Clock synchronization over networks

using sawtooth models

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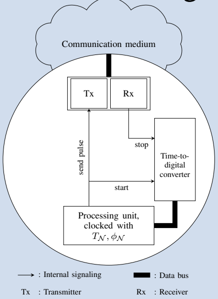
https://github.com/poldap/clock_sync_and_range

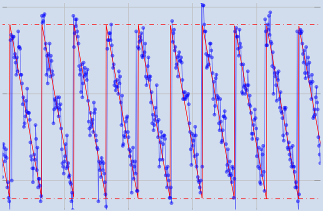
Work done at the KTH Royal Institute of Technology

Clock synchronization

identifiability, estimation theory, sawtooth signals, wireless networks, ranging, synchronization

OJSP2020 ICASSP2020





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- Introduction to the sawtooth model
- Cramér-Rao Lower Bounds
- Estimation
- Results

Authors and funding

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S. Dwivedi

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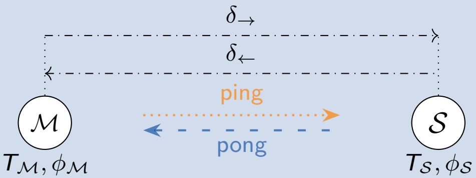
SRA ICT TNG: PITA

T. Oechtering, A. Proutiere, J. Jaldén

Introduction to the sawtooth model - RTT for ranging (I)



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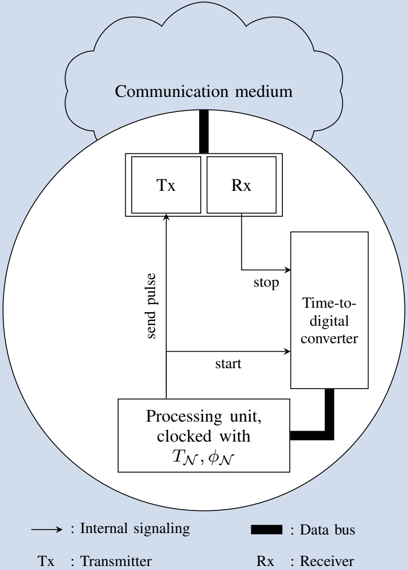

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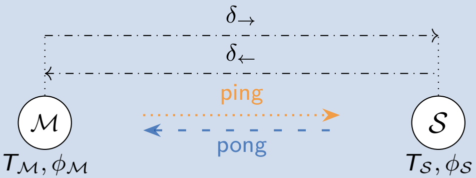

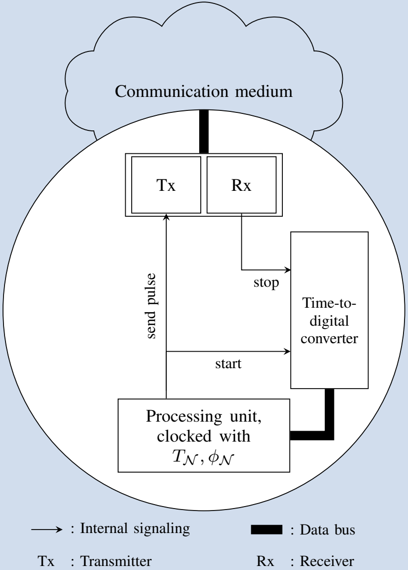

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- Low communication overhead

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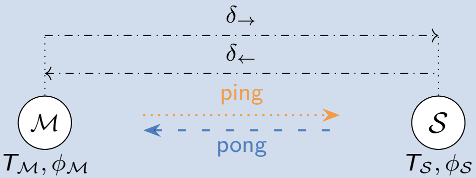

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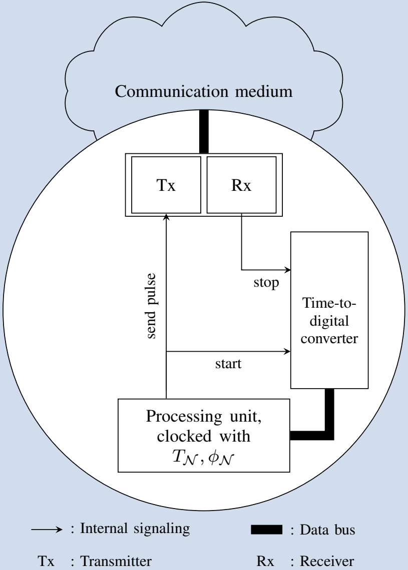
- Low communication overhead
- Low power consumption



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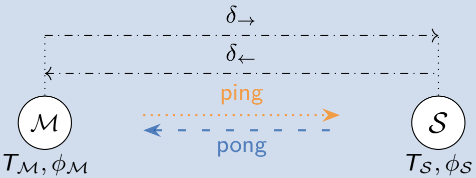

