Convex Quantization Preserves Logconcavity

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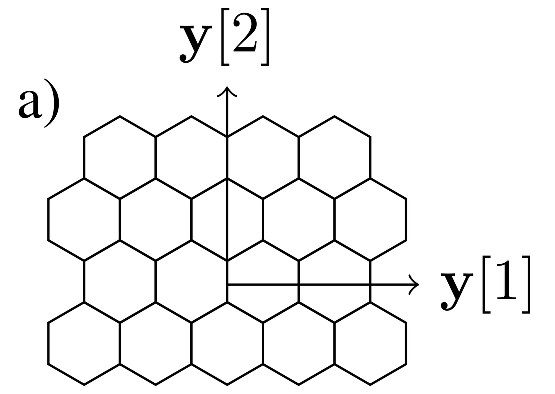
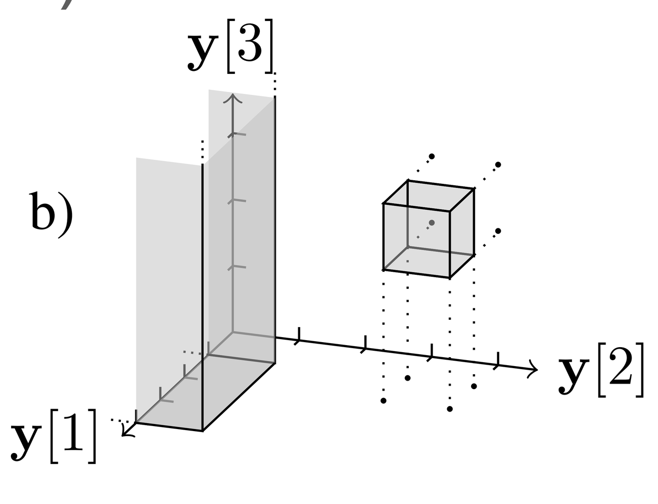
Motivation

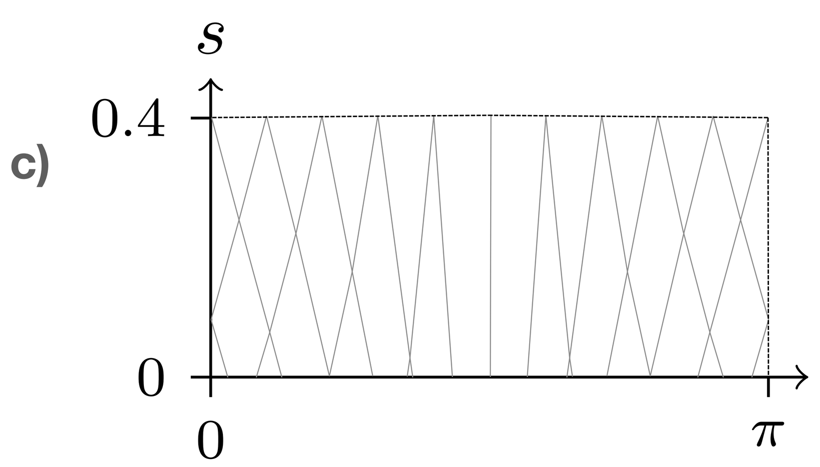
- Reality (quantization)



