Example 1

Calculate the dot product of a=(1,2,3) and b=(4,−5,6). Do the vectors form an acute angle, right angle, or obtuse angle?

Example 2

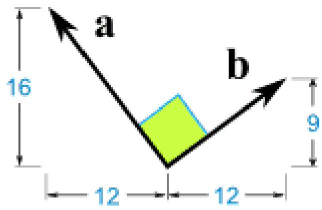
Calculate the dot product of c=(−4,−9) and d=(−1,2). Do the vectors form an acute angle, right angle, or obtuse angle?

Example 3

If a=(6,−1,3), for what value of c is the vector b=(4,c,−2) perpendicular to a?

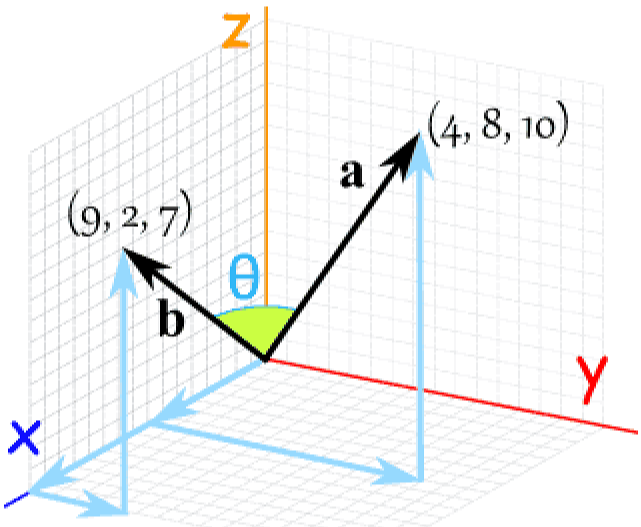
Example 4

Calculate the Dot Product for:



Example 5

Sam has measured the end-points of two poles, and wants to know the angle between them:



Answer 1

Using the component formula for the dot product of three-dimensional vectors,

$a \cdot b = a1b1 + a2b2 + a3b3$,

we calculate the dot product to be

$a \cdot b = 1(4) + 2(-5) + 3(6) = 4 - 10 + 18 = 12$.

Since $a \cdot b$ is positive, we can infer from the geometric definition, that the vectors form an acute angle.

Answer 2

Using the component formula for the dot product of two-dimensional vectors,

$a \cdot b = a1b1 + a2b2$,

we calculate the dot product to be

$c \cdot d = -4(-1) - 9(2) = 4 - 18 = -14$.

Since $c \cdot d$ is negative, we can infer from the geometric definition, that the vectors form an obtuse angle.

Answer 3

For a and b to be perpendicular, we need their dot product to be zero. Since

$a \cdot b = 6(4) - 1(c) + 3(-2) = 24 - c - 6 = 18 - c$,

the number c must satisfy $18 - c = 0$, or $c = 18$

You can double-check that the vector $b = (4, 18, -2)$

is indeed perpendicular to a by verifying that $a \cdot b = (6, -1, 3) \cdot (4, 18, -2) = 0$.

Answer 4

$a \cdot b = |a| \times |b| \times \cos(\theta)$

$a \cdot b = |a| \times |b| \times \cos(90°)$

$a \cdot b = |a| \times |b| \times 0$

$a \cdot b = 0$

or we can calculate it this way:

$$a \cdot b = ax \times bx + ay \times by$$

$$a \cdot b = -12 \times 12 + 16 \times 9$$

$$a \cdot b = -144 + 144$$

$$a \cdot b = 0$$

Answer 5

We have 3 dimensions, so don't forget the z-components:

$$a \cdot b = ax \times bx + ay \times by + az \times bz$$

$$a \cdot b = 9 \times 4 + 2 \times 8 + 7 \times 10$$

$$a \cdot b = 36 + 16 + 70$$

$$a \cdot b = 122$$

Now for the other formula:

$$a \cdot b = |a| \times |b| \times \cos(\theta)$$

But what is $|a|$? It is the magnitude, or length, of the vector a. We can use Pythagoras:

$$|a| = \surd(42 + 82 + 102)$$

$$|a| = \surd(16 + 64 + 100)$$

$$|a| = \surd 180$$

Likewise for $|b|$:

$$|b| = \surd(92 + 22 + 72)$$

$$|b| = \surd(81 + 4 + 49)$$

$$|b| = \surd 134$$

And we know from the calculation above that $a \cdot b = 122$, so:

$$a \cdot b = |a| \times |b| \times \cos(\theta)$$

$$122 = \surd 180 \times \surd 134 \times \cos(\theta)$$

$$\cos(\theta) = 122 / (\surd 180 \times \surd 134)$$

$$\cos(\theta) = 0{,}7855...$$

$$\theta = \cos\text{-}1(0{,}7855...) = 38{,}2...^\circ$$