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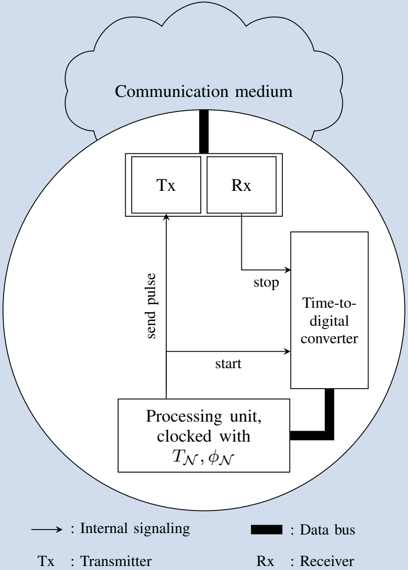
- Low communication overhead
- Low power consumption
- High measurement accuracy



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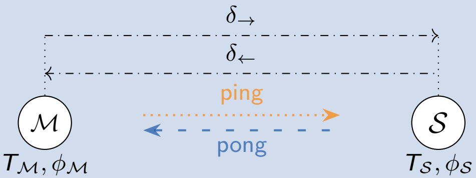

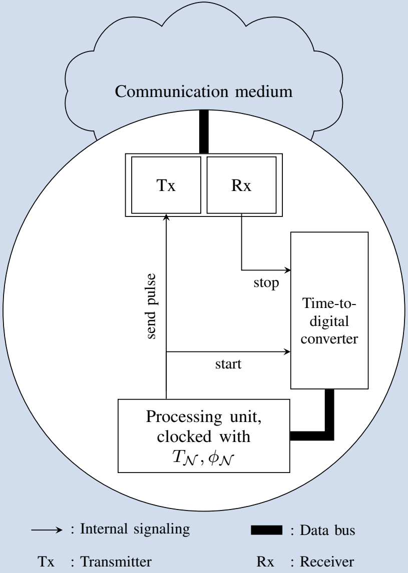

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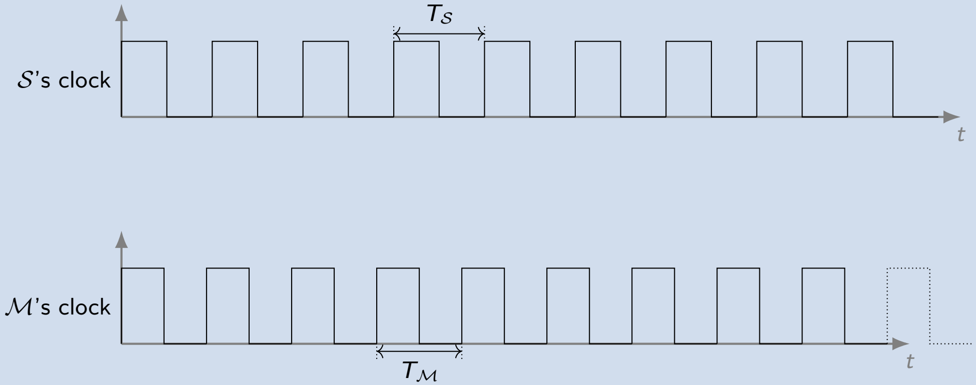
- Low communication overhead
- Low power consumption
- High measurement accuracy
- Nodes can measure their own clock period

Introduction to the sawtooth model - RTT for ranging (II)



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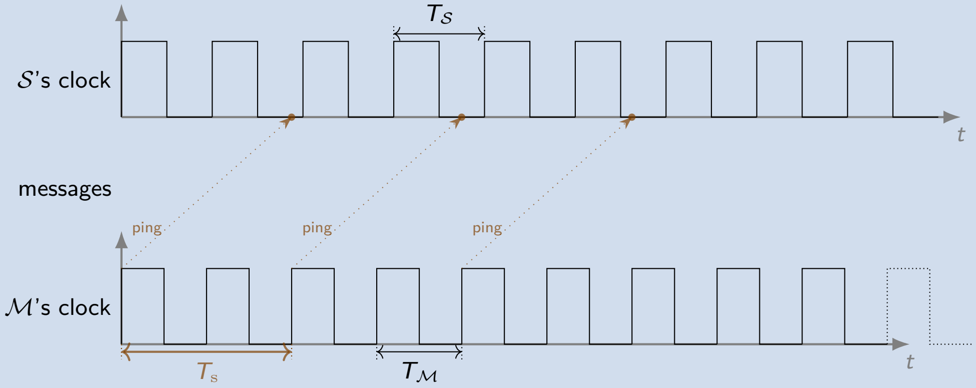

