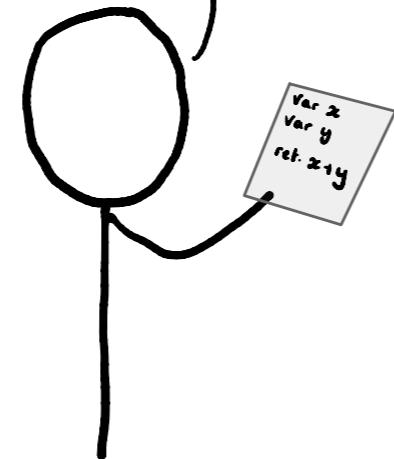


Satisfiability and Synthesis Modulo Oracles

Elizabeth Polgreen, Andrew Reynolds, Sanjit A. Seshia

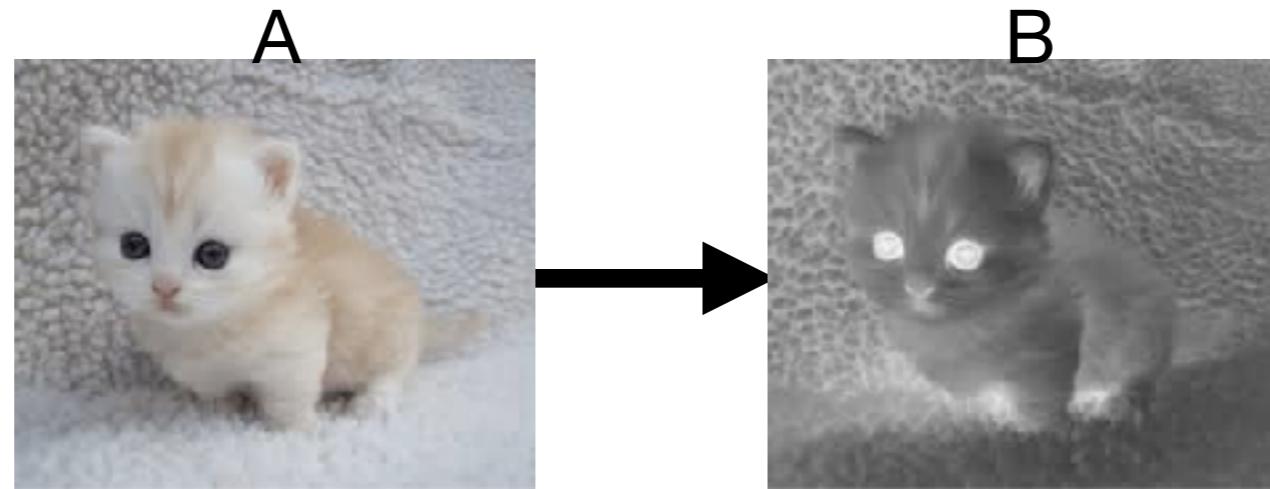
Is my program correct?



No, but I can tell you that
on input 7 it should return 13.



Find a function that transforms Cat A into Cat B?



Can I use program synthesis?

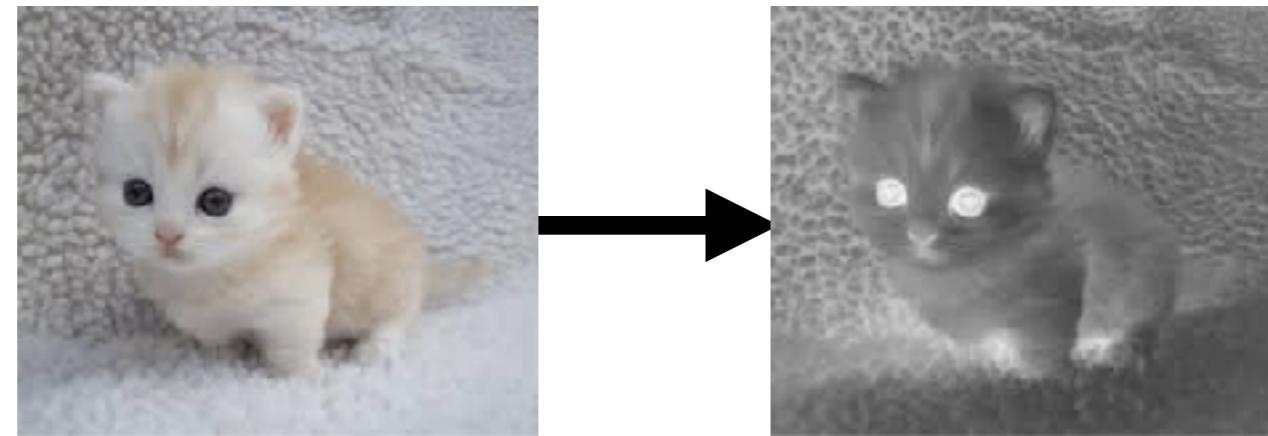
$$\exists f.f(CatA) = CatB$$

!! PROBLEM !!

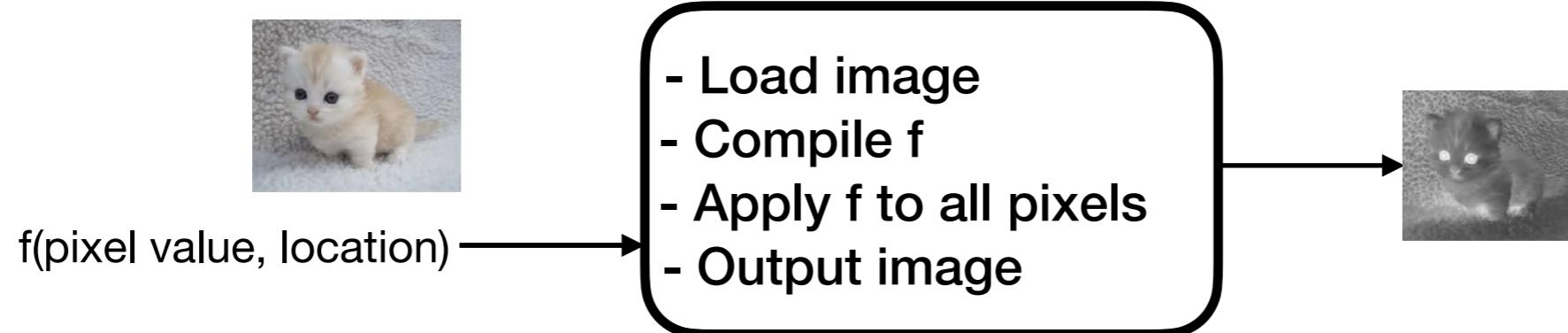
f is complicated! Needs to:

- load the JPG image
- apply transformation to all the pixels
- output a new image

Find a function that transforms Cat A into Cat B?



Can I use program synthesis?

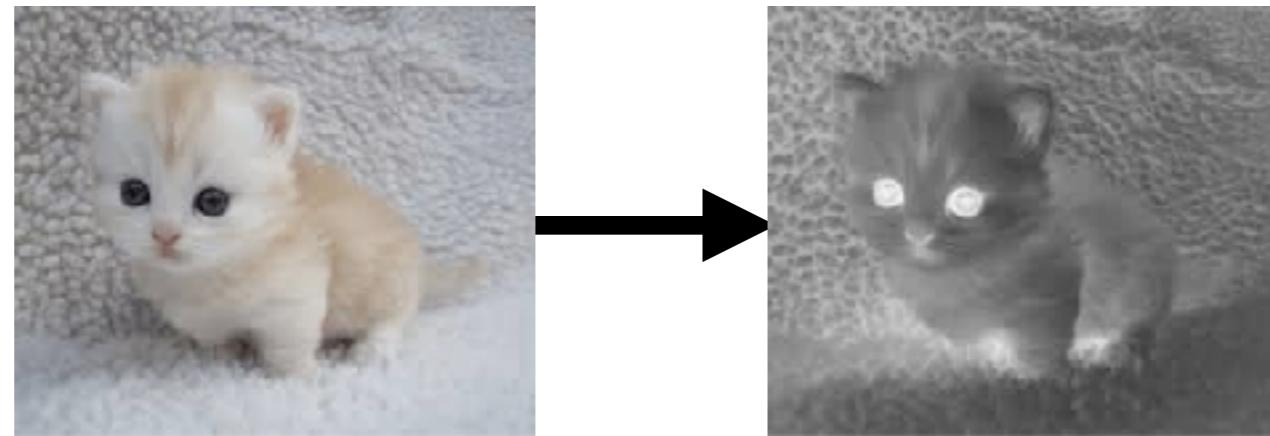


$$\exists f \forall pixels, locs . f(CatA(loc), loc) = CatB(loc)$$

!! PROBLEM !!

Synthesiser needs to interpret CatA(loc)

Find a function that transforms Cat A into Cat B?



Can I use program synthesis?

Give all pixels input-output examples prior to solving?

$$\exists f. f(122,0) = 141 \wedge \dots \wedge f(133,65536) = 144$$

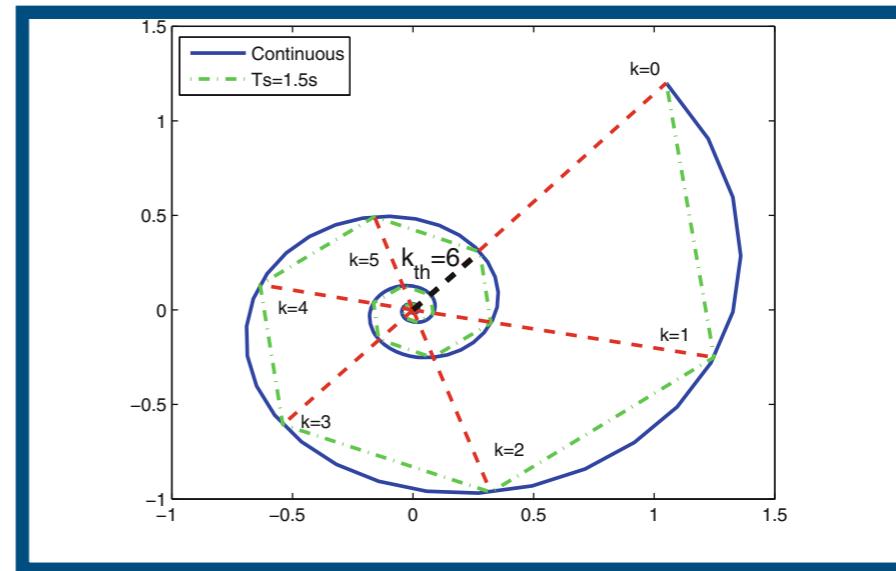
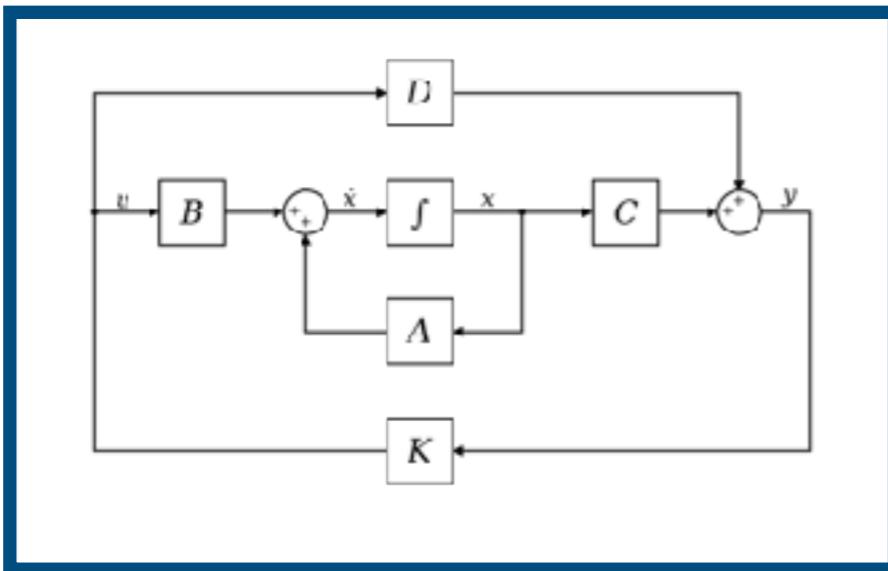
!! PROBLEM !!

We tried this!

- 65536 input-output examples
- Assumes that compiler/image processing library is all bug-free.

Do I need to build a new solver?

Find a digital controller K for this LTI system?



Can I use program synthesis?

$$\exists k. \text{Stable}(A - Bk) \wedge \forall x. \text{Safe}(x)$$

!! PROBLEM !!

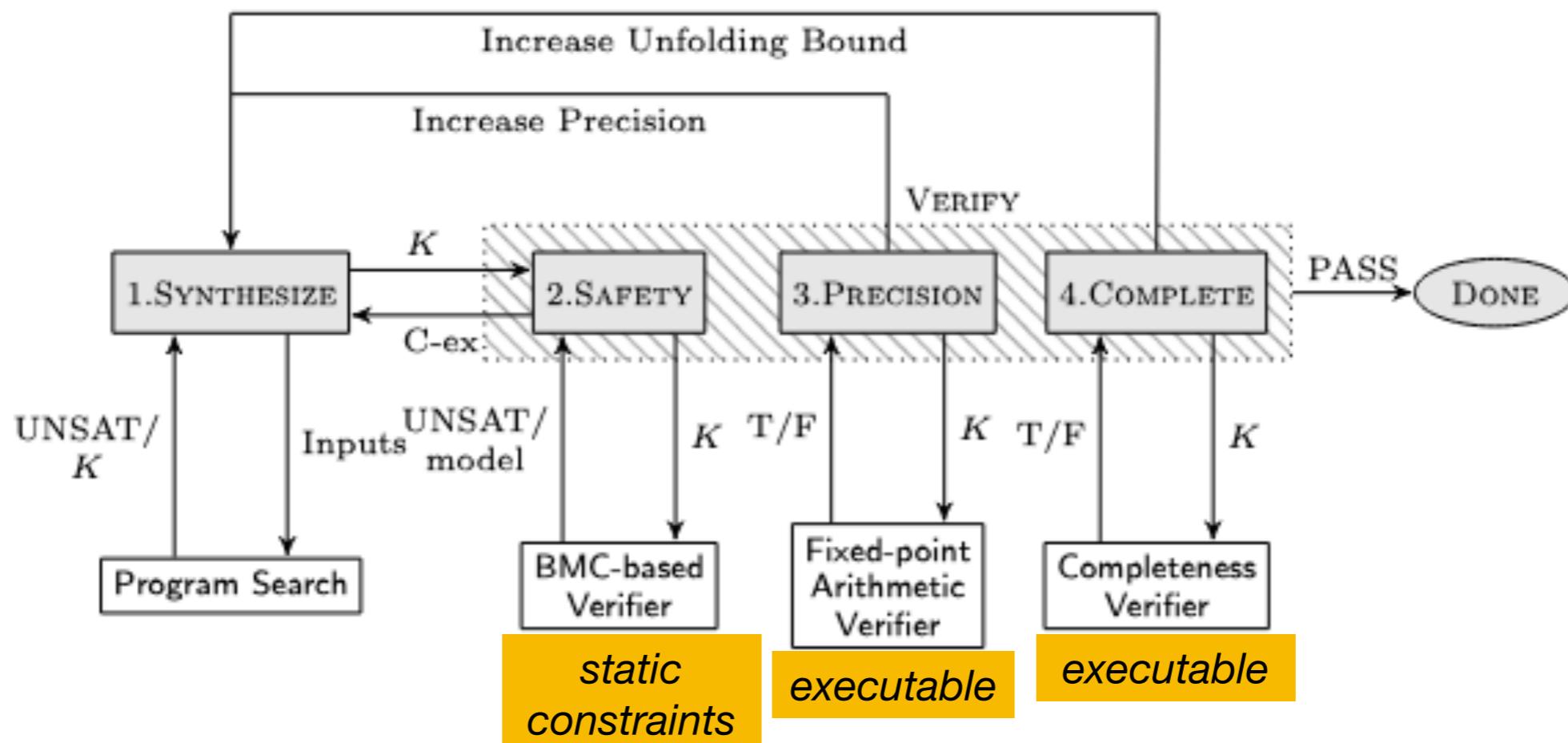
We tried this!

