# Reinforcement Learning for Language Model Training

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## Reinforcement Learning

RL4
LMT

# LMs: Recap

## Transformers,Training & Inference

- $LM \ : \ X \mapsto \Delta(S)$
- transformers use self-attention to offer and retrieve relevant information
  - stacked transformer blocks and multi-head attention increase capacity
- LMs are trained to predict the next word using cross-entropy loss (via teacher-forcing)
- decoding schemes are used for inference given a trained LM
  - different stochastic sampling regimes
- SOTA models exhibit ‘in-context learning’
- advanced prompting techniques might improve LLMs’ generalization performance

# Making LLMs useful

## Adaptation

- training a task-specific head on top of a model
  - e.g., span prediction layer on top of BERT with frozen BERT
  - on a dataset of ground truth input-output pairs for a particular task
- fine-tuning the model
  - further training part or entire model for a shorter time
  - on a dataset of ground truth input-output pairs for a particular task
- practical problem
  - training with standard supervision is impractical (data collection)
  - and inefficient (restricting "ground truth" to finite set of answers for open-ended tasks)
- **RL is the solution:** learn to achieve goal based on feedback from environment rather than direct demonstration of correct behaviour



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# Reinforcement learning

# Flavors of machine learning

- Unsupervised learning
  - e.g., clustering
- discover patterns in unlabeled data
  - 'given my inductive bias, what is the likely structure of the data?'

- Supervised learning
  - also self-supervised learning
  - aka behavioural cloning
- learn to output Y, given X, from labeled data
  - 'do as I show you'
- learning from demonstration

- Reinforcement learning
  - trial-and-error learning
- learning from interaction / experience
  - 'how do I optimally behave in order to maximize reward?'
  - or, 'how do i optimally achieve my goal?'
    - most natural way of learning?
    - tightly connected to the way organisms behave ("pleasure maximizers")

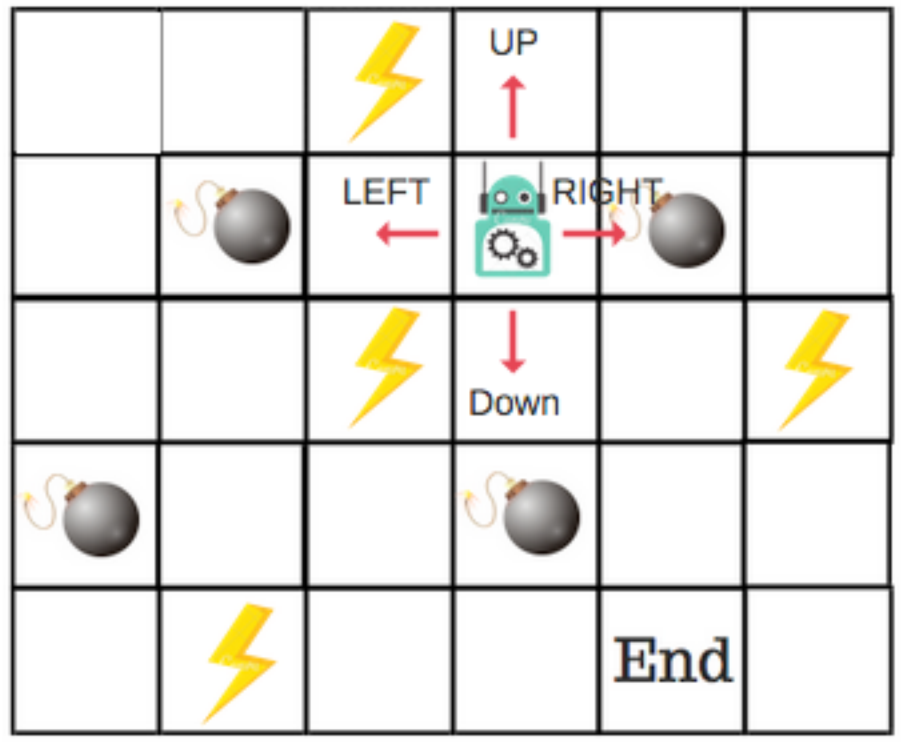


Christian (2020), Bishop (2009)

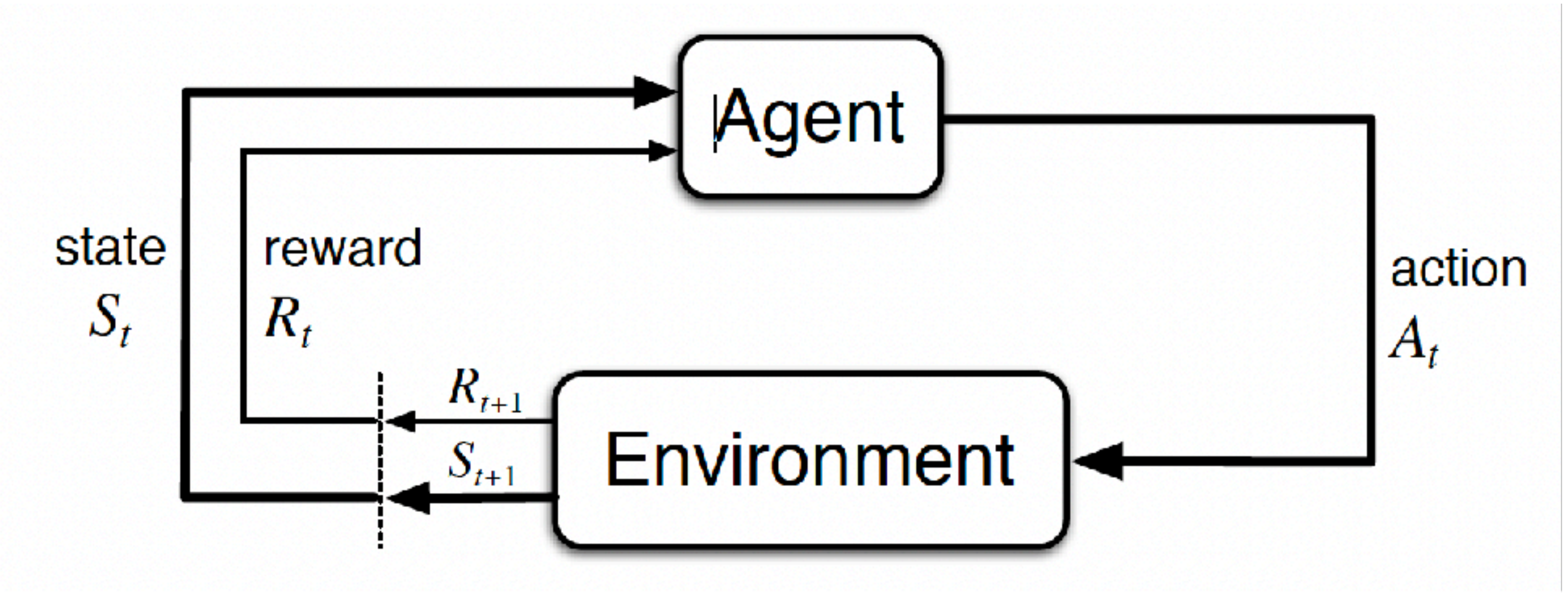
# Reinforcement Learning: Overview

## Introduction

- **Reinforcement Learning:** Computational formalisation of goal-directed learning and decision making under uncertainty
- **Goal:** Maximize rewards (by learning optimal behavior)
- **Basic building blocks:**
  - Agent
  - States
  - Actions
  - Transition function $P$
  - Reward
  - Policy




