Modern programming language translation to the theoretical model of Minsky Machine (Counter Machine).

(Tłumaczenie współczesnego języka programowania
do Maszyny Minsky'ego.)

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17 maja 2018

Abstract

Counter Machine (a machine with finite state set, two counters and input/output tape) is able to express any computations done by modern programming languages. This is the well-known theorem, just like computations performed using Turing Machine and means that anything written in a modern programming language is possible to translate into the theoretical model of Minsky Machine (as well as into Turing Machine).

My goal is to build automatic translation from a modern programming language (C++) to Minsky Machine.

TODO: polishabstract

Contents

Chapter 1

Introduction

1.1 Turing Machine

Modern computers and their abilities nowadays are complex and depend on many different factors to perform given computations, like processor technology, software optimizations within given processor model, input/output communication protocols and many others. This is why if we want to state or proof anything about the behaviour of computer programs then we need to find a common language and general theoretical model representing any type of "computer".

The model is known since 1936 when Alan Turing proposed something called **Turing Machine** to represent a machine that is able to execute any computations expressible in terms of computer program. It has all positive and negative properties which belong to modern programs, i.e. it is possible to create a program which will never stop or consume infinite amount of memory. The model does not allow to solve all the problems – it has got the same limitations in sense of creating algorithms like a regular computer.

Turing Machine is a finite description of an algorithm and uses potentially infinite tape as its memory – we have some amount of tape at the beginning, but we may extend this tape (glue additional tape cells) to the right end if necessary. All cells are either blank or contain a single letter – when machine starts its work there is an input word written on the tape, during computations the machine is allowed to change all the available cells and after it finishes (if it finishes at all) it can either *accept* or *reject* original input word.

As an example we can try to construct Turing Machine recognizing all binary palindromes of even length, which is formally set $A = \{ww^R : w \in \{0,1\}^*\}$ (where ^{R} means reversing given word).

Because we need to use finite description to define behaviour we say that the machine should check first letter a_1 with last one a_n – if these cells contain different let-

ters machine should reject the whole word, otherwise it should mark them as checked (i.e. write # symbol in their cells) and continue with a_2 and a_{n-1}. At the end we should have tape full of #s and the machine should accept given word. If there is just one unchanged cell left then it was a palindrome of odd length and we should reject it.

Turing Machine is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, P)$ where:

- Q is a finite set of states the machine can be within,
- Σ is a finite input alphabet (does not contain blank),
- Γ is a finite tape alphabet and $\Sigma \subset \Gamma$,
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is a finite transition function, L and R mean the machine should move one cell left or right respectively,
- q_0 is the initial state the machine starts within,
- P is a finite set of states which accept input word.

Configuration of a Turing Machine consists of $(w,\ q,\ k)$, where:

- $w \in \Sigma^*$ is a word initially written on the tape,
- q is a state of the machine is currently within,
- k is a cell number the machine is currently looking at.

Having a configuration we can precisely say what is the program execution progress. One configuration is a precise step of computation, so a sequence of configurations is called a **run** of a Turing Machine. This sequence may be finite or infinite the same way like a program may loop forever or finish within some number of steps. Each such run begins with a configuration where some word w is an input, $k = 0$ which is the leftmost cell on the tape and $q = q_0$ which is initial state defined in the description of given machine. If a run is finite and ends with configuration $(w_n,\ q_n,\ k_n)$ then we define the input word to be *accepted* if $q_n \in P$ (q_n is one of accepting states) and *rejected* otherwise.

1.2 Church-Turing thesis and Turing-completeness

Computers are used to perform calculations for us, more precicely we want them to execute some procedures or to perform some process and we call it an *algorithm* in its intuitive meaning. A good example could be a mathematical problem of generating prime numbers – it is fairly simple to think of a method of generating prime

numbers by checking lager and larger numbers and checking their all possible divisors. We are able to precisely say what this algorithm looks like and if we think for a while we would probably be able to express it in terms of Turing Machines. That is exactly what **Church-Turing thesis** states:

Hypothesis. *Intuitive definition of any algorithm is equivalent to some description of Turing Machine.*

This thesis allows us to think about defining algorithms in intuitive way while still staying within the world of Turing Machine programs.