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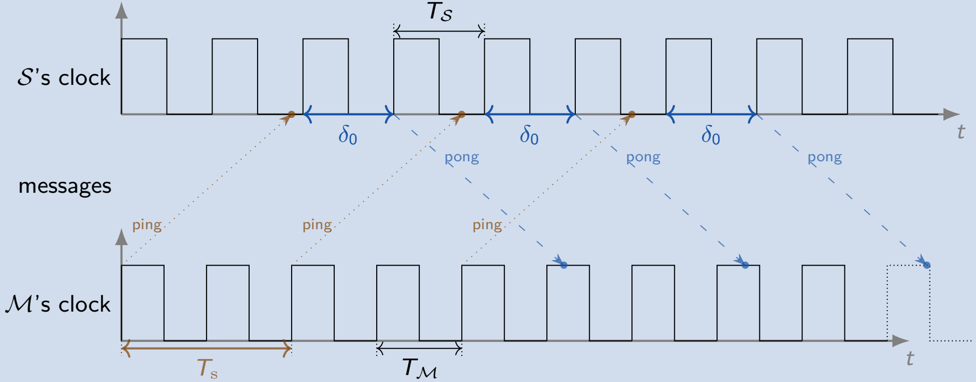

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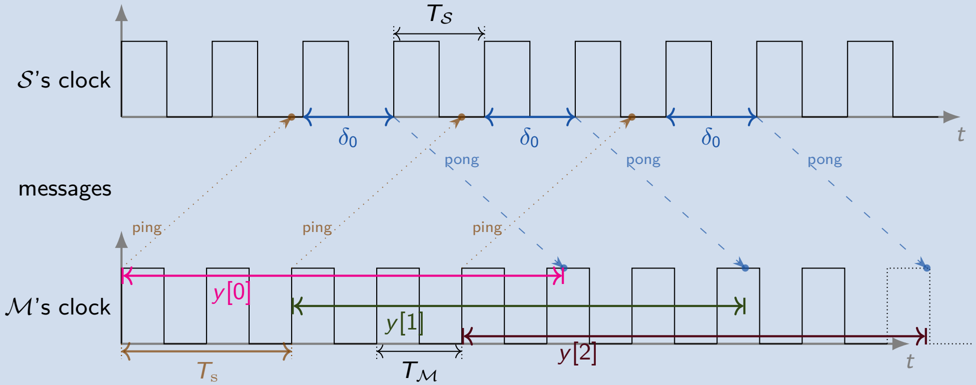

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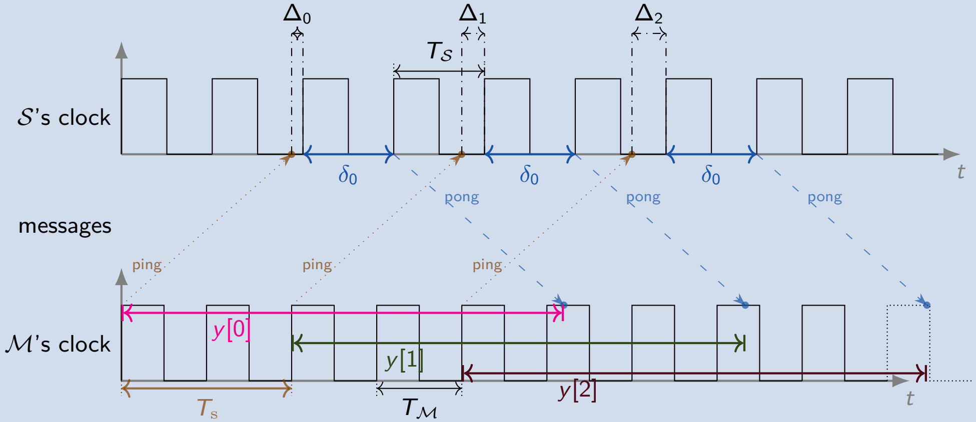

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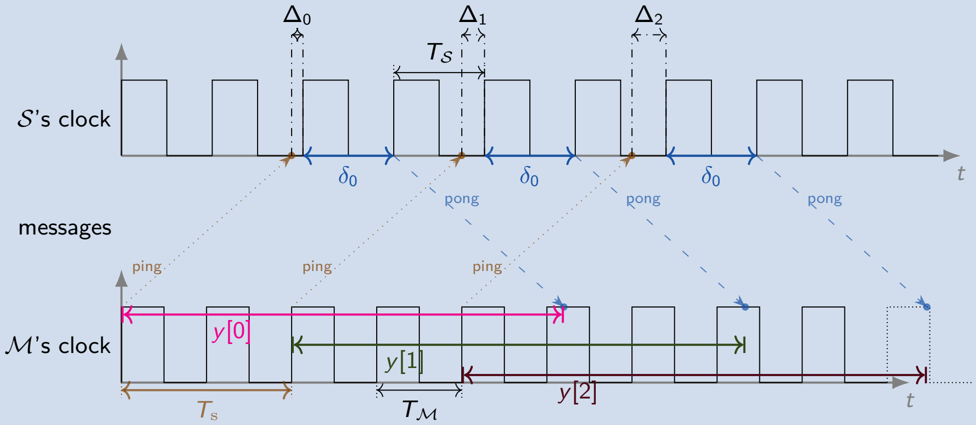

