Cálculo Diferencial e Integral en Varias Variables

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Ejercicio 1

Consigna

Hallar la solución general de las siguientes ecuaciones diferenciales de variables separables:

1. $y' = y^2 - 1$
2. $(1 + y^2)yy' + (1 + y^2) = 0$
3. $xe^{2y}y' - (1 + e^{2y}) = 0$

Resolución

Parte 1

- $y' = y^2 - 1$

$$y' = y^2 - 1$$
$$\iff$$
$$\frac{y'}{y^2-1} = 1$$
$$\iff$$
$$\int \frac{y'}{y^2-1}dx = \int 1dx$$
$$\iff \text{(u=y(x),du=y'(x)dx)}$$
$$\int \frac{1}{u^2-1}du = x + k_1$$
$$\iff \text{(fracciones simples)}$$
$$\int \frac{1}{2}\left(\frac{1}{u-1} - \frac{1}{u+1}\right) du = x + k_1$$
$$\iff$$
$$\frac{1}{2}\left(\int \frac{1}{u-1}du - \int \frac{1}{u+1}du\right) = x + k_1$$
$$\iff$$
$$\frac{1}{2}(ln|u-1| + k_2 - ln|u+1| + k_3) = x + k_1$$
$$\iff (C_1 = \tfrac{1}{2}(k_2+k_3))$$
$$\frac{1}{2}ln\left|\frac{u-1}{u+1}\right| + C_1 = x + k_1$$
$$\iff (C_2 = 2k_1 - C_1)$$
$$ln\left|\frac{u-1}{u+1}\right| = 2x + C_2$$
$$\iff$$
$$\frac{u-1}{u+1} = \pm e^{2x+C_2}$$
$$\iff (K = \pm e_2^C)$$
$$\frac{u-1}{u+1} = Ke^{2x}$$
$$\iff$$
$$u - 1 = Ke^{2x}(u+1)$$
$$\iff$$
$$u - 1 = Ke^{2x}u + Ke^{2x}$$
$$\iff$$
$$u - Ke^{2x}u = Ke^{2x} + 1$$
$$\iff$$
$$u(1 - Ke^{2x}) = Ke^{2x} + 1$$
$$\iff \text{(suponemos que } 1 - Ke^{2x} \neq 0\text{)}$$
$$u = \frac{1+Ke^{2x}}{1-Ke^{2x}}$$
$$\iff \text{(deshago cambio de variable)}$$
$$y(x) = \frac{1+Ke^{2x}}{1-Ke^{2x}}$$

Parte 2

- $(1+y^2)yy' + (1+y^2) = 0$

$$
\begin{aligned}
&(1+y^2)yy' + (1+y^2) = 0\\
&\iff_{(\forall x \in \mathbb{R} : (1+y^2)>0)}\\
&y'y = \frac{-(1+y^2)}{(1+y^2)}\\
&\iff\\
&y'y = -1\\
&\iff\\
&\int y'y dx = \int -1 dx\\
&\iff_{(u=y(x); du=y'(x)dx)}\\
&\int u du = -x + k_1\\
&\iff\\
&\frac{u^2}{2} = -x + k_1\\
&\iff_{(C=k_1-k_2)}\\
&u = \sqrt{2}\sqrt{-x+C}\\
&\iff_{(\text{deshago cambio de variable})}\\
&y(x) = \sqrt{2}\sqrt{-x+C}
\end{aligned}
$$

Parte 3

- $xe^{2y}y' - (1+e^{2y}) = 0$

$$xe^{2y}y' - (1+e^{2y}) = 0$$

$$\iff$$

$$y' = \frac{1+e^{2y}}{e^{2y}} \cdot \frac{1}{x}$$

$$\iff$$

$$\frac{e^{2y}y'}{1+e^{2y}} = \frac{1}{x}$$

$$\iff$$

$$\int \frac{e^{2y}y'}{1+e^{2y}}dx = \int \frac{1}{x}dx$$

$$\iff \scriptstyle (u=y(x);du=y'(x)dx)$$

$$\int \frac{e^{2u}}{1+e^{2u}}du = ln|x| + k_1$$

$$\iff \text{(preparando otro cambio de variable)}$$

$$\frac{1}{2}\int \frac{2e^{2u}}{1+e^{2u}}du = ln|x| + k_1$$

$$\iff \scriptstyle (v=1+e^{2u};dv=2e^{2u}du)$$

$$\frac{1}{2}\int \frac{dv}{v} = ln|x| + k_1$$

$$\iff$$

$$\frac{1}{2}ln|v| + k_2 = ln|x| + k_1$$

$$\iff \text{(deshago cambios de variable)}$$

$$\frac{1}{2}ln|1+e^{2y}| + k_2 = ln|x| + k_1$$

$$\iff \scriptstyle (C=2(k_1-k_2))$$

$$ln|1+e^{2y}| = 2ln|x| + C$$

$$\iff \scriptstyle (n\cdot ln(x)=ln(x^n) \text{ y } (1+e^{2y})>0)$$

$$1+e^{2y} = e^{ln|x^2|+C}$$

$$\iff \scriptstyle (x^2>0 \text{ y } K=e^C)$$

$$1+e^{2y} = Kx^2$$

$$\iff$$

$$e^{2y} = Kx^2 - 1$$

$$\iff$$

$$2y = ln(Kx^2-1)$$

$$\iff$$

$$y = \frac{1}{2}ln(Kx^2-1)$$