An intro to SAT and SMT: a user's perspective

Elizabeth Polgreen

https://github.com/polgreen/SPLV-sat-smt-intro

By the end of these lectures you will know ...

- What the satisfiability problem is
- How to encode problems into SAT/SMT
- Which SAT/SMT solvers are available and how to use them, and some useful tools to generate SAT/SMT queries.
- How SAT/SMT solvers are deployed in the wild

You will not know ...

- The detail of how SAT/SMT solvers work

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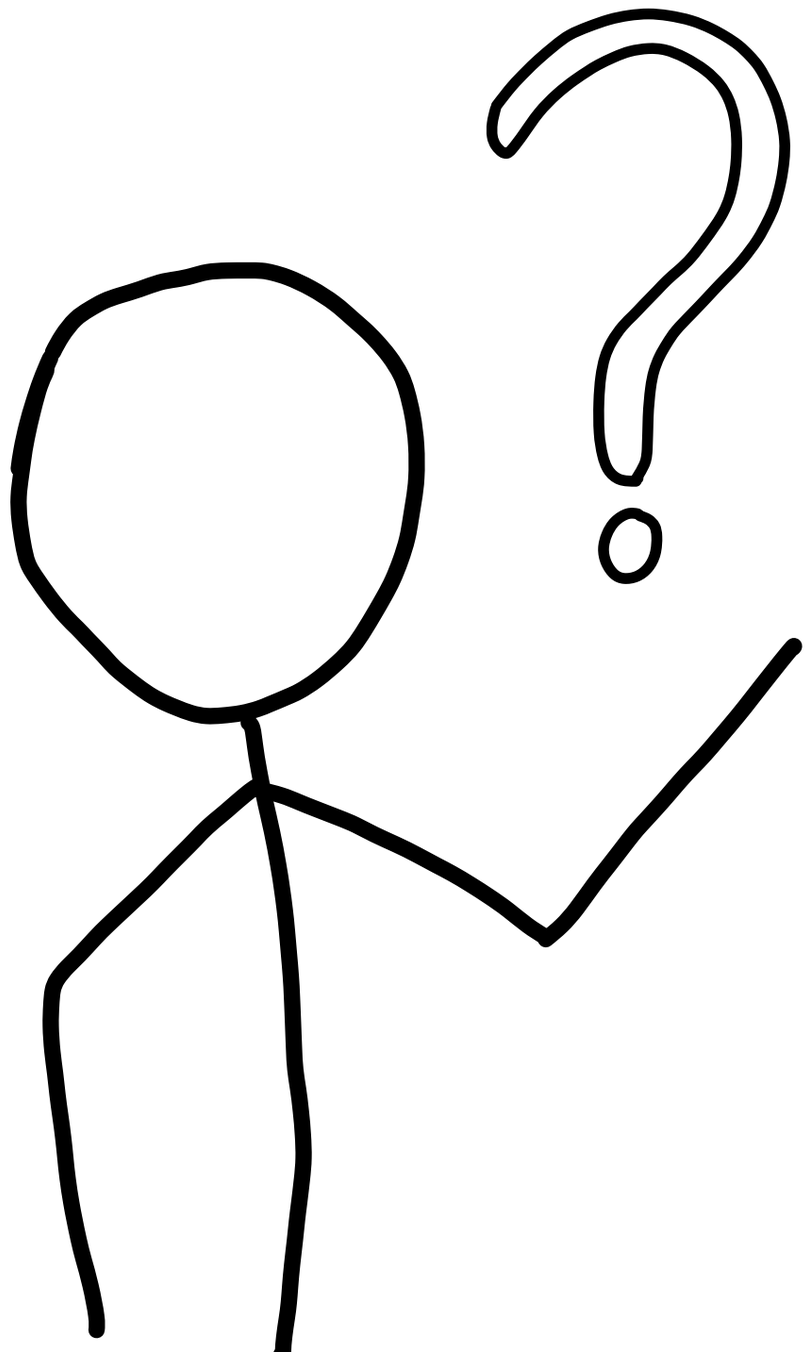
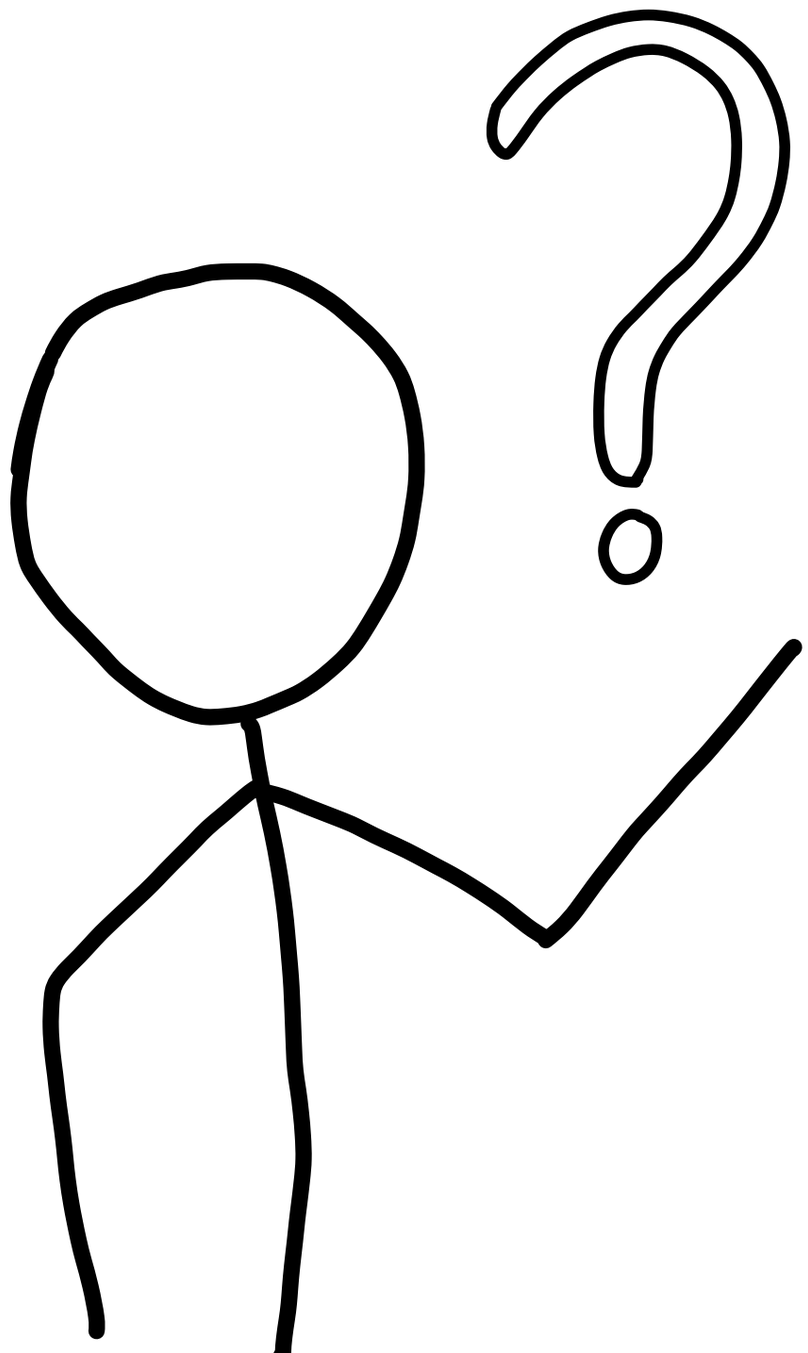
Bonus: how to synthesise programs, using SMT solvers

Who am I?



- Lecturer in LFCS at Edinburgh
- A big user of SAT and SMT solving
- Developer of synthesis algorithms

Who am I?



UCLID5: Modelling, verification and synthesis

CBMC: bounded model checking for C programs

SAT

A : **Boolean**

B : **Boolean**

$\exists A, B$

$A \wedge \neg B$

A : **true**

B : **false**

SAT

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$\exists A, B$

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A : **true**

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SMT

A : **Integer**

B : **Integer**

$\exists A, B$

$A > 0 \wedge B < 0$

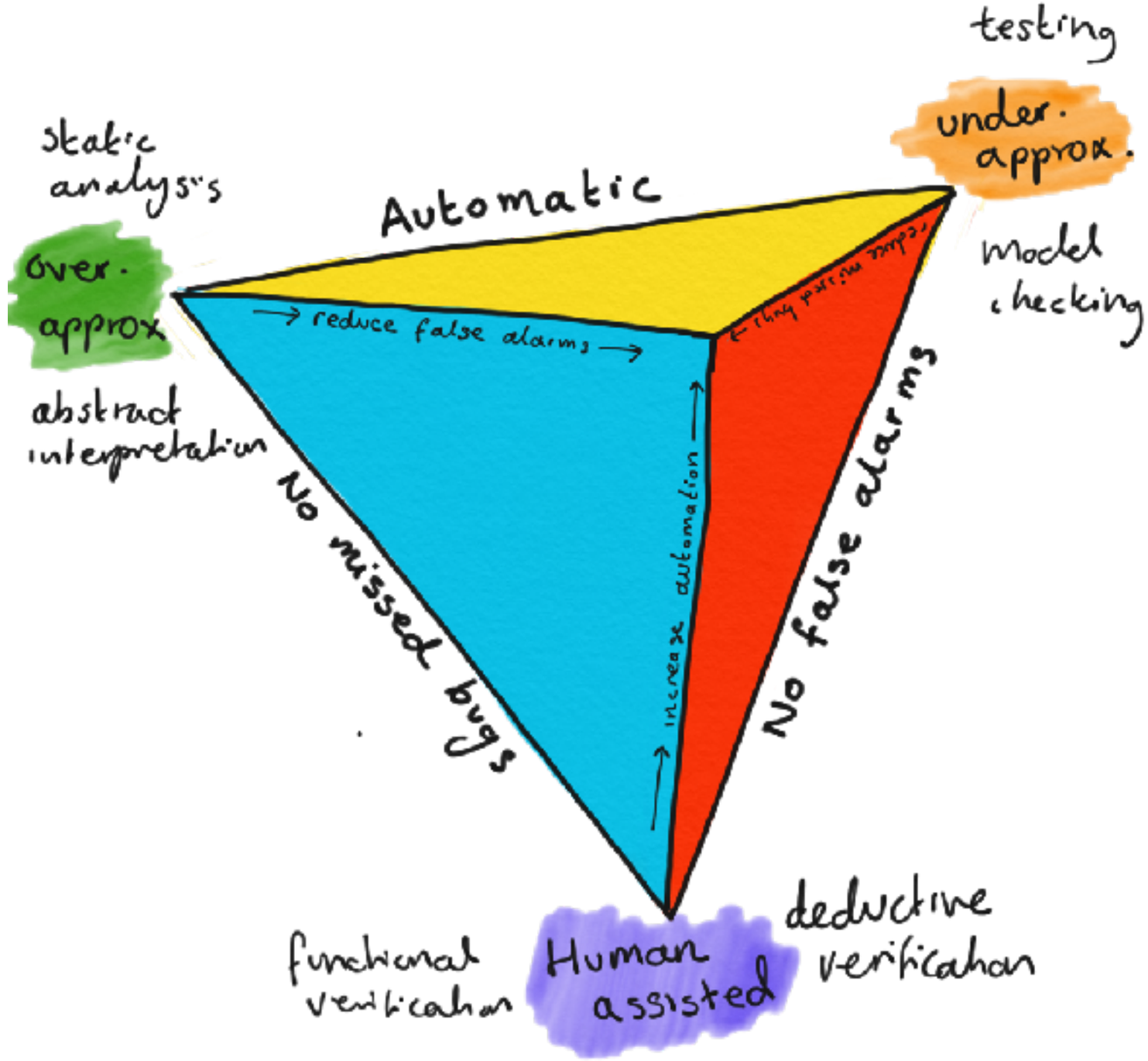
A : **10**

B : **-3**

Why are SAT/SMT important?

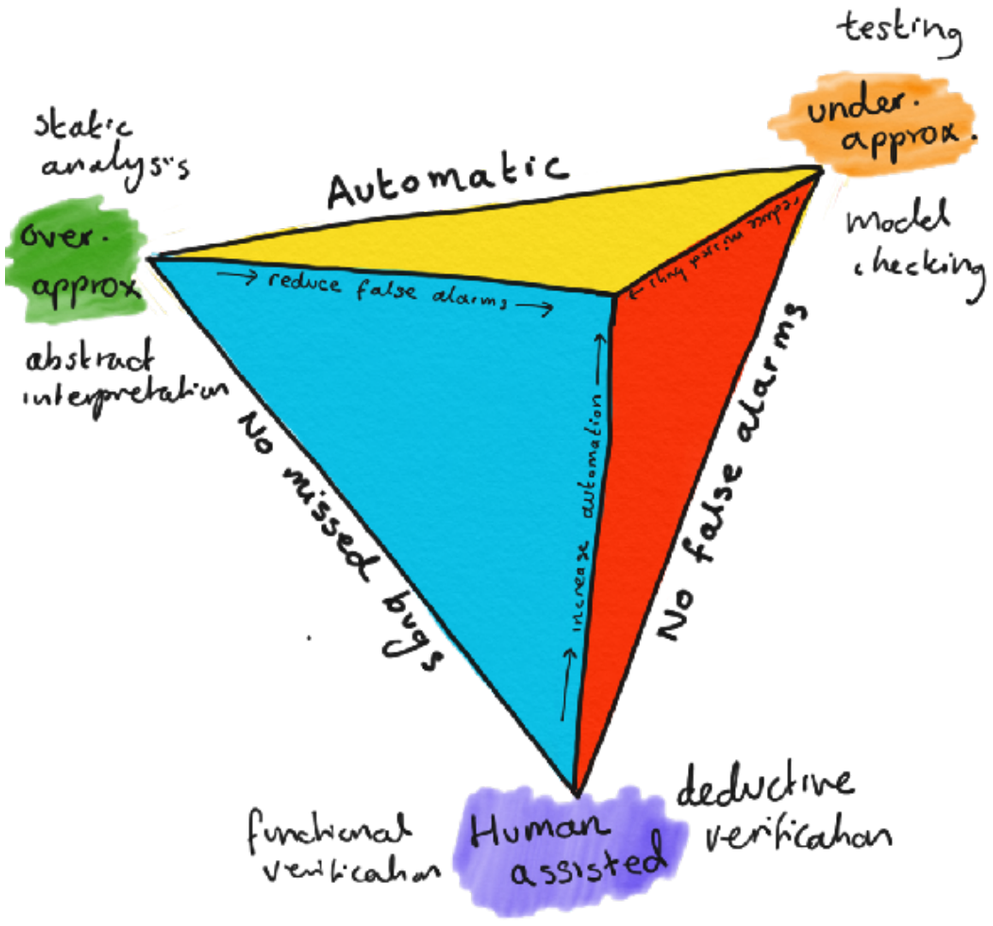
They are used in so many different verification tools!

Why are SAT/SMT important?



(the pyramid of verification - Martin Brain)

(The pyramid of verification)

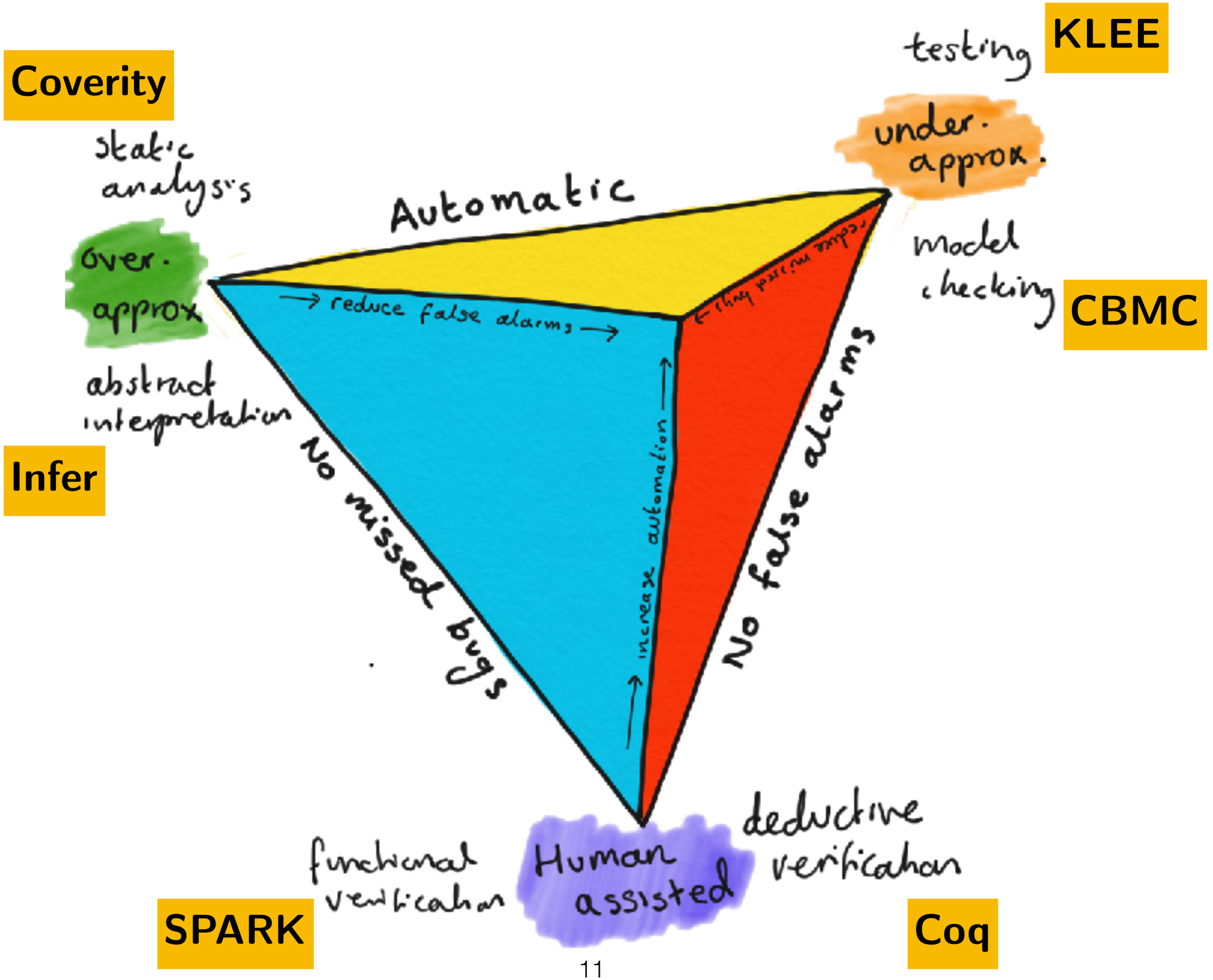


The perfect tool:

- Never misses a bug,
- Never gives a false alarm,
- And is fully automated for all specifications and all programs

This is impossible!

Why are SAT/SMT important?



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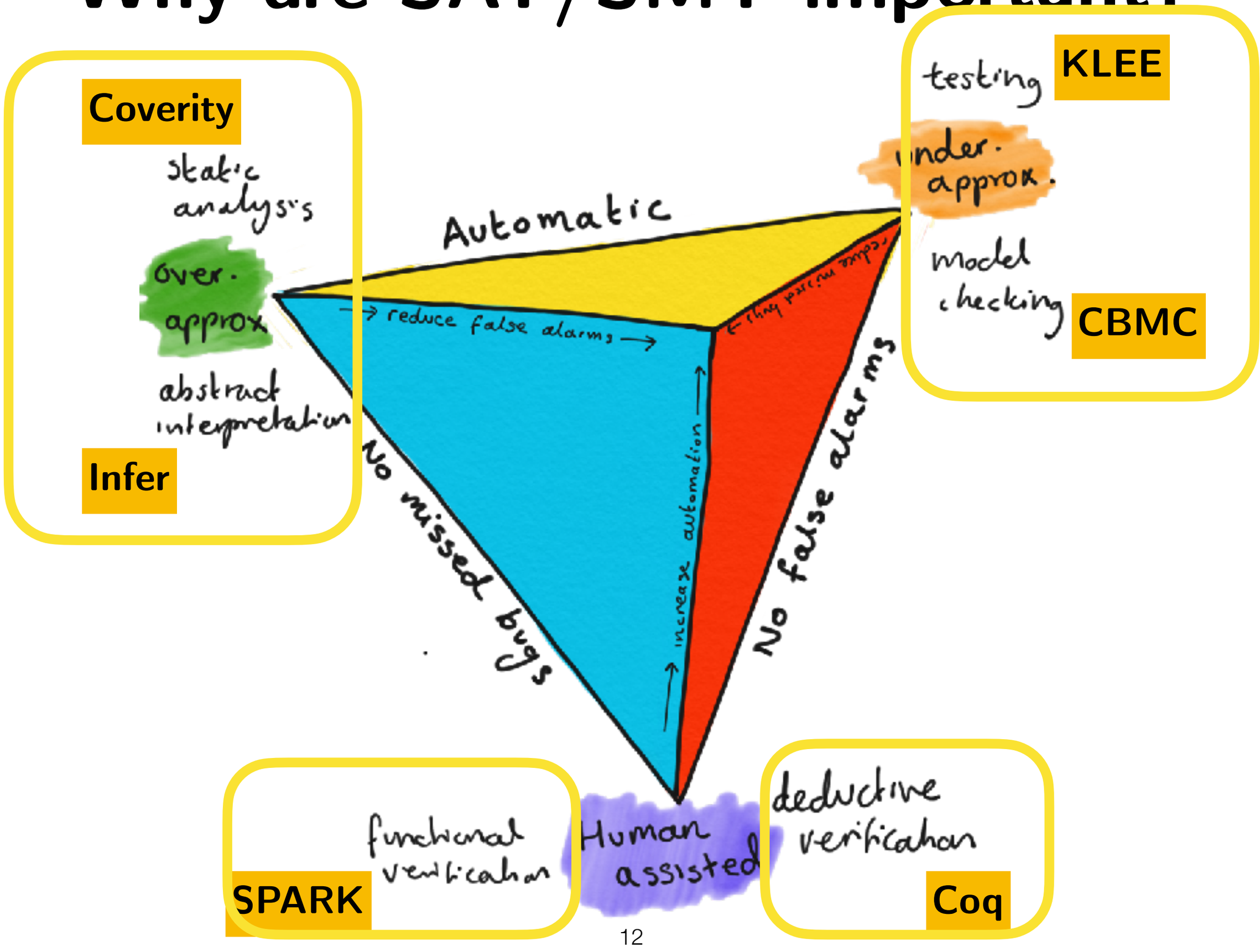


Propositional SAT

A propositional formula is composed from Boolean variables and the operators:

- $\neg$ (negation, 'not', sometimes written '!')
- $\vee$ (disjunction, 'or', sometimes written '||')
- $\wedge$ (conjunction, 'and', sometimes written '&&')
- $\rightarrow$ (implication, $p \rightarrow q \equiv \neg p \vee q$)

Propositional SAT

A formula is:

- Satisfiable if there exist values to the variables such that the formula evaluates to true
- Unsatisfiable if the formula evaluates to false **for all** assignments to the variables
- Valid if the formula evaluates to true **for all** assignments to the variables

Propositional SAT

Given a propositional formula $F(x_1, x_2, x_3, \ldots x_n)$

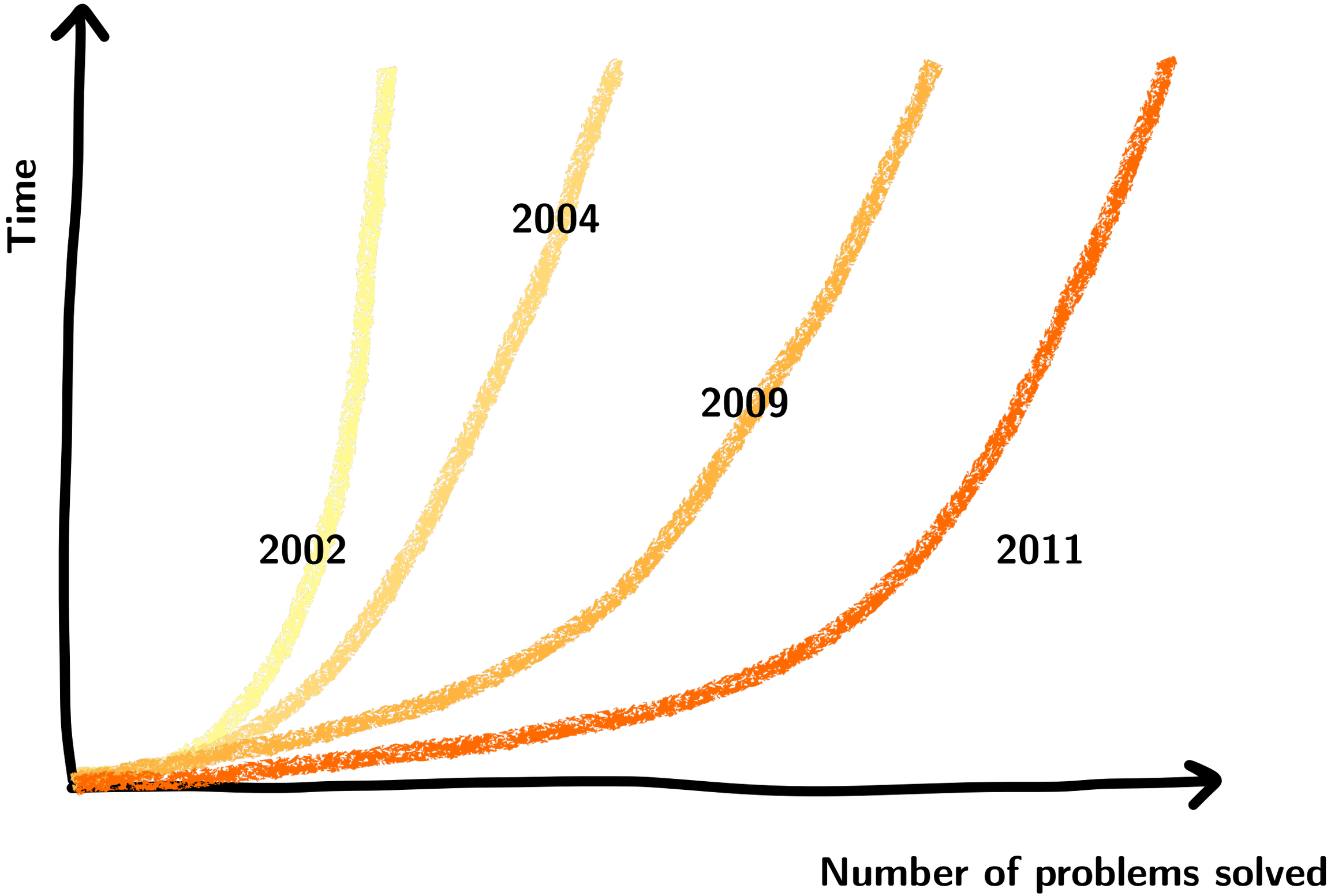
Is F satisfiable?

If yes, return the values to $x_1 \ldots x_n$ that make F true

Complexity

- SAT is the canonical NP-complete problem (no polynomial time algorithm)
- If you can reduce a problem to SAT, the problem is in NP
- Including Mario..

Progress of SAT solvers in SAT competition



Example 1

```
if(!a && !b) h();
else
    if(!a) g();
    else f();
```

```
if(a) f();
else
    if(b) g();
    else h();
```

Are these two code fragments the same?

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Are these two code fragments the same?

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if ¬a ∧ ¬b then h
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Are these two code fragments the same?

$$\texttt{if } \neg a \wedge \neg b \texttt{ then } h \texttt{ else if } \neg a \texttt{ then } g \texttt{ else } f$$

$$\texttt{if } a \texttt{ then } f \texttt{ else if } b \texttt{ then } g \texttt{ else } h$$

Is this formula valid:

$$\begin{array}{ll} & (\neg a \wedge \neg b) \wedge h \ \vee \ \neg(\neg a \wedge \neg b) \wedge (\neg a \wedge g \ \vee \ a \wedge f) \\ \Longleftrightarrow & a \wedge f \ \vee \ \neg a \wedge (b \wedge g \ \vee \ \neg b \wedge h)\ . \end{array}$$

$$(\neg a \wedge \neg b) \wedge h \;\vee\; \neg(\neg a \wedge \neg b) \wedge (\neg a \wedge g \;\vee\; a \wedge f)$$
$$\iff \quad a \wedge f \;\vee\; \neg a \wedge (b \wedge g \;\vee\; \neg b \wedge h)\,.$$

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Look for a counterexample?