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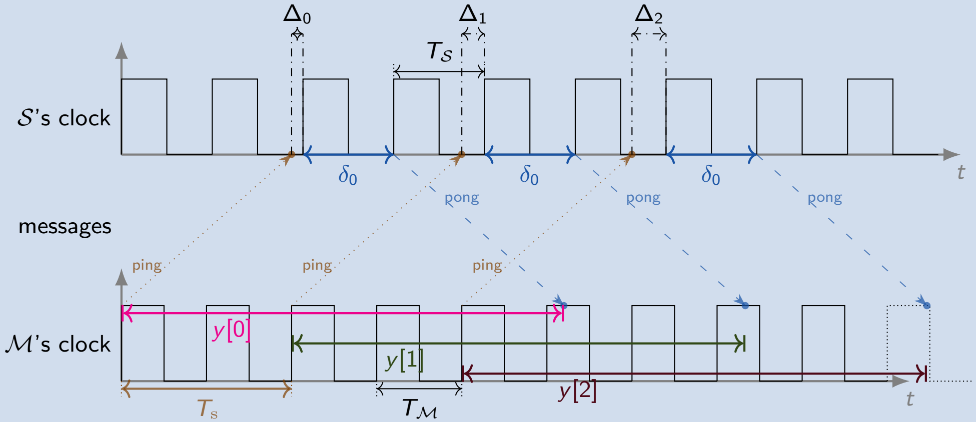
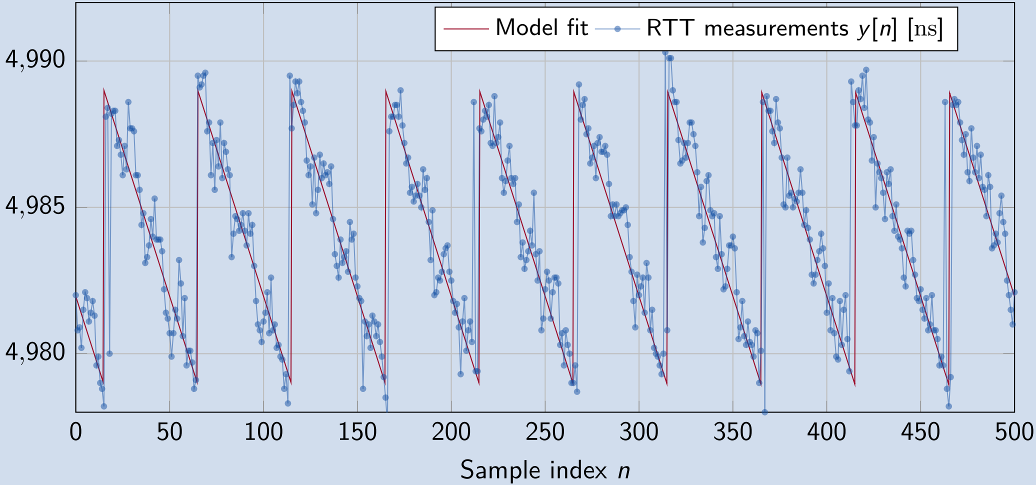


$$y_{\mathrm{det}}[n] = \delta_0 + \delta_{\leftrightarrow} + \Delta_n, \text{ with } \delta_{\leftrightarrow} = \delta_{\rightarrow} + \delta_{\leftarrow} \approx 2\delta_{\rightarrow}, \text{ and}$$

$$\Delta_n = T_{\mathcal{S}} \left(1 - \mathrm{mod}_1 \left[T_{\mathrm{s}} f_{\mathrm{d}} n + \frac{\delta_{\rightarrow}}{T_{\mathcal{S}}} + \frac{\phi_{\mathcal{S}}}{2\pi} \right] \right), \text{ with } f_{\mathrm{d}} = \frac{1}{T_{\mathcal{S}}} - \frac{1}{T_{\mathcal{M}}}.$$

Introduction to the sawtooth model (III)

$$y_{\mathrm{det}}[n] = \delta_{\leftrightarrow} + \delta_0 + T_{\mathcal{S}} \left(1 - \mathrm{mod}_1 \left[T_{\mathrm{s}} f_{\mathrm{d}} n + \frac{\delta_{\rightarrow}}{T_{\mathcal{S}}} + \frac{\phi_{\mathcal{S}}}{2\pi} \right] \right)$$