honeycomb electrode detectors



independent ADCs for each dimension



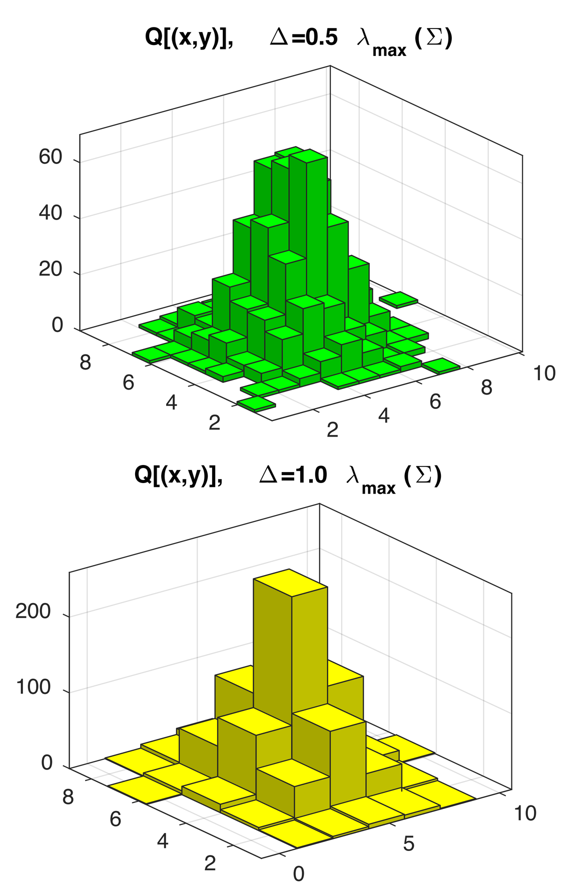
sinogram-domain quantization due to PET scanner geometry

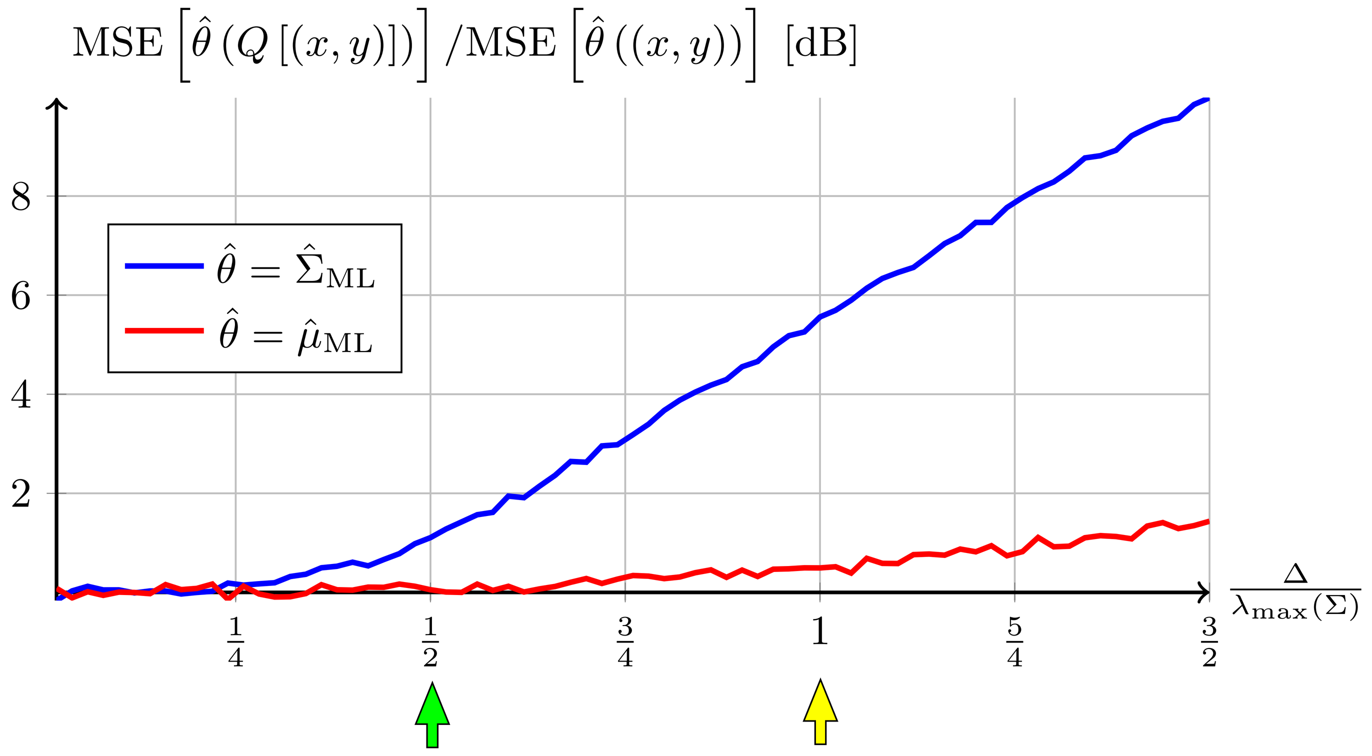
- Our models (continuous)