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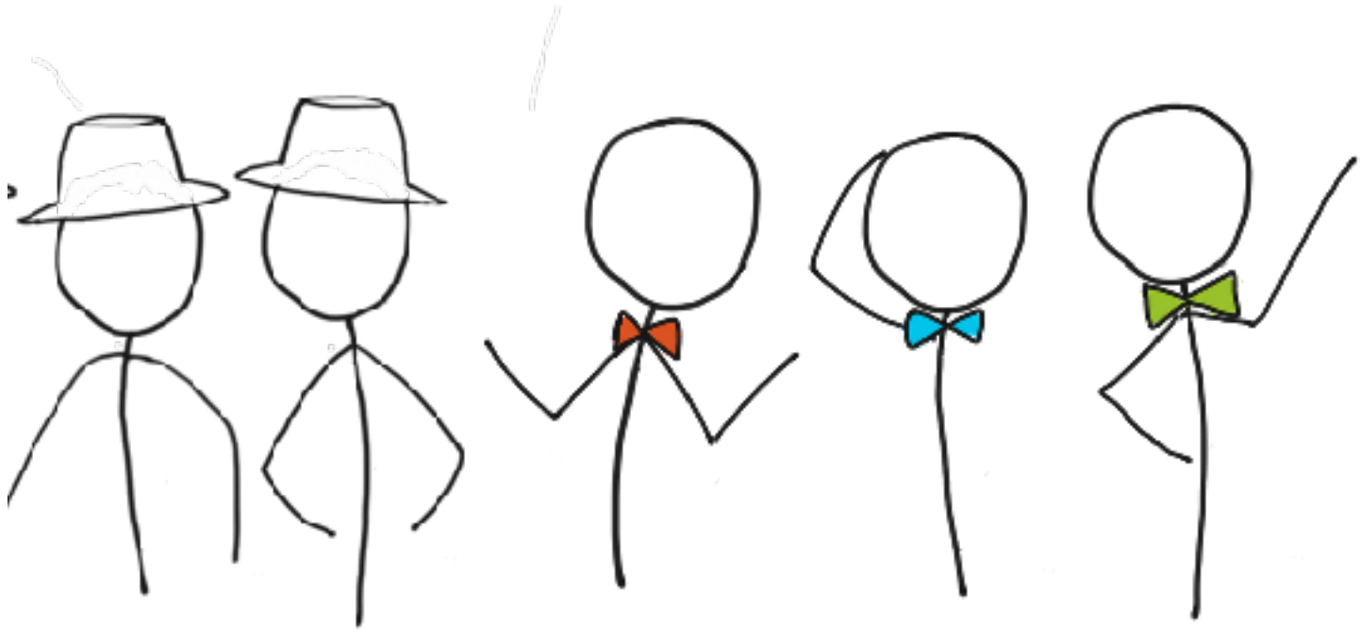
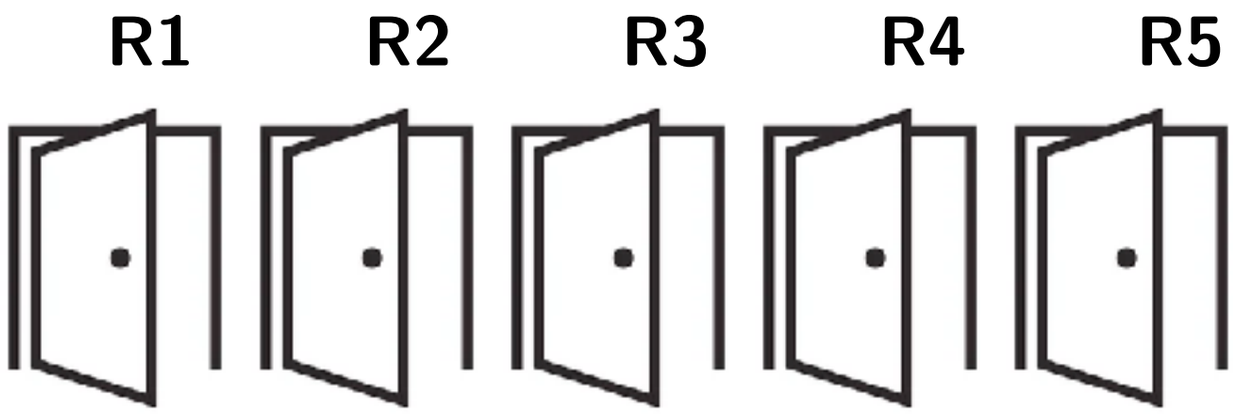


Dr A Dr B Prof C Prof D Prof E

Dr A hates everyone except Dr B

Professor D only wants to be neighbours with other professors



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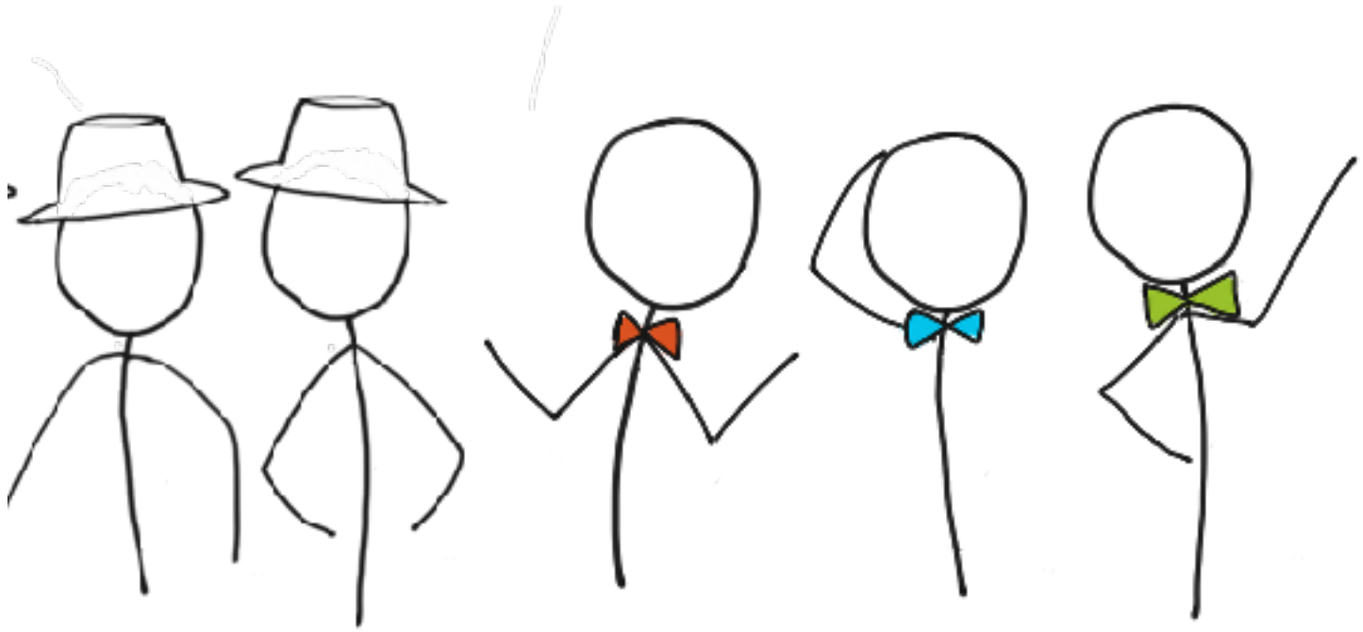

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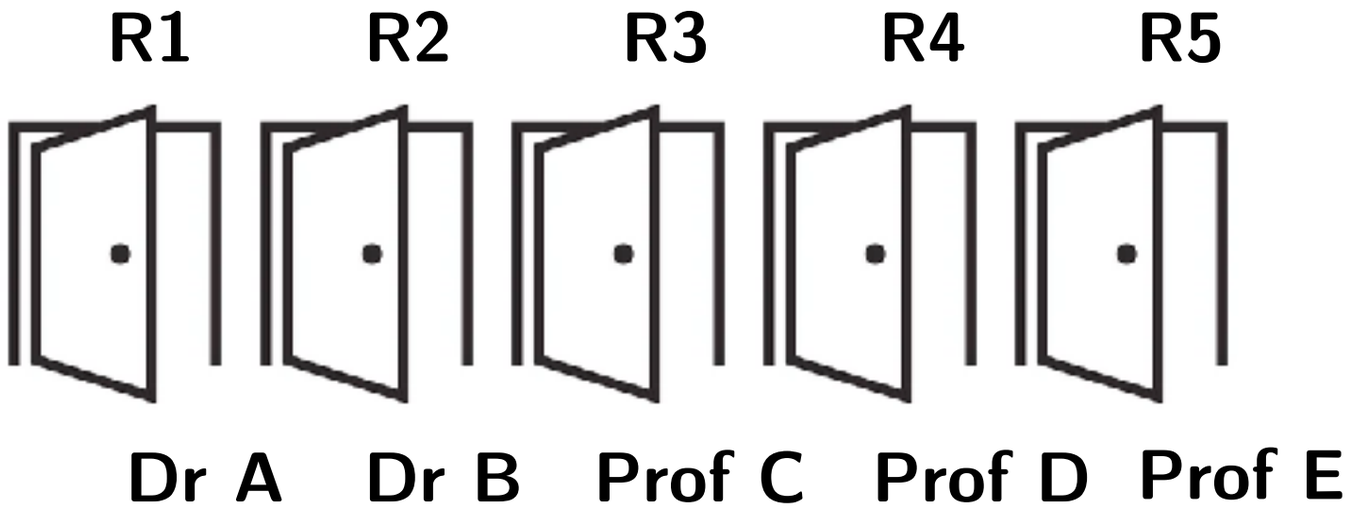
SAT



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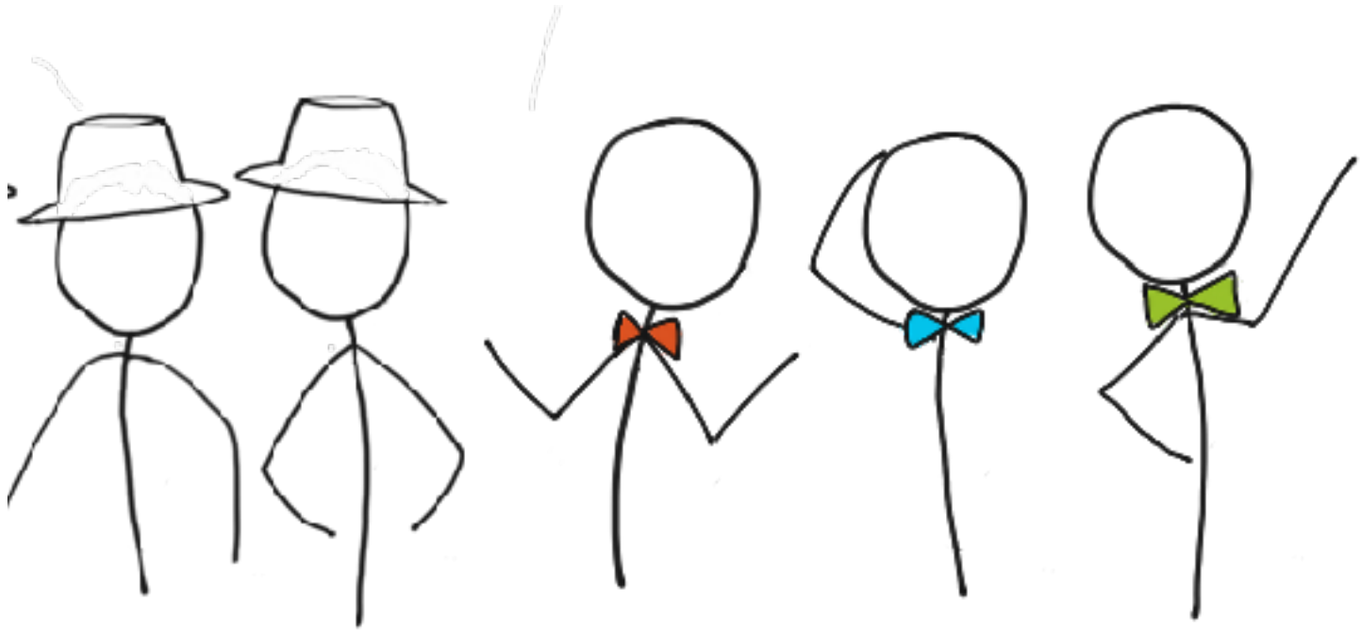

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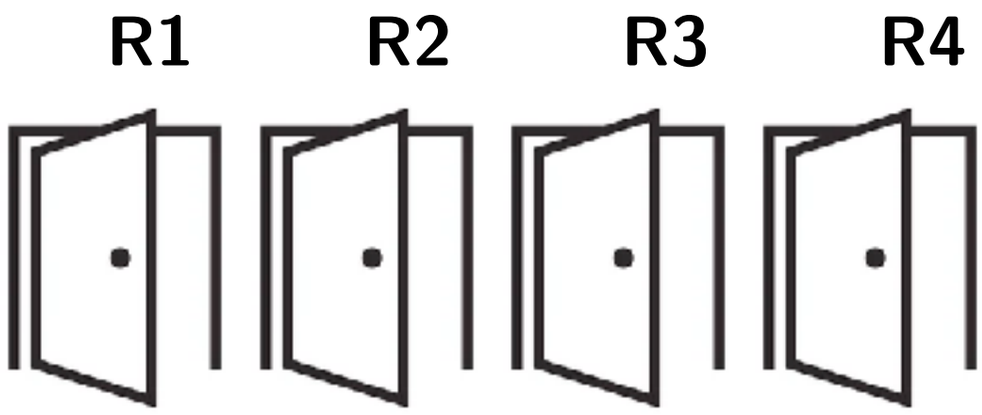
UNSAT



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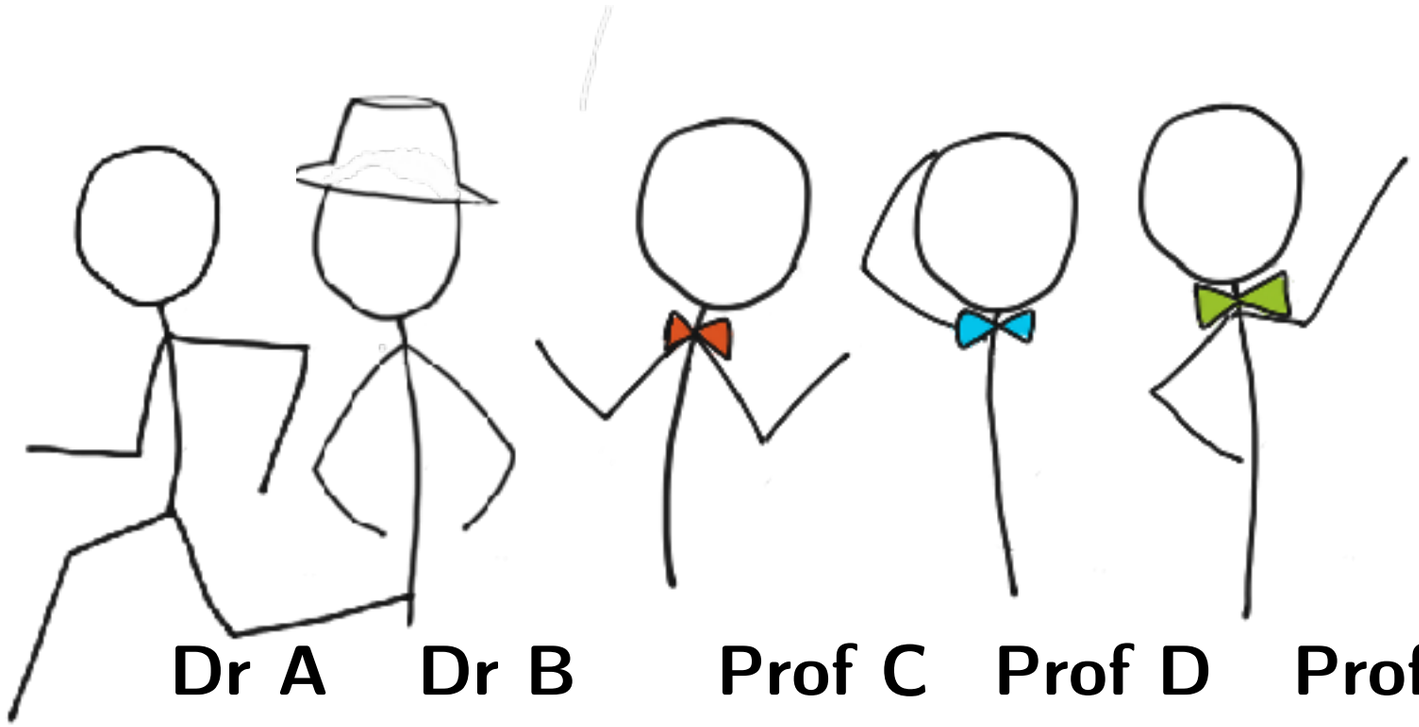

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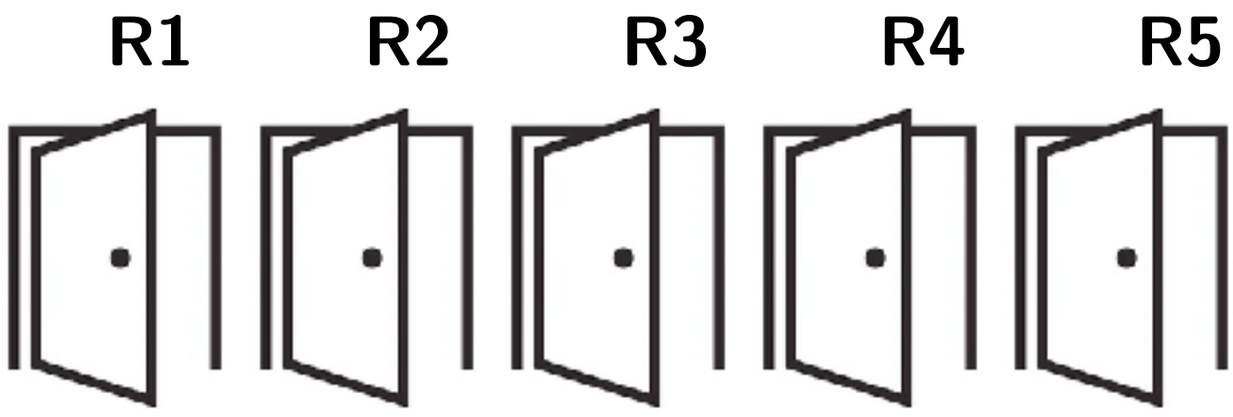

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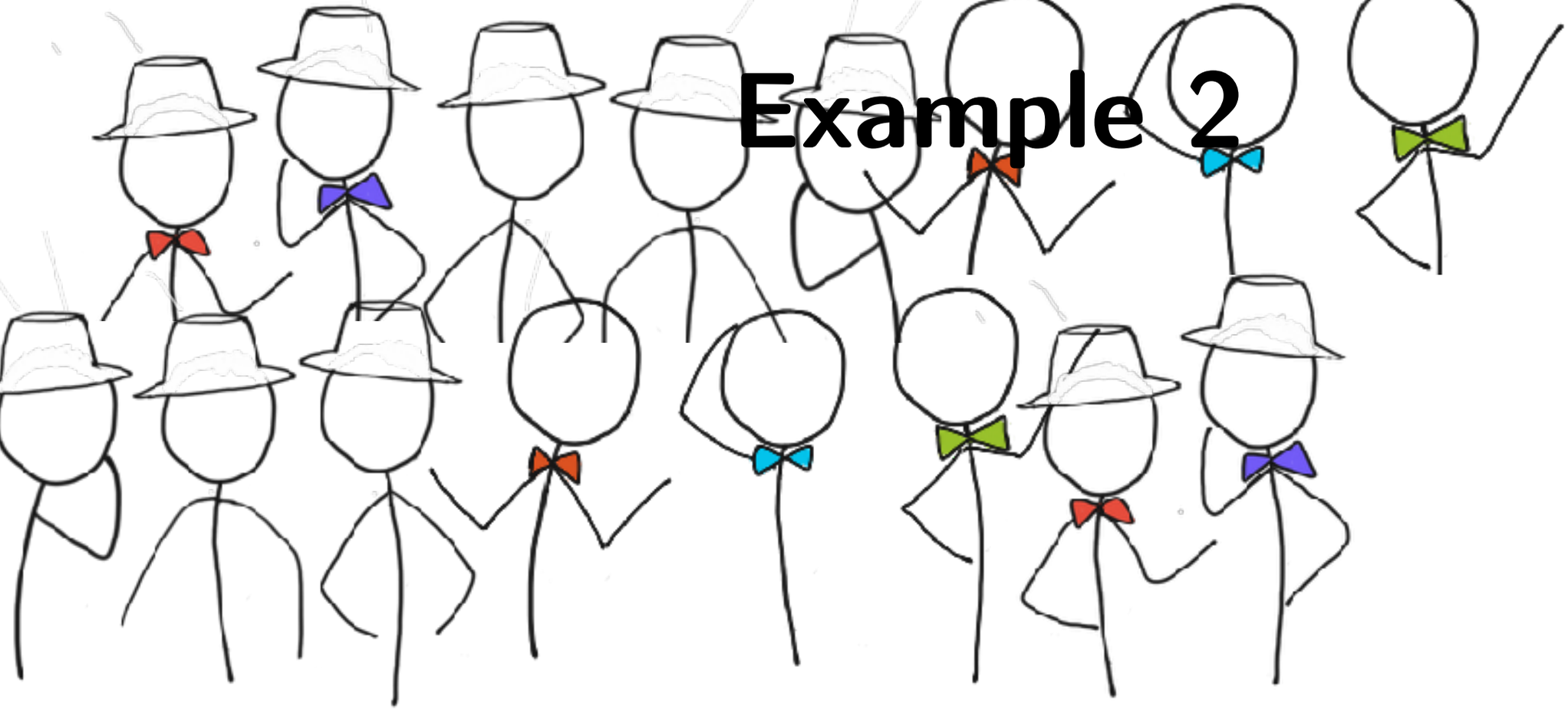


Dr A hates ev

Dr B will only

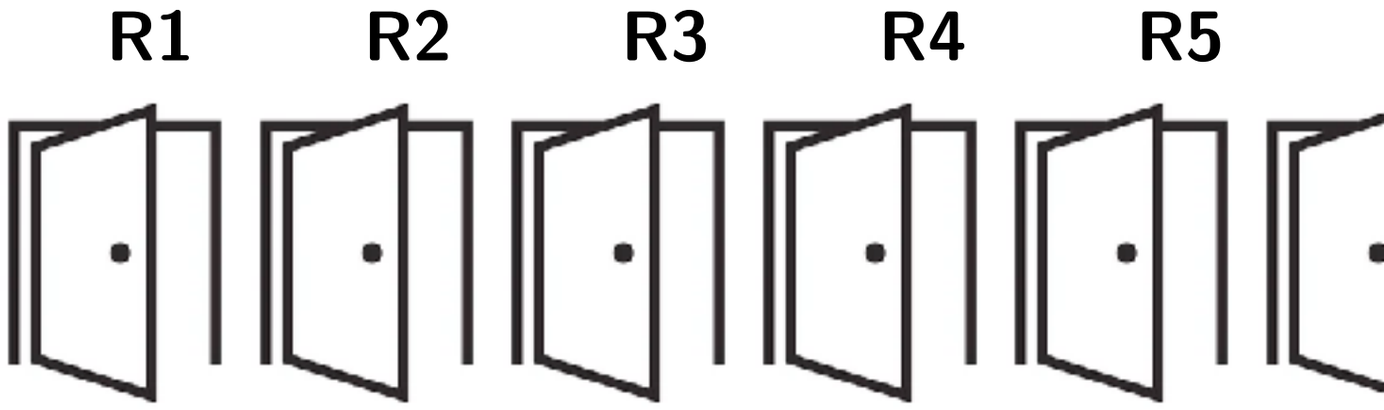
Professor C or
neighbours wit

....

Dr X hates ev

Dr Y only wan

?



Example 2

$A = \{a_1, \ldots a_n\}$ is a set of academics.

$R = \{r_1 \ldots r_k\}$ is a set of offices.

E is a set of pairs of academics who refuse to be office neighbours with each other.

Find an allocation of offices so that all the academics are happy.

Example 2

Define $X = \{x_{ij} \mid i \in \{1,\ldots n\}, j \in \{1,\ldots k\}\}$.

x_{ij} is true iff academic a_i is allocated to office r_j.

$$\bigwedge_{i=1}^{n} \bigvee_{j=1}^{k} x_{ij}$$ every academic is allocated at least one office

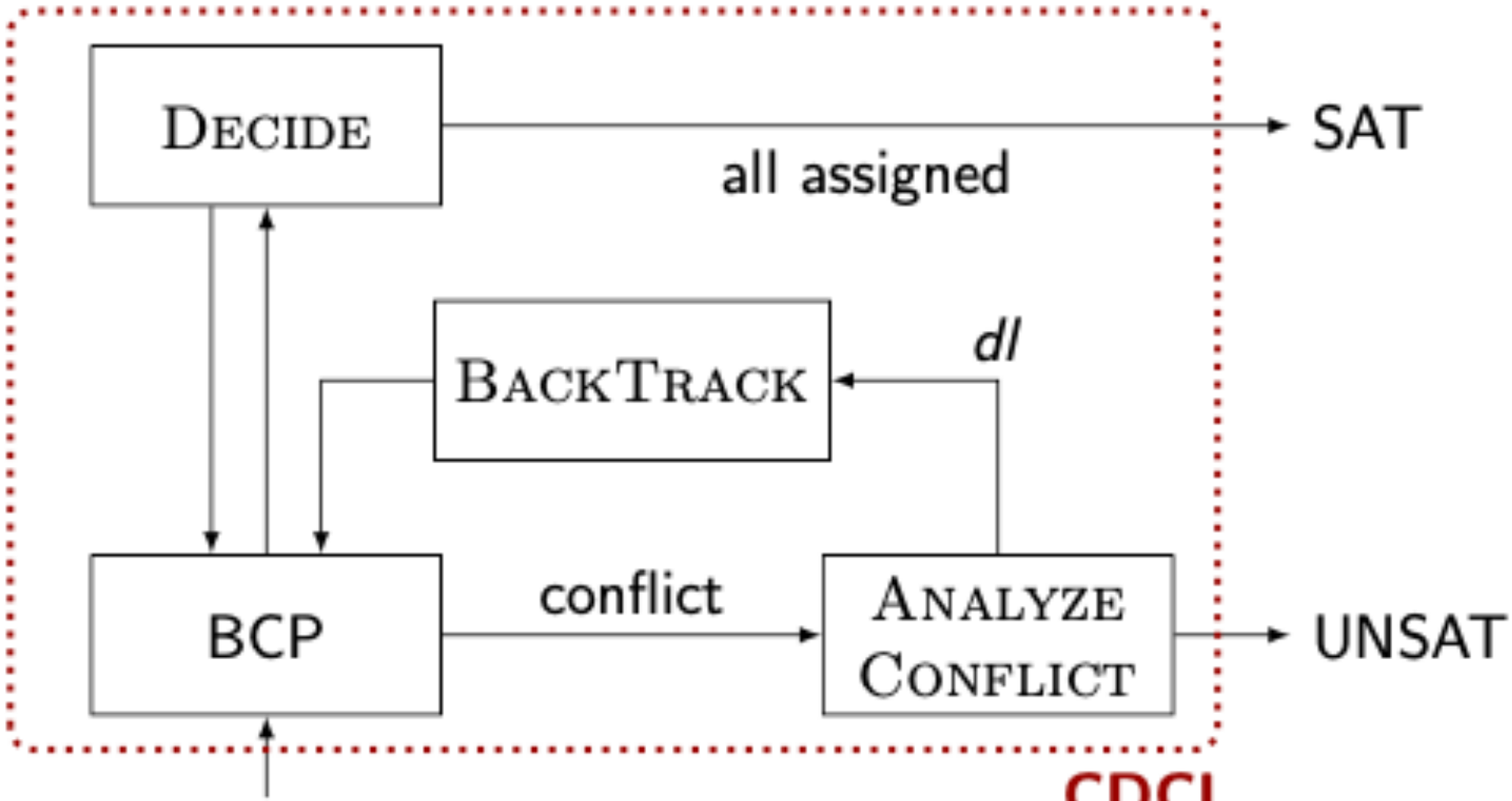
$$\bigwedge_{i=1}^{n} \bigwedge_{j=1}^{k-1} (x_{ij} \implies \bigwedge_{j<t\leq k} \neg x_{it})$$ but no more than one!

For each $(i,j) \in E$, $\bigwedge_{t=1}^{n} (x_{it} \implies (\neg x_{jt+1} \wedge \neg x_{jt-1}))$

And even professor D's preferences are taken into account

CDCL (high-level)

SAT solvers use Conflict Driven Clause Learning. We won't cover this in detail here.



How do we use a SAT solver?