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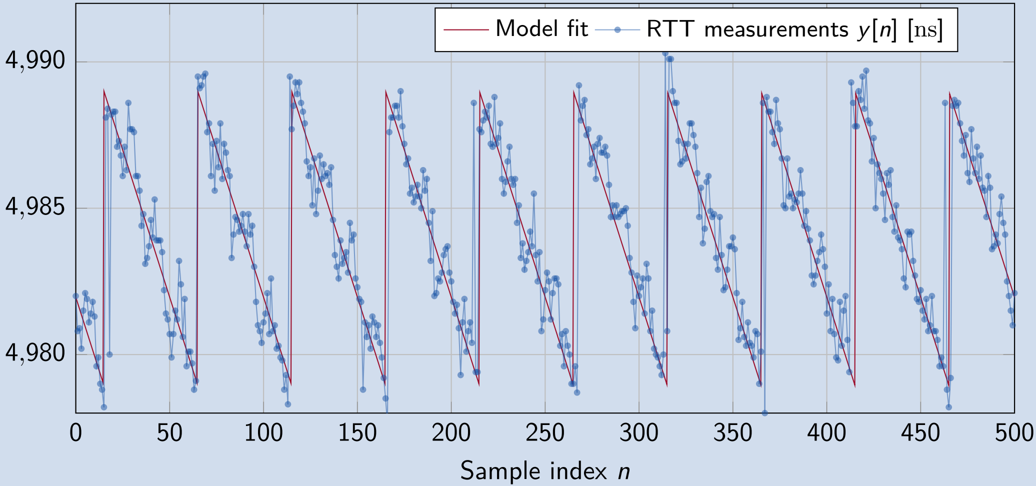

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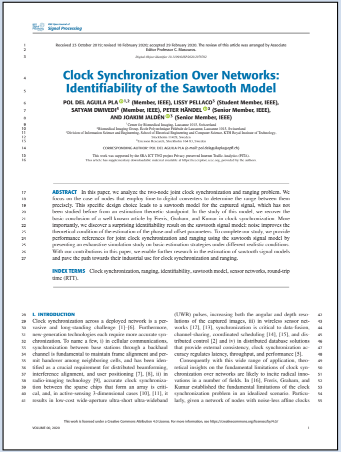
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Clock Synchronization Over Networks: Identifiability of the Sawtooth Model

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ABSTRACT In this paper, we analyze the two-node joint clock synchronization and ranging problem. We focus on the case of nodes that employ time-to-digital converters to determine the range between them precisely. This specific design choice leads to a sawtooth model for the captured signal, which has not been studied before from an estimation theoretic standpoint. In the study of this model, we recover the basic conclusion of a well-known article by Freris, Graham, and Kumar in clock synchronization. More importantly, we discover a surprising identifiability result on the sawtooth signal model: noise improves the theoretical condition of the estimation of the phase and offset parameters. To complete our study, we provide performance references for joint clock synchronization and ranging using the sawtooth signal model by presenting an exhaustive simulation study on basic estimation strategies under different realistic conditions. With our contributions in this paper, we enable further research in the estimation of sawtooth signal models and pave the path towards their industrial use for clock synchronization and ranging.

INDEX TERMS Clock synchronization, ranging, identifiability, sawtooth model, sensor networks, round-trip time (RTT).

I. INTRODUCTION

Clock synchronization across a deployed network is a pervasive and long-standing challenge [1]–[6]. Furthermore, new-generation technologies each require more accurate synchronization. To name a few, i) in cellular communications, synchronization between base stations through a backhaul channel is fundamental to maintain frame alignment and permit handover among neighboring cells, and has been identified as a crucial requirement for distributed beamforming, interference alignment, and user positioning [7], [8], ii) in radio-imaging technology [9], accurate clock synchronization between the sparse chips that form an array is critical, and, in active-sensing 3-dimensional cases [10], [11], it results in low-cost wide-aperture ultra-short ultra-wideband (UWB) pulses, increasing both the angular and depth resolutions of the captured images, iii) in wireless sensor networks [12], [13], synchronization is critical to data-fusion, channel-sharing, coordinated scheduling [14], [15], and distributed control [2] and iv) in distributed database solutions that provide external consistency, clock synchronization accuracy regulates latency, throughput, and performance [5].

