Inside-Out: STL

How to use it wisely?

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Presentation plan

1. Introduction
   - Time complexity
   - Vector
   - Sort?
2. Balanced trees
   - BST
   - AVL
3. Hash tables
   - Idea
   - Properties and limitations
   - Workarounds

Time complexity

Purpose

Time complexity is a tool to measure the efficiency of our algorithm.

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Usually defined with *big-O* notation:

- $O(N)$,
- $O(N^2)$,
- $O(\log N)$,
- $O(N \cdot \log N)$,
- $O(\sqrt{N})$,

Task: find not paired item

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Given an array of integers, find the only number that does **not** have a pair.

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Example

For array $[2, 3, 7, 7, 2, 3, 2]$ the answer is 2.

Task: find not paired item

```
int f(const vector<int>& t) {
    for (int selected_item : t) {
        int cnt = 0;
        for (int item : t)
            if (item == selected_item)
                cnt++;
        if (cnt % 2 == 1)
            return selected_item;
    }
    throw "All items have pairs!";
}
```

What is the time complexity?

Task: find not paired item

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    }
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```

What is the time complexity? $O(N^2)$

Task: find not paired item

```
int f(const vector<int>& t) {
    sort(t.begin(), t.end());
    int cnt = 0, prev = -1;
    for (int i = 0; i < t.size(); ++i) {
        if (prev == t[i]) cnt++;
        else {
            if (cnt % 2 == 1) return prev;
            prev = t[i];
            cnt = 0;
        }
    }
    if (cnt % 2 == 1) return prev;
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}
```

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What is the time complexity? $O(N \cdot \log N)$

Task: find not paired item

```
int f(const vector<int>& t) {
    int result = 0;
    for (int item : t)
        result ^= item;
    if (result > 0)
        return result;
    throw "All items have pairs!";
}
```

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}
```

What is the time complexity? $O(N)$

Task: find not paired item

```
int f(const vector<int>& t) {
    map<int,int> m;
    for (int item : t)
        m[item]++;
    for (auto it : m) {
        if (it.second % 2 == 1)
            return it.first;
    }
    throw "All items have pairs!";
}
```

What is the time complexity?

Task: find not paired item

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```

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Task: find not paired item

```
int f(const vector<int>& t) {
    unordered_map<int,int> m;
    for (int item : t)
        m[item]++;
    for (auto it : m) {
        if (it.second % 2 == 1)
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What is the time complexity? $O(N)$

Vector – time complexity

Where do I find the information about the time complexity?

Vector – time complexity

Where do I find the information about the time complexity?

Documentation!

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C++ / Containers library / std::vector

std::vector

Defined in header <vector>

```
template<
    class T,
    class Allocator = std::allocator<T>
> class vector;
```
(1)

```
namespace pmr {
    template <class T>
    using vector = std::vector<T, std::pmr::polymorphic_allocator<T>>;
}
```
(2) (since C++17)

1) `std::vector` is a sequence container that encapsulates dynamic size arrays.

2) `std::pmr::vector` is an alias template that uses a polymorphic allocator

The elements are stored contiguously, which means that elements can be accessed not only through iterators, but also using offsets to regular pointers to elements. This means that a pointer to an element of a vector may be passed to any function that expects a pointer to an element of an array. (since C++03)

The storage of the vector is handled automatically, being expanded and contracted as needed. Vectors usually occupy more space than static arrays, because more memory is allocated to handle future growth. This way a vector does not need to reallocate each time an element is inserted, but only when the additional memory is exhausted. The total amount of allocated memory can be queried using `capacity()` function. Extra memory can be returned to the system via a call to `shrink_to_fit()`. (since C++11)

Reallocations are usually costly operations in terms of performance. The `reserve()` function can be used to eliminate reallocations if the number of elements is known beforehand.

The complexity (efficiency) of common operations on vectors is as follows:

- Random access - constant $O(1)$
- Insertion or removal of elements at the end - amortized constant $O(1)$
- Insertion or removal of elements - linear in the distance to the end of the vector $O(n)$

`std::vector` (for T other than `bool`) meets the requirements of *Container*, *AllocatorAwareContainer*, *SequenceContainer*, *ContiguousContainer* (since C++17) and *ReversibleContainer*.

Vector – time complexity

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C++ / Containers library / std::vector

std::vector<T,Allocator>::erase

`iterator erase( iterator pos );`	(1)	(until C++11)
`iterator erase( const_iterator pos );`		(since C++11)
`iterator erase( iterator first, iterator last );`	(2)	(until C++11)
`iterator erase( const_iterator first, const_iterator last );`		(since C++11)

Erases the specified elements from the container.

1) Removes the element at pos.

2) Removes the elements in the range `[first, last)`.

Invalidates iterators and references at or after the point of the erase, including the `end()` iterator.

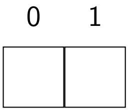
The iterator pos must be valid and dereferenceable. Thus the `end()` iterator (which is valid, but is not dereferencable) cannot be used as a value for pos.

The iterator `first` does not need to be dereferenceable if `first==last`: erasing an empty range is a no-op.