- Formula needs to be in CNF
- Formula is translated to DIMACS

CNF

SAT solvers need formula in Conjunctive Normal Form (CNF), i.e., a disjunction of literals

Terminology:

- An atom p is a propositional symbol
- A literal l is an atom p or its negation $\neg p$
- A clause C is a disjunction of literals $l_1 \vee \ldots \vee l_n$
- A CNF formula is a conjunction of clauses $C_1 \wedge \ldots \wedge C_m$

Tseitsin Transformation

Translates a formula into an equisatisfiable CNF formula in linear time by :

- introducing a fresh variable for every non-atomic sub-formula
- Add a constraint that gives equivalence of new variable with subformula

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Transformation rules for three basic operators

formula	$p \leftrightarrow$ formula	rewritten in CNF
$\neg A$	$(\neg A \rightarrow p) \wedge (p \rightarrow \neg A)$	$(A \vee p) \wedge (\neg A \vee \neg p)$
$A \wedge B$	$(A \wedge B \rightarrow p) \wedge (p \rightarrow A \wedge B)$	$(\neg A \vee \neg B \vee p) \wedge (A \vee \neg p) \wedge (B \vee \neg p)$
$A \vee B$	$(p \rightarrow A \vee B) \wedge (A \vee B \rightarrow p)$	$(A \vee B \vee \neg p) \wedge (\neg A \vee p) \wedge (\neg B \vee p)$

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$A \vee B$	$(p \rightarrow A \vee B) \wedge (A \vee B \rightarrow p)$	$(A \vee B \vee \neg p) \wedge (\neg A \vee p) \wedge (\neg B \vee p)$

$$\phi = ((p \vee q) \wedge r) \rightarrow (\neg s)$$

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$x_1 \leftrightarrow \neg s$

$x_2 \leftrightarrow p \vee q$

$x_3 \leftrightarrow x_2 \wedge r$

$x_4 \leftrightarrow x_3 \rightarrow x_1$

Introduce fresh variables

Tseitsin Transformation

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$T(\phi) =$

$(x_1 \leftrightarrow \neg s) \wedge$

$(x_2 \leftrightarrow p \vee q) \wedge$

$(x_3 \leftrightarrow x_2 \wedge r) \wedge$

$(x_4 \leftrightarrow x_3 \rightarrow x_1) \wedge x_4$

Conjunct all substitutions

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Convert to CNF using rules above and simplify

Tseitsin Transformation

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$$T(\phi) = (\neg p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$$

Maybe you should check this...?

Dimacs

- header line: p cnf <variables> <clauses>,
  where <variables> <clauses> are decimal numbers for the number of variables and clauses in the formula respectively

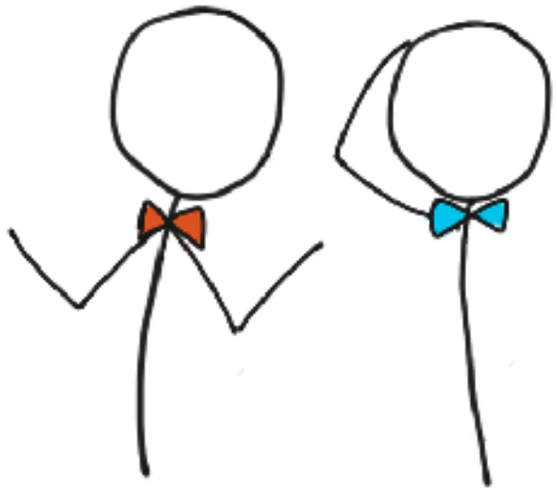
- one clause per line of file with a 0 at the end:

- each variable has an decimal number, and – indicates the negation of that variable.

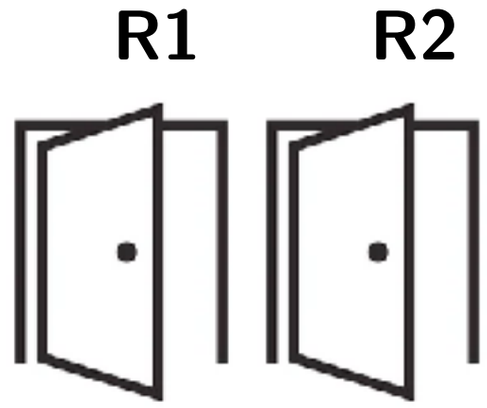
$$T(\phi) = (\neg p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$$

```
p cnf 4 2
-1 -3 -4 0
-2 -3 -4 0
```

Example 2



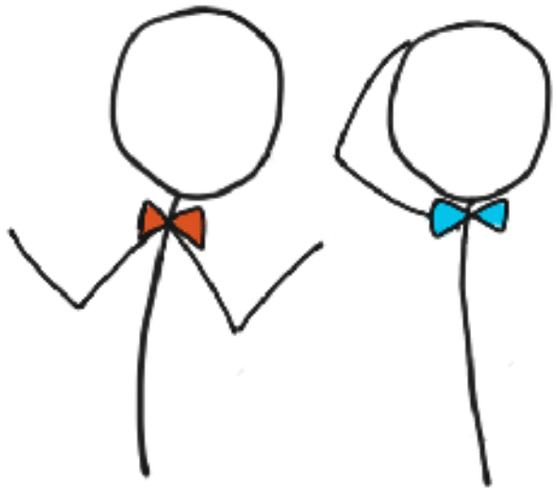
Prof C Prof D



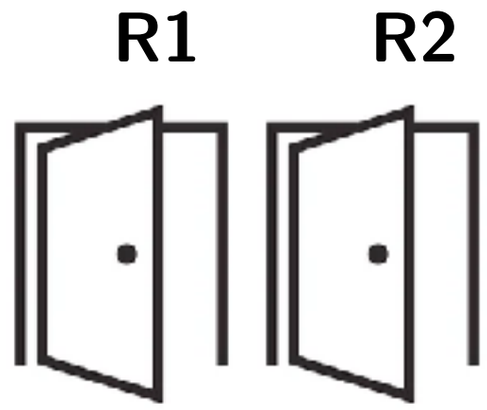
$$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge$$

$$(x_{11} \rightarrow \neg x_{12}) \wedge (x_{12} \rightarrow \neg x_{11}) \wedge (x_{21} \rightarrow \neg x_{22}) \wedge (x_{22} \rightarrow \neg x_{21})$$

Example 2



Prof C Prof D



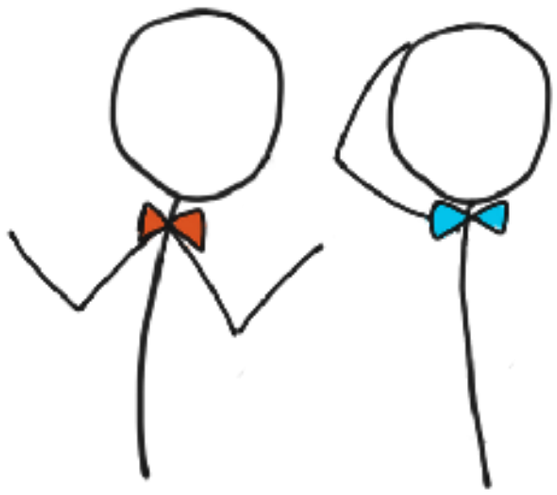
$$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge$$

$$(x_{11} \rightarrow \neg x_{12}) \wedge (x_{12} \rightarrow \neg x_{11}) \wedge (x_{21} \rightarrow \neg x_{22}) \wedge (x_{22} \rightarrow \neg x_{21})$$

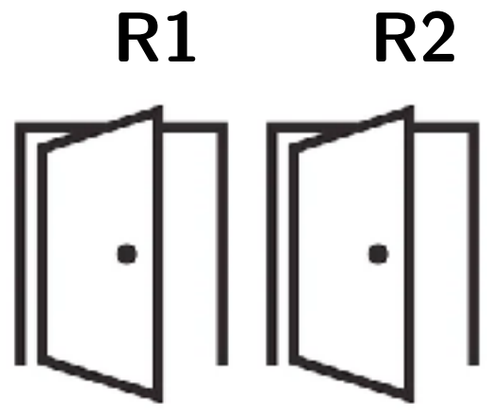
$$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge$$

$$(\neg x_{11} \vee \neg x_{12}) \wedge (\neg x_{12} \vee \neg x_{11}) \wedge (\neg x_{21} \vee \neg x_{22}) \wedge (\neg x_{22} \vee \neg x_{21})$$

Example 2



Prof C Prof D



$$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge$$

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$$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge$$

$$(\neg x_{11} \vee \neg x_{12}) \wedge (\neg x_{12} \vee \neg x_{11}) \wedge (\neg x_{21} \vee \neg x_{22}) \wedge (\neg x_{22} \vee \neg x_{21})$$

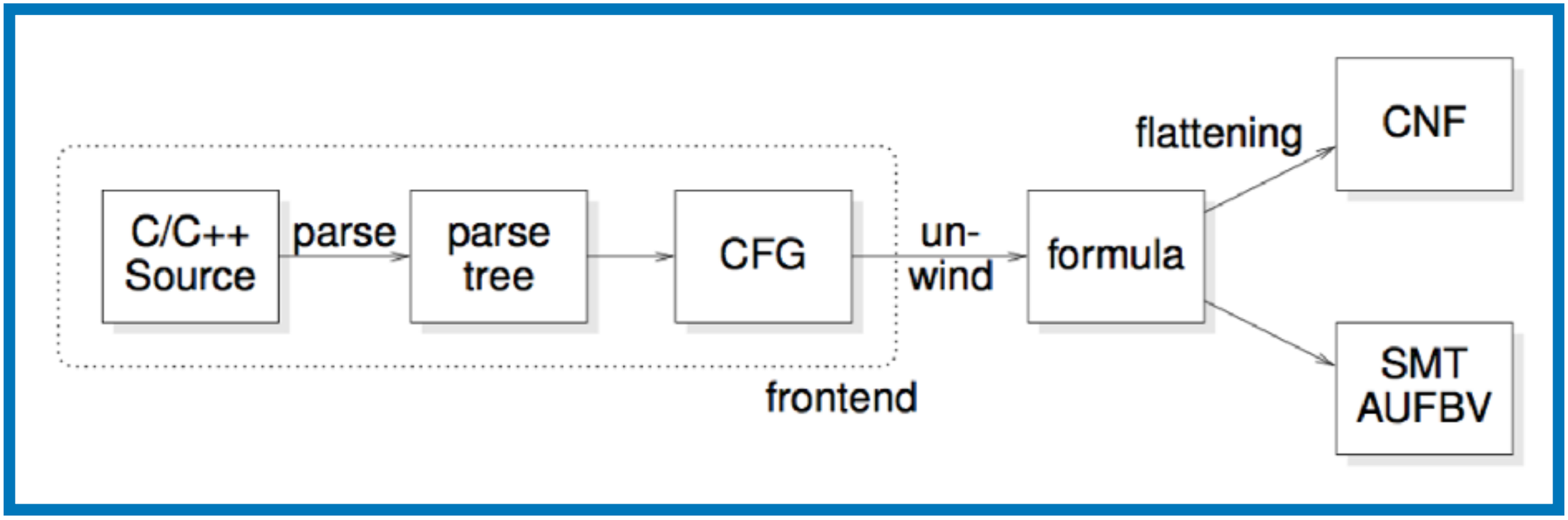
```
p cnf 4 6
1 2 0
3 4 0
-2 -1 0
-1 -2 0
-4 -3 0
-3 -4 0
```

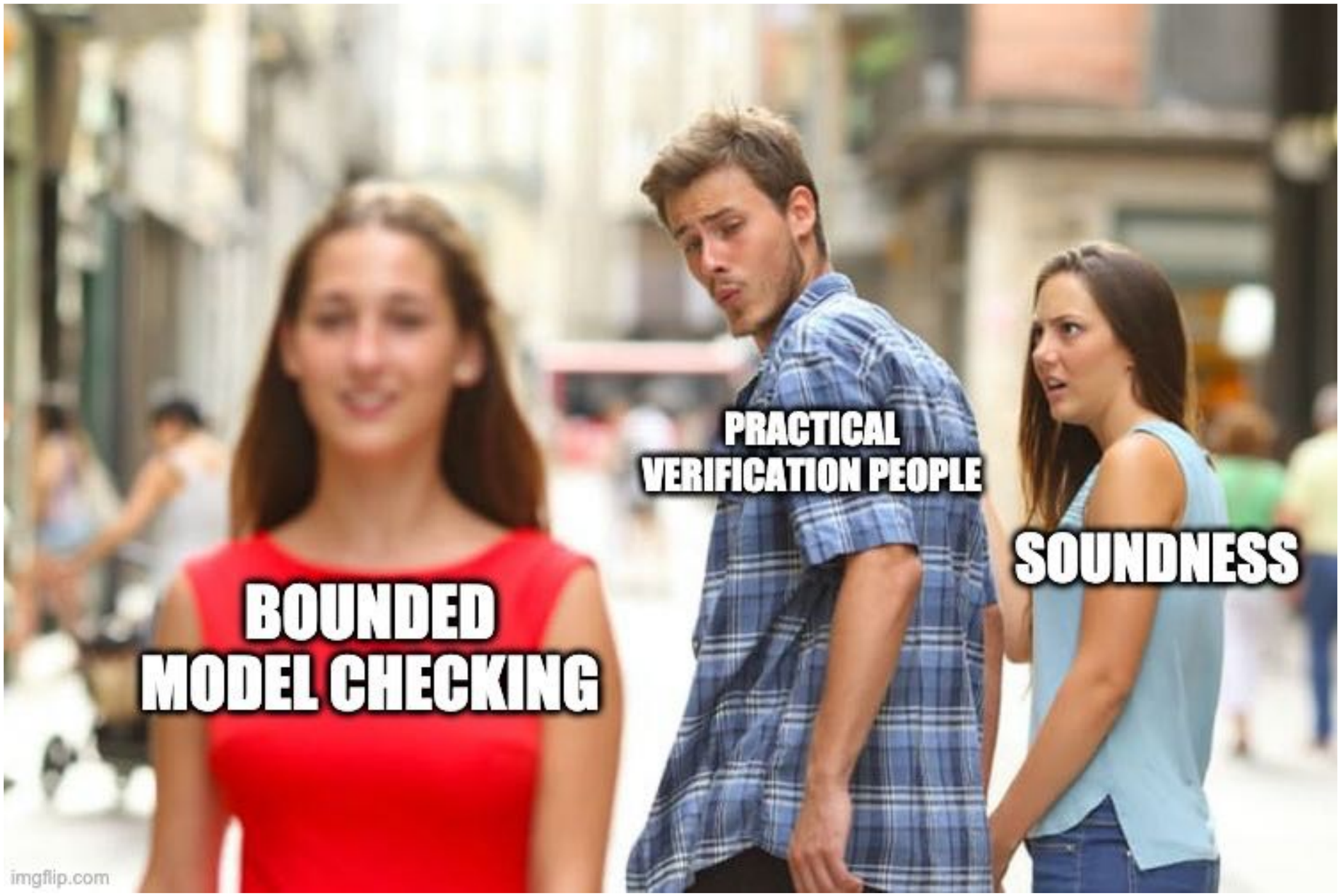
But realistically, no-one does these transformations and writes these dimacs files by hand...

CBMC

- Bounded Model Checking tool for C programs
- Based on producing a SAT formula for all possible paths through a program (with loops unwound to a bound N), and then asking a SAT solver if there is a path that violates an assertion
- Industrial users: Toyota, AWS

CBMC


C/C++
Source
parse
parse
tree
CFG
un-
wind
formula
flattening
CNF
SMT
AUFBV
frontend


PRACTICAL
VERIFICATION PEOPLE
BOUNDED
MODEL CHECKING
SOUNDNESS
imgflip.com

CBMC - example 3

```
bool x;
char y=8, z=0, w=0;

if(x)
  z = y-1;
else
  w = y+1;

assert(z==7 || w==9);
```

CBMC - example 3

```
bool x;
char y=8, z=0, w=0;

if(x)
  z = y-1;
else
  w = y+1;

assert(z==7 || w==9);
```

$$
\begin{aligned}
&(y = 8) \ \wedge \\
&(z = x\,?\,y-1 : 0) \ \wedge \\
&(w = x\,?\,0 : y+1) \ \wedge \\
&(z \neq 7) \ \wedge \\
&(w \neq 9)
\end{aligned}
$$

Example 1 - in CBMC

```
if(!a && !b) h();
else
    if(!a) g();
    else f();
```

```
if(a) f();
else
    if(b) g();
    else h();
```

Example 1 - in CBMC

```
typedef __CPROVER_bool bool;
bool a,b,g,f,h;

bool f1(bool a, bool b, bool f, bool g, bool h)
{
  if(!a && !b)
    return h;
  else if(!a)
    return g;
  else
    return f;
}

bool f2(bool a, bool b, bool f, bool g, bool h)
{
  if(a)
    return f;
  else if(b)
    return g;
  else
    return h;
}

void main(void)
{
  a = nondet();
  b = nondet();
  g = nondet();
  f = nondet();
  h = nondet();
  __CPROVEr_assert(f1(a,b,f,g,h)==f2(a,b,f,g,h),"check equivalent");
}
```

CBMC - example 4

- That was a simple example.. there are much harder ones

Using model checking to triage the severity of security bugs in the Xen hypervisor.

Should we wake the developer up?

Byron Cook[1,2], Björn Döbel[1], Daniel Kroening[1,3], Norbert Manthey[1],
Martin Pohlack[1], Elizabeth Polgreen[5,6], Michael Tautschnig[1,4], Pawel Wieczorkiewicz[1]

[1] Amazon Web Services
[2] University College London
[3] University of Oxford
[4] Queen Mary University of London
[5] UC Berkeley
[6] Edinburgh University

Problem:

- Most systems have layers of security
- Most bugs are not critical security issues
- BUT determining which ones are is a difficult, manual task

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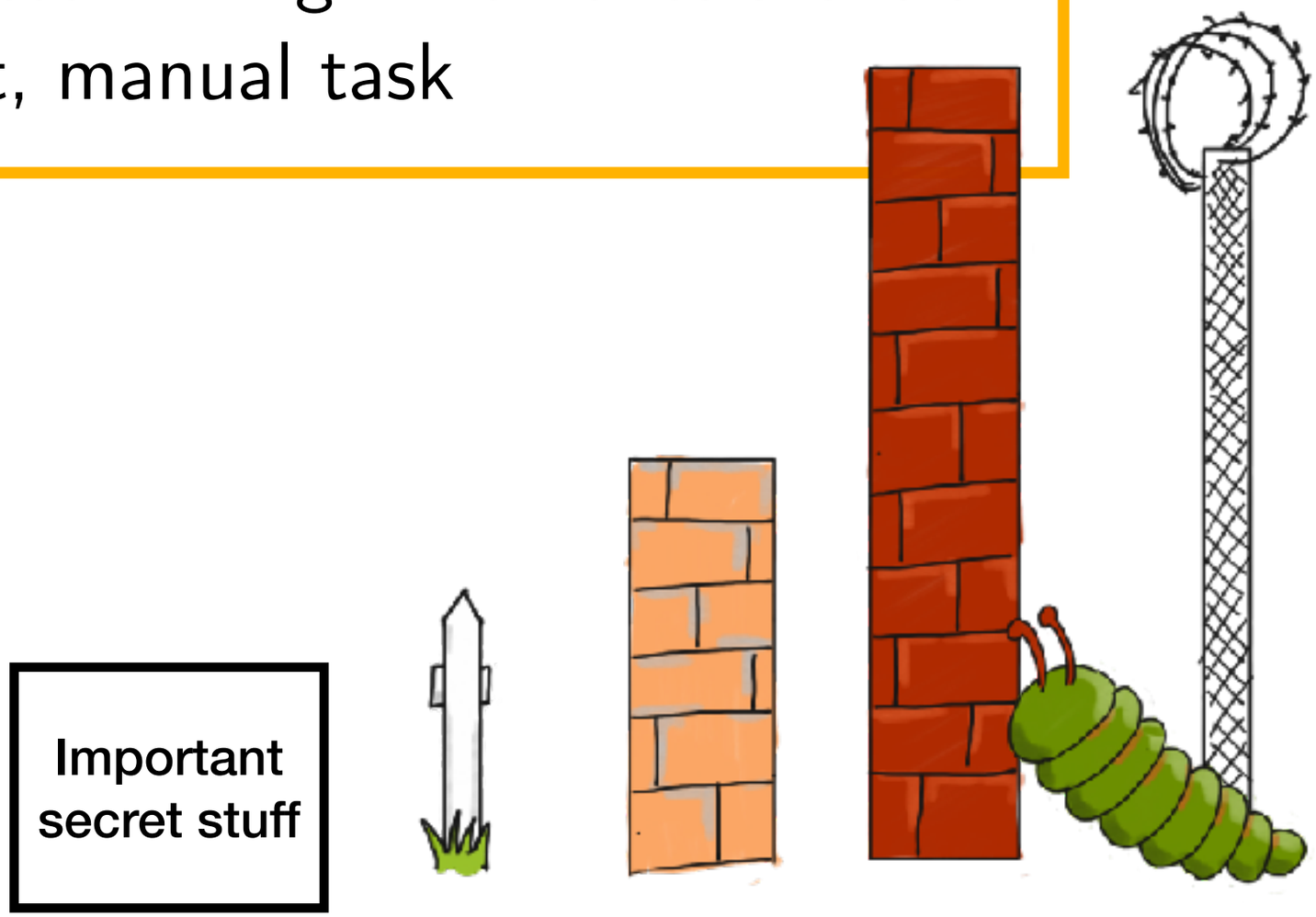


Problem:

- Most systems have layers of security
- Most bugs are not critical security issues
- BUT determining which ones are is a difficult, manual task

Solution:

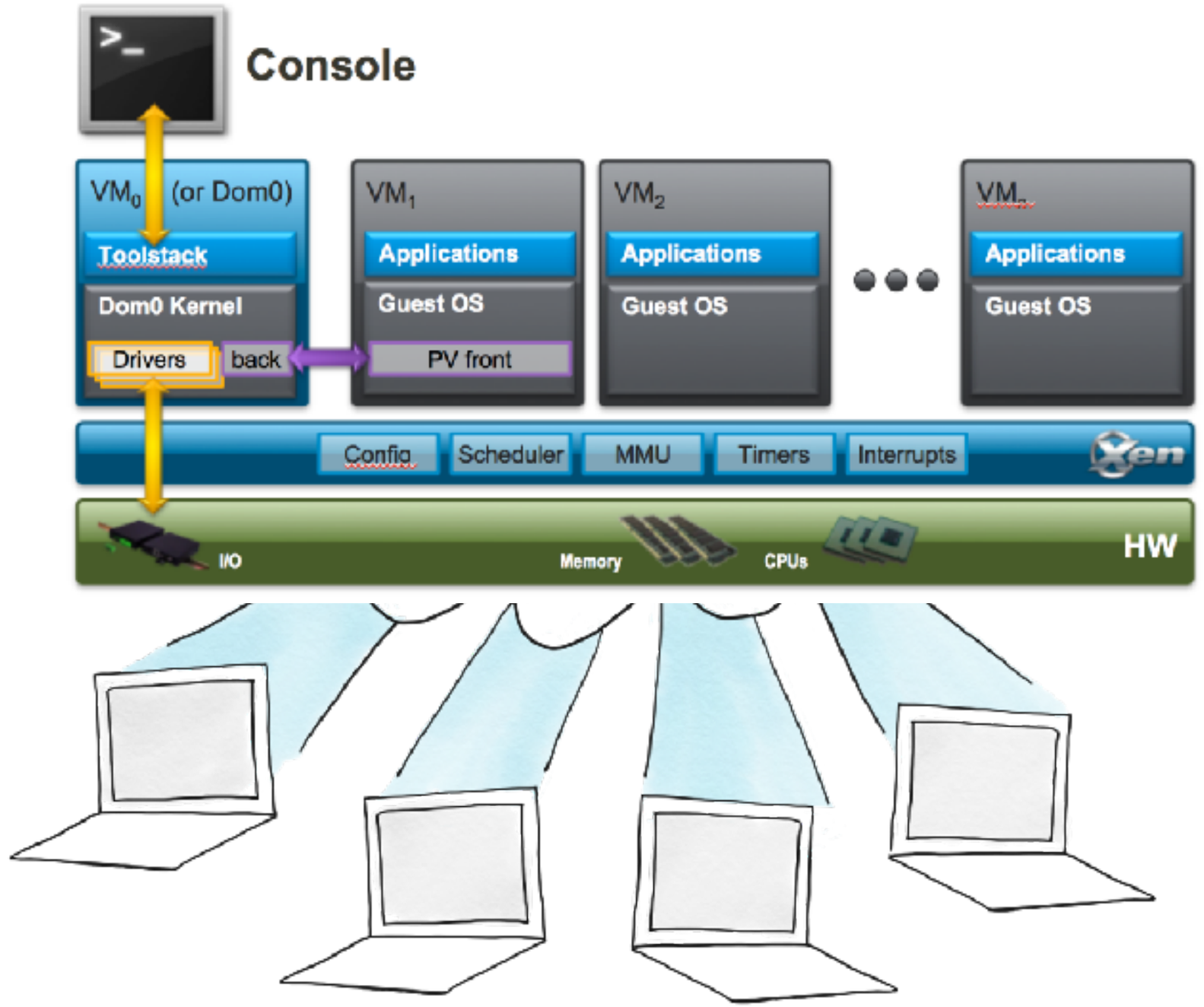
- We show how to use model checking to triage the severity of security bugs
- We make adaptations to CBMC, a bounded model checker for C programs, so that it scales to big code bases
- Case study: Xen

What is Xen?

Hypervisor: creates and runs virtual machines

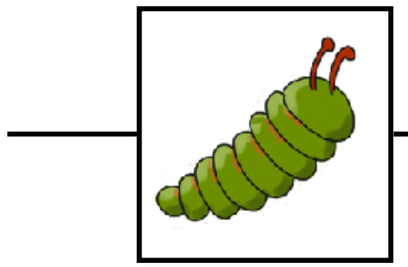
Amazon use a custom version of Xen on some EC2 servers

What is Xen?


Console
VM0 (or Dom0)
Toolstack
Dom0 Kernel
Drivers
back
VM1
Applications
Guest OS
PV front
VM2
Applications
Guest OS
VMn
Applications
Guest OS
Config
Scheduler
MMU
Timers
Interrupts
I/O
Memory
CPUs
HW

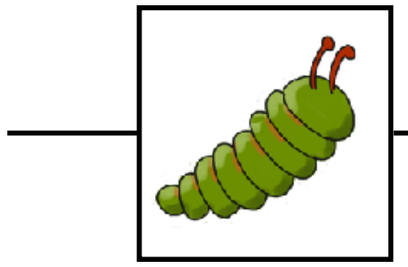
What happens when a bug is discovered?

What happens when a bug is discovered?

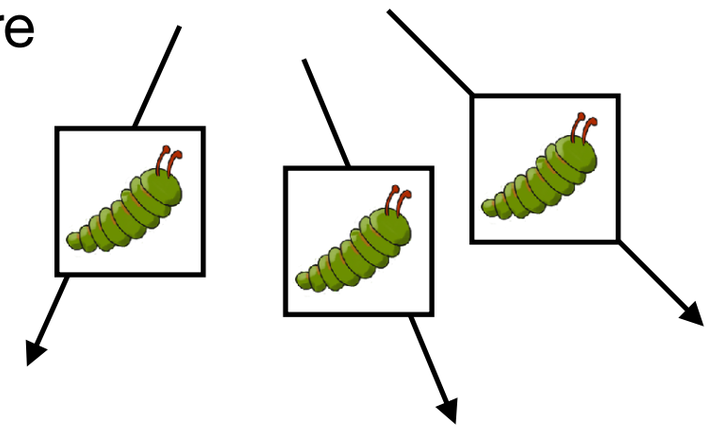


Responsible disclosure

What happens when a bug is discovered?