Consequently with this wide range of application, theoretical insights on the fundamental limitations of clock synchronization over networks are likely to incite radical innovations in a number of fields. In [16], Freris, Graham, and Kumar established the fundamental limitations of the clock synchronization problem in an idealized scenario. Particularly, given a network of nodes with noise-less affine clocks

- Model derivation
- Estimation theoretic analysis
- Conditions for identifiability
- Exhaustive empirical results
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Cramér-Rao Lower Bounds of an unwrapped / linear model

Our model is not differentiable, so one can not define the CRLBs.

$$Y[n] = \delta_{\leftrightarrow} + \delta_0 + W[n] + T_{\mathcal{S}} \left[1 - \mathrm{mod}_1 \left(T_{\mathrm{s}} f_{\mathrm{d}} n + \frac{\delta_{\rightarrow}}{T_{\mathcal{S}}} + \frac{\phi_{\mathcal{S}}}{2\pi} + V[n] \right) \right]$$

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We work on an unwrapped / linearized model $Z[n]$,

$$Z[n] = \delta_0 + \frac{\delta_{\leftrightarrow}}{2} + T_{\mathcal{S}} \left(1 - \frac{\phi_{\mathcal{S}}}{2\pi} \right) - T_{\mathcal{S}} T_{\mathrm{s}} f_{\mathrm{d}} n + U[n] \,,$$

with $U[n]$ a white Gaussian process such that $U[n] \sim \mathcal{N}\left(0, \sigma^2\right)$ with $\sigma^2 = \sigma_w^2 + T_{\mathcal{S}}^2 \sigma_v^2$.

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Caveats of the approach

- Not the same model (but provides a linearized intuition)

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- Non-identifiability of $\phi_{\mathcal{S}}$ and $\delta_{\leftrightarrow}$ (but we can suppose we know the respective other when deriving the lower bound)

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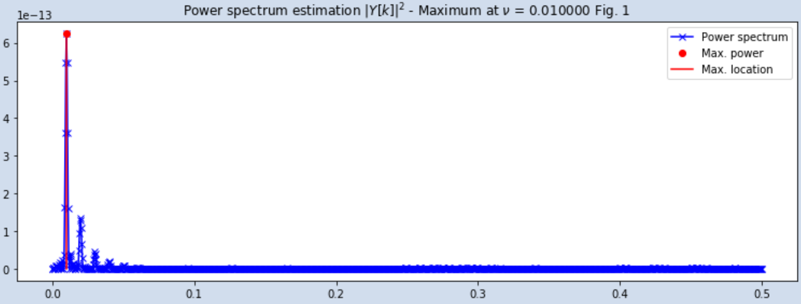
Caveats of the approach

- Not the same model (but provides a linearized intuition)
- Non-identifiability of $\phi_{\mathcal{S}}$ and $\delta_{\leftrightarrow}$ (but we can suppose we know the respective other when deriving the lower bound)
- Dependence of σ^2 on $f_\mathrm{d} = \frac{1}{T_{\mathcal{S}}} - \frac{1}{T_{\mathcal{M}}}$ (but that is covered by standard results)

Estimation - Periodogram and correlation peaks (PCP)

$$Y[n] = \alpha + W[n] + \psi \,\mathrm{mod}_1(\beta n + \gamma + V[n]), \text{ with } \psi < 0 \text{ known when } \beta \text{ is known}$$

- Estimate $|\beta|$ as the highest peak in the periodogram of the $L-1$-times zero-padded, zero-mean data, i.e., $|\hat{\beta}| = \arg\max_k \left|\mathrm{DFT}_{NL}\left(\tilde{y}[n]\right)[k]\right|^2/(NL)$



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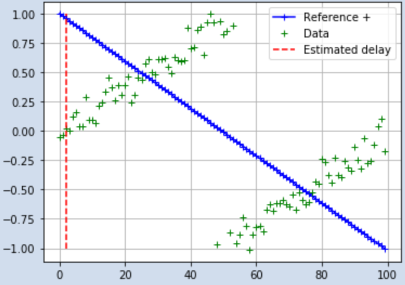
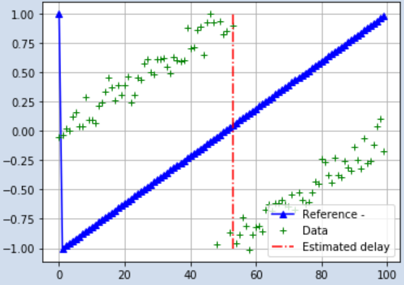
- Generate two single-period templates $p_\pm[n]$ for $\pm\beta$ and circularly correlate them with the first period of the max-normalized zero-mean data
- Estimate the sign of β by the largest correlation and the phase γ from the index at which it happens, i.e., $\hat{\gamma} = \mathrm{mod}_1(\hat{\beta} n^{\mathrm{opt}})$