- Verifier needs to find eigenvalues
- Extremely non-linear

Do I need to build a new solver?

Automated formal synthesis of provably safe digital controllers for continuous plants

Alessandro Abate²  · Iury Bessa³  · Lucas Cordeiro⁴  · Cristina David⁵  ·
Pascal Kesseli⁶  · Daniel Kroening²  · Elizabeth Polgreen¹ 



Find prime factors of a number?



Can I use SMT?

$\exists f_1, f_2 . \text{isPrime}(f_1) \wedge \text{isPrime}(f_2) \wedge f_1 * f_2 = x$

!! PROBLEM

isPrime is a recursive function!

Do I need to build a new solver?

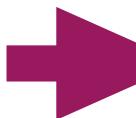
Why don't off-the-shelf solvers work?

- Parts are hard to model with static constraints e.g., the image processing library.
- Parts are hard to reason about e.g., eigenvalues, primes.
- Too many constraints, which ones are important? e.g., which pixels matter?



Solution: Use executable “oracles”

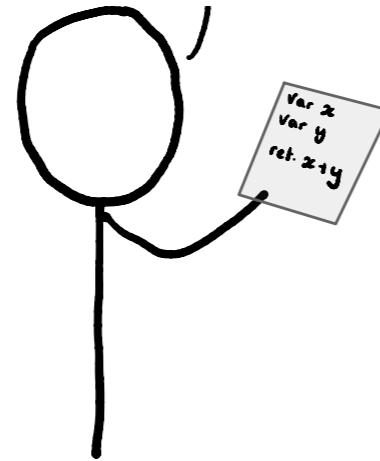
Coming up

- 
- Existing use of oracles
 - Formal definition of oracle interfaces
 - SMT with oracles (**SMTO**):
 - when is it satisfiable/unsatisfiable
 - algorithm
 - Synthesis with oracles (**SyMO**):
 - when are solutions correct
 - **unifying** algorithm for solving
 - More cat pictures
 - Prototype evaluation

What is an oracle?

Query

Is this number prime?



Response

No, it is not prime.



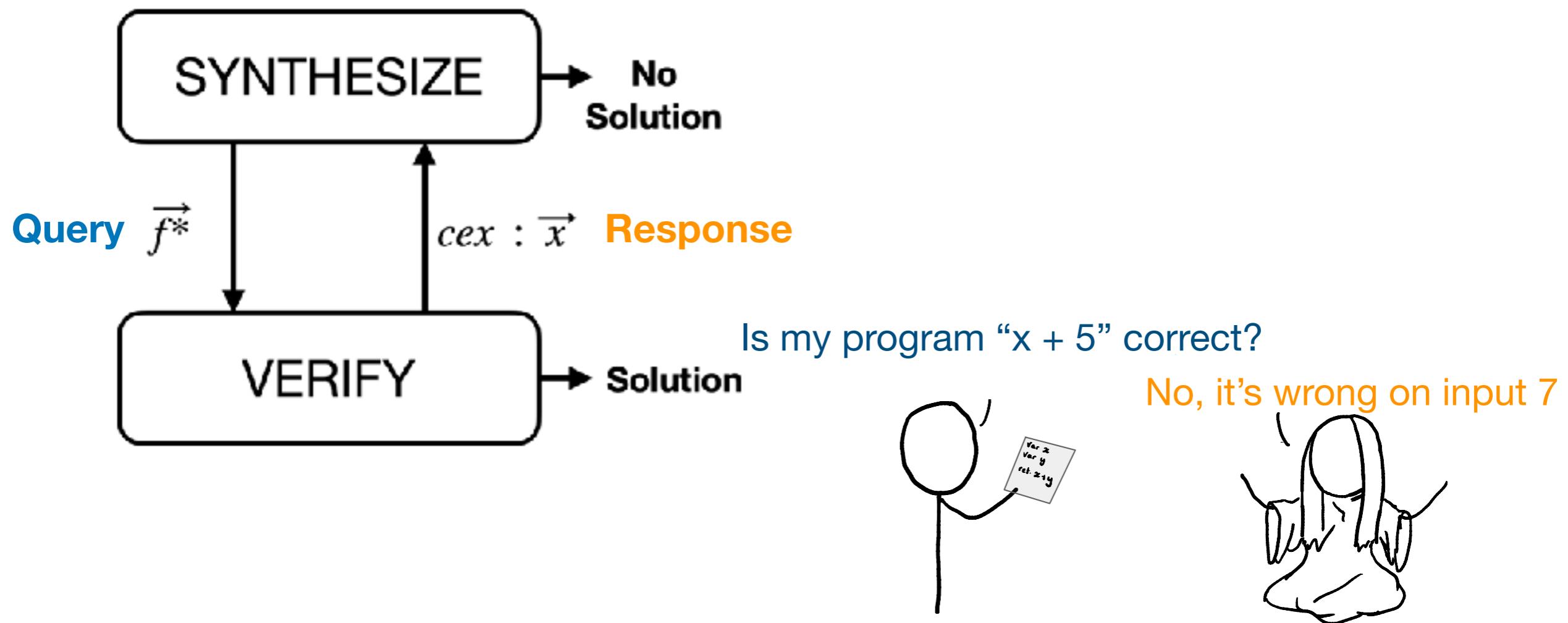
Existing use of oracles:

- Existing synthesis algorithms are using oracles!
- Jha and Seshia[1] set out the theory behind this.
- Our contribution: defining satisfiability/synthesis modulo oracles problem and proposing a unifying algorithm

[1] A theory of formal synthesis via inductive learning - Jha and Seshia

Existing use of oracles:

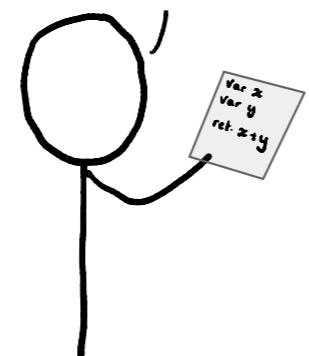
- Counterexample Guided Inductive Synthesis [2]



Existing use of oracles:

- CEGIS(T) [3]

Is my program “ $x + 5$ ” correct?

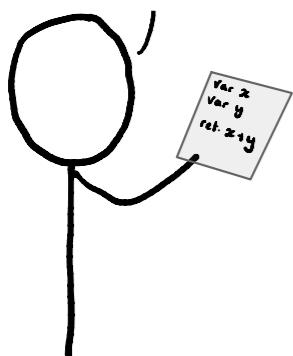


No, but if you replace 5 with 1, it would be



- ICE learning [4]

Is my invariant
“ $x > 5$ ” correct?



No, and
 $inv(6) \implies inv(5)$



No, and
 $inv(100) = false$



No, and
 $inv(0) = true$



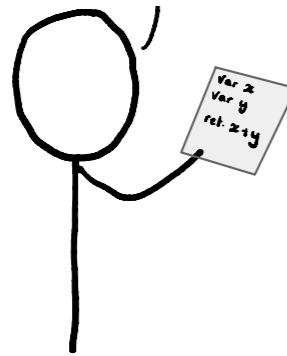
[3] Counterexample Guided Inductive Synthesis modulo Theories - Abate et al

[4] ICE: A robust framework for learning invariants - Garg et al

Existing use of oracles:

- Program lifting [5]

Is my program correct?



No, it's wrong on
this input



it should access
these memory
locations



the runtime
complexity should
be linear



Examples

People use oracles, but they build their own custom solver.
Why?

- Solver needs custom information about the oracles (what does the response from the oracle mean?)

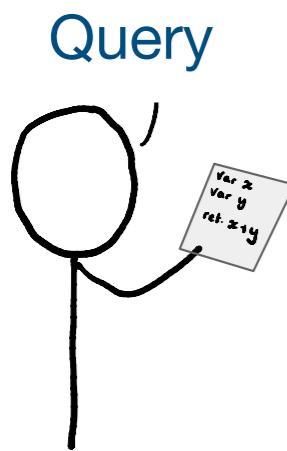
What if we could communicate this information to an off-the-shelf SMT or SyGuS solver?

Coming up

- Existing use of oracles
- Formal definition of oracle interfaces
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What is an oracle?

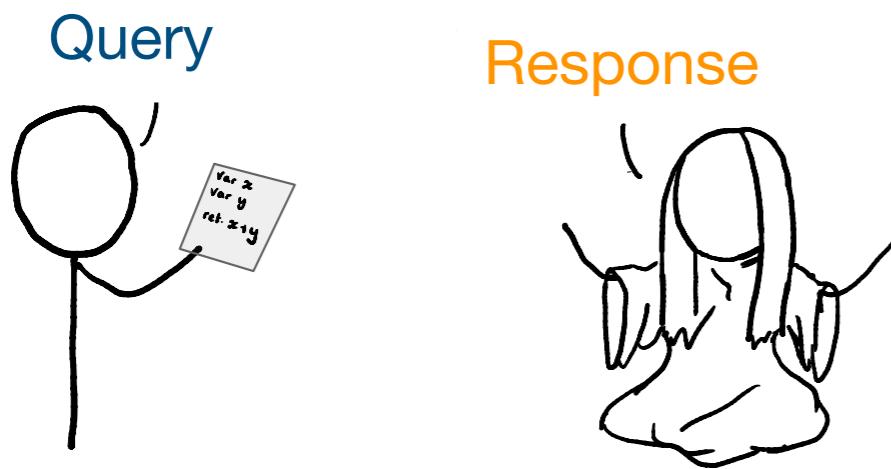
- We define how the oracle is queried by defining an interface



\vec{y} : query domain
 \vec{z} : response co-domain

What is an oracle?

- We define how the oracle is queried by defining an interface

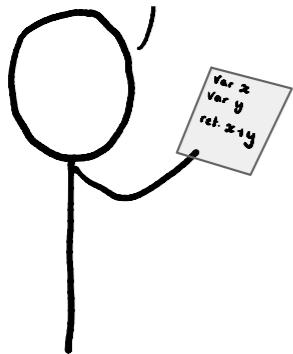


\vec{y} : query domain
 \vec{z} : response co-domain
 α_{gen} : assumption generator
 β_{gen} : constraint generator

- and **assumption** and **constraint** generators, which generate:
 - **assumptions** the solver is allowed to make
 - and **constraints** the solver must abide by

Example oracle: \mathcal{O}_{prime}

Is this number y prime?



No, z=false, it is not prime.

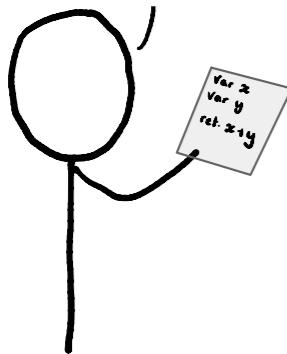


Solve: $\exists x . prime(x) \wedge (x \% 2 = 1)$

\vec{y}	: $(y : integer)$
\vec{z}	: $(z : bool)$
α_{gen}	: $prime(y) = z$
β_{gen}	: \emptyset

Example oracle: \mathcal{O}_{prime}

Is this number y prime?

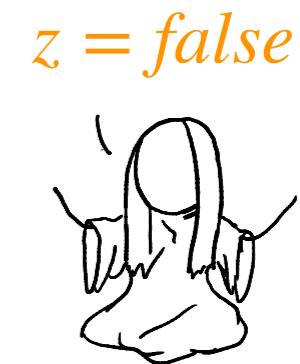
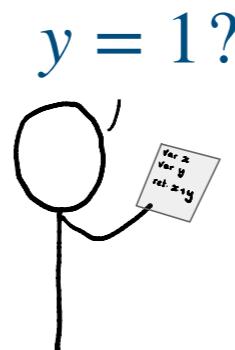


No, z=false, it is not prime.

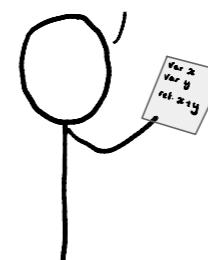


\vec{y}	: $(y : integer)$
\vec{z}	: $(z : bool)$
α_{gen}	: $prime(y) = z$
β_{gen}	: \emptyset

Solve: $\exists x . prime(x) \wedge (x \% 2 = 1)$

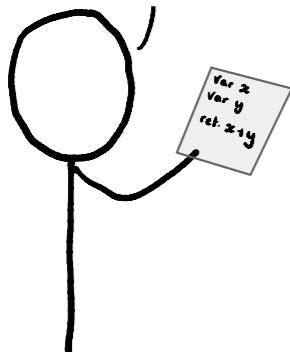


Okay, now I know I can assume
 $prime(1) = false$



Example oracle: $\mathcal{O}_{positive-witness}$

Is my program f^* correct?



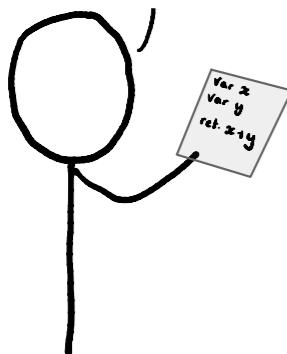
I can tell you that on input 7 it should return 13



\vec{y}	: $(f^* : int \rightarrow int)$
\vec{z}	: $(z_1 : int, z_2 : int)$
α_{gen}	: \emptyset
β_{gen}	: $f(z_1) = z_2$

Example oracle: $\mathcal{O}_{positive-witness}$

Is my program f^* correct?

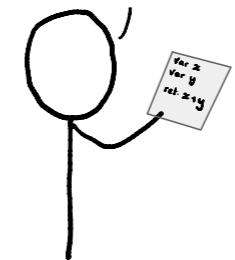


I can tell you that on input 7 it should return 13

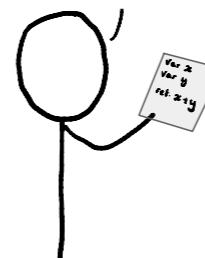


\vec{y}	: $(f^* : int \rightarrow int)$
\vec{z}	: $(z_1 : int, z_2 : int)$
α_{gen}	: \emptyset
β_{gen}	: $f(z_2) = z_3$

$f^* = x + 1$ $z_1 = 7, z_2 = 13$



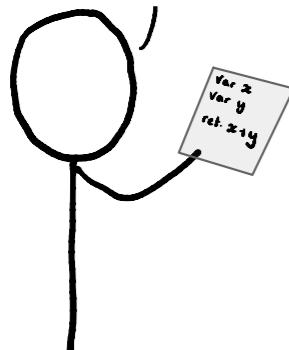
Okay, now I know that a valid program f must satisfy $f(7) = 13$



Oracle function symbols

Is this number y prime?

No, z=false, it is not prime.



An oracle function symbol is a symbol whose behaviour is defined to be the **same** as an external oracle.

Note: oracle must be **functional**

$\vec{y} : (y : \text{integer})$

$\vec{z} : (z : \text{bool})$

$\alpha_{gen} : \text{prime}(y) = z$

$\beta_{gen} : \emptyset$

prime is an oracle function symbol

Coming up

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Satisfiability Modulo Theories and Oracles (SMTO)

An SMTO problem is a tuple:

\vec{f} : a set of ordinary function symbols

$\vec{\theta}$: a set of oracle function symbols

ρ : a formula in a background theory

$\vec{\mathcal{O}}$: a set of oracle interfaces

$\vec{f} : \{f_1, f_2\}$

$\vec{\theta} : \{prime\}$

$\rho : prime(f_1) \wedge prime(f_2) \wedge (f_1 * f_2 = 24)$

$\vec{\mathcal{O}} : \{\mathcal{O}_{prime}\}$

\mathcal{O}_{prime}

$\vec{y} : (y : integer)$

$\vec{z} : (z : bool)$

$\alpha_{gen} : prime(y) = z$

$\beta_{gen} : \emptyset$

Is this satisfiable? What is a valid assignment to f_1 and f_2 ?