It turns out that when designing any modern algorithms we use the same way of intuitive description of a procedure and then we are converting it into *implementation* which is a representation of given algorithm using a modern programming language. From thesis above we claim that most probably it would be possible to express it as a Turing Machine (but probably require lots of thought).

To avoid a painful process of expressing algorithms in an unhandy theoretical model we have a concept of **Turing-completeness**. All popular modern programming languages are proven to be Turing-complete which means that we are able to implement our algorithm in such language if and only if there exists some equivalent Turing Machine description.

1.3 Counter Machine

We have so far theoretical model – Turing Machine – which has the same power of expression as modern languages and it turns out there are many others we can describe and prove they are equivalent to Turing Machine (so to modern languages as well). One of such alternatives is **Counter Machine** (sometimes called **Minsky Machine**) which is really primitive in its construction.

Similarly to Turing Machine, Counter Machine has finite description – finite set of states Q and transition function δ defining when the machine may move from one state to another. Instead of the infinite tape there are 2 counters – each counter works like a glass of coins, the machine may throw a coin (or several coins) into the glass, check whether the glass is empty or take out a coin (only one and only if glass is not empty). Each counter is basically a non-negative integer, but the machine is not allowed to explicitly check its value, it just gets boolean information about emptiness.

There are a few alternative models – if we allow the machine to use 3 counters instead of 2 then we get equivalent model, the same for 4 or more counters. According to this fact, we can choose how many counters it is convenient for us to use. The only exception is Counter Machine with 1 counter which is not able to express everything a 2 Counter Machine can – TODO: jaką klasę języków możemy wyrazić?

1.4 Brainfuck

As we defined above Counter Machine is a minimalistic model of computations, the same way **Brainfuck** is a minimalistic programming language which recalls the definition of Turing Machine – program operates on a tape containing numbers, it is possible to explicitly read and change values in cells. A program operates on a data pointer which points to the current focus cell, so that it is able to freely move around. The main difference between Turing Machine and Brainfuck is lack of states and transitions, instead a program is a sequence of intructions that are executed one by one.

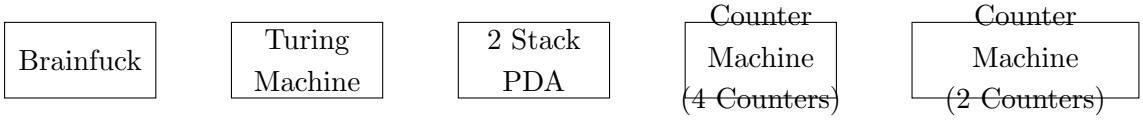
Usually Brainfuck implementations assume that tape length is around 30 000 cells which is enough in most cases. If we want to prove Turing-completeness though, this tape needs to be potentially infinite. Of couse, no modern machine is equipped with infinite memory, but we are able to extend it (with buying additional hardware), so this factor is omitted when proving Turing-completeness. It is not a problem in this thesis at all since the main goal is to show that anything written in modern programming language is possible to automatically translate into theoretical model.

1.5 Purpose of this thesis

This thesis should be a proof of theoretical concept contained in Church-Turing thesis – any algorithm or mathematical method we are able to come up with (as long as it is expressible in any programming language) is possible to execute on the most primitive theoretical model which is Counter Machine.

Implementation translates Brainfuck into Counter Machine model and consists of a few separate parts which are completely separate translations between more and more primitive theoretical models. It is possible to access any transitional (temporary) programs created during the whole process.

The plan for translation that shows what is generated as transitional models:



TODO: Co to są ~~maszyny Turinga, Mińskiego, Brainfuck, Turning-zupełność, hipoteza Churcha~~

TODO: O czym jest ta praca - lista kontrybucji

TODO: ~~Po co jest ta praca — „proof of concept”; „walory dydaktyczne”,~~ ...

TODO: „Related work”

TODO: Plan pracy

...

Chapter 2

Preliminaries

2.1 Brainfuck

Brainfuck is a programming language with only 8 statements and execution of any program is done using a finite sequence of memory cells. Statements operate on these cells using data pointer — initially the pointer is set on the leftmost cell in the sequence.