Reference waveform with positive $+\beta$ vs the data - Fig. 2



Reference waveform with negative $-\beta$ vs the data - Fig. 3



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- Estimate the sign of β by the largest correlation and the phase γ from the index at which it happens, i.e., $\hat{\gamma} = \mathrm{mod}_1(\hat{\beta} n^{\mathrm{opt}})$
- Estimate ψ through its known relation with β, and α by the closed-form minimum prediction mean squared error solution assuming $\hat{\beta}$, $\hat{\psi}$ and $\hat{\gamma}$ are correct, i.e.,

$$\hat{\alpha}_{\hat{\beta},\hat{\gamma}} = \sum_{n=0}^{N-1} y[n] - \sum_{m=0}^{N-1} \hat{\psi}_{\hat{\beta}} \,\mathrm{mod}_1\left[\hat{\beta} m + \hat{\gamma}\right].$$

Estimation - Grid Search, either local (LGS), or global (GGS)

Define a grid $\mathcal{G} \times \mathcal{B}$ in $\left[-\frac{1}{2}, \frac{1}{2}\right) \times [0, 1)$, and estimate the point in the grid that minimizes the prediction mean squared error, i.e.,

$$\min_{(\beta,\gamma)\in\mathcal{G}\times\mathcal{B}} \left\{ \sum_{n=0}^{N-1} \left(y[n] - \hat{\alpha}_{\beta,\gamma} - \hat{\psi}_{\beta} \operatorname{mod}_1[\beta n + \gamma] \right)^2 \right\},$$

where $\hat{\alpha}_{\beta,\gamma}$ is the closed form solution as above, and $\hat{\psi}_{\beta}$ is the known amplitude given the frequency.

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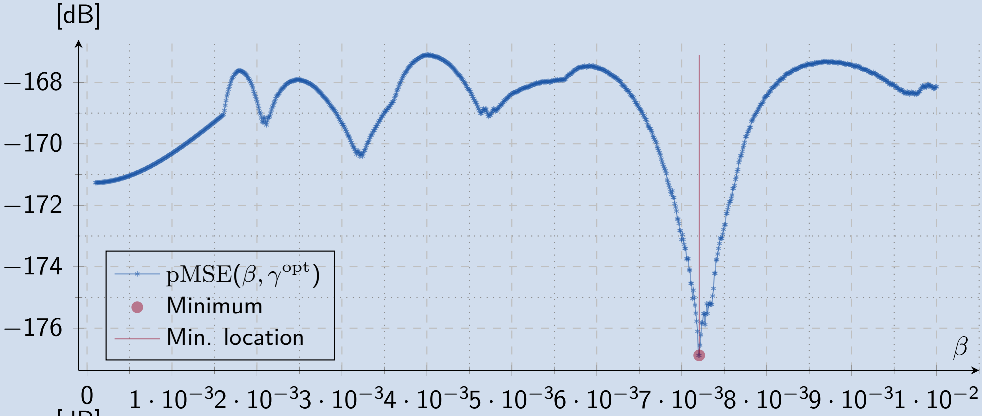

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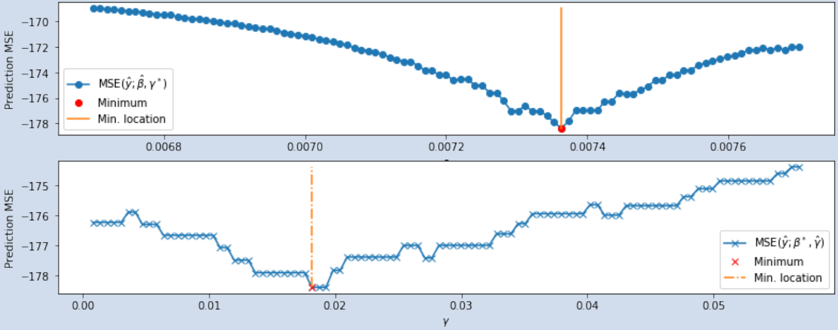
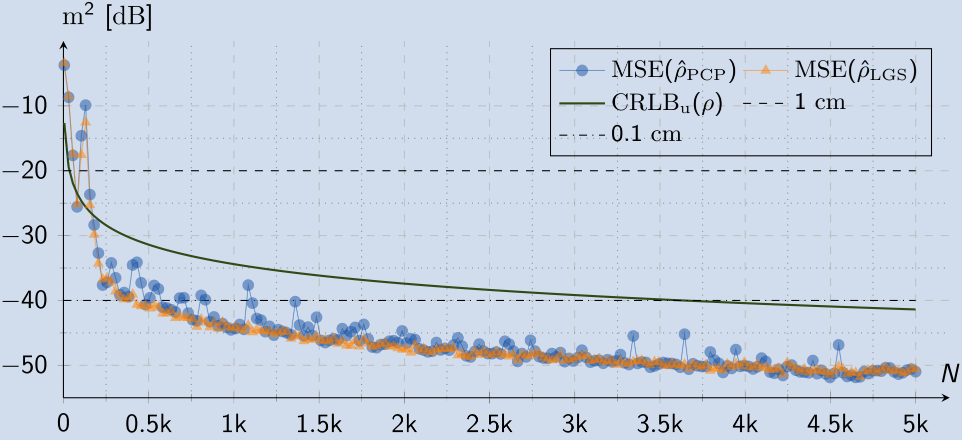
- $\mathcal{G} \times \mathcal{B}$ can be chosen global, resulting on a very irregular function landscape
- or local, around the result of PCP, where smoother behavior is expected and better estimates are likely due to a finer gridding if the PCP was close to the right solution