Satisfiability Modulo Theories and Oracles (SMTO)

SAT?

$$\text{prime}(f_1) \wedge \text{prime}(f_2) \wedge (f_1 * f_2 = 24)$$

$\mathcal{O}_{\text{prime}}$

$$\begin{aligned}\vec{y} &: (y : \text{integer}) \\ \vec{z} &: (z : \text{bool}) \\ \alpha_{\text{gen}} &: \text{prime}(y) = z \\ \beta_{\text{gen}} &: \emptyset\end{aligned}$$

- If prime does what we expect, then yes! But we don't know that
- If we haven't called the oracle, an assignment must work for ALL possible behaviours of prime
- When we call $\mathcal{O}_{\text{prime}}$, we get **assumptions** about prime
- If the assumptions rule out all possible assignments: UNSAT

Satisfiability Modulo Theories and Oracles (SMTO)

\mathcal{O}_{prime}

SAT?

$$prime(f_1) \wedge prime(f_2) \wedge (f_1 * f_2 = 24)$$

$\vec{y} : (y : integer)$
$\vec{z} : (z : bool)$
$\alpha_{gen} : prime(y) = z$
$\beta_{gen} : \emptyset$

Conjunction of assumptions. True if no assumptions

Satisfiable iff $\exists f_1, f_2. \forall prime . A \implies \rho$ is satisfiable

Unsatisfiable iff $\exists f_1, f_2. \exists prime . A \wedge \rho$ is unsatisfiable

Unknown otherwise

Satisfiability Modulo Theories and Oracles (SMTO)

\mathcal{O}_{prime}

SAT?

$$prime(f_1) \wedge prime(f_2) \wedge (f_1 * f_2 = 24)$$

$\vec{y} : (y : integer)$
 $\vec{z} : (z_1 : bool, z_2 : integer)$
 $\alpha_{gen} : prime(y) = z$
 $\beta_{gen} : f_1 < z_2$

Constraints must be obeyed by the solver:

Conjunction of constraints. True if no constraints.

Satisfiable iff $\exists f_1, f_2 . \forall prime . A \implies (\rho \wedge B)$ is satisfiable

Unsatisfiable iff $\exists f_1, f_2 . \exists prime . A \wedge \rho \wedge B$ is unsatisfiable

Unknown otherwise

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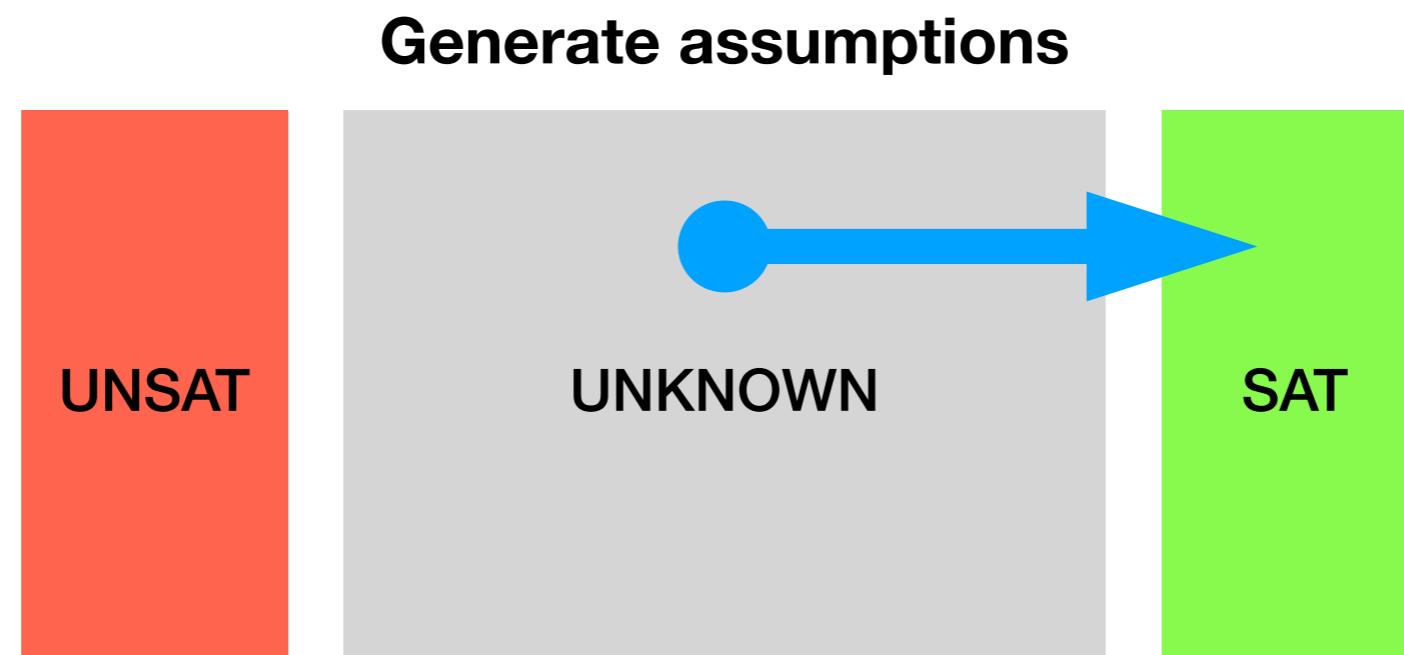


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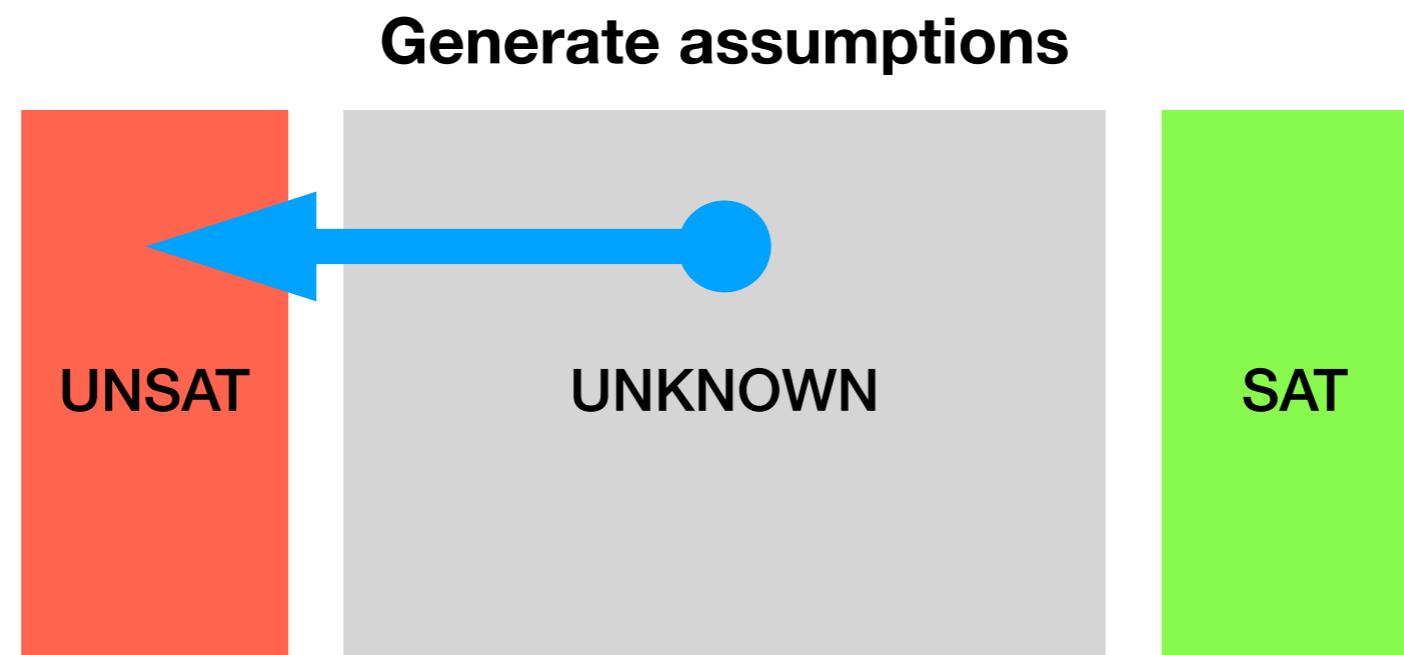


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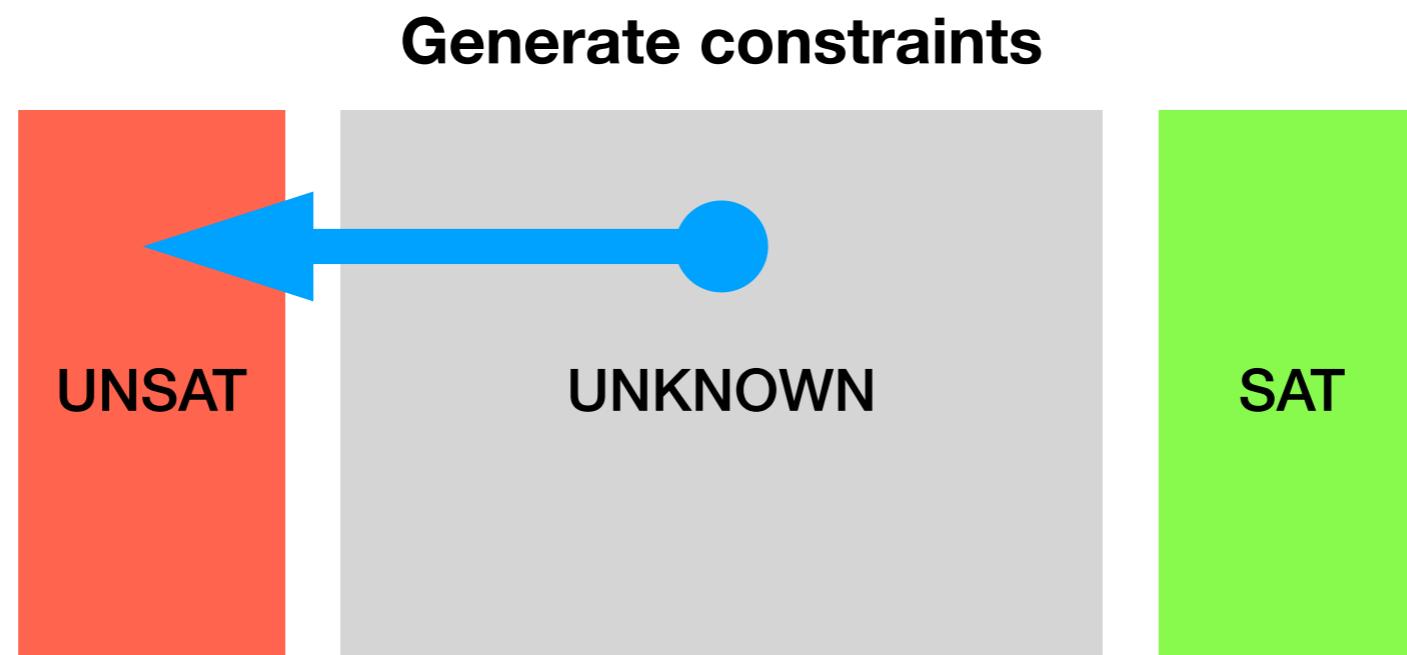


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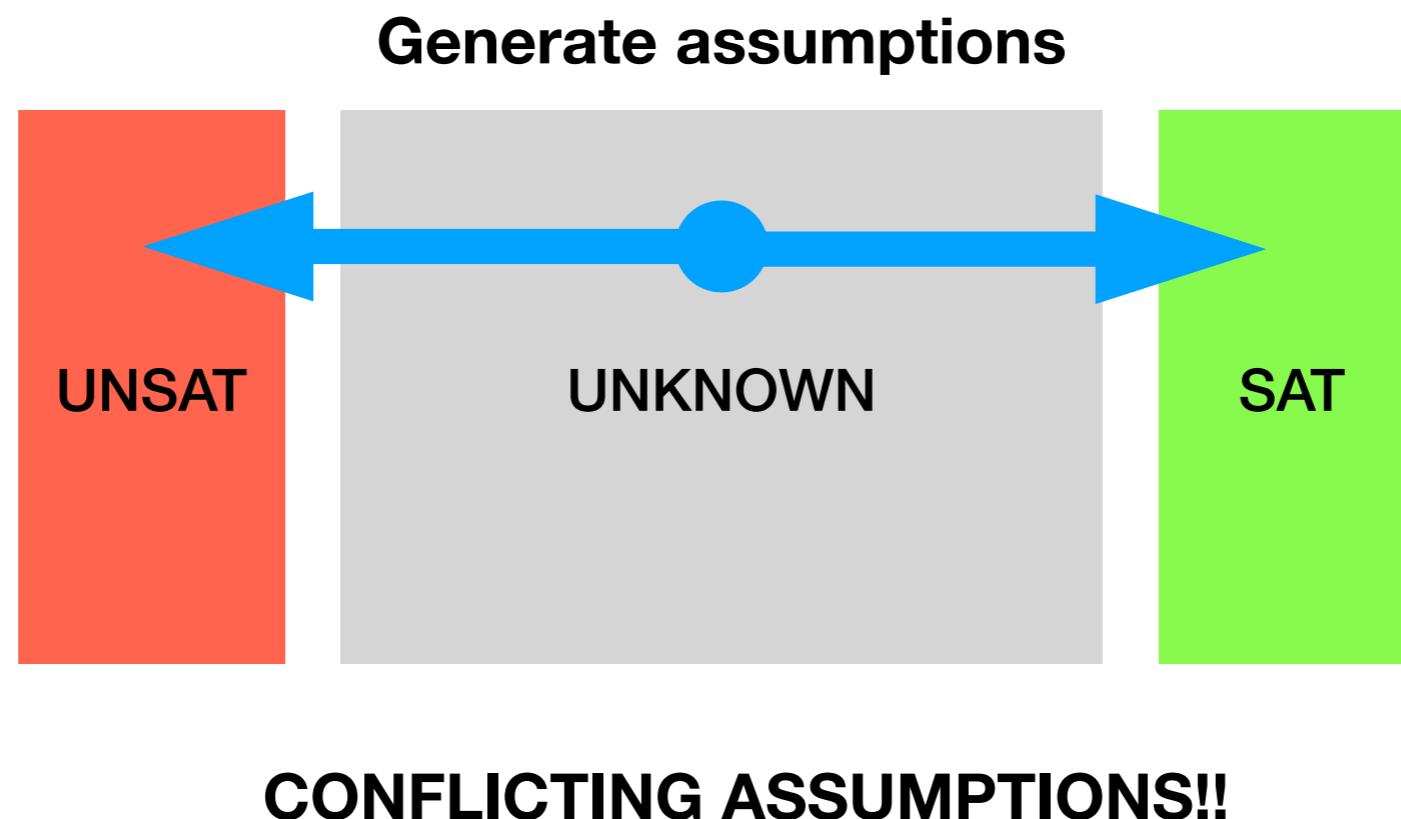


