CMLS: First Homework
Disteacher

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Abstract

Our assignment was to develop in Supercollider a system devoted to learning the use of different types of audio distortion.

1 Introduction

We decided to implement a tool that could help the user in visualizing the input waveform, the distortion function and the output waveform after the distortion, in order to make the user understand how does the distortion affect sound.

To reach our goal, we created a user-friendly application that plots the input, the transfer function and the distorted output. Along with this, we gave the user the possibility to manipulate the main parameters of the distortion: in this way it's possible to make some experiments and to understand what is the influence of those parameters on the resulting output. The parameters with which the user can interact are Input Gain, Gain, Harshness and Output.

In our application we also prepared three different distortion presets, to make the user interact with some typical distortion functions used by famous musicians. In our application we considered three of the main types of distortions: Overdrive, Distortion and Fuzz, and we associated them to three famous artists to make it easy to understand what we're talking about.

These three types of distortion show different features that will be better explained also from a mathematical point of view later in the report.

2 Mathematical theory of distortion

Distortion is deeply correlated with the mathematical concept of non-linearity. The property of linearity is associated to those functions that obey to this rule:

$$f(ax_1[n] + bx_2[n]) = af(x_1[n]) + bf(x_2[n]) \tag{1}$$

where f is a linear function, a and b are two coefficients and x_1 and x_2 are the inputs of the function.

In the introduction we cited three types of distortion: all these three are characterized to be *non-linear* functions. Anyway, there are some differences between them:

- *Overdrive*: an almost linear function that becomes progressively non-linear at higher levels;
- *Distortion*: non-linear for almost every input signal;
- *Fuzz*: completely non-linear function, that causes abrupt changes in the output signal, leading to a harsher sound.

2.1 Hard vs Soft Clipping

Both in analog and digital systems, a physical constraint prevents the output to exceed a certain threshold. In analog systems, this limit is caused by the system architecture and by the nature of the amplifiers, while in digital systems it is usually determined by the number of bits used for the signal representation. When a signal exceeds these limits, *clipping* occurs: this means that regardless of how much you increase the input, you won't get any increase in the output.

Then we can distinguish between hard and soft clipping, depending on the type of function: this is one of the main characteristics used to distinguish between different types of distortion. Intuitively, we differentiate between them by saying that *hard* clipping causes an abrupt transition between the unclipped and the clipped regions, while *soft* clipping is smoother, generating a brighter and harsher sound for the hard clipping and a smoother and warmer sound for the soft one. Then, by modifying the shape of the knee between the two regions, you can obtain lots of different types of soft clipping distortions and you can have a more aggressive sound or a cleaner one.

Two examples are shown in figure 1 and 2.

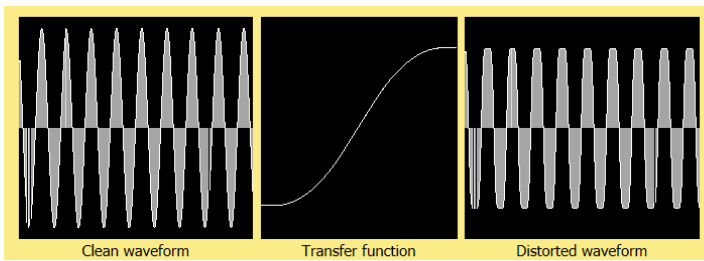


Figure 1: An example of soft clipping extracted from the GUI of our application.

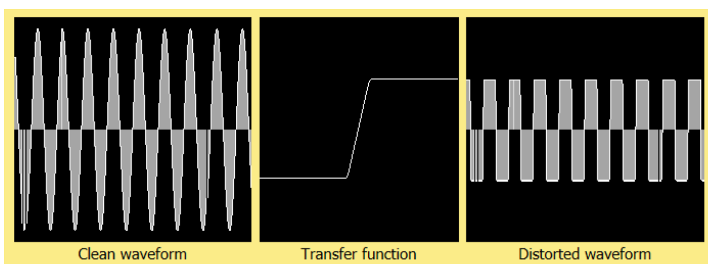


Figure 2: An example of hard clipping extracted from the GUI of our application.

2.2 Transfer function

The mathematical equations of the transfer functions are commented in this section. The plots of the various transfer functions are implemented in `https://www.geogebra.org/m/x5bhsngv`. At this link it's possible to see how does each one of the transfer function react to the changing of the parameters, in particular changing G and T, effecting respectively the gain and the harshness. We have implemented the quadratic, the hyperbolic tangent, the foldback and the linear transfer functions.

2.2.1 Quadratic transfer function

The quadratic transfer function is a non-linear type of function that can be divided into two zones, as shown in figure 3. Here the green part represents the linear part and the red part represents the quadratic one. The following describes how to get this mathematical expression. The three knobs of GAIN, HARSHNESS and OUTPUT are mapped by the three variables G, H and P in the transfer function.