Associative RL




Sutton & Barto (2018, p. 48, Fig 3.1), image source

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Introduction

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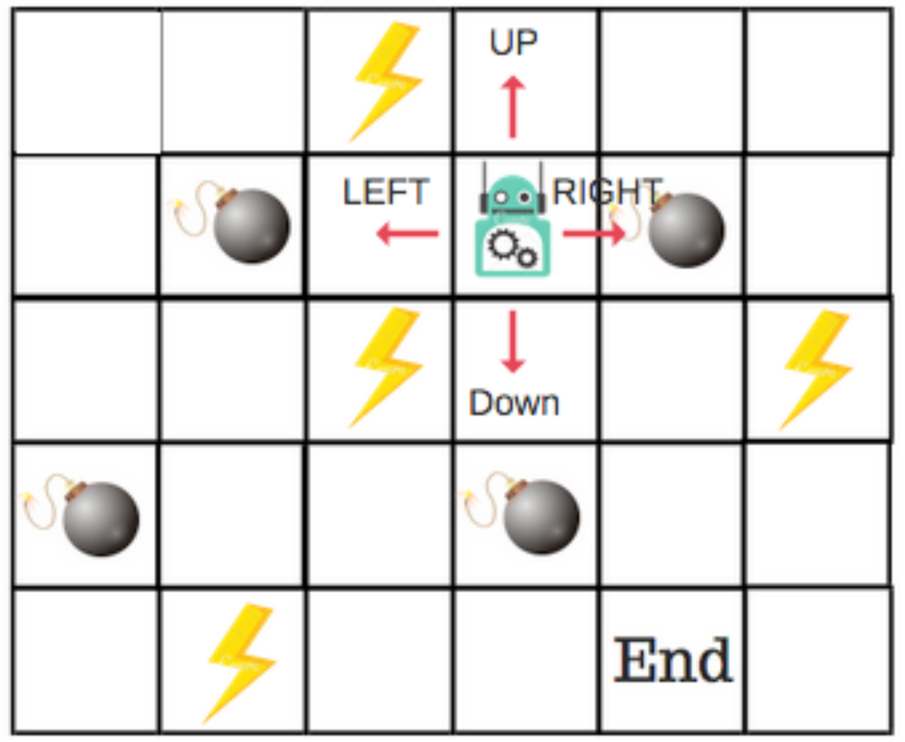
- **Goal:** Maximize rewards (by learning optimal behavior)
- **Basic building blocks:**
  - Agent
  - States: $S_t \in \mathcal{S}$ for $t = 0, 1, 2, 3, \ldots$
  - Actions: $A_t \in \mathcal{A}(s)$
  - Transition function: $P(s' \mid s, a)$
  - Reward: $R_{t+1} \in \mathcal{R}$
  - Policy: $\pi(S_t) = P(A_t \mid S_t)$

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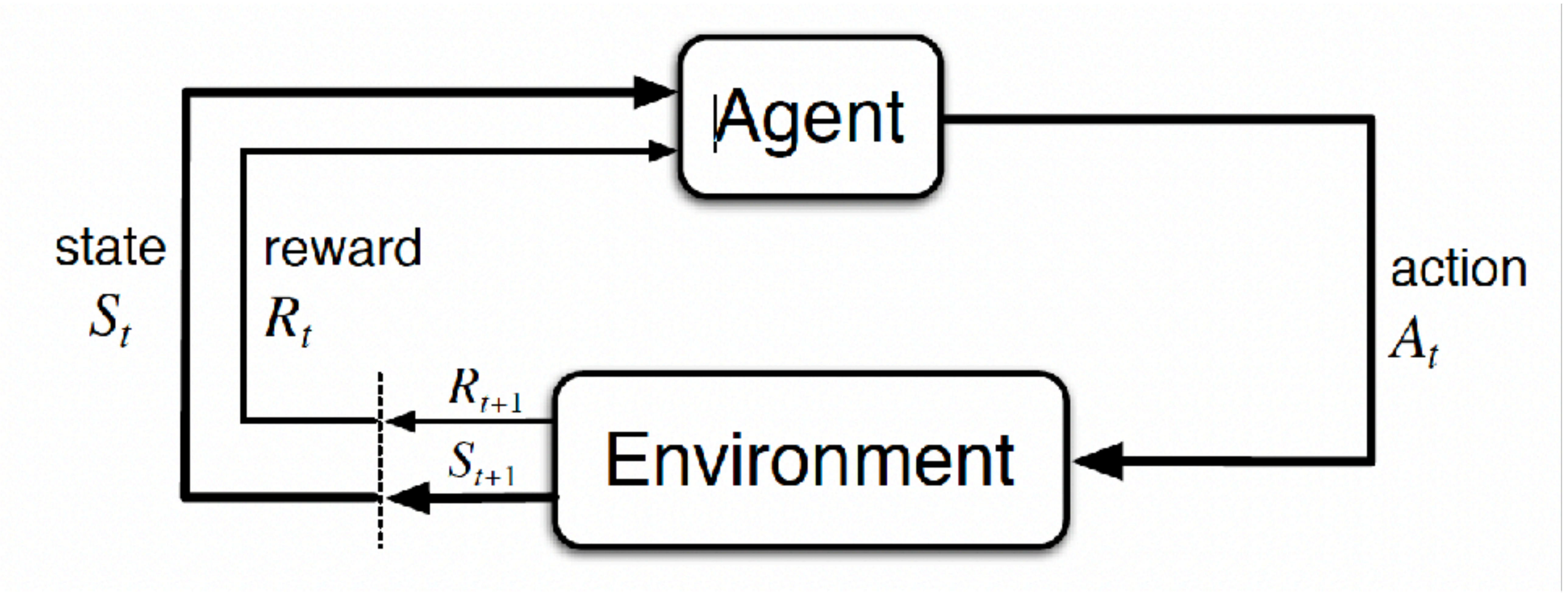
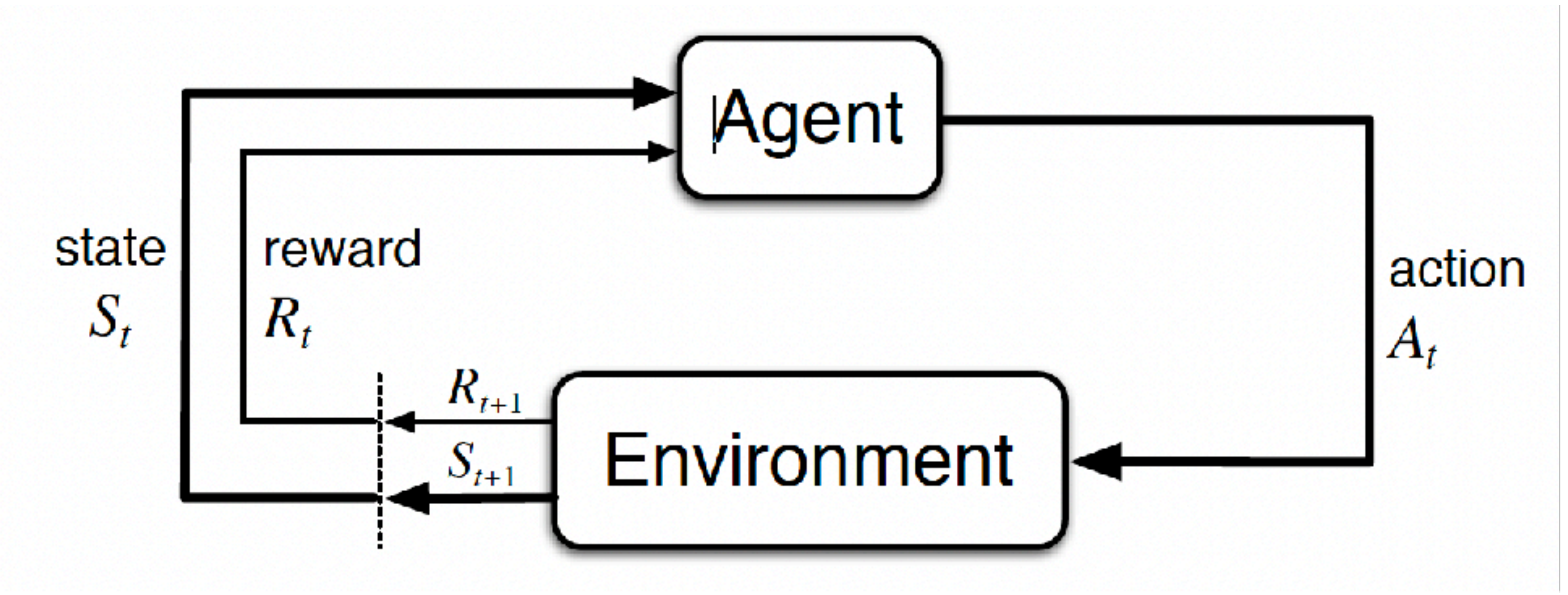