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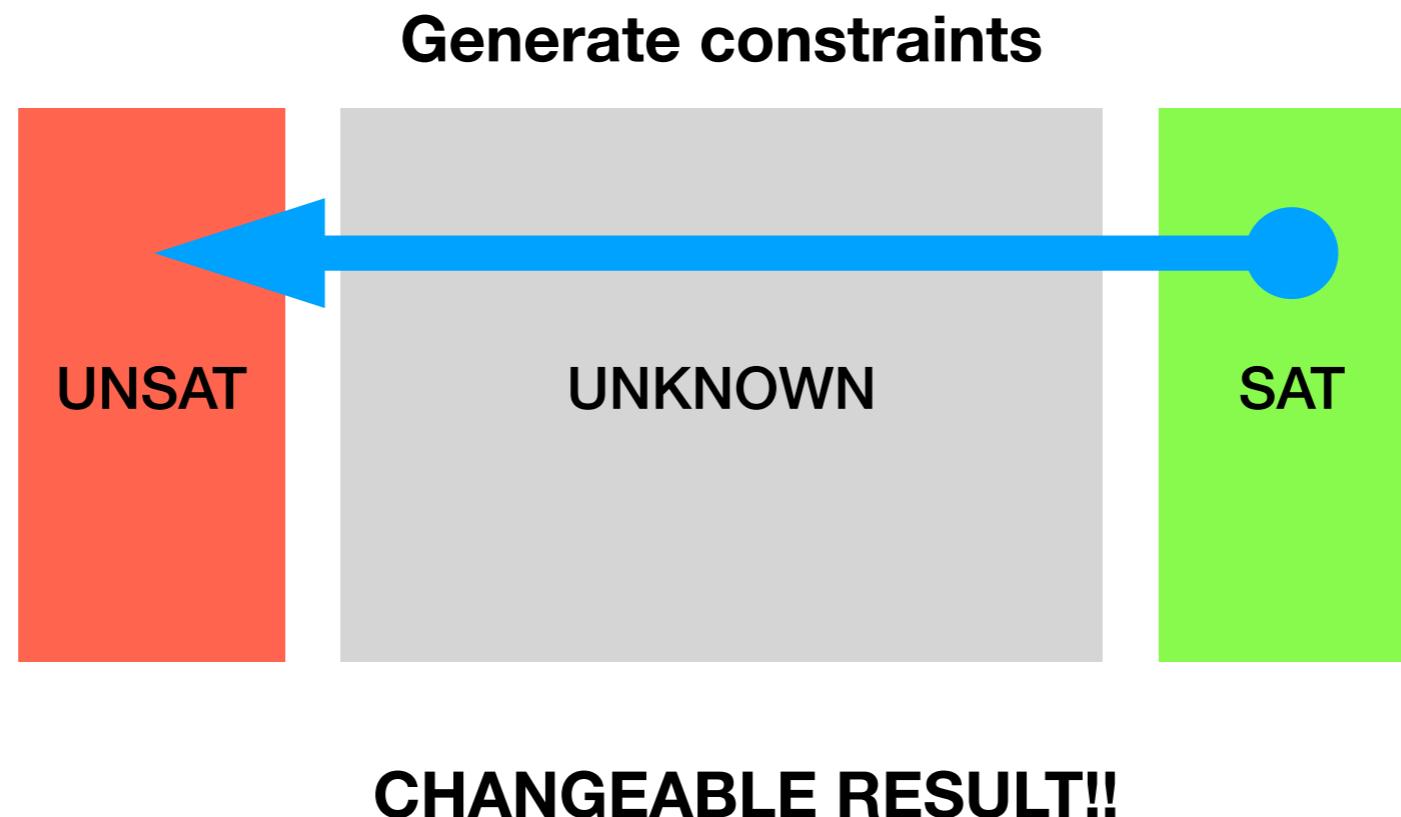


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Definitional

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A **definitional** SMTO problem is a tuple:

\vec{f} : a set of ordinary function symbols

$\vec{\theta}$: a set of oracle function symbols

ρ : a formula in a background theory

$\vec{\mathcal{O}}$: a set of oracle interfaces

And:

- All oracle interfaces define oracle functions
- There are no constraint generators

→ No conflicting assumptions (only functional oracles)

→ No changeable results (no constraints)

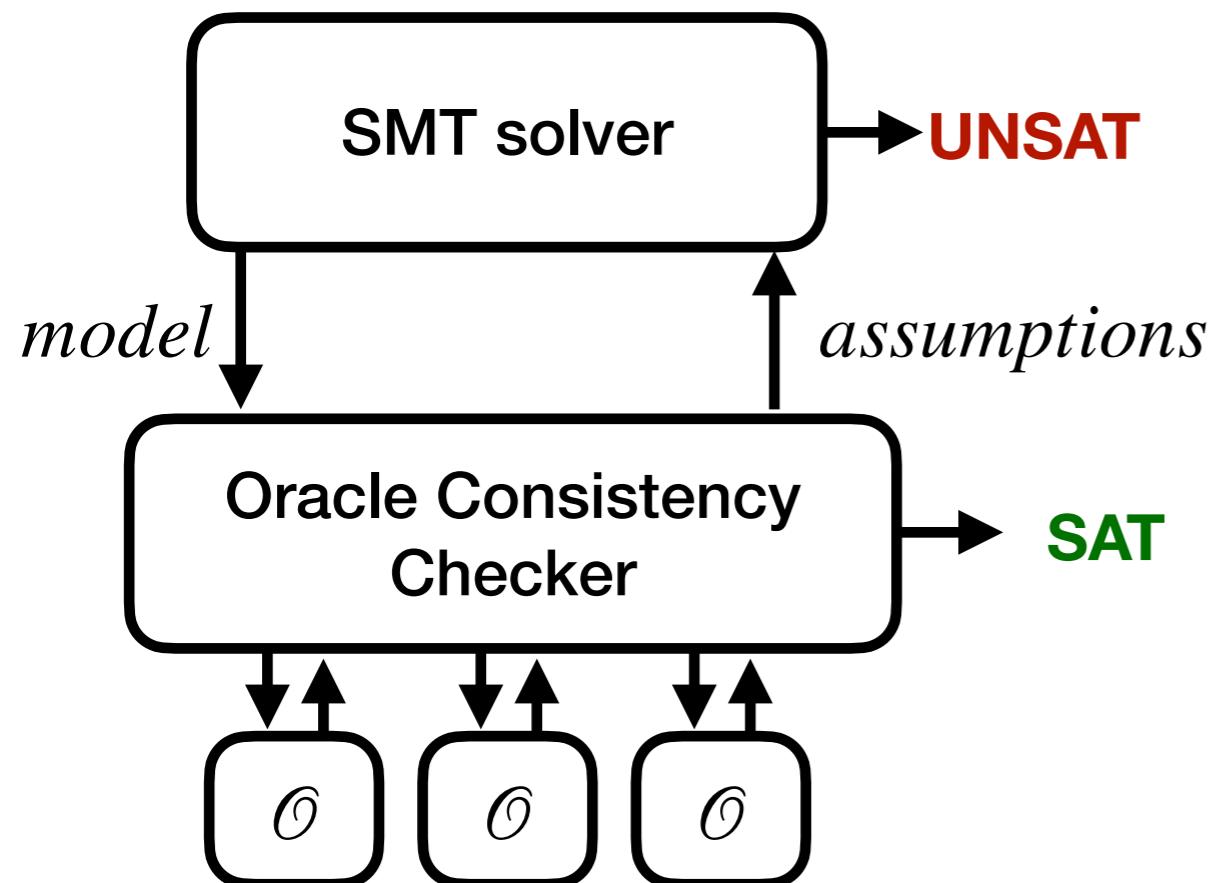
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Solving (definitional) SMTO

Satisfiable iff $\exists f_1, f_2 . \forall prime . A \implies \rho$ is satisfiable

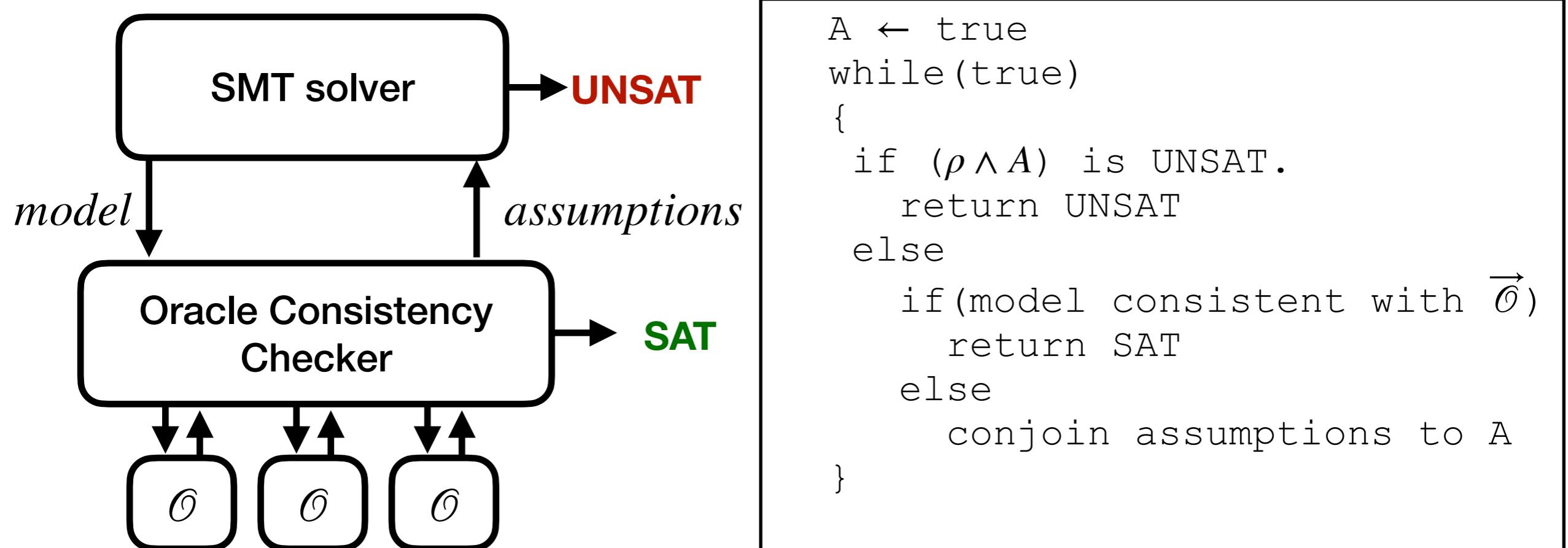
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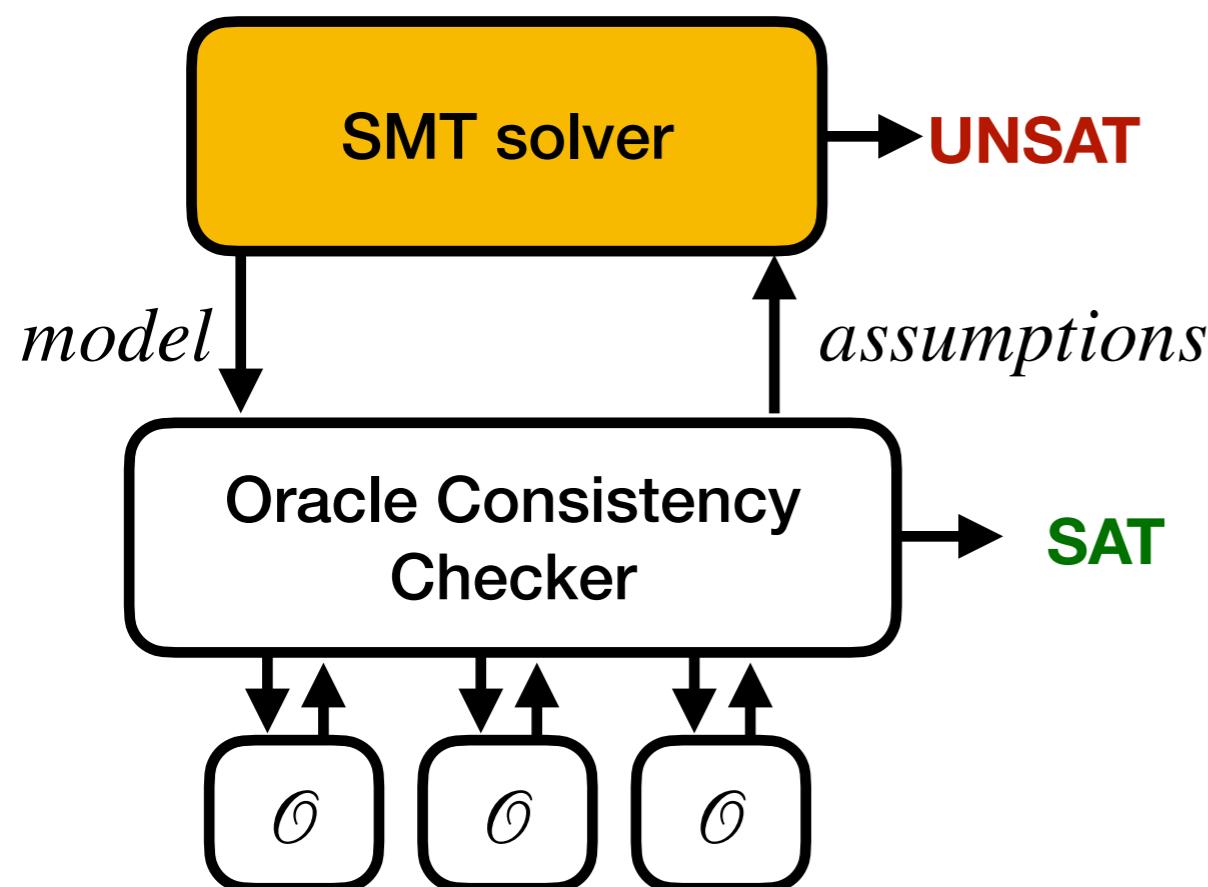
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Solving (definitional) SMTO

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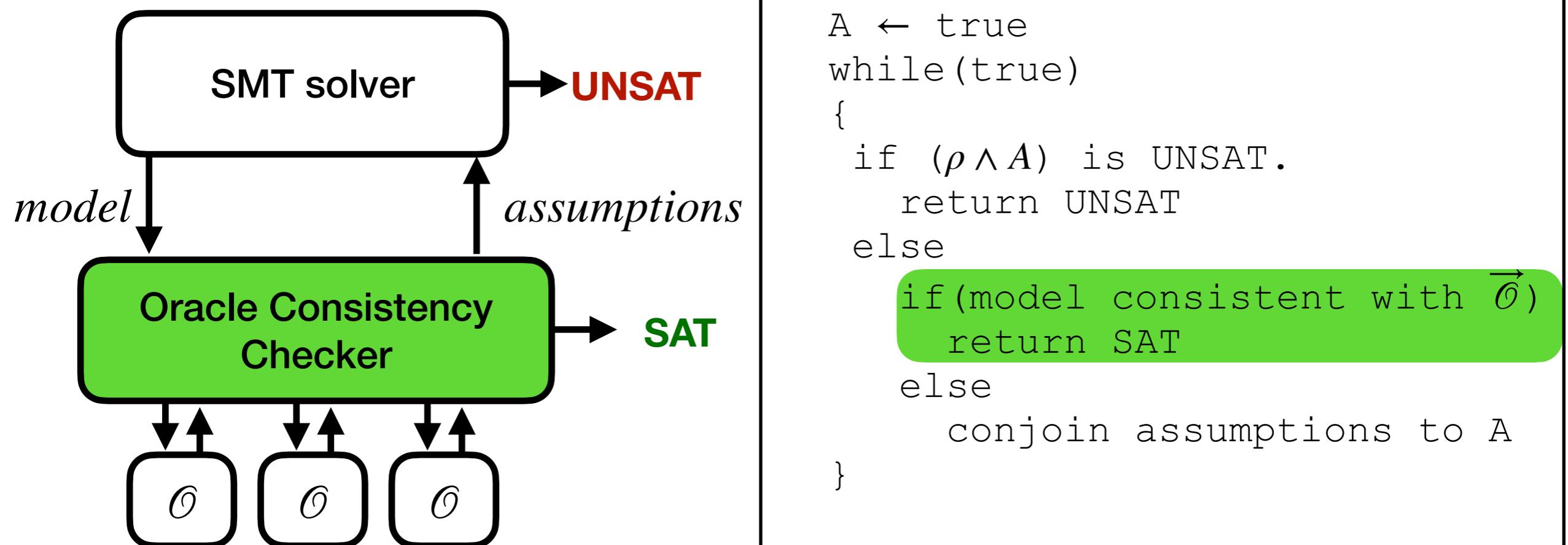
```

A ← true
while(true)
{
    if ( $\rho \wedge A$ ) is UNSAT.
        return UNSAT
    else
        if (model consistent with  $\vec{O}$ )
            return SAT
        else
            conjoin assumptions to A
}
  
```

Solving (definitional) SMTO

Satisfiable iff $\exists f_1, f_2. \forall prime . A \implies \rho$ is satisfiable

Unsatisfiable iff $\exists f_1, f_2. \exists prime . A \wedge \rho$ is unsatisfiable



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Synthesis Modulo Oracles (SyMO)

A SyMO problem is a tuple:

- \vec{f} : a tuple of functions to synthesise
- $\vec{\theta}$: a set of oracle function symbols
- $\forall \vec{x} . \phi$: a formula in a background theory,
where ϕ is quantifier-free
- $\vec{\mathcal{O}}$: a set of oracle interfaces

$$\begin{aligned}\vec{f} &: \{f\} \\ \vec{\theta} &: \{corr\} \\ \forall \vec{x} . \phi &: \forall x . corr(f) \wedge f(x) < 256 \\ \vec{\mathcal{O}} &: \{\mathcal{O}_{corr}\}\end{aligned}$$

$$\begin{aligned}\mathcal{O}_{corr} \\ \vec{y} &: (f^* : int \rightarrow int) \\ \vec{z} &: (z_1 : Bool, z_2 : int, z_3 : int) \\ \alpha_{gen} &: corr(f^*) = z_1 \\ \beta_{gen} &: f(z_2) = z_3\end{aligned}$$

What is a valid f ?