Complexity

Linear: the number of calls to the destructor of T is the same as the number of elements erased, the assignment operator of T is called the number of times equal to the number of elements in the vector after the erased elements

Vector – internal implementation



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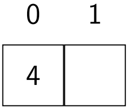

Vector – internal implementation

0	1
4	12

Vector – internal implementation

0	1
4	12

0	1	2	3

Vector – internal implementation

0	1
4	12

0	1	2	3
4			

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0	1
4	12

0	1	2	3
4	12		

Vector – internal implementation

0	1	2	3
4	12		

Vector – internal implementation

0	1	2	3
4	12	1	

Vector – internal implementation

0	1	2	3
4	12	1	5

Vector – internal implementation

0	1	2	3	4	5	6	7

0	1	2	3
4	12	1	5

Vector – internal implementation

0	1	2	3	4	5	6	7
4							

0	1	2	3
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0	1	2	3	4	5	6	7
4	12						

0	1	2	3
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4	12	1					

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0	1	2	3	4	5	6	7
4	12	1	5	87			

Vector – time complexity

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Vector – time complexity

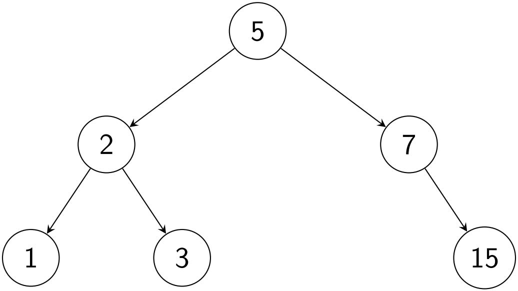
- *insert (back)* – $O(1)$ (expected),
- *delete (back)* – $O(1)$ (expected),
- *lookup (index)* – $O(1)$,
- *insert (middle)* – $O(N)$,
- *delete (middle)* – $O(N)$,
- *find (value)* – $O(N)$.

Binary search tree

Long long ago, before c++11 ...

Binary search tree

Long long ago, before c++11 . . . all sets were based on the binary search trees.



AVL vs. BST

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AVL vs. BST – time complexity

BST

- *insert* – $O(N)$,
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- *lookup* – $O(N)$.

AVL vs. BST – time complexity

BST

- *insert* – $O(N)$,
- *delete* – $O(N)$,
- *lookup* – $O(N)$.

AVL

- *insert* – $O(\log N)$,
- *delete* – $O(\log N)$,
- *lookup* – $O(\log N)$.

Hashing – why do we need it?

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What if $|U|$ is small (i.e. 1 000 000)?

What if $|U|$ is large (i.e. 10^{18})?

...then we need *hashing*!

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- f needs to be fast to compute,
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- what if we have a *collision* ($f(x) = f(y)$)?
- what if f does **not** distribute elements universally over the available cells?

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Example hash function for `std::pair<T1,T2>`:

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struct pair_hash
{
    template <class T1, class T2>
    std::size_t operator()
        (const std::pair<T1, T2> &pair) const
    {
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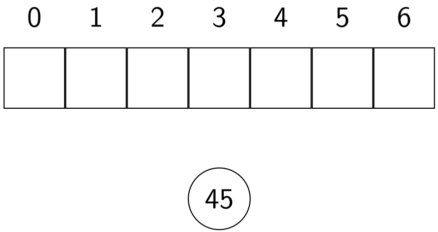
```
pokorska@thinkpad:~/wrocpp/wrocpp$ make blow
g++     blow.cpp    -o blow
pokorska@thinkpad:~/wrocpp/wrocpp$ ./blow
x = 107897: 361.175 seconds, sum = 2666686666700000
x = 126271: 0.078 seconds, sum = 2666686666700000
pokorska@thinkpad:~/wrocpp/wrocpp$
```

Open addressing

0	1	2	3	4	5	6

Open addressing



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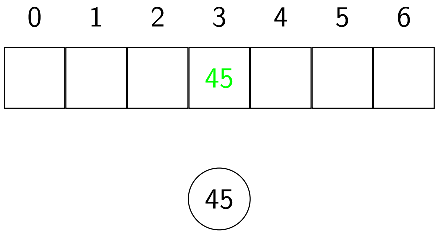
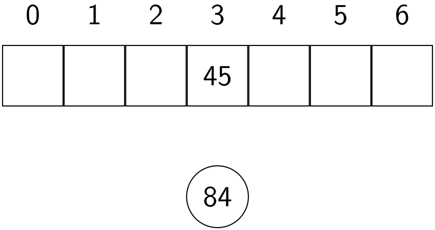


Open addressing

0	1	2	3	4	5	6
			45			

Open addressing



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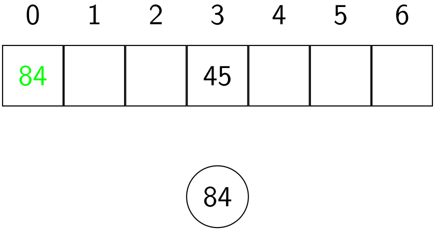


Open addressing

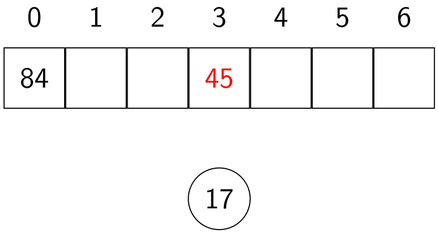
0	1	2	3	4	5	6
84			45			

Open addressing

0	1	2	3	4	5	6
84			45			

17

Open addressing



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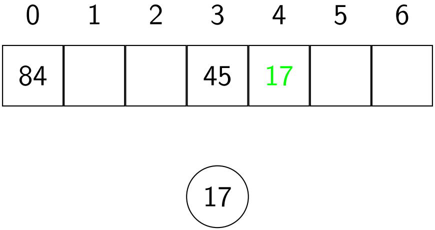

Open addressing

0	1	2	3	4	5	6
84			45	17		

Open addressing

0	1	2	3	4	5	6
84			45	17		

3

Open addressing

0	1	2	3	4	5	6
84			45	17		

3

Open addressing

0	1	2	3	4	5	6
84			45	17		

3

Open addressing

0	1	2	3	4	5	6
84			45	17	3	

3

Open addressing

0	1	2	3	4	5	6
84			45	17	3	

Cuckoo-hashing