Sutton & Barto (2018, p. 48, Fig 3.1), image source

# Markov Decision Processes

## Formal definition




**Finite MDPs**: $(S, A, T, R)$

1. $S_t \in S$ for $t = 0, 1, 2, 3, \ldots$
2. $A_t \in A(s)$
3. $R_{t+1} \in R$
4. $T(s' \mid s, a) = \sum_{r' \in R} P(s', r' \mid s, a)$
5. Horizon H, discount factor $0 \leq \gamma \leq 1$

Expected rewards for state $s$ and action $a$: $r(s, a) = \mathbb{E}(R_t \mid S_{t-1} = s, A_{t-1} = a) = \sum_{r \in R} r \sum_{s' \in S} P(s', r \mid s, a)$

**Markov property**: $S_{t-1}, A_{t-1}$ include all information about past agent-environment interactions that are relevant for $S_t, R_t$



Sutton & Barto (2018, p. 48, Fig 3.1)

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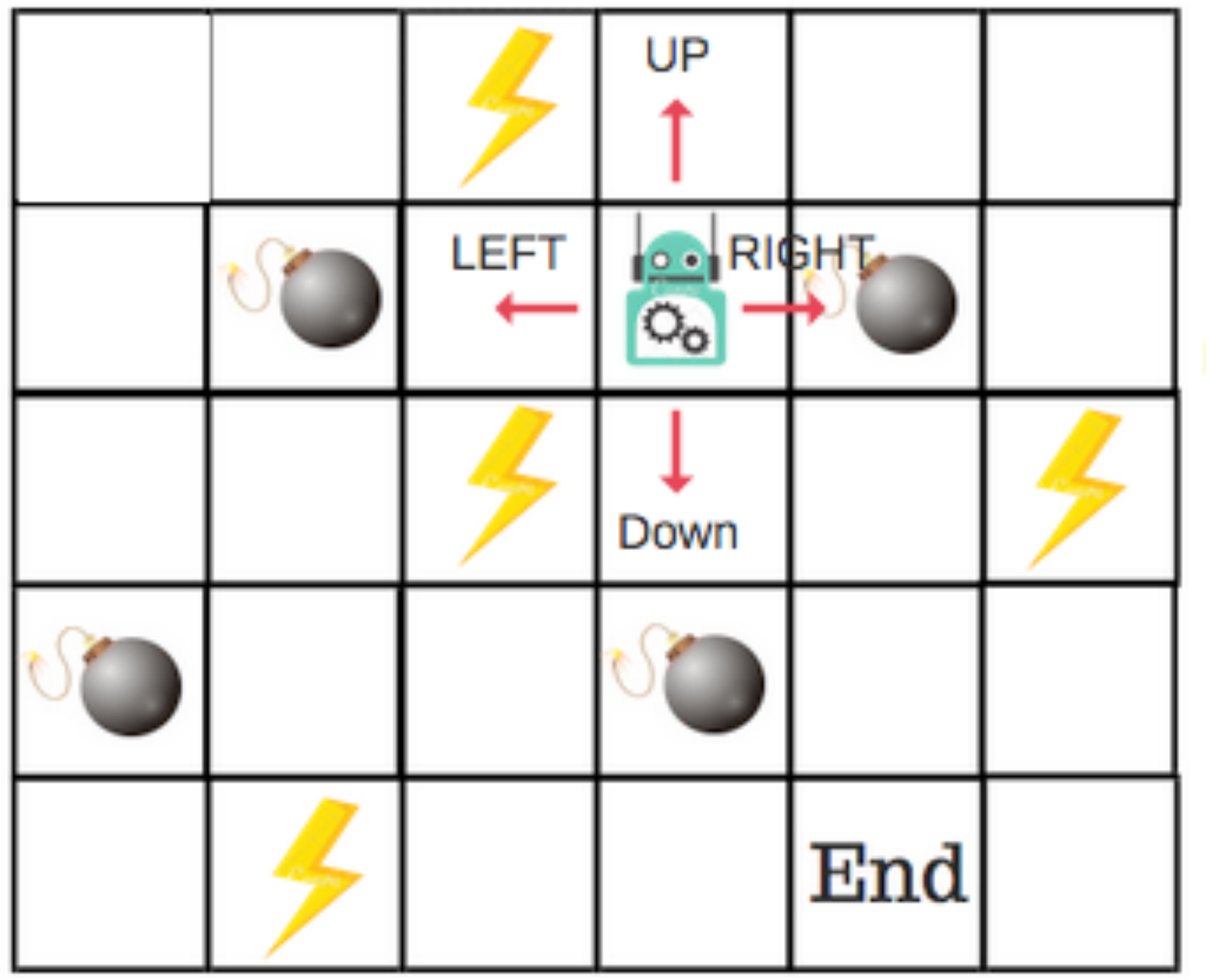


Goal: maximize **returns** until goal achieved

$G_t = R_{t+1} + R_{t_2} + \ldots + R_T$

Formally: maximize expected **discounted** rewards over **episode**

$$G_t = R_{t+1} + \gamma R_{t_2} + \gamma^2 R_{t+3} + \ldots + \gamma^{T-t-1} R_T = \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$



Sutton & Barto (2018, p. 48, Fig 3.1)

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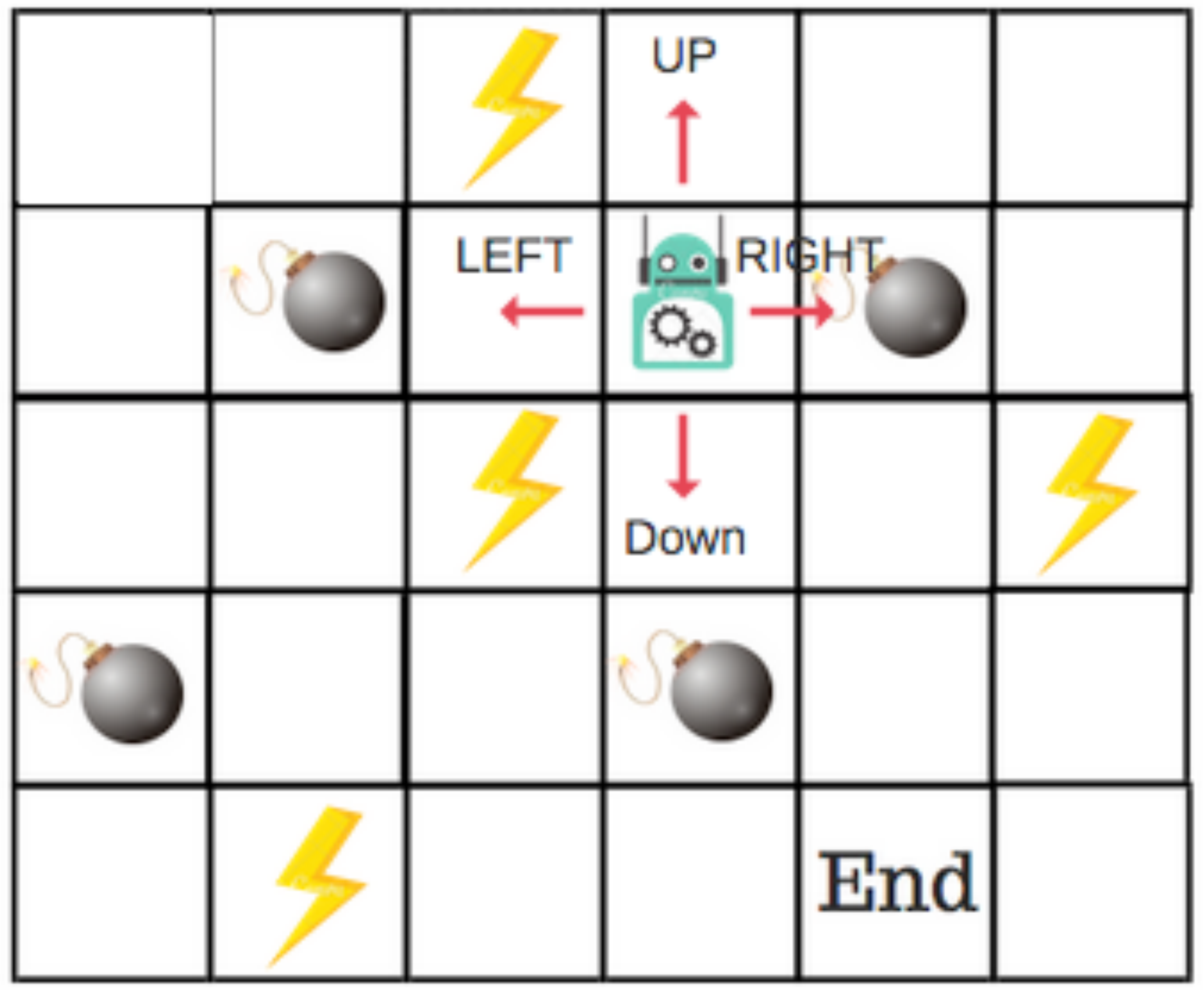
$$= R_{t+1} + \gamma G_{t+1}$$