Synthesis Modulo Oracles (SyMO)

What is a valid f ?

$$\forall x . \phi : \forall x . \text{corr}(f) \wedge f(x) < 256$$

f is **VALID** if **there is no** x such that ϕ is false

i.e., if the SMTO problem $(\vec{x}, \vec{\theta}, \neg\phi\{\vec{f} \rightarrow \vec{f}^*\}, \vec{\mathcal{O}})$ is **UNSAT**

f is **INVALID** if **there is an** x such that ϕ is false

i.e., if the SMTO problem $(\vec{x}, \vec{\theta}, \neg\phi\{\vec{f} \rightarrow \vec{f}^*\}, \vec{\mathcal{O}})$ is **SAT**

Synthesis Modulo Oracles (SyMO)

A SyMO problem is a tuple:

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- $\forall \vec{x} . \phi$: a formula in a background theory,
where ϕ is quantifier-free
- $\vec{\mathcal{O}}$: a set of oracle interfaces

And:

- All **assumption** generators define **oracle function symbols**
- All oracles are **functional**

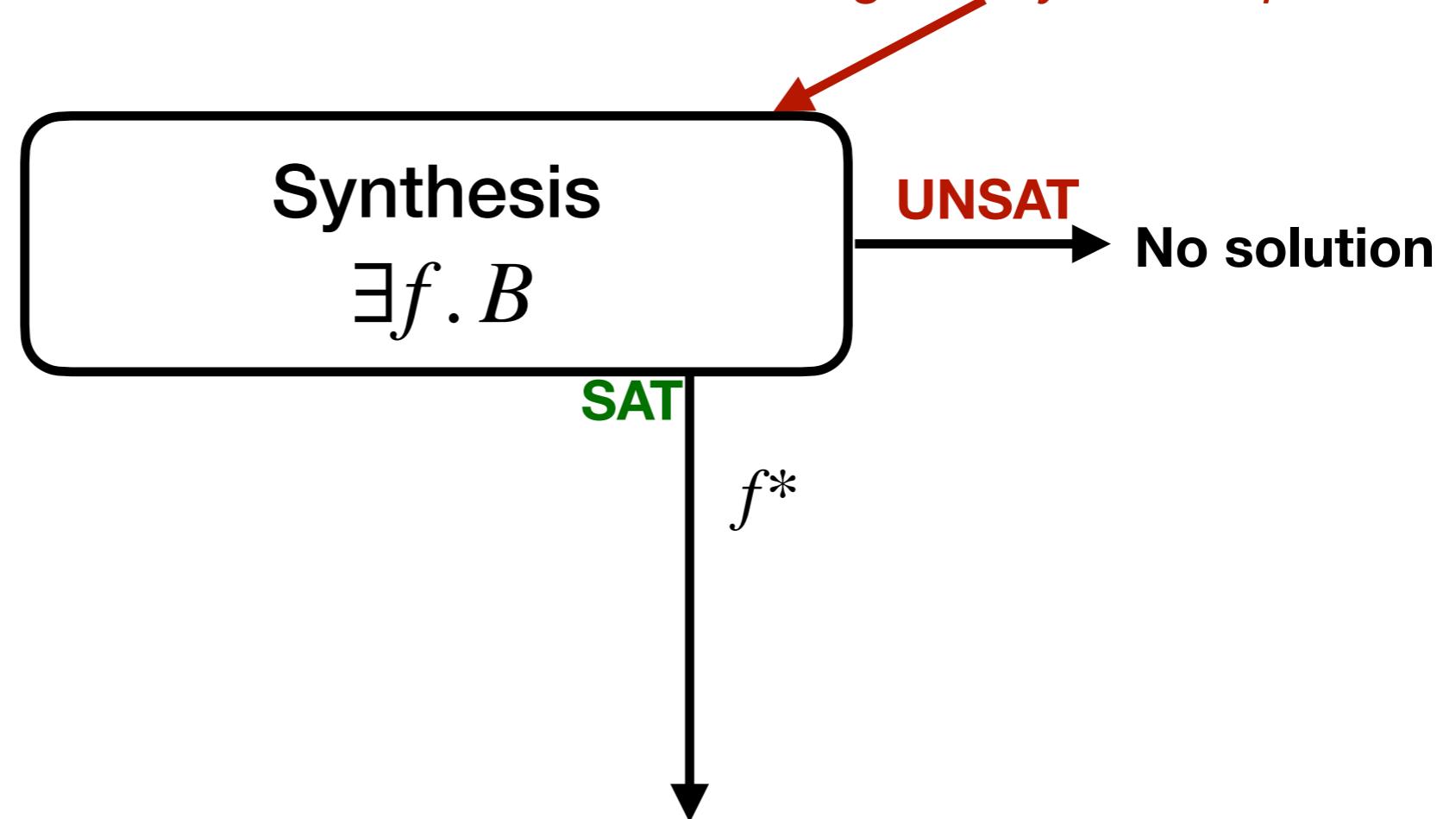
\implies checking \vec{f} is valid is now **definitional SMT**

Coming up

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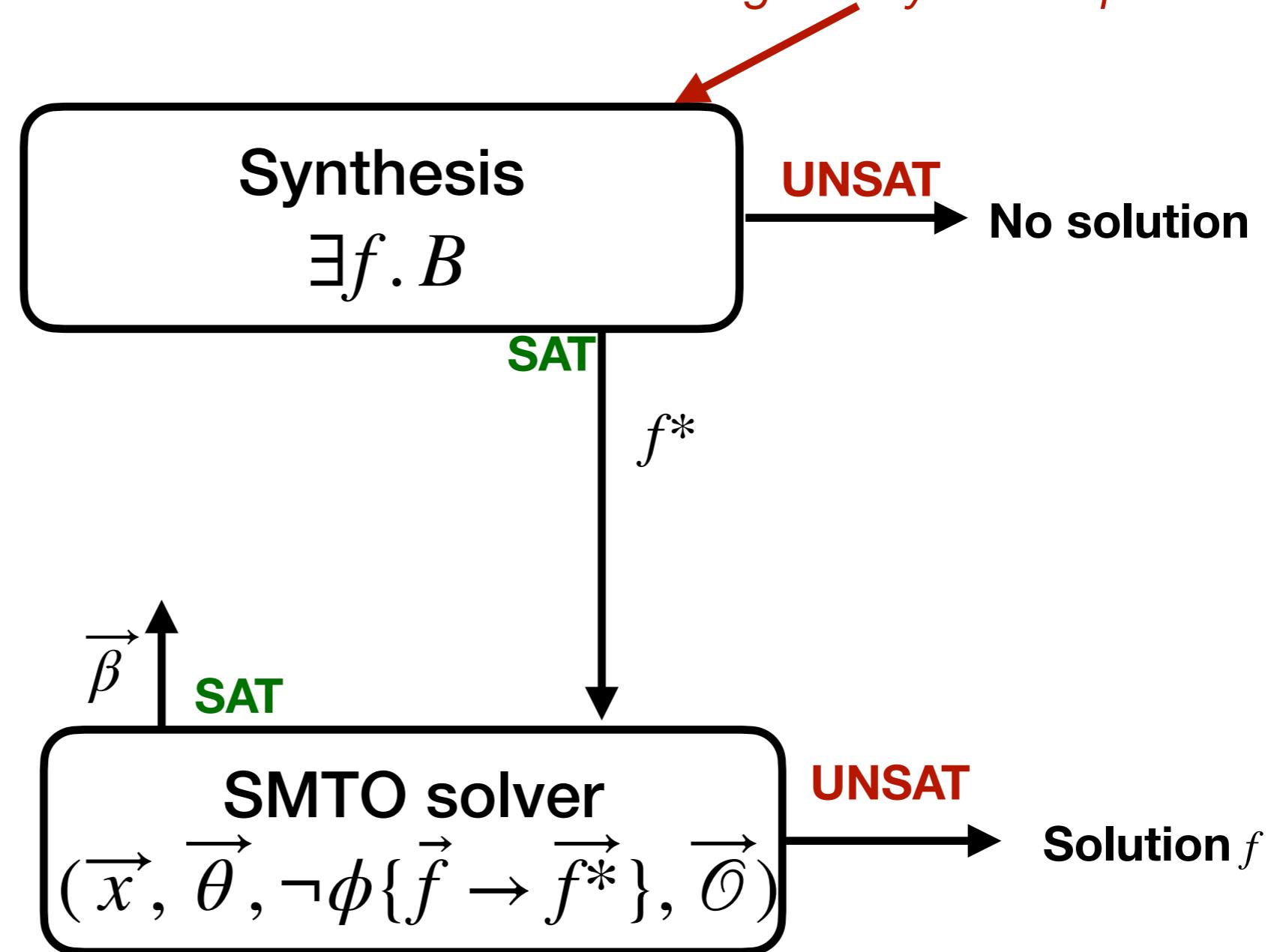
Solving SyMO

Constraints used to guide synthesis phase



Solving SyMO

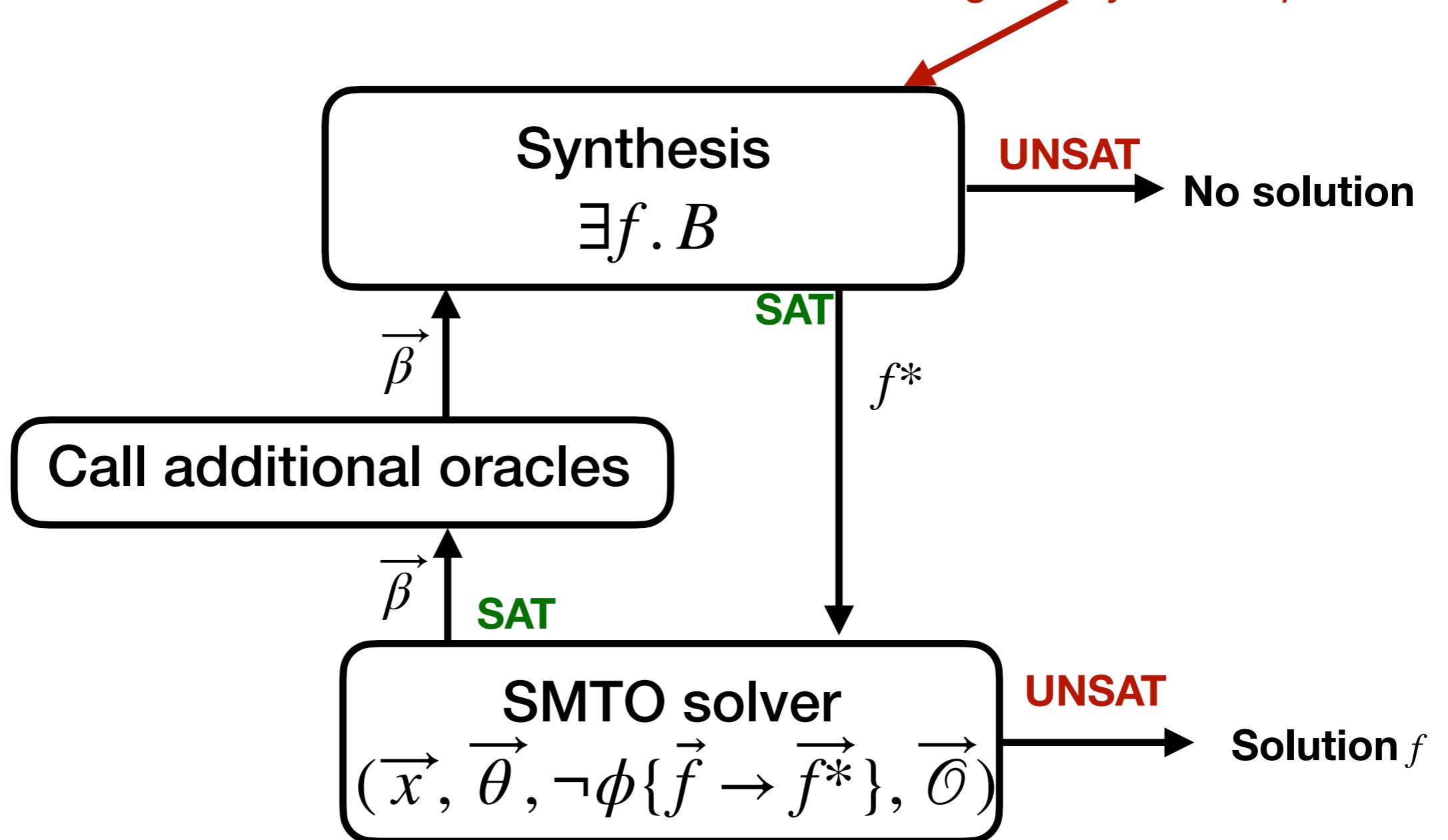
Constraints used to guide synthesis phase