Statements:

- `>` moves the data pointer one cell to the right,
- `<` moves the data pointer one cell to the left,
- `+` increase by 1 the value held in the cell under the data pointer,
- `-` decrease by 1 the value held in the cell under the data pointer,
- `.` print to stdout the character that is under the data pointer,
- `,` read from stdin a character and write it to the cell under the data pointer,
- [beginning of a loop with condition checking whether value under the data pointer is zero. If it is then execution jumps to the matching],
-] closing symbol of a loop — execution jumps to the beginning of the loop (matching [) and then check for zero value under the data pointer is performed.

It is allowed to use any other characters within the code, but anything else than the 8 listed above are ignored — it is useful for creating comments in the code.

An example code printing "Hello World!":

```
++++++++++
```

```
[
>+++++++>++++++++++>+++>+<<<<-
] We set up the values in few cells for future use.
>++.               prints 'H'
>+.                prints 'e'
+++++++.           prints 'l'
.                  prints 'l'
+++.               prints 'o'
>++.               prints space
<<+++++++++++++++. prints 'W'
>.                 prints 'o'
+++.               prints 'r'
------.            prints 'l'
--------.          prints 'd'
>+.                prints '!'
>.                 prints newline character
```

2.2 Turing Machine

2.2.1 Instructions

Turing Machine consists of a finite number of states, changes between them and potentially infinite tape, on which it is able to write and read symbols. It has no code sequence, a program is a set of state changes and definition of an initial state. There is special final state `END` that does not need to be defined to be used.

There is a predefined set of symbols used on the tape which is whole ASCII character set extended with the same amount of additional (non-ASCII) symbols. Regular ASCII characters have 7 bits (numbers 0-127), so tape symbols have 8 bits allowing to hold regular ASCII and special characters (numbers 128-255).

There are few special definitions (so far using regular ASCII, but it will be changed to use special characters instead):

- `BLANK` which represents empty field on the tape
- `*` (ALL) defines all characters (both ASCII and special)
- `#` (NOTHING) means no character which is used to say that we do not want to write anything on tape during this state change
- `&` (NON-ZERO) defines all characters except 0
- `0` (ZERO) represents zero that is used for conditional jumps (jump zero)

- `>` (NEXT-CHAR) allows writing on the tape next character (increased by one), i.e. writes `g` if on the tape was `f`
- `<` (PREV-CHAR) similarly as above but writes the previous character, i.e. writes `e` when seen `f`

Defining initial state:

```
START: state_name
```

Each state change looks as follows:

```
state_name symbol(s) -> target_state head_move symbol_to_write
```

`symbol(s)` is ASCII character including special definitions (currently all special definitions use regular ASCII so that it is easy to see in a standard text file), in future it will allow getting non-ASCII special characters as well.

`head move` is one of `L`, `R` or `-` which steer the head to go left, right or stay in place, respectively.

`symbol to write` is any (currently ASCII) symbol. Currently, there is no mechanism to prevent writing any symbol from special definitions, but it will cause an undefined behaviour of the machine. The only symbol from special definitions that is allowed to appear as `symbol_to_write` is `#` (NOTHING) meaning that symbol on the tape should not change.

2.2.2 Extensions of the theoretical model

Standard input/output handling is done by allowing to read or write single ASCII character from/to stdin/stdout. It is possible to add additional reading or writing before moving the head. It is done by modifying change symbol `->` in the state change definition.

- Reading is done with `->*`, i.e. `state1 A ->* state2 R NOTHING` which means when seen symbol `A` in state `state1` we read from stdin one character, overwrite `A` to read value and move head one field right.
- Writing is done similarly with `->^`, i.e. `state1 A ->^state2 R NOTHING` which will print symbol `A` on stdout and move the head to the right.

