Problem Set 2

AA279D: Dyn, Nav, Ctrl of DSS
Spring Quarter 2022/2023
Due: April 19, 2023, Wed, 3:00PM PT
Prof. Simone D'Amico

Submission Instructions

Please briefly document all tasks outlined below in a report which will grow during the course. You should include a table with change logs since the last submission, and an index for sections at the beginning. Please submit your report as a single PDF file to the course Canvas website. It should include narrative, plots, tables, code, and interpretations. You should use typesetting software like LaTeX or Microsoft Word to produce your document. Do not submit extra files.

Topics

Week 2. Continuation of project. Unperturbed relative orbit motion.

Problem 1: Everything is Relative

We would like to propagate the general nonlinear equations of relative motion and visualize the results in the RTN or LVLH coordinate frame. No perturbations are applied yet. To this end, you are asked to do the following:

a) Let the chief orbit be determined through the initial conditions from Problem Set 1. The deputy orbits have the same initial orbit elements except for small variations depending on the mission you have selected. If you are uncertain on the variations to apply, remember the following rules of thumb:
    1. Except the semi-major axis, differences in orbital elements (measured in radians or non-dimensional for eccentricity) produce a baseline given by the given difference multiplied by the semi-major axis (e.g., in meters). You may want to apply differences to at least 3 orbital elements.
    2. A difference in semi-major axis corresponds to a difference in specific mechanical energy, thus to a difference in orbital velocity at the same orbit radius, thus to a locally unbounded relative motion (drift). You may want to setup an identical semi-major axis for chief and deputy.

b) Perform a numerical integration of the nonlinear equations of relative motion and plot the resulting relative position and velocity in the rotating RTN frame with origin at the chief spacecraft. *Hint*: recall the meaning of the relative position and velocity contained in the equations of relative motion.

c) Compute the relative orbit by using the fundamental orbital differential equations of absolute motion (see Problem Set 1). First compute the inertial deputy and chief orbits, and then difference the results. Plot the resulting relative motion on the same plot generated in (b). *Hint*: Recall that the RTN frame is rotating w.r.t. inertial frame, thus the velocity transformation has to be treated with care. Also, since we have an analytical solution for the absolute motion, you might decide to use it instead or as verification of a numerical integration of individual orbits.

d) Compare the results of (b) and (c) and verify that they are identical except for numerical errors. This should hold for any choice of the initial conditions of deputy relative to chief. Check this correspondence for a second set of initial conditions where a larger (non-zero) difference in semi-major axis is introduced.

e) We would like to re-establish a bounded periodic relative motion between deputy and chief, i.e. remove the drift introduced in (d). What is the most fuel-efficient impulsive maneuver to be performed by the deputy in order to re-establish bounded periodic motion? Both size and location of the maneuver are unknown. *Hint*: try to integrate analytically the Gauss Variational Equations over an impulsive maneuver delta-v with constant orbit elements during maneuver execution, you are interested in one orbit element specifically.

f) Apply the maneuver computed in (e) by repeating simulation (b) and introducing a discontinuity in the inertial velocity of the deputy (no change in its position at the maneuver time). Is the resulting relative motion according to expectations? *Hint*: a numerical integrator can be re-initialized in Simulink by adding an extra state input to the integrator block that describes the new initial conditions (i.e., changed velocity, same position).

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