Inside-Out: STL

How to use it wisely?

Jadwiga Pokorska

TietoEvry

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Presentation plan

1. Introduction
   - Time complexity
   - Vector
2. Balanced trees
   - BST
   - AVL
3. Hash tables
   - Idea
   - Properties and limitations
   - Workarounds

Time complexity

Purpose

Time complexity is a tool to measure the efficiency of our algorithm.

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Usually defined with *big-O* notation:

- $O(N)$,
- $O(N^2)$,
- $O(\log N)$,
- $O(N \cdot \log N)$,
- $O(\sqrt{N})$,

Task: find not paired item

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Given an array of integers, find the only number that does **not** have a pair.

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Given an array of integers, find the only number that does **not** have a pair.

Example

For array $[2, 3, 7, 7, 2, 3, 2]$ the answer is 2.

Task: find not paired item

```
int find_not_paired(const vector<int>& t) {
    for (int selected_item : t) {
        int cnt = 0;
        for (int item : t)
            if (item == selected_item)
                cnt++;
        if (cnt % 2 == 1)
            return selected_item;
    }
    return 0;
}
```

What is the time complexity?

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        if (cnt % 2 == 1)
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    }
    return 0;
}
```

What is the time complexity? $O(N^2)$

Task: find not paired item

```
int find_not_paired(const vector<int>& t) {
    sort(t.begin(), t.end());
    int cnt = 0, prev = -1;
    for (int i = 0; i < t.size(); ++i) {
        if (prev == t[i]) cnt++;
        else {
            if (cnt % 2 == 1) return prev;
            prev = t[i];
            cnt = 0;
        }
    }
    if (cnt % 2 == 1) return prev;
    return 0;
}
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            prev = t[i];
            cnt = 0;
        }
    }
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    return 0;
}
```

What is the time complexity? $O(N \cdot \log N)$

Task: find not paired item

```
int find_not_paired(const vector<int>& t) {
    int result = 0;
    for (int item : t)
        result ^= item;
    if (result > 0)
        return result;
    return 0;
}
```

What is the time complexity?

Task: find not paired item

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int find_not_paired(const vector<int>& t) {
    int result = 0;
    for (int item : t)
        result ^= item;
    if (result > 0)
        return result;
    return 0;
}
```

What is the time complexity? $O(N)$

Task: find not paired item

```
int find_not_paired(const vector<int>& t) {
    map<int,int> m;
    for (int item : t)
        m[item]++;
    for (auto it : m) {
        if (it.second % 2 == 1)
            return it.first;
    }
    return 0;
}
```

What is the time complexity?

Task: find not paired item

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int find_not_paired(const vector<int>& t) {
    map<int,int> m;
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What is the time complexity? $O(N \cdot \log N)$

Task: find not paired item

```
int find_not_paired(const vector<int>& t) {
    unordered_map<int,int> m;
    for (int item : t)
        m[item]++;
    for (auto it : m) {
        if (it.second % 2 == 1)
            return it.first;
    }
    return 0;
}
```

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What is the time complexity? $O(N)$

Vector – time complexity

Where do I find the information about the time complexity?

Vector – time complexity

Where do I find the information about the time complexity?

Documentation!

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C++ / Containers library / std::vector

std::vector

Defined in header <vector>

```
template<
    class T,
    class Allocator = std::allocator<T>
> class vector;
```
(1)

```
namespace pmr {
    template <class T>
    using vector = std::vector<T, std::pmr::polymorphic_allocator<T>>;
}
```
(2) (since C++17)

1) `std::vector` is a sequence container that encapsulates dynamic size arrays.

2) `std::pmr::vector` is an alias template that uses a polymorphic allocator

The elements are stored contiguously, which means that elements can be accessed not only through iterators, but also using offsets to regular pointers to elements. This means that a pointer to an element of a vector may be passed to any function that expects a pointer to an element of an array. (since C++03)

The storage of the vector is handled automatically, being expanded and contracted as needed. Vectors usually occupy more space than static arrays, because more memory is allocated to handle future growth. This way a vector does not need to reallocate each time an element is inserted, but only when the additional memory is exhausted. The total amount of allocated memory can be queried using `capacity()` function. Extra memory can be returned to the system via a call to `shrink_to_fit()`. (since C++11)

Reallocations are usually costly operations in terms of performance. The `reserve()` function can be used to eliminate reallocations if the number of elements is known beforehand.

The complexity (efficiency) of common operations on vectors is as follows:

- Random access - constant $O(1)$
- Insertion or removal of elements at the end - amortized constant $O(1)$
- Insertion or removal of elements - linear in the distance to the end of the vector $O(n)$

`std::vector` (for T other than `bool`) meets the requirements of *Container*, *AllocatorAwareContainer*, *SequenceContainer*, *ContiguousContainer* (since C++17) and *ReversibleContainer*.

Vector – time complexity

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C++ / Containers library / std::vector

std::vector<T,Allocator>::erase

`iterator erase( iterator pos );`	(1)	(until C++11)
`iterator erase( const_iterator pos );`		(since C++11)
`iterator erase( iterator first, iterator last );`	(2)	(until C++11)
`iterator erase( const_iterator first, const_iterator last );`		(since C++11)

Erases the specified elements from the container.

1) Removes the element at pos.

2) Removes the elements in the range `[first, last)`.

Invalidates iterators and references at or after the point of the erase, including the `end()` iterator.