- We can identify optimal way to behave if we know what good particular states and/or actions are:

Optimal **state-value function:**

$$V_\pi^*(s) = \max_\pi \mathbb{E}[G_t \mid S_t = s] = \max_\pi \mathbb{E}[R_{t+1} + \gamma G_{t+1} \mid S_t = s]$$

$$= \max_a \sum_{s', r} P(s', r \mid s, a)[r + \gamma G_{t+1} \mid S_t = s] \text{ for all } s$$

- Optimization problem: (computationally) find **optimal policy** $\pi * (S_t) = P(A_t \mid S_t)$



Sutton & Barto (2018, p. 48, Fig 3.1)

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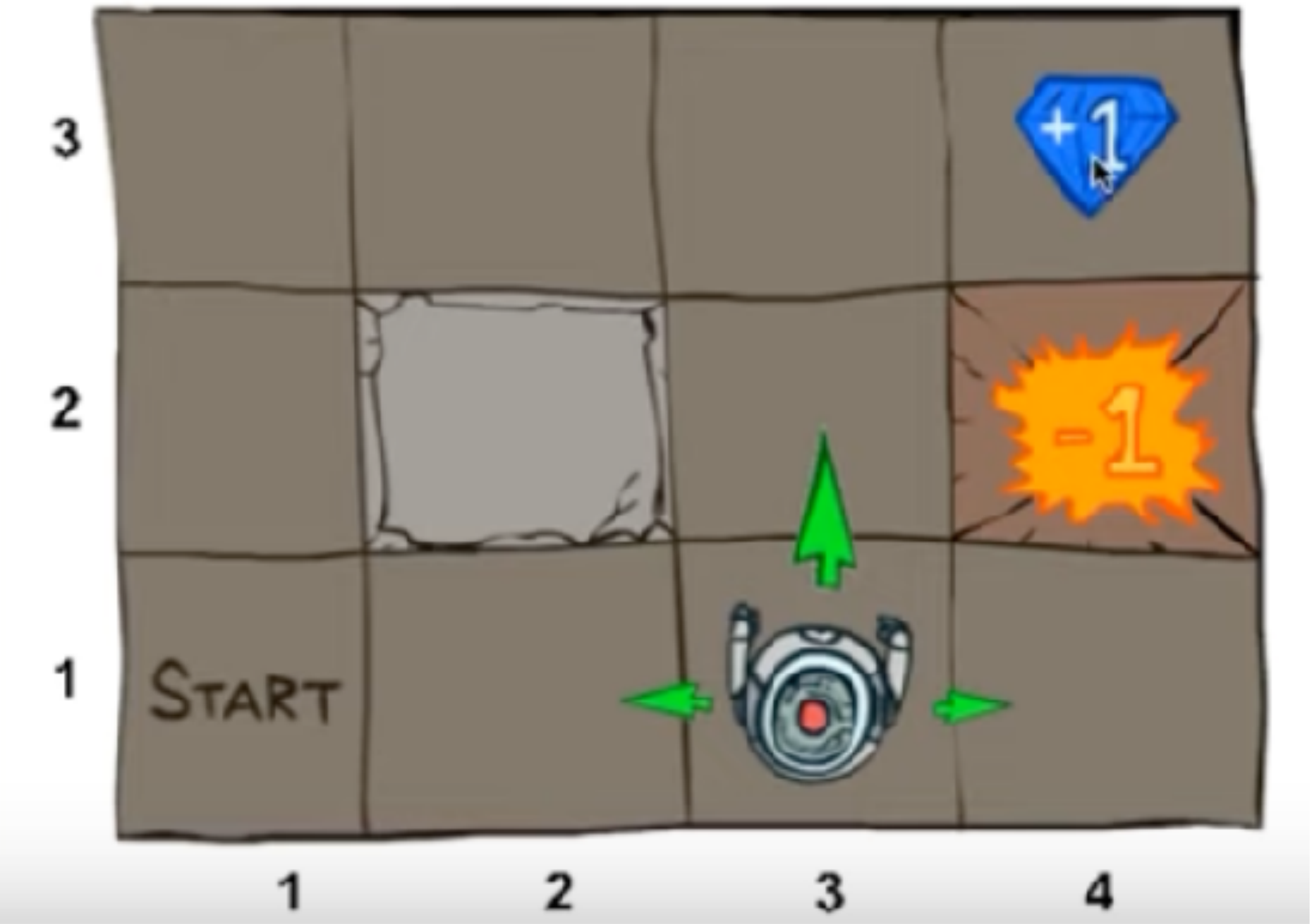
4. $T(s' \mid s, a) = \sum_{r' \in R} P(s', r' \mid s, a)$

Optimal state-value function:

$$V^*_\pi(s) = \max_\pi \mathbb{E}[G_t \mid S_t = s] = \max_\pi \mathbb{E}[\sum_{k=t+1}^{T} \gamma^{k-t-1} R_k \mid S_t = s]$$