We also pass the assignment model back from the SMT solver as a constraint

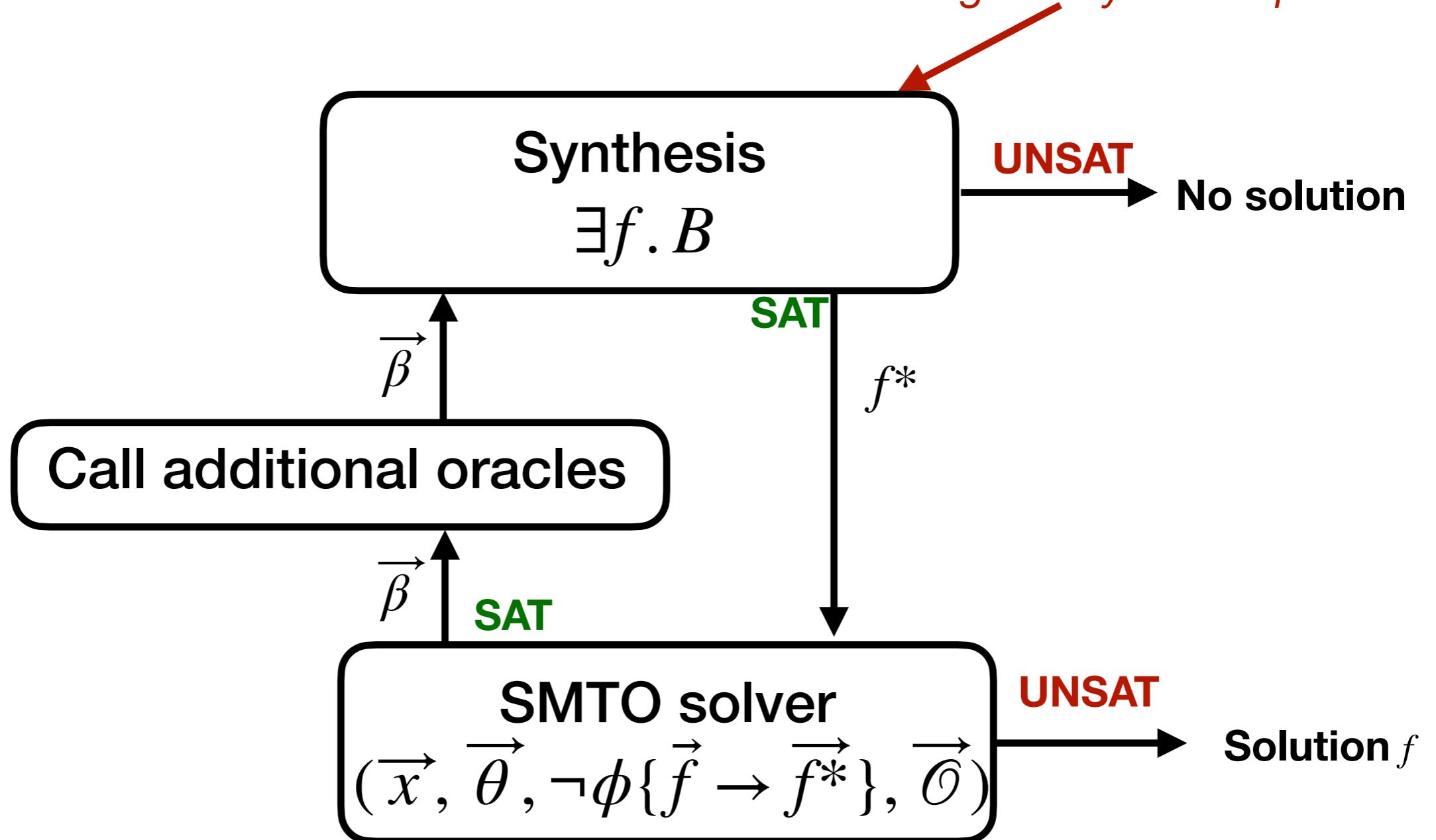
Solving SyMO

Constraints used to guide synthesis phase



Solving SyMO

Constraints used to guide synthesis phase



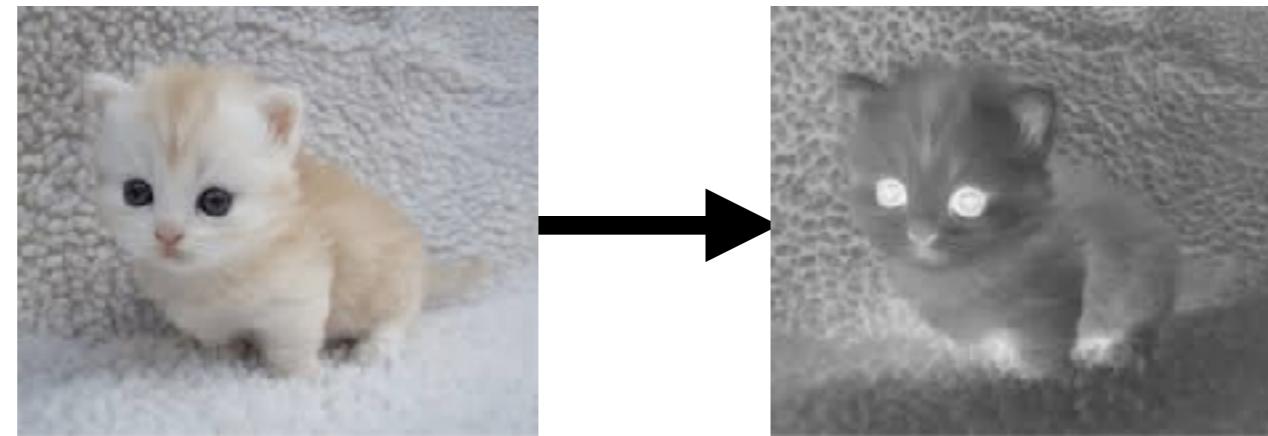
Note: f^* is guaranteed to satisfy B so the SMT solver doesn't need to consider B

Thus definitional SMT!

Coming up

- Existing use of oracles
 - Formal definition of oracle interfaces
 - SMT with oracles (**SMTO**):
 - when is it satisfiable/unsatisfiable
 - algorithm
 - Synthesis with oracles (**SyMO**):
 - when are solutions correct
 - **unifying** algorithm for solving
-
- More cat pictures
 - Prototype evaluation

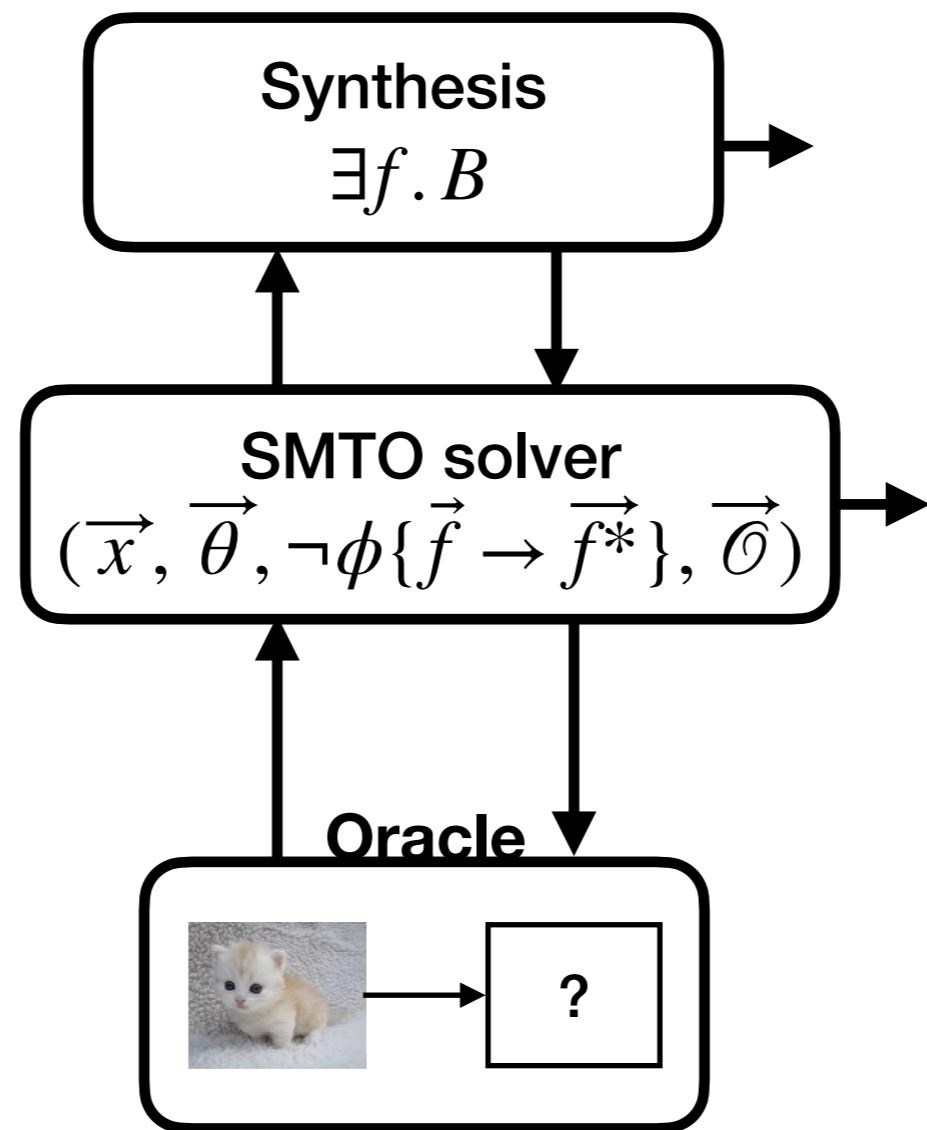
Find a function that transforms Cat A into Cat B?

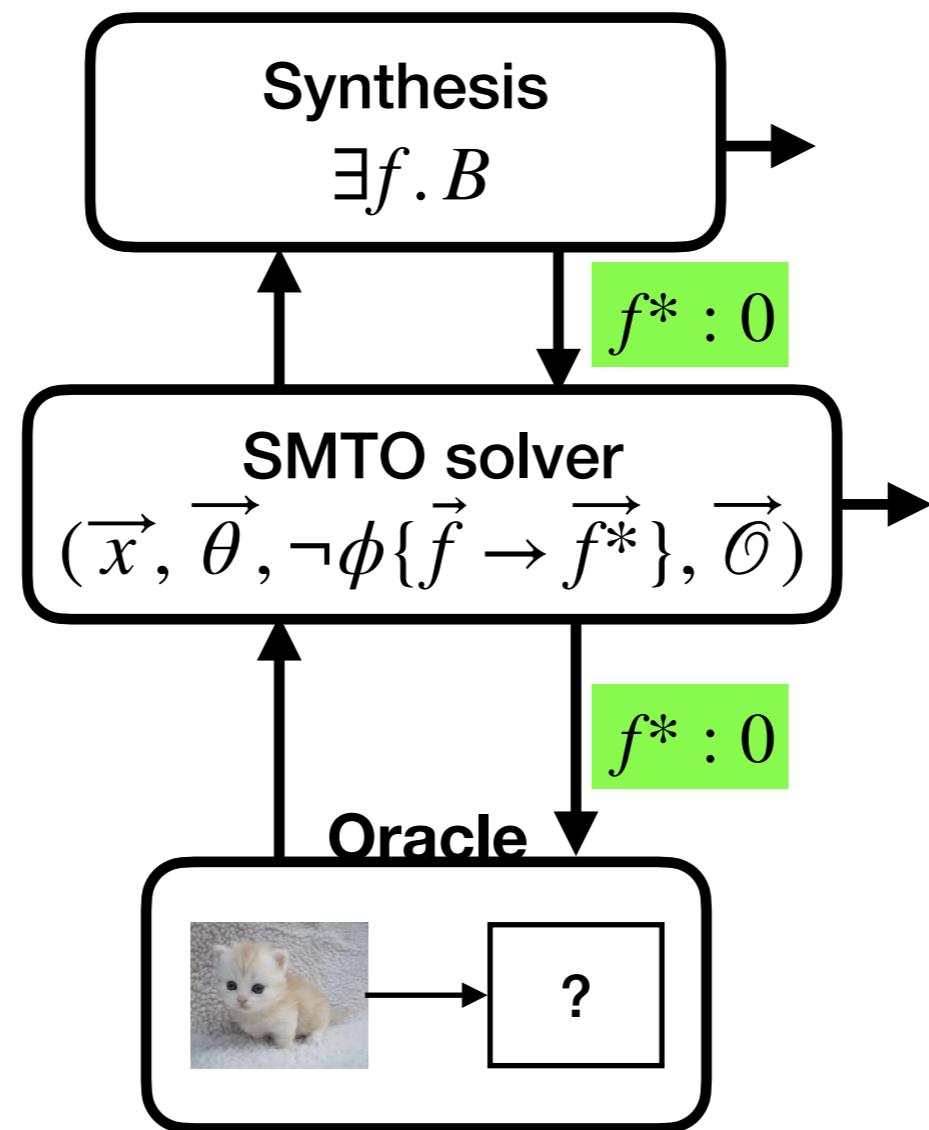


$$\exists f. \forall x. corr(f)$$

Oracle:



