Problem Set 3

AA279D: Dyn, Nav, Ctrl of DSS
Spring Quarter 2022/2023
Due: April 26, 2023, Wed, 12:00AM PT
Prof. Simone D'Amico

Submission Instructions

Please briefly document all tasks outlined below in a report which will grow during the course. You should include a table with change logs since the last submission, and an index for sections at the beginning. Please submit your report as a single PDF file to the course Canvas website. It should include narrative, plots, tables, code, and interpretations. You should use typesetting software like LaTeX or Microsoft Word to produce your document. Do not submit extra files.

Topics

Week 3. Continuation of project. Linearized relative orbit motion in Cartesian coordinates.

Problem 1: We are Close in Near-Circular Orbits

We would like to propagate the relative spacecraft motion according to the HCW equations. To this end, you are asked to perform the following analyses.

a) Come up with a suitable set of initial conditions for your orbit. You should start with the initial conditions for absolute and relative states determined in the previous problem set and modify them to accommodate the assumptions made by the HCW equations. In other words, the separations should be small relative to the distance from the primary attractor's center (e.g., $\rho \approx 0.001\ r_0$), with near-zero eccentricity (e.g. $e \approx 0.001$). Note that if your previous set of initial conditions satisfies these assumptions, there is no need to change them. Otherwise, be sure to justify your final selection.
b) Use your new set of initial conditions to compute the following at $t_0 = 0$:
   i. Inertial position and velocity (e.g., ECI), and orbital elements of chief and deputy
   ii. Relative position and velocity (expressed in the chief RTN frame, with time-derivatives taken in the RTN frame), and orbit element differences between deputy and chief
c) Using the parameters from part (b), compute the six integration constants of the HCW equations (e.g., notated as c_1 to c_6)
d) Propagate the relative state in Simulink using the standard solution of HCW expressed in rectilinear coordinates over 15 orbit periods. Plot the resulting relative position and velocity in 3D and in the TR, NR, and TN planes (first letter indicates x-axis, second letter indicates y-axis, use axis equal and grid).
e) Discuss whether the general behavior matches your expectations given the initial conditions, the integration constants, and the applied orbit element differences. Is

the relative motion bounded as expected from $\delta a = 0$ (energy matching condition). If not, why? How would you mitigate the phenomenon?

Problem 2: We are Close in Eccentric Orbits

Now that we have developed some intuition via mathematical analysis, let us proceed with a simulation to further understand these dynamics models:

a) Let the initial conditions for absolute and relative states be determined by Problem 1, however now you should increase the eccentricity of the reference orbit to a value of at least 0.1. Make sure that the initial conditions lie within the range of validity of the Tschauner-Hempel equations and that the resulting motion is bounded. In other words, the maximum separation between spacecraft should be small relative to the minimum distance from the primary attractor center (e.g., $\rho \approx 0.001\, r_0$) and the chief and deputy spacecraft should have equal semi-major axes. Justify your final selection of initial conditions.

b) Based on the initial conditions chosen in part (a), compute the exact values of the 6 integration constants of the YA solution (i.e. do not use an approximation). *Hint*: Section 5 of Alfriend may offer some insight into this problem.

c) Propagate the relative position and velocity using the YA solution over 15 orbits using true anomaly as the independent variable. Plot the resulting relative position and velocity in 3D and in the TR, NR, and TN planes (first letter indicates x-axis, second letter indicates y-axis, use axis equal and grid).

d) Are the trends obtained in part (c) according to expectations given the initial conditions and the integration constants? Is the relative motion bounded as expected from $\delta a = 0$ (energy matching condition). If not, why? How would you mitigate the phenomenon?

e) Compute quasi-nonsingular relative orbit elements (i.e., based on eccentricity vector and mean argument of latitude)

f) Propagate the relative position and velocity using the geometric linear mapping with relative orbit elements for arbitrary eccentricity. Plot the resulting relative position and velocity in the same plots as those in part (c).

g) Are the results of the previous simulation as expected? How do the relative orbit elements from part (e) and the integration constants from part (b) compare numerically?

h) Produce the true relative position and velocity from analytical or numerical propagation of the nonlinear equations and compare with the results from parts (c) and (f) on the same plots. Plot the propagation errors of the simulations from parts (c) and (f) in the RTN frame as usual. Which analytical solution is more accurate, the solution of the differential equation or the geometric mapping? Why?

i) Repeat this exercise two more times, always starting with the definition of initial conditions and proceeding with brief comments on results and expectations. You should investigate the following cases:

    i. Difference in semi-major axis between deputy and chief. Justify your choice of δa by commenting on the expected along-track drift/orbit.

    ii. Highly eccentric orbit of the chief spacecraft ($e > 0.5$).

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