At this stage, the coefficient P is not taken into account, since it is a scaling factor. The linear part of the transfer function can be expressed by $f(x) = Gx$. In this case, G stands for the slope of

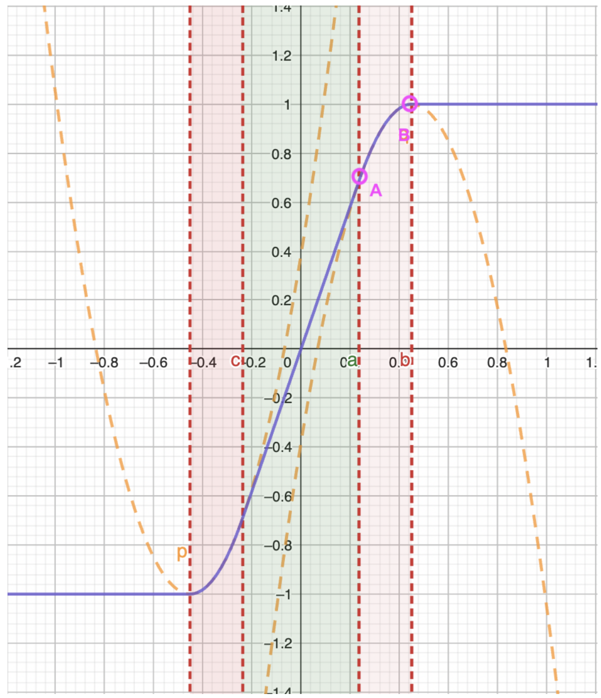


Figure 3: The quadratic transfer function.

this linear function. In order to make the ordinate value of point A in figure 3 as T, the range of this linear part should be $[-\frac{T}{G}, \frac{T}{G}]$, which ensures that $|f(\pm\frac{T}{G})| = T$.

The general expression for a quadratic function is $f(x) = ax^2 + bx + c$, where a, b and c are coefficients. To get the expression of those unknown coefficients by G, H and P, three conditons are needed (taking the positive area as an example):

1. $f(\dfrac{T}{G}) = G \cdot \dfrac{T}{G}$,
2. $f'(\dfrac{T}{G}) = G$,
3. $f(-\dfrac{b}{2a}) = 1$.

The first and second conditions satifiy that at point A, the function and its derivative are both continuous. However, the third condition ensures that at point B, which is the vertex of the quadratic function, the ordinate value is 1. This satisfies the continuity of both the function itself and its derivative. The solution of a, b, and c are

- $a = \dfrac{G^2}{4(T-1)}$,
- $b = \dfrac{G(T-2)}{2(T-1)}$,
- $c = \dfrac{(T-2)^2}{4(T-1)} + 1$.

After getting all of the coefficients, the scaling factor P can be applied. In conclusion, the quadratic transfer function is (function 2):

$$f(x) = \begin{cases} -P & x \le \dfrac{T-2}{G} \\ P\left[\dfrac{G^2}{4(1-T)}(x - \dfrac{T-2}{G})^2 - 1\right] & \dfrac{T-2}{G} < x < -\dfrac{T}{G} \\ PGx & |x| \le \dfrac{T}{G} \\ P\left[\dfrac{G^2}{4(T-1)}(x - \dfrac{2-T}{G})^2 + 1\right] & \dfrac{T}{G} < x < \dfrac{2-T}{G} \\ P & x \ge \dfrac{2-T}{G} \end{cases} \tag{2}$$

The type of transfer function of figure 4 can be described as an Overdrive effect, as it has a linear region for values of the input between $-\frac{1}{3}$ and $\frac{1}{3}$, a clipping region for values of the input of magnitude greater than $\frac{2}{3}$ and a gradual transition between the two.

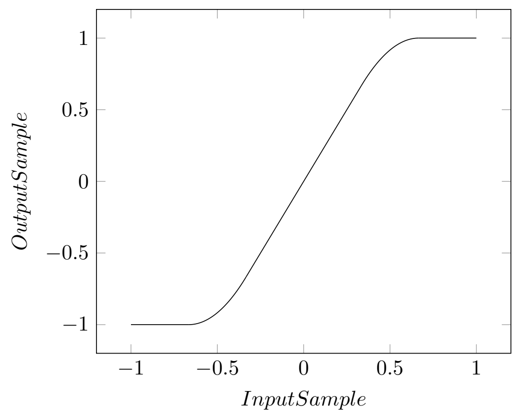


Figure 4: The overdrive transfer function.

2.2.2 Linear transfer function

The linear transfer function could be regarded as a particular case from the quadratic transfer function, corresponding to having the parameter $T = 1$, as shown in figure 5.

