Solving the Koshi problem of the system of first-order equations using Runge-Kutta method

Here we have a system of equations

$$J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} = T(t),$$

$$L_a\frac{di_a(t)}{dt} + R_a i_a(t) + e_m(t) = e_a(t),$$

$$e_m(t) = K_1\frac{d\theta(t)}{dt},$$

$$T(t) = K_2 i_a(t).$$

That could be introduced like

$$\frac{x(t)}{dt} = Ax + f(t)$$

$x = (x_1, x_2, x_3) = (\theta, \frac{d\theta}{dt}, i_a)$, $f(t) = (0, 0, \frac{e_a(t)}{L})$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{K_2}{J} \\ 0 & -\frac{K_1}{L} & -\frac{R}{L} \end{pmatrix}$$

1. $(e_a(t) = 0)$

   Stability conditions:

$$\begin{cases} \frac{R}{L} + \frac{B}{J} > 0 \\ \frac{BR + K_1K_2}{JL} > 0 \end{cases}$$

For example

$$B = 1, \quad R = 1, \quad J = 2, \quad L = 8, \quad K_1 = 5, \quad K_2 = \frac{1}{16}$$

Koshi problems: x(0) = (0,0,30) (1), x(0) = (5,-4,6) (2), x(0) = (10, 0, -16) (3)

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T = 0
X = [0,0,30]
h = 1/1000

t_points = [T]
x1_points = [X[0]]
x2_points = [X[1]]
x3_points = [X[2]]