2.2.3 Example

A program that reads a letter from stdin, writes this letter into stdout and if the written letter was `B` or `b` then prints `.` at the end as well, otherwise finishes.

```
START: s1
s1 ALL ->* s2 - NOTHING
s2 ALL ->^ s3 - NEXT_CHAR
s3 ALL -> s4 - NOTHING
s4 b ->^ s5 R NOTHING
s4 B ->^ s5 R NOTHING
s5 ALL -> s6 - .
s6 ALL ->^ s7 - NOTHING
s7 ALL -> END - NOTHING
```

Note: If there is no state change defined for given configuration (nothing matches) then it is assumed that machine gets to `END` state. Because of this in the above example, last instruction is not necessary.

2.3 2 Stack Pushdown Automaton

2 Stack Pushdown Automaton consists of two stacks, input tape and definition of states and transitions between them. Each transition looks as follows:

```
state_name left_pattern right_pattern ->
    target_state left_stack_items right_stack_items
```

Explanation:

- `left_pattern` is symbol or pattern that should be matched for the symbol at the top of the left stack
- `right_pattern` is the same as above, but for right stack
- `left_stack_items` are the list of items that should be pushed to the left stack before moving to `target_state`. It might be a single letter `"a"`, a sequence of letters `"abc"` or sequence of letters and references, i.e. `("a" + ORIG_LEFT + "b")` where `ORIG_LEFT` means the letter we read from the left stack (the one matched in `left_pattern`). Note: If `+` is used it is required to put the whole sequence in parenthesis
- `right_stack_items` are the same as above, but for items to be pushed into right stack

Special references and definitions in transitions:

- `ORIG_LEFT` is the letter taken from the left stack
- `ORIG_RIGHT` is the letter taken from right stack

- `INPUT_CHAR` is the letter taken from input tape (only in input transition type - see section below)
- `NOTHING` may be used as `left_stack_items` or `right_stack_items` and means that nothing is pushed into left/right stack
- `END` is special state name into which the transition is made if no other transition is specified
- `$` is the symbol of the empty stack

2.3.1 Extensions of the theoretical model

Input/Output handling is done similarly to Turing Machine - We change `->` in transition to `->*` or `->^`.

When defining input transition it is allowed to use `INPUT_CHAR` in any items to be pushed into left/right stack. Here is an example that takes a character from input tape and pushes it into the left stack (and ignores symbols taken from both stacks).

```
state1 ALL ALL ->* state2 INPUT_CHAR NOTHING
```

When defining output transition we **must** specify what character is printed with adding `Output: <letter>` at the very back of transition definition. An example that prints letter taken from the left stack (and ignores what was taken from right stack):

```
state1 ALL ALL ->^ state2 NOTHING NOTHING Output: ORIG_LEFT
```

An example that prints letter "a" (and ignores what was taken from stacks):

```
state1 ALL ALL ->^ state2 NOTHING NOTHING Output: "a"
```

Important note: Order of defining transition matters. If patterns do not match a distinct set of letters then the transition that appeared first is applied.

2.3.2 Example

An example (equivalent to the example from Turing Machine):

```
START: init_state
init_state $ $ -> s1 BLANK NOTHING
s1 ALL ALL ->* s2 INPUT_CHAR ORIG_RIGHT
```

```
s2 ALL ALL ->^ s3 ORIG_LEFT ORIG_RIGHT Output: ORIG_LEFT
s3 b $ -> s4 (ORIG_LEFT + BLANK) $
s3 b ALL -> s4 (ORIG_LEFT + ORIG_RIGHT) NOTHING
s3 B $ -> s4 (ORIG_LEFT + BLANK) $
s3 B ALL -> s4 (ORIG_LEFT + ORIG_RIGHT) NOTHING
s4 ALL ALL -> s5 "." ORIG_RIGHT
s5 ALL ALL ->^ s6 ORIG_LEFT ORIG_RIGHT Output: ORIG_LEFT
s6 ALL ALL -> END ORIG_LEFT ORIG_RIGHT
```

2.4 Counter Machine (4 counters)

Counter Machine consists of 4 counters each holding non-negative integer, a finite number of states and transitions between them. Each transition looks as follows:

```
state_name (pattern pattern pattern pattern) ->
    target_state (number number number number)
```

Where:

- `state_name` is the state in which counter machine needs to be within for this transition to be applied,
- `pattern` is one of values `0`, `1` or `_` meaning empty counter, non-empty counter and any counter state and defines what is expected state of the given counter - the transition may be applied only when all counter states are matched (Notice that symbol `_` is matched with any state of the counter),
- `target state` is the state in which machine will be after applying the transition,
- `number` is an integer from the range `[-1, MAX_INT]` specifying what should be added to given counter - it is allowed to decrease counter by 1 only, but it is possible to increase it by any number that can be stored in regular integer type.

