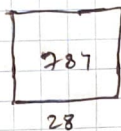


MNIST dataset.



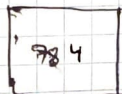
28

28

28x28

m training images

$$X = \begin{bmatrix} - & x^{(1)} & - \\ - & x^{(2)} & - \\ & \vdots & \\ - & x^{(n)} & - \end{bmatrix}^T$$



28x28

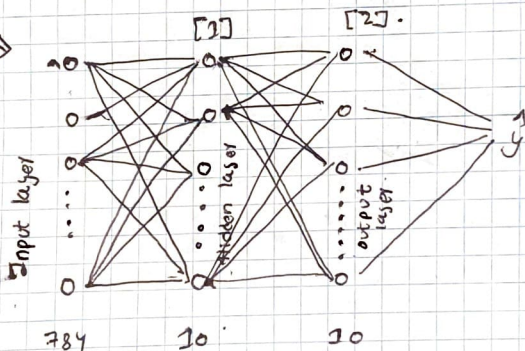


0, 1, 2, ..., 9

10 classes

$$= \begin{bmatrix} | & | & \dots & | \\ x^{(1)} & x^{(2)} & \dots & x^{(n)} \\ | & | & \dots & | \end{bmatrix}$$

Forward propagation



$$A^{[0]} = X \quad (284 \times m) \quad \dots (i)$$

$$A^{[1]} =$$

$$Z^{[1]} = W^{[1]} A^{[0]} + b^{[1]} \quad \dots (ii)$$

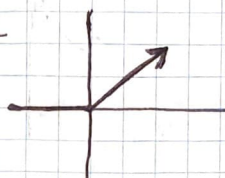
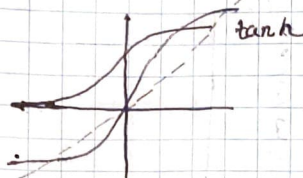
$$g(Z^{[1]}) =$$

$$\text{ReLU}(Z^{[1]}) \quad \dots (iii)$$

10	10	284	10
x	x	x	x
m	284	m	1

2nd layer is linear combination of input layers

rectified linear unit.



ReLU(x)

$$\rightarrow \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]} \dots (V)$$

$$\begin{matrix} 10 & 0 & 0 & 10 \Rightarrow 10 \times m \\ x & x & x & x \\ m & 10 & m & 1 \end{matrix}$$

$$A^{[2]} = \text{softmax}(Z^{[2]}) \dots (V)$$

Each of the 10 nodes correspond to each of the 10 digits that could be recognized. need a probabilities



output layer

softmax activation.

$$\begin{bmatrix} 1.3 \\ 5.1 \\ 2.2 \\ 0.7 \\ 1.1 \end{bmatrix}$$



$$\frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$

(v_i)

$$\begin{bmatrix} 0.2 \\ 0.90 \\ 0.05 \\ 0.01 \\ 0.02 \end{bmatrix}$$

Backward propagation

on (iv)

$$\delta z^{[2]} = A^{[2]} - y$$

$10 \times m$ $10 \times m$ $10 \times m$

one hot encode

$$y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\delta W^{[2]} = \frac{1}{m} \delta z^{[2]} A^{[1]T}$$

10×10 $10 \times m$ $m \times 10$

$$\delta W^{[2]} = \frac{1}{m} \sum \delta z^{[2]}$$

10×1 10×1

$$\delta z^{[1]} = W^{[2]T} \delta z^{[2]} * g'(z^{[1]})$$

$10 \times m$ 10×10 $10 \times m$ $10 \times m$

$$\delta W^{[2]} = \frac{1}{m} \delta z^{[2]} x^T$$

10×784 $10 \times m$ $m \times 784$

$$\delta b^{[2]} = \frac{1}{m} \sum \delta z^{[2]}$$

10×1 10×1

$\alpha = \text{learning rate}$

$$W^{[2]} = W^{[1]} - \alpha \delta W^{[2]}$$

$$b^{[2]} = b^{[1]} - \alpha \delta b^{[2]}$$

$$W^{[2]} = W^{[2]} - \alpha \delta W^{[2]}$$

$$b^{[2]} = b^{[2]} - \alpha \delta b^{[2]}$$

Forward propagation

Backward propagation

