**Skolemizare (discutie)**

**Ex1:** *Every philosopher writes at least one book.*

∀x(Philo(x) → ∃y(Book(y) ∧ Write (x, y)))

eliminam implicatia: ∀x(¬Philo(x) ∨ ∃y(Book(y) ∧ Write (x, y)))

skolemizare (y =g(x)): ∀x(¬Philo(x) ∨ (Book(g(x)) ∧ Write (x, g(x))))

**Ex2:** *All students of a philosopher read one of their teacher’s books.*

∀x∀y(Philo(x) ∧ StudentOf (y, x) → ∃z(Book(z) ∧ Write (x, z) ∧ Read (y, z)))

eliminam implicatia: ∀x∀y(¬Philo(x) ∨ ¬StudentOf (y, x) ∨ ∃z(Book(z) ∧ Write (x, z) ∧ Read (y, z)))

skolemizare (z = h(x, y)): ∀x∀y(¬Philo(x) ∨ ¬StudentOf (y, x) ∨ (Book(h(x, y)) ∧ Write(x, h(x, y)) ∧ Read(y, h(x, y))))

**Ex3:** *There exists a philosopher with students.*

∃x∃y[Philo(x) ∧ StudentOf (y, x)]

skolemizare (x = a, y = b): Philo(a) ∧ StudentOf (b, a)

**P3**. Consider the following axioms:

1. *Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.*∀ x (BUY(x) → ∃ y (OWNS(x,y) ∧ (RABBIT(y) ∨ GROCERY(y))))
2. *Every dog chases some rabbit.*  
   ∀ x (DOG(x) → ∃ y (RABBIT(y) ∧ CHASE(x,y)))
3. *Mary buys carrots by the bushel.*  
   BUY(Mary)
4. *Anyone who owns a rabbit hates anything that chases any rabbit.*  
   ∀x∀y(¬(OWNS(x, y) ∧ RABBIT(y)) ∨ ∀z∀t(¬(CHASE(z, t) ∧ RABBIT(t)) ∨ HATE(x, z)))
5. *John owns a dog.*  
   ∃ x (DOG(x) ∧ OWNS(John,x))
6. *Someone who hates something owned by another person will not date that person.*∀x∀y∀z(¬(HATE(x, y) ∧ OWNS(z, y)) ∨ ¬DATE(x, z))
7. *(Conclusion) If Mary does not own a grocery store, she will not date John.*  
   ¬ (∃x (OWNS (Mary, x) ∧ GROCERY(x)) ∨ ¬DATE (Mary, John))

Eliminam implicatiile:

1. ∀x(¬BUY(x) ∨ ∃y (OWNS (x, y) ∧ (RABBIT(y) ∨ GROCERY(y))))
2. ∀x(¬DOG(x) ∨ ∃y (CHASE (x, y) ∧ RABBIT(y)))
3. BUY(Mary)
4. ∀x∀y(¬(OWNS(x, y) ∧ RABBIT(y)) ∨ ∀z∀t(¬(CHASE(z, t) ∧ RABBIT(t)) ∨ HATE(x, z)))

∀x∀y(¬OWNS(x, y) ∨ ¬RABBIT(y) ∨ ∀z∀t(¬CHASE(z, t) ∨ ¬RABBIT(t) ∨ HATE(x, z)))

1. ∃x (OWNS (John, x) ∧ DOG(x))
2. ∀x∀y∀z(¬(HATE(x, y) ∧ OWNS(z, y)) ∨ ¬DATE(x, z))

∀x∀y∀z(¬HATE(x, y) ∨ ¬OWNS(z, y) ∨ ¬DATE(x, z))

1. ¬ (∃x (OWNS (Mary, x) ∧ GROCERY(x)) ∨ ¬DATE (Mary, John))

¬∃x (OWNS (Mary, x) ∧ GROCERY(x)) ∧ DATE (Mary, John)

∀x (¬OWNS (Mary, x) ∨ ¬GROCERY(x)) ∧ DATE (Mary, John)

Skolemizare:

1. y = f (x) ⇒ ∀x(¬BUY(x) ∨ (OWNS (x, f (x)) ∧ (RABBIT (f (x)) ∨ GROCERY (f (x)))))
2. y = g(x) ⇒ ∀x(¬DOG(x) ∨ (CHASE (x, g(x)) ∧ RABBIT(g(x))))
3. BUY(Mary)
4. ∀x∀y(¬OWNS(x, y) ∨ ¬RABBIT(y) ∨ ∀z∀t(¬CHASE(z, t) ∨ ¬RABBIT(t) ∨ HATE(x, z)))
5. x = a ⇒ OWNS (John, a) ∧ DOG(a)
6. ∀x∀y∀z(¬HATE(x, y) ∨ ¬OWNS(z, y) ∨ ¬DATE(x, z))
7. ∀x (¬OWNS (Mary, x) ∨ ¬GROCERY(x)) ∧ DATE (Mary, John)

Forma normala prenex:

1. ¬BUY(x) ∨ (OWNS (x, f (x)) ∧ (RABBIT (f (x)) ∨ GROCERY (f (x))))
2. ¬DOG(x) ∨ (CHASE (x, g(x)) ∧ RABBIT(g(x)))
3. BUY(Mary)
4. ¬OWNS (x, y) ∨ ¬RABBIT(y) ∨ ¬CHASE (z, t) ∨ ¬RABBIT(t) ∨ HATE (x, z)
5. OWNS (John, a) ∧ DOG(a)
6. ¬HATE (x, y) ∨ ¬OWNS (z, y) ∨ ¬DATE(x, z)
7. (¬OWNS (Mary, x) ∨ ¬GROCERY(x)) ∧ DATE(Mary, John)

Rezolutie:

1. ¬BUY(x) ∨ OWNS(x, f (x))
2. ¬BUY(x) ∨ RABBIT(f (x)) ∨ GROCERY(f (x))
3. ¬DOG(x) ∨ CHASE(x, g(x))
4. ¬DOG(x) ∨ RABBIT(g(x))
5. BUY(Mary)
6. ¬OWNS(x, y) ∨ ¬RABBIT(y) ∨ ¬CHASE(z, t) ∨ ¬RABBIT(t) ∨ HATE(x, z)
7. OWNS(John, a)
8. DOG(a)
9. ¬HATE(x, y) ∨ ¬OWNS(z, y) ∨ ¬DATE(x, z)
10. ¬OWNS(Mary, x) ∨ ¬GROCERY(x)
11. DATE(Mary, John)
12. 9 + 11 ⇒ ¬HATE(Mary, y) ∨ ¬OWNS(John, y)
13. 7 + 12 ⇒ ¬HATE(Mary, a)
14. 6 + 13 ⇒ ¬OWNS(Mary, y) ∨ ¬RABBIT(y) ∨ ¬CHASE(a, t) ∨ ¬RABBIT(t)
15. 4 + 14 ⇒ ¬OWNS(Mary, y) ∨ ¬RABBIT(y) ∨ ¬CHASE(a, g(x)) ∨ ¬DOG(x)
16. 3 + 15 ⇒ ¬OWNS(Mary, y) ∨ ¬RABBIT(y) ∨ ¬DOG(a)
17. 8 + 16 ⇒ ¬OWNS(Mary, y) ∨ ¬RABBIT(y)
18. 2 + 17 ⇒ ¬BUY(x) ∨ GROCERY(f (x)) ∨ ¬OWNS(Mary, f (x))
19. 10 + 18 ⇒ ¬BUY(x) ∨ ¬OWNS(Mary, f (x))
20. 1 + 19 ⇒ ¬BUY(Mary)
21. (21) 5 + 20 ⇒ ∅

**Tema:**

 Consider the following axioms:

1. *Anyone whom Mary loves is a football star.*∀ x (LOVES(Mary,x) → STAR(x))
2. *Any student who does not pass does not play.*∀ x (STUDENT(x) ∧ ¬ PASS(x) → ¬ PLAY(x))
3. *John is a student.*  
   STUDENT(John)
4. *Any student who does not study does not pass.*∀ x (STUDENT(x) ∧ ¬ STUDY(x) → ¬ PASS(x))
5. *Anyone who does not play is not a football star.*∀ x (¬ PLAY(x) → ¬ STAR(x))
6. *(Conclusion) If John does not study, then Mary does not love John.*¬ STUDY(John) → ¬ LOVES(Mary,John)

Eliminam implicatiile:

1. ∀ x (¬LOVES(Mary,x) ∨ STAR(x))
2. ∀ x (¬STUDENT(x) ∨ PASS(x) ∨ ¬ PLAY(x))
3. STUDENT(John)
4. ∀ x (¬STUDENT(x) ∨ STUDY(x) ∨ ¬ PASS(x))
5. ∀ x (PLAY(x) ∨ ¬ STAR(x))
6. STUDY(John) ∨ ¬ LOVES(Mary,John)

Negatie: ¬STUDY(John) ∧ LOVES(Mary,John)

Forma normala prenex:

1. ¬LOVES(Mary,x) ∨ STAR(x)
2. ¬STUDENT(x) ∨ PASS(x) ∨ ¬ PLAY(x)
3. STUDENT(John)
4. ¬STUDENT(x) ∨ STUDY(x) ∨ ¬ PASS(x)
5. PLAY(x) ∨ ¬ STAR(x)
6. ¬STUDY(John) ∧ LOVES(Mary,John)

Rezolutie:

1. ¬LOVES(Mary,x) ∨ STAR(x)
2. ¬STUDENT(x) ∨ PASS(x) ∨ ¬ PLAY(x)
3. STUDENT(John)
4. ¬STUDENT(x) ∨ STUDY(x) ∨ ¬ PASS(x)
5. PLAY(x) ∨ ¬ STAR(x)
6. A. ¬STUDY(John)

B. LOVES(Mary,John)

1. [2+4] => ¬STUDENT(x) ∨ ¬ PLAY(x) ∨ STUDY(x)
2. [5+7] => ¬STUDENT(x) ∨ STUDY(x) ∨ ¬ STAR(x)
3. [1+8] => ¬STUDENT(x) ∨ STUDY(x) ∨ ¬LOVES(Mary,x)
4. [3+9] => STUDY(John) ∨ ¬LOVES(Mary,John)
5. [6A+10) => ¬LOVES(Mary,John)
6. [6B+11] => Ø