Responsible disclosure



Members of the X

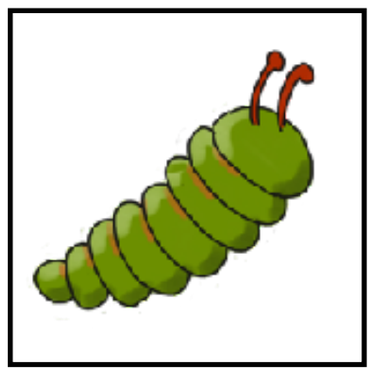
XSA: Xen Security Announcement

```
ISSUE DESCRIPTION
=================

The x86 instruction CMPXCHG8B is supposed to ignore legacy operand
size overrides; it only honors the REX.W override (making it
CMPXCHG16B).  So, the operand size is always 8 or 16.

When support for CMPXCHG16B emulation was added to the instruction
emulator, this restriction on the set of possible operand sizes was
relied on in some parts of the emulation; but a wrong, fully general,
operand size value was used for other parts of the emulation.

As a result, if a guest uses a supposedly-ignored operand size prefix,
a small amount of hypervisor stack data is leaked to the guests: a 96
bit leak to guests running in 64-bit mode; or, a 32 bit leak to other
guests.
```



Advisories, publicly released or pre-released

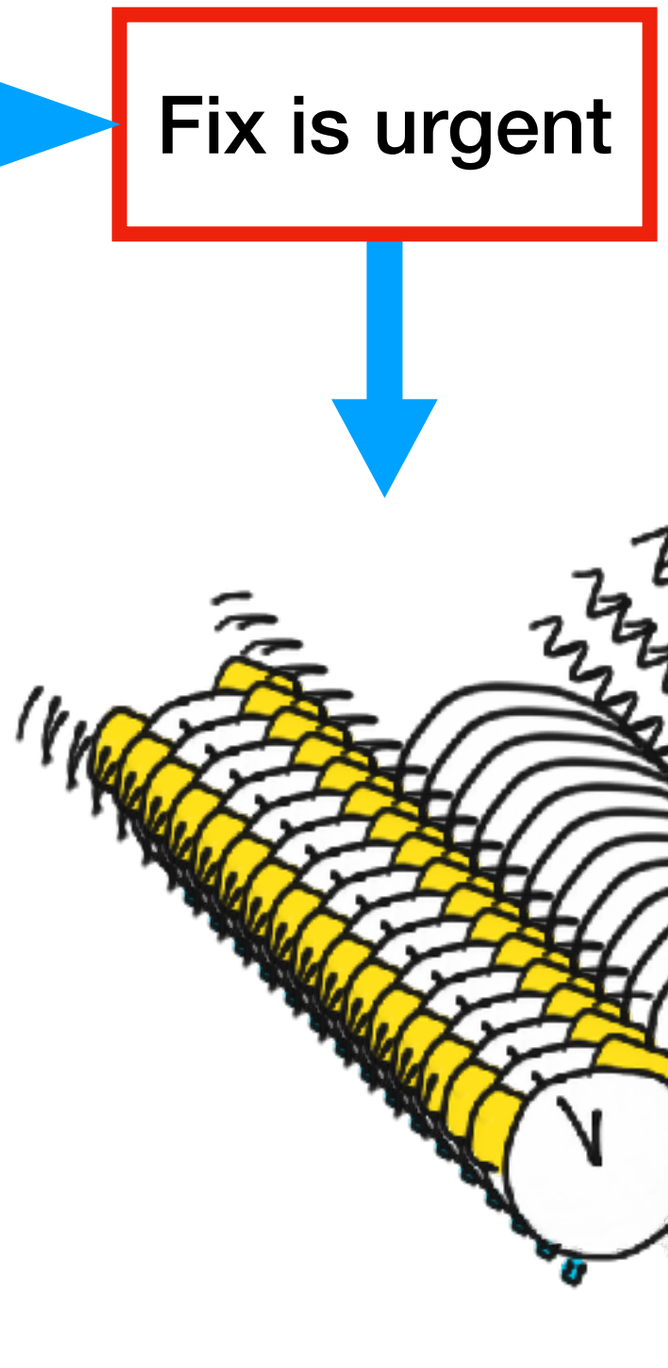
All times are in UTC. For general information about Xen and security see the Xen Project website and security policy. A JSON document listing advisories is also available.

Advisory	Public release	Updated	Version	CVE(s)	Title
XSA-344	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-343	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-342	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-341	2020-09-08 15:35		-	-	Unused Xen Security Advisory number
XSA-340	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-339	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-338	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-337	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-336	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-335	2020-08-24 12:00	2020-08-24 12:17	2	CVE-2020-14364	QEMU: usb: out-of-bounds r/w access issue
XSA-334	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-333	2020-09-22 12:00			none (yet) assigned	(Prereleased, but embargoed)
XSA-329	2020-07-16 12:00	2020-07-21 11:00	3	CVE-2020-15852	Linux iopermbitmap context switching issues
XSA-328	2020-07-07 12:00	2020-07-07 12:23	3	CVE-2020-15567	non-atomic modification of live EPT PTE
XSA-327	2020-07-07 12:00	2020-07-07 12:23	3	CVE-2020-15564	Missing alignment check in VCPUOP_register_vcpu_info
XSA-321	2020-07-07 12:00	2020-07-07 12:21	3	CVE-2020-15565	insufficient cache write-back under VT-d
XSA-320	2020-06-09 16:33	2020-06-11 13:09	2	CVE-2020-0543	Special Register Buffer speculative side channel
XSA-319	2020-07-07 12:00	2020-07-07 12:18	3	CVE-2020-15563	inverted code paths in x86 dirty VRAM tracking
XSA-318	2020-04-14 12:00	2020-04-14 12:00	3	CVE-2020-11742	Bad continuation handling in GNTTABOP_copy
XSA-317	2020-07-07 12:00	2020-07-07 12:18	3	CVE-2020-15566	Incorrect error handling in event channel port allocation
XSA-316	2020-04-14 12:00	2020-04-14 12:00	3	CVE-2020-11743	Bad error path in GNTTABOP_map_grant

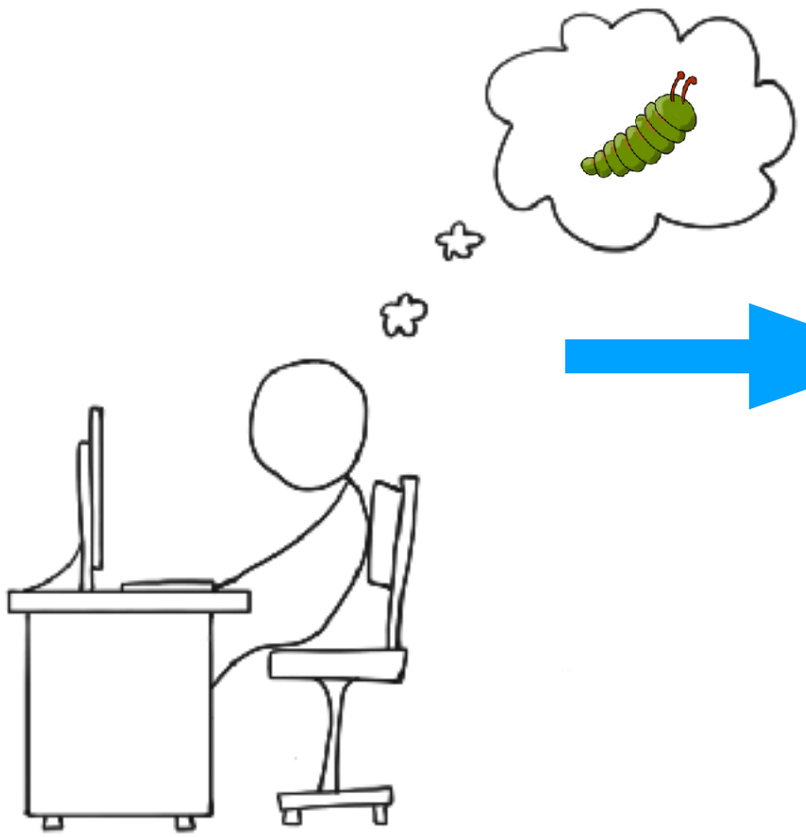
Fix is urgent

Fix is not time-critical

Fix is urgent

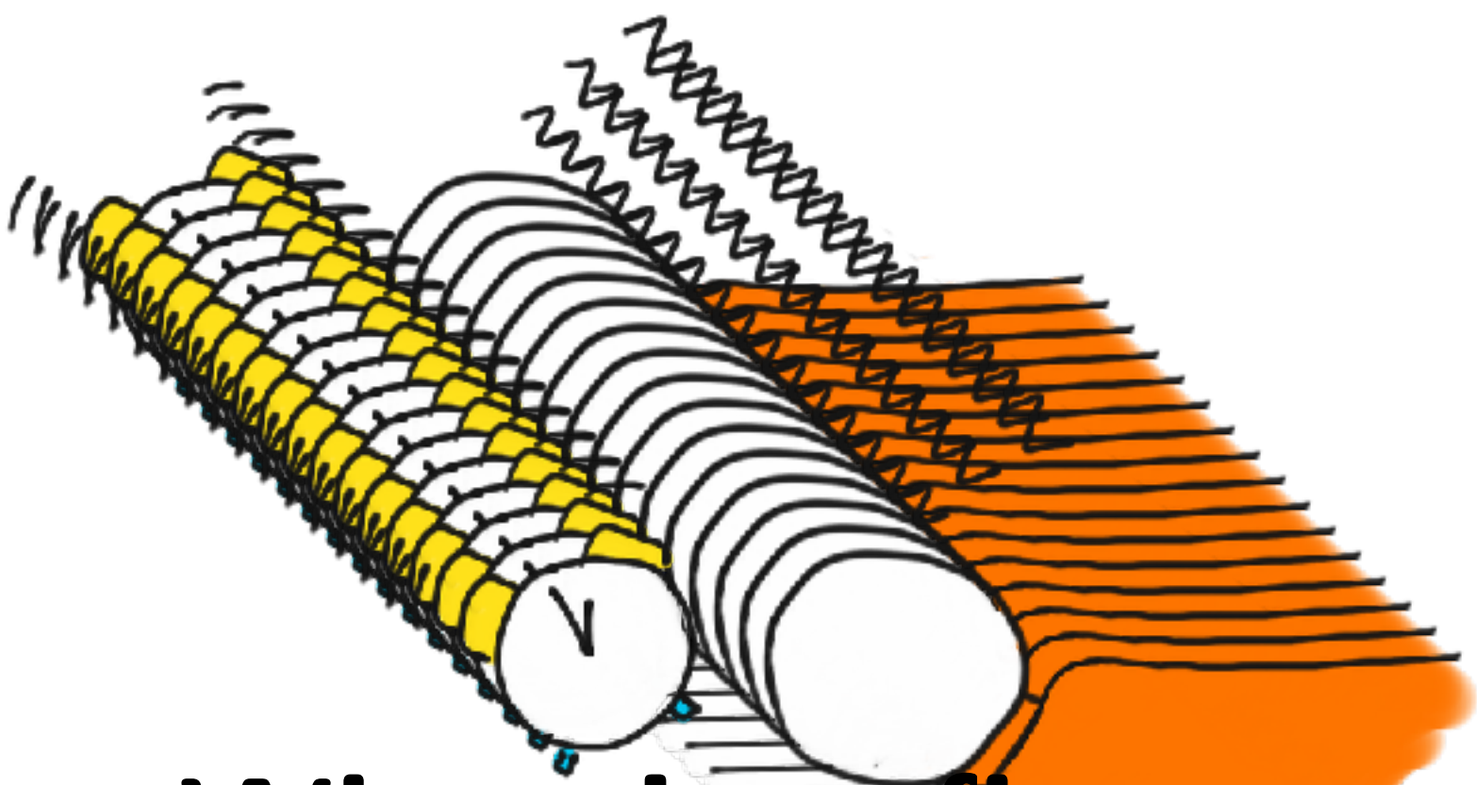


Fix is not time-critical



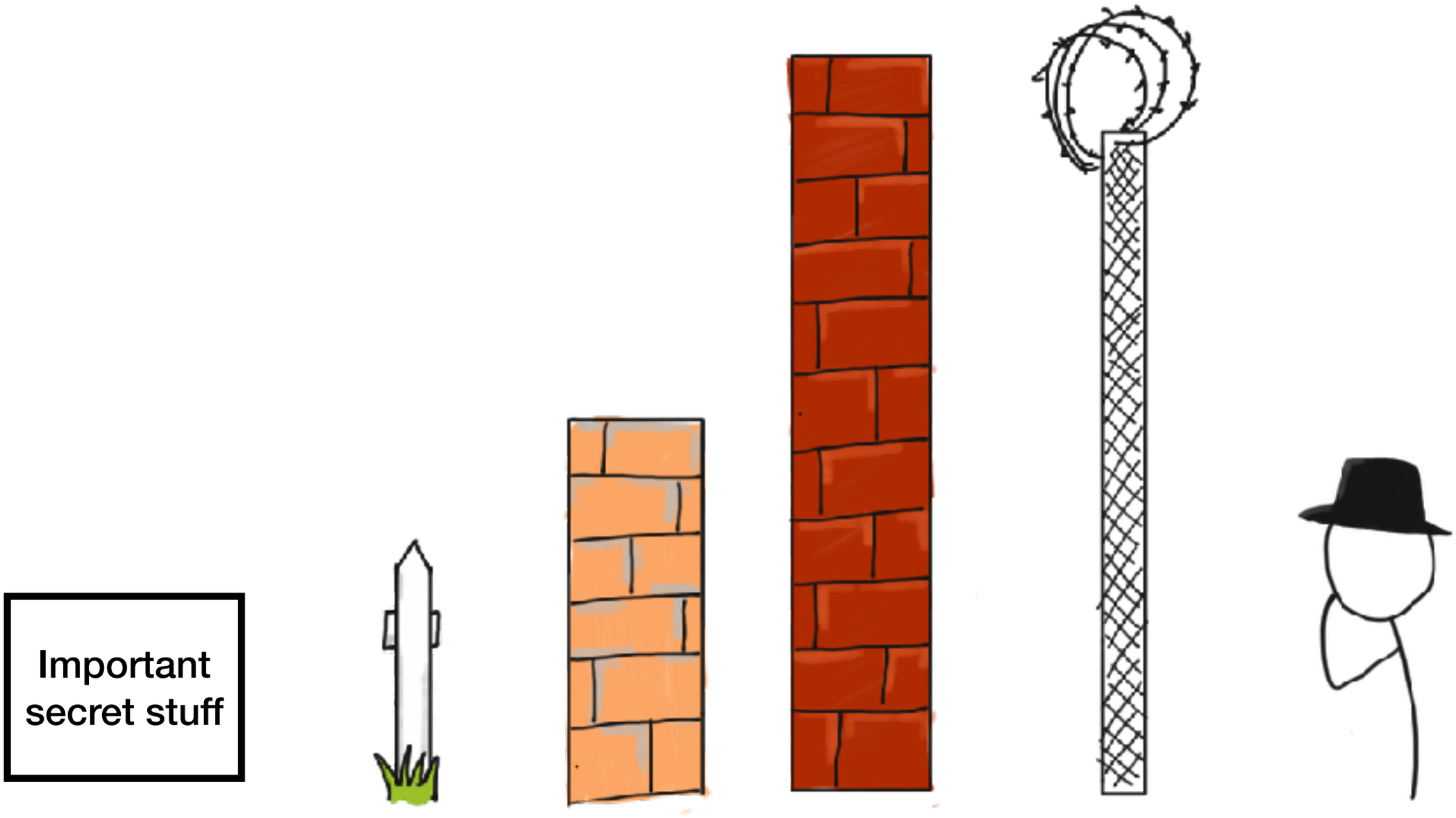
Fix is urgent

Fix is not time-critical



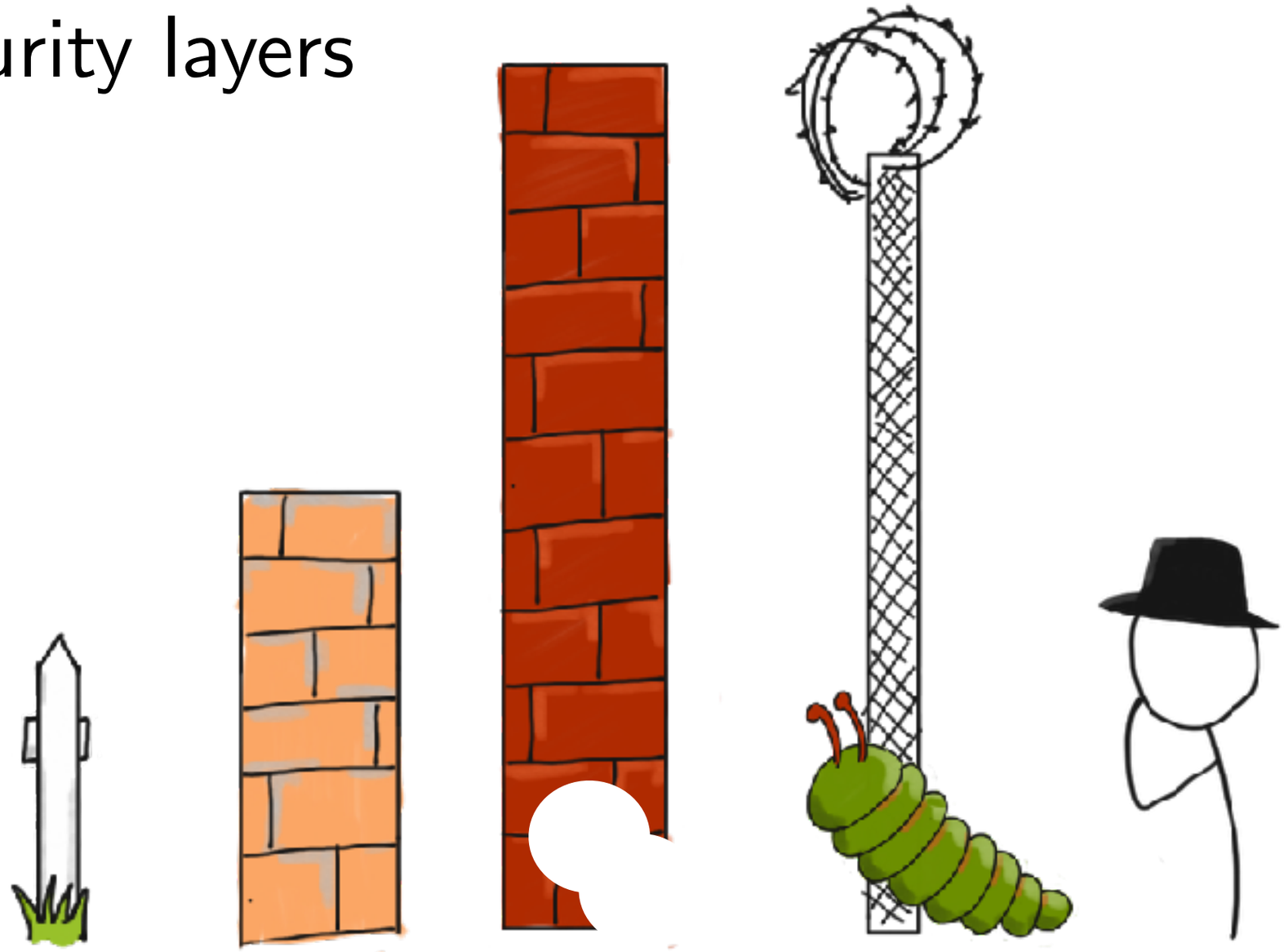
When is a fix urgent?

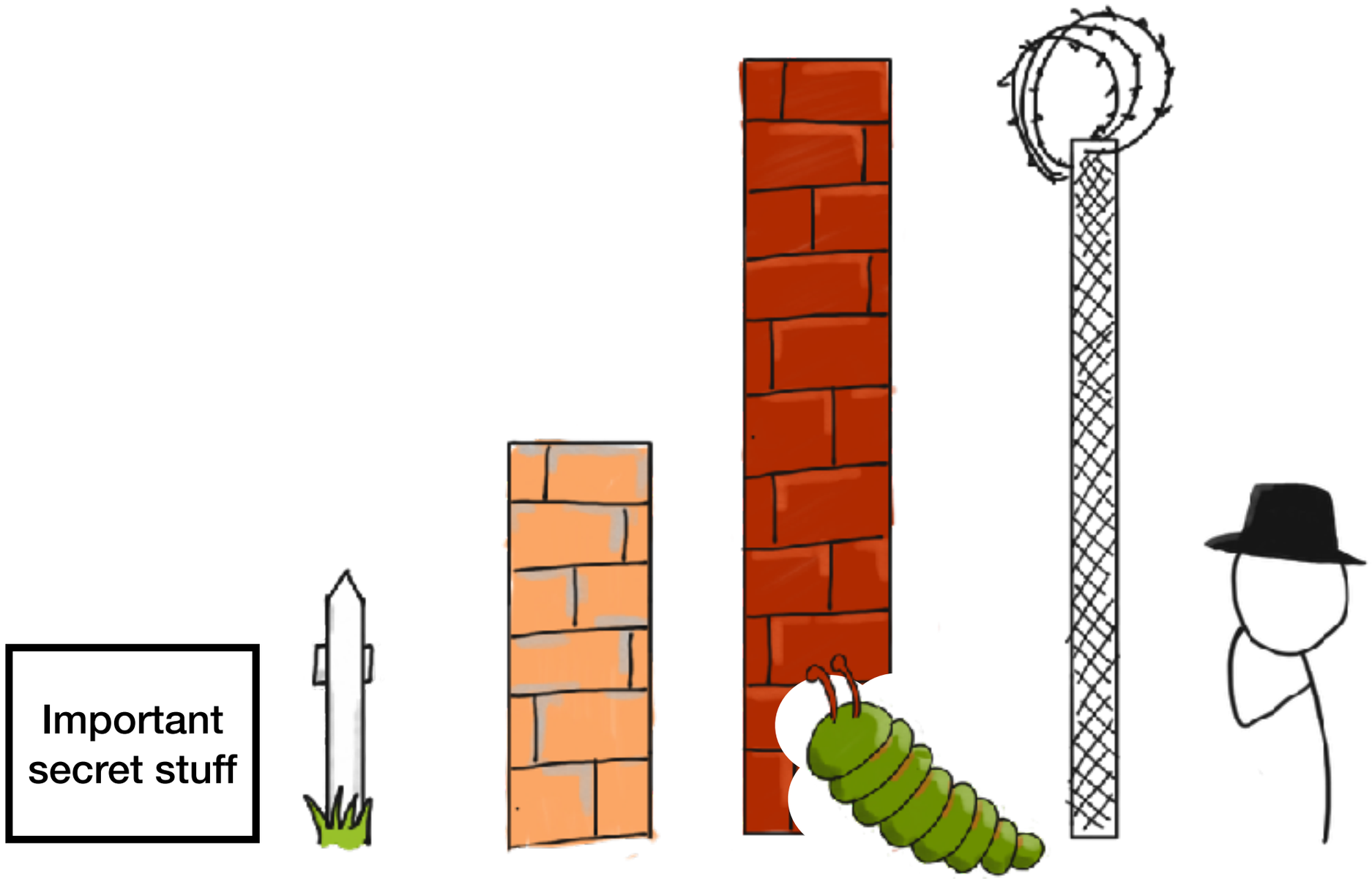
- Well-engineered systems are built with defence in depth



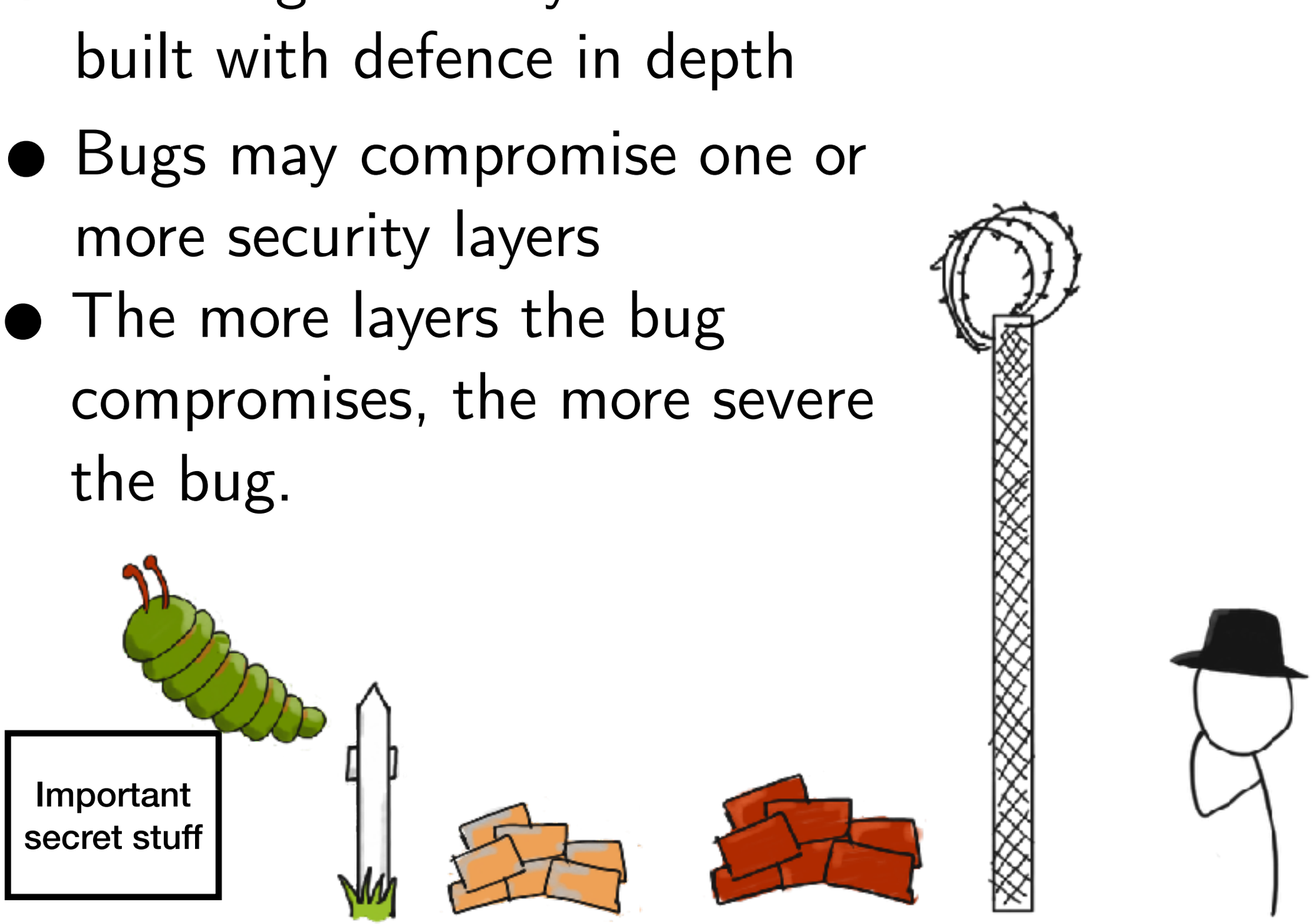
- Well-engineered systems are built with defence in depth
- Bugs may compromise one or more security layers

Important secret stuff



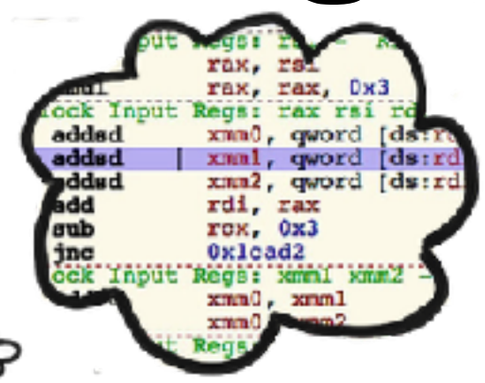

Important
secret stuff

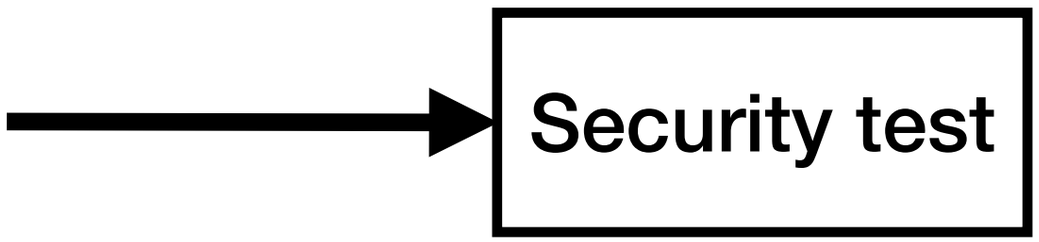
- Well-engineered systems are built with defence in depth
- Bugs may compromise one or more security layers
- The more layers the bug compromises, the more severe the bug.



How do we determine if a fix is urgent?

How do we determine if a fix is urgent?