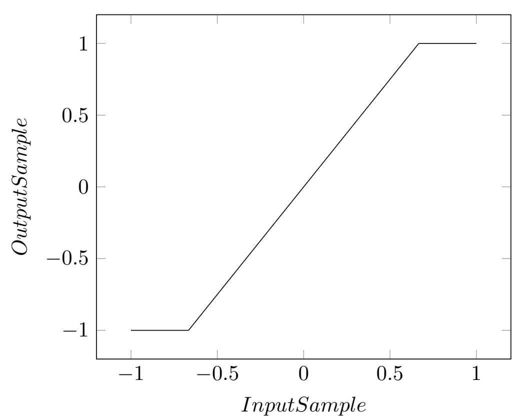


Figure 5: The linear transfer function.

Considering a linear transfer function, such as that in the figure, we can clearly see that the transition from the unclipped to the clipped region is quite abrupt: this implies that with this kind of transfer function we will obtain a harsher output sound, richer in terms of generated harmonics.

The linear transfer function, considering it as a particular case of the quadratic one, can be expressed in this way (function 3):

$$f(x) = \begin{cases} -P & x \leq -\dfrac{1}{G} \\ PGx & |x| < \dfrac{1}{G} \\ P & x \geq \dfrac{1}{G} \end{cases} \tag{3}$$

2.2.3 Hyperbolic tangent transfer function

The hyperbolic tangent transfer function is

$$h(x) = P\tanh(Gx), \tag{4}$$

as shown in figure 6.

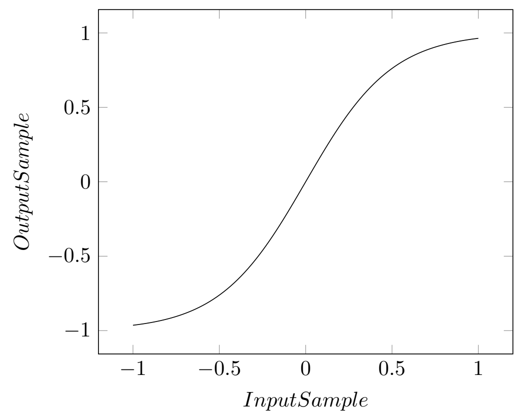


Figure 6: The hyperbolic tangent transfer function.

In the shown image, the parameter P is $P = 1$ and the parameter G is $G = 2$.

This type of curve is very smooth, even more than the quadratic curve especially for high gain values, and this will then lead to a softer sound.

2.2.4 Foldback transfer function

The foldback transfer function is:

$$g(x) = Gsin(Gx)e^{-2G|x|} \tag{5}$$

As can clearly be seen in the figure 7, foldback distortion is particularly aggressive and is therefore a very special effect to use, which has nothing to do with the other types of distortion mentioned above. This type of distortion therefore allows for very original sounds, which leave room for creativity.

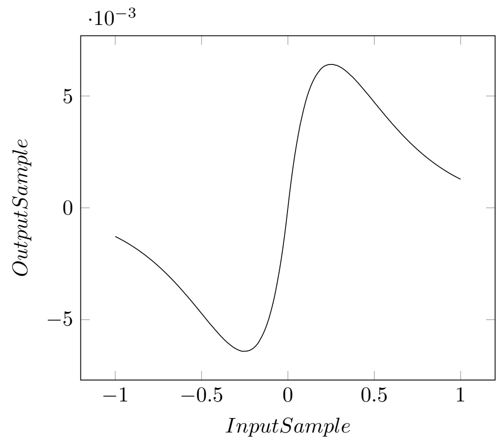


Figure 7: The foldback transfer function.

3 Analog Emulation

The sound produced by vacuum-tube analog amplifiers is still nowadays the favourite one of many guitarists. To satisfy their needs, during the digital era some kinds of *analog emulation* amplifiers are diffused, as of course they are a lot cheaper and easier to handle than the real vacuum-tube amplifiers.

Analog emulation techniques might be very complex, mathematically speaking, even when implementing the simplest effects. Anyway there are some actions that can be taken:

1. rounding the corners of the waveform as it approaches the clipping level - this will give the sound an *analog* feel, as electronic circuites aren't as abrupt in changes of the waveform as the digital systems;
2. the curve could be slightly asymmetrical - this will produce both odd and even harmonics, generating a richer sound;
3. use oversampling.

In order to reach our goal, we introduced the second one of these tricks in our application, giving the possibility to listen both to the normal digital distortion and to the analog-emulated distortion. In order to do that, we introduced some randomness in the parameters, in particular adding or subtracting a little random number to the gain and the threshold. This is performed separately for the upper positive part of the transfer function and the negative part, to be even more realistic. Doing so, every time the user introduces the analog behaviour by clicking on the relative button, a different response is triggered and a slightly different effect can be heard.