deterministic optimal policy

$\gamma = 1$

$V^*(4,3) = 1$
$V^*(3,3) = 1$
$V^*(2,3) = 1$
$V^*(1,1) = 1$
$V^*(4,2) = -1$

deterministic optimal policy

$\gamma = 0.9$

$V^*(4,3) = 1$
$V^*(3,3) = 0.9$
$V^*(2,3) = 0.81$
$V^*(1,1) = 0.9^5 = 0.59$
$V^*(4,2) = -1$



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# Markov Decision Processes

## Formal definition

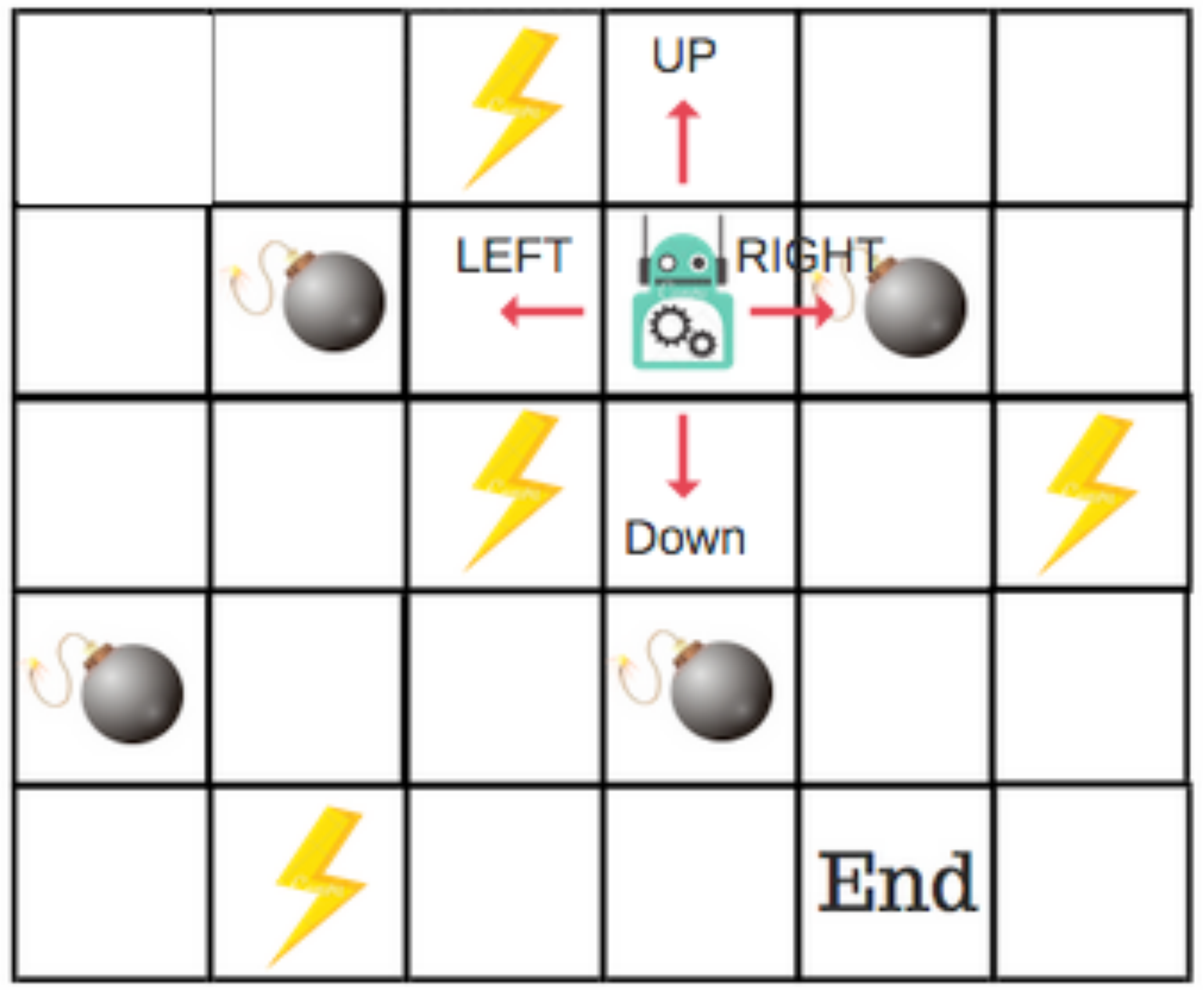
**Finite MDPs:** $(S, A, T, R)$

1. $S_t \in S$ for $t = 0, 1, 2, 3, \ldots$
2. $A_t \in A(s)$
3. $R_{t+1} \in R$
4. $T(s' \mid s, a) = \sum_{r' \in R} P(s', r' \mid s, a)$

Goal: maximize discounted **returns**

$$G_t = R_{t+1} + \gamma R_{t_2} + \gamma^2 R_{t+3} + \ldots + \gamma^{T-t-1} R_T = \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

$$= R_{t+1} + \gamma G_{t+1}$$




- We can identify optimal way to behave if we know what good particular states and/or actions are:

Optimal **action-value function:**

$$Q^*_\pi(s, a) = \max_\pi \mathbb{E}[G_t \mid S_t = s, A_t = a] = \max_\pi \mathbb{E}[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a]$$

$$= \sum_{s', r} P(s', r \mid s, a)[r + \gamma \max_{a'} Q^*(s', a') \mid S_t = s, A_t = a] \text{ for all } s, a$$



Sutton & Barto (2018, p. 48, Fig 3.1)

# Markov Decision Processes
## Formal definition

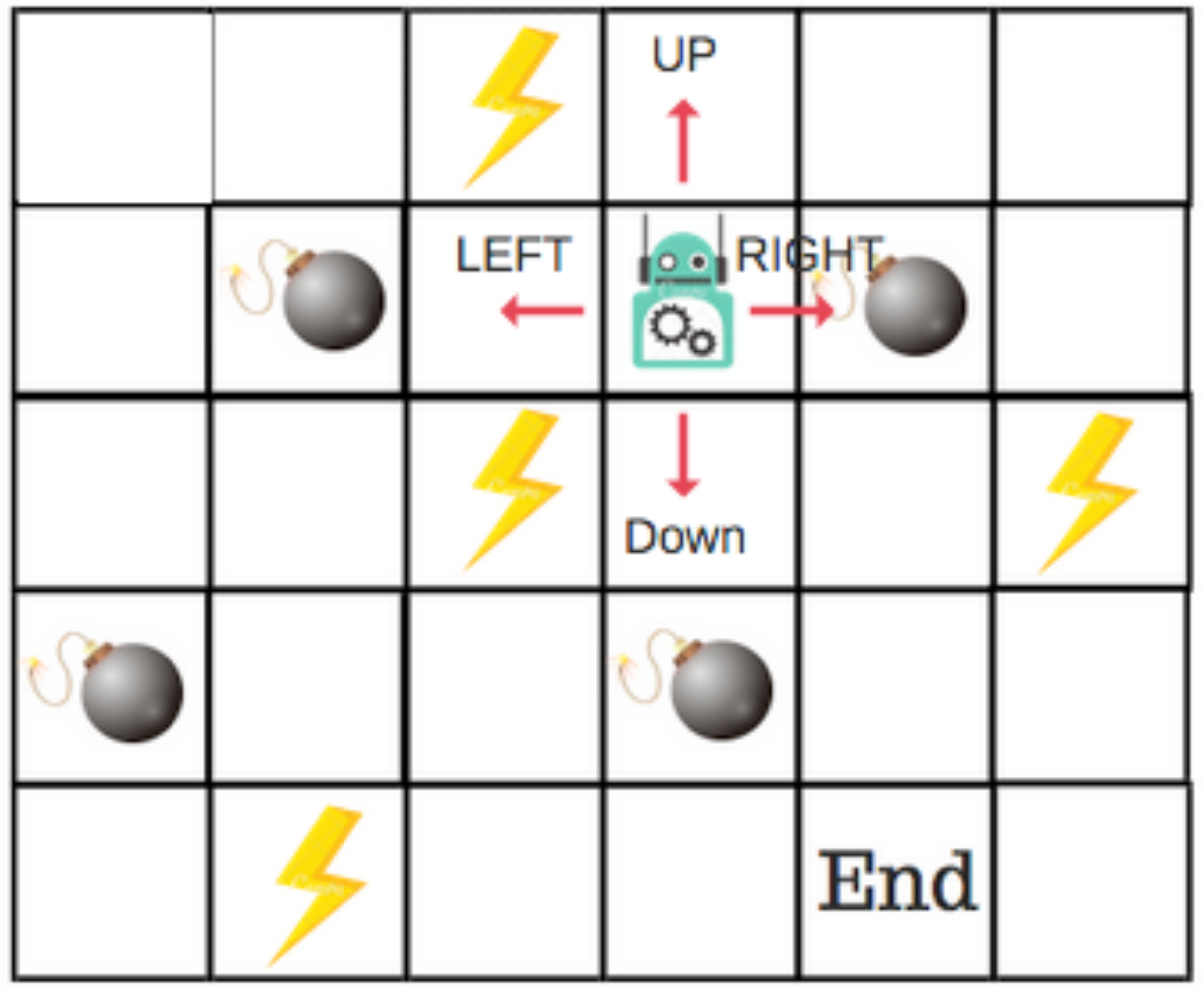
**Finite MDPs:** $(S, A, T, R)$

1. $\mathrm{S_t} \in S$ for t $= 0, 1, 2, 3, \ldots$
2. $\mathrm{A_t} \in A(\mathrm{s})$
3. $\mathrm{R_{t+1}} \in R$
4. $T(s'|s,a) = \sum_{r' \in R} P(s', r'|s,a)$