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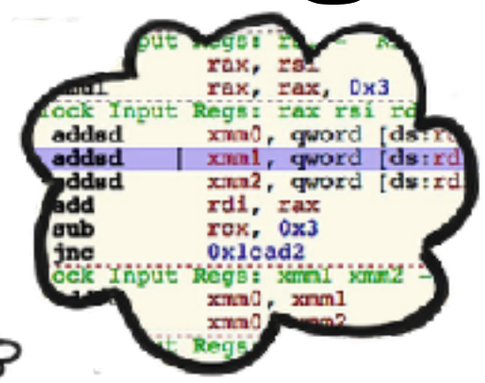
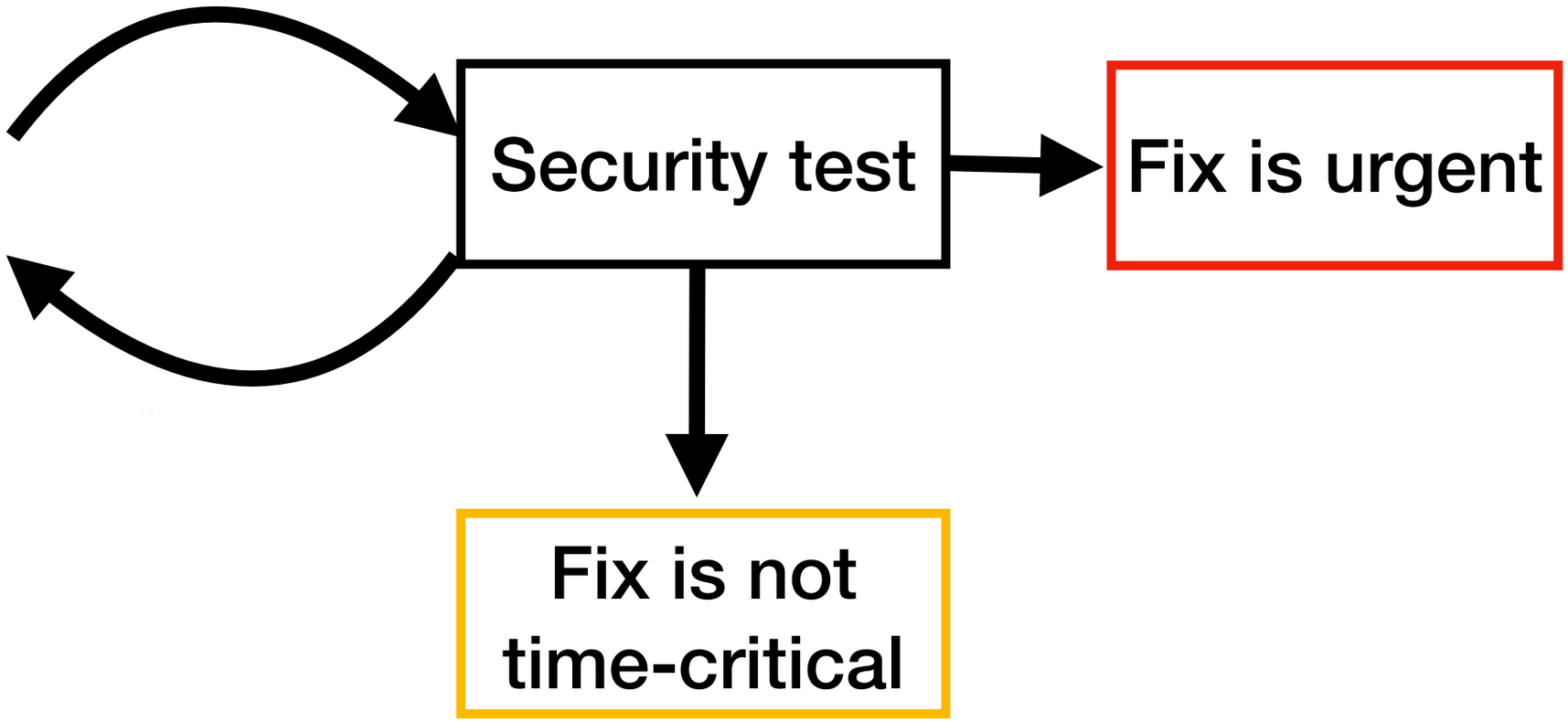
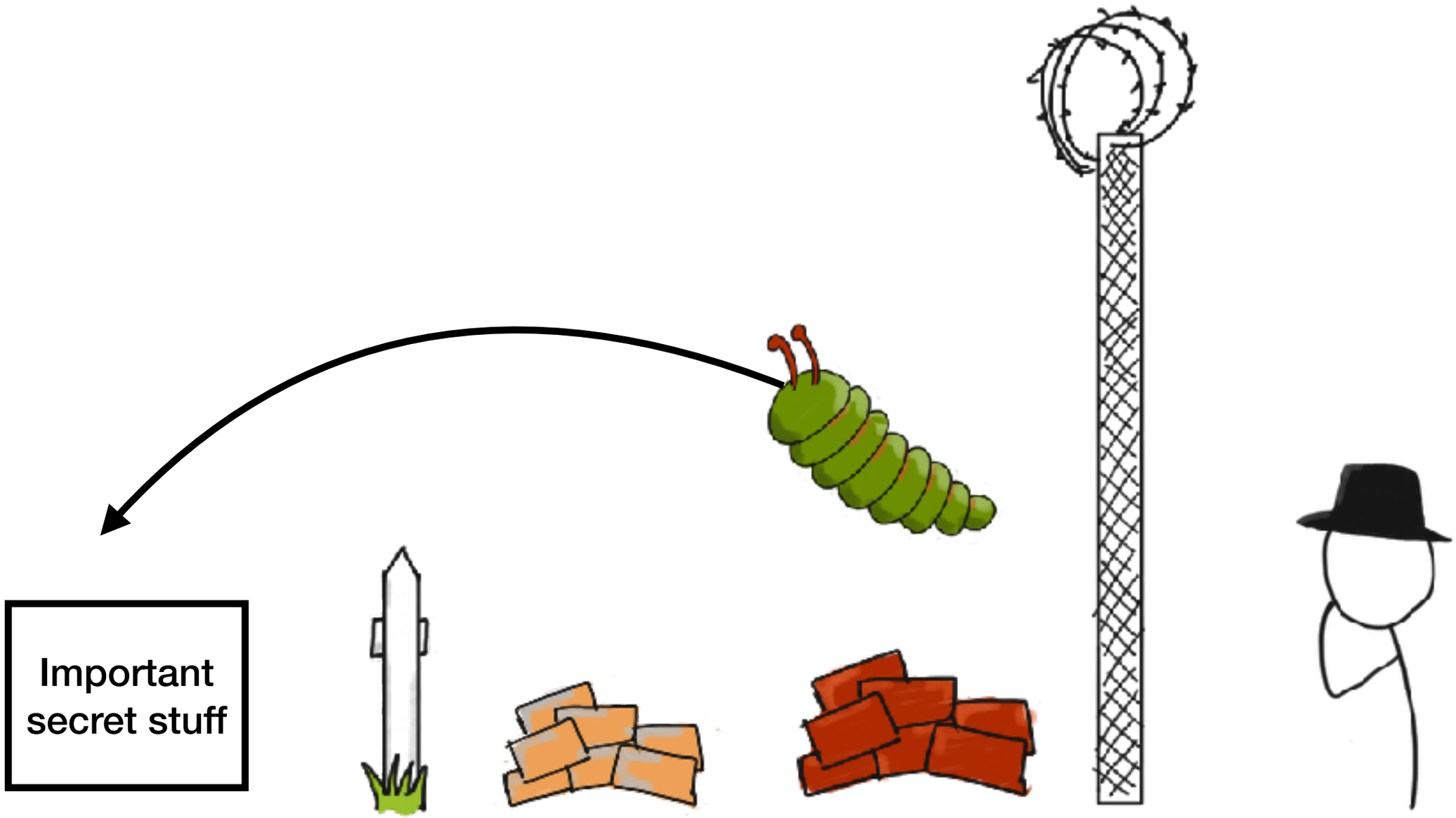
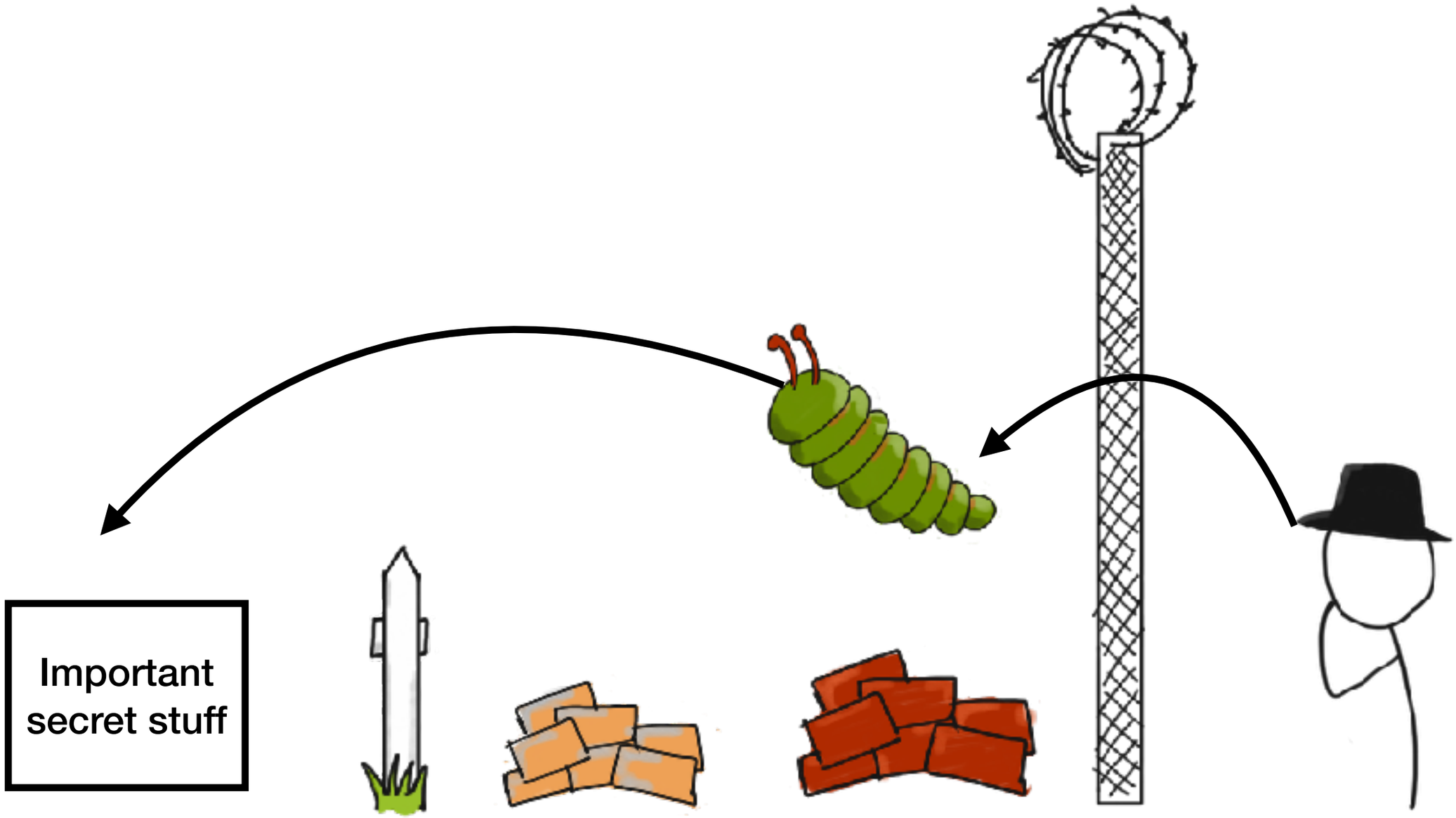




Using SAT-based model checking


Important
secret stuff

Security tests establish reachability of the bug



Reachability assertion

```
ISSUE DESCRIPTION
=================

The x86 instruction CMPXCHG8B is supposed to ignore legacy operand
size overrides; it only honors the REX.W override (making it
CMPXCHG16B).  So, the operand size is always 8 or 16.

When s                                                          on
emulat                                                          was
relied                                                          eral,
operan
```

```
assert(op_bytes==8 || op_bytes==16);
```

```
As a result, if a guest uses a supposedly-ignored operand size prefix,
a small amount of hypervisor stack data is leaked to the guests: a 96
bit leak to guests running in 64-bit mode; or, a 32 bit leak to other
guests.
```

SAT take-aways

- SAT solvers are surprisingly good at NP
- If you have an NP problem, don't solve it yourself, translate it into SAT!
- Don't do the translation yourself! Use a tool.

SAT

A : **Boolean**

B : **Boolean**

$$\exists A, B$$

$$A \wedge \neg B$$

A : **true**

B : **false**

SMT

A : **Integer**

B : **Integer**

$$\exists A, B$$

$$A > 0 \wedge B < 0$$

A : **10**

B : **-3**

Satisfiability Modulo Theories

- SMT solvers solve formula in some quantifier-free fragment of a first-order theory T

- Formulas use propositional connectives and a set Σ of additional function and predicate symbols that uniquely define the theory T

- Σ is called the signature of T

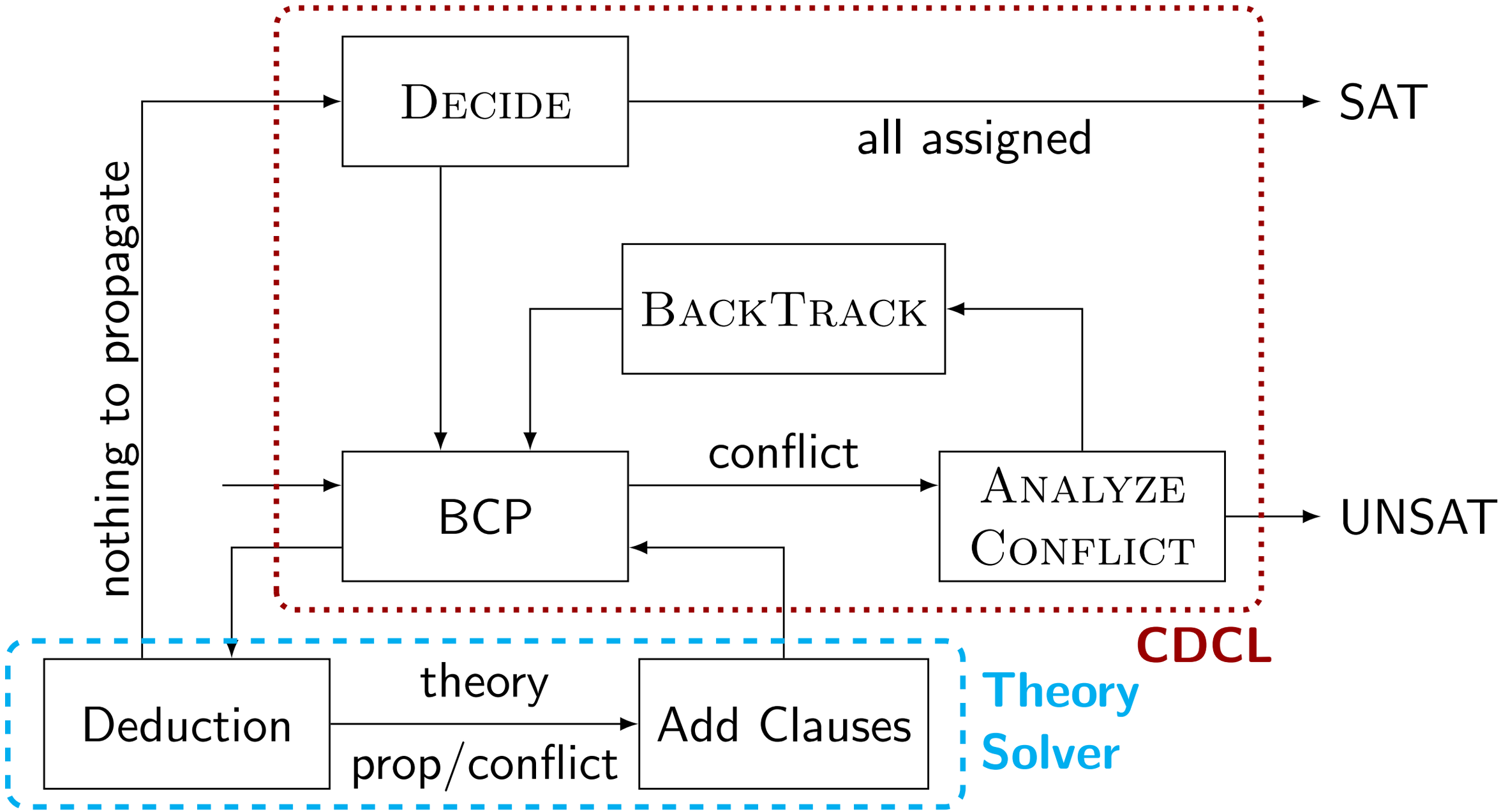
- SMT solvers determine whether the formula is T-valid, T-satisfiable or T-unsatisfiable

What theories?

- Arrays
- BitVectors
- Floating Point
- Integers
- Reals
- Strings
- Uninterpreted Functions

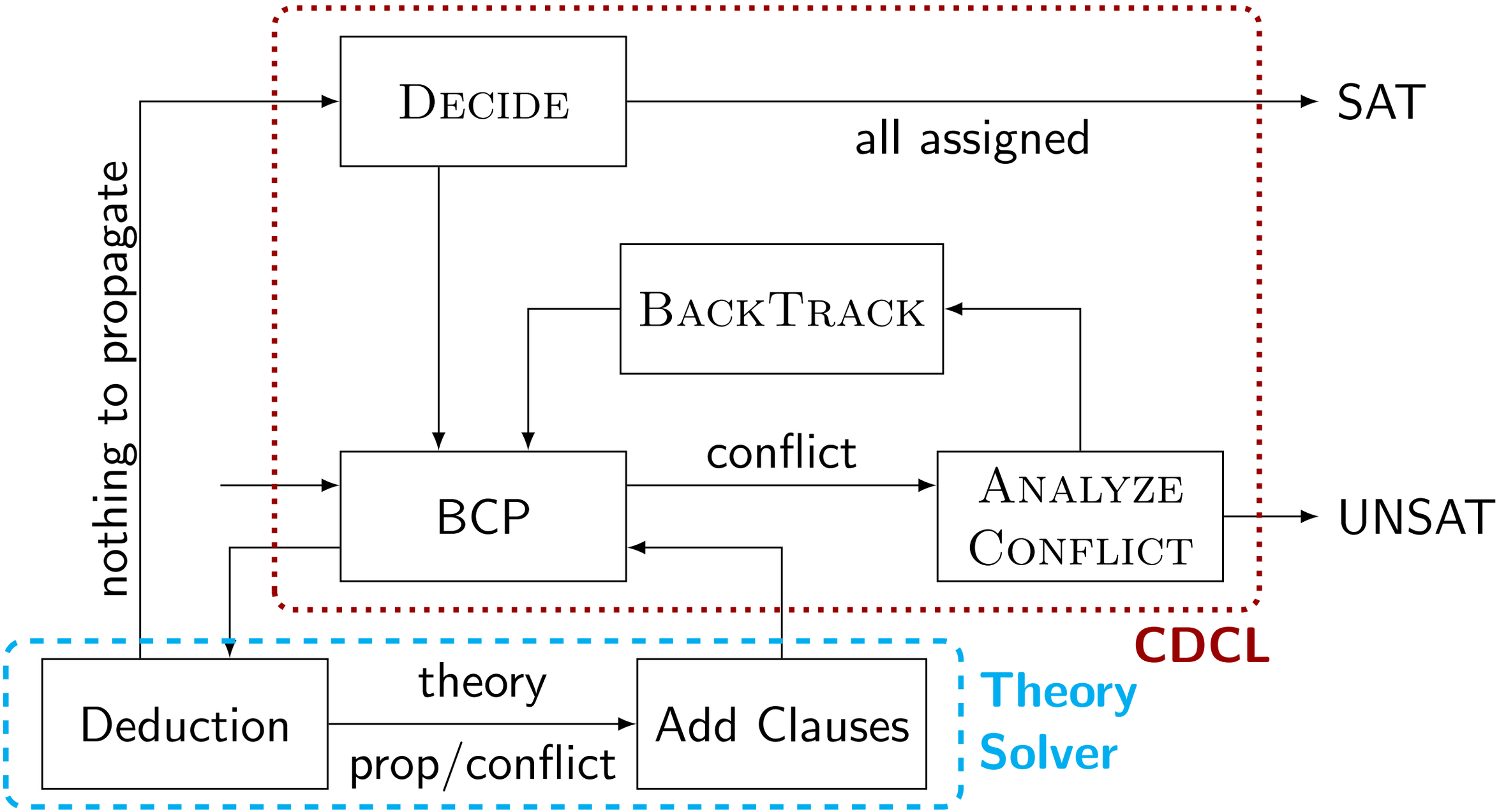
$$(x > 10) \wedge (y < 7) \wedge (x = y)$$

CDCL(T)



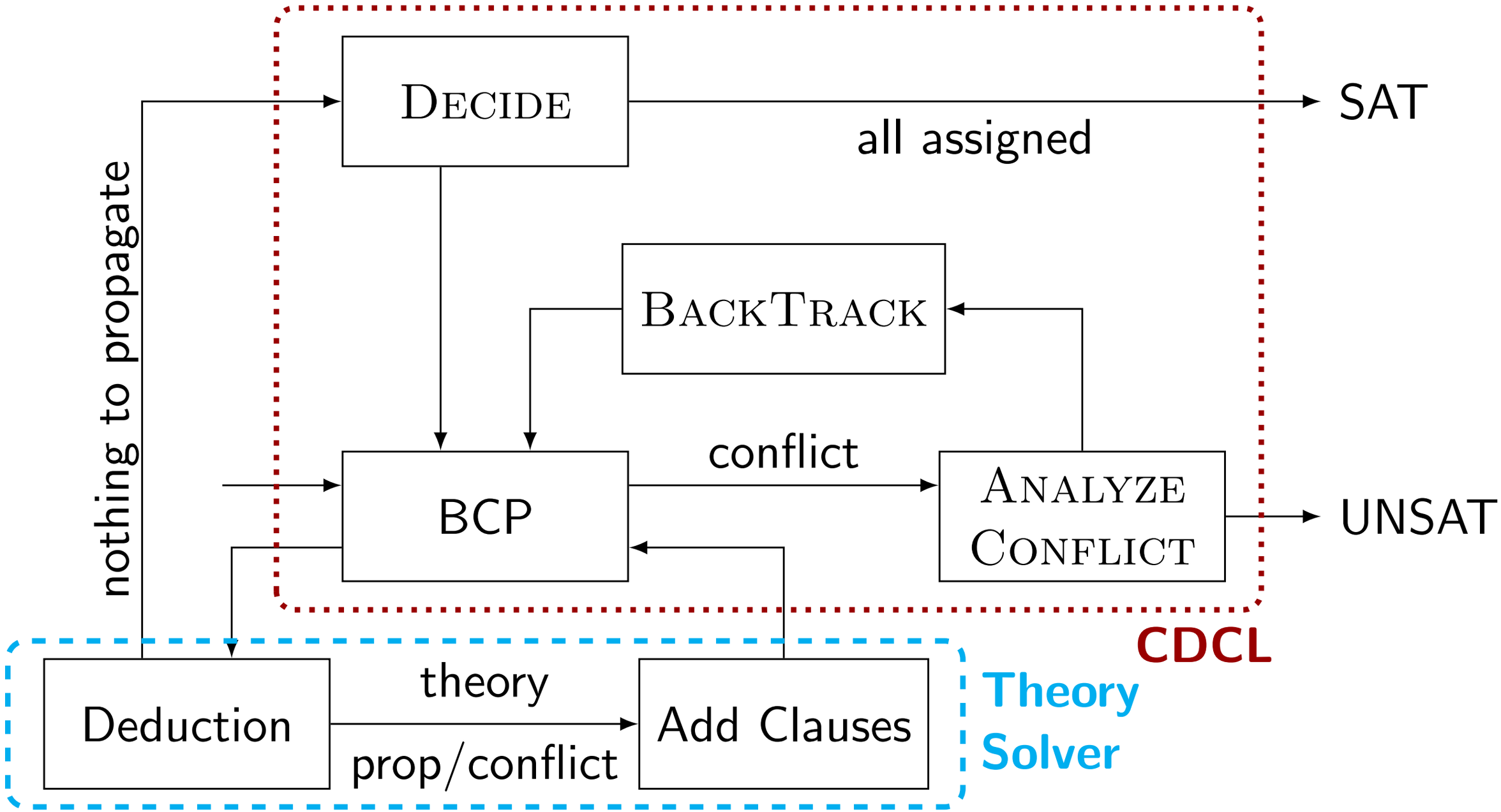
Step 1: make a boolean skeleton of your formula

CDCL(T)



Step 2: give the boolean skeleton to the SAT solver

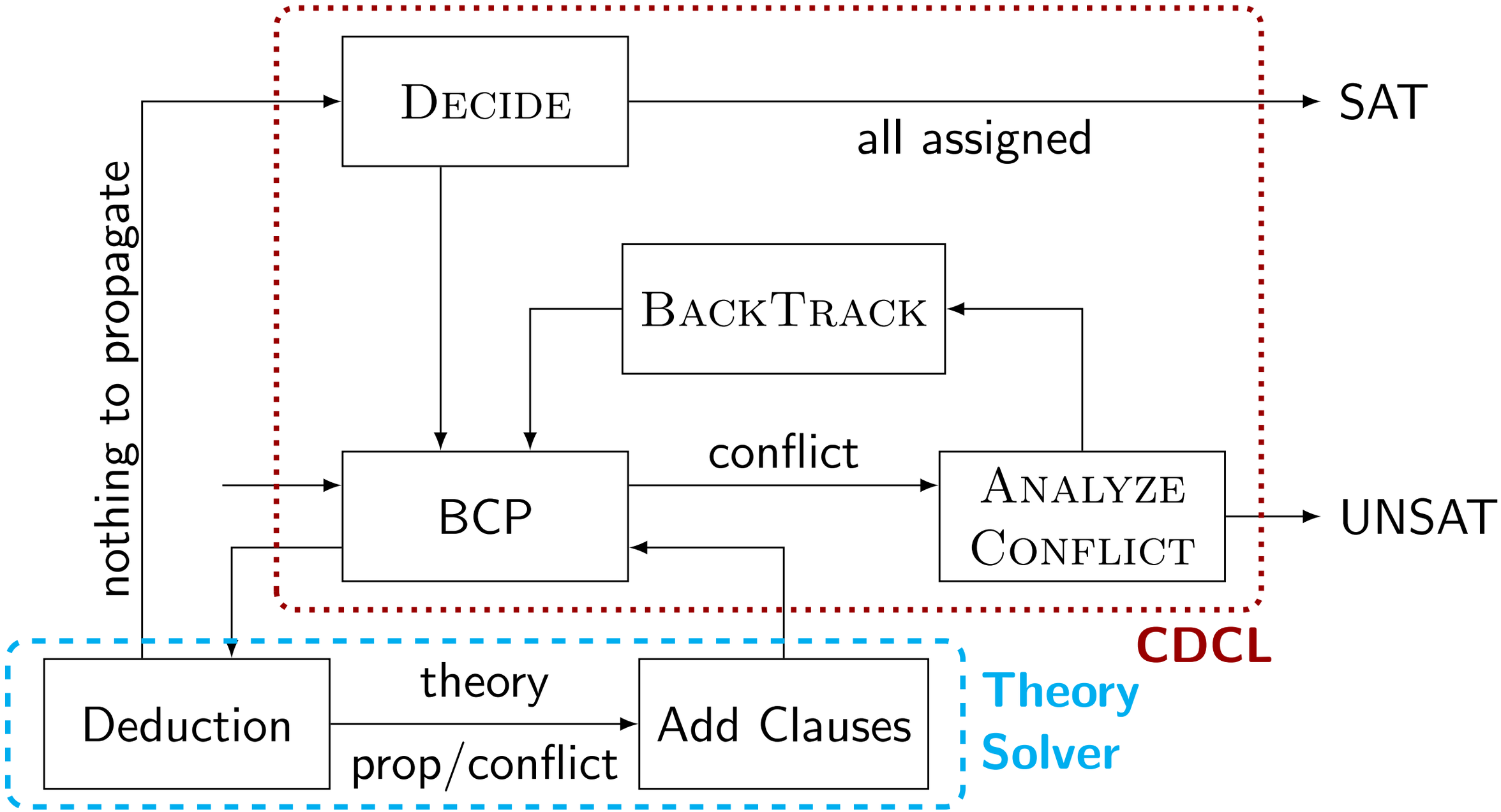
CDCL(T)



Step 2: give the boolean skeleton to the SAT solver

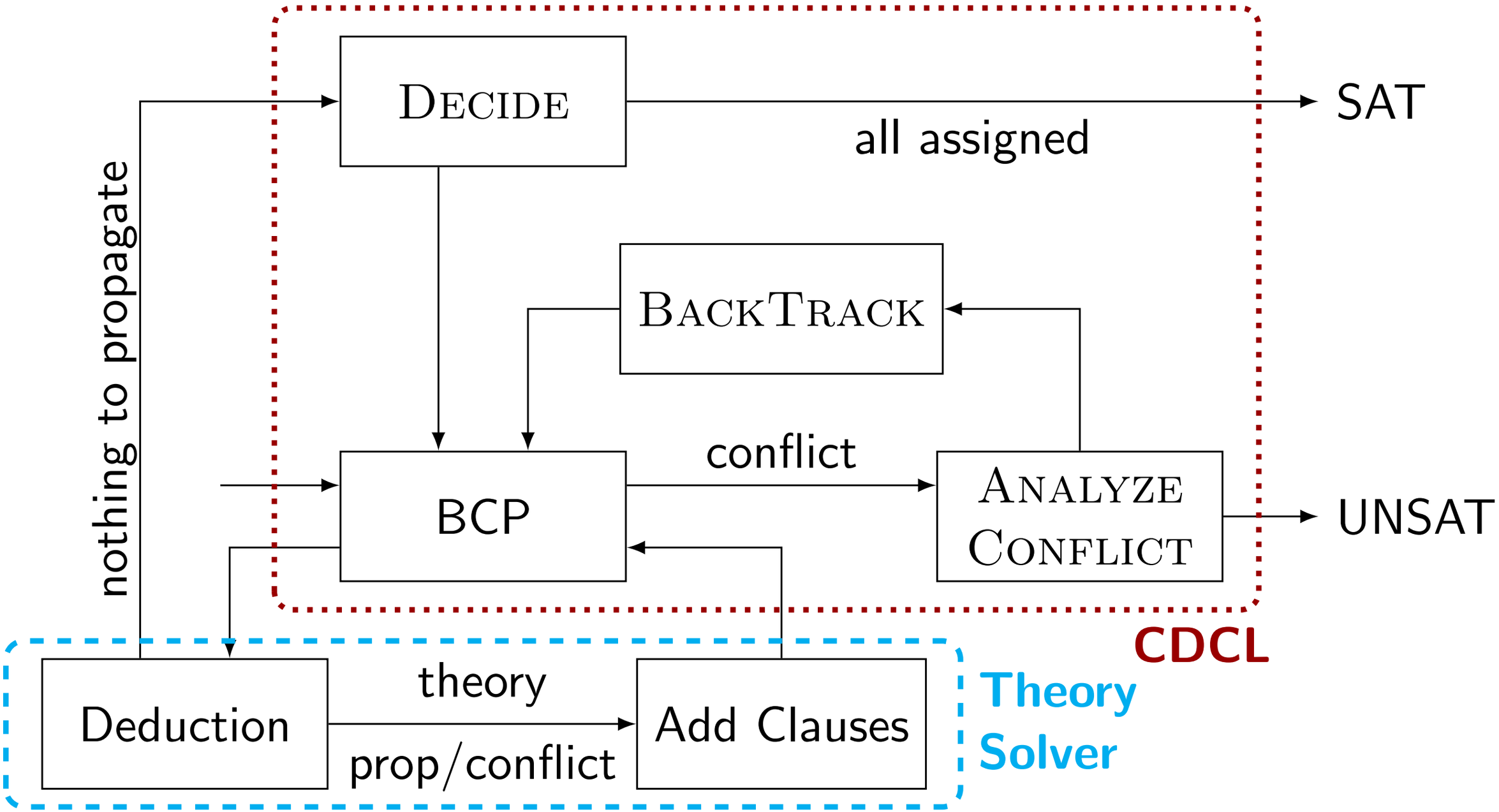
If it's UNSAT, we're done!