while T < 80:
  k1 = firstSDE(T,X)
  k2 = firstSDE (T+h/2, addseq(X, [k1[0] * h/2, k1[1] * h/2, k1[2] * h/2]))
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k3 = firstSDE (T+h/2, addseq(X, [k2[0] * h/2, k2[1] * h/2, k2[2] * h/2]))
k4 = firstSDE (T+h, addseq(X, [k3[0] * h, k3[1] * h, k3[2] * h]))
X[0] += h/6*(k1[0]+2*k2[0]+2*k3[0]+k4[0])
X[1] += h/6*(k1[1]+2*k2[1]+2*k3[1]+k4[1])
X[2] += h/6*(k1[2]+2*k2[2]+2*k3[2]+k4[2])
T += h
t_points.append(T)
x1_points.append(X[0])
x2_points.append(X[1])
x3_points.append(X[2])
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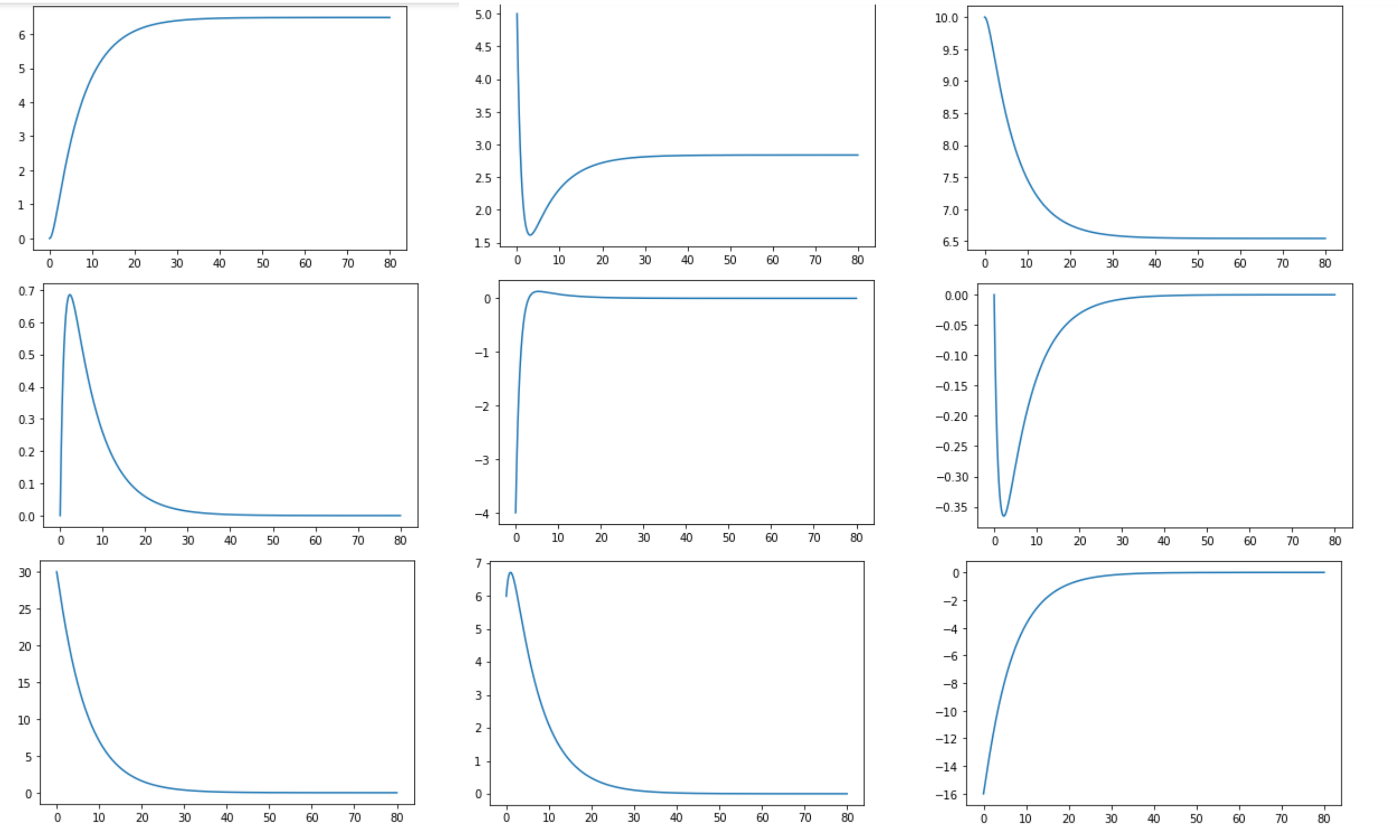
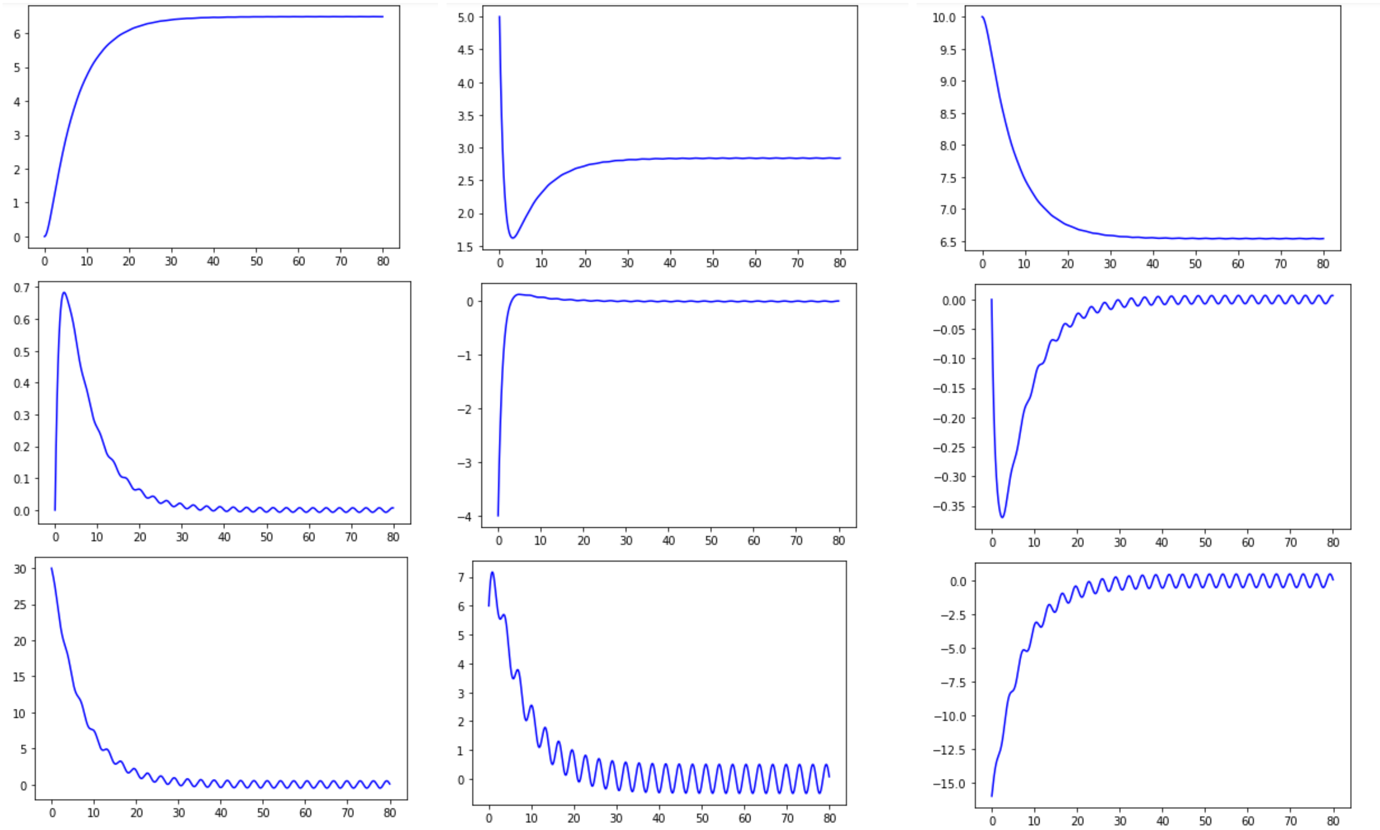
Results:

x(0) = (0,0,30) x(0) = (5,-4,6) x(0) = (10, 0, -16)