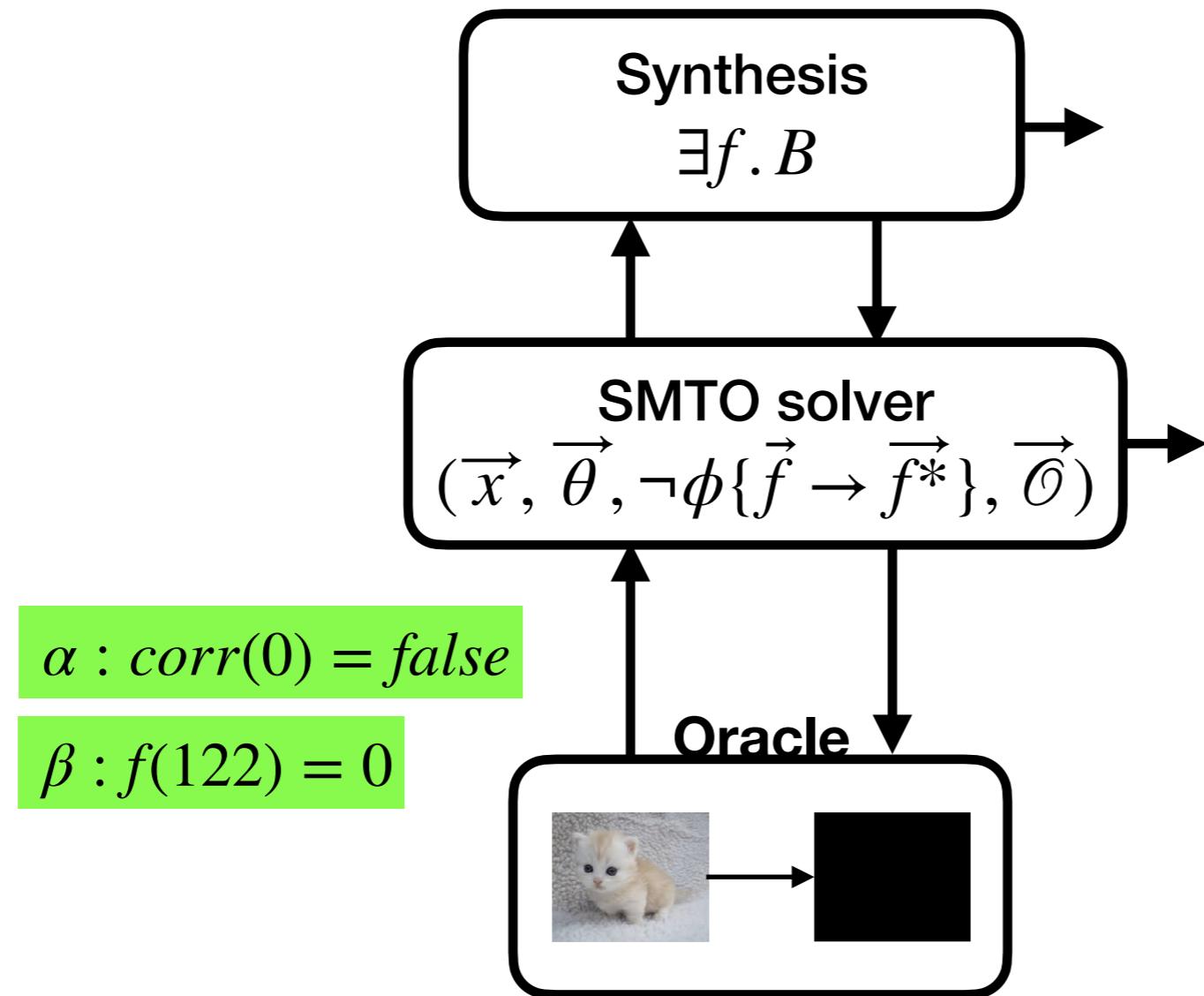
$\phi : corr(f)$ $A : \top$ $B : \top$ 

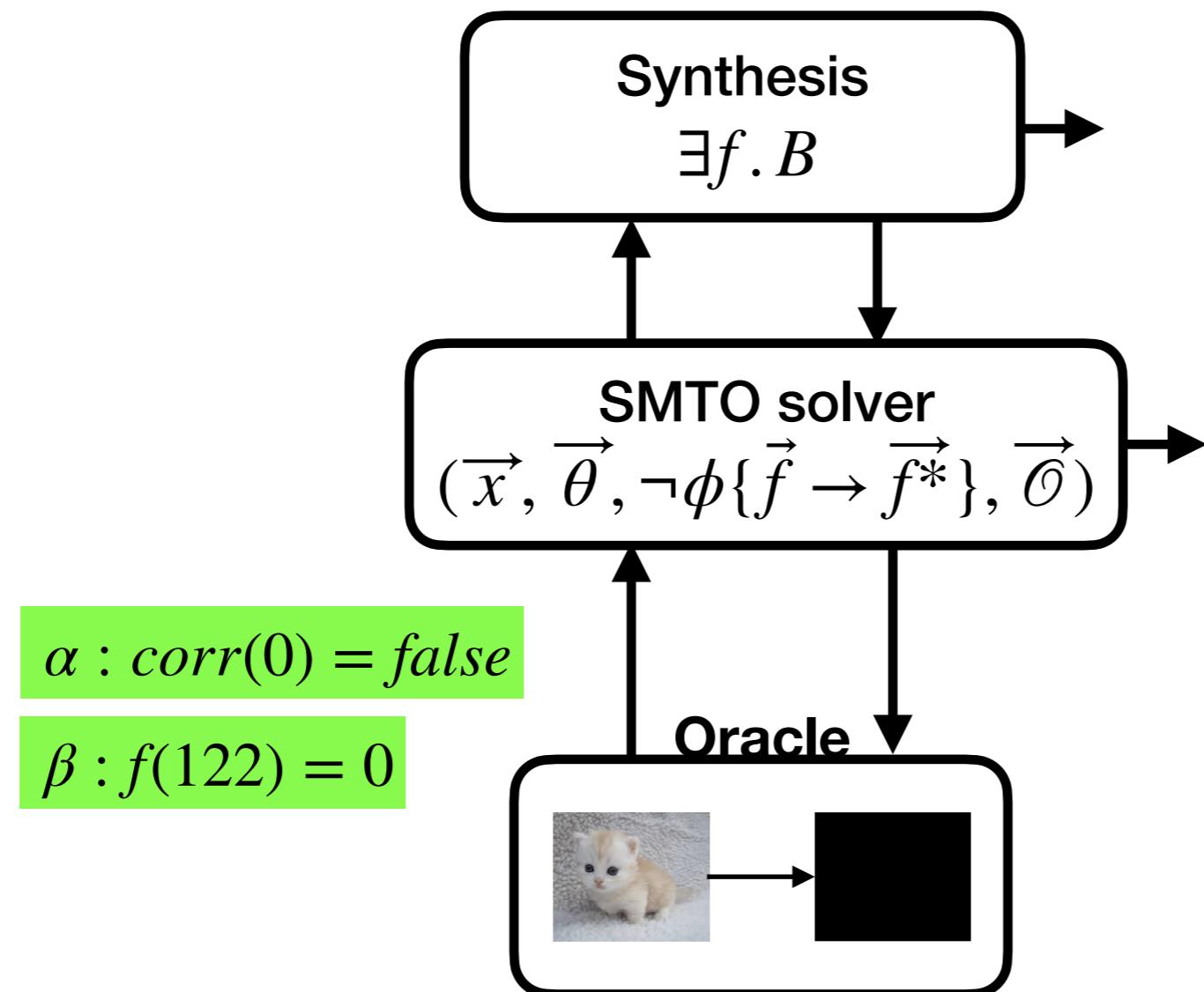
$\phi : corr(f)$ $A : \top$ $B : \top$ 

$\phi : corr(f)$

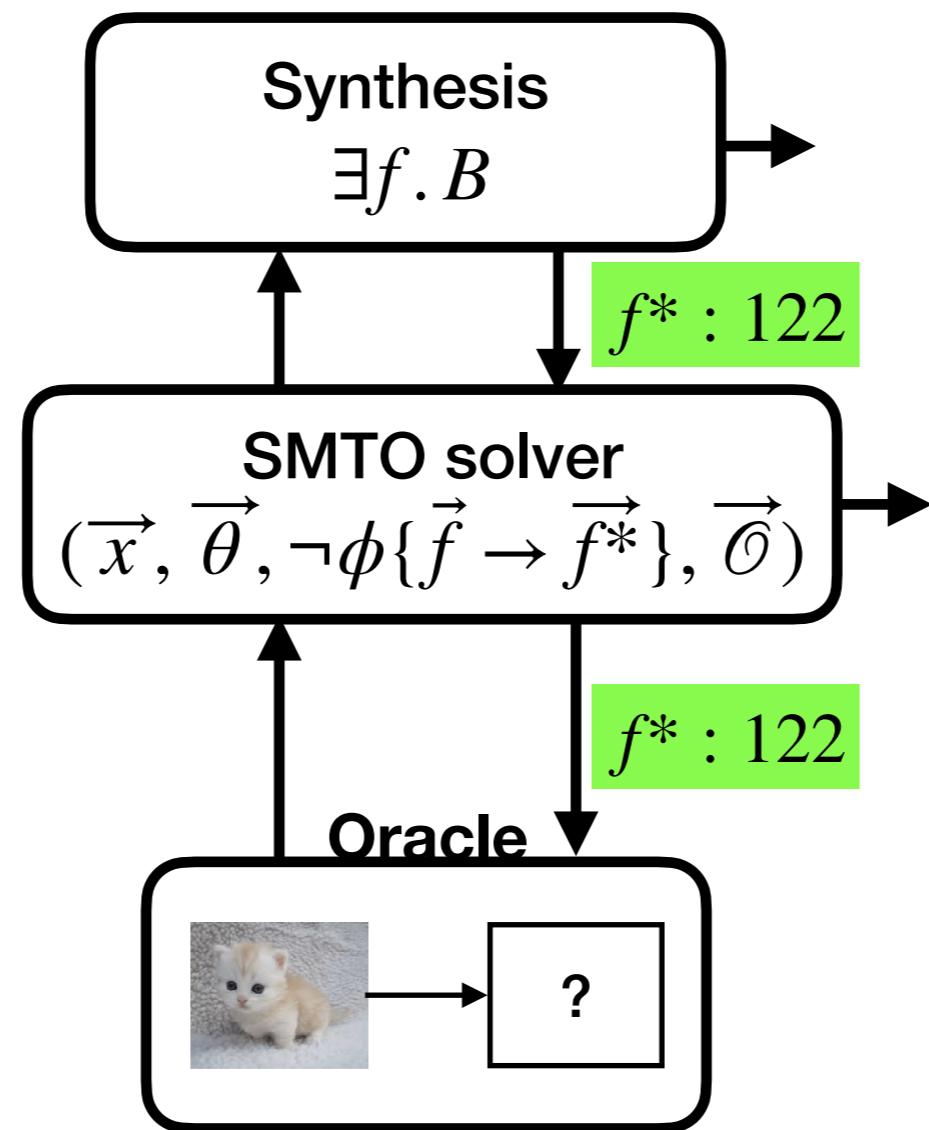
$A : \top$

$B : \top$

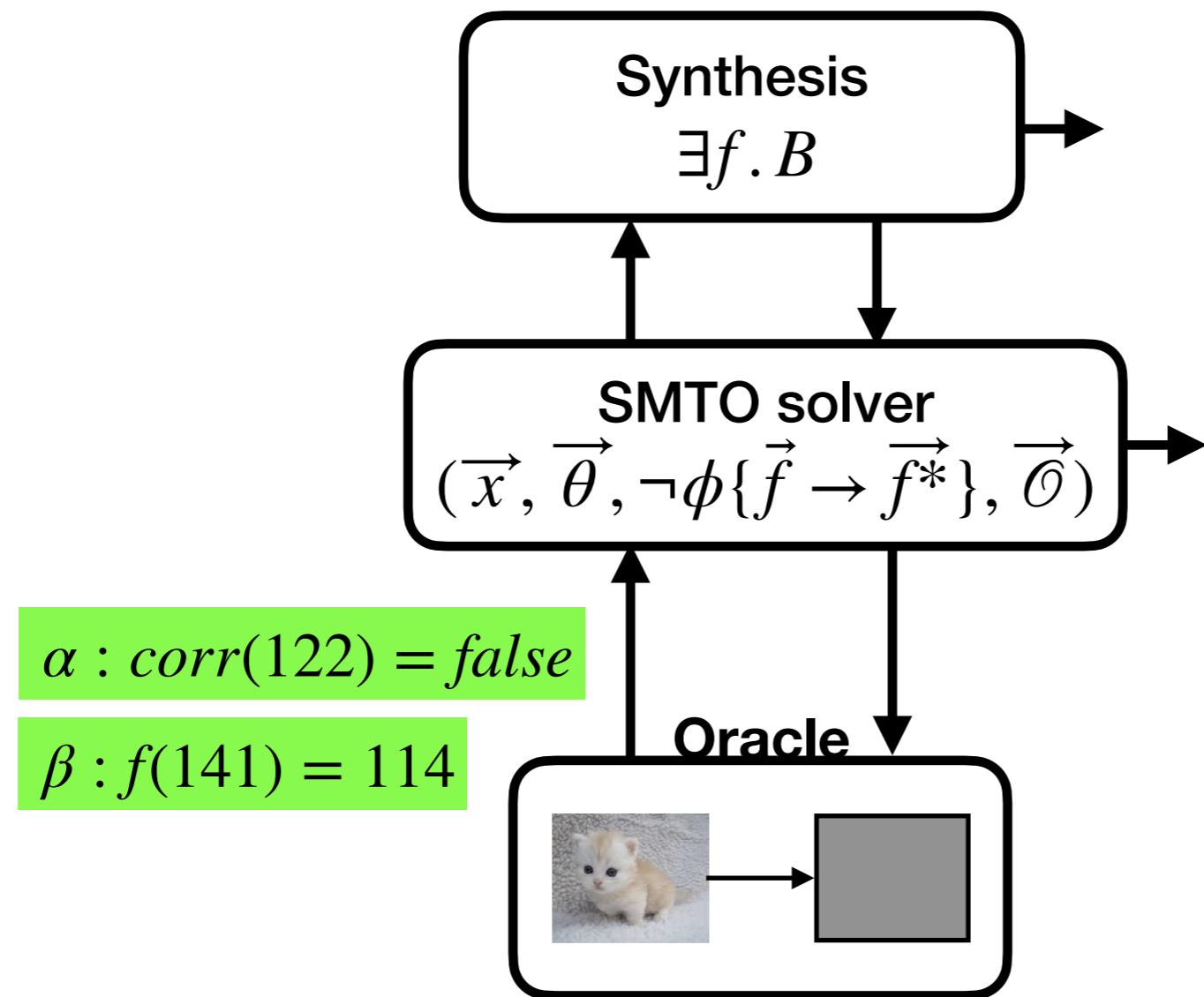


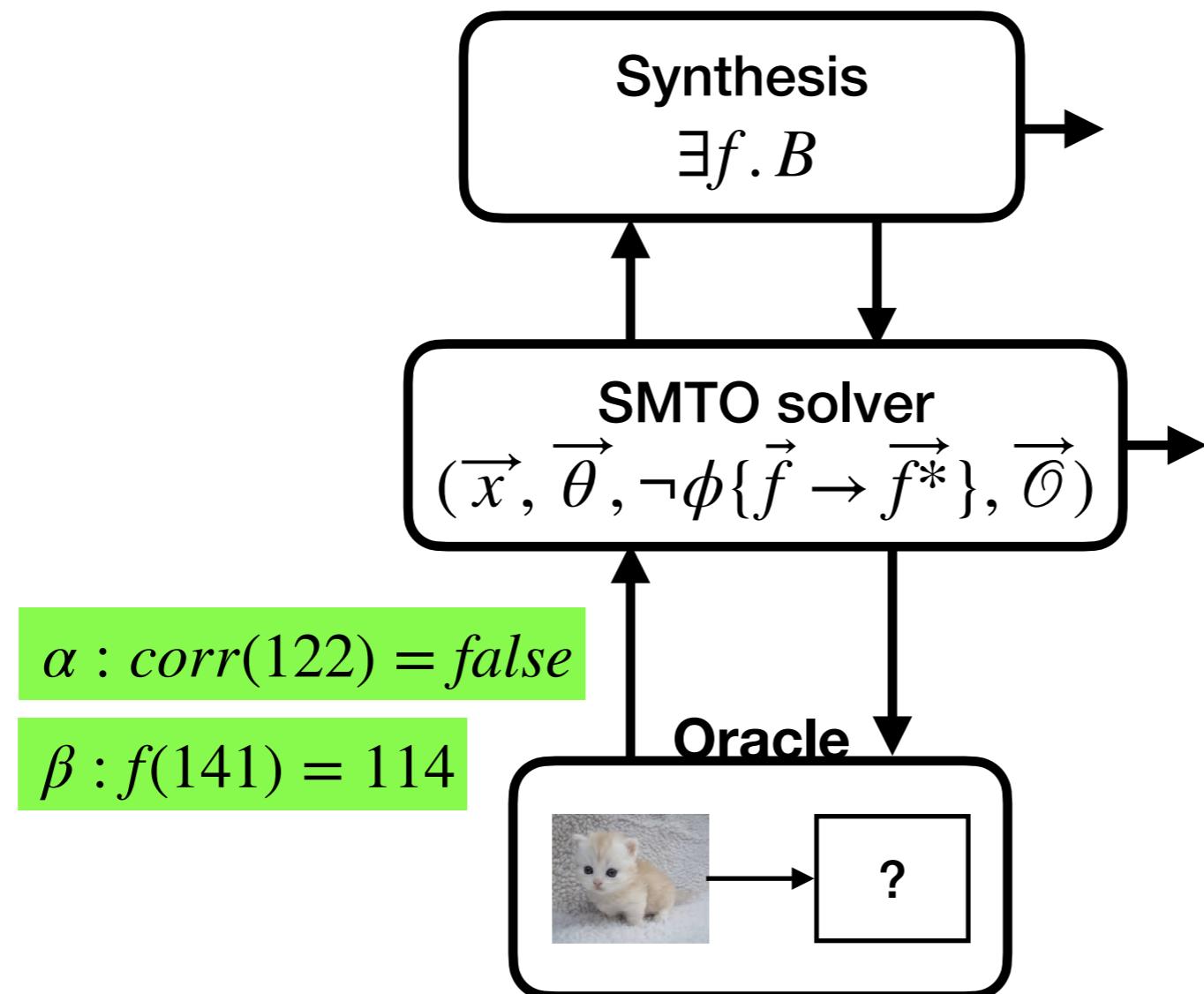
$\phi : corr(f)$
 $A : \top \wedge corr(0) = \text{false}$
 $B : \top \wedge f(122) = 0$


$\phi : corr(f)$
 $A : \top \wedge corr(0) = \text{false}$
 $B : \top \wedge f(122) = 0$



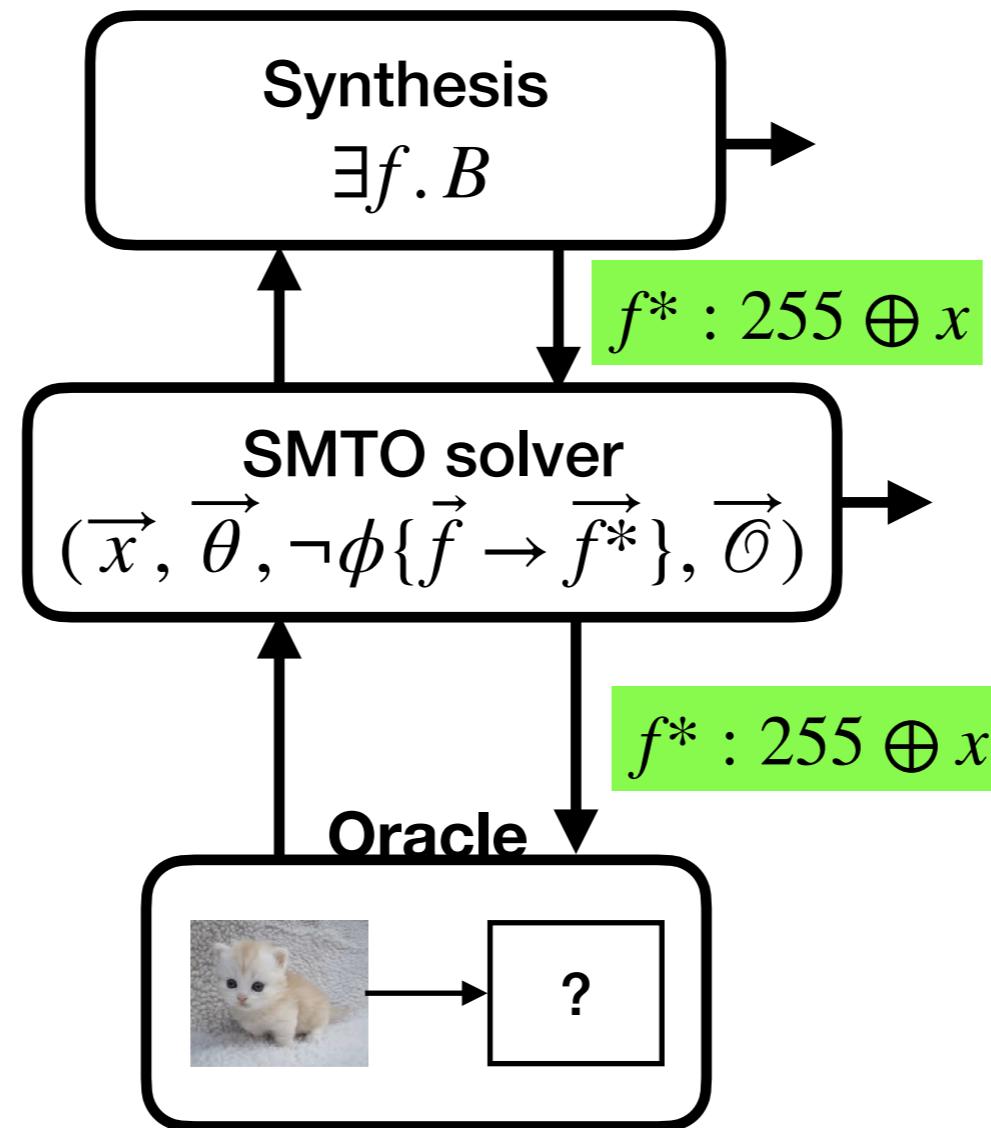
$\phi : corr(f)$
 $A : \top \wedge corr(0) = \text{false}$
 $B : \top \wedge f(122) = 0$