CDCL(T)



Step 3: If it's SAT, check with the theory solver

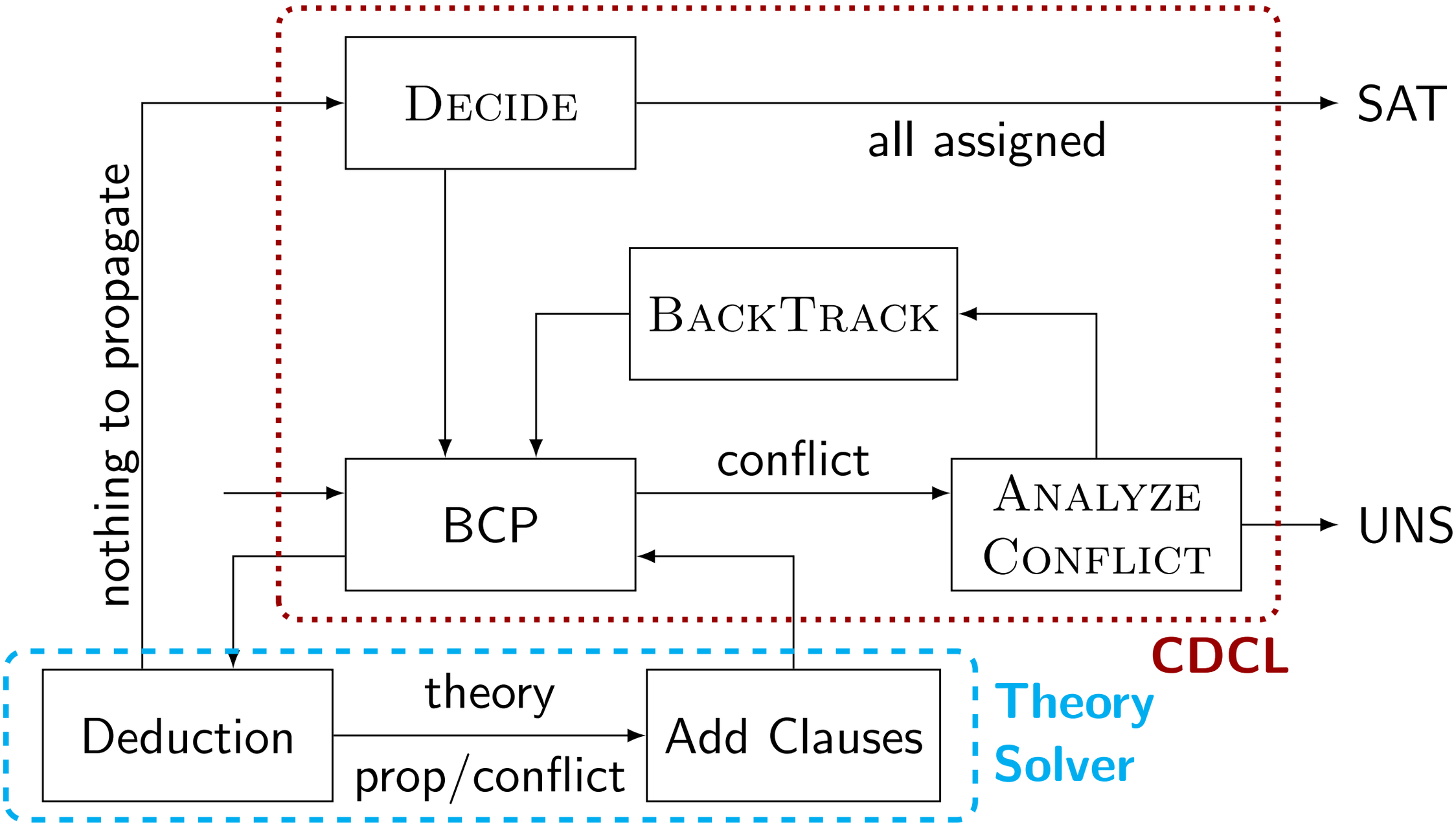
CDCL(T)



Step 3: If it's SAT, check with the theory solver

If theory solver agrees: return assignment

CDCL(T)



Step 3: If it's SAT, check with the theory solver

Otherwise, return clause to block assignment

Notes

- SMT solvers combine theories with Nelsen-Oppen
- Combinations of theories may be undecidable, even if the individual theories are decidable.
- Not covering details: read "Decision Procedures" by Strichman and Kroening.

SMT-LIB

- SMT-LIB is the standard input format (Python API's exist for specific solvers)
- An SMT-LIB file must include:
  - Set-logic
  - Function declarations (variables are 0-ary functions)
  - Assertions
  - Check-sat command (and optionally get-model)

SMT-LIB

```
(set-logic LIA)
(declare-fun a () Int)
(declare-fun f (Int Bool) Int)
(assert (> a 10))
(assert (< (f a true) 100))
(check-sat)
(get-model)
```

SMT-LIB - example 5

```
(set-logic LIA)
(declare-fun a () Int)
(declare-fun f (Int Bool) Int)
(assert (> a 10))
(assert (< (f a true) 100))
(check-sat)
(get-model)
```

```
sat
(
  (define-fun a () Int
    11)
  (define-fun f ((x!0 Int) (x!1 Bool)) Int
    0)
)
```

Examples

- Solving Sudokus
- Verifying Code
- Verifying access policies for S2 buckets at Amazon
- Synthesising code!

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

SMT-LIB - uninterpreted functions

- Functions have no side effects and are total (defined on all input values). No exceptions!

```
(declare-fun A (Int Int) Int)
```

SMT-LIB - arrays

- Functions have no side effects and are total (defined on all input values). No exceptions!

```
(declare-fun A (Int Int) Int)
```

Could also use nested arrays instead:

```
(declare-fun A () (Array Int (Array Int Int)))
```

Arrays are not like arrays in C! More like functions!

SMT-LIB - constants

- Functions have no side effects and are total (defined on all input values). No exceptions!

```
(declare-fun A (Int Int) Int)
```

"declare-const" is syntax sugar for declaring a nullary symbol:

```
(declare-const A (Array Int (Array Int Int)))
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
(assert (and (<= 1 (A 1 2))(>= 9 (A 1 2))))
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
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9				6				5
5	3	6	2					7
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All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
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```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
(assert (and (<= 1 (A 1 2))(>= 9 (A 1 2))))
…
(assert (distinct (A 1 1)(A 1 2)(A 1 3)(A 1
4)(A 1 5)(A 1 6)(A 1 7)(A 1 8)(A 1 9)))
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
(assert (and (<= 1 (A 1 2))(>= 9 (A 1 2))))
…
(assert (distinct (A 1 1)(A 1 2)(A 1 3)(A 1
4)(A 1 5)(A 1 6)(A 1 7)(A 1 8)(A 1 9)))
…
(assert (= 9 (A 1 3)))
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
(assert (and (<= 1 (A 1 2))(>= 9 (A 1 2))))
…
(assert (distinct (A 1 1)(A 1 2)(A 1 3)(A 1
4)(A 1 5)(A 1 6)(A 1 7)(A 1 8)(A 1 9)))
…
(assert (= 9 (A 1 3)))
(check-sat)(get-model)
```

Example 6 - Sudoku

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

All rows and columns contain the numbers 1 to 9, only once each

Each box contains the numbers 1 to 9, only once each

Solves the conjuction of all assertions!

```
(declare-fun A (Int Int) Int)
(assert (and (<= 1 (A 1 1))(>= 9 (A 1 1))))
(assert (and (<= 1 (A 1 2))(>= 9 (A 1 2))))
…
(assert (distinct (A 1 1)(A 1 2)(A 1 3)(A 1
4)(A 1 5)(A 1 6)(A 1 7)(A 1 8)(A 1 9)))
…
(assert (= 9 (A 1 3)))
(check-sat)(get-model)
```

SMT-LIB - push/pop

```
(declare-const x Int)
(declare-const y Int)
(declare-const z Int)
(push)
(assert (= (+ x y) 10))
(assert (= (+ x (* 2 y)) 20))
(check-sat)
(pop) ; remove the two assertions
(push)
(assert (= (+ (* 3 x) y) 10))
(assert (= (+ (* 2 x) (* 2 y)) 21))
(check-sat)
```

SMT-LIB - push/pop

```
(declare-const x Int)
(declare-const y Int)
(declare-const z Int)
(push)
(assert (= (+ x y) 10))
(assert (= (+ x (* 2 y)) 20))
(check-sat)
(pop) ; remove the two assertions
(push)
(assert (= (+ (* 3 x) y) 10))
(assert (= (+ (* 2 x) (* 2 y)) 21))
(check-sat)
(declare-const p Bool)
(pop)
(assert p) ; error
```

SMT-LIB - bitvectors

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

- No notion of signed-ness
- Overflow wraps around
- Divide by zero gives FFFFF

```
(declare-fun A ((_ BitVec 3)(_ BitVec 3))(_ BitVec 3))
```

SMT-LIB - bitvectors

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

- No notion of signed-ness
- Overflow wraps around
- Divide by zero gives FFFFF

```
(declare-fun A ((_ BitVec 3)(_ BitVec 3))(_ BitVec 3))

(assert (bvuge (_ bv9 3)(A (_ bv1 3) (_ bv1 3)))
```

CBMC - example 3

```
bool x;
char y=8, z=0, w=0;

if(x)
  z = y-1;
else
  w = y+1;

assert(z==7 || w==9);
```

$$(y = 8) \wedge (w = 0) \wedge (z = 0) \ \wedge$$
$$(z = x\,?\,y - 1 : 0) \ \wedge$$
$$(w = x\,?\,0 : y + 1) \ \wedge$$
$$(z \neq 7) \ \wedge$$
$$(w \neq 9)$$

How do we represent char in SMT?

SMT-LIB - quantifiers

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

- Compact
- SMT solvers support first-order quantification
- Careful not to write things that are trivially false!

```
(declare-fun A (Int Int) Int)

(assert (forall ((i Int)(j Int)) (=> (and (<= i 9)(>=
i 0)(<= j 9)(>= j 0)) (and (<= (A i j) 9)(>= (A i j)
1)))))
```

SMT-LIB - define-fun

		9	8	5	6			
	8				9			
2					7			
7					1	3	9	6
9				6				5
5	3	6	2					7
			9					1
			3				6	
			6	8	2	4		

- Functions can be defined, useful to make things more compact

```
(define-fun inRange ((x Int)) Int
 (and (<= 1 x)(>= 9 x)))

(declare-fun A (Int Int) Int)
(assert (and (inRange (A 1 1)) (inRange (A 1
2) …))
```

Other ways to build SMT files

- Python API's available for specific solvers

  https://ericpony.github.io/z3py-tutorial/guide-examples.htm

- Using other tools, e.g., UCLID5, CBMC

What next?

- SMT for synthesis
- UCLID5: useful modeling tool to generate SMT-and synthesis queries
- SMT for verifying permissions at Amazon (if time)

Safety Invariant

```
int a = 0;
int b = 1;

while (*)
  local_b=b;
  b=b+a;
  a=local_b;

assert(a<=b);
```

$$init(x) \iff a = 0 \wedge b = 1$$

$$trans(x, x') \iff a' = b \wedge b' = a + b$$

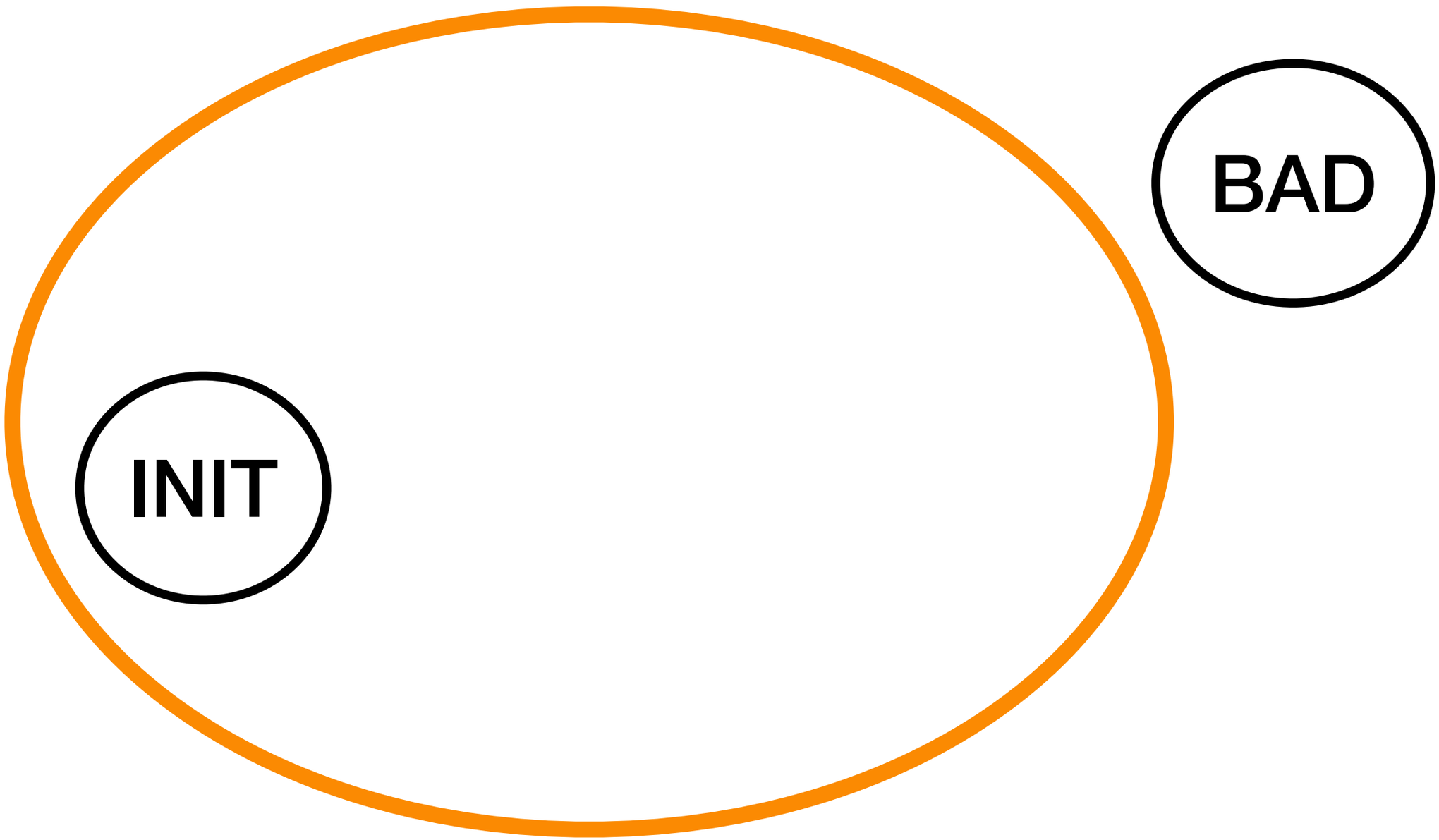
find $inv(x)$ **such that:**

$$init(x) \implies inv(x)$$

$$inv(x) \wedge (a \leq b) \wedge trans(x, x') \implies inv(x')$$

$$inv(x) \implies (a \leq b)$$

Safety Invariant



Safety Invariant - Example 8

```
int a = 0;
int b = 1;

while (*)
  local_b=b;
  b=b+a;
  a=local_b;

assert(a<=b);
```

$$init(x) \iff a = 0 \wedge b = 1$$

$$trans(x, x') \iff a' = b \wedge b' = a + b$$

find $inv(x)$ **such that:**

$$init(x) \implies inv(x)$$

$$inv(x) \wedge (a \leq b) \wedge trans(x, x') \implies inv(x')$$

$$inv(x) \implies (a \leq b)$$

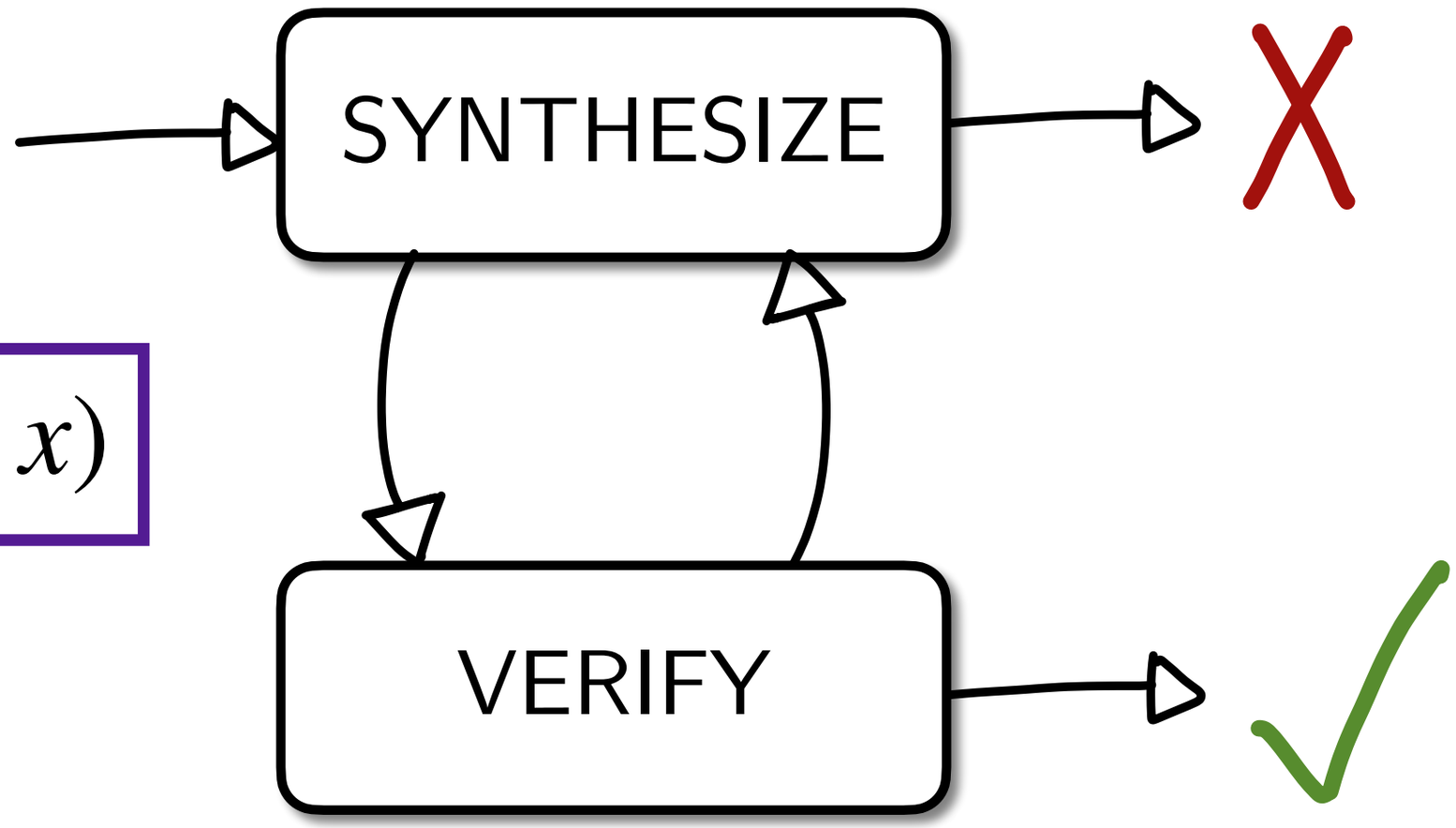
Using SMT for synthesis

$$\exists P \,.\, \forall x \,.\, \sigma(P, x)$$

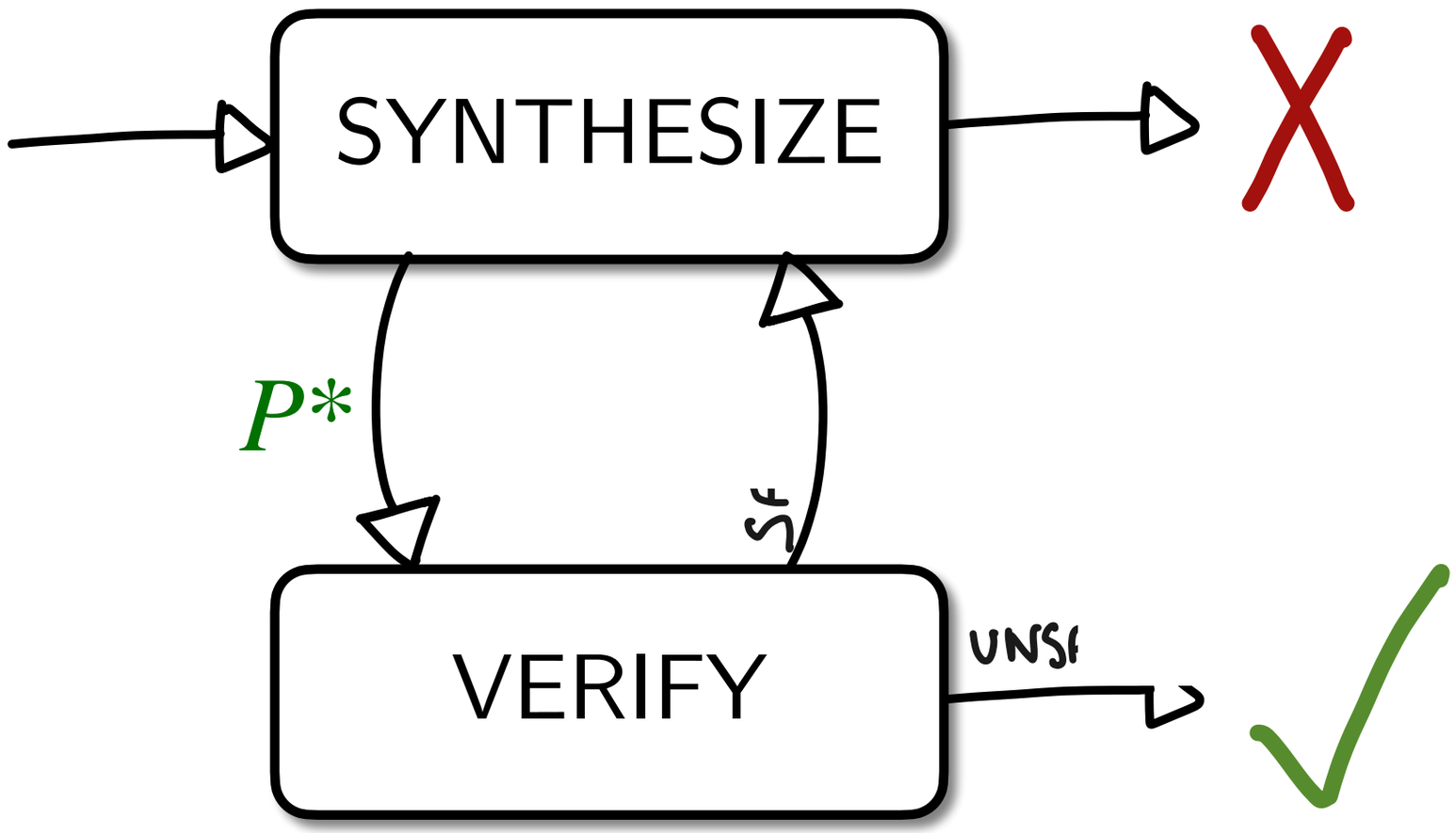
2nd-order logic

CEGIS

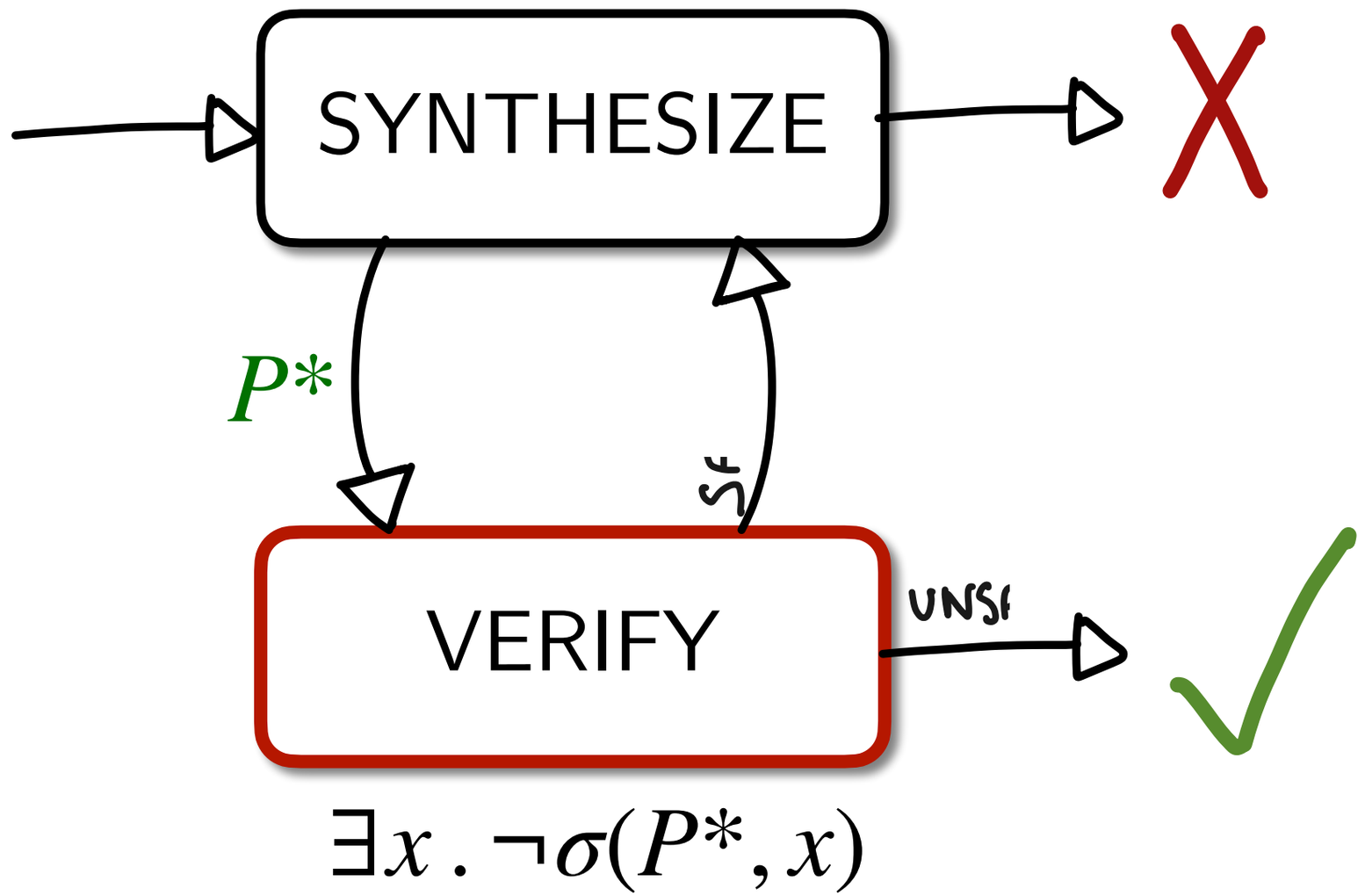
$$\exists P \forall x \,.\, \sigma(P, x)$$