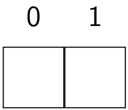
The iterator pos must be valid and dereferenceable. Thus the `end()` iterator (which is valid, but is not dereferencable) cannot be used as a value for pos.

The iterator `first` does not need to be dereferenceable if `first==last`: erasing an empty range is a no-op.

Complexity

Linear: the number of calls to the destructor of T is the same as the number of elements erased, the assignment operator of T is called the number of times equal to the number of elements in the vector after the erased elements

Vector – internal implementation



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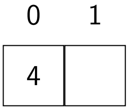

Vector – internal implementation

0	1
4	12

Vector – internal implementation

0	1
4	12

0	1	2	3

Vector – internal implementation

0	1
4	12

0	1	2	3
4			

Vector – internal implementation

0	1
4	12

0	1	2	3
4	12		

Vector – internal implementation

0	1	2	3
4	12		

Vector – internal implementation

0	1	2	3
4	12	1	

Vector – internal implementation

0	1	2	3
4	12	1	5

Vector – internal implementation

0	1	2	3	4	5	6	7

0	1	2	3
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Vector – internal implementation

0	1	2	3	4	5	6	7
4							

0	1	2	3
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4	12						

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0	1	2	3	4	5	6	7
4	12	1	5	87			

Vector – time complexity

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- *insert (back)* – $O(1)$ (expected),
- *delete (back)* – $O(1)$,
- *lookup (index)* – $O(1)$,
- *insert (middle)* – $O(N)$,
- *delete (middle)* – $O(N)$,
- *find (value)* – $O(N)$.

Vector – time complexity

- *insert (back)* – $O(1)$ (expected),
- *delete (back)* – $O(1)$,
- *lookup (index)* – $O(1)$,
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- *delete (middle)* – $O(N)$,
- *find (value)* – $O(N)$.

Note: vector does not shrink by itself.

Insert complexity proof.

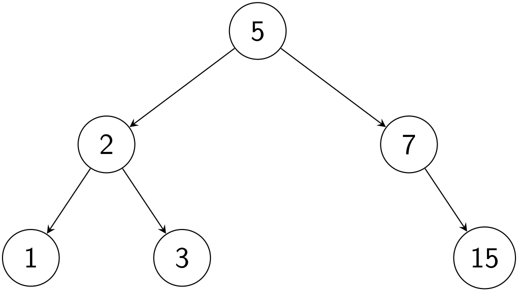
Proof!

Binary search tree

Long long ago, before c++11 . . .

Binary search tree

Long long ago, before c++11 ... all sets were based on the binary search trees.



AVL vs. BST

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AVL vs. BST – worst case time complexity

BST

- *insert* – $O(N)$,
- *delete* – $O(N)$,
- *lookup* – $O(N)$.

AVL vs. BST – worst case time complexity

BST

- *insert* – $O(N)$,
- *delete* – $O(N)$,
- *lookup* – $O(N)$.

AVL

- *insert* – $O(\log N)$,
- *delete* – $O(\log N)$,
- *lookup* – $O(\log N)$.

AVL vs. BST – worst case time complexity

BST

- *insert* – $O(N)$,
- *delete* – $O(N)$,
- *lookup* – $O(N)$.

AVL

- *insert* – $O(\log N)$,
- *delete* – $O(\log N)$,
- *lookup* – $O(\log N)$.

Note

`std::set` and `std::map` are internally using Red-Black Trees which have the same time complexity as AVL, but different internal constraints to ensure balancing.

Hashing – why do we need it?

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U – universe of numbers that may appear in the data.

What if $|U|$ is small (i.e. $1\,000\,000$)?

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...then we need *hashing*!

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What could possibly go wrong?

- what if we have a *collision* $(f(x) = f(y))$?
- what if f does **not** distribute elements uniformly over the available cells?

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Example hash function for `std::pair<T1,T2>`:

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struct pair_hash
{
    template <class T1, class T2>
    std::size_t operator()
        (const std::pair<T1, T2> &pair) const
    {
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    {
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Code!

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Code!

```
pokorska@thinkpad:~/wrocpp/wrocpp$ make blow
g++     blow.cpp   -o blow
pokorska@thinkpad:~/wrocpp/wrocpp$ ./blow
x = 107897: 361.175 seconds, sum = 2666686666700000
x = 126271: 0.078 seconds, sum = 2666686666700000
pokorska@thinkpad:~/wrocpp/wrocpp$ 
```

Set vs. unordered set in practice

Code!

Set vs. unordered set in practice

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```
pokorjad:~/wrocpp$ ./set_vs_unordered_set
set: 66.966 seconds
set lookup: 49.879 seconds, elems found: 10005884
unordered set: 43.651 seconds
set lookup: 15.091 seconds, elems found: 10005884
pokorjad:~/wrocpp$
```

Open addressing

Hash function: $hash(x) = x \bmod 7$.

0	1	2	3	4	5	6

Open addressing

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0	1	2	3	4	5	6

45

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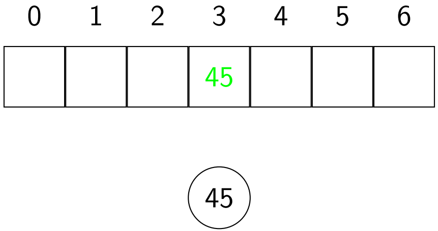

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84

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17

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3

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84			45	17		

3

Open addressing

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Cuckoo-hashing (optional)

Idea of how to make the lookup in $O(1)$ in worst case (not expected).