Empirical results

300 Monte Carlo repetitions, $f_{\mathrm{d}} = 73$ Hz, $\phi_{\mathcal{S}} = 3\pi/4$, $\delta_{\leftrightarrow} = 2\rho/c$, $\rho = 2$ m, $1/\sigma_v^2 = 40$ dB, $\psi^2/\sigma_w^2 = 20$ dB, $T_{\mathcal{M}} = 10$ ns, $T_{\mathrm{s}} = 0.1$ ms, and $\delta_0 = 5$ μs.



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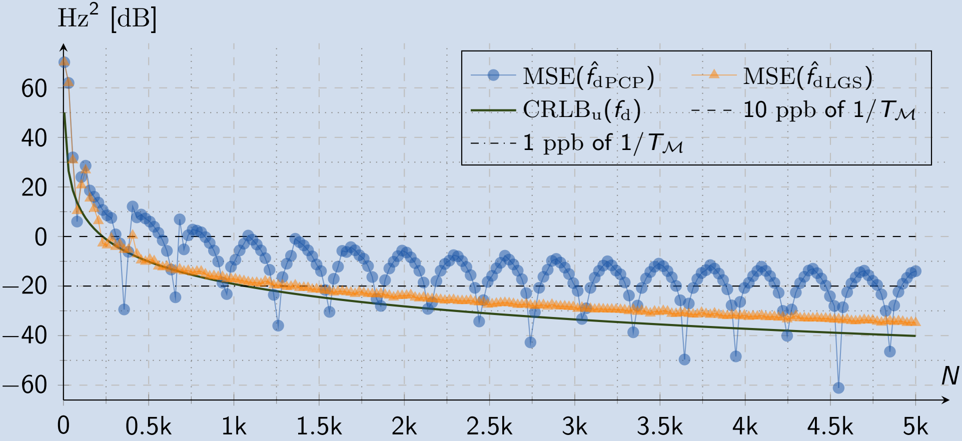
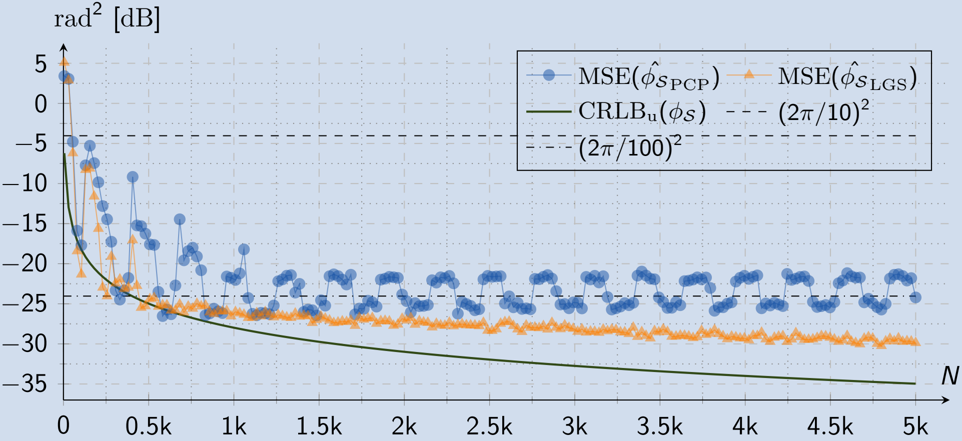


Empirical results

300 Monte Carlo repetitions, $f_{\mathrm{d}} = 73$ Hz, $\phi_{\mathcal{S}} = 3\pi/4$, $\delta_{\leftrightarrow} = 2\rho/c$, $\rho = 2$ m, $1/\sigma_v^2 = 40$ dB, $\psi^2/\sigma_w^2 = 20$ dB, $T_{\mathcal{M}} = 10$ ns, $T_{\mathrm{s}} = 0.1$ ms, and $\delta_0 = 5\ \mu$s.



Clock synchronization over networks

using sawtooth models

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https://github.com/poldap/clock_sync_and_range

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