Goal: maximize discounted **returns**

$$G_t = R_{t+1} + \gamma R_{t_2} + \gamma^2 R_{t+3} + \ldots + \gamma^{T-t-1} R_T = \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

$$= R_{t+1} + \gamma G_{t+1}$$




- Optimization problem: (computationally) find **optimal policy** $\pi^*(S_t) = P(A_t|S_t)$
- Optimal policy $\pi^*: \pi \geq \pi' \Leftrightarrow v^*_{\pi^*}(s) \geq v_{\pi'}(s)$ for all $s$ and $q^*_{\pi^*}(s,a) = \max_{\pi'} q_{\pi'}(s,a)$
- **Can be estimated from experience!**
  - e.g., value iteration methods for episode tasks



Sutton & Barto (2018, p. 48, Fig 3.1)

# Multi-Armed Bandits

## Finding the best restaurant

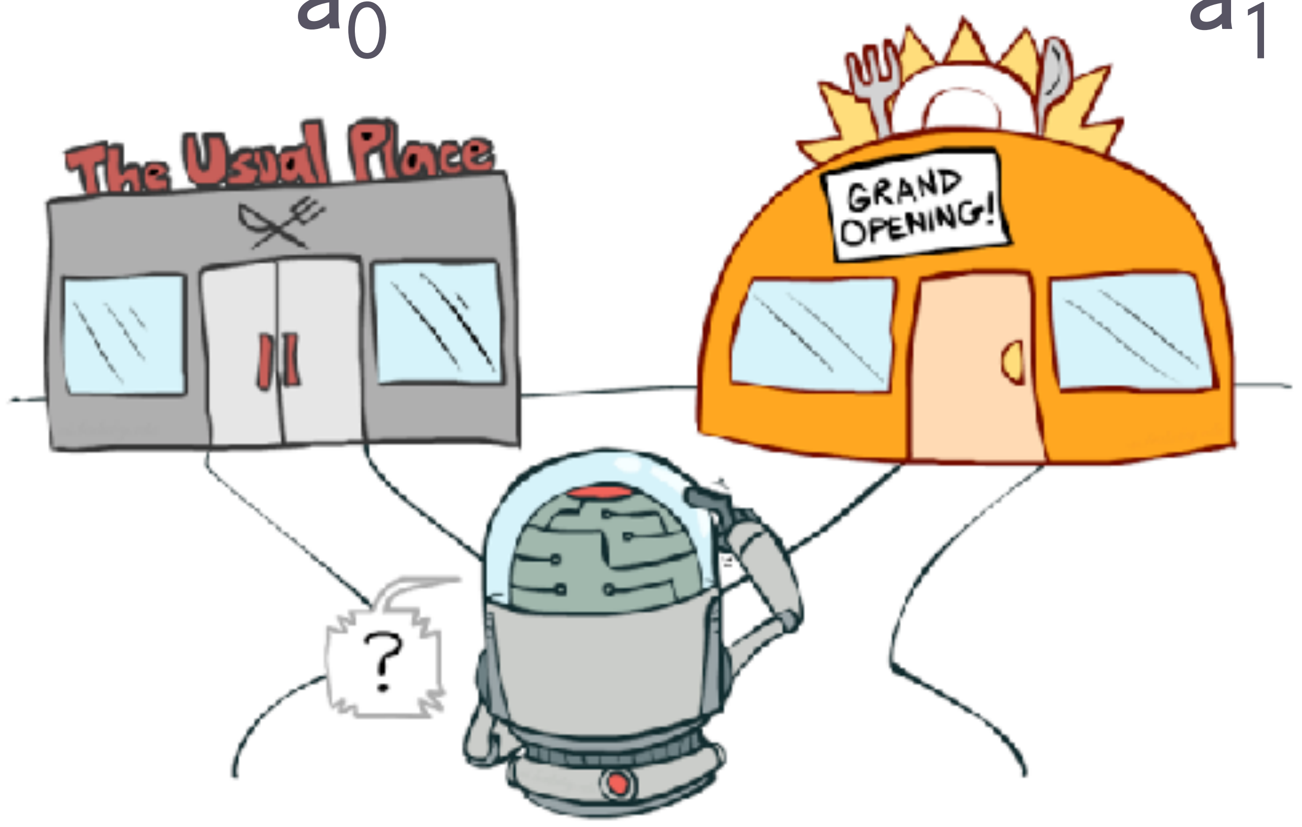
**Finite MDPs:** $(S, A, T, R)$

1. $S_t \in S$ where $S = \{s\}$

$t$ is iteration over repeated choice now

2. $A(s) = \{a_0, a_1, \dots\}$
3. $R_t \in R$
4. $T(s' \mid s, a) = \sum_{r' \in R} P(s', r' \mid s, a)$

$A_t = a_i$ $a_0$ $a_1$




$s$

Goal: maximize reward over repeated action selection

Optimal **action-value function:**

$$Q^*_\pi(s, a) = \max_\pi \mathbb{E}[G_t \mid S_t = s, A_t = a]$$

$$\downarrow$$

$$Q^*_\pi(a) = \max_\pi \mathbb{E}[R_t \mid A_t = a]$$

- Optimization problem: (computationally) find optimal policy $\pi^*(S_t) = P(A_t \mid S_t)$
- **Can be estimated from experience over repeated decision-making!**
  - e.g., with action-value methods like the sample-average method
- central problem: **exploration vs. exploitation**