$\phi : corr(f)$
 $A : \top \wedge corr(0) = \text{false} \wedge corr(122) = \text{false}$
 $B : \top \wedge f(122) = 0 \wedge f(141) = 114$


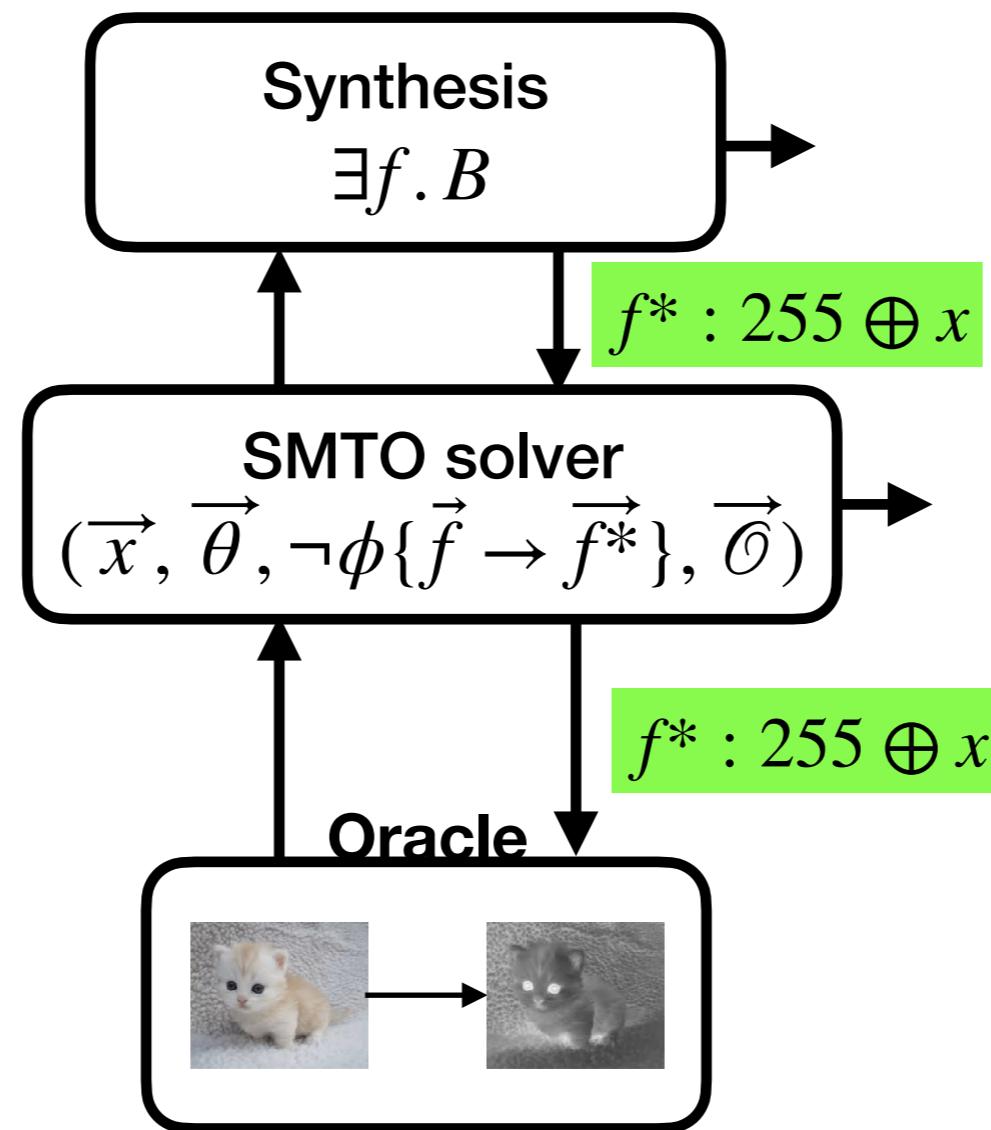
$$\phi : \text{corr}(f)$$

$$A : \top \wedge \text{corr}(0) = \text{false} \wedge \text{corr}(122) = \text{false}$$

$$B : \top \wedge f(122) = 0 \wedge f(141) = 114$$


$$\phi : \text{corr}(f)$$

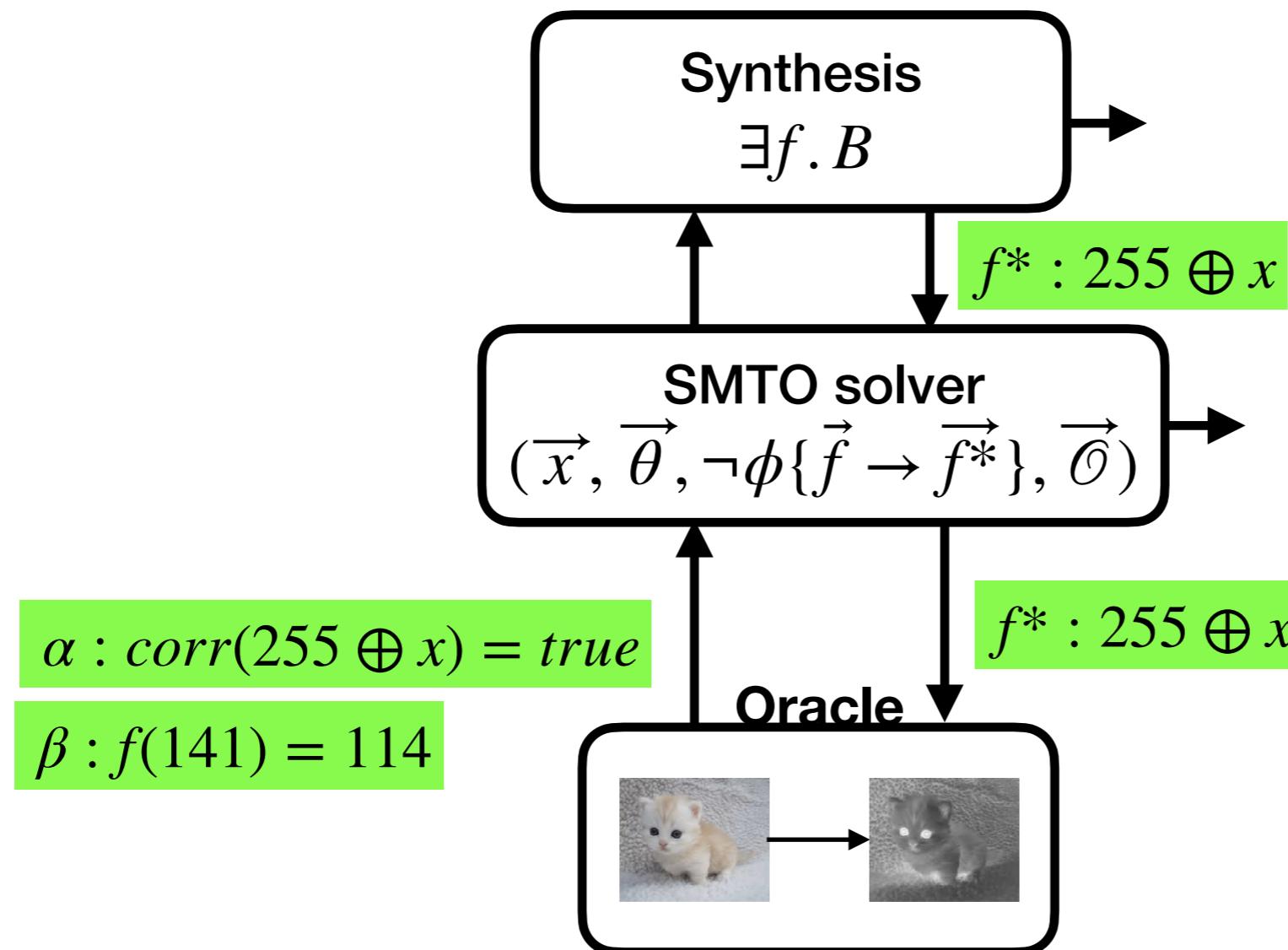
$$A : \top \wedge \text{corr}(0) = \text{false} \wedge \text{corr}(122) = \text{false}$$

$$B : \top \wedge f(122) = 0 \wedge f(141) = 114$$


$\phi : corr(f)$

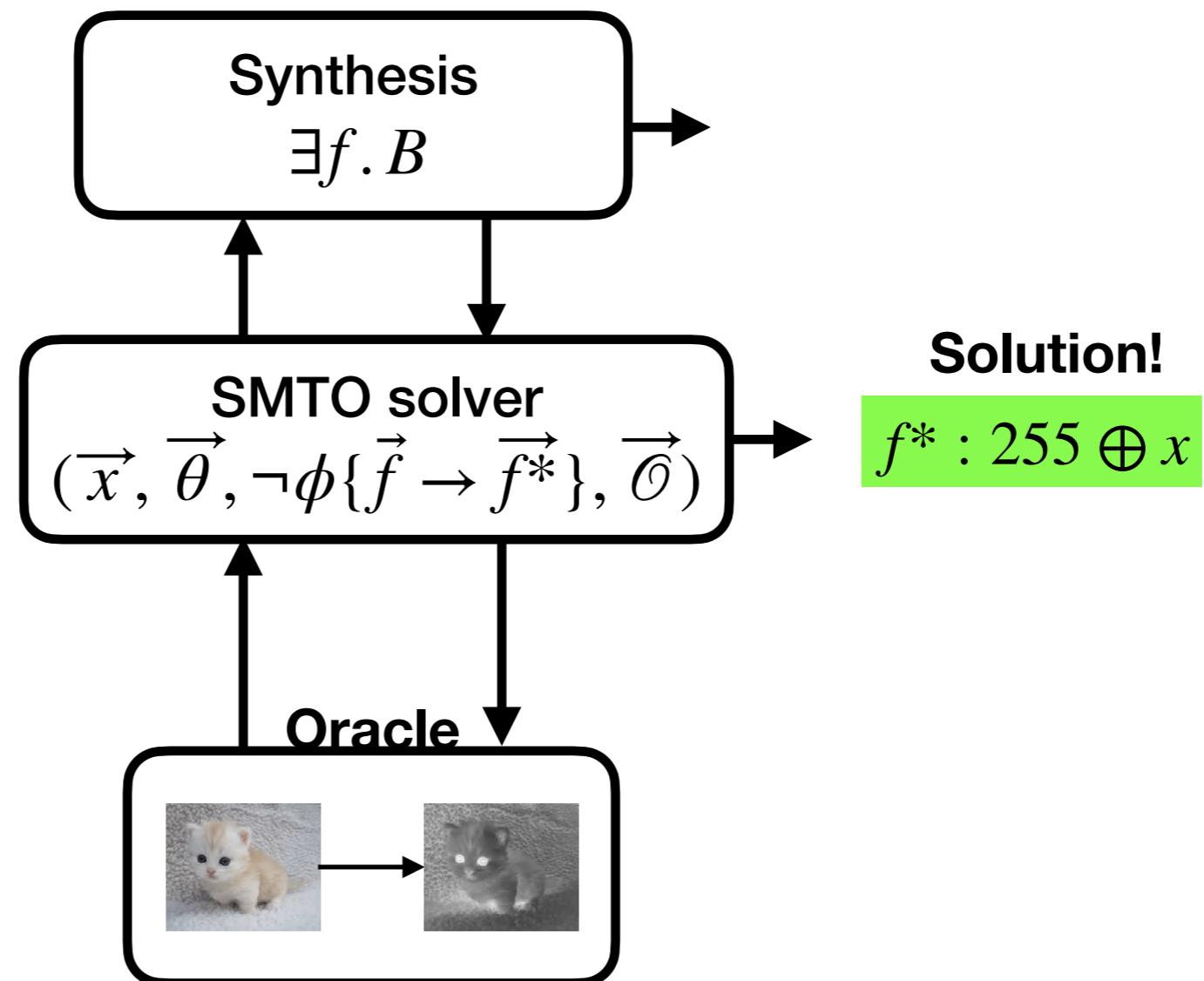
$$A : \top \wedge \text{corr}(0) = \text{false} \wedge \text{corr}(122) = \text{false}$$

$$B : \top \wedge f(122) = 0 \wedge f(141) = 114$$



$$\phi : \text{corr}(f)$$

$$A : \top \wedge \text{corr}(0) = \text{false} \wedge \text{corr}(122) = \text{false}$$

$$B : \top \wedge f(122) = 0 \wedge f(141) = 114$$


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Case studies

		Problem	#	Delphi (oracles)		CVC5 (no oracles)		Approximate model
			#	#	s	#	s	
SyMO	Images	10	9	21.6s		0		
	Control stability	112	104	29.3s		16	19.4s	
	Control safety	112	31	59.9s		0		
	PBE	150	148	0.5s		150	<0.5s	
SMT0	Math	12	9	<0.5s		5	2.2s	

<https://github.com/polgreen/delphi>

<https://github.com/polgreen/oracles>

SyGuS-IF extension

- Syntax for declaring oracle constraints, oracle assumptions and oracle functions

```

⟨OracleCmd⟩ ::= (oracle-assume (⟨SortedVar⟩*) (⟨SortedVar⟩*) ⟨Term⟩ ⟨Symbol⟩ )
| (oracle-constraint (⟨SortedVar⟩*) (⟨SortedVar⟩*) ⟨Term⟩ ⟨Symbol⟩ )
| (declare-oracle-fun ⟨Symbol⟩ (⟨Sort⟩*) ⟨Sort⟩ ⟨Symbol⟩ )
| (oracle-constraint-io ⟨Symbol⟩ ⟨Symbol⟩ )
| (oracle-constraint-cex ⟨Symbol⟩ ⟨Symbol⟩ )
| (oracle-constraint-membership ⟨Symbol⟩ ⟨Symbol⟩ )
| (oracle-constraint-poswitness ⟨Symbol⟩ ⟨Symbol⟩ )
| (oracle-constraint-negwitness ⟨Symbol⟩ ⟨Symbol⟩ )
| (declare-correctness-oracle ⟨Symbol⟩ ⟨Symbol⟩ )
| (declare-correctness-cex-oracle ⟨Symbol⟩ ⟨Symbol⟩ )

```

Conclusion

- Defined oracle interfaces, grouping responses into **assumptions** and **constraints**
- Presented algorithms for SMTO and SyMO
- Performs synthesis with complex oracles without building new solvers

Conclusion

Oracles can be (almost) anything!

- Must be able to say “yes” or “no” for correctness
- Must provide semantic guidance to the learner
- Can be used with traditional SMT constraints

Do you have oracles? Talk to us!

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sseshia@berkeley.edu