Note: If there are many transitions that may be applied in given state matching all counters **the first one** is applied. It means that order of defining transitions matters.

It is required to give an initial state of the machine with the following statement:

```
START: state_name
```

2.4.1 Extensions of the theoretical model

Input/Output operations fit in the schema of using counters - input and output are additional counters which transitions may use in a similar way as counters are used.

Input transition is defined as follows:

```
state_name (counters) input pattern ->*
    target_state (numbers) input_operation
```

Where:

- `input_pattern` is one of `0`, `1` or `_` (same as counter pattern) and specifies what should be the state of input counter for this transition to be applied,
- `input_operation` specifies what action should be performed on input counter and is one of `LOAD`, `-1` or `NOOP` meaning respectively loading a character from stdin into the input counter, decrease input counter by `1` and leaving input counter untouched.

This transition reads from stdin into input counter:

```
state1 (_ _ _ _) _ ->* state2 (0 0 0 0) LOAD
```

These transitions read the value from input counter and store it in first counter:

```
state1 (_ _ _ _) 1 ->* state1 (1 0 0 0) -1
state1 (_ _ _ _) 0 ->* state2 (0 0 0 0) NOOP
```

Note: It is assumed that transition may just decrease the input counter and is not allowed to increase its value directly.

Output transition is defined as follows:

```
state_name (counters) ->^ target_state (numbers) Output: output_operation
```

Where:

- `output_operation` specifies what should be performed on output counter and may be one of `FLUSH` or non-negative number, meaning respectively pushing counter to stdout and modifying the value stored in the counter by the given number.

These transitions print character stored in first counter:

```
state1 (1 _ _ _) ->^ state1 (-1 0 0 0) Output: 1
state1 (0 _ _ _) ->^ state2 (0 0 0 0) Output: FLUSH
```

Note: It is assumed that transition may just increase the output counter and is not allowed to decrease its value directly.

2.4.2 Example

Code that reads a character from stdin doubles its ASCII value and prints the result.

```
START: s1
s1 (_ _ _ _) _ ->* s2 (0 0 0 0) LOAD
s2 (_ _ _ _) 1 ->* s2 (1 0 0 0) -1
s2 (_ _ _ _) 0 ->* s3 (0 0 0 0) NOOP
s3 (1 _ _ _) -> s3 (-1 2 0 0)
s3 (0 _ _ _) -> s4 (0 0 0 0)
s4 (_ 1 _ _) ->^ s4 (0 -1 0 0) Output: 1
s4 (_ 0 _ _) ->^ END (0 0 0 0) Output: FLUSH
```

2.5 Counter Machine (2 counters)

Counter Machine with 2 counters has the same definition as Counter Machine with 4 counters, but it is allowed to use only 2 counters, so transitions become of the form:

```
state_name (pattern pattern) -> target_state (number number)
```

Input/Output is handled the same way it is handled in Counter Machine with 4 counters.

Chapter 3

Theoretical underpinnings

TODO: O tym, jak te redukcje działają

Chapter 4

Implementation

4.1 Ogólny wstęp

TODO: Uzasadnienie doboru narzędzi

TODO: Najciekawsze wyzwania

TODO: Wszystko, co było ciekawe

4.2 Szczegóły techniczne

TODO: Co jest gdzie, jak to się uruchamia, itd. / skrócona instrukcja użytkownika

TODO: Ograniczenia implementacji

4.3 Przykłady

Dużo przykładów...

4.4 Ewaluacja

Porównanie czasów

Tabelka

4.5 Optymalizacje

Chapter 5

Podsumowanie, wnioski, możliwości kontynuowania

Appendix A

Szczegółowy sposób instalacji i konfiguracji lub dostępu do działającego systemu oraz podręcznik użytkownika systemu.