

7.2 a)

$$I = x_1 + x_2 L_1 + x_3 L_1 L_2 + x_4 L_1 L_2 L_3 + \cdots + x_d \prod_{j=1}^{d-1} L_j = x_1 + \sum_{i=2}^d (x_i \prod_{j=1}^{i-1} L_j)$$

$$\left\{ \begin{array}{l} x_1 = I \bmod \prod_{j=1}^{d-1} L_j \bmod \prod_{j=1}^{d-2} L_j \bmod \dots \bmod \prod_{j=1}^2 L_j \bmod L_1 \\ x_2 = \left\lfloor \frac{I \bmod \prod_{j=1}^{d-1} L_j \bmod \prod_{j=1}^{d-2} L_j \bmod \dots \bmod \prod_{j=1}^2 L_j}{L_1} \right\rfloor \\ x_i = \left\lfloor \frac{I \bmod \prod_{j=1}^{d-1} L_j \bmod \prod_{j=1}^{d-2} L_j \bmod \dots \bmod \prod_{j=1}^i L_j}{\prod_{j=1}^{i-1} L_j} \right\rfloor \\ x_{d-2} = \left\lfloor \frac{I \bmod \prod_{j=1}^{d-1} L_j \bmod \prod_{j=1}^{d-2} L_j}{\prod_{j=1}^{d-3} L_j} \right\rfloor \\ x_{d-1} = \left\lfloor \frac{I \bmod \prod_{j=1}^{d-1} L_j}{\prod_{j=1}^{d-2} L_j} \right\rfloor \\ x_d = \left\lfloor \frac{I}{\prod_{j=1}^{d-1} L_j} \right\rfloor \end{array} \right.$$

$$\therefore x_i = \left\{ \begin{array}{ll} I \bmod \prod_{j=1}^{d-1} L_j \bmod \prod_{j=1}^{d-2} L_j \bmod \dots \bmod \prod_{j=1}^2 L_j \bmod L_1 & i = 1 \\ \left\lfloor \frac{I \bmod \prod_{j=1}^{d-1} L_j \bmod \dots \bmod \prod_{j=1}^i L_j}{\prod_{j=1}^{i-1} L_j} \right\rfloor & 1 < i < d \\ \left\lfloor \frac{I}{\prod_{j=1}^{d-1} L_j} \right\rfloor & i = d \end{array} \right.$$