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CSE 2320-002

Homework 2

1.

a) is $2^{n+1} = O(2^n)$

=>

Let $f(n) = 2^{n+1}$, $g(n) = 2^n$

Computing limits of $\frac{f(n)}{g(n)}$ at infinity,

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2^{n+1}}{2^n} = \lim_{n\to\infty} 2^{n+1-n}$$

$$= \lim_{n\to\infty} 2^1$$

$$= \lim_{n\to\infty} 2 = 2$$; which is a constant

$\therefore$ Yes, $2^{n+1} = O(2^n)$ because there exists a positive constant $c\,(=2)$ such that $f(n) \le c\, g(n)$; for all $n \ge n_0$.

b) is $2^{2n} = O(2^n)$

=>

Let $f(n) = 2^{2n}$, $g(n) = 2^n$

Computing limits of $\frac{f(n)}{g(n)}$ at infinity,

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2^{2n}}{2^n} = \lim_{n\to\infty} \frac{(2^n)^2}{(2^n)^1}$$

$$= \lim_{n\to\infty} (2^n)^{2-1} = \lim_{n\to\infty} 2^n$$

Using exponent rule, "$2^n = e^{n\ln(2)}$"

$$= \lim_{n\to\infty} e^{n\ln(2)}$$

$$= \infty \ne 0 \text{ or constant}$$

$\therefore$ No, $2^{2n} \ne O(2^n)$ because the limit computed was valued at ∞ which does not fulfill the condition : $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$ or c.

2.

let $f(n) = \left(\frac{4}{9}\right)^0 + \left(\frac{4}{9}\right)^1 + \left(\frac{4}{9}\right)^2 + \cdots + \left(\frac{4}{9}\right)^n$. find θ for $f(n)$.

=> Here,

$f(n)$ represents a geometric series where the common ratio $(r) = \frac{4}{9}$ (i.e $0 < r < 1$)

$\therefore$ $f(n)$ can be written as the summation of the following:

$$f(n) = \sum_{k=0}^{n} r^k \quad (k = 0, 1, 2, 3 \ldots n).$$

$$= \sum_{k=0}^{n} \left(\frac{4}{9}\right)^k$$

$$= \frac{1}{1-r} \quad \left[\because \sum_{k=0}^{n} r^k \leq \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}\right]$$

$$= \frac{1}{1-(4/9)}$$

$$= \frac{9}{5}$$

$$= \theta(1)$$

$\therefore T(f(n)) = \theta(1)$