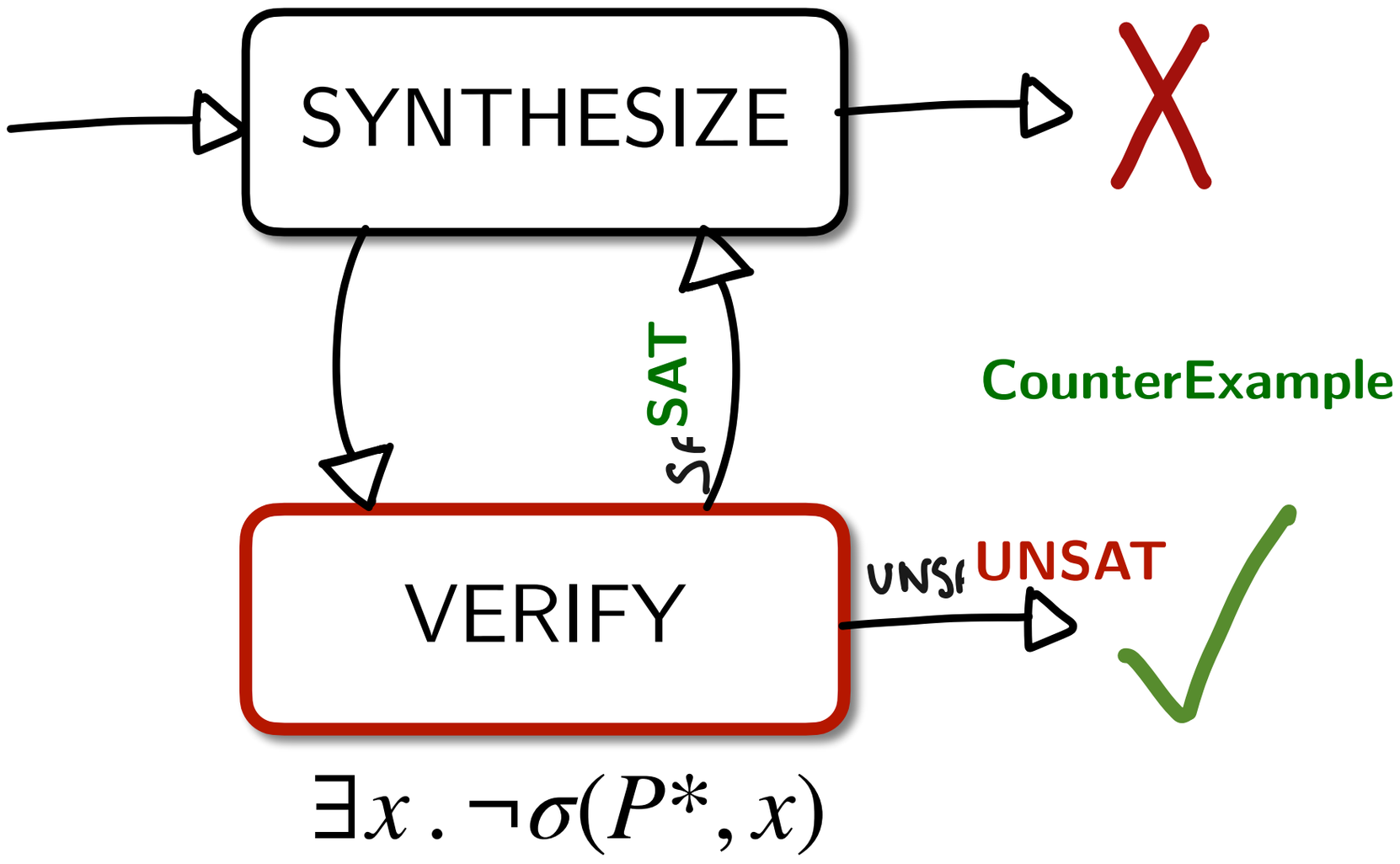
CEGIS



CEGIS

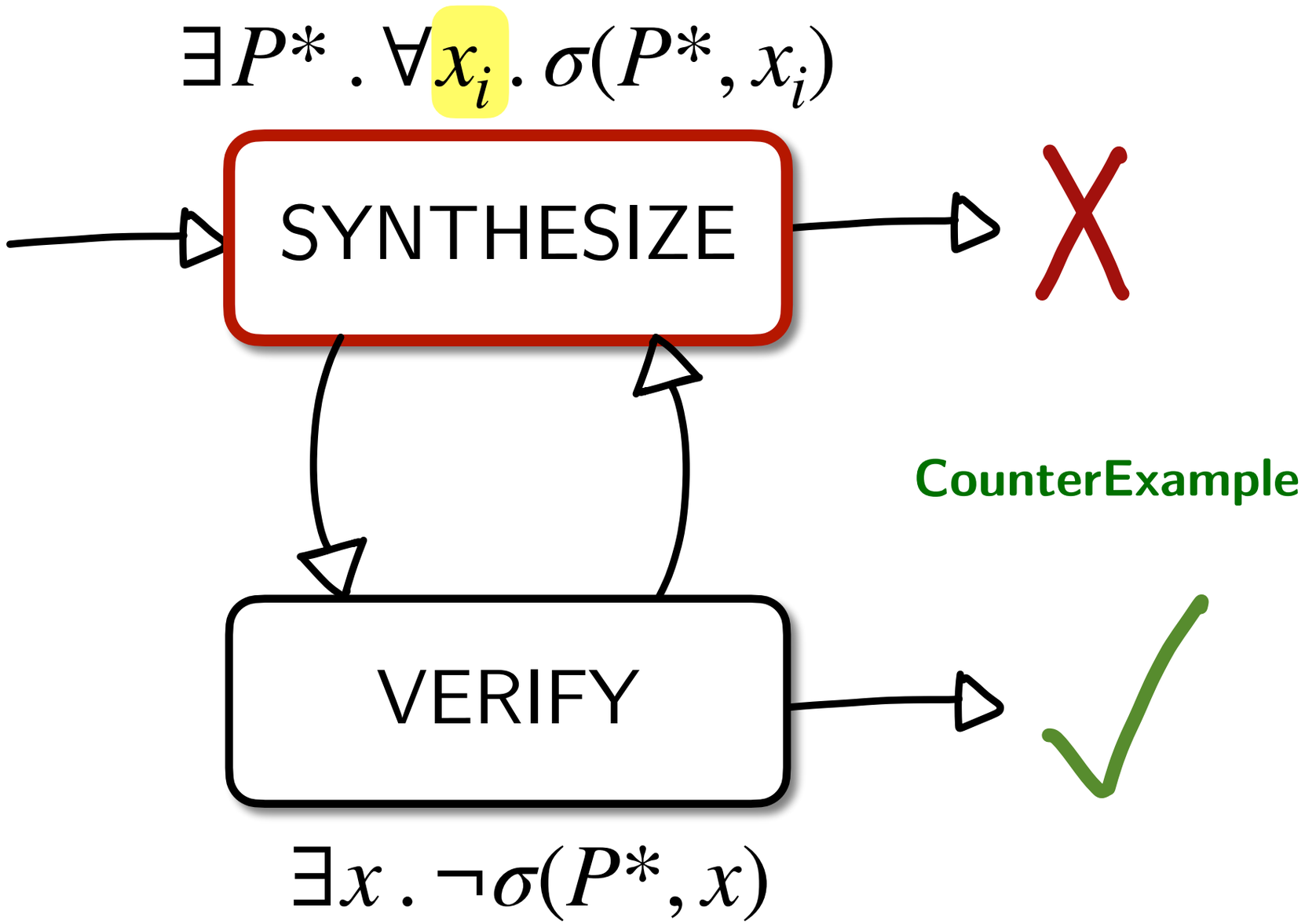


CEGIS

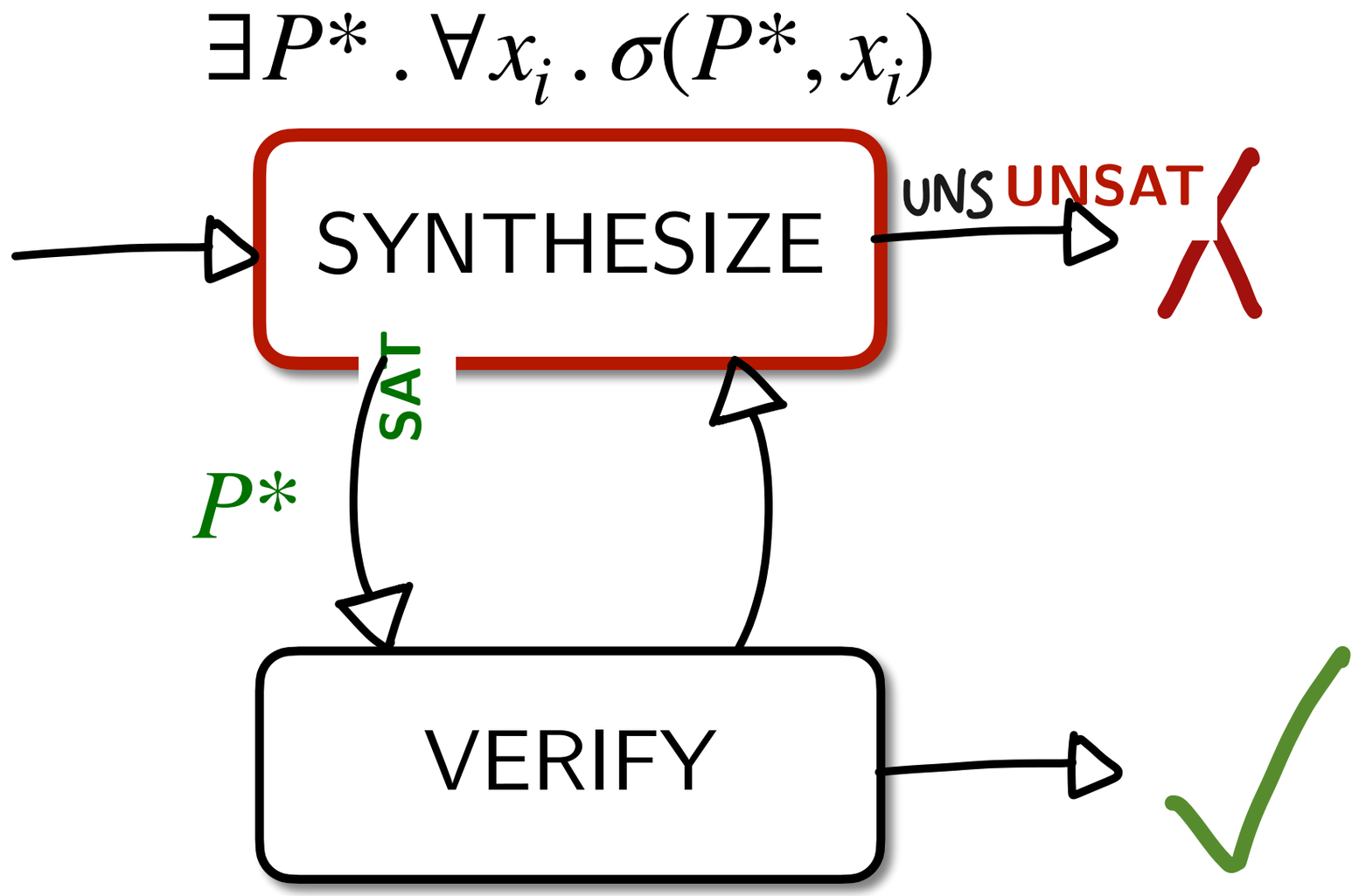


$$\exists x \,.\, \neg\sigma(P^*, x)$$

CEGIS

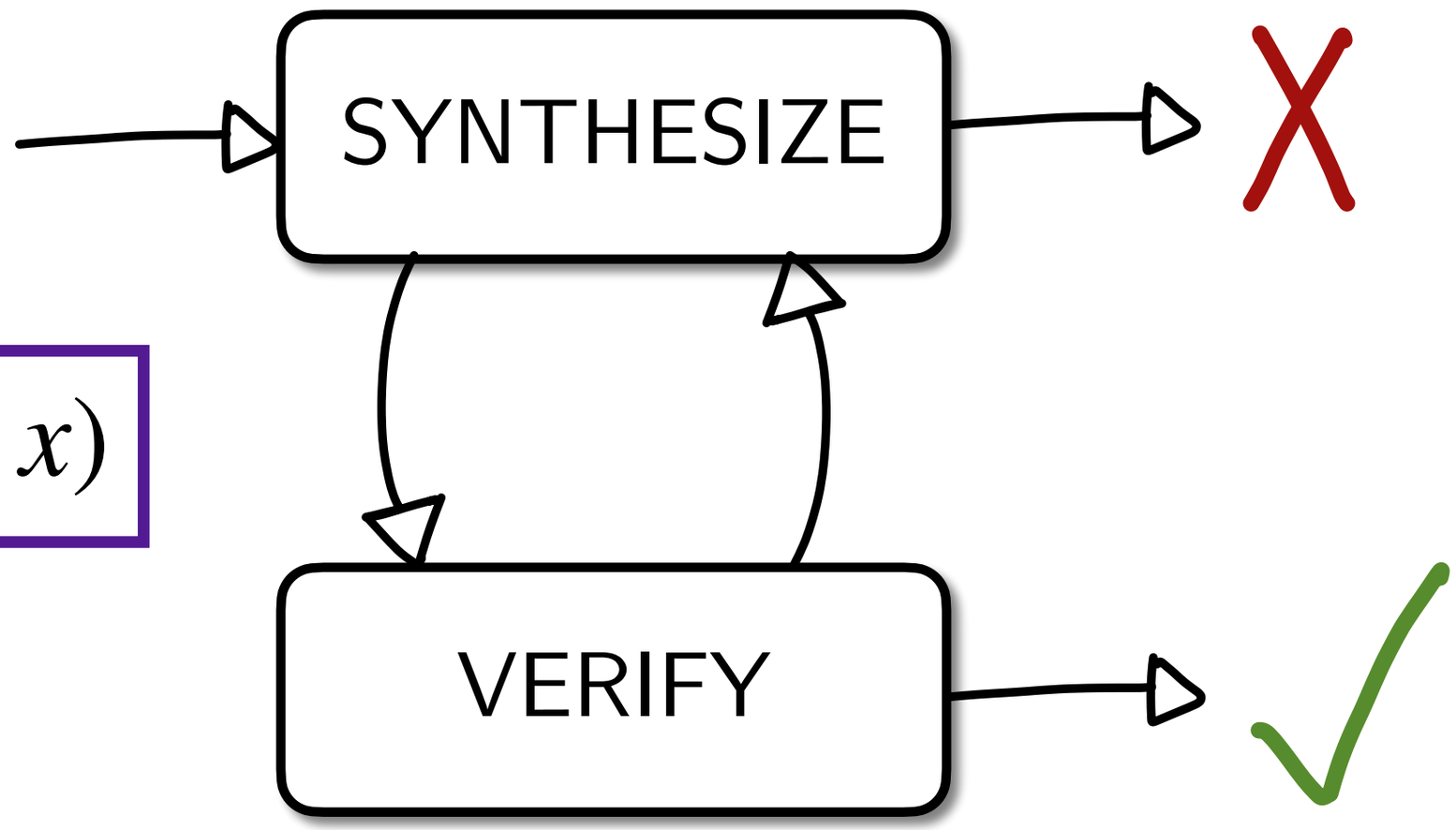

$\exists P^* . \forall x_i . \sigma(P^*, x_i)$
SYNTHESIZE
X
CounterExample
VERIFY
✓
$\exists x . \neg\sigma(P^*, x)$

CEGIS


$\exists P^* . \forall x_i . \sigma(P^*, x_i)$
SYNTHESIZE
UNS UNSAT
SAT
P^*
VERIFY

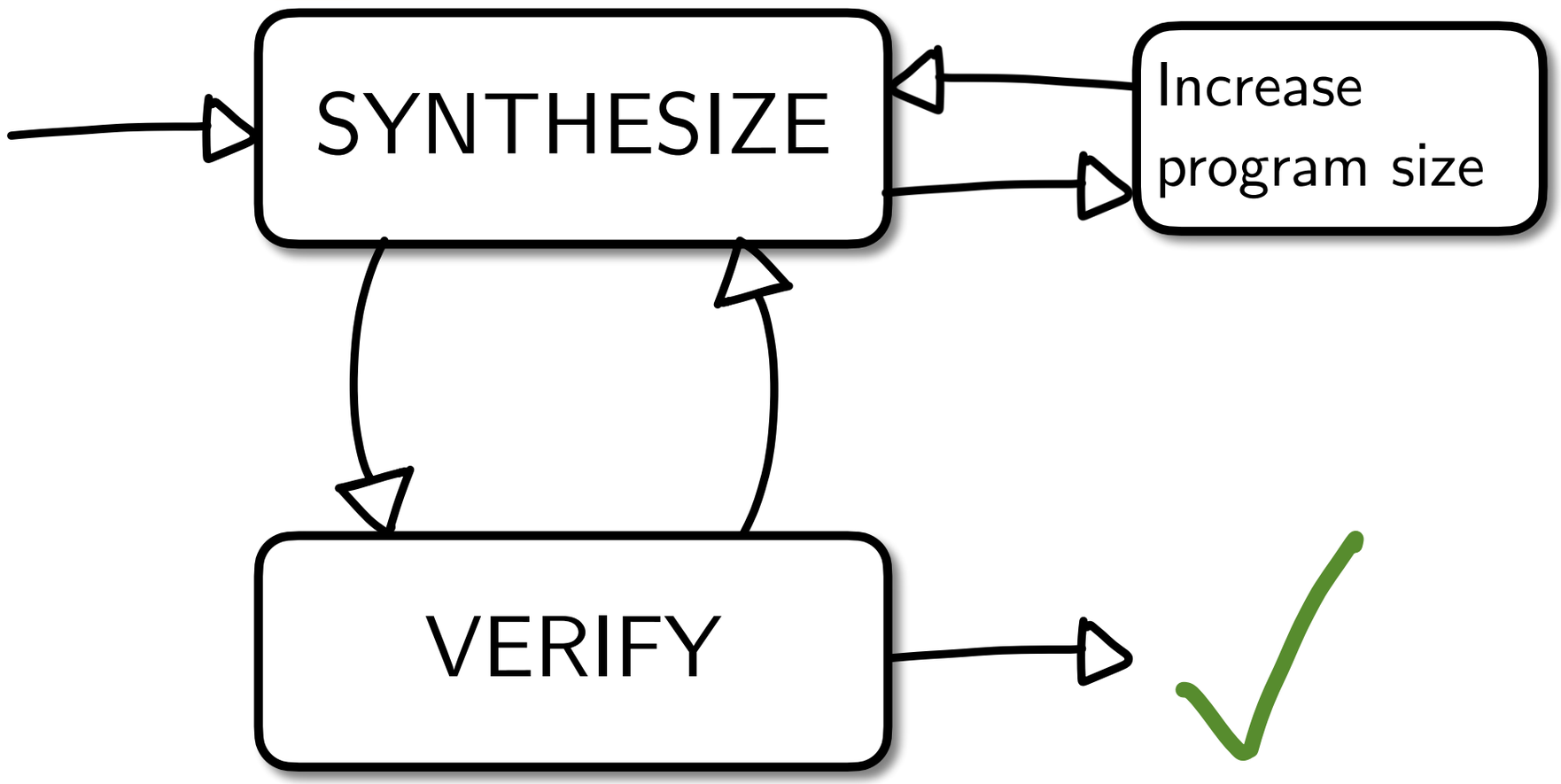
Variations on CEGIS: what goes in the synthesis box

$$\exists P \forall x \,.\, \sigma(P, x)$$



Symbolic Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$



Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$

$C_0 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$

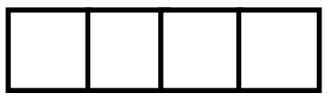
$P_1 ::= arg_1 \,|\, arg_2 \,|\, C_0$

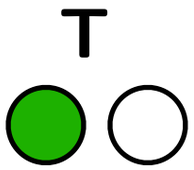
Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$

$C_0 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$

$P_1 ::= arg_1 \,|\, arg_2 \,|\, C_0$





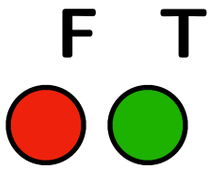
$$P_1 = arg_1$$

Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$

$C_0 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$

$P_1 ::= arg_1 \,|\, arg_2 \,|\, C_0$



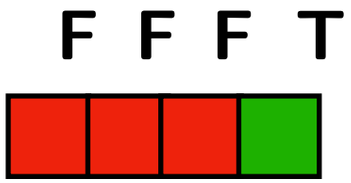
$$P_1 = arg_2$$

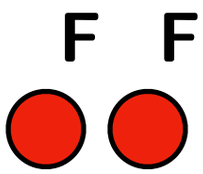
Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$

$$C_0 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$$

$$P_1 ::= arg_1 \,|\, arg_2 \,|\, C_0$$





$$P_1 = 1$$

Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$

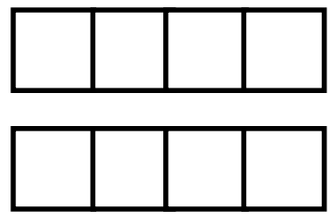
$$C_0 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$$

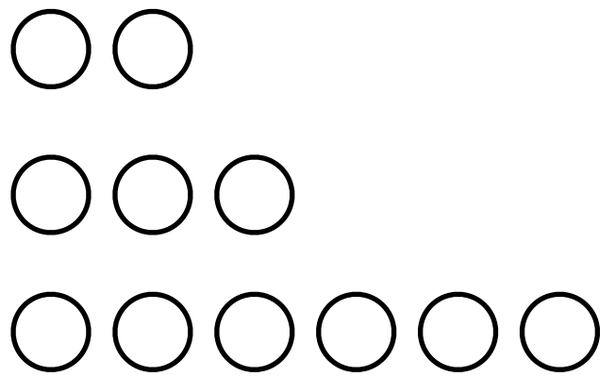
$$C_1 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$$

$$P_1 ::= arg_1 \,|\, arg_2 \,|\, C_0$$

$$P_2 ::= P_1 + P_1 \,|\, arg1 \,|\, arg_2 \,|\, C_1$$

$$P_3 ::= P_2 + P_1 \,|\, P_2 - P_1 \,|\, \ldots$$





Synthesis Encoding

$$\exists P^* \,.\, \forall x_i \,.\, \sigma(P^*, x_i)$$

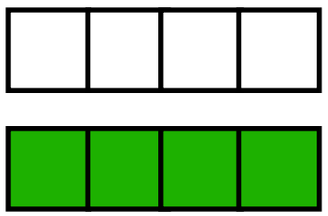
$$C_0 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$$

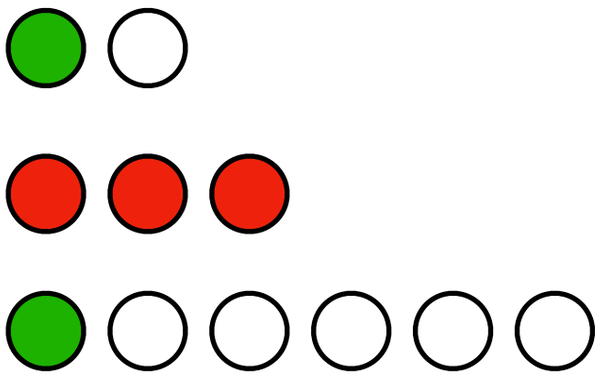
$$C_1 ::= 0000 \,|\, 0001 \,|\, \ldots \,|\, 1111$$

$$P_1 ::= arg_1 \,|\, arg_2 \,|\, C_0$$

$$P_2 ::= P_1 + P_1 \,|\, arg1 \,|\, arg_2 \,|\, C_1$$

$$P_3 ::= P_2 + P_1 \,|\, P_2 - P_1 \,|\, \ldots$$





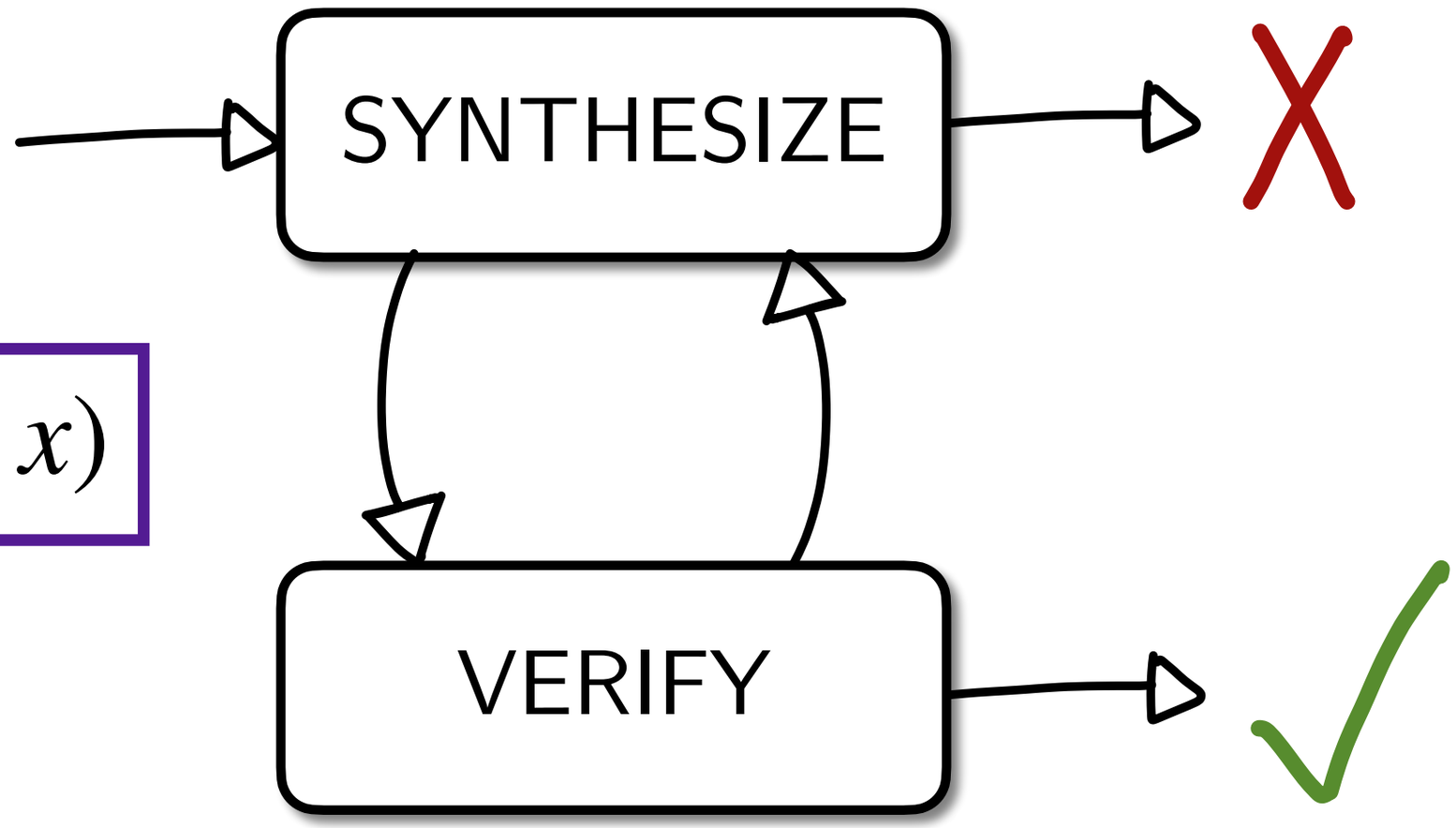
$$P_3 = 15 + arg_1$$

Variations on CEGIS: what goes in the synthesis box

- Enumeration (bottom-up, top-down, type-directed)
- Partial evaluation
- Stochastic
- Anything that can make guesses!

CEGIS

$$\exists P \forall x \,.\, \sigma(P, x)$$



UCLID5: verification, modeling and synthesis

- Build models based on transition systems and axioms
- Generate SMT queries for bounded model checking and (k)-induction
- Or synthesis queries

https://github.com/uclid-org/uclid

UCLID5 - example 9

```
module main {
 // System description.
 var a, b : integer;
 init {
   a = 0;
   b = 1;
 }
 next {
   a', b' = b, a + b;
 }
 // System specification.
 invariant a_le_b: a <= b;
 // Proof script.
 control {
   induction;
   check;
 }
}
```

UCLID5

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```

The induction procedure checks:

UCLID5

```
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 invariant a_le_b: a <= b;
 // Proof script.
 control {
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   check;
 }
}
```

The induction procedure checks:

1. That a_le_b holds at init

UCLID5

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module main {
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 }
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 invariant a_le_b: a <= b;
 // Proof script.
 control {
   induction;
   check;
 }
}
```

The induction procedure checks:

1. That a_le_b holds at init
2. That if a_le_b holds on entry to next, it will hold on exit

UCLID5

```
module main {
 // System description.
 var a, b : integer;
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```

The induction procedure checks:

1. That a_le_b holds at init
2. That if a_le_b holds on entry to next, it will hold on exit

a_le_b does actually hold but it is not inductive! Verification fails!

