal
- Right angle:** angle that equals 90°
- Right-angled triangle:** triangle that has one of its angles equal to 90° (a right angle). A small square is placed in that angle to mark the right angle.
- Rotation:** object that is turned about a certain point. This point (centre of rotation) needs to be identified, along with the size of the angle of rotation and whether the object is to be turned clockwise or anticlockwise.
- Rounding:** technique used to give an approximate answer to a question. If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same. If the second is 5, 6, 7, 8 or 9, the first digit is rounded up to the next value. This also applies to decimals. For example, 2.456 rounded to 2 decimal places is 2.46.
- Rule:** pattern that links a set of input numbers to a set of output numbers in algebra; for example, 'adding 5' is a rule
- Sample space (probability):** the complete set of outcomes or results obtained from an experiment. It is shown as a list enclosed in a pair of braces (curly brackets) {}.

- Scale (in relation to maps):** ratio that states the actual distance represented by a distance on a map; for example,  $1 \text{ cm} \leftrightarrow 500 \text{ m}$  and  $1 : 50\,000$  mean the same thing, that is, 1 cm on a map represents an actual distance of 500 m (or 50 000 cm).
- Scalene triangle:** triangle that has no equal sides. The angles will all be unequal too.
- Scatterplot:** graphical representation of bivariate data
- Sequence:** a number pattern
- Side elevation:** view observed when standing directly to one side of an object. The left view or the right view of the object can be drawn.
- Side view:** see side elevation
- Simulation (probability):** experiment that uses simple devices such as coins, dice, a pack of cards and spinners to model or mimic a real life situation
- Solution:** the answer obtained when solving an equation
- Solving equations:** finding a value for a pronumeral that makes one side of an equation equal to another
- Speed:** special rate that tells us how fast something is moving; that is, how quickly distance changes over a period of time: speed =  $\frac{\text{distance}}{\text{time}}$ . Common units are kilometres per hour (km/h) and metres per second (m/s).
- Square:** parallelogram in which all sides are equal and all angles are equal to  $90^\circ$
- Square numbers:** numbers that have two equal factors; for example,  $9 = 3 \times 3$  and  $36 = 6 \times 6$
- Square root ( $\sqrt{\cdot}$ ):** number that multiplies by itself to equal the original number. Finding a square root is the opposite of squaring a number; for example,  $\sqrt{49} = 7$ .
- Straight angle:** angle that equals  $180^\circ$
- Substitution:** process by which a number replaces a pronumeral in a formula
- Successful trial:** a trial that results in the outcome you wanted
- Summary statistics:** calculations made with data in order to summarise the information or get meaning from them
- Supplementary angles:** sum (add) to  $180^\circ$
- Synoptic maps:** maps that show the weather conditions of an area
- Term:** part of an equation or expression. Terms can be expressed as one or more than one pronumeral, or a number only. Examples include  $5x$ ,  $3ab$ ,  $xyz$ , 8 and so on.
- Terminating decimals:** decimals that have a fixed number of decimal places; for example, 0.6 and 2.54
- Tessellations:** repeated patterns that cover a flat surface without any gaps
- Tetrahedron:** polyhedron with 4 faces. It is also called a triangular-based pyramid because it has a triangle at the bottom and a point at the top.
- Theoretical probability:** probability determined by predicting outcomes rather than by gathering data
- Time:** measurement used to work out how long we have been doing things or how long something has been happening. Common time units are:  
 $1 \text{ minute} = 60 \text{ seconds}$ ,  $1 \text{ hour} = 60 \text{ minutes}$ ,  
 $1 \text{ day} = 24 \text{ hours}$  and  $1 \text{ year} = 365 \text{ days}$  (366 in a leap year).
- Timelines:** lists that show when events occur
- Time zones:** regions on the Earth's surface that run from the North Pole to South Pole. The time everywhere in one zone is the same. The time of day in zones next to each other differs by 1 hour. There are 24 time zones around the Earth, giving 24 hours of time or 1 day. Generally, travelling east means you would set your watch ahead and travelling west means setting your watch back.
- Timetable:** lists that show when events are scheduled to happen; for example, the train timetable shows when and where trains should be each day.
- Top view:** see bird's eye view
- Topographic maps:** maps that show the elevations of features of an area such as mountains and valleys by the use of contour lines
- Transformation:** ordered set of directions that enables a shape to be moved from one place to another. At the start the shape is called the original object and after transformation it is called the image. Examples of transformations are translations, reflections and rotations.
- Translation:** horizontal (left/right) or vertical (up/down) movement
- Transversal:** a line that intersects a pair (a set) of parallel lines
- Trapezium:** quadrilateral in which one pair of opposite sides is parallel
- Traversable (networks):** networks that can be traced over without lifting the pen
- Trial:** number of times a probability experiment is conducted
- Triangle:** 2-dimensional, closed shape formed by three straight lines
- True bearings:** directions that are written as the number of degrees (3 digits) from north in a clockwise direction, followed by the word 'true' or 'T'; for example, due east would be  $090^\circ$  true or  $090^\circ$  T.
- Two-way table (probability):** displays the sample space of 2-step experiments in a logical manner
- Undecagon:** 11-sided polygon. It also has 11 angles.
- V (Roman numeral):** represents five
- Variable:** a symbol in an equation or expression that may take many different values
- Vertex (angles) vertices (plural):** point where two lines meet
- Vertex (shapes) vertices (plural):** point where 3 or more edges of a polyhedron meet
- Vertical:** upright. A vertical line is drawn straight up and down the page; that is, it is drawn at right angles (perpendicular) to the horizontal.
- Vertical axis:** the y-axis
- Vertically opposite angles:** special angles formed when two straight lines intersect. The two non-adjacent angles are called vertically opposite angles. These angles are equal in size.
- Vinculum:** horizontal bar separating the numerator from the denominator
- Volume:** amount of space a 3-dimensional object occupies. The units used are cubic units, such as cubic metres,  $\text{m}^3$ , and cubic centimetres,  $\text{cm}^3$ . A volume of  $1 \text{ m}^3$  is the space occupied by a cube of side length 1 m.
- Whole numbers:** counting numbers: 0, 1, 2, 3, 4, ...
- X (Roman numeral):** represents 10
- x-axis:** horizontal line with a scale that is part of the Cartesian plane
- y-axis:** vertical line with a scale that is part of the Cartesian plane

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