$$\mathbf{y} = \mathbf{\Psi}^{-1}(\mathbf{S}\mathbf{x} + \mathbf{w}),$$

with $\mathbf{\Psi} \in \mathcal{M}_n^+(\mathbb{R})$, $\mathbf{S} \in \mathcal{M}_{n,m}(\mathbb{R})$, and $\mathbf{x} \in \mathbb{R}^m$

- Why is that a problem? (wrong model)





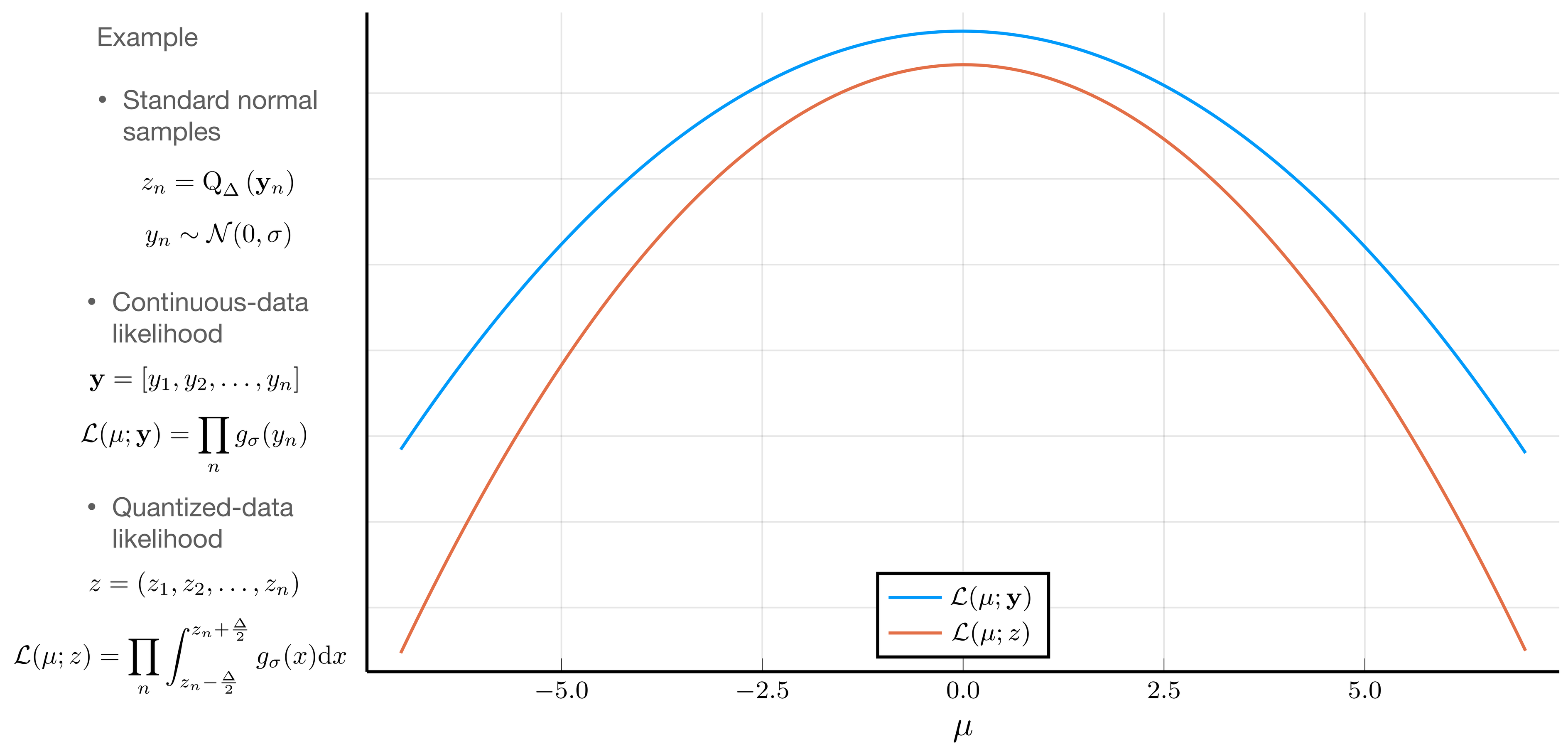
Central Question

- Likelihood Logconcavity

$$z = \mathrm{Q}(\mathbf{y}) = \mathrm{Q}\left(\mathbf{\Psi}^{-1}(\mathbf{S}\mathbf{x} + \mathbf{w})\right), \qquad (1)$$

$\mathrm{Q}\colon \mathbb{R}^n \to \mathcal{Z}$, where $\mathcal{Z}$ is a countable set.

$\boxed{\text{Is } \mathcal{L}(\mathbf{x}, \mathbf{\Psi}; z) \text{ logconcave w.r.t. } \mathbf{x} \text{ and } \mathbf{\Psi}?}$



Our Answer

Same condition needed in cont. models

