

**Example 1**

Calculate the dot product of  $\vec{a} = (2, 3, 5)$  and  $\vec{b} = (4, 1, -2)$ . Do the vectors form an acute angle, right angle, or obtuse angle?

**Example 2**

Calculate the dot product of  $\vec{c} = (6, -5)$  and  $\vec{d} = (8, 2)$ .

Do the vectors form an acute angle, right angle, or obtuse angle?

**Example 3**

If  $\vec{a} = (2, -1, 9)$ , for what value of  $c$  is the vector  $\vec{b} = (1, c, -3)$  perpendicular to  $\vec{a}$ ?

**Example 4**

If  $\vec{a} = (8, -11, 3)$ , for what value of  $c$  is the vector  $\vec{b} = (0, c, 4)$  perpendicular to  $\vec{a}$ ?

**Example 5**

Calculate the dot product of  $\vec{c} = (8, -4)$  and  $\vec{d} = (3, 1)$ .

Do the vectors form an acute angle, right angle, or obtuse angle?

**Answer 1**

Using the component formula for the dot product of three-dimensional vectors,

$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ , we calculate the dot product to be

$$\vec{a} \cdot \vec{b} = 2 \cdot 4 + 3 \cdot 1 + 5 \cdot (-2) = 8 + 3 - 10 = 1.$$

Since  $\vec{a} \cdot \vec{b}$  is positive, we can infer from the geometric definition, that the vectors form an acute angle.

**Answer 2**

Using the component formula for the dot product of two-dimensional vectors,  $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$ ,

we calculate the dot product to be  $\vec{c} \cdot \vec{d} = 6 \cdot 8 + (-5) \cdot 2 = 48 - 10 = 38$

Since  $\vec{c} \cdot \vec{d}$  is negative, we can infer from the geometric definition, that the vectors form an acute angle.

**Answer 3**

For  $\vec{a}$  and  $\vec{b}$  to be perpendicular, we need their dot product to be zero. Since

$$\vec{a} \cdot \vec{b} = 2 \cdot 1 + (-11) \cdot c + 9 \cdot (-3) = 2 - c - 27, \text{ the number } c \text{ must satisfy } -25 - c = 0, \text{ or } c = -25$$

You can double-check that the vector  $\vec{b} = (1, -25, -3)$  is indeed perpendicular to  $\vec{a}$  by verifying that  $\vec{a} \cdot \vec{b} = 2 \cdot 1 + (-1) \cdot (-25) + 9 \cdot (-3) = 2 + 25 - 27 = 0$ .

**Answer 4**

For  $\vec{a}$  and  $\vec{b}$  to be perpendicular, we need their dot product to be zero. Since

$$\vec{a} \cdot \vec{b} = 8 \cdot 0 + (-1) \cdot c + 4 \cdot 3 = 0 - 11c + 12, \text{ the number } c \text{ must satisfy } 12 - 11c = 0, \text{ or } c = 12/11.$$

**Answer 5**

Using the component formula for the dot product of two-dimensional vectors,  $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$ ,

we calculate the dot product to be  $\vec{c} \cdot \vec{d} = 3 \cdot 8 + (-4) \cdot 1 = 24 - 4 = 20$ .

Since  $\vec{c} \cdot \vec{d}$  is negative, we can infer from the geometric definition, that the vectors form an acute angle.