4 Usage

The plug-in has two modes of usage:

- **Tutorial**: In this first mode, the user is only allowed to reproduce predefined audio files that he can choose from the *Input Source* menu. While the audio is playing, the user can switch between 3 main presets of distortion (Overdrive, Distortion and Fuzz) to understand how this affects the sound and the scopes. He can also change all the parameters' value, in order to build a certain confidence with the plug-in. The audio file[1] we have chosen are typical examples of musical instruments on which distortion effects are commonly applied: guitar, bass and voice; we also added a sinusoidal signal at 20 Hz[2] and an entire musical piece to offer a wider perspective on the effects that distortion can have on sound and also on waves' shapes.

[1] At the moment, the files are saved locally on our computers. You can download the files from the GitHub repository and replace the path in the very first lines of code with your own.

[2] We chose such a low frequency for the sine wave to allow the user to visualise the effect of distortion on the waveform more clearly.

- **I can do it**: In this modality the user can start experiment, playing some sounds by his own. In order to do that, the user can choose the sound source from the *Input Source* menu: he can use the sound coming from the computer microphone or the sound coming from an external soundcard[3]. This modality gives the user the possibility to squeeze his creativity and find new sounds he enjoys.

5 Interface

- **Mode**: *"Tutorial" mode* or *"I can do it" mode.*

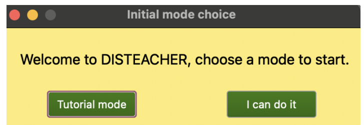
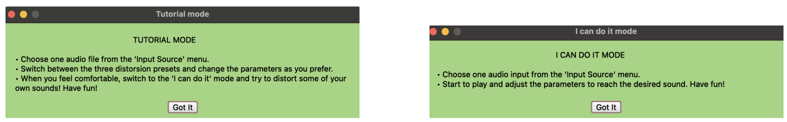


Figure 8: Initial mode choice.



(a) (b)

Figure 9: Mode instruction: (a) "tutorial" mode and (b) "I can do it" mode.

- **Knobs roles**: Instructions of the roles of different knobs. By clicking "i".

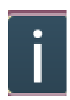


Figure 10: "i".

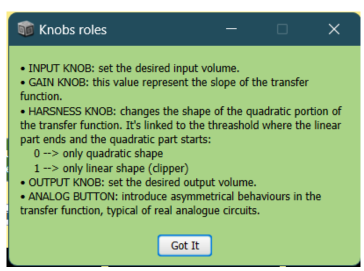


Figure 11: Knobs roles.

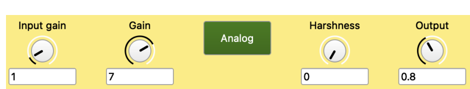


Figure 12: Knobs.

[3] In this case there could be some problems due to the numbering of the bus in Supercollider. It could be useful to check the input and the output buses when booting the server, to be sure you are selecting the correct one. In any case, you can always manually change this two indexes in the very first rows of the code.

- **Default effects**: *"Steve Ray Vaughan (Overdrive)"*, *"AC/DC (Distortion)"*, or *"Matt Bellamy (Fuzz)"*.

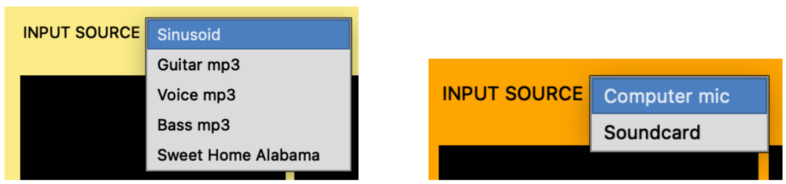
  Overdrive: Gain 2, Harshness 0.3, Output 0.8

  Distortion: Gain 7, Harshness 0, Output: 0.8

  Fuzz: Gain 9, Harshness 0.8, Output: 0.5

Figure 13: Default effects.

- **Input Source**:



(a) (b)

Figure 14: Input source: (a) "tutorial" mode and (b) "I can do it" mode.

- **Curve Type**:

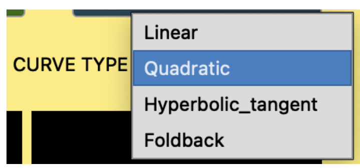


Figure 15: Curve types.

- **Play/stop**:

(a) (b)

Figure 16: Play/stop button.

- **Diagrams**:

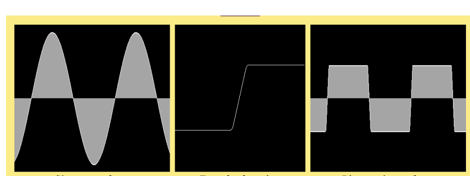


Figure 17: Diagrams.