UCLID5: under the hood

```
module main {
 // System description.
 var a, b : integer;
 init {
   a = 0;
   b = 1;
 }
 next {
   a', b' = b, a + b;
 }
 // System specification.
 invariant a_le_b: a <= b;
 // Proof script.
 control {
   induction;
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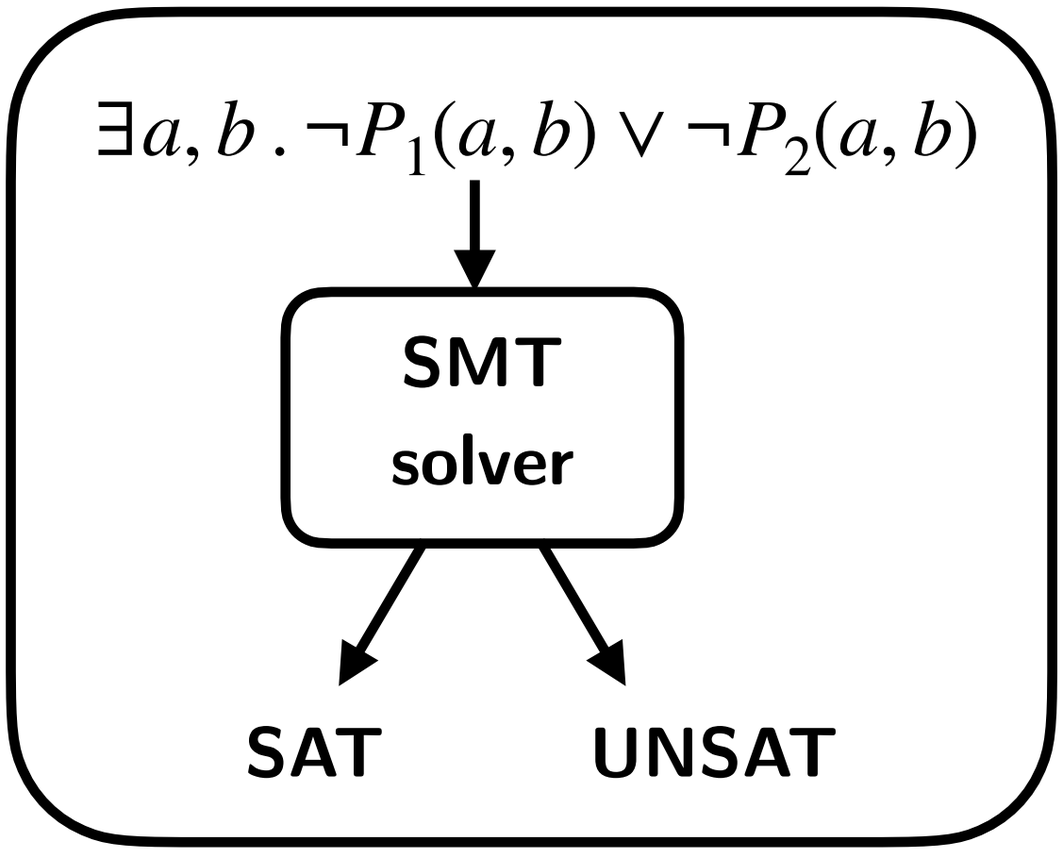
 }
}
```

The induction procedure checks:

1. That a_le_b holds at init
2. That if a_le_b holds on entry to next, it will hold on exit



UCLID5: under the hood

```
module main {
 // System description.
 var a, b : integer;
 init {
   a = 0;
   b = 1;
 }
 next {
   a', b' = b, a + b;
 }
 // System specification.
 invariant a_le_b: a <= b;
 // Proof script.
 control {
   unroll(3);
   check;
 }
}
```

The bmc/unroll procedure checks:

1. That a_le_b holds at init
2. That a_le_b holds for 2 steps
3. That a_le_b holds for 3 steps

UCLID5: synthesis

```
module main {
 synthesis function h(x : integer, y : integer): boolean;
 var a, b : integer;
 init {
   a = 0;
   b = 1;
 }
 next {
   a', b' = b, a + b;
 }
 // System specification.
 invariant a_le_b: a <= b && h(a, b);
 // Proof script.
 control {
   induction;
   check;
 }
}
```

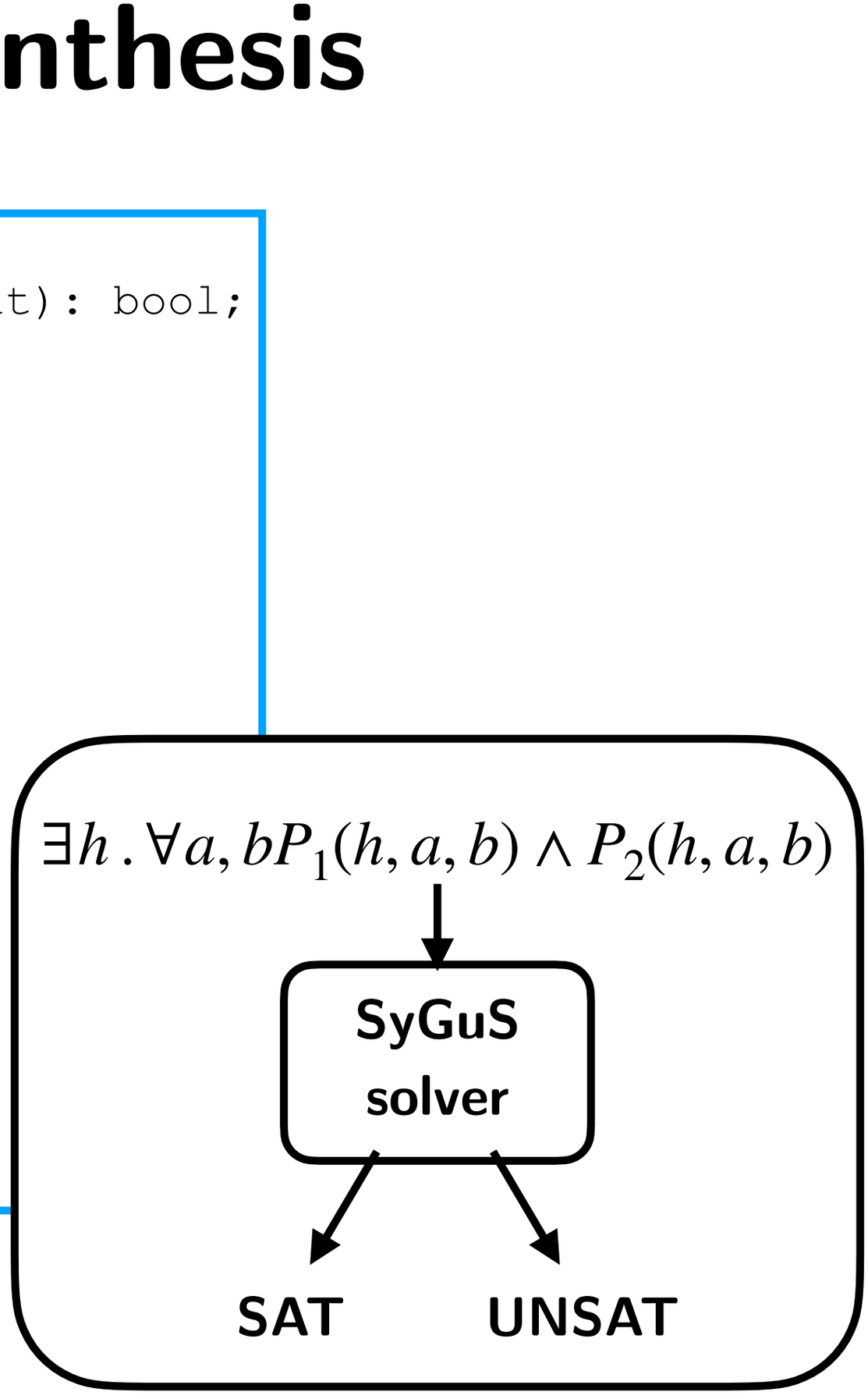
Synthesize h!

UCLID5: synthesis

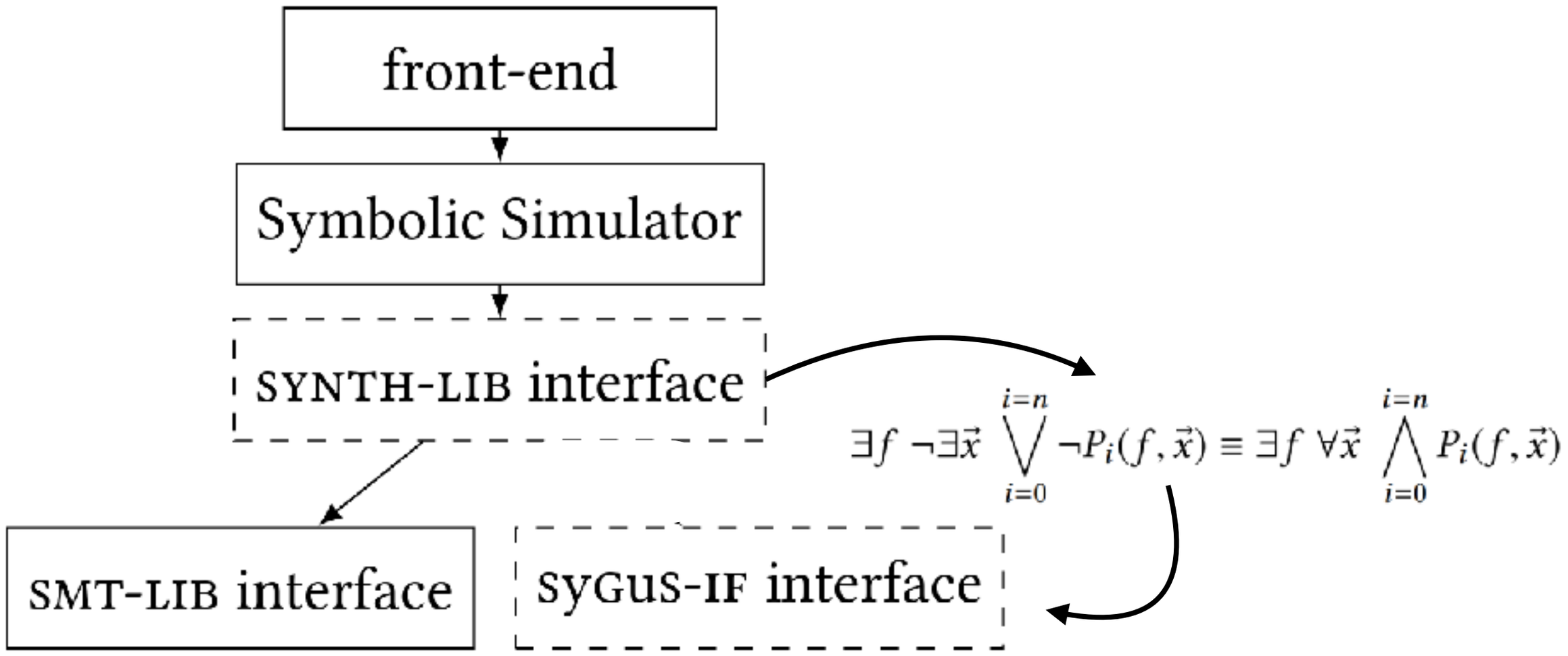
```
module main {
 synthesis function h(x : integer, y : integer): boolean;
// h(x, y) := x >= 0
 var a, b : integer;
 init {
   a = 0;
   b = 1;
 }
 next {
   a', b' = b, a + b;
 }
 // System specification.
 invariant a_le_b: a <= b && h(a, b);
// invariant a_le_b: a <= b && a >= 0;
 // Proof script.
 control {
   induction;
   check;
 }
}
```

UCLID5: synthesis

```
module main {
 synthesis function h(x : int, y : int): bool;
 var a, b : integer;
 init {
   a = 0;
   b = 1;
 }
 next {
   a', b' = b, a + b;
 }
 // System specification.
 invariant a_le_b: a <= b && h(a, b);
 // Proof script.
 control {
   induction;
   check;
 }
}
```

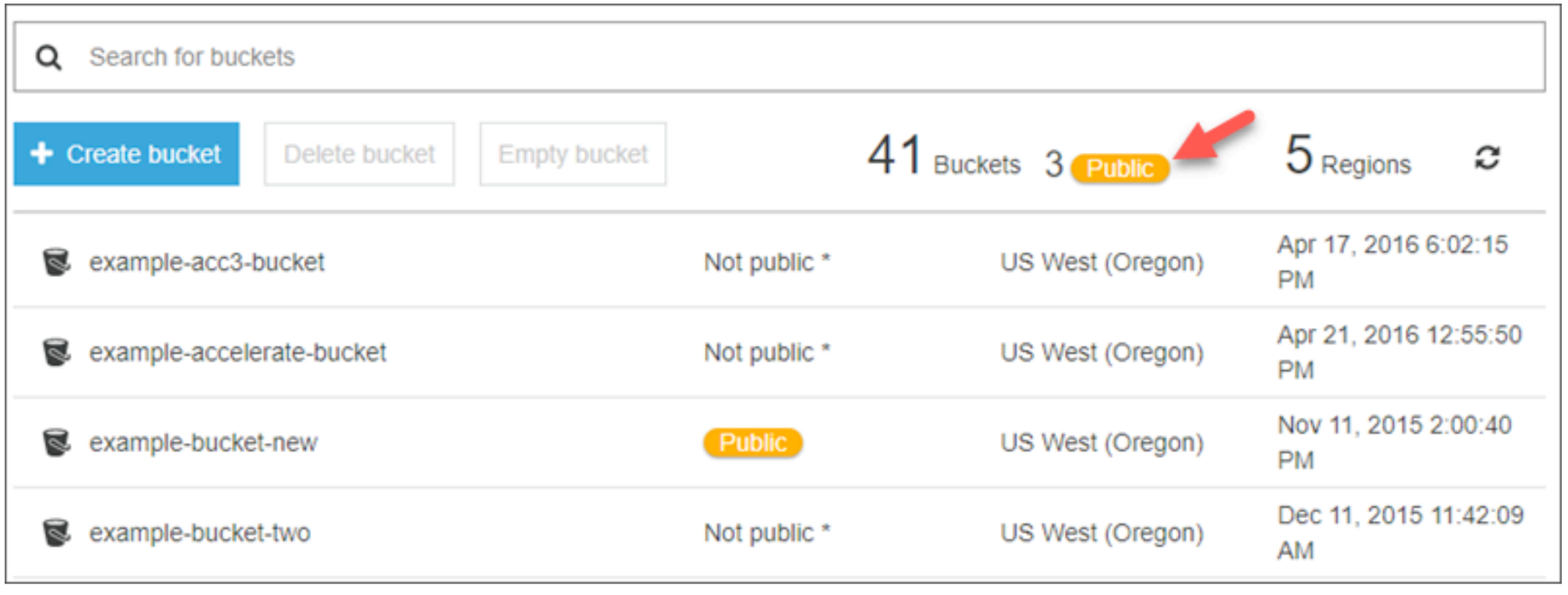


UCLID5: synthesis



Zelkova

- Uses SMT to verify access policies for AWS S3 buckets
- Makes millions of SMT calls daily



Zelkova

- S3 access policies are surprisingly hard for users to write
- Public buckets are a real security issue..

https://businessinsights.bitdefender.com/worst-amazon-breaches

Zelkova

Files

2.491 Of 8.657 Billion

AWS Buckets

141121 Of 440268

Azure Blobs

7961 Of 75103

Digital Ocean Spaces

3952 Of 7126

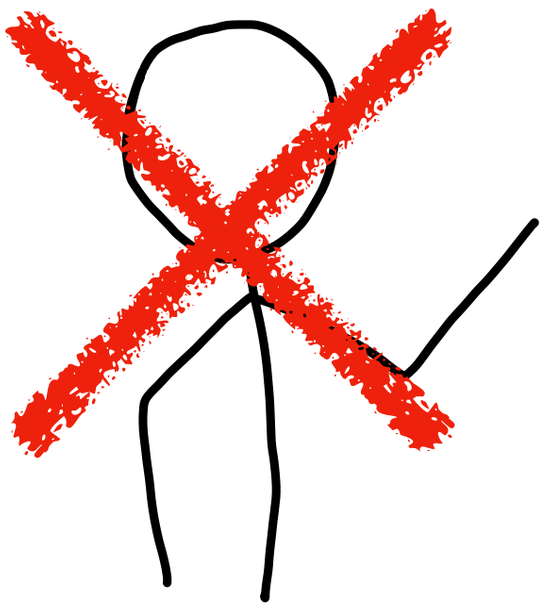
Last Update

04 June 2022

https://buckets.grayhatwarfare.com/

Zelkova

```
{
    "Effect": "Allow",
    "NotPrincipal": { "AWS": "111122223333" },
    "Action": "*",
    "Resource": "arn:aws:s3:::test-bucket"
}
```



Zelkova

```
{
    "Effect": "Allow",
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    "Action": "*",
    "Resource": "arn:aws:s3:::test-bucket"
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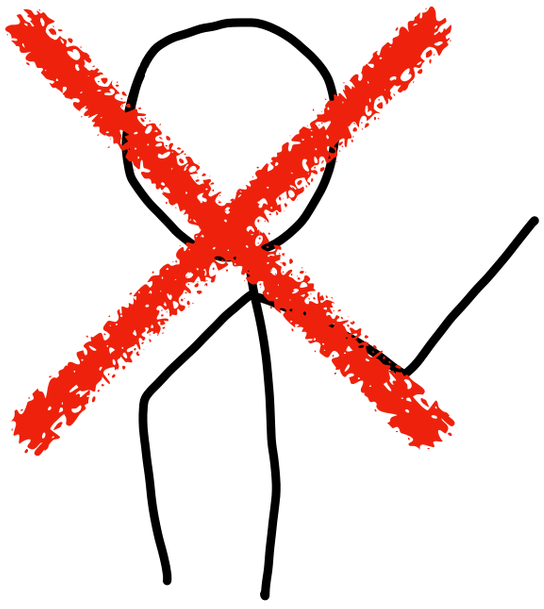

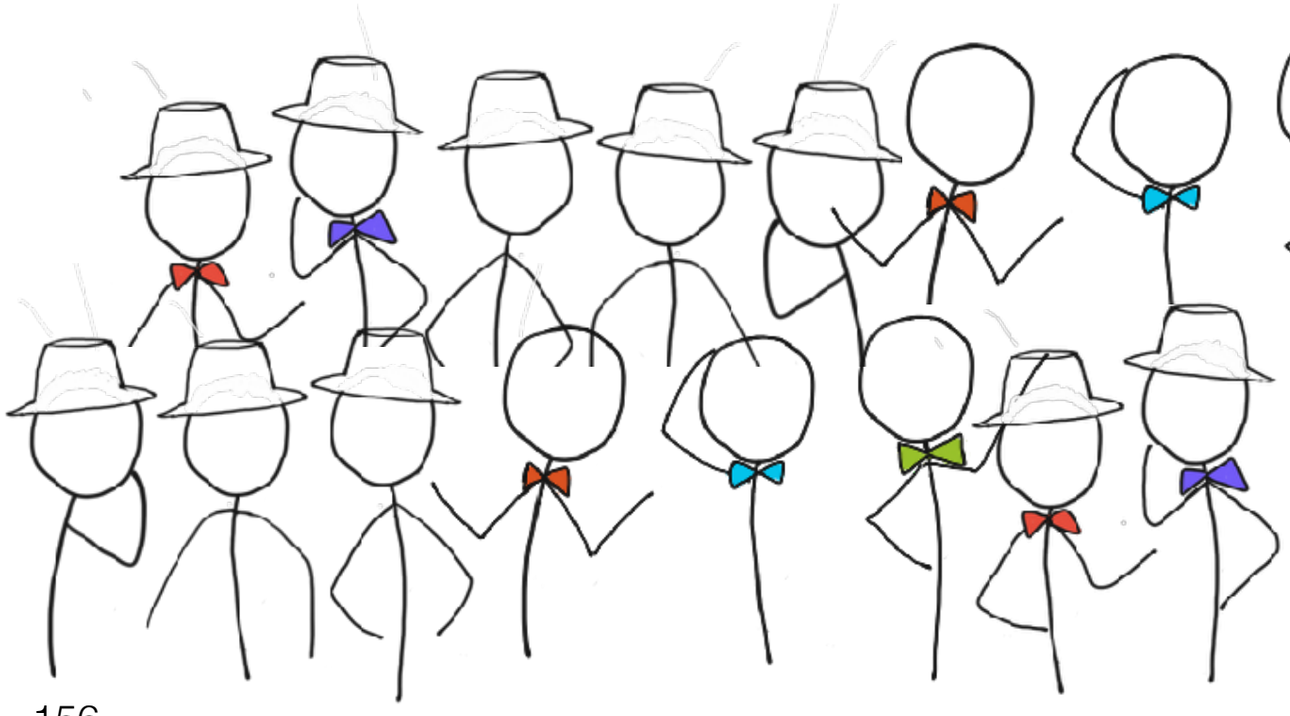

Zelkova

```
{
    "Effect": "Allow",
    "NotPrincipal": { "AWS": "111122223333" },
    "Action": "*",
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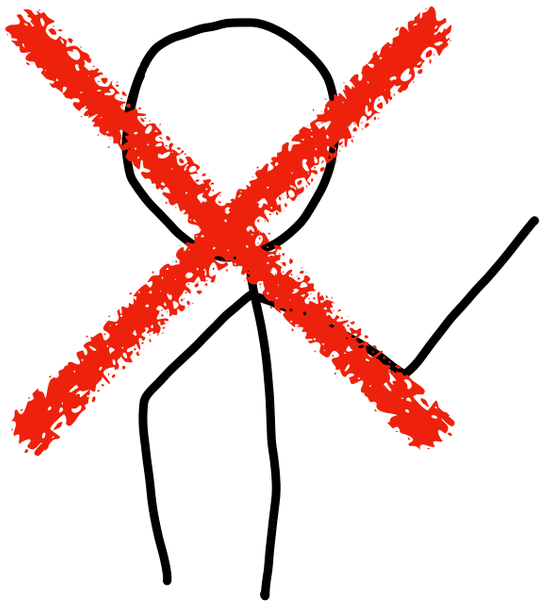
}
```



Zelkova

Comparing policies:

- P_1
- P_2
- $P_1 \implies P_2$: Policy 2 is at least as permissive as Policy 1

Zelkova

Comparing policies:

- P_1 : a policy that allows unauthenticated users
- P_2 : your policy
- $P_1 \implies P_2$: Policy 2 is at least as permissive as Policy 1

Zelkova

Turning policies into formula:

```
{
    "Effect": "Allow",
    "NotPrincipal": { "AWS": "111122223333" },
    "Action": "*",
    "Resource": "arn:aws:s3:::test-bucket"
}
```

(Resource = “arn:aws:s3:::test-bucket”) ∧ (Principal ≠ 11112222333)

Zelkova

Fixed:

```
{
    "Effect": "Deny"
    "Principal": { "AWS": "111122223333" },
    "Action": "*",
    "Resource": "arn:aws:s3:::test-bucket"
}
```

Zelkova: Example 7

```
(( allow,
   principal   :   students,
   action      :   getObject,
   resource    :   cs240/Exam.pdf),
 ( allow,
   principal   :   tas,
   action      :   getObject,
   resource    :   (cs240/Exam.pdf,
                    cs240/Answer.pdf)))
```

(a) Policy X

- everybody can access all the contents of *cs240/*
- students cannot access the answers.

```
(( allow,
   principal   :   *,
   action      :   getObject,
   resource    :   cs240/*),
 ( deny,
   principal   :   students,
   action      :   getObject,
   resource    :   cs240/Answer.pdf))
```

(b) Policy Y

- students can read the exam
- TAs can read both the exam and its answers.

Zelkova: Example 7

```
(( allow,
   principal   :   students,
   action      :   getObject,
   resource    :   cs240/Exam.pdf),
 ( allow,
   principal   :   tas,
   action      :   getObject,
   resource    :   (cs240/Exam.pdf,
                    cs240/Answer.pdf)))
```

(a) Policy X

$$X_0 : a = \text{“}getObject\text{”} \wedge p = \text{“}students\text{”} \wedge r = \text{“}cs240/Exam.pdf\text{”}$$

$$X_1 : a = \text{“}getObject\text{”} \wedge p = \text{“}tas\text{”} \wedge (r = \text{“}cs240/Exam.pdf\text{”} \vee r = \text{“}cs240/Answer.pdf\text{”})$$

$$X : X_0 \vee X_1$$

2 allow statements gives disjunction

Zelkova: Example 7

Y_0 : $a =$ *"getObject"* $\wedge\ r =$ *"cs240/*"*

Y_1 : $a =$ *"getObject"* $\wedge\ p =$ *"students"* $\wedge\ r =$ *"cs240/Answer.pdf"*

Y : $Y_0 \wedge \neg Y_1$

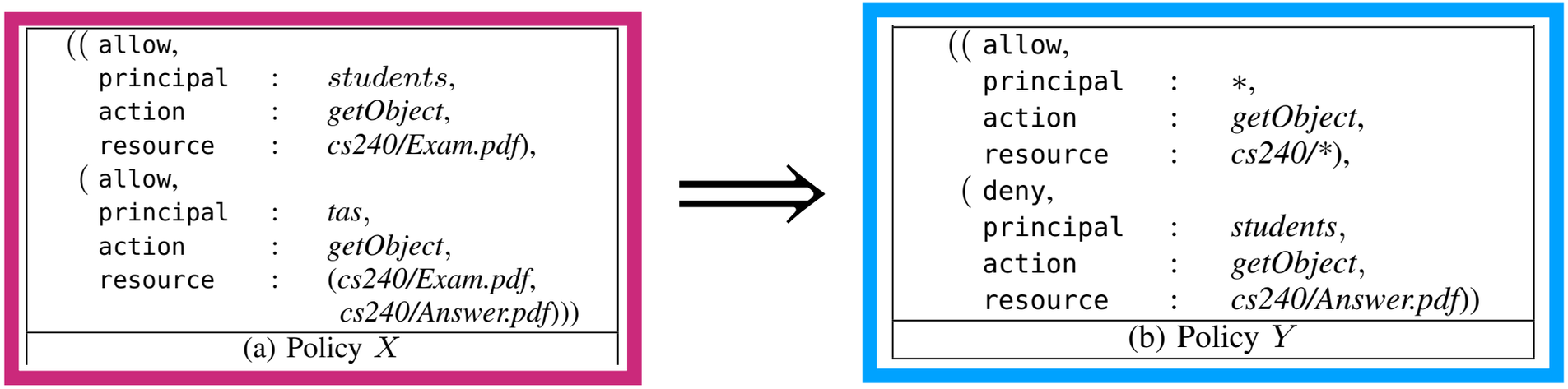
Allow and deny statement are conjuncted together

```
(( allow,
   principal   :   *,
   action      :   getObject,
   resource    :   cs240/*),
 ( deny,
   principal   :   students,
   action      :   getObject,
   resource    :   cs240/Answer.pdf))
```

(b) Policy Y

Zelkova: Example 7

Check:



((allow,
  principal : *students*,
  action : *getObject*,
  resource : *cs240/Exam.pdf*),
 (allow,
  principal : *tas*,
  action : *getObject*,
  resource : (*cs240/Exam.pdf*,
  cs240/Answer.pdf)))

(a) Policy X

$\Longrightarrow$

((allow,
  principal : *,
  action : *getObject*,
  resource : *cs240/**),
 (deny,
  principal : *students*,
  action : *getObject*,
  resource : *cs240/Answer.pdf*))

(b) Policy Y

$$X \implies Y$$

True: All requests allowed by X are allowed by Y

Zelkova: Example 7

Check:

((allow,
  principal : *students*,
  action : *getObject*,
  resource : *cs240/Exam.pdf*),
 (allow,
  principal : *tas*,
  action : *getObject*,
  resource : (*cs240/Exam.pdf*,
  cs240/Answer.pdf)))

(a) Policy X

$\Longleftarrow$

((allow,
  principal : *,
  action : *getObject*,
  resource : *cs240/**),
 (deny,
  principal : *students*,
  action : *getObject*,
  resource : *cs240/Answer.pdf*))

(b) Policy Y

$$Y \implies X$$

False: Y allows requests that X does not

Zelkova: Example 7

```
(( allow,
   principal   :   *,
   action      :   getObject,
   resource    :   cs240/*),
 ( deny,
   principal   :   students,
   action      :   getObject,
   resource    :   cs240/Answer.pdf))
```

(b) Policy Y

Anyone can access files in cs240

Zelkova: Example 7

Check:

((allow,
  principal : *students*,
  action : *getObject*,
  resource : *cs240/Exam.pdf*),
 (allow,
  principal : *tas*,
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(b) Policy Y

$$Y \implies X$$

False: Y allows requests that X does not

Note: Amazon developed their own regex extension of Z3 to solve this

By the end of these lectures you will know ...

- What the satisfiability problem is
- How to encode problems into SAT/SMT
- Which SAT/SMT solvers are available and how to use them, and some useful tools to generate SAT/SMT queries.
- How SAT/SMT solvers are deployed in the wild

You will not know ...

- The detail of how SAT/SMT solvers work