Theorem 1 ($\boxed{\text{Logconcave Noise}}$ **and** $\boxed{\text{Convex Quantizers}}$) *Consider a sample $z \in \mathcal{Z}$ drawn from model (1). Assume that Q is a convex quantizer and that the pdf $\mathrm{f}_{\mathbf{w}}(\cdot)$ of the noise is logconcave. Then,*

a. *for a given scale parameter $\mathbf{\Psi}_0 \in \mathcal{M}_n^+(\mathbb{R})$, the likelihood $\mathcal{L}(\mathbf{x}, \mathbf{\Psi}_0; z)$ is logconcave with respect to $\mathbf{x}$,*

b. *for scale parameters of the form $\mathbf{\Psi} = \psi\mathbf{I}$ with $\psi > 0$, the likelihood $\mathcal{L}(\mathbf{x}, \psi\mathbf{I}; z)$ is jointly logconcave with respect to $\mathbf{x}$ and ψ,*

c. *for diagonal positive-definite scale parameters $\mathbf{\Psi} = \mathbf{\Lambda}$, the likelihood $\mathcal{L}(\mathbf{x}, \mathbf{\Lambda}; z)$ is jointly logconcave with respect to $\mathbf{x}$ and $\mathbf{\Lambda}$ if Q is composed of independent ADCs for each dimension.*

How?