2. $(e_a(t) = A\sin(b(t-d))$

x(0) = (0,0,30) x(0) = (5,-4,6) x(0) = (10, 0, -16)



3. $e(t) = A\theta(t) + Ci_a(t)$

$$A' = \begin{pmatrix} A & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{K_2}{J} \\ 0 & -\frac{K_1}{L} & -\frac{R}{L}+C \end{pmatrix}$$

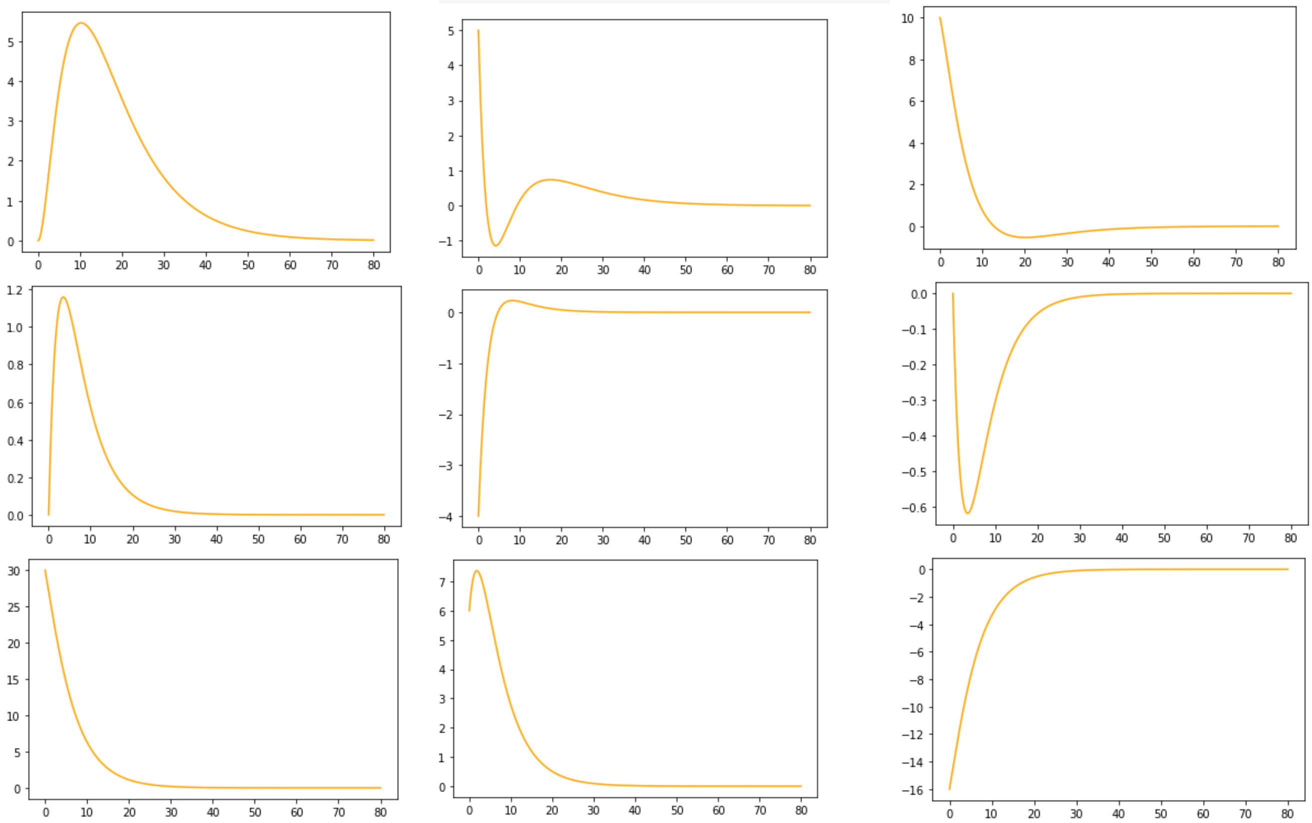
Stability conditions:

$$\begin{cases} \frac{R}{L}+\frac{B}{J}-A-C>0 \\ \left(\frac{R}{L}+\frac{B}{J}-A-C\right)\left(\frac{BR+K_1K_2}{JL}-\frac{RA}{L}-\frac{AB}{J}+AC-\frac{BC}{J}\right)-\left(\frac{ABC}{J}-\frac{ABR+K_1K_2}{JL}\right)>0 \\ \frac{ABC}{J}-\frac{ABR+K_1K_2}{JL}>0 \end{cases}$$

For example, there are parameters:

$$B=1, \quad R=1, \quad J=2, \quad L=8, \quad K_1=5, \quad K_2=\frac{1}{16}, \quad A=-0.1, \quad C=0.01$$

x(0) = (0,0,30) x(0) = (5,-4,6) x(0) = (10, 0, -16)



Comparing all the Koshi problems and systems of equations:

x(0) = (0,0,30) x(0) = (5,-4,6) x(0) = (10, 0, -16)