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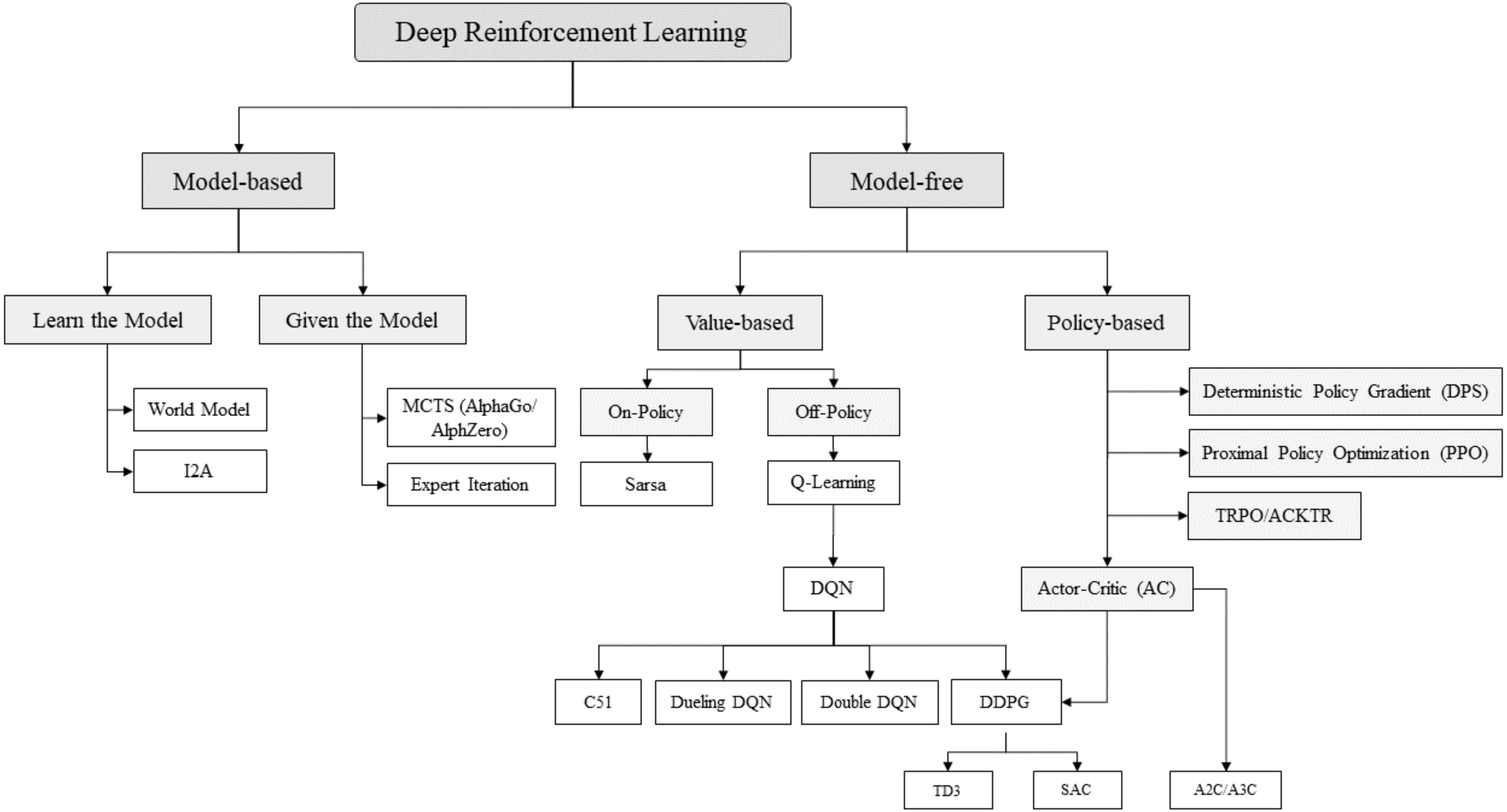
# RL Algorithms
## Approximating Optimal Policy



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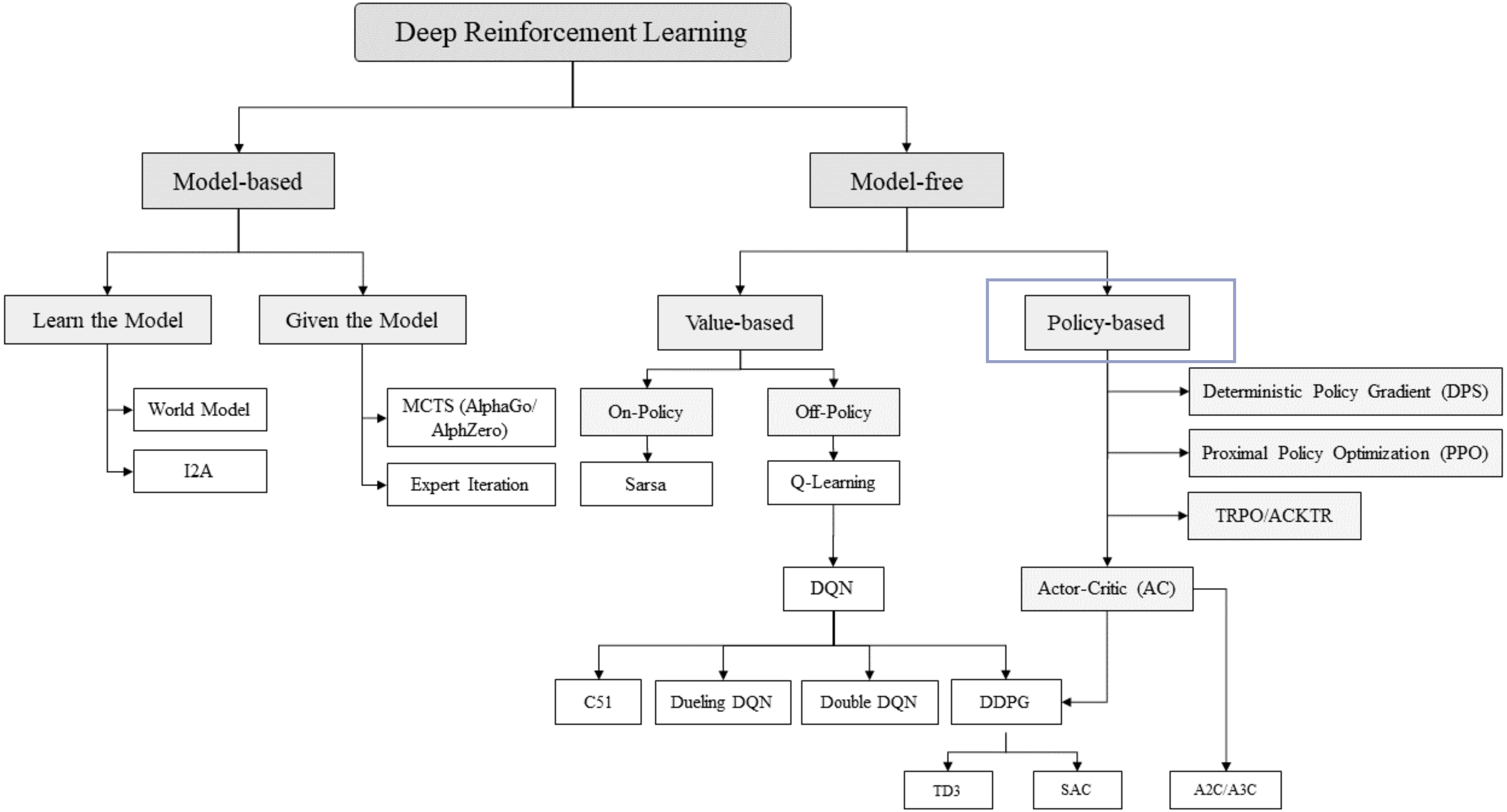
# RL Algorithms
## Approximating Optimal Policy



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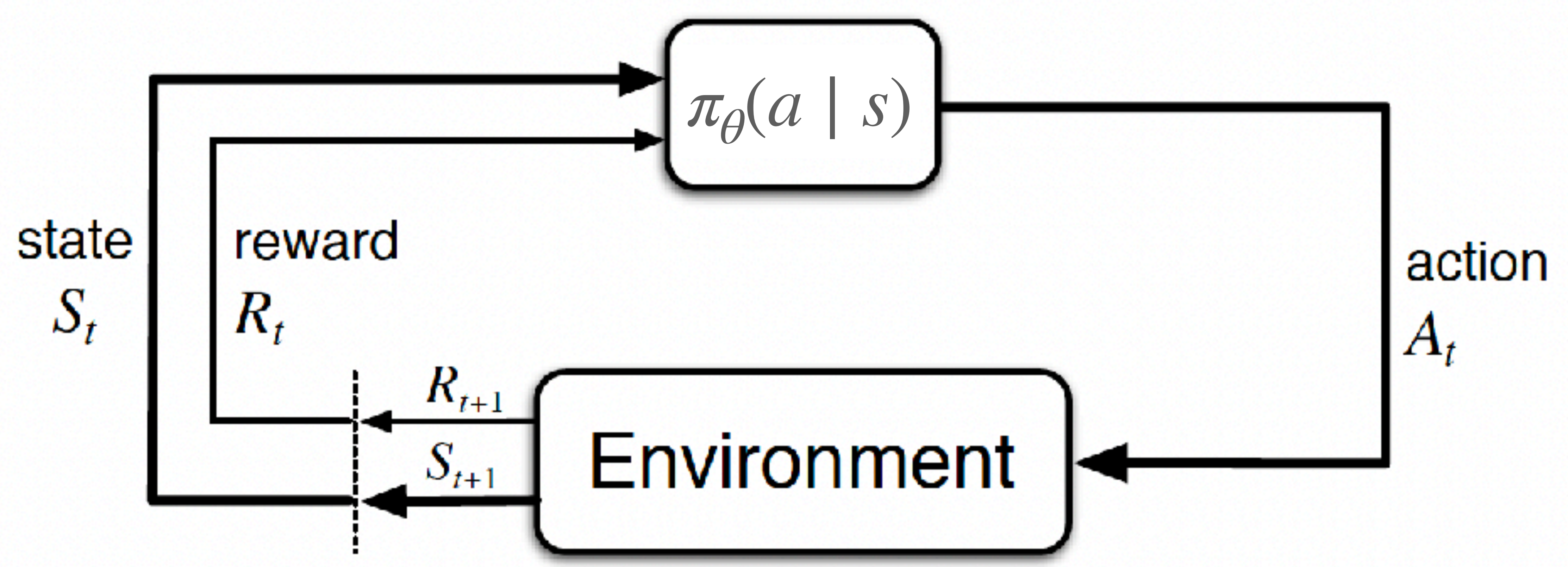
# Policy-Gradient Methods