- Study of the sets

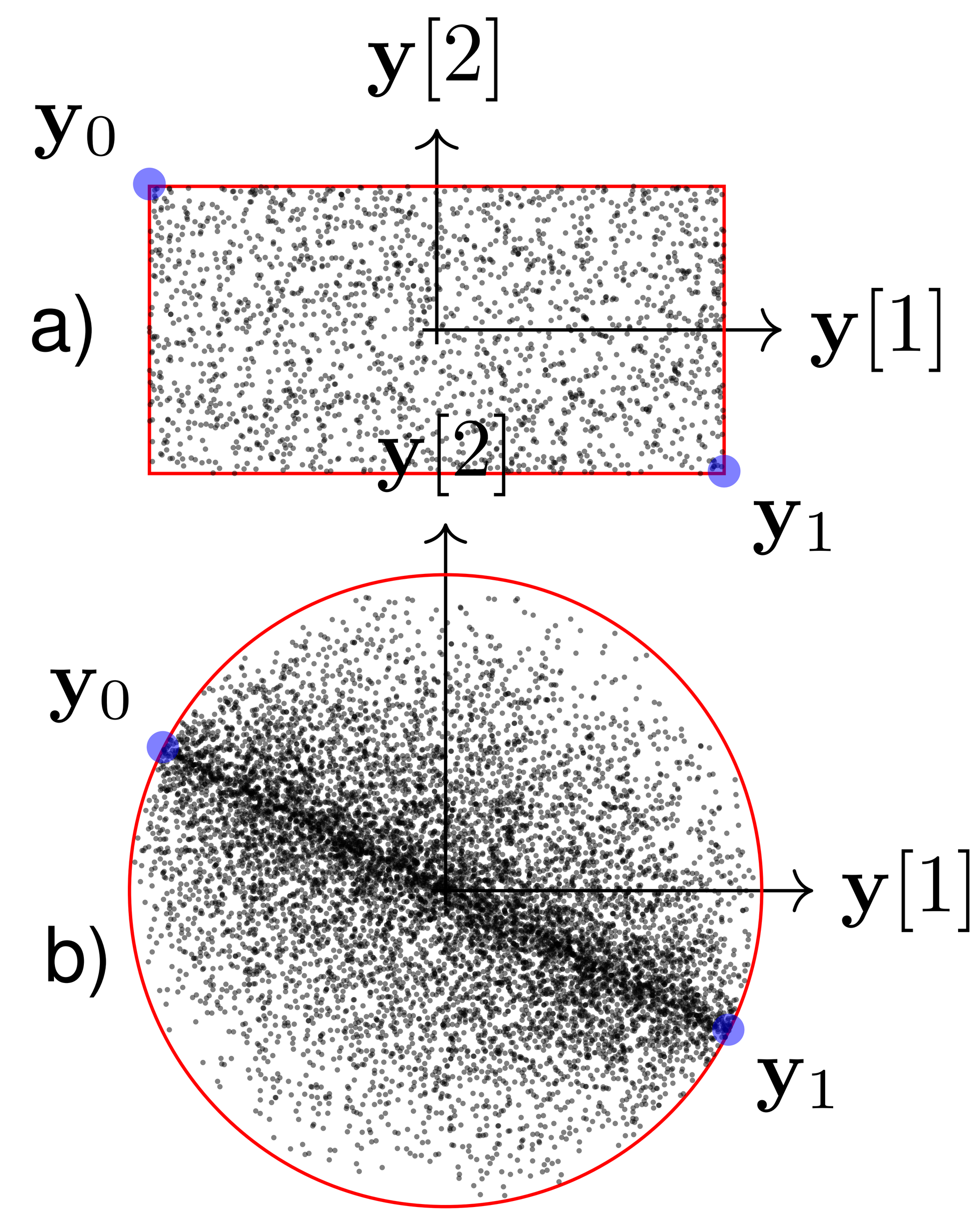
$$\{\mathbf{y} : \mathbf{C}\mathbf{y}_1 + (\mathbf{I} - \mathbf{C})\mathbf{y}_0\}$$

when $\mathbf{C}$ are

a) random diagonal matrices

b) random p.s.d. matrices

with $\rho(\mathbf{C}) \leq 1$.



- Prékopa's Theorem

Logconcave probability laws on Minkowski averages of convex sets.

- Key realisation

$$\mathcal{L}(\mathbf{x}, \mathbf{\Psi}; z) = \mathrm{P}_{\mathbf{w}}\left[\mathcal{W}_z(\mathbf{x}, \mathbf{\Psi})\right],$$

with $\mathcal{W}_z(\mathbf{x}, \mathbf{\Psi}) = \left\{\mathbf{w} \in \mathbb{R}^n : \mathbf{y} \in \mathrm{Q}^{-1}(z)\right\}$.