## Introduction

- so far: deriving optimal policy from estimated value function
  - coming up with value functions might be difficult
  - state-value function doesn't prescribe actions
  - action-value functions require argmax
- idea: **optimize policy directly, such that expected reward is maximized**
  - think: optimize model with respect to objective function
- goal: find optimal $\theta$
  - $\max_{\theta} \mathbb{E}_{\pi_\theta}[G_t]$
- recall LM optimization: tweak $\theta$ so as to minimize loss
  - Gradient descent: $\theta_{new} = \theta_{old} - \alpha \nabla L_\theta$
  - Now: gradient ascent: $\theta_{new} = \theta_{old} + \alpha \nabla L_\theta$



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# Policy-Gradient Methods

## Policy-gradient theorem

- goal: find optimal $\theta$
  - Now: gradient ascent: $\theta_{new} = \theta_{old} + \alpha \nabla L_\theta$
- we write $\tau$ for a sequence of states, actions, rewards and $R(\tau)$ for (discounted) return
  - $L(\theta) = \sum_\tau P(\tau, \theta)\ R(\tau)$
- sample-based policy gradient estimation

$$
\begin{aligned}
\nabla L(\theta) &= \nabla \sum_\tau P(\tau, \theta)\ R(\tau) = \sum_\tau \nabla_\theta P(\tau, \theta)\ R(\tau) \\
&= \sum_\tau \frac{P(\tau, \theta)}{P(\tau, \theta)} \nabla_\theta P(\tau, \theta) R(\tau) \\
&= \sum_\tau P(\tau, \theta) \frac{\nabla_\theta P(\tau, \theta)}{P(\tau, \theta)} R(\tau) = \sum_\tau P(\tau, \theta) \nabla_\theta \log P(\tau, \theta) R(\tau) \\
&\approx \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^i, \theta) R(\tau^i)
\end{aligned}
$$

∇log(f(x)) = ∇f(x)/f(x)

# Policy-Gradient Methods

## REINFORCE

Policy gradient estimation: $\nabla L(\theta) = \sum_{\tau} P(\tau,\theta)\,\nabla_\theta \log P(\tau,\theta) R(\tau) \;\approx \frac{1}{m}\sum_{i=1}^{m} \nabla_\theta \log P(\tau^i,\theta) R(\tau^i)$

**REINFORCE** update rule:

for each episode:
  generate $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$ following $\pi(\,.\,|\,.\,,\theta)$
  for t in {0, 1, …. T-1}:

$$G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

$$\theta_{t+1} \leftarrow \theta_t + \alpha\, G \nabla log\ \pi(A_t \,|\, S_t, \theta_t)$$



Sutton & Barto (2018)

# Actor-Critic Algorithms
## REINFORCE

(Advantage) Actor-Critic (A2C)

$$\theta_{t+1} = \theta_t + \alpha(G_t - \hat{v}(s, \mathbf{w}))\ \nabla log\ \pi(A \mid s, \theta_t))$$

- Actor-critic methods evaluate the value of the action taken in a state:

$$\theta_{t+1} = \theta_t + \alpha\underbrace{\boxed{(G_{t:t+1}}}_{\text{One-step return}} - \underbrace{\boxed{\hat{v}(s, \mathbf{w}))}}_{\text{Critic}}\ \nabla log\ \underbrace{\boxed{\pi(A \mid s, \theta_t))}}_{\text{Actor}}$$

-



Sutton & Barto (2018)

# Policy-Gradient Methods

## TRPO & PPO

$$\nabla L(\theta_t) \propto \mathbb{E}_\pi[G_t \nabla log\ \pi(A \mid s, \theta)]$$

- Rewriting policy-gradient theorem:

  $L_\theta = \mathbb{E}_t[\hat{A}_t\ log\ \pi_\theta(a_t \mid s_t)]$, where **advantage** $\hat{A}_t = G_t - b(S_t)$

- Improve PG with Trust Region Policy Optimization (TRPO):

  Surrogate objective

  $\boxed{L^{CPI}(\theta) = \mathbb{E}_t[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta\_old}(a_t \mid s_t)}\hat{A}_t]}$ such that $\hat{\mathbb{E}}_t[KL[\pi_{\theta\_old}(\,.\mid s_t)\pi_\theta(\,.\mid s_t)]] \leq \delta$

  Justification actually suggests penalty instead of constraint: $\mathbb{E}_t[r_t(\theta)A_t - \beta KL[\pi_{\theta\_old}(\,.\mid s_t)\pi_\theta(\,.\mid s_t)]]$



Schulman et al. (2015), Schulman et al. (2017)

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$$\nabla J(\theta_t) \propto \mathbb{E}_\pi[G_t \nabla log\ \pi(A \mid s, \theta)]$$

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  Justification actually suggests penalty instead of constraint: $\mathbb{E}_t[r_t(\theta)A_t - \beta KL[\pi_{\theta\_old}(\,.\mid s_t)\pi_\theta(\,.\mid s_t)]]$

- Clip updates with PPO:

Expectation over batch

Boundaries

$$L^{CLIP}(\theta) = \boxed{\mathbb{E}_t}[\boxed{min}(\boxed{r_t(\theta)\hat{A}_t}, clip(r_t(\theta), \boxed{1-\epsilon, 1+\epsilon})\hat{A}_t)]$$

Lower bound on unclipped objective

Standard probability ratio

Schulman et al. (2015), Schulman et al. (2017)

# Policy-Gradient Methods

## Language models as policies

Policy gradient estimation: $\nabla L(\theta) = \sum_{\tau} P(\tau, \theta) \nabla_{\theta} \log P(\tau, \theta) R(\tau) \approx \frac{1}{m} \sum_{i=1}^{m} \nabla_{\theta} \log P(\tau^i, \theta) R(\tau^i)$

- policy $P$: language model
- trajectories $\tau$: generations from language model
- $\log P(\tau^i, \theta)$: log probability of a generation $\tau^i$ under the language model
- **$R(\tau^i)$: reward for generation $\tau^i$**



Sutton & Barto (2018)

# Summary

## Reinforcement learning

- the central framework for formalizing RL problems are Markov Decision Processes (MDPs)
- task of RL is to solve MDP such that the expected return is maximized
  - and to find the optimal policy
- classical solution methods for MDPs include estimation of optimal state- and action-value functions
- policy gradient methods directly optimize the policy such that the expected return is maximized
  - can be applied to LMs!

# Announcements

**Solutions to exercises will be on Moodle on November 15th!**

**Next class (November 15th) online only!**